

## ARTICLES

### A PRIORI KNOWLEDGE IN PERSPECTIVE: MATHEMATICS, METHOD, AND PURE INTUITION

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#### I

**W**ITH A STROKE OF THE PEN Kant instituted in one famous passage a distinction which has probably attracted more attention in the intervening two hundred years than any other philosophical distinction.

In what follows . . . we shall understand by *a priori* knowledge, not knowledge independent of this or that experience, but knowledge absolutely independent of all experience. Opposed to it is empirical knowledge, which is knowledge possible only *a posteriori*, that is, through experience.<sup>1</sup>

Before Kant, the *a priori*-*a posteriori* distinction was assumed to be virtually synonymous with the analytic-synthetic distinction: *a priori* (or analytic) reasoning proceeds from a cause to its effect, or from a universal to a particular instantiation, while a *a posteriori* (or synthetic) reasoning proceeds in the reverse direction (*CPR*, B2). Kant himself sometimes used these terms in their traditional sense, as referring to the logical *methods* of analysis and synthesis.<sup>2</sup> But his genius was to have transposed these distinctions into the center of *epistemology* by differentiating between them and applying them to types of knowledge as well as to methods of reasoning. Ever

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<sup>1</sup> Immanuel Kant, *Critique of Pure Reason*, trans. Norman Kemp Smith (London: The Macmillan Press, 1933), B2-3. Subsequent references will be included in the text and abbreviated '*CPR*'.

<sup>2</sup> See, e.g., *CPR*, B395n and Kant's *Prolegomena to Any Future Metaphysics* (New York: Bobbs-Merrill, 1950), p. 276n (Berlin Academy numbering). I discuss the difference between analytic and synthetic "methods" and "judgments" in detail in "Knowledge and Experience: An Examination of the Four Reflective 'Perspectives' in Kant's Critical Philosophy," *Kant-Studien* 78 (1987): 170-200. See especially section 2.

since then, a fundamental task for virtually every metaphysician has been to determine what, if any, meaning can be assigned to the terms in these two distinctions, especially the former.

The trouble is that many philosophers, particularly in the modern Anglo-American tradition, fail to distinguish clearly between the logical and epistemological uses of the four terms mentioned above. As a result, muddled accounts of the distinctions, and premature rejections of one or more terms or combinations of terms, are all too common, to the extent that they have now become the norm.<sup>3</sup> This is the first of two articles in which I will try to clear up such muddles by revealing the inadequacies of modern accounts as opposed to a sympathetic interpretation and reconstruction of the account given by Kant. In this paper I will use the work of Phillip Kitcher as a springboard,<sup>4</sup> and in the sequel I will use the work of Saul Kripke in a similar fashion. My conclusion in these studies will be that the views of the former rest on an equivocation and deserve to be rejected or thoroughly revised, whereas important aspects of the views expressed by the latter can be accepted and, given some important terminological modifications, used to correct some shortcomings in Kant's version of these distinctions.

Kitcher begins his analysis in *The Nature of Mathematical Knowledge* with a chapter on "Epistemological Preliminaries" and four more chapters in which he criticizes various versions of the apriorist understanding of mathematical knowledge. The remainder of the book develops his main thesis, that in order to understand the epistemological order of mathematics one must understand the historical order" (*NMK*, 5). In virtue of the fact that "we can identify a perceptual basis for mathematical knowl-

<sup>3</sup> Even Kant implicitly rejected one combination of terms, viz., "analytic a posteriori," as senseless (see, e.g., *CPH*, B11; cf. *Prolegomena*, 268). In the sequel to this article, "A Priori Knowledge in Perspective: Naming, Necessity and the Analytic A Posteriori" (forthcoming in *The Review of Metaphysics*), I will argue that he was mistaken to do so.

<sup>4</sup> Especially his recent book, *The Nature of Mathematical Knowledge* (Oxford: Oxford University Press, 1984), references to which will hereafter be included in the text, using the abbreviation '*NMK*'. Three articles by Kitcher will also be referred to in this way: "Kant and the Foundations of Mathematics" (hereafter '*KFM*'), *The Philosophical Review* 84 (1975): 23-50; "A Priori Knowledge" (hereafter '*APK*'), *The Philosophical Review* 89 (1980): 3-23; and "Apriority and Necessity" (hereafter '*AN*'), *Australian Journal of Philosophy* 58 (1980): 89-101.

edge" (*NMK*, 11), he rejects the apriorist program, and suggests instead that "the mathematical knowledge of someone at the present day [can] be explained by reference to a chain of prior knowers" (*NMK*, 5). Mathematical knowledge is neither analytic nor a priori, according to Kitcher, but (apparently) synthetic a posteriori:

A full account of what knowledge is and of what types of inferences should be counted as correct is not to be settled in advance. Rather, it must emerge from considerations of the ways in which humans actually infer and from the knowledge claims which we actually make. (*NMK*, 97)

In this paper I will not discuss the extent to which Kitcher's attempt to provide a "complete explanation" of mathematical knowledge (*NMK*, 10) is successful; rather I will limit my criticism to the more basic issue of the extent to which he has properly understood the phrase "a priori knowledge." In Section II, I shall use Kitcher's argument as an example of how confusion ensues when a philosopher conflates the logical and epistemological versions of the a priori-a posteriori or the analytic-synthetic distinctions, mentioned above. In Section III, I shall focus on one of the most common manifestations of such confusion by showing how Kitcher's misunderstanding of the phrase "a priori knowledge" is traceable not to an incorrect understanding of "a priori," but to a one-sided understanding of "knowledge." Finally, in Section IV, I will examine Kitcher's crude interpretation of Kant's doctrine of "pure intuition." The overall point of the paper is to show that an accurate interpretation of Kant's epistemological framework does not contradict the (less philosophically interesting) position developed by Kitcher concerning the nature of mathematical (or any other) knowledge; for the two can coexist as compatible perspectives, though neither tells the whole story.

## II

Kitcher's entire argument rests on a bogus distinction, introduced at the beginning of the first chapter of *The Nature of Mathematical Knowledge*, between what he calls the "psychologistic" and the "apsychologistic" approaches to epistemology.<sup>5</sup> The former

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<sup>5</sup> *NMK*, 13-14. See also *APK*, 6-7, 21-23, and *AN*, 89-90.

"focuses on processes which produce belief, processes which will always contain . . . psychological events" (NMK, 13), whereas the latter concentrates on "the character of the subject's belief system, the nature of the propositions believed, and their logical interconnections" (NMK, 14). Such a distinction is illegitimate, however, for the following reasons.

The first is that these two options are not necessarily mutually exclusive. That is, there is no reason—at least, none given by Kitcher in his four-page discussion of this distinction (NMK, 13-7)—why someone could not adopt both approaches at once, or even a third approach which does not fit into either category.<sup>6</sup> Since Kitcher tends to argue from the inadequacy of an apychological account to the necessity and sufficiency of a psychological account (see, e.g., NMK, 17, 91), the lack of any argument demonstrating that they are comprehensive and mutually exclusive alternatives is inexcusable.

A second deficiency in Kitcher's distinction is his tendency to equate the apychological approach with "positivism," which he believes lies at the "root" of all apychological approaches (NMK, 15-16; see also APK, 21-23). He does discuss other versions of apriorism in subsequent chapters. But the only view he considers before rejecting the apychological approach is "the positivist thesis that mathematics is analytic" and hence a priori (NMK, 15). This is extremely misleading because it assumes that by demonstrating the falsity of a view which everyone agrees is false, one can reach a true conclusion. "If the positivist is wrong, then psychology is right," is no more logically necessary than: "If the earth is not flat, then it is round," or "If it is not raining outside, then it is not cloudy." In each case the conclusion may or may not be true, but its truth cannot be deduced merely from the falsity of the premise. Nevertheless, after just a few pages of argument, directed primarily to the positivist, Kitcher announces:

<sup>6</sup> Kant's epistemology could be regarded as an example of one which has a foot in both camps: much of his first *Critique* deals with transcendental "processes" which appear at times to be just such "psychological events," yet throughout the work he is also clearly interested in discerning "the character of the subject's belief system." Kitcher's belief that Kant's conception of apriority is psychological (see APK, 21) is based, as we shall see in Section IV, on a very shallow interpretation of Kant's transcendental processes.

"I conclude that an adequate theory of knowledge will be psychologistic" (*NMK*, 17). From this point on he never looks back. The argument has been settled in advance because, as we shall see shortly, he has assumed his answer from the beginning.

My third criticism of Kitcher's distinction can be made by connecting it with the ancient distinction between analytic and synthetic methods of argumentation.<sup>7</sup> This way of using these terms refers not to the fixed epistemological status of a single judgment, viewed, as it were, as a snapshot of a given situation, but rather to the various logical patterns exhibited by a flowing series of judgments in their complex interrelationships. Whereas analytic and synthetic *methods* both have to do with the logical structure of a proposition's proof, the same distinction applied to *judgments* (according to Kant) has to do with the epistemological grounds for its truth. Kitcher's psychologistic epistemology follows the *synthetic* (a posteriori) method, inasmuch as he begins with the concrete (perceptual processes and beliefs) and proceeds to the abstract (the knowledge so produced).<sup>8</sup> The apsychologistic epistemology which he rejects, by contrast, follows the *analytic* (a priori) method, for it begins with the abstract (a given item of knowledge) and proceeds to the concrete (the processes and beliefs upon which it is based). Both methods typically employ a whole series of interconnected judgments or propositions, but the former examines the chronological evolution of knowledge as it is constructed, while the latter concentrates on the logical structure of a body of knowledge which is assumed to be given. Kitcher fails to recognize the compatibility of these two methods; instead he asks us to choose one or the other. Yet the two methods are compatible, as long as we regard them as two perspectives, or ways of thinking, each of which provides a legitimate *systematic context* for raising epistemological questions.

The fourth and fatal blow to Kitcher's bogus distinction is that

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<sup>7</sup> A good example of the way in which these reciprocal methods can be used is found in Euclid's *Elements*, and is discussed by Heath in the commentary to his translation, *The Thirteen Books of Euclid's Elements*, vol. 1 (Cambridge: Cambridge University Press, 1956), 133.

<sup>8</sup> Kitcher explicitly states in *NMK*, 100 that his goal is to explain the rational "method" or "pattern" which determines the historical development of mathematical knowledge. Such a method is clearly both synthetic and a posteriori.

he assumes his synthetic a posteriori method is capable on its own of determining the ultimate epistemological status of a priori judgments, such as those made by mathematicians.<sup>9</sup> Yet this is impossible. For the synthetic method (at least in its psychological version) is by definition an a posteriori enterprise, while its analytic counterpart is a priori. Propositions used in the former method will naturally appear to be a posteriori, and those in the latter method, a priori. But Kant introduced a new, synthetic a priori method, according to which the status of a judgment depends on its epistemological source (i.e., its purity or impurity, and its basis in intuition or conceptualization). From this perspective alone, Kant would claim, can the ultimate status of a judgment be determined. By failing to distinguish between method and judgment and by limiting himself to an exclusively empirical version of the synthetic method, it seems that Kitcher begs the question from the beginning, for the a priori as such is not to be found among the psychological processes and empirical beliefs to which he chooses to limit his attention.

Clearing up such inaccuracies in Kitcher's account of a priori knowledge is important because his views represent the tendencies of a whole school of philosophy which is rapidly growing (thus providing a social context in which such erroneous views appear to be true—to borrow one of Kitcher's own favorite idioms). At the very least I hope I have discouraged those who favour this sort of approach from adopting the narrow exclusivism exemplified by Kitcher. For, as it stands, Kitcher's work merely replaces the old "metaphysical positivism" with the newfangled "sociological positivism" which is so much in vogue today, but which suffers from many of the same faults when taken to an extreme. I have not yet

<sup>9</sup> If Kitcher does not intend to determine the fixed epistemological status of such judgments, then he is just boxing his shadow in the arguments given in *NMK*, 36–100. For no one would disagree that the status of such judgments *as experienced*—that is, when viewed from the perspective of the synthetic a posteriori method—is a posteriori. The debate over the status of mathematical knowledge concerns not how to classify the (a posteriori) experience of *learning* mathematical truths, but rather whether to classify such truths as analytic a priori or synthetic a priori, once they are learned. Kitcher might have failed to recognize this, since at one point he actually criticizes a psychological approach on the grounds that they do not pay sufficient attention to "the reasons for which a person believes" an item of a priori knowledge (*APK*, 23); yet such reasons are clearly irrelevant if the distinction is taken in its static sense, as it generally is.

addressed the issue which is central both to this paper and to the epistemological chapters of *The Nature of Mathematical Knowledge*: what is a priori knowledge? Now that we have clarified the important difference between a methodological perspective and the *epistemological* status of a particular judgment, we can safely proceed to tackle this important issue.

### III

We saw in Section II that 'a priori knowledge', viewed as a *logical* concept, refers to knowledge which arises out of following an analytic method. Kant's special, *epistemological* definition of a priori knowledge, by contrast, was quoted at the beginning of Section I. I have accused Kitcher of failing to give a fair hearing to a priori knowledge by not paying sufficient attention to this distinction and by limiting his perspective to the synthetic (a posteriori) method. The defender of such an approach might counter this attack by pointing out that Kitcher's *definition* of 'a priori knowledge' is explicitly based on Kant's own definition (in *CPR*, B2-3). Kant himself adopts a synthetic method in his examination of a priori knowledge in the first *Critique*, so what is to prevent Kitcher from doing the same? If this is true—if Kitcher really has defined a priori knowledge in a thoroughly Kantian way—then perhaps he has not begged the question after all, for he has every right to view with empirical, psychological spectacles what Kant views with transcendental spectacles, so long as the difference in perspective is recognized.<sup>10</sup> In this section, therefore, I will examine and criticize what Kitcher actually says about 'a priori knowledge'.

Kitcher provides a clear and concise formal analysis of 'a priori knowledge' on page 24 of *The Nature of Mathematical Knowledge*: "X knows a priori that  $p$  if and only if X knows that  $p$  and X's belief that  $p$  was produced by a process which is an a priori warrant for it."<sup>11</sup> He then goes on to define what it means for a process to be "an a priori warrant for X's belief that  $p$ ": such a process will always "produce warranted *true* belief independently of experience" (i.e., no matter what the person's background experience has been like). Throughout the remainder of the book Kitcher's argu-

<sup>10</sup> I elaborate on the compatibility of the empirical and transcendental perspectives in my article, "Six Perspectives on the Object in Kant's Theory of Knowledge", *Dialectica* 40 (1986): 121-51; see especially section 4.

<sup>11</sup> See also *APK*, 9-10.

ments with the apriorist center around the notion of an "a priori warrant," the assumption being that this is the only way to discount Kitcher's understanding of a priori knowledge. This is particularly evident in the first section of chapter 5, where, after attempting to cut off the last few lines of escape for the apriorist, Kitcher concludes that the apriorist has simply failed to recognize "the social character of most of our knowledge" (NMK, 91). Once this is realized and the term *a priori* is analyzed accordingly, "apriorist doctrines no longer look attractive" (NMK, 89n).

There is, however, a line of escape which Kitcher never considers—probably as a direct result of his attachment to the psychologistic school. He rightly analyzes 'a priori knowledge' as being fulfilled by two conditions: (1) "X knows that *p*" and (2) "X's belief that *p* was produced by a process which is an a priori warrant for it." He spends very little time discussing this step in his analysis, because its validity is so obvious. Instead, he devotes almost his entire effort to a more detailed analysis of (2), and, unfortunately, he never bothers to analyze (1) in greater detail. He merely assumes that "knowledge" of any sort—synthetic, analytic, a posteriori or a priori—can be adequately defined as justified (or "warranted") true belief (NMK, 23–24).<sup>12</sup> The popularity of this exclusively empirical definition of knowledge these days would perhaps excuse Kitcher for this neglect if it were not for the fact that he uses these terms explicitly as a way of trying to explain what *Kant* meant by "knowledge" (NMK, 23). By slipping in a significantly different meaning for the word *knowledge*, then devoting his attention solely to the meaning of *a priori* (or "a priori warrant"), Kitcher believes he has succeeded in destroying the traditional apriorist position once and for all and in giving it "the burial it deserves" (AN, 101), when in fact he has grossly misrepresented it (or at least, Kant's version of it) from the beginning.

As I have demonstrated elsewhere,<sup>13</sup> Kant uses the word *knowledge* (*Erkenntnis*) in several distinct ways, depending on whether he is viewing it from a transcendental, empirical, logical or practical perspective. He would probably not be opposed to the practice of describing a *posteriori* knowledge in terms of psychological be-

<sup>12</sup> See also *APK*, 6–8, and *AN*, 90.

<sup>13</sup> See my article, "Knowledge and Experience," section 3.



liefs. When he announces that "empirical knowledge is experience" (*CPR*, B165-66; see also B147, B218), he may have something of this sort in mind. But he would cringe at the thought of describing *a priori* knowledge in this way. To do so is to miss altogether the point of the phrase "independent of experience." For a person to have a priori knowledge of some sort (or indeed for a proposition to have an "a priori" status attached to it) it is not necessary for that person to entertain a conscious "belief" of any sort (nor for the proposition to be uttered by some person), as Kitcher assumes (see *NMK*, 21). Moreover, the a priori knowledge which Kant seeks to establish in the first *Critique* is possessed by every person who has ever had any empirical knowledge whatsoever (i.e., it has "strict universality" [*CPR*, B4]), whether or not the person is *conscious* of possessing such knowledge. This is because a priori knowledge in Kant's transcendental sense refers only to that which serves as the necessary and universal *ground* of all empirical knowledge: it is the "knowledge" which tacitly guides the original confrontation of subject and object (i.e., immediate experience) so that empirical knowledge (mediate experience) eventually results.

The transcendental question, the answer to which determines whether any given item of knowledge is a priori or a posteriori, is not how such knowledge *actually* came to be known, but whether it has its source in the nature of the subject or in the nature of the object.<sup>14</sup> This is precisely Kant's point when he says "though all

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<sup>14</sup> Thus it is unnecessary for the transcendental philosopher to "distinguish cases of empirical knowledge of propositions which could be known a priori from cases of actual a priori knowledge" (*NMK*, 24). Everything that is *actually* known is known empirically; but some such knowledge turns out to have an a priori status when regarded from the transcendental perspective. Kitcher's claim that attributing an analytic status to a mathematical knowledge-claim "forces us to attribute knowledge from the beginning" (*NMK*, 15; see also *APK*, 22)—that is, before an adequate proof is worked out—is therefore irrelevant. The epistemological status of an item of mathematical (or any other) knowledge is analytic or synthetic, and a priori or a posteriori, whether or not it is actually known to be such. The "knowledge" known "from the beginning" is the empirical *belief* that *p*, which must be carefully distinguished from one's subsequent knowledge of some *proof* (and hence, of the truth, analyticity or apriority) of *p*. Given this dual sense of knowledge, we can say that the Critical philosopher seeks to know empirically that which all knowers know transcendently. Kitcher comes close to realizing this in *KFM*, 24-25, but falls short.

our knowledge [i.e., our temporal awareness of it] begins with experience, it does not follow that it all arises out of [i.e., has its ultimate source in] experience."<sup>15</sup> If a priori knowledge does not "arise out of experience," then obviously, we have it whether or not we have empirical knowledge (i.e., conscious awareness) of its role in structuring our experience. For Kant, therefore, a priori knowledge is independent of the very "justified true belief" (i.e., empirical knowledge) in terms of which Kitcher defines it.

Once Kant's special notion of a priori knowledge is clearly understood, it is easy to extricate the apriorist from Kitcher's hold. Consider the following passage taken from *The Nature of Mathematical Knowledge*:

If apriorists are to escape this criticism on grounds that the analysis of apriority is too strong, then they must allow that it is not necessary for an a priori warrant to belong to a type of process members of which could warrant the belief in question given any sufficient experience. To make this concession is to abandon the fundamental idea that a priori knowledge is knowledge which is independent of experience. (NMK, 88)

As we have seen, the apriorist need not criticize Kitcher's analysis of apriority, and is therefore not forced to make such a concession. All the apriorist needs to do is to abandon Kitcher's exclusively empiricist definition of knowledge, and with it, the supposition that a priori knowledge needs to be warranted by some psychological "process" before it can be called a priori knowledge.<sup>16</sup> What remains of Kitcher's criticism of the apriorist, once his incorrect use of Kant's terms is adjusted, is merely the platitude that there is no such thing as empirical knowledge which is independent of empirical knowledge.

The vacuity of Kitcher's favourite way of arguing is now evident. He argues time and again from the fact "that experience could mislead us by suggesting that some (true) mathematical statements are false" (NMK, 64) to the conclusion that mathematical

<sup>15</sup> CPR, B1. Another way of saying the same thing is: although all our knowledge is synthetic a posteriori as we experience it, it does not follow that it all has this same status when we view it from other, more abstract, perspectives.  
<sup>16</sup> Also to be abandoned, of course, are other empiricist aberrations which creep into Kitcher's account, such as his suggestion "that we can introduce approximations to a priori knowledge" (APK, 20).

cal knowledge cannot possibly be a priori, since we can never even be certain that our mathematical beliefs are true.<sup>17</sup> He does mention (in *NMK*, 42) that some apriorists would say "that it is possible for us to know a proposition a priori without knowing it for certain." However, he rejects this possibility straightaway, insisting that such uncertainties "indicate that our knowledge is not a priori." Yet this is like arguing that because we might be mistaken, or uncertain, about the *cause* of a specific *effect* in the physical world, the transcendental *law of causality* cannot be a priori. Given Kant's understanding of his terms, such suggestions are ludicrous, since that which is a priori is in some sense "known" by *every* knower, even if a particular knowing subject has no opinion at all, or (mistakenly) *thinks* he knows otherwise. For example, a person might truly believe that  $7 + 5 = 11$ , and a whole society of uneducated companions might grow up with the same belief; but Kant would still maintain that they all have within them, ready to be discovered empirically, the a priori knowledge that  $7 + 5 = 12$ , whether or not they are "certain" of the truth of their former belief.

The source of the kind of misunderstanding of 'a priori knowledge' exemplified by Kitcher is not a failure to understand 'a priori', but a failure to recognize the radically perspectival character of knowledge.<sup>18</sup> In order to understand any one part of Kant's System it is crucial to adopt the perspective—the set of assumptions and rules—which guides its development.<sup>19</sup> Likewise, in order to classify any proposition as a priori or a posteriori, it is necessary to understand its relation to its context—the epistemological perspective which is assumed. A priori knowledge, properly construed, is in no sense intended to *replace* the truths of empirical science with a priori counterparts, as Kitcher accuses some (unnamed) "concep-

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<sup>17</sup> Thus he says in *NMK*, 83: "Exercising our linguistic ability is not an a priori warrant if experience can undermine the use of the language." It is not an a priori warrant in his empirical sense because no such animal exists!

<sup>18</sup> An example of Kitcher's failure in this respect is his lack of concern for distinguishing between the *empirical* items of knowledge which occupy much of his concern and the *logical* items of knowledge which "certain types of thought-experiments may generate" (*NMK*, 31).

<sup>19</sup> I have discussed the implications of this requirement more fully in "Knowledge and Experience," and "Six Perspectives." See also my article, "The Architectonic Form of Kant's Copernican Logic," *Metaphilosophy* 17 (1986): 266–88.

tualists" of attempting (NKK, 84). Rather, it provides the very *foundation* upon which empirical science rests and flourishes. Consequently, conscious knowledge of a priori knowledge has no role whatsoever in empirical science *as such*, and cannot be revised by its discoveries. Thus, the Kantian apriorist does not "discard the wrong processes as irrelevant props" (NKK, 93), but regards each process as relevant to the perspective to which it belongs: empirical processes are relevant to empirical inquiries and transcendental processes, to transcendental inquiries.

Kitcher rightly insists that "we must allow [at least] a minimal role to experience, even in a priori knowledge" (NKK, 21; APK, 5). But, whereas for Kant this role is limited to the consideration of "experience in general," Kitcher wrongly assumes that this gives him the right to discuss transcendental questions about a priori knowledge by adopting an empirical perspective. Much of his analysis of how mathematical (and other) knowledge comes to be known is quite helpful, and sheds considerable light on important empirical questions. Moreover, his arguments do demonstrate the inconsistency of those who try to uphold an apriorist position while adopting the same empirical perspective on knowledge. For it is a fundamental mistake to assume that such empirical inquiries can yield transcendental conclusions—to say nothing of "a full account of what knowledge is" (NKK, 97). How can an investigation of the a posteriori development of knowledge establish anything about its a priori roots? If only Kitcher had recognized the perspectival remedy for this error, the truly valuable aspects of his psychology of knowledge could have been emphasized more clearly. Such intellectual blindness to different but compatible perspectives is the primary cause of the unhealthy philosophical extremism which has plagued the history of Western philosophy. Its exposure is the primary task of the Critical philosopher.

#### IV

Kant himself was not always entirely consistent in his use of the terms we have been considering. A good example is in his brief discussion of the synthetic a priori status of mathematical knowledge (in *CPR*, 14-17), where he argues that  $7 + 5 = 12$  is synthetic because we cannot *discover* the concept '12' merely by analyzing the

concepts of '7', '5', '+', and '='. Here he too is confusing the *method* according to which an item of knowledge is learned with its *status* once it is given, just as Kitcher and so many others have done. It seems that Kitcher is right in thinking that Kant's view of the nature of mathematical knowledge needs to be revised, though, as we have seen, the revision he himself suggests is far too extreme. In the sequel to this essay I will use Kripke's analysis of naming as a springboard for offering a more balanced revision of Kant's doctrine. As a conclusion to the present paper, however, it will be helpful to expose one further fault in Kitcher's understanding of Kant.

Kant's <sup>main</sup> criterion for analyticity is that a proposition be true *solely* in virtue of its conformity to fundamental logical laws (given the meanings of the terms involved); and his corresponding criterion for syntheticity is that an intuition also be required for verification (see *CPR*, B16, 749). An a priori proposition can be synthetic only if its validity rests on some *pure* intuition. In his explanation of what is involved in such "pure intuition," Kitcher falls prey to another common misconception as to what Kant has in mind. His criticism of "Kant's thesis that our psychological constitution dictates the geometrical structure of experience" (*NMK*, 55) assumes that "pure intuition" refers to a literal construction of mental images, on the basis of which mathematical knowledge is obtained. Pure intuition is thus interpreted as a "sensuous" process (*NMK*, 53), according to which "pictorial representation takes place in the mind's eye" (*NMK*, 95); as such it is equated with "mental perception" (*NMK*, 51).<sup>20</sup> Kitcher summarizes his understanding of "the picture behind Kant's theory" most clearly in "Kant and the Foundations of Mathematics": "That picture presents the mind bringing forth its own creations and the naive eye of the mind scanning those creations and detecting their properties with absolute accuracy" (*KFM*, 50).

This crude "constructivist" interpretation of pure intuition<sup>21</sup>

<sup>20</sup> See also *KFM*, 30-31, and *APK*, 8.

<sup>21</sup> A similar interpretation is put forward by H. A. Prichard in *Kant's Theory of Knowledge* (Oxford: Clarendon Press, 1909; see pp. 233-35, 240-41). Graham Bird's book by the same title (London: Routledge and Kegan Paul, 1962) presents a thoroughgoing refutation of this way of reading Kant (see pp. 2-11).

gives rise to misguided statements such as the following, from *The Nature of Mathematical Knowledge*:

For Kant not only claims that mathematical truths are necessarily true but that they must necessarily appear to be true. The very point of [Kant's] thesis that mathematical truths describe the structure which the mind imposes on experience is to deny that experience could mislead us about mathematics. (NMK, 55)

Unfortunately, nowhere in this discussion of Kant's position (in NMK, 49-57) does Kitcher support his interpretation with even a single reference to Kant's text. (Instead he refers the reader to KFM, where he does make use of occasional quotations from CPR.) No doubt, in the present case, the reason is that this comment is a fabrication, a dummy argument set up in such a way as to be easily cut down. For as we saw in Section III, any answer to the empirical question of whether experience can mislead us is irrelevant to the transcendental question as to the status of mathematical knowledge.

This crude constructivist reading of Kant is probably based on a confusion between pure intuition and the faculty Kant calls "inner sense." When Kant refers to the construction of a triangle, for instance, he is not, as Kitcher wrongly assumes (see NMK, 23-4, 49-53), claiming that our geometrical knowledge of the triangle's properties is a priori in virtue of the fact that certain properties can be read off of our mental images. The process of inspecting our mental images with "the mind's eye" would be regarded by Kant as a function of *inner sense*. Geometrical knowledge is no more a priori because of how we picture certain figures to ourselves than arithmetical knowledge is a priori because of how we experience objects presented to our "outer sense": in both cases such specific experiences could be misleading, so they are not used by Kant as "a priori warrants." On the contrary, Kant's purpose in pages B137-38 of the *Critique of Pure Reason* (a passage improperly used by Kitcher in KFM, 30-1) is to clarify that, although such geometrical constructions can provide (empirical) knowledge because they are empirical, pure intuition on its own "is not yet [empirical] knowledge."

What then does Kant mean by the term "pure intuition"? He regards pure intuition as the very source of time and space (the "two forms of sensible intuition" [CPR, B36]), so its influence "must

be found in us prior to any perception of the object" (*CPR*, B41); hence, any reference to pure intuition as a "process" must clearly be taken metaphorically. It has to do not with our experience (i.e., empirical knowledge), but with its source in the *immediate* encounter between subject and object (*CPR*, B41). Moreover, it is crucial to note that Kant always brings geometry into the picture when discussing *space* as the form of *outer* sense, not time as the form of inner sense.<sup>22</sup> This indicates that when he suggests that, in order to verify geometrical propositions, "you are constrained to have recourse to intuition. . . . You therefore give yourself an object in intuition" (*CPR*, B65), the "process" to which he is referring is more likely to be exemplified by drawing a figure on a piece of paper than by scanning a mental image. However, both of these are *empirical* processes. What he is really getting at is something deeper: the synthetic a priori status of geometrical propositions can be verified only by acknowledging a *pre-empirical process* (viz., pure intuition) in which every knowing subject participates. Thus when he says, "You must therefore give yourself an object *a priori* in intuition, and ground upon this your synthetic proposition" (*CPR*, B65), he is not necessarily referring to a *conscious* process, but to the need to acknowledge that geometrical structures are rooted in the process by which the subject imposes a spatial form upon experience.<sup>23</sup>

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<sup>22</sup> See *CPR*, A24, B40-41, B64-66, B137-38, B271.

<sup>23</sup> *CPR*, B24, B41, B66, B662. Kant makes the same point in *CPR*, Bxi-xii: "A new light flashed upon the mind of the first man . . . who demonstrated the properties of the isosceles triangle. The true method, so he found, was not to inspect what he discerned either in the figure, or in the bare concept of it, and from this, as it were, to read off its properties; but to bring out what was necessarily implied in the concepts that he had himself formed *a priori*, and had put into the figure in the construction by which he presented it to himself." Here the a priori process of constructing concepts and putting them into the figure is clearly distinguished from the a posteriori process of actually drawing a sample figure and reading off its properties. Since the triangle constructed by pure intuition is "an object which lies entirely outside the sphere of our understanding" (*CPR*, B621; see also B271), and since "No image could ever be adequate to the concept of a triangle in general" (*CPR*, B180), it is often helpful to synthesize the intuition and the concept by actually constructing the figure empirically (*CPR*, B271, B744-45). (Kant elaborates on his understanding of "construction" in *CPR*, B741-50, where he is admittedly not always entirely clear on the distinction between a priori and a posteriori construction.)

Kitcher's most rigorous argument against Kant's theory of the a priori foundation of mathematics comes in "Kant and the Foundations of Mathematics," "Pure intuitions," on his reading of Kant, "are supposed to teach us *general* truths which describe the structure of space *exactly*" (*KFM*, 42). Using this as his interpretive starting point, Kitcher then easily demonstrates (in *KFM*, 42-46) the absurdity of Kant's argument. His refutation runs something like this:

1. Kant's theory is that we come to know general geometrical truths by imagining or drawing a particular figure and then distinguishing between its necessary (synthetic a priori) properties and its accidental (synthetic a posteriori) properties
2. To make such a distinction assumes that we already "recognize the structure of space" (*KFM*, 46)
3. Therefore, we cannot come to know general geometrical truths merely by Kant's method of inspecting a particular figure in pure intuition

This attempted refutation fails because it rests on a complete misunderstanding of Kant's doctrine of pure intuition. Kitcher's interpretive starting point is fallacious. For Kant portrays pure intuitions as the transcendental *source* of our *possible* empirical knowledge of space; such knowledge does not become actual (and hence should not be described in terms of "truths") until it is used in conjunction with some conceptual framework, and in accordance with some rule, or schema, governing its application. Once Kant's distinction between transcendental construction in pure intuition and empirical construction in space is recognized, the true course of his argument can be charted:

1. Kant asks us to reflect transcendentially on what is happening when we draw a sample geometrical figure in space and inspect its properties
2. The more we perform this empirical exercise the better we get at judging which properties are accidental and which are universal and necessary (though we may sometimes judge wrongly, especially at first)
3. The transcendental philosopher accounts for the difference by tracing the accidental properties to the (synthetic a posteriori) idiosyncrasies of the particular example, while accounting for



the (nonanalytic) universal and necessary properties by describing them as (synthetic a priori) features which we unconsciously impose upon the figure (i.e., construct in pure intuition) in the very act of constructing its empirical representation

Kitcher's main mistake is to fail to see that pure intuition is Kant's transcendental *explanation* of how we are able to "recognize the structure of space" (cf. Kitcher's second step), and *not* his explanation of how we distinguish empirically between different types of properties. The root cause of this misunderstanding, no doubt, is that Kitcher expects Kant to be providing reasons (i.e., "a priori warrants") for believing geometrical propositions to be true, whereas Kant, uninterested in such psychological matters, is offering a transcendental explanation of what is happening when such a warranted true belief is, in fact, formed.

"Kant and the Foundations of Mathematics" presents a second argument against Kant (see *KFM*, 46-50). Although it is really just a restatement of the first argument, it may be helpful to address it separately. Kitcher summarizes the supposed circularity of Kant's argument as follows:

We can know that space has a property only by knowing that a series of intuitions is possible. But we can know that that series of intuitions is possible only if we know that space has the original property. We begin by trying to discover the limitations of experience; we end up by assuming them. (*KFM*, 49)

The noncircularity of this argument can ~~only~~ be exposed <sup>simply</sup> by explicating the perspective from which the word *know* is viewed in each of its occurrences. Accordingly, the first two sentences can be restated as:

We can know empirically that space has a property only by knowing transcendently a series of possible intuitions. Likewise, we can know empirically that that series of intuitions is possible only if we know transcendently that space has the original property.

The argument is not circular, but perspectival: it illustrates the thoroughgoing reciprocity of the transcendental and empirical perspectives. Once this is understood, the third sentence is also deprived of its sting: "trying to discover the limitations of experience"

is just how Kant characterizes his transcendental task: "assuming them" is then ~~all that is necessary to carry out empirical tasks~~ <sup>requirement for obtaining empirical knowledge</sup>

The two examples Kitcher uses in discussing his second criticism are the propositions: "that all line segments are infinitely divisible" (*KFM*, 46) and "that there is one and only one straight line joining two given points" (*KFM*, 49). Kitcher states: "We can, according to Kant, know by means of pure intuition," the truth of these propositions (*KFM*, 49). Kant's actual position, however, is that we do know these propositions transcendentially (thanks to our faculty of pure intuition), whether or not we know that we know them, and that we can gain empirical knowledge of their truth by performing various experiments on paper or in imagination. Indeed, even if the recalcitrance of the physical world prevents us at some point from carrying out such experiments—e.g., because some subatomic particle turns out to be indivisible, or because space itself turns out to be curved—Kant's point remains defensible. Perhaps it is empirically impossible to divide some particle; but to assume that it is therefore not transcendentially divisible (i.e., that pure intuition does not motivate us to keep trying) would be to assume that it had somehow reached the status of nonspatiality. For as long as something is regarded as spatial, we can at least *imagine* its division (even if we can't actually divide it). Likewise, when space and time are considered together, as empirical companions, it turns out that the shortest distance between two points is not a straight line. Nevertheless, when they are considered transcendentially as distinct "forms of intuition," the truth of the Euclidean proposition holds true for any given "snapshot" of plane space. (Imagination separates the spatial and temporal aspects of reality in this transcendental way, which is why we cannot actually picture to ourselves a curved line being the shortest distance between two points.) Thus Kitcher entirely misses Kant's point when he assumes (in *KFM*, 50) that Kant "must think that we can distinguish the one straight line from curves which are 'nearly straight.' But we cannot." For *all* such empirical details, whether they be caused

<sup>24</sup> Kitcher's argument here closely parallels Jacob's famous criticism of the "thing in itself" as being a necessary assumption of Kant's System, and yet rendered useless by the System itself. Likewise, my rebuttal of the former here takes the same form as my rebuttal of the latter, as given in "Six Perspectives," section 4.

by the physical limitations of the knowing subject's body or by those of the known object's structure, are irrelevant to ("independent of") Kant's transcendental (synthetic a priori) concerns.

The legitimacy of Kant's doctrine of pure intuition as the a priori warrant for geometrical propositions is a question too complex to be discussed in any greater detail in this paper. Instead, it will suffice to urge those who wish to criticize Kant's doctrine to criticize the *deep* view he presents, and not some shallow misunderstanding which is doomed from the start.<sup>25</sup> Kitcher's misunderstanding of Kant's doctrine of pure intuition, as it is found in the Aesthetic, may be due to the fact that certain passages in the Analytic of Principles, which seem to be referring to similar processes, *are* actually dealing with empirical thought-experiments, such as when he compares his perception of a ship moving down stream to that of the parts of a house (see *CPR*, B237-38). However, in such passages he is dealing solely with the principles governing the *empirical* perspective, hence with the actual structure of the phenomenal world, not with the transcendental foundations on which experience in general is based. So if this is the source of Kitcher's error, it simply illustrates once again that there is no excuse for confusing the empirical openness of the synthetic a posteriori processes which characterize ordinary experience with the transcendental mystery of the synthetic a priori processes rooted in pure intuition.

In this paper I have argued that, although Kitcher's view of the empirical nature of mathematical knowledge would deserve respect if presented in the context of a balanced, perspectival framework, it is unacceptable in the extreme form in which he presents it. I argued in Section II that his distinction between apsychologistic and psychologistic epistemology, and his claim that only the latter is valid, is bogus because (1) they are not mutually exclusive, (2) the positivism against which he argues is not the best version of the former, (3) he fails to distinguish between judgments and methods

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<sup>25</sup> Kitcher acknowledges in *KFM*, 46n the possibility of finding such a deeper doctrine in the Aesthetic of *CPR*; but he says he prefers "to concentrate on the more interesting (i.e., easily refutable?) and detailed approach which Kant favors throughout the *Critique*." To justify the claim that Kant favours the interpretation of pure intuition given by Kitcher would require considerably more textual evidence than he gives (or could give!).

of argumentation, and (4) he tries to discover the a priori by using a posteriori tools alone. I argued in Section III that Kitcher's empiricist definition of knowledge as "warranted true belief" blinds him from the very start to Kant's more sophisticated, perspectival understanding of knowledge, with the result that the Kantian apriorist could accept most aspects of Kitcher's view of mathematical knowledge without jeopardizing his own position. And in this section I pointed out the shallowness of the crude constructivist interpretation of Kant's "pure intuition," and urged that the true depth of Kant's meaning be understood before his doctrine is criticized. Now that some of the misinterpretations which muddy the water of Kant's terminology have been cleared away, the foundation has been laid for a more important task, which I will take up in the sequel. That task is to examine how Kant's epistemological framework can be used to criticize certain more important modern developments in epistemology, and how the latter can be used to reconstruct the former in a more up-to-date form.

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