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CHAPTER 17

PREHISTORIC NUMERACY
Approaches, Assumptions, and Issues

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Currently, there are two approaches to investigating prehistoric numeracy. The first emerged in the 1980s and is focused on interpreting Palaeolithic artifacts using archaeological methods and criteria: "what" things mean. It was pioneered by archaeologists Alexander Marshack (1985, 1989, 1991) and Francesco d'Errico (1991, 1998, 2001), often in dialogue with or reaction to one another. D'Errico's version interprets artifacts by examining, characterizing, and comparing their marks to experimentally created engravings and ethnographically attested notations. These studies have established reliable methods and criteria for discerning the direction of the movement of the engraving point, the order in which the marks were produced, and whether they were made by the same or different tools. Assessments of how many tools were used has been used as the basis for inferring whether marks were accumulated at one time or over time, assuming that the same tool means at-one-time, different tools over-time accumulation. The latter, temporally distributed accumulation, is assumed to be more consistent with counting, particularly when marks are not otherwise differentiated. The possibility that subsequent marks might have ritual significance, which Marshack (1985) postulated might be the case, appears to have been discounted along the way, though the grounds for this are unclear.

A different approach to investigating prehistoric numeracy has been developed over the past decade by cognitive archaeologists Lambros Malafouris (2010a, 2013, 2021) and Karenleigh Overmann (2016b, 2019, 2023). It focuses on the role of counting devices in numerical thinking: "how" things mean. The approach adopts the perspective that cognition is *extended* in including material forms as a component (Clark, 2008; Clark & Chalmers, 1998) and *enacted* in consisting of the interaction between brain, body, and world (Hutto, 2013; Hutto & Myin, 2013). In the extended/enacted approach (hereafter, the extended approach), material devices are an integral part of the cognitive system for numbers, as well as an active participant in the process whereby numerical meaning emerges. The extended approach does not seek to identify or interpret numerical purpose in specific artifacts, but rather, to understand the role of material devices in realizing and elaborating numerical concepts in general. However, the approach has generated insights with the potential to characterize prehistoric number systems, and perhaps inform artifactual interpretations as well.

Discussion is organized as follows: Longstanding concerns with inferential arguments in archaeology are introduced to state the rationale for the analysis, along with its methodology. Each approach is presented in detail and analyzed for its assumptions, strengths, and shortfalls, followed by an examination of something they share, the use of ethnographic data. Inferences are then explored, followed by a concluding section that suggests there may be potential benefits in combining and expanding the two methodologies.

Inferential Arguments in Archaeology

At least since Hawkes (1954) offered his degrees of anthropological inference and noted the difficulties inherent in arguing for the higher levels, archaeologists have been concerned with the inferences they make. The need for argumentative rigor is particularly true when investigating the evolution of a cognitive ability, as cognition can be witnessed and tested only in extant populations. Recognizing that we have no way to gain direct insight into the evolutionary process itself, we infer something about what the process would have been like from data about a phenomenon distinct from the evolution of the cognitive ability in question but which bears on it in some fashion.

The method used to analyze and compare the two approaches to prehistoric numeracy was inspired by the "Windows Approach" developed by Rudolph Botha (2006, 2010, 2016, 2020) for analyzing arguments about the evolution of language. As Botha notes, a distinct phenomenon (X) is said to offer a window on the evolution of a cognitive ability (Y) on the grounds that "'looking at' X makes it possible 'to see' something of Y" (Botha, 2016, p. 4). Such inferences are said to be sound when they are grounded in factual evidence, warranted so their conclusions are very probably true, and pertinent to the conclusions being drawn (Botha, 2016). Crucially, Botha's technique was developed to analyze specific arguments, not approaches like those presented here. Accordingly, the present analysis draws from Botha's technique to explicate the assumptions, strengths, and shortfalls of the two approaches, rather than analyzing specific arguments made by either.

In a deductive argument, the soundness of an argument is a matter of all the premises being true and the form being valid; by this metric, either an argument is sound, or it is not. In contrast, in an inferential argument, soundness lacks the "all-or-nothing" status it has in deduction (Botha, 2016, p. 24) and might be compared instead to the legal standards "a preponderance of the evidence" and "beyond a reasonable doubt." The former is the minimal standard an inferential argument should meet, the latter a desirable goal to attain, with the certainty of a conclusion judged by where it falls on the spectrum created by the two. In both inferential and deductive arguments, warranting means establishing that the evidence supports the conclusion. Doing so involves an inferential step, with a *bridge theory* providing grounds for believing that a distinct phenomenon relates to the evolution of a particular cognitive ability and which specifically links their properties (Botha, 2016).

Numbers, the present concern, and language, the cognitive ability on which Botha's analyses have focused, differ significantly in their archaeological signature. Language produces no material traces before the advent of writing, and thus is archaeologically invisible before that point. Arguments about the evolution of language supported with archaeological evidence other than writing face a large evidentiary gap and are particularly vulnerable to any weaknesses regarding why and how such other evidence might be relevant. In contrast, numbers can and do have the potential to leave archaeological traces, if not in their initial emergence as behaviors, perishable materials, and language, then at least in whatever portion of their subsequent expression involves artifacts made of non-perishable materials. This means that archaeological evidence is not as unrelated to the evolutionary development of numbers as it necessarily is for language. This has freed research in prehistoric numeracy to address questions such as whether artifacts represented numbers or some other social purpose (the focus of the traditional approach) and what artifacts have to do with numerical thinking (the focus of the extended approach).

The Traditional Approach

The traditional approach is concerned with interpreting prehistoric artifacts containing marks, especially linear incisions (Figure 17.1, A_1). These artifacts are ambiguous regarding any numerical use, particularly when the marks are undifferentiated, as indistinguishable marks lack explicit clues to their meaning, numerical or otherwise. Given their Palaeolithic origin, there are no observational data to use as a basis for characterizing their social purpose or use, and the limitations of archaeological insight into prehistoric culture means there is often insufficient cultural context to support a particular interpretation. Prehistoric marks are instead analyzed in terms of their morphology and surface disposition, qualities that are then compared to ethnographically attested notations whose encoding factors have been analyzed and identified (d'Errico, 1991, 1998). Prehistoric marks are also examined microscopically for what their characteristics might suggest about direction, order, and the number of tools involved in their production. These qualities are compared to experimentally produced marks, whose controlled manufacture reliably correlates marks with the tools and methods used to produce them (d'Errico, 1991, 2001).

D'Errico's artifactual analyses (B_1) extended work by Marshack (Bednarik, 1991) and identified two critical shortfalls in Marshack's interpretations. The first concerned Marshack's analytical methods, which d'Errico criticized as having "never been clearly described nor validated by replicative experiments" (d'Errico, 1998, p. 20; also see Elkins, 1996; Robinson, 1992). In comparison, d'Errico and his colleagues closely examine marks on prehistoric artifacts, including microscopically, and carefully describe them in terms of their morphology and surface disposition. They also experimentally produce and exhaustively describe marks and the tools and movements used to make them. These efforts have yielded

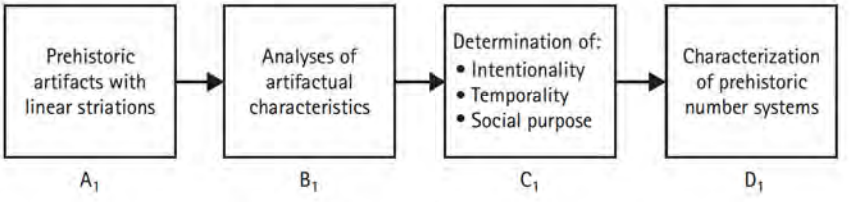

FIGURE 17.1 The traditional approach to prehistoric numeracy.

both a comprehensive dataset of prehistoric marks and a reliable standard of comparison for making inferences about the tools used to produce them. These studies and products, in turn, provide a solid, empirical basis for replicating the results obtained. These efforts have the requisite scientific rigor and credibility needed to ground the results and conclusions (B_1) in a satisfactory manner.

The second shortfall concerned Marshack's correlations of artifactual marks to lunar phenomena. These varied from artifact to artifact to such an extent that d'Errico deemed them subjective and unreplicable (d'Errico, 2001). D'Errico's approach has differed accordingly: Rather than interpreting artifacts based on perceived conformances to lunar phenomena, he compiled ethnographic data on notational systems from Africa, Europe, and the Americas, identifying four encoding factors used to differentiate notational marks from each other: element quantity, morphology, and spatial distribution, as well as temporal accumulation as inferred from the tools used (d'Errico, 1998). Morphological regularity was deemed to provide a basis for differentiating intentionality from unintentionality in mark production, a reasonable and plausible criterion that appears adequately warranted, given the ethnographic and experimental data that serve as comparisons. The data collected, standards used, and analytical criterion thus adequately ground inferences of intentionality (C_1) drawn from the artifactual analyses.

Temporally distributed accumulation, as inferred from the use of different tools to produce otherwise indistinguishable marks, is used to support interpretations of marks as notations—possibly numerical in intent, perhaps as representing counting—rather than as reflecting non-notational purposes like decoration, musical instruments, grip improvement, or tools of unknown types and functions. This inference is less certain, since it is not clear from published descriptions whether such non-notational marks were included and analyzed as part of the ethnographic data forming the comparative standard; the emphasis on notations suggests they were not.[1] Further, the validity of temporal distribution as a basis for inferring numerical intention (C_1) becomes questionable when the results are visually indistinguishable. That is, indistinguishability in final appearance suggests the prioritization of regularity, an overdetermination of the form needed for accumulation.

The final part of the inference (C_1) is weaker still. For artifacts used as notational systems in extant societies, d'Errico associates his four encoding factors with six social purposes: prayer aids, memory aids, records, calendars, messages, and hunting marks[2] (d'Errico, 1998). However, he has not correlated the encoding factors with the social purposes in ways that would enable the latter to be inferred from the former, whether extant or prehistoric. In addition, he has not specified notations categorized as numerical in terms of any characteristic organizational encoding (e.g., value bundling). This possibly indicates that the sample contains none. Neither has he specified any of the extant notational systems in terms of the associated number systems (e.g., extent of counting, organizing base, etc.), data critical to making inferences about a number system from the marks it produces.

These shortfalls suggest that the traditional approach makes three assumptions. The first assumption is that concepts of numbers are either absent or present, and that when present, resemble those of today's Western tradition, the "backward appropriation" that makes us think the numbers we know are what numbers are, regardless of time or place (Rotman, 2000, p. 40). Given the variability observed between extant cultural number systems, this assumption is, minimally, risky. The assumed risk is even higher for recently emerged systems, as they are the ones with the greatest variability. Since prehistoric numbers are more likely to have emerged recently within their time period, they are likely to have been highly variable as well. This suggests that the prehistoric numbers being looked for archaeologically may differ significantly from what those numbers are likely to have been.

The second assumption is that marks, including the specifically numerical marks on a tally, are reliable indices of numbers and/or numeracy. This, however, is unlikely to be the case. Three Australian "message sticks" about a century old, each with seven notches, illustrate the point. Their meaning was provided by knowledgeable informants: a list of men killed, possibly a mnemonic use rather than counting per se; a warning to allies that an abandoned campsite had been laced with concealed poisoned bones, a conventional meaning; and a notification to a man that his wife had died, another conventional message (Carroll, 1896, Figs 3 and 5; Howitt, 1889, Fig. 7; also see Kelly, 2020). These meanings show that seven notches do not necessarily mean seven of something. Neither do the marks provide insight into the associated number system. Australian languages might count no higher than "three" or "ten" (Bowern & Zentz, 2012), but message sticks can contain quantities of marks that fall within this range or exceed it, and artifacts with upwards of 30 notches are not unknown (e.g., Hamlyn-Harris, 1918, Figs 8 and 27; Howitt, 1889, Figs 1, 9, 10, 11, and 12).

The third assumption is that marks were meaningful in the past in the same way they would be today. When marks are regularly spaced and indistinguishable except under the microscope, they cannot be said to comprise a code of any kind (d'Errico, 1998). As a result, interpreting such marks has tended to revert to Marshack's technique of matching their quantity to natural phenomena like the lunar cycle (e.g., the 29 notches on the Lebombo bone; see d'Errico et al., 2012). However, "without the right cognitive ecology or skillful intentionality ... meaning does not emerge even in the case w[h]ere visual resemblance or indexicality is inherent" (Malafouris, 2021, pp. 10–11). On this final step of interpretation, the inferences from artifactual analyses become susceptible to the same issues for which Marshack was criticized, with the result that the interpretations of numerical intent (C_1) are weak.

Even if the determinations of (C_1) had the desired soundness, characterizing the associated number system (D_1) would be constrained because the traditional approach is necessarily limited to the devices found in the material record. These are likely to be only a fraction of the devices used, since the ethnographic literature shows that most devices used for numbers are made of perishable materials or consist of behaviors like finger-counting and sorting. If prehistoric numeracy emerged in similar fashion—and there is no compelling reason to believe otherwise—then emerging numbers would have been indicated gesturally, described verbally, and/or represented with perishable materials; all of these behaviors and forms are unlikely to produce an archaeological signature. This means that the traditional approach is unlikely to find evidence of the initial emergence of numbers, and in fact must infer numerical development either from material forms used for numbers subsequent to their emergence (e.g., tallies of non-perishable materials) or material forms marked for secondary, non-numerical purposes (e.g., musical instruments or personal ornaments). These considerations are missing from the stages of numerical emergence outlined in d'Errico et al. (2018). Complicating this further is the present murkiness of characteristics and criteria by which numerical developments might be inferred from subsequent/numerical or secondary/non-numerical material forms.[3]

A further constraint is that of the artifacts known archaeologically, only those with potentially notational markings—particularly incisions and cupules—have been subject to analysis as potential numerical implements. Other material forms, including hand stencils

(Leroi-Gourhan, 1967; Rouillon, 2006) and strung beads (d'Errico et al., 2005), have potential numerical meaning and utility (Coolidge & Wynn, 2011; Overmann, 2014), but attempts to interpret them as numerical artifacts have faced difficulties similar to those encountered with undifferentiated linear marks. For example, while wear marks can show that shells were strung (d'Errico et al., 2005), they cannot determine whether they were used as a rosary (d'Errico, 1998). Since many cultures make and wear beads but few count with them, strung shells are more likely to have been personal ornaments (Overmann, 2013). However, this cultural observation does not help identify the possible exceptions.

Beyond the methodological constraints just noted, the traditional approach does not perceive a need to characterize the associated number systems (D_1), perhaps assuming prehistoric numbers to be essentially no different from the Western numbers familiar to most researchers. Yet cultural numbers differ non-trivially in terms of their content, structure, and organization. Beyond characteristics like extent and organizing base, numbers can be distinguished in qualities like discreteness and relatedness. In Amazonian Brazil, Mundurukú numbers count up to "about three or four" and are fuzzy rather than discrete at their upper limit (Rooryck et al., 2017). In Papua New Guinea, Oksapmin numbers are related to each other by their order within the counting sequence; as the relations between them are ordinal, they are not easily manipulated with the same arithmetical algorithms used with Western numbers (Saxe, 2012), which require more relations than are present in an ordinally incremented sequence. Minimally, such differences suggest that Palaeolithic numbers would differ, perhaps significantly, from today's Western numbers. This highlights the need to contextualize the ethnographic data used as a comparative standard for notations with details of the associated number systems, and to ensure the comparative data reflect the full spectrum of numerical elaboration.

Notably, the traditional approach has begun to incorporate the number sense (or *numerosity*) that humans share with other species (d'Errico et al., 2018). Numerosity lets us recognize or *subitize* small quantities up to "about three or four"; above that range, we see "many" and appreciate magnitude—larger and smaller—in the quantity of groups, assuming the difference lies above a threshold of noticeability (Dehaene, 2011; Piazza, 2011). This functionality constrains the appreciability and thus the usability of large quantities of marks. These must be recounted, if numbers are available; if numbers are not available, they must be compared to a known standard, judged by the relative size of their total, or remain uncounted. This, in turn, implies that undifferentiated linear marks, especially in large quantities, might represent accumulation but not counting, at least not counting as understood within an organized system of numbers.

The traditional approach has not yet considered how the perceptual system for quantity interacts with different quantities of artifactual marks, or the role that artifacts might have in numerical elaboration. Given the stages of numerical emergence presented in d'Errico et al. (2018), the traditional approach may also assume either that tally marks are the first manifestation of number (a possibility the ethnographic data contradict in showing the use of the hand for this purpose), or that they directly prefigure symbolic signs for numbers (a possibility the ethnographic data and literature on numerical notations suggest is unlikely). These matters are discussed further in conjunction with the extended approach. And as was mentioned above, inferring numerical emergence from subsequent/numerical and secondary/non-numerical artifacts is not well supported, theoretically or methodologically. However, traditional analyses do provide useful evidence of whether and when the requisite cognitive ecology (i.e., artifacts with intentional marks) might be in place, estimated by D'Errico and colleagues (2018) at about 40,000 years ago. What is needed to develop these insights further is an overarching theory about the role of material forms in numerical realization and elaboration, something the traditional approach lacks but the extended approach provides.

The Extended Approach

The extended approach uses Material Engagement Theory (MET) (Malafouris, 2010b, 2013, 2019; Renfrew & Malafouris, 2008) as its theoretical framework. MET draws on philosophical concepts like extension, the idea that material forms are a constitutive component of cognition (Clark, 2008; Clark & Chalmers, 1998), to understand the role of material forms in numerical cognition. From this perspective, the archaeological record can be treated as an integral part of cognition, rather than as externalized content or behavioral traces of mental processes (Wynn et al., 2021). The initial application of MET to numeracy yielded the insights that material forms make the perceptual experience of quantity tangible and manipulable (Malafouris, 2010a; also see Coolidge & Overmann, 2012), and that separating the conjoined representation of commodity and quantity in early Mesopotamian signs facilitated their later development as systems of writing and mathematics (Malafouris, 2010a, 2013). The approach was later expanded to include insights from cultural, behavioral, neuropsychological, and linguistic data to view numeracy as a materially mediated system, rather than a purely mental or linguistic one (Overmann, 2016b, 2018, 2019, 2023). To explain why material devices are essential to numerical realization and elaboration, the approach analyzes counting devices in terms of their *affordances* (Overmann, 2016b, 2019, 2023), relations between what material forms are and what agents can do with them (Gibson, 1977, 1979). It also views the material forms used as counting devices as anchoring and stabilizing numerical concepts (Hutchins, 2005), and as a source of material properties that become numerical properties (Frege, 1953) through processes like conceptual blending (Fauconnier & Turner, 2002; Lakoff & Núñez, 2000).

The extended approach redraws the boundaries of numerical cognition to include material devices and then asks how a cognitive system composed of interacting psychological, physiological, behavioral, and material elements changes over time, beginning with the perceptual experience of quantity in the physical world, to achieve highly elaborated constructs represented with symbolic notations. This analysis synthesizes interdisciplinary data related to extant number systems. These include psychological processes like numerosity in humans and other species; cross-cultural behaviors before and with numbers; social purposes and uses for numbers; material devices used to represent and manipulate numbers; numerical structure and organization; historical change in numerical and mathematical ideas; properties of notational systems; differences between numerical and non-numerical signs in early writing systems; how numerical and non-numerical signs change over time and across linguistic groups; geographic and temporal distributions of number systems; and numerical language.

These data (Figure 17.2, A_2) are analyzed as clues to a holistic cognitive system, yielding a naturalistic, non-neurocentric explanation of how numbers emerge as concepts

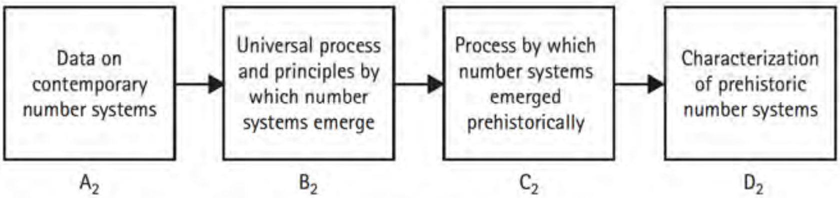

FIGURE 17.2 The extended approach to prehistoric numeracy.

mediated and informed by the material forms used to represent and manipulate them. Numerical conceptualization starts with the perceptual ability to appreciate quantity and a world with appreciable quantities. That is, numerosity is viewed as an integral system composed of brain, body, and world, whose material dimension can be altered in ways that bring forth meaning. Expressed another way, symbolic notations are understood to structure and organize numerical concepts in a way that informs both how they are acquired and what they are understood to be (Schlimm, 2018); the extended approach simply expands this insight to include precursor technologies like tallies and fingers. Numbers then emerge under two specific conditions. The first emergence condition is the social need to manage internal and external complexity, a context that is differentially timed by global migration and informed by local factors and conditions (Divale, 1999; Epps, 2006; Epps et al., 2012; Hammarström, 2015; Heine, 1997; Overmann, 2015, 2019, 2023; Stampe, 1976; Winter, 1999). The second emergence condition is the use of material forms to make numerosity tangible and thus tractable to expression and manipulation. Numbers also elaborate under two specific conditions. The first elaborational condition is an increased social need for numbers, which motivates numerical use, potentializing their extension and efficiency. The second elaborational condition is the recruitment of additional material forms into the cognitive system for numbers, whose properties act as proxies for numerical properties, thereby becoming an implicit part of how numbers are conceptualized.

Inferring that numbers emerge in a universal manner (B_2) involves multiple, converging lines of evidence (A_2). If not individually dispositive, their cumulative weight must also be considered, an essential aspect of inferential argumentation:

- First, numerosity functions the same across and despite significant differences in numerical elaboration (Gordon, 2004; Henrich et al., 2010; Pica et al., 2004), providing a common starting point for the realization process, along with pentadactyl limbs, material culture, and behavioral strategies like pairing and one-to-one correspondence. As numbers elaborate—and independent of the degree of elaboration attained—numerosity continues to underpin numerical thinking (Nieder, 2017a, 2017b), plausibly helping to influence elaboration toward the commonality observed cross-culturally in extant number systems.
- Second, lexical numbers emerge across languages and cultures as “one” and “two,” perhaps “three” or “about three,” and “many,” often subdivided as “big many” and “small many,” while other numerical features of language like grammatical number show similar patterning. The cross-linguistic, cross-cultural distribution of numerical features of language suggest both the influence of numerosity (Overmann, 2015, 2019,

2023; also see Franzon et al., 2019; Rinaldi & Marelli, 2019) and a common realization process.

- Third, the hand, whose five-fingered form is a species trait, is so commonly used for counting, and at such an early stage of numerical emergence, that most of the world's number systems use quantities like five, ten, or twenty based on the digits or quantities like four, six, eight, twelve, or fourteen based on the segments, joints, or interstitial spaces, as their organizing base (Comrie, 2011, 2013). The involvement of the hand also means that numbers are not particularly dependent on language as they emerge, since they can be represented gesturally and understood visually. This common material mediation precedes, overlaps, and influences how numbers are expressed linguistically (Overmann, 2019, 2023).
- Fourth, counting devices are not just ubiquitous (e.g., Flegg, 1983; Ifrah, 2000; Menninger, 1992; Owens, 2018), their types are also common cross-culturally: devices that accumulate, including fingers, tallies, knotted strings, torn leaves, groups of pebbles or corn, the body, and marks on surfaces; devices that accumulate and group, like fingers and toes, groups of pebbles or corn, the abacus, counting boards, and the Mesopotamian tokens; and handwritten notations. Device sequences suggest that material forms create habits and expectations that inform the selection of new devices, limitations that motivate the incorporation of new devices, and opportunities for properties of new devices to become conceptual properties of numbers (Overmann, 2016b, 2018, 2019, 2023). That is, from the common starting point provided by numerosity and the hands, the recruitment of new devices is systematized by device affordances and limitations, yielding highly similar outcomes in numerical elaboration.
- Fifth, differences in numerical content, structure, and organization reliably correspond to the material forms used to represent and manipulate numbers (Overmann, 2018, 2019, 2023). In extant societies, cross-cultural numerical similarities are the result of common psychological, physiological, and behavioral capacities and capabilities, while cross-cultural numerical variability becomes a function of whether material devices are used, which ones are used, and how they are used.
- Finally, although systems of numerical signs are invented independently, they are organized in only a handful of ways, perhaps as few as five (Chrisomalis, 2010). As signs, their forms remain highly conserved (particularly for subitizable numbers), both across time within any particular culture and as borrowed between cultures, because they are *semasiographic*,[4] semantically meaningful as numbers without any phonetic component (Overmann, 2016a, 2021), and *translinguistic*, meaningful across and without language. The observed organizational commonality highlights the origin of numerical signs from systematized precursor technologies, while the evident continuity of form suggests that numerical signs experience and respond similarly to common factors governing and influencing their function and use.

A universal process and principles for numerical emergence (B_2) appears to hold for prehistoric numbers as well (C_2). This inference is based on the following evidence:

- The phylogenetic distribution of numerosity within extant primates supports the conclusion that ancestral species had numerosity (Baum, 2008; Delsuc et al., 2005), so archaic humans in the Palaeolithic would have had it as well. Within primates, numerosity has similar form and function: The intraparietal sulcus subserves the ability to perceive quantity (Dehaene et al., 2003; Orban, 2016; Viswanathan & Nieder, 2013), functionally differentiated as subitization and magnitude appreciation (Beran et al., 2011, 2013; Beran & Parrish, 2016; Cutini & Bonato, 2012; Jordan & Brannon, 2006). The phylogenetic distribution across species and shared form and function within primates means that archaic humans would have had the same perceptual experience of quantity as their starting point for realizing numbers, along with pentadactyl limbs and material culture. However, the degree to which they would have shared behavioral strategies like pairing and one-to-one correspondence is an open question.

- Language would have been in place by the Upper Palaeolithic, something on which even the accounts proposing that it emerged only very recently (e.g., Bolhuis et al., 2014) agree. Insights into early number-words come from proto-languages, hypothetical ancestral languages reconstructed from shared descendent vocabularies. Proto-languages typically provide insight into the last 10,000 years, far short of the Palaeolithic period of interest. However, they do show the use of the hand in numbers. For example, in Proto-Semitic, estimated as emerging around 3750 BCE, the word for "five" is related to the word for "hand," and terms for "ten," "hundred," "thousand," and "ten thousand" show the number system was organized decimally (Kitchen et al., 2009; Lipínski, 2001; Miller & Shipp, 2014), "ten-ness" that reflects the use of the hand. Similarly, in Proto-Indo-European, thought to have been spoken between 6,500 and 4,500 years ago, the words for "five" and "ten" suggest terms for one or both hands, and the term for "hundred," which is analyzable as "ten tens," shows decimal organization (Beekes, 2011; Blažek, 1999; Bomhard, 2008). These characteristics suggest that Neolithic behaviors, material forms, and processes of realization and elaboration were similar to those observed in extant number systems. This in turn implies that these processes would likely have been similar in earlier periods as well.

These psychological, behavioral, material, and linguistic commonalities provide grounds for inferring that Palaeolithic numbers would have emerged and elaborated (C_2) by means of behaviors and counting devices similar to those observed in extant cultural number systems. The likelihood of a universal process, in turn, has implications for characterizing prehistoric numeracy (D_2), and perhaps for interpreting prehistoric artifacts as well, something that will be explored in the next section. Notably, artifactual interpretation has not been a goal of the extended approach, which has rather sought to understand how material forms function generally in numerical emergence and elaboration (Overmann, 2016b, 2019, 2023), or in forming the requisite conditions and "cognitive ecology" for meaning to emerge (Malafouris, 2021).

Calling the model "extended" raises the issue of whether it is necessary to commit to the idea that cognition is extended to accept the insights generated by the model. Here we might recognize that some cognitive states (e.g., reading) do not and cannot exist without interacting with a material form (in this case writing, of which numerals form a part). The continuity of function (i.e., instantiation) between numerical notations and precursor forms (e.g., tokens, tally marks, fingers) suggests that the latter forms can act as the material component of extended states in the way that the former does (Overmann, 2023). All these technologies differ in the details of their material substance, but arguably very little in their actual form or in *how* they mean (i.e., by *instantiating* quantity, as three fingers, three beads, and three vertical strokes on a tally or in a numeral mean three by *being* three). Then, if written numerals that are part of a written text can be recognized as part of an extended cognitive state, near-identical forms like tallies and tokens might reasonably be part of an extended state as well. Certainly, the question of whether cognition is extended cannot and will not be settled here. However, redrawing the boundaries of cognition lets us examine how material forms act as a component of the cognitive system for numbers, both in the moment and over ontogenetic, cultural, and evolutionary spans of time. We can recognize material forms as influencing change in the behaviors and psychological processing of their users, which in the case of writing is learning to read. Then we can ask how and why material forms can influence such change and, moreover, accumulate and distribute it between individuals and generations.

The commitment to an extended model of cognition raises several issues. One is whether a universal process for numeracy depends on cognition being extended; if the extended hypothesis is unsound, a universal process based on it might be as well. Arguably, the situation is just the reverse: A universal process does not depend on cognition being extended as much as it supports the idea that it is, especially given the many correspondences between numerical content, structure, and organization and the material devices used to represent and manipulate numbers. Another issue is whether the extended hypothesis must be adopted for either the approach to succeed or researchers to accept its findings. Regarding the former, all approaches to investigating cognition make choices regarding what to include or exclude. Redrawing the boundaries of numerical cognition to include material forms, as the extended approach does, has shown initial success in generating novel insights. Regardless of any success, however, the inclusion of material forms for analytical purposes does not establish that cognition is, in fact, extended. Nor is that necessarily the goal, which is instead to see whether redrawing the boundaries has value in generating novel insights (Malafouris, 2013). Regarding researcher acceptance, the insights generated by the extended approach can alternatively be framed as a universal process inferred from multiple sources of evidence or a visual epistemology for numbers, both of which are consistent with, but not dependent on, the idea of extended cognition.

While a visual epistemology seems uncontroversial in geometry (Dehaene et al., 2006; Giaquinto, 2007; Izard et al., 2011), the idea of a manuovisual route for accessing numerical intuitions—one that works in conjunction with language but is phylogenetically, diachronically, and ontogenetically prior to it—has been contentious. Objections often resist the notions that the hand is a material structure and that it comprises as such a critical element of the material component of numbers. Certainly, the visual epistemology in geometry does not require that the hand be included as a material form. Excluding the hand in numbers, however, has consequences: Numerical conceptualization becomes sequestered to the brain, and language becomes its first overt expression. Connecting the perceptible experience of quantity to both language and material artifacts then becomes much more difficult (Núñez, 2017a, 2017b). Material artifacts like tallies become passive repositories of mental content. And finally, explaining why some but not all societies develop highly elaborated numbers becomes quite challenging (Chomsky, 2004a, 2004b; Hurford, 1987; also see criticism in Verran, 2000). In contrast, a manuovisual route for accessing numerical intuitions connects the number sense to symbolic notations through the hand and intermediate forms like the tally; recognizes material artifacts as actively influencing numerical properties; and explains the variability between number systems as a function of whether devices are used, which devices are used, and how the devices are used.

Ethnographic Data

Both approaches assume that ethnographic data can provide insight into prehistoric numeracy, with the expanded approach using such data more broadly than the traditional approach, which has confined its use to cross-cultural notations. Using ethnographic data to gain insight into past cultural practices has a contentious and justly criticized history in archaeology (Ascher, 1961; Wylie, 2002). Typically, the use of ethnographic data is questioned on the basis of its legitimacy and trustworthiness. Arguments against the data's legitimacy assert that using "the present of one society simply to interpret the past of another, especially as the present is often seen as a latter-day survival of [a] stage passed elsewhere in the world," is "immoral" (Gosden, 1999, p. 9). Arguments against the data's trustworthiness include the incomparability of cultures and the unreliability of the data, discussed below. A final question, one pertinent to an analysis of inferential strength, emerges from the circumstance that analogies require the use of inference in forming arguments.

Against the argument that cultures, especially complex ones, are incomparable is the insight that this does not seem to be true when it comes to numbers. While it is true that, in general, "an incredible variety of codes of behaviour in fact actuate human conduct" (Smith, 1955, p. 5), numbers demonstrate an unusual cross-cultural uniformitarianism, as well as a strong continuity of form even across significant spans of time, place, and cultural change, as systematized by a common starting point and elaborational mechanisms and as canalized by common psychological, physiological, and behavioral capacities. Numbers also appear to be independent of specific environmental conditions, as attempts to tie numerical elaboration to specific resourcing strategies have been unsuccessful (e.g., Divale, 1999). What the pattern of global distribution appears to indicate instead is a differential timing of numerical emergence according to continental arrival, with numerical emergence representing a common and consistent response to the need to manage internal and external complexity as imposed by factors like population growth and increased interactions with other social groups.

The idea that ethnographic data are inherently unreliable recognizes that they tend to be "(1) time-limited, (2) based on unreliable informants, (3) based on ambiguous and biased data collation" (Currie, 2016, p. 90), (4) ill-defined in their temporal and geographic extent, (5) not randomly sampled, and (6) non-independent because of cultural relatedness or geographic proximity (Ember & Ember, 2000; Kaplan & Manners, 1972; Pelto & Pelto, 1978). Against this argument is the fact that for numbers, cumulative unreliability might actually work in their favor. That is, when even the most problematic of instruments find the same thing, time and time again, then what they report seems likely to be a case of what is actually there.

But it is legitimate to use such data? It has been observed that "one of the main purposes of archaeology is to study the variability of human societies and understand cultural processes" (Politis, 2002, p. 63, as translated). Here too, numbers appear to be a special case. They not only demonstrate a species capacity and emphasize our common humanity, but cross-cultural comparisons can occasion the insights that Western numbers are one cultural system among many and that cultural number systems are inherently neither advantageous nor privileged, just different in what they provide and respond to. For example, simultaneously keeping track of two ordinal sequences in order to subtract them, as Oksapmin participants did when asked to perform Western-style arithmetic with their traditional numbers (Saxe, 2012), involves demands on working memory that a person enculturated into Western numbers would likely find difficult to negotiate. A Westerner might also find it difficult to find ten different ways of forming a number like 16,669—let alone construct it with negatives (e.g., as 70 is formed by taking 10 from 80) and keep track of negative subtraction, as can be done in Yoruba[5] (Akinadé & Ọdẹ́jọbí, 2014; Ẹkundayọ, 1977). If the metric is mental manipulation, Western numbers seem undemanding in comparison, almost to the point of impoverishment. Rigidly excluding the use of comparative ethnographic data precludes opportunities for both realizing such insights and genuinely admiring traditional methods.

Implications of the Ethnographic Data

Given the likelihood of universal processes for numerical emergence and elaboration, even in prehistory, the ethnographic data raise several issues in interpreting prehistoric artifacts, detailed below. These do not constitute, nor are they intended to provide, a complete list. However, they do suggest considerations and constraints that should inform artifactual interpretations, as well as hypotheses that could be tested, especially if a comprehensive and robust ethnographic standard were available to act as a comparison.

First, the prehistoric material record is unlikely to provide insight into the earliest numbers to emerge, as these would have consisted of gestural or linguistic references to material objects. In addition, the material record is unlikely to contain the majority of counting devices used, as these would not preserve because they consisted of perishable materials, ephemeral forms, or behaviors. Investigations of emerging numbers thus arrive at much the same position previously mentioned for language: Arguments must show how and why the archaeological evidence is relevant, since artifacts, if numerical, are most likely to be associated with subsequent developments in cultural numeracy (e.g., the incorporation of non-perishable materials to represent numbers that were realized and elaborated previously with perishable and ephemeral forms), and if non-numerical, are indirectly related at best to numerical developments. Neither implication excludes the possibilities that artifacts with marks were made and used before, during, and after numbers began to emerge and elaborate, and that at least some of them might have been implicit to numerical conceptualization or represented actual counting devices. This, in turn, raises the question of how well artifacts, whether subsequent/numerical or secondary/non-numerical, might provide accurate and reliable insight into aspects of the associated cultural number system.

Second, prehistoric number systems would have differed from today's Western numbers, perhaps significantly. They may have been highly variable as well, perhaps to the extent found today in Papua New Guinea. Indigenous number systems there include restricted systems that count no higher than "two" or "three"; digit-tally systems (quinary, quinary–decimal, quinary–vigesimal, decimal, and vigesimal); systems with binary, trinary, quaternary, and senary organization; and body-counting systems with cycles ranging from 12 to 74

(Bowers & Lepi, 1975; Evans, 2009; Lean, 1992; Owens, 2001; Saxe, 2012). The understanding that emerging numbers can vary considerably, and that societies can use more than one method of counting, should inform what we look for in the material record as potential signs of emerging numeracy. Here too a revised and expanded ethnographic standard would be highly beneficial (Hayden, 2021). Ideally, the standard would be informative as to the kinds of material traces left by different counting behaviors and devices as used within different types of number systems; it would also be informative on matters such as when, why, and how societies incorporate non-perishable material forms to record their numbers.

Finally, undifferentiated linear marks are likely to remain a challenge, both in interpreting their meaning and in trying to use them to gain insight into the associated number system. The experimental evidence, to the degree that undifferentiated linear marks can illuminate aspects of any number system, should indicate only the modern numerical tradition(s) of the researchers who produced it. Regarding the ethnographic evidence, if it is to act as a comparative standard for identifying characteristics of the number systems associated with specific artifacts, the data set should be expanded to encompass the full range of variability in cultural number systems, associate notational marks with characteristics of the number systems producing them, and include subsequent/numerical and secondary/non-numerical notations, as well as marks made for purposes other than notations. Ultimately, even if new methods, criteria, and standards can be created, the challenge of interpreting the meaning of undifferentiated linear marks is likely to persist, not only because unencoded forms are ambiguous as to their intent, purpose, and use, but also because they can be produced by societies with no numbers, few numbers, and highly elaborated numbers.

Conclusion

Neither the traditional approach nor the extend approach currently delivers a satisfactory or complete picture of prehistoric numeracy. The traditional approach lacks an overarching theory about what material forms have to do with numbers, and the extended approach is not concerned with the meaning of specific artifacts. Combining the two approaches might yield greater insight into prehistoric numeracy, with the extended approach suggesting enhancements to the current ethnographic standard as compiled by the traditional approach, and with the traditional approach incorporating insights about the role of material forms in numeracy from the extended approach.

The lack of archaeological evidence (especially for emerging numbers), the possible differences between prehistoric and extant numbers, and the potential interaction between numerical elaboration and undifferentiated marks all highlight the need for a standard that describes and differentiates notations and non-notational marks according to characteristics of the associated number systems, and which includes specimens across the full spectrum of numerical elaboration. Such a standard is not currently known to exist. Even with a reliable standard, it will remain difficult to discern whether artifacts with undifferentiated marks are subsequent/numerical or secondary/non-numerical.

Further, answering the question of when numbers first emerged may require "windows" other than archaeological, ethnographic, and linguistic ones, perhaps pointing the inquiry back to the kinds of things used to investigate the evolution of language, such as ontogenetic acquisition and the numerical capacities of other species. Another potential window might be migration and population analyses, given that the geographic distribution of elaboration in cultural number systems corresponds globally to the planetary migration of the human species and regionally to local factors and conditions (Overmann, 2019, 2023). A third window might consider marks, regardless of their meaning, as forming the necessary cognitive ecology for systems of meaningful marks to emerge, particularly those made within the past 150,000 years, the period coextensive with anatomically modern humans.

Finally, the need for strong inference and judicious use of ethnographic analogies remains paramount and was the motivation for the present analysis. Hopefully, this effort will lead to further explorations, not only of what ethnographic data can reveal about number systems, but also of a combined, expanded research agenda and analyses of specific arguments as formulated by the two approaches.

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Notes

1. Using ethnographic data as a basis for comparison, Brian Hayden (2021) recently proposed new criteria for distinguishing between notational, decorative, and utilitarian marks on prehistoric artifacts. While Hayden characterizes this work as exploratory, it is a welcome step in the right direction.
2. Wayfinding is a significant omission, given cultural systems that use undifferentiated marks to depict or record series of landmarks (see e.g., Hugh-Jones, 2016).
3. Recently, "tokenism" was suggested as a potential way to infer numerical developments from secondary (non-numerical) artifacts like "symbols, cave paintings, portable art, [and the] ornamental decoration of tools" (Schlaudt, 2020, p. 634). Drawing on Peirce's type–token distinction, tokenism proposes the utilitarian use of artifacts may provide evidence of signs being reduced to "mere instantiations of a type," thereby reflecting developments in numerical thinking (p. 633). Tokenism may assume an independent and prior development of a type (in this case, a number concept), which may indeed be the case for both subsequent/numerical and secondary/non-numerical artifacts by definition. Putting tokenism into archaeological practice will require, minimally, developing criteria for inferring tokenism in artifacts, differentiating it from mere physical skill in productive movements, and connecting it to numerical thinking. Specific lacunae will also need to be addressed, as for example, there is no indication of how tokenism might fit with numerical emergence through the use of the fingers as attested in extant cultures, or the way in which numerical signs develop and change over time (Chrisomalis, 2010; Overmann, 2016a, 2019, 2021; see further discussion in Overmann, 2023).
4. Numerical signs differ from non-numerical signs because they instantiate quantity—three marks *are* three—rather than depict through resemblance and convention. Depiction is

ambiguous regarding the words intended, so non-numerical signs either depend on memorization and interpretation, or they must specify word types and sound values, which complicates their visual appearance. In comparison, instantiation and bundling make numerical signs unambiguous and concise, improving their semantic intelligibility; adding visual elements to specify sounds (e.g., "seven" instead of "7") detracts from this concision and degrades the usability it provides (Overmann, 2016a, 2019, 2021, 2023).

5. The ten ways to form 16,669 in Yoruba are glossed as follows (see Akinadé & Ọdẹ́jọbí, 2014; Ẹkundayọ, 1977):

   (20,000 × 1) – 300 – (1 + 30)
   (20,000 × 1) – 400 + (1 from (10 from (20 × 4)))
   (20,000 × 1) – ((20 × 2) + 300) + 9
   (1,000 from (2,000 × 10)) + ((20 × 2) + (200 × 3)) + (1 from 30)
   (1,000 from (2,000 × 10)) + ((20 × 3) + (200 × 3)) + 9
   (1,000 from (2,000 × 10)) + ((20 × 4) + (200 × 3)) – (1 + 10)
   (1,000 from (2,000 × 10)) + (100 from (200 × 4)) – (1 + 30)
   (1,000 from (2,000 × 10)) + (200 × 3) + (1 from (10 from (20 × 4)))
   (100 from (200 × (1 from (20 × 5)))) – (1 + 30)
   (200 × (2 from (20 × 5))) + (1 from (10 from (20 × 4)))

References

Akinadé, O. O., & Ọdẹ́jọbí, Ọ. A. (2014). Computational modelling of Yorùbá numerals in a number-to-text conversion system. *Journal of Language Modelling*, *2*(1), 167–211.

Ascher, R. (1961). Analogy in archaeological interpretation. *Southwestern Journal of Anthropology*, *17*(4), 317–325.

Baum, D. (2008). Trait evolution on a phylogenetic tree: Relatedness, similarity, and the myth of evolutionary advancement. *Nature Education*, *1*(1), 1–6.

Bednarik, R. G. (1991). Comment on d'Errico, "Microscopic and statistical criteria for the identification of prehistoric systems of notation." *Rock Art Research*, *8*(3), 89–91.

Beekes, R. S. P. (2011). *Comparative Indo-European linguistics: An introduction* (2nd ed.). John Benjamins.

Beran, M. J., Decker, S., Schwartz, A., & Schultz, N. (2011). Monkeys (*Macaca mulatta* and *Cebus apella*) and human adults and children (*Homo sapiens*) compare subsets of moving stimuli based on numerosity. *Frontiers in Psychology*, *2*, 1–7.

Beran, M. J., McIntyre, J. M., Garland, A., & Evans, T. A. (2013). What counts for 'counting'? Chimpanzees, *Pan troglodytes*, respond appropriately to relevant and irrelevant information in a quantity judgment task. *Animal Behaviour*, *85*(5), 987–993.

Beran, M. J., & Parrish, A. E. (2016). Capuchin monkeys (*Cebus apella*) treat small and large numbers of items similarly during a relative quantity judgment task. *Psychonomic Bulletin & Review*, *23*(4), 1206–1213.

Blažek, V. (1999). *Numerals: Comparative etymological analyses and their implications: Saharan, Nubian, Egyptian, Berber, Kartvelian, Uralic, Altaic and Indo-European languages*. Masarykova Univerzita.

Bolhuis, J. J., Tattersall, I., Chomsky, N., & Berwick, R. C. (2014). How could language have evolved? *PLoS Biology*, *12*(8), 1–6.

Bomhard, A. R. (2008). Some thoughts on the Proto-Indo-European cardinal numbers. In J. D. Bengtson (Ed.), *In hot pursuit of language in prehistory. Essays in the four fields of anthropology in honor of Harold Crane Fleming* (pp. 213–222). John Benjamins.

Botha, R. (2006). On the windows approach to language evolution. *Language & Communication*, *26*(2), 129–143.

Botha, R. (2010). On the soundness of inferring modern language from symbolic behaviour. *Cambridge Archaeological Journal*, *20*(3), 345–356.

Botha, R. (2016). *Language evolution: The Windows Approach*. Cambridge University Press.

Botha, R. (2020). *Neanderthal language: Demystifying the linguistic powers of our extinct cousins*. Cambridge University Press.

Bowern, C., & Zentz, J. (2012). Diversity in the numeral systems of Australian languages. *Anthropological Linguistics*, *54*(2), 133–160.

Bowers, N., & Lepi, P. (1975). Kaugel Valley systems of reckoning. *Journal of the Polynesian Society*, *84*(3), 309–324.

Carroll, A. (Ed.). (1896). The message sticks. *The Australasian Anthropological Journal*, *1*(1), 10–11.

Chomsky, N. (2004a). *Biolinguistics and the human capacity* (pp. 1–9). Magyar Tudományos Akadémia.

Chomsky, N. (2004b). *The generative enterprise revisited: Discussions with Riny Huybregts, Henk van Riemsdijk, Naoki Fukui and Mihoko Zushi* (2nd ed.). Mouton de Gruyter.

Chrisomalis, S. (2010). *Numerical notation: A comparative history*. Cambridge University Press.

Clark, A. (2008). *Supersizing the mind: Embodiment, action, and cognitive extension*. Oxford University Press.

Clark, A., & Chalmers, D. J. (1998). The extended mind. *Analysis*, *58*(1), 7–19.

Comrie, B. (2011). *Typology of numeral systems*. Max Planck Institute. https://mpi-lingweb.shh.mpg.de/numeral/TypNumCuhk_11ho.doc

Comrie, B. (2013). Numeral bases. In M. S. Dryer & M. Haspelmath (Eds.), *World atlas of language structures online*. Max Planck Institute for Evolutionary Anthropology. http://wals.info/chapter/131

Coolidge, F. L., & Overmann, K. A. (2012). Numerosity, abstraction, and the emergence of symbolic thinking. *Current Anthropology*, *53*(2), 204–225.

Coolidge, F. L., & Wynn, T. (2011). Commentary on Henshilwood and Dubreuil, The Still Bay and Howiesons Poort, 77–59 Ka: Symbolic material culture and the evolution of the mind during the African Middle Stone Age. *Current Anthropology*, *52*(3), 380–382.

Currie, A. (2016). Ethnographic analogy, the comparative method, and archaeological special pleading. *Studies in History and Philosophy of Science, Part A*, *55*, 84–94.

Cutini, S., & Bonato, M. (2012). Subitizing and visual short-term memory in human and non-human species: A common shared system? *Frontiers in Psychology*, *3*, 1–2.

D'Errico, F. (1991). Microscopic and statistical criteria for the identification of prehistoric systems of notation. *Rock Art Research*, *8*, 83–93.

D'Errico, F. (1998). Palaeolithic origins of artificial memory systems: An evolutionary perspective. In C. Renfrew & C. Scarre (Eds.), *Cognition and material culture: The archaeology of symbolic storage* (pp. 19–50). McDonald Institute for Archaeological Research.

D'Errico, F. (2001). Memories out of mind: The archaeology of the oldest memory systems. In A. Nowell (Ed.), *In the mind's eye: Multidisciplinary approaches to the evolution of human cognition* (pp. 33–49). International Monographs in Prehistory.

D'Errico, F., Backwell, L., Villa, P., Degano, I., Lucejko, J. J., Bamford, M. K., Higham, T. F. G., Colombini, M. P., & Beaumont, P. B. (2012). Early evidence of San material culture represented by organic artifacts from Border Cave, South Africa. *Proceedings of the National Academy of Sciences of the United States of America*, *109*(33), 13214–13219.

D'Errico, F., Doyon, L., Colagé, I., Queffelec, A., Le Vraux, E., Giacobini, G., Vandermeersch, B., & Maureille, B. (2018). From number sense to number symbols. An archaeological perspective. *Philosophical Transactions of the Royal Society of London. Series B, Biological Sciences*, *373*(1740), 1–10.

D'Errico, F., Henshilwood, C. S., Vanhaeren, M., & van Niekerk, K. L. (2005). *Nassarius kraussianus* shell beads from Blombos Cave: Evidence for symbolic behaviour in the Middle Stone Age. *Journal of Human Evolution*, *48*(1), 3–24.

Dehaene, S. (2011). *The number sense: How the mind creates mathematics* (Revised). Oxford University Press.

Dehaene, S., Izard, V., Pica, P., & Spelke, E. S. (2006). Core knowledge of geometry in an Amazonian indigene group. *Science*, *311*(5759), 381–384.

Dehaene, S., Piazza, M., Pinel, P., & Cohen, L. (2003). Three parietal circuits for number processing. *Cognitive Neuropsychology*, *20*(3), 487–506.

Delsuc, F., Brinkmann, H., & Philippe, H. (2005). Phylogenomics and the reconstruction of the tree of life. *Nature Reviews Genetics*, *6*(5), 361–375.

Divale, W. (1999). Climatic instability, food storage, and the development of numerical counting: A cross-cultural study. *Cross-Cultural Research*, *33*(4), 341–368.

Ẹkundayọ, S. A. (1977). Vigesimal numeral derivational morphology: Yoruba grammatical competence epitomized. *Anthropological Linguistics*, *19*(9), 436–453.

Elkins, J. (1996). On the impossibility of close reading: The case of Alexander Marshack. *Current Anthropology*, *37*(2), 185–226.

Ember, M., & Ember, C. R. (2000). Testing theory and why the "units of analysis" problem is not a problem. *Ethnology*, *39*, 349–363.

Epps, P. (2006). Growing a numeral system: The historical development of numerals in an Amazonian language family. *Diachronica*, *23*(2), 259–288.

Epps, P., Bowern, C., Hansen, C. A., Hill, J. H., & Zentz, J. (2012). On numeral complexity in hunter-gatherer languages. *Linguistic Typology*, *16*(1), 41–109.

Evans, N. (2009). Two *pus* one makes thirteen: Senary numerals in the Morehead-Maro region. *Linguistic Typology*, *13*(2), 321–335.

Fauconnier, G., & Turner, M. (2002). *The way we think: Conceptual blending and the mind's hidden complexities*. New York.

Flegg, G. (1983). *Numbers: Their history and meaning*. Dover Publications.

Franzon, F., Zanini, C., & Rugani, R. (2019). Do non-verbal number systems shape grammar? Numerical cognition and number morphology compared. *Mind & Language*, *34*(1), 37–58.

Frege, G. (1953). *The foundations of arithmetic: A logical-mathematical investigation into the concept of number* (J. L. Austin, Trans.; 2nd rev.). Harper & Brothers.

Giaquinto, M. (2007). *Visual thinking in mathematics: An epistemological study*. Oxford University Press.

Gibson, J. J. (1977). The theory of affordances. In R. Shaw & J. Bransford (Eds.), *Perceiving, acting, and knowing: Toward an ecological psychology* (pp. 127–143). Lawrence Erlbaum Associates.

Gibson, J. J. (1979). *The ecological approach to visual perception*. Houghton Mifflin.

Gordon, P. (2004). Numerical cognition without words: Evidence from Amazonia. *Science, 306*(5695), 496–499.

Gosden, C. (1999). *Anthropology & archaeology: A changing relationship*. Routledge.

Hamlyn-Harris, R. (1918). On messages and 'message sticks' employed among the Queensland Aborigines. *Memoirs of the Queensland Museum, 6*, 13–36.

Hammarström, H. (2015). *Restricted numeral systems and the hunter-gatherer connection*. Max Planck Institute for Psycholinguistics. https://www.eva.mpg.de/fileadmin/content_files/linguistics/conferences/2015-speaking-of-Khoisan/P8b_Linguistics_Hammarstroem_Numeral_systems.pdf

Hawkes, C. (1954). Archeological theory and method: Some suggestions from the Old World. *American Anthropologist, 56*(2), 155–168.

Hayden, B. (2021). Keeping count: On interpreting record keeping in prehistory. *Journal of Anthropological Archaeology, 63*, 101304.

Heine, B. (1997). *Cognitive foundations of grammar*. Oxford University Press.

Henrich, J., Heine, S. J., & Norenzayan, A. (2010). The weirdest people in the world? *Behavioral and Brain Sciences, 33*(2-3), 61–135.

Howitt, A. W. (1889). Notes on Australian message sticks and messengers. *The Journal of the Anthropological Institute of Great Britain and Ireland, 18*, 314–332.

Hugh-Jones, S. (2016). Writing on stone; writing on paper: Myth, history and memory in NW Amazonia. *History and Anthropology, 27*(2), 154–182.

Hurford, J. R. (1987). *Language and number: The emergence of a cognitive system*. Blackwell.

Hutchins, E. (2005). Material anchors for conceptual blends. *Journal of Pragmatics, 37*(10), 1555–1577.

Hutto, D. D. (2013). Radically enactive cognition in our grasp. In Z. Radman (Ed.), *The hand, an organ of the mind: What the manual tells the mental* (pp. 227–252). MIT Press.

Hutto, D. D., & Myin, E. (2013). *Radicalizing enactivism: Basic minds without content*. MIT Press.

Ifrah, G. (2000). *The universal history of numbers: From prehistory to the invention of the computer* (D. Bellos, E. F. Harding, I. Monk, & S. Wood, Trans.). John Wiley & Sons.

Izard, V., Pica, P., Spelke, E. S., & Dehaene, S. (2011). Flexible intuitions of Euclidean geometry in an Amazonian indigene group. *Proceedings of the National Academy of Sciences of the United States of America, 108*(24), 9782–9787.

Jordan, K. E., & Brannon, E. M. (2006). A common representational system governed by Weber's law: Nonverbal numerical similarity judgments in 6-year-olds and rhesus macaques. *Journal of Experimental Child Psychology, 95*(3), 215–229.

Kaplan, D., & Manners, R. A. (1972). *Culture theory*. Prentice-Hall.

Kelly, P. (2020). Australian message sticks: Old questions, new directions. *Journal of Material Culture, 25*(2), 133–152.

Kitchen, A., Ehret, C., Assefa, S., & Mulligan, C. J. (2009). Bayesian phylogenetic analysis of Semitic languages identifies an Early Bronze Age origin of Semitic in the Near East. *Proceedings of the Royal Society B, Biological Sciences, 276*(1668), 2703–2710.

Lakoff, G., & Núñez, R. E. (2000). *Where mathematics comes from: How the embodied mind brings mathematics into being*. Basic Books.

Lean, G. A. (1992). *Counting systems of Papua New Guinea and Oceania* [Unpublished doctoral dissertation]. Papua New Guinea University of Technology.

Leroi-Gourhan, A. (1967). Les mains de Gargas: Essai pour une étude d'énsemble. *Bulletin de La Société Préhistorique Française, 64*(1), 107–122.

Lipínski, E. (2001). *Semitic languages: Outline of a comparative grammar* (2nd ed.). Peeters.
Malafouris, L. (2010a). Grasping the concept of number: How did the sapient mind move beyond approximation? In C. Renfrew & I. Morley (Eds.), *The archaeology of measurement: Comprehending heaven, earth and time in ancient societies* (pp. 35–42). Cambridge University Press.
Malafouris, L. (2010b). Metaplasticity and the human becoming: Principles of neuroarchaeology. *Journal of Anthropological Sciences, 88*, 49–72.
Malafouris, L. (2013). *How things shape the mind: A theory of material engagement*. MIT Press.
Malafouris, L. (2019). Mind and material engagement. *Phenomenology and the Cognitive Sciences, 18*, 1–17.
Malafouris, L. (2021). Mark making and human becoming. *Journal of Archaeological Method and Theory, 28*(1), 95–119.
Marshack, A. (1985). Theoretical concepts that lead to new analytic methods, modes of enquiry and classes of data. *Rock Art Research, 2*, 95–111.
Marshack, A. (1989). Evolution of the human capacity: The symbolic evidence. *American Journal of Physical Anthropology, 32*(S10), 1–34.
Marshack, A. (1991). *The roots of civilization: The cognitive beginnings of man's first art, symbol and notation* (Revised). McGraw-Hill.
Menninger, K. (1992). *Number words and number symbols: A cultural history of numbers* (P. Broneer, Trans.). Dover Publications.
Miller, D. B., & Shipp, M. R. (2014). *An Akkadian handbook: Helps, paradigms, glossary, logograms, and sign list* (2nd ed). Eisenbrauns.
Nieder, A. (2017a). Evolution of cognitive and neural solutions enabling numerosity judgements: Lessons from primates and corvids. *Philosophical Transactions of the Royal Society of London. Series B, Biological Sciences, 373*(1740), 20160514.
Nieder, A. (2017b). Number faculty is rooted in our biological heritage. *Trends in Cognitive Sciences, 21*(6), 403–404.
Núñez, R. E. (2017a). Is there really an evolved capacity for number? *Trends in Cognitive Sciences, 21*(6), 409–424.
Núñez, R. E. (2017b). Number—Biological enculturation beyond natural selection. *Trends in Cognitive Sciences, 21*(6), 404–405.
Orban, G. A. (2016). Functional definitions of parietal areas in human and non-human primates. *Proceedings of the Royal Society B, Biological Sciences, 283*(1828), 1–9.
Overmann, K. A. (2013). Material scaffolds in numbers and time. *Cambridge Archaeological Journal, 23*(01), 19–39.
Overmann, K. A. (2014). Finger-counting in the Upper Palaeolithic. *Rock Art Research, 31*(1), 63–80.
Overmann, K. A. (2015). Numerosity structures the expression of quantity in lexical numbers and grammatical number. *Current Anthropology, 56*(5), 638–653.
Overmann, K. A. (2016a). Beyond writing: The development of literacy in the Ancient Near East. *Cambridge Archaeological Journal, 26*(2), 285–303.
Overmann, K. A. (2016b). The role of materiality in numerical cognition. *Quaternary International, 405*, 42–51.
Overmann, K. A. (2018). Constructing a concept of number. *Journal of Numerical Cognition, 4*(2), 464–493.
Overmann, K. A. (2019). *The material origin of numbers: Insights from the archaeology of the Ancient Near East*. Gorgias Press.

Overmann, K. A. (2021). Writing system transmission and change: A neurofunctional perspective. In G. Gabriel, K. A. Overmann, & A. Payne (Eds.), *Signs – sounds – semantics. Nature and transformation of writing systems in the Ancient Near East* (pp. 93–116). Ugarit-Verlag.

Overmann, K. A. (2023). *The materiality of numbers: Emergence and elaboration from prehistory to present*. Cambridge University Press.

Owens, K. (2001). The work of Glendon Lean on the counting systems of Papua New Guinea and Oceania. *Mathematics Education Research Journal*, *13*(1), 47–71.

Owens, K. (2018). *History of number: Evidence from Papua New Guinea and Oceania*. Springer International Publishing.

Pelto, P. J., & Pelto, G. H. (1978). *Anthropological research: The structure of inquiry* (2nd ed.). Cambridge University Press.

Piazza, M. (2011). Neurocognitive start-up tools for symbolic number representations. In S. Dehaene & E. Brannon (Eds.), *Space, time and number in the brain: Searching for the foundations of mathematical thought* (pp. 267–285). Academic Press.

Pica, P., Lemer, C., Izard, V., & Dehaene, S. (2004). Exact and approximate arithmetic in an Amazonian indigene group. *Science*, *306*(5695), 499–503.

Politis, G. G. (2002). Acerca de la etnoarqueología en América del Sur. *Horizontes Antropológicos*, *8*(18), 61–91.

Renfrew, C., & Malafouris, L. (2008). Steps to a "neuroarchaeology" of mind. *Cambridge Archaeological Journal*, *18*(3), 381–385.

Rinaldi, L., & Marelli, M. (2019). The use of number words in natural language obeys Weber's law. *Journal of Experimental Psychology: General*, *149*(7), 1215–1230.

Robinson, J. (1992). Not counting on Marshack: A reassessment of the work of Alexander Marshack on notation in the Upper Palaeolithic. *Journal of Mediterranean Studies*, *2*(1), 1–17.

Rooryck, J., Saw, J., Tonda, A., & Pica, P. (2017). *Mundurucú number words as a window on short-term memory*. The Royal Society of London. https://halshs.archives-ouvertes.fr/halshs-01497577

Rotman, B. (2000). *Mathematics as sign: Writing, imagining, counting*. Stanford University Press.

Rouillon, A. (2006). Au Gravettien, dans la grotte Cosquer (Marseille, Bouches-du-Rhône), l'Homme a-t-il compté sur ses doigts? *Anthropologie*, *110*(4), 500–509.

Saxe, G. B. (2012). *Cultural development of mathematical ideas*. Cambridge University Press.

Schlaudt, O. (2020). Type and token in the prehistoric origins of numbers. *Cambridge Archaeological Journal*, *30*(4), 629–646.

Schlimm, D. (2018). Numbers through numerals: The constitutive role of external representations. In S. Bangu (Ed.), *Naturalizing logico-mathematical knowledge: Approaches from psychology and cognitive science* (pp. 195–217). Routledge.

Smith, M. A. (1955). The limitations of inference in archaeology. *The Archaeological News Letter*, *6*, 3–7.

Stampe, D. (1976). Cardinal number systems. In S. S. Mufwene, C. A. Walker, & S. B. Steever (Eds.), *Papers from the twelfth regional meeting, Chicago Linguistic Society, April 23–25, 1976* (Vol. 12, pp. 594–609). Chicago Linguistic Society.

Verran, H. (2000). Aboriginal Australian mathematics: Disparate mathematics of land ownership. In H. Selin (Ed.), *Mathematics across cultures: The history of non-Western mathematics* (pp. 289–311). Kluwer Academic.

Viswanathan, P., & Nieder, A. (2013). Neuronal correlates of a visual "sense of number" in primate parietal and prefrontal cortices. *Proceedings of the National Academy of Sciences of the United States of America*, *110*(27), 11187–11192.

Winter, W. (1999). When numeral systems are expanded. In J. Gvozdanović (Ed.), *Numeral types and changes worldwide* (pp. 43–53). Mouton de Gruyter.
Wylie, A. (2002). The reaction against analogy. In *Thinking from things: Essays in the philosophy of archaeology* (pp. 136–153). University of California Press.
Wynn, T., Overmann, K. A., & Malafouris, L. (2021). 4E cognition in the Lower Palaeolithic: An introduction. *Adaptive Behavior*, *29*(2), 99–106.