The Geometrical Solution of The Problem of Snell's Law of Reflection Without Using the Concepts of Time or Motion

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Abstract

During 17th century a scientific controversy existed on the derivation of Snell's laws of reflection and refraction. Descartes gave a derivation of the laws, independent of the minimality of travel time of a ray of light between two given points. Fermat and Leibniz gave a derivation of the laws, based on the minimality of travel time of a ray of light between two given points. Leibniz's calculus method became the standard method of derivation of the two laws. We demonstrate in this article that Snell's law of reflection follows from simple results of geometry. We do not use the concept of motion or the time of travel in our demonstration. That is, time plays no role in our demonstration.

Key words

Reflection, Snell's law of reflection, Least time principle, Geometry, Harmonic Conjugates, Harmonic Ratio, Internal Division, External Division, Apollonius circle, Orthogonal circle, Ptolemy's Theorem

Introduction

A bitter scientific battle raged between two groups one led by Descartes and his followers – the Cartesians – and the other led by Fermat, during the 17th century[1-3]. Neither side won, but the battle came to an end with time as the warriors left the field with passage of time. The issue around which the fight took place was law of refraction of light. Descartes published the law of constancy of the ratio of sines of angles of incidence and of refraction in 1637, though it was discovered by Willebrord Snell as early as 1621 but remained unpublished. Descartes based his result on the motion of a tennis ball hit towards the surface of separation of the two media. This model required the speed of light be greater in the denser medium and lower in the rarer medium. Fermat rejected Descartes' result as it was counter intuitive to think of such a speed relation.

Fermat developed a method for the study of maxima and minima[4,5]. He applied it to the problem of refraction of light. He assumed the speed of light is greater in the rarer medium and lower in the denser medium and derived the law of refraction assuming that light chooses the path of least time between any two points. His result also gave the sine law but the value of the constant of the ratio of speeds had the inverse value to that of Descartes'. Descartes rejected the result and severely criticized it stating that it was obtained by pure luck and not hard work [1-3]. Fermat did not concentrate on reflection of light; he thought his least time principle explains reflection.

Leibniz took a conciliatory approach and assumed light had greater speed in a denser medium than in a rarer medium and that light travelled along a path of maximum 'ease'. Ease being defined as the product of speed and the resistance of the medium of travel[1-3]. With these assumptions he derived the laws of reflection and reflection using his method of calculus[6]. He also arrived at the least time principle for the path of a ray of light between any two points. Leibniz's calculus method became the standard method of proof ever since. We demonstrate in this article that Snell's law of reflection is a consequence of fundamental results of geometry. Time and motion play no role in our demonstration.

Statement of the problem of Snell's law of reflection

We may pose the problem of Snell's law of reflection as follows.

Given the end points of the path of a reflected ray of light and the surface of reflection, find the point of incidence on the surface of reflection.

We show that the given data uniquely fixes the point of incidence on the surface of reflection that satisfies the equal angles law of reflection – the Snell's law of reflection. That proves the law of reflection.

Let F, G be the given end points of the path of ray of light in a vertical plane and AB be the horizontal plane of reflection (see Fig. 1).

Fig. 1 Figure shows the end points F, G of the path of a ray of light reflected on the surface AB

Construction

Draw the line joining F, G. Let it intersect the plane AB at P' (see Fig. 2).

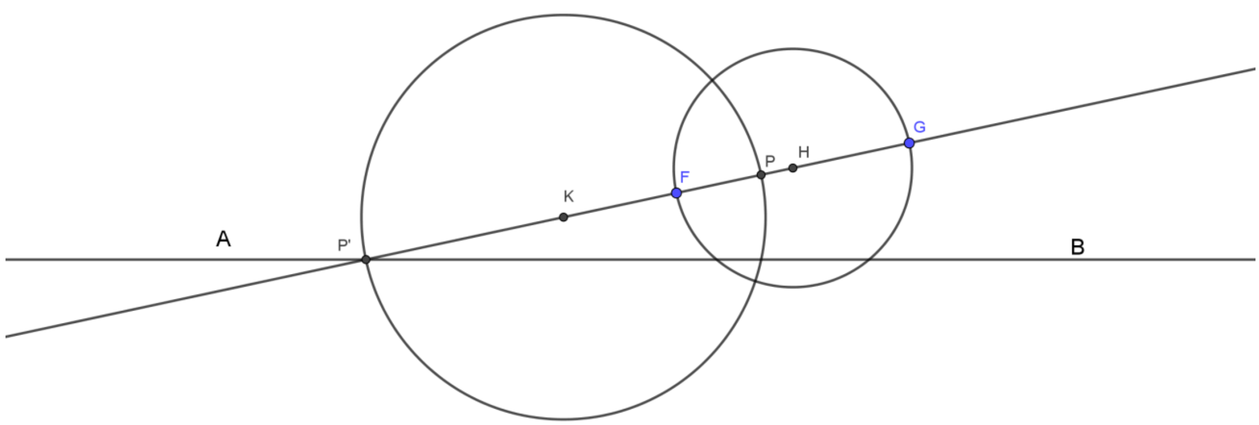

Fig. 2 Line through F, G intersects the plane AB at P'. H is the midpoint of FG. K is the midpoint of PP'.

Let P' divide the line segment GF externally in the ratio, k > 1. For k< 1 similat arguments hold.

$$GP' : FP' = k : 1 \qquad k > 1 \qquad (1)$$

Let P be the harmonic conjugate[7] of P'. Then P and P' divide the line segment GF in the same ratio internally and externally, respectively,

$$GP : PF = k : 1 \qquad (2)$$

Draw a circle with PP' as diameter. This is the Apollonius circle decided by the ratio, k. According to the definition of Apollonius circle, every point Q on this circle (see Fig. 3) is such that

$$GQ : QF = k : 1 \qquad (3)$$

Draw the circle with FG as diameter. This circle (green) is decided by the end points of the path of the ray of light. According to the principles of geometry, it intersects the Apollonius circle (red) orthogonally.

Join Q, F and Q, G. Draw the angle bisectors of the angle FQP. These bisectors form the orthogonal chords QP and QP' of the Apollonius circle (Fig. 3).

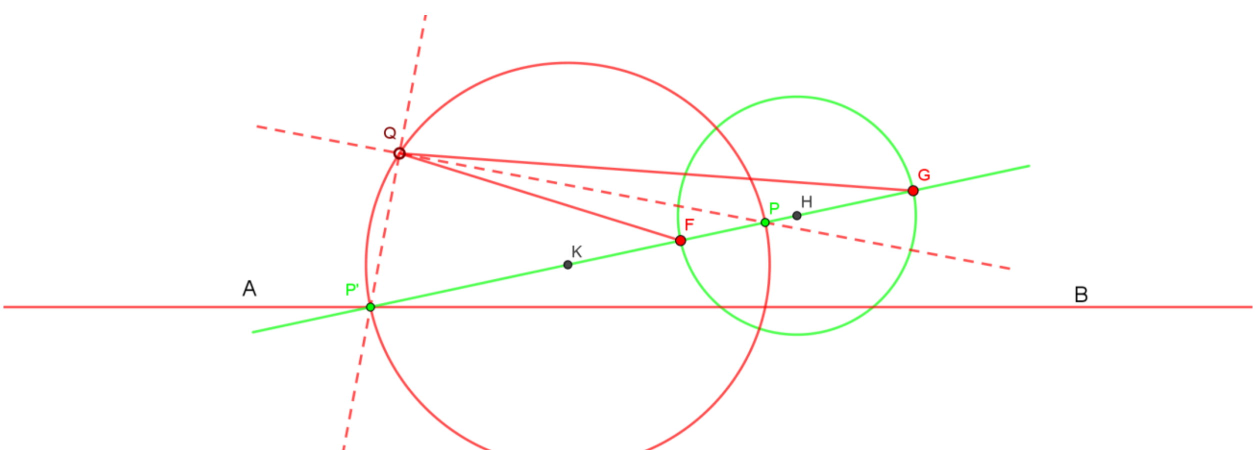

Fig. 3 Q is an arbitrary point on Apollonius circle. Bisectors of the angle FQG form the orthogonal chords of the Apollonius circle.

Since Q is an arbitrary point on this result applies to every point on the circle. Therefore it also applies to the point of intersection C of the Apollonius circle and the plane of reflection AB (Fig. 4). Join F, C and G, C. Draw the angle bisectors of the angle FCG. The orthogonal chords of the Apollonius circle CP and CP' fall along these bisectors. Therefore, if a ray of light from F is incident on the plane surface AB at point C it is reflected along CG, making equal angles with the normal CP.

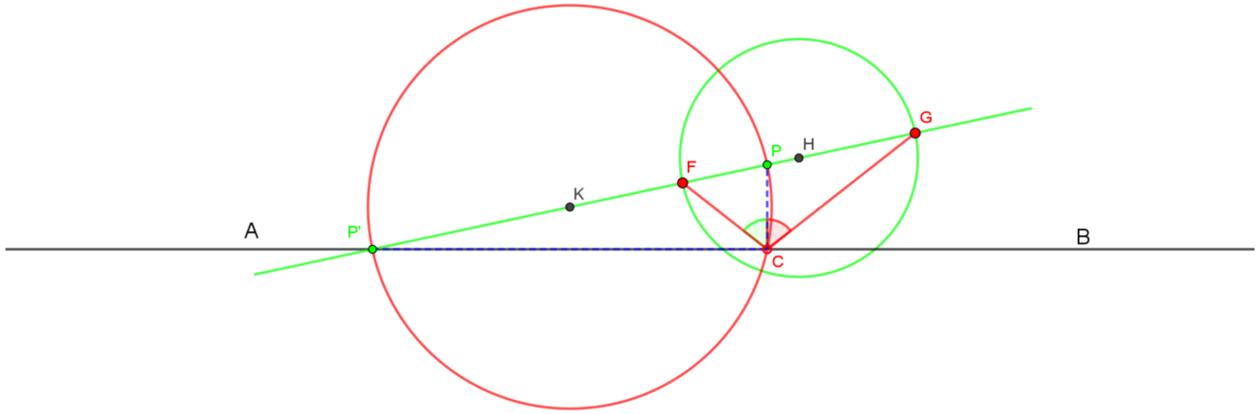

Fig. 4 Orthogonal chords CP, CP' bisect the angle FCG. angle FCP is equal to angle PCG

We call the attention of the reader to note that it is the orthogonal chords of the circle that act as the surface and the normal for the reflection. It is *not* the tangent and the radial line that act as the surface and the normal for the reflection, as is the case in conventional treatments.

Is the path of reflection the minimal distance/time path?

To see if the path of the reflected ray FCG corresponds to the minimal distance /time path, we construct the equilateral triangle with FG as side; we also construct its circum circle FMG (Fig. 5). According to Ptolemy's theorem the sum of the distances of any point on the minor arc FG, such as N, to the points F, G is a minimum. That is (FN + NG) is a minimum.

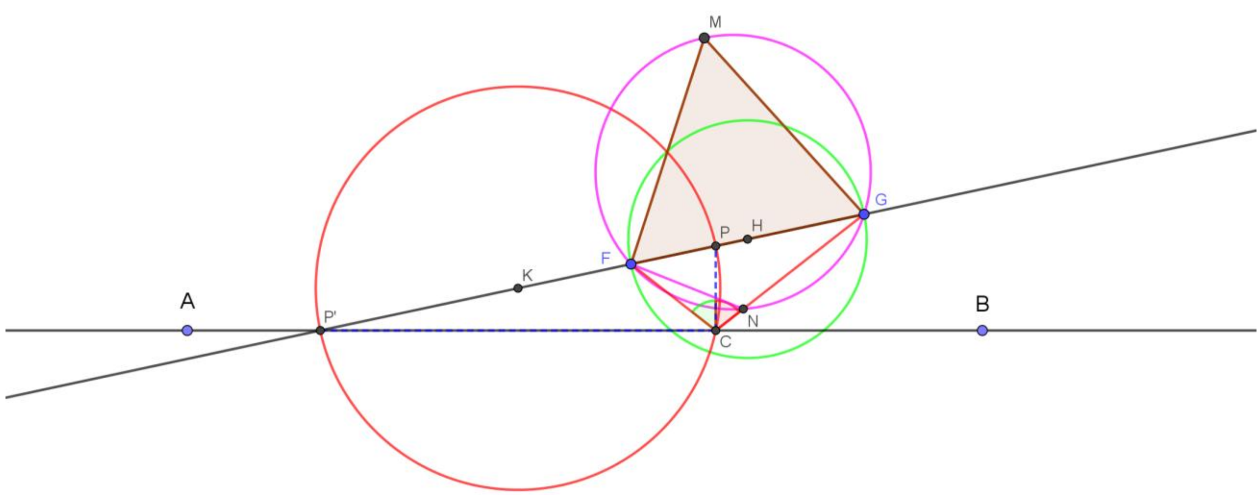

Fig. 5 Figure shows the equilateral triangle with FG as side and its circumcircle FMG. The circle FMG does not pass through since angle FCG is an arbitrary angle.

However, since C does not lie on the circumcircle (FC + CG) is not a minimum.

Minimality of the distance of the path is not a criterion for reflection of a light ray following Snell's law of reflection.

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References

1. Richard Lamborn, "Thinking Nature, Pierre Maupertuis and the Charge of Error Against Fermat and Leibniz". Ph. D. thesis, Univ. South Florida (2015).

2. McDonogh J. K., Leibniz on Natural Teleology and the Laws of Optics, http://nrs.harvard.edu/urn-3:HUL.InstRepos:5130440 (2009). doi: 10.1111/j.1933-1592.2009.00254.x

3. McDonogh J. K., Leibniz and Optics,The Oxford Handbook of Leibniz, Ch. 23, Ed. Maria Rosa Antognazza, Oxford Univ. Press. (2012),

4. Fermat, *Study of Maxima and Minima*, Originally written in Latin (1638). Translated into English from the French Translation in the CEvres de Fermat vol.3, pp. 121-156.

5. Per Stromholm, *"Fermat's Methods of Maxima and Minima and of Tangents, A Reconstruction",* Arch. Hist. Exact Sci., Vol. 5 (1968) 47-69.

6. W. Leibniz, Tentamen Anagogicum: An Anagogical Essay in The Investigation of Causes, Acta Eruditorum (1682), Philosophical Papers and Letters a Selection Translated and Edited by Leroy E Loemkar, Holland. Boston, D Reidel Pub. Co. (1976).

7. C. Stanley Ogilvy. *Excursions in geometry*, Dover Pub. Inc. New York, (1990)