# A Critique of Meillassoux's Reflections on Mathematics from the Perspective of Bunge's Philosophy

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**Résumé** — Quentin Meillassoux est l'un des principaux philosophes français d'aujourd'hui. Son premier livre, *Après la finitude. Essai sur la nécessité de la contingence*, publié pour la première fois en 2006 et traduit en anglais en 2008, est déjà devenu un classique culte. Il comporte une préface de son ancien mentor, Alain Badiou. L'un des principaux objectifs de Meillassoux est de réhabiliter la distinction entre qualités premières et qualités secondes, typique des philosophies prékantiennes. Plus précisément, il affirme que les mathématiques sont capables de révéler les qualités premières de tout objet : « tous les aspects de l'objet qui peuvent être formulés en termes mathématiques peuvent être considérés de manière significative comme des propriétés de l'objet en soi. » Ici, nous allons utiliser la philosophie mathématique de Bunge pour remettre en question l'hypothèse précédente.

**Abstract** — Quentin Meillassoux is one of the leading French philosophers of today. His first book, *Après la finitude: Essai sur la nécessité de*

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*la contingence*, first published in 2006 and translated into English in 2008, has already become a cult classic. It features a *préface* by his former mentor, Alain Badiou. One of Meillassoux's main goals is to rehabilitate the distinction between primary and secondary qualities, typical of pre-Kantian philosophies. Specifically, he claims that mathematics is capable of disclosing the primary qualities of any object: "all those aspects of the object that can be formulated in mathematical terms can be meaningfully conceived as properties of the object in itself." (Meillassoux, 2008: 3, emphasis removed). Here we will use Bunge's philosophy of mathematics in order to challenge the preceding assumption.

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## 1 Meillassoux's Philosophy of Mathematics in *After Finitude*

First, it will be necessary to indicate that Meillassoux rejects a thesis which he calls "Pythagorean". Whether or not this has anything to do with what Pythagoras actually upheld, Meillassoux uses that term to refer to the thesis that mathematical statements, such as formulas and equations, are as real as any object in the Universe. Contrary to this point of view, he claims that mathematical statements are not real but ideal instead. This is found in his discussion of the accretion of the Earth, where he says:

> Consequently, our Cartesian physicist will maintain that those statements about the accretion of the earth which can be mathematically formulated designate actual properties of the event in question (such as its date, its duration, its extension), even when there was no observer present to experience it directly. In doing so, our physicist is defending a Cartesian thesis about matter, but not, it is important to note, a Pythagorean one: the claim is not that the being of accretion is inherently mathematical—that the numbers or equations deployed in the ancestral statements exist in themselves. For it would then be necessary to say that accretion is a reality every

> bit as ideal as that of number or of an equation. Generally speaking, statements are ideal insofar as their reality is one of signification. But their referents, for their part, are not necessarily ideal (the cat is on the mat is real, even though the statement "the cat is on the mat" is ideal). In this particular instance, it would be necessary to specify: the referents of the statements about dates, volumes, etc., existed 4.56 billion years ago as described by these statements—but not these statements themselves, which are contemporaneous with us[2].

Nevertheless, there is some ambiguity in the preceding distinction between statements and their referents. This was noted by Graham Harman in his book on Meillassoux's philosophy. Harman explains this ambiguity in the following way:

> Meillassoux says that the Cartesian position towards physics (and he takes the side of Descartes on most issues) must be distinguished from the Pythagorean position that the mathematical is reality itself. The Cartesian position is supposedly different in so far as it is the *referent* of equations which has existence independent of humans, not the equations themselves. This sounds plausible enough in Descartes's case, given the explicit role in his philosophy of physical substance. But assuming that Meillassoux means to take an anti-Pythagorean line in this passage (which he probably does), it remains unclear what *his* residual "referent" would be beyond the mathematical other than the "dead matter" that we have already found lacking[3].

Meillassoux's philosophy of mathematics is ambiguous on this point because, on the one hand, he claims that mathematical statements can disclose the primary qualities of an object, such as its length, height, figure, and so forth. These primary qualities are properties that the object has in itself, independently of the presence of human beings. So, for example, an object that has a triangular shape has a mathematical property independently of the presence of human beings. But on the other hand, the rejection of the "Pythagorean" thesis entails that the object in question cannot have a triangular shape by itself, since the concept of "triangle" is a term used in the statements of geometry, understood as a branch of mathematics. Having indicated this ambiguity, we will assume that

[2] Meillassoux, *After Finitude*, 2008 [2006], p. 12.

[3] Harman, *Quentin Meillassoux*, 2015 [2011], p. 207.

Meillassoux's view on this issue is that objects in themselves have primary qualities, which are inherently mathematical. These properties are real, while the mathematical statements that disclose them are ideal. Such a view is at odds with Bunge's philosophy of mathematics. Consider the following statement:

> There is no reason to expect that pure mathematics is capable of disclosing, without further ado, the structure of reality[4].

Why not? Because pure mathematics, by itself, only deals with constructs. In order to study reality, we need empirical science; pure mathematics alone is insufficient for that task. To be sure, Meillassoux is aware of this: "For what is at stake here", he says, "is the nature of scientific discourse, and more particularly of what characterizes this discourse, i.e. its *mathematical* form[5]." And, later on, he says, "it is the discourse of empirical science as such that we are attempting to understand and to legitimate[6]". Thus, Meillassoux recognizes that there is a difference between pure mathematics and empirical science. Furthermore, he believes that one of the salient features of empirical science is that it relies heavily on mathematics; not entirely, but to a large extent. Of course, Bunge does not have any qualms with this. The decisive issue here is: What do the statements of empirical science refer to, especially those that rely heavily on mathematics? Meillassoux seems to believe, in agreement with Descartes and Locke, that properties such as length, height, figure, among others, are not merely technical terms of the vocabulary of geometry, but real properties that can be found in external objects instead. We will see that this is not the case according to Bunge.

But before we do so, and in order to understand Bunge's mathematical fictionalism, it will be necessary to take a quick look at the history of non-Euclidean geometries, and the consequences that their development had for philosophy.

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[4] Bunge, *Ontology I: The Furniture of the World*, 1977, p. 150.

[5] Harman, *Quentin Meillassoux*, 2015 [2011], p. 26.

[6] *Ibid.*, p. 28.

## 2 A Brief History of Non-Euclidean Geometries

In the fourth chapter of *After Finitude*, Meillassoux makes some scarce comments on the history of mathematics[7]; specifically, he refers to the development of non-Euclidean geometries during the nineteenth century, stating that "we are all familiar" with their history, and then he summarizes Lobachevsky's work. Although Meillassoux's target audience may be familiar with that history, it seems to us that it must be recounted here.

But before we present that history, it will be convenient to note that according to Meillassoux, philosophers have recently become modest, and even prudent, when discussing scientific issues[8]. It seems to us that this has been especially true after the Sokal affair. Unlike previous generations, today's continental philosophers have learned to be cautious about topics such as non-Euclidean geometries, Einstein's theories of special and general relativity, quantum physics and Gödel's theorems, among others.

This was never a problem for analytic philosophers. For example, Ernest Nagel and James Newman wrote a book on Gödel's proof[9], and Thomas Kuhn wrote a book on quantum physics[10]. None of these authors have been criticized by Alan Sokal or Jean Bricmont for misusing scientific concepts. Kuhn has been criticized by Sokal and Bricmont in *Fashionable Nonsense* for fostering philosophical relativism, but not for misunderstanding physics[11]. The point is that philosophers may be knowledgeable enough to write on topics such as Gödel's work and quantum physics without falling into charlatanry. That some philosophers do fall into charlatanry when discussing these topics does not mean that all of them do so. Of course, neither Sokal nor Bricmont claim the contrary. They specifically criticize a group of thinkers, those that they regard as postmodern intellectuals. But to step into that discussion exceeds the purposes of this article. We have only advanced these remarks in order to clearly state that we are fully aware of the perils surrounding the philosophical discussions of complicated scientific issues.

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[7] *Ibid.*, p. 92.

[8] *Ibid.*, p. 13.

[9] Nagel & Newman, *Gödel's Proof*, 1958.

[10] Kuhn, *Black-Body Theory and the Quantum Discontinuity, 1894–1912*, 1987.

[11] Sokal & Bricmont, *Fashionable Nonsense*, 1998 [1997], p. 51.

Thus, our presentation of the history of non-Euclidean geometries will follow Meillassoux's remark about modesty and prudence. In order to do so, we will use a well-known Argentine textbook on the philosophy of mathematics by Gregorio Klimovsky and Guillermo Boido, *Las desventuras del conocimiento matemático* ("The Misadventures of Mathematical Knowledge")[12]. Klimovsky was a mathematician and philosopher of science who introduced set theory in Argentina. Boido was a physicist and historian of science, who wrote a popular history book on Galileo. A more detailed presentation of non-Euclidean geometries and their history can be found in Richard Trudeau's book, *The non-Euclidean revolution*[13]. Several quotes and definitions by philosophers and mathematicians of the past can also be found in Trudeau's book.

It will be necessary to begin by considering Euclid's *Elements*, which has certain similarities with Aristotle's way of conceiving axioms and theorems. For Aristotle, axioms are self-evident principles, which are undeniably true. From them, theorems can be deduced, and which are also undeniably true. Thus, he says in the *Posterior Analytics*:

> That which is an indispensable antecedent to the acquisition of any knowledge I call an Axiom; for there are some principles of this kind, and "axiom" is the name generally applied to them[14].

And later on, he highlights the self-evidence that characterizes axioms, when he says:

> There are three elements in demonstrations: (1) the conclusion which is demonstrated, i. e., an essential attribute of some genus; (2) axioms or self-evident principles from which the proof proceeds; (3) the genus in question whose properties, i. e. essential attributes, are set forth by the demonstrations[15].

Euclid's postulates apparently were more or less similar to Aristotle's axioms; that is, they were true statements which do not need to be demonstrated. Klimovsky and Boido say the following:

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[12] Klimovsky & Boido, *Las desventuras del conocimiento matemático*, 2005.

[13] Trudeau, *The Non-Euclidean Revolution*, 2008 [1987].

[14] Aristotle, *Aristotle's Posterior Analytics*, 1901, p. 6.

[15] *Ibid.*, p. 20.

> The statements that Euclid calls *postulates* are assumptions that we must accept without demonstration and that concern geometry itself. They are roughly equivalent to Aristotle's axioms, although our geometer does not make any philosophical considerations about their evidence and merely asks the reader to accept them[16].

This being so, let us examine the history of non-Euclidean geometries, which has its roots in the attempts to prove Euclid's fifth postulate. These roots go far back to Antiquity. Philosophers like Posidonius and Geminus had the suspicion that the fifth postulate was not really a postulate, but a theorem. There were more or less solid reasons for this doubt. First of all, the grammatical expression of the fifth postulate is much more complicated and extensive than the other four. In its original formulation, it says nothing about parallels. Let us cite Euclid's five postulates, in order to see how "strange" the fifth looks, at least from a grammatical point of view:

> Let the following be postulated:
> 1. To draw a straight line from any point to any point.
> 2. To produce a finite straight line continuously in a straight line.
> 3. To describe a circle with any center and distance.
> 4. That all right angles are equal to one another.
> 5. That, if a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles[17].

The fifth postulate looked grammatically "strange" when one compared it to the other four. But this was not the only problem. If it was, then there would not be any other reasons, other than grammar, to suspect that this was a theorem. In other words, it would have been a postulate which was poorly written, but a postulate nonetheless. There was another source of doubt, more problematic than grammar. It was the fact that the fifth postulate was explicitly used only once in Euclid's book. On the other hand, the first, second, third and fourth postulates are frequently used throughout the book, in order to deduce many different theorems. It seemed

[16] Klimovsky & Boido, *Las desventuras del conocimiento matemático*, 2005, p. 78-79.
[17] Euclid, *The Thirteen Books of Euclid's Elements*, 1908, p. 154-55.

suspicious that there was a postulate whose only role was to deduce one specific theorem. In the words of Klimovsky and Boido:

> It is striking that Euclid has placed among the postulates of his system one that is used explicitly only once, as if some aversion on the part of the author of the *Elements* lies behind it. We would say that everything happens as if in a certain religion we found a god of rain, another of fire, a third of the earth and a fourth of the sea, but also a god whose specific purpose is to cure a particular cold to a certain king. A divinity destined exclusively to that seems a bit excessive[18].

This is why philosophers like Posidonius and Geminus suspected that the fifth "postulate" was a theorem, and they attempted to prove this. Even more so, they succeeded. They really did deduce the fifth postulate, therefore proving that it was a theorem. But there was a catch: they introduced an additional postulate in order to do this. Thus, Posidonius, whose work we know from the commentaries of Proclus, apparently proposed the following additional postulate:

> Parallel straight lines are equidistant[19].

Now this is much more concise and elegant than Euclid's formulation of the fifth postulate, as far as grammar is concerned. And with it, one can deduce Euclid's fifth "postulate" as a theorem. The problem is that the postulate that Posidonius introduces is actually equivalent to Euclid's. They say the same thing, even if this is not immediately evident. But it can be proved. If one takes the first four postulates of Euclid's *Elements*, together with the postulate that Posidonius introduces, it is possible to deduce, as a theorem, Euclid's fifth postulate. But the converse is also true. If one takes all of Euclid's postulates, then Posidonius' "postulate" can be deduced as a theorem. So Posidonius did not really prove that Euclid's fifth postulate was a theorem. In order to do so, he would have had to either deduce it using only the first four postulates of the *Elements*, or he would have had to introduce a new postulate which would not be logically equivalent to Euclid's fifth. He believed that he had succeeded in pursuing this second option, but later it was shown that this had not been

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[18] Klimovsky & Boido, *Las desventuras del conocimiento matemático*, 2005, p. 90.

[19] Trudeau, *The Non-Euclidean Revolution*, 2008 [1987], p. 128.

the case. The other philosopher of that time, Geminus, had a similar experience.

At the beginning of the Middle Ages, Proclus summarized most of the earlier attempts at proving the fifth postulate. All of them had the same thing in common: they introduced an additional postulate, which was shown later to be equivalent to Euclid's fifth. Proclus himself attempted an additional proof. He did so by surreptitiously introducing a statement that is equivalent to Euclid's fifth postulate.

This kept going on and on during the Middle Ages and later during the Renaissance as well. At the same time, mathematics in general had been marching forward, especially in the works of Copernicus, Galileo, and later in Descartes. Mathematics, says Meillassoux, began to describe a "glacial world", one that was independent of human experience, and even of human existence:

> It is this *glacial* world that is revealed to the moderns, a world in which there is no longer any up or down, centre or periphery, nor anything else that might make of it a world designed for humans. For the first time, the world manifests itself as capable of subsisting without any of those aspects that constitute its concreteness for us[20].

Yet, the map of this glacial landscape would remain incomplete until Euclid's fifth postulate could finally be proven. It seemed like an almost impossible task, since there had been numerous attempts during the past centuries, and all of them had failed. By the 18th century, the situation was scandalous. While Kant claimed in a footnote to the *Critique of Pure Reason* that the lack of a solid proof for the existence of external things was "the scandal of philosophy"[21], D'Alembert claimed in the *Essays on the Elements of Philosophy* that the problem of the parallel postulate was "the scandal of geometry"[22]. Euclid's fifth postulate came to be known as "the parallel postulate" because it could be written in a more elegant and concise way by using the notion of parallels. So, for example, it became customary to use the following equivalent formulation, which was popularized by John Playfair at the end of the 18th century:

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[20] Meillassoux, *After Finitude*, 2008 [2006], p. 115.

[21] Kant, *Critique of Pure Reason*, 2000 [1781–1787], p. 121.

[22] Le Lionnais, « Beauty in Mathematics », 2004, p. 133.

> Through a given point not on a given straight line, and not on that straight line produced, no more than one parallel straight line can be drawn[23].

According to Klimovsky and Boido, during the early decades of the 19th century, a small group of mathematicians:

> [...] had the firm suspicion that the postulate of the parallels is unprovable from the previous four and that it is possible to obtain new conclusions, without finding any contradiction, admitting these four postulates and the *negation* of the fifth[24].

Among this group was Gauss. He developed a new geometry, a non-Euclidean one, but he did not publish his results immediately. Gauss did not publish his manuscripts because he feared that his colleagues would consider his work to be "the result of an insane lucubration, worthy of an eccentric"[25].

However, Gauss received a book from an old friend of his, a mathematician called Wolfgang Bolyai. It was a two-volume work on geometry. This treatise included an appendix written by his son, Johann Bolyai. In this appendix, Johann Bolyai had developed a non-Euclidean geometry by accepting Euclid's first four postulates and this additional one: "from a point exterior to a strait line there is more than one parallel that passes through that point". Previously, Johann had told his father that he had "created a universe out of nothing". When Gauss received this book, he wrote a letter to Wolfgang. He praised Johann's work, and felt relieved that other people had reached similar results by negating Euclid's fifth postulate. He now had more confidence in the idea that he was not a lone eccentric, but a serious researcher who, despite having produced a geometry which seemed "strange", had no logical errors. Gauss decided to encourage other mathematicians to investigate these possibilities. Yet the atmosphere of the time was rather uncertain, many mathematicians still felt that they could be making fools of themselves if they insisted too much on this issue. Johann Bolyai decided to stop publishing, in part due to the reason just mentioned, and in part because he felt that Gauss could rob him of his

[23] Trudeau, *The Non-Euclidean Revolution*, 2008 [1987], p. 128.

[24] Klimovsky & Boido, *Las desventuras del conocimiento matemático*, 2005, p. 94.

[25] *Ibid.*, p. 95.

merits if the community of mathematicians were to fully accept the idea that it was possible, and legitimate, to develop non-Euclidean geometries.

Johann Bolyai was not entirely wrong in his suspicions. He was wrong to suppose that Gauss would try to steal his merit. But he was not wrong in supposing that the community of mathematicians would not accept the possibility of non-Euclidean geometries. This last point was to be corroborated when a third figure emerged on the scene, Nikolai Lobachevksy. He had developed a non-Euclidean geometry very similar to that of Bolyai, and he presented it in conferences and in publications. Lobachevsky had been urged by a friend of Gauss to publish these results; apparently because Lobachevsky himself felt rather uneasy about it, just like Gauss and Bolyai had felt. None of them were wrong on this point, because when the community of mathematicians started to pay attention to what they had written, they were accused of fabricating "caricatures of geometry" and even "morbid manifestations of geometry"[26].

What were the characteristics of these early non-Euclidean geometries? Why did they seem so "repugnant", or hard to accept? Neither Gauss, Bolyai nor Lobachevsky reached any contradictions by denying Euclid's fifth postulate. Instead, what they obtained was a series of "weird" theorems, which nonetheless were perfectly valid from a logical point of view. They were so "weird" that they defied intuition, and even common sense. For example, "the sum of the angles of a triangle is less than 180 degrees". Or they included statements like this one: "from a point exterior to a straight line, an infinite number of parallels pass through that point". As if this was not enough, another mathematician, Bernhard Riemann, developed a non-Euclidean geometry which claimed that "from a point exterior to a straight line, no parallels pass through that point". While Gauss, Bolyai and Lobachevsky developed different versions of what was later to be called "hyperbolic geometry", Riemann developed what would later be known as "elliptic geometry". It was Felix Klein who introduced these terms to describe the new geometries developed by his colleagues.

When the community of mathematicians began to pay sufficient attention to these new geometries, their initial rejection gave way to a more sophisticated way of resisting them. Instead of using terms like

[26] *Ibid.*, p. 94-96.

"caricature" and "morbid" to describe these geometries, the idea that began to gain acceptance was that these new geometries were perfectly logical, but that, unlike Euclid's, they did not refer to anything in the real world. In other words, it was claimed that Euclid's geometry is the only one that correctly describes physical space, while these other geometries do not describe anything. They were, in a sense, "imaginary", while Euclid's, on the other hand, was real.

Since that was supposedly the case, this gave way to the idea that those mathematicians who were working on non-Euclidean geometries were more or less wasting their time. Or, at best, they were simply entertaining themselves with a "game", as if they were inventing new rules for playing chess. Of course, one can invent any alternative rules for chess and have fun playing with those rules, no matter how bizarre they may be. But if one wanted to do serious research as a mathematician, then the efforts had to be made in the only geometry which was not purely imaginary, the only one that can describe physical space, that is, Euclidean geometry.

David Hilbert did not share the preceding opinion. For him, the invention of non-Euclidean geometries was not a waste of time. On the contrary, he claimed that a sharp distinction must be drawn between the development of a purely formal system, on the one hand, and the task of finding applications for that formal system, on the other. In other words, one must distinguish between "pure" and "applied" mathematics. That a mathematical system, such as a non-Euclidean geometry, has no immediate applications in the real world, does not mean that there are no applications in principle. Because it could be the case that there are such applications, but that we have simply not found them yet. Thus, it is hastily and inadvisable to condemn research in pure mathematics just because it has no immediate applications.

Hilbert maintained that pure mathematics was the study of formal systems, and that the only thing that matters in these formal systems is their syntax. Applied mathematics, on the other hand, is the task of finding semantic interpretations of those formal systems. It is only at this point that semantics enters the scene; in purely formal systems, all that matters are their syntax. This distinction between pure and applied mathematics began to gain acceptance within the community of mathematicians, but there was still some reticence to the idea that non-Euclidean geometries could have a physical interpretation. They were too weird; their most basic statements went against common sense. The tide finally turned when Einstein described physical space in 1916 using an interpretation of Riemann's elliptic geometry. This showed that non-Euclidean geometries could indeed have a relation to the real world, and that they could be used to describe physical space just as good, if not better, than Euclidean geometry.

Profound consequences ensued. Some of them were even quite disturbing. First of all, intuition and common sense were no longer a guarantee of what kind of mathematical research qualifies as "legitimate". In other words, one cannot dismiss a work of mathematics simply because it runs contrary to intuition and common sense. Second, it was no longer clear that Euclidean geometry was the only "true" or "real" geometry, and it was not clear that there could even be such a thing, Euclidean or not. Instead, Hilbert's distinction between pure and applied mathematics became the new cornerstone of mathematical research. All purely formal systems are equally legitimate; Euclid's geometry is not "better" or "worse" than non-Euclidean geometries. As long as they are treated in a purely formal way, all of them are on an equal footing. Regarding Hilbert's work, Klimovsky and Boido say the following:

> Hilbert himself claimed that, while we are somehow obliged to use words from everyday language to speak *in* (or *within*) a formal axiomatic system, instead of "point", "line", and "plane" we could well use "table", "chair" and "beer glass" without altering in the least the system itself: "point" or "table", here, are mere empty labels without any meaning[27].

Shocking, isn't it? At least it was shocking to those mathematicians that still adhered to the Aristotelian notion that axioms must be "true" and "self-evident". What Hilbert showed was that an axiom does not necessarily have to be "true" or "self-evident" in the Aristotelian sense. Rather, it is a meaningless expression, composed of meaningless signs, which is arbitrarily formulated by the mathematician, in order to see what can be deduced from it. The theorems, which are deduced from the axioms, are no longer "true" either, as Aristotle thought. Instead, they are meaningless expressions, composed of meaningless signs, which are derived from

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[27] *Ibid.*, p. 106.

the axioms simply by following a set of accepted, arbitrary rules. In this sense, a formal axiomatic system can be compared to a game of chess:

> Actually, such a structure really looks like a logical game with some resemblance to chess. In chess we do not know exactly what we are referring to with the pieces (what we do know is how to move them), and no one in their right mind will believe that they are executing monarchical politics because they move the king, the queen and their pawns. Calling the pieces "king", "bishop" or "tower" is a tribute to tradition; in the same way, in a non-Euclidean geometry the words "point", "line", "plane", etc., have no meaning. Such a methodology is known as *formal axiomatic method*, or simply *axiomatic method*, and the game we have described in particular is an example of what is called a *formal axiomatic system*[28].

And later on, they say:

> And if one were to ask here, from a purely theoretical, non-historical or practical point of view, which one of these is the legitimate chess, the answer would be: they are all equally legitimate, once it is accepted, for each of them, their corresponding pieces, initial positions, rules, etc. The same applies to axiomatic systems. From a purely logical perspective, we can understand Euclid's geometry as a formal axiomatic system, since it has its vocabulary, the categories of that vocabulary, and it has its starting points, the axioms, and what is deduced from them, the theorems. Both the Euclidean geometry and the non-Euclidean geometries would be, on an equal level, formal axiomatic systems, that is, "games" that, as with the different variants of chess that we have mentioned, would have to be considered, all of them, perfectly legitimate[29].

Having said this, we are ready to examine Bunge's philosophy of mathematics, which draws upon the philosophical consequences of the history of non-Euclidean geometries.

## 3 BUNGE'S PHILOSOPHY OF MATHEMATICS

We saw that for Meillassoux, mathematical statements are ideal, but their referents are not. We also saw that Harman noticed an ambiguity in this

[28] *Ibid.*, p. 104.

[29] *Ibid.*, p. 115.

seemingly unproblematic position. In Bunge's work, we find a solution to this ambiguity. He unequivocally states that numbers are not found in the Universe among objects such as rocks, trees and mountains. Numbers, according to him, are brain processes:

> Although thinking of the number 3 is a brain process, hence one located in space-time, the number 3 is nowhere because it is a fiction existing by convention or fiat, and this pretense does not include the property of spatiotemporality. What holds for the number 3 holds for every other idea—concept, proposition, or theory. In every case we abstract from the neurophysiological properties of the concrete ideation process and come up with a construct that, by convention, has only conceptual or ideal properties[30].

According to Bunge, the number 3 is a fiction, and so is every other mathematical entity. There is more to be said, because not only does he consider mathematical entities to be fictional, he says that every concept, proposition and theory are fictional as well. He calls these "constructs", and they include even the most complex scientific theories. So, for instance, a scientific theory about gravity is *not* gravity itself. For one thing, gravity is a fundamental force of nature, while a *theory* about gravity is not: it is a brain process. And brain processes are not fundamental forces of nature. So far, this is in agreement with Meillassoux's distinction between statements and their referents. But it seems that Meillassoux would be inclined to believe that an iron sphere, for example, is spherical in itself. It would be a sphere even if there was no one to look at it, since its spherical shape is understood here as a primary quality. Bunge would disagree:

> Concrete objects (things) have no intrinsic conceptual properties, in particular no mathematical features. This last statement goes against the grain of objective idealism, from Plato through Hegel to Husserl, according to which all objects, in particular material things, have ideal features such as shape and number. What is true is that some of our ideas about the world, when detached from their factual reference, can be dealt with by mathematics. (For example, by analysis and abstraction we can extract the constructs "two" and "sphere" from the proposition "That iron sphere is composed of two halves".) In particular, mathematics helps us to study the

[30] Bunge, *Ontology II: A World of Systems*, 1979, p. 146.

> (mathematical) form of substantial properties. In short, not the world but some of our ideas about the world are mathematical[31].

Material things, therefore, do not have shapes, at least strictly speaking. We can, of course, talk about material things *as if* they had shapes, for example when we say that a certain iron object is spherical. But that object, in itself, is not spherical. This may seem hard to accept. Jean-Pierre Marquis, in his appraisal of Bunge's philosophy of mathematics, expresses his concern regarding the clarity of this point, and offers some comments on Bunge's example of the iron sphere:

> I must admit that this is not entirely clear to me. Needless to say, the iron sphere is not, strictly speaking, a sphere in the mathematical sense. The sensory impression of the sphere presumably gives us an approximation of what a sphere in the strict sense would look like. One could perhaps say that we treat the iron sphere *as if* it were a sphere. But in order to do this, we already need to have the mathematical concept of sphere. The *mathematical concept* of sphere is not in the iron sphere. The concept of sphere is given in a certain language, be it geometric, analytic or algebraic, thus in a certain context. It is, in Bunge's terminology, a *construct*[32].

In order to clarify Bunge's example of the iron sphere, it will be useful to remember what happened to the concept of triangles during the development of non-Euclidean geometries in the nineteenth century. In Euclidean geometry, the sum of the angles of a triangle is equal to 180 degrees. For millennia, this seemed to be an absolute truth. However, in some non-Euclidean geometries it is possible to prove, without contradiction, that the sum of the angles of a triangle is greater than 180 degrees; this is the case of elliptic geometry. In others, such as hyperbolic geometry, the sum of the angles of a triangle can be less than 180 degrees. One cannot say that the triangles of Euclidean geometry are the "real" triangles and that the triangles of non-Euclidean geometries are "not real". What holds for triangles also holds, in general, for all other shapes: spheres, squares, rectangles, and so forth: there is no reason to believe that there is such a thing as a "real" sphere as characterized by this or that geometry, as opposed to other "non-real" spheres characterized by other geometries. The

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[31] Bunge, *Ontology I: The Furniture of the World*, 1977, p. 118.

[32] Marquis, « Mario Bunge's Philosophy of Mathematics », 2012, p. 1574.

preceding point can be clarified further by considering some of Bunge's comments on cultural objects:

> I submit that the same holds, *mutatis mutandis*, for all cultural objects. Thus, a sculpture that nobody looks at is just a chunk of matter—and so is a philosophical treatise that nobody reads. There is no immortality in cultural creations just because they can be externalized ("embodied") and catalogued[33].

Initially, one could argue that a certain sculpture is a chunk of matter that has a specific shape. But, just like the property of "being spherical" is not a primary quality of an iron sphere, neither is "having a specific shape" in the case of a sculpture that no one is looking at. Suppose we are considering a sculpture of a horse, or of Pegasus. The sculpture itself, without observers, would not look like a horse or Pegasus, because there would not be anyone looking at it. If this is so, then it would not only apply to cultural objects, but to natural ones as well. A waterfall would not look like a waterfall when nobody is looking at it, the Moon would not look round or spherical, on the contrary, both of them would just be chunks of matter, without any visual appearance.

Bunge traces a distinction between attributes and properties. Attributes, according to him, are characteristics that we ascribe to things, but the things in question, by themselves, do not have those attributes. Properties, on the contrary, do belong to things in themselves, independently of human existence. Attributes are constructs, while properties are real. Thus, when we say that a sculpture looks like a horse, this is something that we are attributing to a chunk of matter. When we say that the sculpture in question is made of iron, this is a property of that chunk of matter itself. Iron has properties that are independent of our scientific hypothesis and theories, although we use the latter in order to understand the former. In this sense, "spherical" or "having a spherical shape" is not a property, it is an attribute. Attributes can be mathematical, but not properties. Whatever properties the object itself has, these are never mathematical.

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[33] Bunge, *Ontology II: A World of Systems*, 1979, p. 168-70.

## 4 Concluding Remarks

One of the most prominent features of French philosophy in the continental tradition is, from a historical perspective, its increasing association with mathematics. It was a prominent topic in the works of Gilles Deleuze, and even more so in those of Alain Badiou. Quentin Meillassoux's work is in line with that tradition, and our wager is that it could greatly benefit from Bunge's philosophy of mathematics. The rationale for this is that Bunge's approach provides an unequivocal solution to the ambiguity that Harman had recognized in Meillassoux's discussion of the "Pythagorean" thesis. Although Bunge advances some ideas which may seem difficult to accept, such as the idea that objects in themselves do not have geometric shapes, he nevertheless also provides reasons for doubting Meillassoux's claim that any property which can be mathematized can be construed as a primary quality. Numbers, algebraic structures, and other mathematical entities are not real objects nor properties of real objects, but useful fictions instead. They are brain processes, and by convention we feign that they have autonomous existence.

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### References

Aristotle, 1901, *Aristotle's Posterior Analytics*, Oxford, Blackwell, traduit par E.S. Bouchier.

Bunge M., 1977, *Treatise on Basic Philosophy : Ontology I, the Furniture of the World*, vol. 3, 8 vol., Dordrecht, Reidel.

— 1979, *Treatise on Basic Philosophy : Ontology II, a World of Systems*, vol. 4, 8 vol., Dordrecht, Reidel.

Euclid, 1908, *The Thirteen Books of Euclid's Elements, Vol. 1 : Introduction and Books I & II*, Cambridge, Cambridge University Press, édité et traduit par T.L. Heath.

Harman G., 2015 [2011], *Quentin Meillassoux : Philosophy in the Making*, Edinburgh, Edinburgh University Press.

Kant I., 2000 [1781–1787], *Critique of Pure Reason*, Cambridge, Cambridge University Press, édité et traduit par P. Guyer & A.W. Wood.

Klimovsky G. & Boido G., 2005, *Las desventuras del conocimiento matemático : filosofía de la matemática: una introducción*, Buenos Aires, AZ Editora.

Kuhn T.S., 1987, *Black-Body Theory and the Quantum Discontinuity, 1894–1912*, Chicago, University of Chicago Press.

Le Lionnais F., 2004, « Beauty in Mathematics », *in* Le Lionnais F. (éd.), Pinter C. & Kline H. (trad.), *Great Currents of Mathematical Thought, Vol. II : Mathematics in the Arts and Sciences*, New York, Dover.

Marquis J.-P., 2012, « Mario Bunge's Philosophy of Mathematics : An Appraisal », *Science & Education*, 21 (10), p. 1567-94.

Meillassoux Q., 2008 [2006], *After Finitude : An Essay on the Necessity of Contingency*, London, Continuum, traduit par R. Brassier, *Après la finitude : essai sur la nécessité de la contingence*, Paris, Le Seuil.

Nagel E. & Newman J.R., 1958, *Gödel's Proof*, New York, New York University Press.

Sokal A.D. & Bricmont J., 1998 [1997], *Fashionable Nonsense : Postmodern Intellectuals' Abuse of Science*, New York, Picador, traduit par A.D. Sokal & J. Bricmont, *Impostures intellectuelles*, Paris, Odile Jacob.

Trudeau R.J., 2008 [1987], *The Non-Euclidean Revolution*, Boston, Birkhäuser.