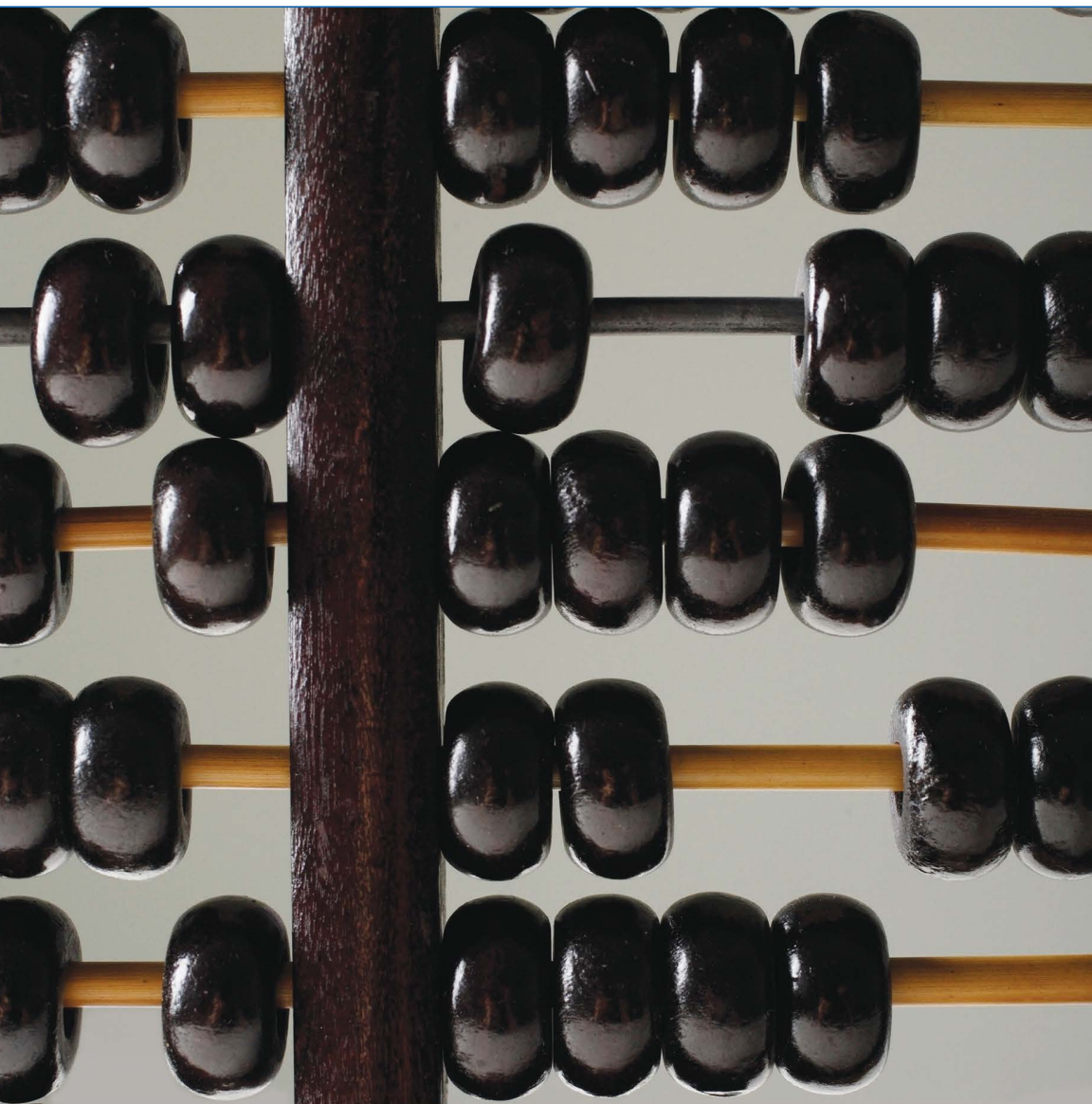


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# THE MATERIALITY OF NUMBERS

EMERGENCE AND ELABORATION  
FROM PREHISTORY TO PRESENT

KARENLEIGH A. OVERMANN

## The Materiality of Numbers

This is a book about numbers – what they are as concepts and how and why they originate – as viewed through the material devices used to represent and manipulate them. Fingers, tallies, tokens, and written notations, invented in both ancestral and contemporary societies, explain what numbers are, why they are the way they are, and how we get them. Cognitive archaeologist Karenleigh A. Overmann is the first to explore how material devices contribute to numerical thinking, initially by helping us to visualize and manipulate the perceptual experience of quantity that we share with other species. She explores how and why numbers are conceptualized and then elaborated, as well as the central role that material objects play in both processes. Overmann's volume thus offers a view of numerical cognition that is based on an alternative set of assumptions about numbers, their material component, and the nature of the human mind and thinking.

KARENLEIGH A. OVERMANN earned her doctorate in archaeology from the University of Oxford as a Clarendon scholar after retiring from twenty-five years of active service in the US Navy. She currently directs the Center for Cognitive Archaeology at the University of Colorado, Colorado Springs.

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Emergence and Elaboration from  
Prehistory to Present

KARENLEIGH A. OVERMANN

*University of Colorado, Colorado Springs*



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

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## Foreword

If, like me, you grew up in the industrialized world, you have been surrounded by numbers almost from birth. Our modern world is immersed in numbers. We encounter them everywhere – on money, calendars, highway signs, airline flights, even our official identities. Numbers lie at the core of mathematics, where they also reveal complex relationships that have enabled our ever more complex hegemony over the natural world. Ability to handle numbers is rivaled only by ability in language when we assess human intelligence. And like language, there are regions of our brains that appear to be dedicated to numerical and mathematical understanding.

Numbers are a way to think about quantity. However, they are not natural phenomena. There are no numbers in the natural world waiting to be discovered. The natural world does have ways to think about quantity that evolved long ago and are shared by many different animal species. One is the ability to distinguish sets of , , , , and sometimes . The other is the ability to compare sets of items to one another and judge which is larger. But these are not numbers. Numbers are ordinal – they extend in an invariant sequence – and they have cardinal value; each successive number advances in value by the same amount ( $1$ ,  $1 + 1 = 2$ ,  $2 + 1 = 3$ ,  $3 + 1 = 4$ , etc.). But how did we acquire this unnatural but very useful way to think about quantity?

This is the question that Karenleigh Overmann tackles in this book, and it is a fascinating account. Dr. Overmann examines an array of evidence, including archaeology (from the Stone Age through the rise of civilization), brain anatomy and function, nonhuman primates, languages (both living and dead), and the geographic distribution of counting systems. But the weight of her reasoning rests on the comparative ethnographic evidence of human counting from around

the world and from a variety of technical and cultural contexts. Not all humans have numbers, or even count. Some societies have words for only *one*, *two*, and *three*; any quantity greater is “many.” Others have systems of counting that match items with digits (fingers and toes), and occasionally other body parts – a hand’s worth of shells or an elbow’s worth of yams. And yet others use tokens, usually small manipulable objects, to expand the range of counting possible with body parts. Dr. Overmann marshals this evidence brilliantly into an account of how numbers must have emerged in the prehistoric and early historic past.

But unlike many other scholars who have examined this evidence, Dr. Overmann does not consider these fingers and tokens to be expressions of a number system; instead, she presents them as the very resources through and by which people created numbers in the first place. Think for a moment about how children learn numbers. Most often they first learn a list of number labels, sometimes extending well past *twenty*. But they do not yet understand the cardinal nature of numbers – the rule that generates a sequence running to infinity. They then use their memorized list as an aid to build an understanding of numbers – first the smaller values (3 is one more than 2), then larger values (14 is one more than 13), until ultimately, they acquire an understanding of the rule itself. Cognitive science labels such aids “scaffolds.” But what about people who possessed no such scaffold, that is, people who had no labels for numbers? How could they reach an understanding of cardinality and ordinality? They couldn’t have “discovered” them because nature does not provide any cardinal or even ordinal sequences. But they did have potential aids in the form of fingers and toes and material objects. Dr. Overmann documents clearly how such manipulation, over hundreds of generations, yielded our understanding of number. Our ancestors invented numbers, and they did it by manipulating material things.

Dr. Overmann’s achievement is arguably the best explicit example of evolutionary extended cognition yet proposed. Models of

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embodied and extended cognition have been very much in vogue in some academic circles for over thirty years now. The idea that the mind is made up not of just gray (really, pink) matter in the head, but also includes bodily resources and cultural extensions is popular because it sidesteps epistemological problems attendant with the classic Cartesian dualism between minds and bodies. However, most of the literature remains primarily philosophical and theoretical, with the occasional thought experiment in support (e.g., Merleau-Ponty's blind man's stick). The approach has made few inroads into the empirical science of human evolution. Dr. Overmann's study of number is thus badly needed, providing a persuasive account of the active role played by materiality in the recent evolution of the human mind.

Dr. Overmann's analysis has significance beyond the narrow domain of numerical cognition. By documenting the active role of materiality in the development of numerical thinking, she opens the door for consideration of materiality as a component of other cognitive systems. Cognitive archaeology – my particular field of expertise – is a recent addition to the corpus of methods that science deploys to investigate the evolution of mind, and it has always been hamstrung by the Cartesian mind-body dualism of traditional cognitive science. Archaeology studies material things and must infer the components of mindedness. If material things played an active role in cognitive evolution, then archaeologists' burden of proof would be lightened considerably. But it would still require close reasoning and careful marshaling of evidence from a variety of domains. Dr. Overmann shows the way in this remarkable volume.

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*September 2022*



## Preface

Early in the sixteenth century, Spanish conquistadors first encountered the *kipu*, the device of knotted strings used by the Inka to record numbers and other information.<sup>1</sup> The *kipu* astonished the Spaniards, who clearly did not understand it as a form of writing – not even its numerical component, which shared many of its qualities with the various forms of abacus prevalent in Europe at the time.<sup>2</sup> Since the sixteenth century, colonial chroniclers and modern authors alike have spoken of the *kipu* as a device for computation. However, *kipus* could not have been used for this purpose, since knots cannot possibly be tied, untied, and retied quickly enough for the effective manipulation that computation requires. This is one of the main differences that the *kipu* has with an abacus, which has the manipulability needed for calculating because its beads are separate, and thus rearrangeable, objects. What follows from this persistent misunderstanding of the *kipu* is this: We generally lack insight into the material devices used for numbers, what they can and cannot do in representing and manipulating numbers, and why this is important to understanding what numbers are and where they come from. This book attempts an answer.

Certainly, a lot of books have already been written about numbers, including their origin, and from a wide variety of perspectives: psychological, developmental, neuroscientific, comparative, linguistic, cultural/ethnographic, historical, philosophical, and of course, mathematical. This particular book differs because it addresses something that all the others mention

<sup>1</sup> De Acosta, 1590; de la Vega, 1609; Guamán Poma, 1615. *Khipu* is the Quechua word for “knot”; the word is spelled *quipu* or *quipo* in Spanish.

<sup>2</sup> Pullan, 1968.

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only in passing, if at all: the material devices used to represent and manipulate numerical concepts, things like fingers and tallies and tokens and notations. These devices are important because they can explain what numbers are and why they are the way they are. They are the material aspect or *materiality* of numbers; they make numbers material and thus tangible and rearrangeable as forms that occasion the emergence of new numerical properties and meanings.<sup>3</sup>

There are three reasons why the materiality of numbers is so critical and yet so frequently invisible or ignored. First, our perceptual system for quantity has a great deal to do with numbers, since it patterns their initial expression and informs their content, visualization, and manipulation as numbers become more elaborate. This perceptual system is known as *numerosity* or the *number sense*.<sup>4</sup> The latter term is misleading because what we perceive is *quantity*, not *number*, a nontrivial distinction between experience and conceptualization.<sup>5</sup> While numerosity is multimodal – that is, we can perceive quantity not just visually but also through hearing<sup>6</sup> and touch<sup>7</sup> – only the visual dimension permits our quantity percepts to be manipulated into new, concept-generating stimuli that can be attended simultaneously. Simply, our auditory and tactile senses lack the width and granularity of focus needed to attend to multiple, parallel instances of number. Not only can material forms enable us to do this, but they also let us take control of the quantity that we visually perceive and enable us to rearrange it until it makes different sense to us. This is one of the reasons that the material forms we use to represent and manipulate numbers are critically important to their realization and elaboration as concepts.

The second reason why the material aspect of numbers is critical but neglected is that the numbers we have today, the numbers

<sup>3</sup> I am indebted to Lambros Malafouris for the idea that materiality makes the ineffable sense of quantity tangible and manipulable. This view was foundational to the argument presented here of how numbers are realized and elaborated.

<sup>4</sup> Dehaene, 2011.   <sup>5</sup> Núñez, 2017a, 2017b.

<sup>6</sup> Jordan et al., 2005, 2008; Beran, 2012.   <sup>7</sup> Davis et al., 1989; Krause et al., 2013.

of the Western tradition, are thousands of years old – likely tens of thousands of years old, and perhaps even 100,000 years old or more. During this impressive cultural persistence, they have not only acquired properties from the material devices used to represent and manipulate them, but they have also become distributed over multiple, differing devices – fingers, tallies, tokens, and numerical notations – along with all the things that display numerical representations, from clocks to coins, and the things to which they are applied, from angels to zebras. This distribution has made numbers functionally independent of any particular device; independence, in turn, makes numbers appear to be an in-the-head phenomenon, something that does not require any material devices.<sup>8</sup> In fact, the idea that numbers are a mental phenomenon is a fundamental assumption that pervades the psychological, developmental, neuroscientific, comparative, linguistic, cultural/ethnographic, historical, philosophical, and mathematical literature on numbers. A related and quite common assumption is the idea that the material devices used for numbers are mere passive repositories for that mental content: The brain thinks something up and then externalizes it onto a material form. In this book, the opposing view is taken: Material devices are not passive repositories, but rather, active participants in the cognitive system for numbers. They are active because they influence and systematize how numbers emerge and become elaborated.

A third reason the material aspect is neglected, one that is not inconsiderable, is that recognizing and then putting the materiality back into numbers requires stretching the definition of what constitutes a material form beyond an easy recognizability. For example, the fingers used in counting can be considered a material device, and so can the temporary structures created by the various sorting and collaborative strategies found in Oceania and Africa, and so too can symbolic notations like the Western numbers 0 through 9.

<sup>8</sup> Overmann, 2019a, 2019b.

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As material forms, such things obviously differ in the nature of their materiality from the actual artifacts used in counting, things like tallies, tokens, counting boards, and calculators. In these unconventional cases, the structures in question can be recognized both as material and as possessing unique attributes. The fingers are material but also composed of living flesh; sorting and collaborative strategies produce temporary material forms whose ephemerality differs from the solidity and persistence of devices like the abacus; and symbolic notations, while material, possess an arbitrary, conventional form. Arguably, these other attributes do not necessitate the exclusion of such forms from being considered for their material qualities. In fact, considering unconventional devices for their material quality is analytically useful. It lets us see how material forms act to structure numbers and infuse them with properties like linearity, stable order, capacity, manipulability, and conciseness. Importantly, it allows us to reconstruct and thus connect the entire span of numerical elaboration, from the perception of quantity shared with other species to symbolic notations. What connects numerosity to notations and the associated concepts of numbers is the manuovisual manipulation of material forms, a behavior exhibited by the human species alone.

The initial premise of the argument presented here is straightforward. Today, symbolic notations are understood as having a role in how we acquire numerical concepts and in what we understand them to be.<sup>9</sup> Symbolic notations – especially early forms like *five* as represented by vertical wedges, as in Babylonian cuneiform – share affinities of form and function with the technologies that preceded them, things like fingers (vertically extended digits) and tally marks (vertical parallel lines). Accordingly, we can ascribe the same roles that notations have in conceptual acquisition and content to their precursors, and these precursors systematically connect symbolic notations to the initial perception of

<sup>9</sup> Schlimm, 2018.

quantity. This is not surprising, since after all, before and without symbolic notations, societies both ancient and contemporary have used devices like fingers, tallies, and abaci to record numbers and calculate with them, and quite capably too. Numbers have been used and taught for thousands of years, and for most of that history and prehistory, they were used and taught without symbolic notations, a form that is relatively recent in the chronology.<sup>10</sup>

From this starting point, this book explains the role of material forms within the cognitive system for numbers, focusing on how numbers emerge and become elaborated. For example, new material forms are systematically incorporated to address limitations of previous forms in a way that is highly similar across cultures; new material forms add new properties to numbers to act thereby as the mechanism of numerical elaboration; and as new forms are added, the explicit representational form becomes more concise, while an implicit component, the knowledge that the user must supply, concomitantly increases.<sup>11</sup>

Seeing numbers as a system realized and elaborated through the use of material devices can require more than an unconventional view of what constitutes a material device for numbers: It may also require an unconventional view of cognition itself. In this book, cognition is considered to be a system that consists of bodies and material forms, in addition to the brain. That is, the mind is seen as embodied, embedded, extended, and enactive (4E). The theoretical framework used to analyze cognition as a 4E system distributed over time is the Material Engagement Theory (MET) of archaeologist Lambros Malafouris.<sup>12</sup> A 4E framework like MET is vital to the field of evolutionary cognitive archaeology, the discipline that seeks to understand how the human mind evolved by examining the material record of ancient societies and extinct hominin species. Treating the mind as 4E opens up a dimension of temporal change that cannot otherwise be easily investigated. The MET framework elevates the

<sup>10</sup> Adkins, 1956, p. 1.    <sup>11</sup> Overmann, 2019b.    <sup>12</sup> Malafouris, 2013.

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material record from passive traces of behaviors to actual components of ancient minds.<sup>13</sup> This in turn permits the examination of a temporal dimension of change over time that disciplines like contemporary neuroscience cannot match in either depth or granularity.

Focusing on the materiality of numbers does not mean that the other domains investigating numbers are unimportant. Far from it, for they are often crucial to understanding not just numbers but also how material forms function in numbers. For example, the neurological interconnections and interactions between the different parts of the brain involved in numbers explain why finger-counting, the use of the hand as a material device for the representation and some manipulation of numbers, is so cross-culturally ubiquitous. Language, considered here to be another means of accessing and expressing our numerical intuitions, is often a source of evidence that the fingers were once used for counting, not just in the global preponderance of *ten*, *five*, and *twenty* as the organizing basis for numbers, but also in formations like *six* that mean *five plus one* and imply that the fingers were used to explicate and then label numerical concepts. Conversely, the hand being used as a material device explains why its patterns of use and associated linguistic labeling differ cross-culturally, since despite the fact that all humans have essentially the same hand and behavior in using it for counting, societies use the hand differently in counting. In other words, despite the common assumption that numbers are purely mental constructs, there does not seem to be any internal (that is, mental or innate) mandate for using the hand for counting in any specific way. This leaves different societies free to make unique choices in how the hand is used, creating a wide and occasionally astonishing cross-cultural variability.

It will also be useful to explore some of the assumptions made by the other disciplines that investigate *numeracy*, the ability to reason with numbers that presumes numerical concepts are available.

<sup>13</sup> Wynn et al., 2021.

For example, psychologists looking at numeracy in humans and cognitive scientists focusing on numerical abilities in animals often see numerosity as all that is needed for conceptualizing numerical concepts and performing simple computations. Accordingly, they find numerical concepts and operations in species as divergent as chimpanzees and honeybees.<sup>14</sup> This is the nativist view that mental capacities and structures are innate or biologically endowed, rather than learned. Be that as it may, if the nativist view were correct, then linguists, who argue that language is the reason we have number concepts, cannot be right, since only humans have language.<sup>15</sup> The exact opposite is true as well, since if language were the source of numbers and numeracy, then as only humans have language, only humans would have numbers. In fact, both positions are problematic because all humans have numerosity and language, but not all human societies have numbers, at least not to the degree found in the Western numerical tradition. Some societies, like the Mundurukú of Amazonian Brazil, have very few numbers<sup>16</sup> – *one, two, about three*, and *about four* – and one of their neighbors, the Pirahã, are said to have no numbers whatsoever.<sup>17</sup> If reports of Mundurukú and Pirahã anumeracy are more severe than their actuality, the degree to which some societies do lack numbers is nonetheless a severe challenge to the positions that numerosity and/or language comprise all that are/is needed for numbers.<sup>18</sup>

Pace those holding such positions, these shortfalls create explanatory gaps that material forms have the potential to answer. Accordingly, it is argued here that material forms are also needed: not just material culture in general, but specific material devices for representing and manipulating numbers – however unconventionally defined – as motivated by specific socio-material conditions. Along the way, many fascinating number systems and devices will be

<sup>14</sup> Nieder, 2017a, 2017b; Howard et al., 2018; Clarke & Beck, 2021.

<sup>15</sup> Overmann, 2021e. <sup>16</sup> Pica & Lecomte, 2008; Rooryck et al., 2017.

<sup>17</sup> Everett, 2005, 2007, 2013; Frank et al., 2008. <sup>18</sup> Overmann, 2021e.

introduced. We will also see how and why devices work, what they can and cannot do as material forms, and why this too is important. The book proceeds as follows:

- Chapters 1 and 2 look at what numbers are as concepts and historical and contemporary ideas about their origin.
- Chapters 3 through 5 examine some of the major things contributing to what numbers are – perceptual functions like numerosity, behavioral strategies like finger-counting and pairing, and language – while focusing on how these domains contribute to our understanding of the material component.
- Chapter 6 examines how number systems are distributed globally and regionally; these nonrandom distributions let us see number systems in Australia and South America as presently emerging, providing critical insight into a panhuman process.
- Chapter 7 introduces key ideas about materiality and delineates the process whereby the incorporation and use of material forms systematically produces and elaborates numerical concepts.
- Chapter 8 reviews the theoretical framework used, Material Engagement Theory, and shows why the extended/enactive model of cognition is both useful and valid.
- Chapter 9 examines how quantity percepts are made tangible and manipulable through the use of *distributed exemplars* (these are features of the environment whose reliable quantity is recruited to express that number), the fingers, and small, manipulable objects.
- Chapter 10 deals with material forms like the tally that accumulate, and examines the implications of *tokenization*, the idea that deindividuated elements are important to numbers and the process whereby elements like notches might become deindividuated.
- Chapter 11 introduces the archaeological techniques currently being used to determine whether prehistoric artifacts were tallies, along with the shortfalls of such techniques.
- Chapter 12 is concerned with highly elaborated number systems that use material devices that accumulate and group, but not symbolic notations; this class of number systems is often not well understood.
- Chapter 13 examines numerical notations, what they add to numerical elaboration, how they differ from other written signs, and why their qualities are important.



- Finally, Chapter 14 sums things up by looking at the way ahead for this fascinating, promising, and unique research into numbers.

The view that numbers are essentially material in their origin and elaboration is a novel perspective, one that the philosopher Karim Zahidi recently called a “naturalistically plausible account of the emergence of the modern natural number concept.”<sup>19</sup> This perspective is new. It has not been tried in at least the 2500 years since Plato pronounced that numbers were mental constructs – eternal but intangible and invisible concepts existing somewhere external to the human minds that somehow still managed to contact and apprehend them. In explaining this new perspective on numerical origin and elaboration, this book draws its conclusions from a different set of assumptions about numbers, their material component, and the nature of the human mind.

<sup>19</sup> Zahidi, 2021, p. 530.