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Arcanum Artis Inveniendi: Leibniz and Analysis

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> "Mathematics is an experimental science. The formulation and testing of hypotheses play in mathematics a part not other than in chemistry, physics, astronomy, or botany" (Wiener 1923, 271).

I Introduction

Leibniz was undoubtedly a many-sided man, and a polymathic mind, if ever there was one. The concept of analysis is notoriously, for its part, a polycephalous monster, and nearly all its meanings are spread through Leibniz's multifarious works, where the philosophical, epistemological, logical, and mathematical receptions of the term seem to be inextricably interwoven. Much the same is true of its counter-term, synthesis, and thus their mutual relation itself presents various aspects.

A thorough survey of these varieties lies far beyond the scope of the present study, and they have already supplied the subject-matter of some very good accounts (in particular Duchesneau 1993, 55-104). Here we'll just try to find some traces of what Goethe would have called a "red thread"—like the one he saw metaphorically twisted throughout the literary cordage of Ottilie's diary in the *Wahlverwandtschaften*. Analysis is introduced by Leibniz in juridical, scientific, mathematical, or philosophical contexts, under different conditions and with different purposes; but even for such manifold uses should exist some common ground and univocal meaning. The analysis of thoughts and that of truths, the analysis of problems and that of things, all imply slightly or consistently different proceedings, and nevertheless they must perform somehow one and the same operation.

In a very general sense, analysis is for Leibniz, like for anyone else, the resolution of something complex into simpler elements. A procedure of this kind is applied, for instance, to physical objects by natural scientists. As Leibniz writes to des Billettes in 1697, they make use of "a certain analysis of sensible bodies, [protracted only to an extent] useful for the practice of their discipline” (Leibniz A, I, 13, 656). Depending on their object, such practices can in principle proceed in perfectly symmetrical manners, either from individual entities to universal features, or from universal concepts to particular instances. Thus Martial Gueroult distinguished two aspects of analysis with respect to Leibniz, one that “goes from the concrete to the abstract; this is the one which tends to ascend indefinitely towards the simple notions”; and another one “which, on the contrary, goes from the abstract to the concrete and, in principle, from the less to the more real” (Gueroult 1946, 251). There are Leibnitian texts on the analysis of physical bodies confirming this interpretation[1], but it is anyway somewhat too vague to be useful outside the immediate terrain of application.

II Truth conditions

A first preciser specification of analysis, and a distinguishing one as for Leibniz’s thought, is its application to truths, that is, as it may also be called, “conceptual” analysis:

> “According to Leibniz, truths of reason in general, and logical truths in particular, are necessary and eternal, true in all possible worlds, provable (i.e. reducible to identical propositions) in a finite number of steps, and hence ’analytic‘ in the strong sense (namely, the conceptual analysis that shows that the concept of the predicate is contained in that of the subject can be actually performed)” (Dascal 1988, 27).

Here a “truth” is the description of a state of fact expressed by one or several propositions in the form “subject-predicate” (substance-state), *i.e.* each proposition specifying a property of a determined substance at a determined instant of time—a property as such or a property acting as a non-relational “requisitum” to a relational state of things (Mugnai 1992). Leibniz writes in the §33 of the *Monadology*:

> “When a truth is necessary, its reason can be found by analysis, resolving it into more simple ideas and truths, until we come to those which are primitive” (Leibniz GP, VI, 612).

In every propositional truth, the predicate is someway contained in the subject, connected by conditions that can be showed by analysis—just like mathematical theorems, adds Leibniz notably, “are reduced by analysis to Definitions, Axioms and Postulates” (*ibid.*).

So there must also be a reason, or a chain of reasons, for all truths of fact, that is to say, for contingent truths. They concern the sequences of events that constitute the universe of created beings, in which “the analysis into particular reasons might go on into endless detail” (*ibid.*, 613), because of the immense variety of things in nature and the infinite division of bodies.

> “There is an infinity of present and past forms and motions which join to make up the efficient cause of my present writing; and there is an infinity of minute tendencies and dispositions of my soul, which contribute to make its final cause” (*ibid*.).

And all this minuteness involves infinite other contingent objects and events, “each of which still requires a similar analysis” (*ibid*.). As Leibniz once briefly condensed his theory of contingency, the root of contingency lies in the infinite (*radix contingentiae est in infinitum*): truths of fact are contingent, because no analysis can exhaust the infinite complexity of their truth conditions.

We are confronted here with the most general sense of the term, in which the concept of analysis is restricted to its fundamental elements. In so far as this is meant, it is true what Rescher maintains: that for Leibniz “‘analysis’ is a logical process of a very rudimentary sort, based on the inferential procedures of *definitional replacement* and *determination of predicamental containment* through explicit use of logical process of inference” (Rescher 1967, 23). But it’s easy to find quite different epistemological conceptions of analysis in Leibniz’s writings, in particular when questions concerning the scientific method are dealt with.

III There is Method in't

Leibniz felt a lively interest in the advancement of medical knowledge and of its methods. In a *De scribendis novis medicinae elementis*, written in 1680-82, we find the following remarks on the difference between analysis and synthesis in the study of pathology:

> “The method is truly analytical when, for every function, we investigate its media, or organs, and their modes of operating; thus we acquire knowledge of the body from [the knowledge of] its parts. After having completed this, we’ll return to the synthesis, coordinating everything to the one, and we’ll describe the prime motor, the instruments of motion (both the liquid and the solid ones), their connections, and the whole economy of the animal” (in Pasini 1996, 214).

The synthesis is then drawn from theoretical principles, namely the Galenic distinction of vessels, humours and spirits, out of which Leibniz’s favourite definition of the animal body as an “hydraulo-pneumatical-pyrobolical engine” can easily be deduced.

Synthesis is here an *a priori* proceeding, while analysis is a method to acquire empirical knowledge. Both contribute to the investigation of physiology, but analysis seems to act as first, being the chief mean to systematically gather information, whereas synthesis represents the correct foundation by which it is possible to gain systematicity for the information collected. This conception, of course, is not in any way peculiar of Leibniz[2].

If we read further in the *De scribendis novis medicinae elementis*, towards the end we encounter again the opposition of analysis and synthesis; this time the matter is not the method of investigation, but the communication of knowledge. Both analysis and synthesis again play a defined role: this is quite relevant, since the idea that analysis pertains mainly to discovery and synthesis to explaining and teaching is at Leibniz's time very close to a commonplace.

"Duplex Methodus tractandi Morbos", he declares, "una Analytica per symptomata, altera Synthetica per causas" (*ibid.*, 217). Disease can be considered analytically, based upon symptoms, or synthetically, based upon causes. It is important to teach first the true analysis of illness, writes Leibniz further, namely "the art both to inquire into the signs, and to identify an illness by means of signs" (*ibid.*). Synthesis will be taught only after giving a specimen of analysis, *i.e.* "a general healing method, which is to the pathological synthesis what algebra is to the elements of geometry" (*ibid.*). Here again we see Leibniz draw a parallel with mathematics, and in particular between the method of analysis in general, and algebra—that is, for a mathematician of his time, analysis in the most proper sense.

IV The Anatomy of Wit

Leibniz maintains, more in general, that inventive people who make discoveries and enlarge knowledge usually proceed in two ways: "per Synthesin sive Combinationem et per analysin" (Leibniz VE, 1362), as we read in a *De arte characteristica et inventoria*. Combination, or synthesis, is a conjunction of thoughts, maybe even arbitrary, so devised as to let some new knowledge arise. Analysis requires to dwell upon the proposed subject, and to resolve its concept into other simpler concepts, or to determinate its requisite elements or components.

Leibniz observes that all inventive spirits are either more combinatory or more analytical in disposition. A combinatory wit can remind of things past and connect them to present needs and experiences. Analytics thoroughly examine present things, but remain so immersed in their object as to limit their power of observation. Combinatory spirits are superior, because their ability is a rare gift: "Combinare vero remota promte, non est cujusvis" (*ibid.*, 1363).

In the second version of a programmatic sketch *De arte combinatoria scribenda*, Leibniz remarks analogously:

> "I must premise a chapter concerning the difference between the analytical and the combinatory method, and the difference between analytical and combinatory wits" (Leibniz VE, 1098).

Analytical wits, according to him, are more short-sighted, so to speak, while combinatory ones are rather long-sighted ("Analytici magis Myopes; Combinatorii magis similes presbites", *ibid.*, 1099): in fact, in analysis it is suitable to pay attention to fewer things, but with more precision, whereas combinatorics considers many things together, and much more perspectively; thus analysis has more in common with miniature painting, and combinatorics with large-scale sculpture.

Analysis is much easier to apply, since it consists of definable procedures:

> "Once a procedure of analysis is detected, it requires only attention, or firmness of mind (...) and indeed there are such people, whose wit isn't vagabond, and who are able to reckon in their imagination, even without paper and pencil" (*ibid.*).

Combinatorics, on the contrary, requires to quickly and promptly browse a manifold of subjects, and to treat them in unexpected ways. Their practical instruments also differ: people with a weaker imagination make use of figures and symbols to better focus questions, while those with a weaker memory and unable to represent many things together, are helped by the use of tables. "Characteristica vera et tabulis et analysi auxiliatur" (*ibid.*).

In the art of discovery, that is in the course of knowledge, both analytic and combinatory spirits, as we read in the *De arte characteristica et inventoria*, will particularly profit of a method. The method is described in general: "Methodus inveniendi consistit in quodam cogitandi filo id est regula transeundi de cogitatione in cogitationem" (*ibid.*). Method means something that provides the thinking processes with a leading thread, *i.e.* with a rule regulating the movement from one thought to the other. The rule must consist in a palpable instrument: as the compass rules the hand in correctly tracing a circle, for correct thinking "instrumentis quibusdam sensibilibus indigemus" (*ibid.*). These palpable instruments of thought are again tables for the combinatorics and characters—symbols—for the analysis[3].

"Characterem voco quicquid rem aliam cogitanti repraesentat" (*ibid.*)—a character is anythings that represents another thing to a thinking person. If we could keep the things themselves before us, we would have less need for such characters. The representation is based on some relation or rule of correspondence between them: so the ellipse represents a circle by being its projection. Models and figures of things can be considered as characters: they too are crafted so as to express the essence of the thing. Characters don't need to be similar to the objects they represent: numerical symbols express correctly the properties of number, but they don't resemble them.

V Thought instruments

This conception of the method as an instrument, or a collection of instruments and techniques, and not as a set of precepts, marks one of the most important differences between Leibniz and the greater part of his contemporaries, notably Descartes. For Leibniz a method "is" an instrument, and an instrument, in the method of analysis, is an algorithm based on characters. Hence, on non-mathematical ground too, analysis is in principle a symbolic operation for Leibniz. Moreover, systems of symbolic operations, *i.e.* algorithms, can be legitimately used, both for the comprehension and organization of existing knowledge and for the creation of new knowledge, also outside their traditional grounds.

The construction of general methods for the acquisition, sharing and transmission of knowledge, in the form of complex algorithmic instruments for logical and conceptual calculi, is an idea that dates back to the young Leibniz. Adolescent, he devised an "alphabet of human thoughts": it will grow into one of Leibniz's greatest projects, that of an art of discovery based on a "characteristic" (art of characters or symbols) of general use for combinatorics and analysis at the same time.

An analysis of our thoughts (*analyse de nos pensées*), states Leibniz in 1684, is "of the greatest importance both for judging and for inventing" (Leibniz A, I, 4, 342). This analysis of thought, he specifies elsewhere, "respondet analysi characterum", corresponds to a symbolic analysis, in that characters can express our thoughts and their relations, thus providing our reasonings with a "mechanical thread" (Leibniz VE, 811) This idea is explained more clearly in many programmatic essays, one of which received the not particularly original title of *Initia et specimina scientiae novae generalis* ("First steps and examples of a new general science"). Leibniz distinguishes between dialectics, or analysis of opinions, and analysis of truths; the latter, he affirms, is the secret for the development of the art of invention and discovery:

> "I shall also add the vulgar analysis of human judgements, *i.e.* the principles on which human opinions are based, that are dialectic and ought not to be despised. It wouldn't be necessary to bring them into surer principles, only with the purpose to confirm something we already know. But since the whole secret of the art of discovery [*totum arcanum artis inveniendi*], by virtue of which human science could make an immense progress, depends on the analysis of truths (that is the emendation of our thoughts), it is convenient to proceed to the highest levels of analysis" (Leibniz VE, 702).

This art will comprehend a method to perform in any field rigorous demonstrations, "equal or even superior to mathematical ones, which suppose many elements that here could be demonstrated" (*ibid.*). It is a wholly new calculus, that, according to Leibniz, is at work in every human reasoning and is nevertheless as accurate as arithmetical or algebraic calculations are.

The same concepts are repeated ever and again in Leibniz's countless manifestoes for this new discipline:

> "Since when I had the pleasure to considerably improve the art of discovery, or analysis, of the mathematicians, I began to have certain new views, that is, to reduce all human reasoning to a sort of calculus, which would be of use in discovering a truth in so far as it is possible *ex datis*, *i.e.* from what is given or known" (Leibniz GP, VII, 25).

A universal writing would also result of it, that "would be like a sort of general algebra, and would give the means to perform reasoning by calculation" (*ibid.*, 26): such a calculus would not only be an instrument for learning and research, but it would be an infallible judge of controversies as well, offering a way to solve disputes by simple reckoning.

Leibniz explains this extended meaning of calculus in a letter he wrote to Tschirnhaus in 1678: "Nihil enim aliud est Calculus, quam operatio per characteres, quae non solum in quantitate, sed et in omni alia ratiocinatione locum habet" (Leibniz GM, IV, 462). A calculus is nothing else than an operation performed by means of characters—that is, an algorithm of symbolic analysis—that takes place not only with quantities, but in any kind of reasoning as well.

VI The place of analysis

The place of analysis in this more general frame is, as one may expect, quite variable. In a short and schematic note, Leibniz lists the chapters for a work to be entitled *Guilielmi Pacidii Plus Ultra sive Initia et specimina scientiae generalis*. There we find among others the following arrangement of analysis and synthesis, combinatory and discovery, mathesis and art of invention:

> "10. De arte inveniendi
> 11. De synthesi seu arte combinatoria
> 12. De Analysi
> 13. De combinatoria speciali, seu scientia formarum, sive qualitatum in genere sive de simili et dissimili
> 14. De Analysi speciali seu scientia quantitatum in genere seu de magno et parvo
> 15. De mathesi generali ex duabus praecedentibus composita" (Leibniz GP, VII, 49-50)

Analysis and combinatorics in general seem to be tied to the art of invention; two more specific versions, that concern quantity and forms, are presented as the two branches that compose universal mathesis[4].

Another more detailed program is rubricated *Initia et specimina scientiae generalis*. It describes at length the structure of a complex work, dedicated to the "instauratione et augmentis scientiarum" (Leibniz GP VII, 57). After a first book dedicated to the logical form of arguments and to the ways to determine the eternal truths, the second book should treat *de arte inveniendi*, the "art of discovery, namely that of the tangible thread by which investigation is ruled", and of its divisions, "ejusque artis speciebus", namely combinatorics and analysis (*ibid.*).

In the *Fundamenta calculi ratiocinatoris* (1688-1689) Leibniz defines the calculus used in the universal art of characters as follows: "A calculus or operation consists in the exhibition of relations, performed by the transmutation of formulas according to some prescribed rule" (Leibniz VE, 1205); again, it might well be an exemplar definition of the analytical proceedings. And anyway, for Leibniz, any analytical calculation is a formal argument: as we read in a latter to the palatine countess Elisabeth of 1678:

> "un calcul d'analyse est un argument *in forma*, puisqu'il n'y a rien qui y manque, et puisque la forme ou la disposition de tout ce raisonnement est cause de l'evidence" (Leibniz A, II, 1, 437).

When Leibniz defines combinatorics in his *De artis combinatoriae usu in scientia generali* (of 1683-84), he states that "Combinatoria agit de calculo in universum", the combinatory art deals with every aspect of the calculus,

> "that is to say, with universal marks or characters (...) and their rules, dispositions and processes, or with formulas universally. Of this general calculus, the algebraic calculus is a species, *i.e.* the one based on the laws of multiplication" (Leibniz VE, 1354).

If even combinatorics reveals blatantly to be framed just like analysis, on the other hand mathematical analysis is clearly, as Leibniz himself often affirms, a specimen of the *ars characteristica*. In 1691 Leibniz writes to Huygens that:

> "the best and most convenient feature in my new calculus is this: that it exhibit truths by means of a sort of analysis, without any of those efforts of the imagination, that often succeed only by chance, and thus gives us the same advantage over Archimedes that Vieta and Descartes let us gain over Apollonius" (Leibniz GM, II, 104).

The infinitesimal calculus, he means, frees the geometer from the need to concentrate on the geometrical situation of the problem in order to devise a helpful construction, such as the insertion of a suitable ad-hoc linear segment.

Three months later Leibniz hammers again the qualities of his calculus in Huygens' mind, and he supports his argument with an example:

> "I remember that, as I once studied the cycloid, my calculus presented to me the greater part of the discoveries that have been made on the subject, nearly without

> any need for meditation. Indeed what I like best in this calculus, is that it gives us the same advantage in the field of Archimedean geometry that Vieta and Descartes have given us in Euclidean and Apollonian geometry, since it exempts us from working with the imagination" (*ibid*., 123).

In fact, from the study of the function it is possible to exhibit numerous geometrical properties of the curve, by way of analysis: "Caeteraque omnia circa cycloidem inventa, pluraque alia similiter ex tali calculo analytice derivantur.» (Leibniz GM, II, 118)

VII Calculus on my mind

Leibniz often intends with "analysis" a particular analytical method or a set of analytical techniques, developed by other mathematicians, and from some writings of his one might imagine that "quot sunt capita, tot sunt analyses". Leibniz is clearly conscious of the novelty and peculiarity of his mathematical discoveries. He writes in 1692:

> "I have developed a new analysis concerning the infinite; it is quite different from Cavalieri's geometry of indivisibles and from Wallis' arithmetic of infinite series, since it doesn't depend on lines as the former, nor on numerical series, as the latter, but it is general, and thus symbolic or Specious. But instead of the vulgar analytical calculus applying to powers and roots, it performs the calculus of differences and summations" (Leibniz GM, V, 263-264).

"Vulgar" analysis (*i.e.* the algebra of Descartes, his mathematical and philosophical *tête de turque*) is often reprehended by Leibniz, since it doesn't comprehend some of the most fascinating concepts of seventeenth century mathematics (infinitesimals, imaginary numbers), nor some of the most important objects of Leibniz's analytic research (transcendent relations, the theory of determinants).

A very important methodological distinction is drawn in a famous letter addressed to Antonio Magliabecchi. There are two forms of analysis, states Leibniz here; first comes the analysis of Vieta and Descartes, that is considered by the moderns to be the only analysis, and "that solves every problem separately, studying the relation of the unknown to the known quantities" (Leibniz GM, VII, 312). The other one has its scope in reducing the problem "to another problem, easier than the first one" (*ibid.*). The latter was known also to the ancients, as it appears, for instance, from the *Data*. In writing to Huygens, Leibniz explains this distinction as that between analysis "per saltum" and "per gradus, cum problema propositum reducimus ad aliud facilius" (Leibniz GM, II, 116-117). The first one is more absolute, but the second often works better.

In a *De methodis synthetica et anagogica applicandis in algebra*, the synthetic method is defined analogously: “cum problema difficile soluturi incipimus a facilioribus” (Leibniz VE, 1095). Leibniz also observes that algebra performs a fake synthesis, in treating the unknown quantities as if they were known. The anagogic method is that of pure analysis, “quae nihil syntheseos habet” (*ibid.*); and the “*Data* veterum” are of pertinence to the anagogic method, that hence appears to be the heir of the method described to Magliabecchi. Here we proceed backwards, “always reducing the problem to another, easier problem. And this is my method” (*ibid.*), adds Leibniz, used for ordinary equations, but also for the resolution of the ordinates of a curve, viz. in transcendent problems.

Another front is to be opened soon. As Leibniz writes to Mélchisedec Thévenot in 1691:

> “Since I believe that geometry and mechanics have now become fully analytical, I have devised to extend the calculus to other subjects, even to subjects that until now nobody thought would have supported it” (Leibniz A, I, 7, 356).

And he adds, as usual: “Here I mean by ’calculus‘ every notation representing a reasoning, even without any relationship to numbers” (*ibid.*).

In 1679, four years after the completion of his work on the fundamentals of the infinitesimal calculus, Leibniz writes to Huygens: “Mais apres tous les progres que j’ay faits en ces matieres, je ne suis pas encor content de l’algebre” (Leibniz GM, II, 18-19), after all the work I did with algebra I think we need something different and more powerful in treating with geometrical entities. It is “une autre analyse proprement geometrique ou lineaire, qui nous exprime directement *situm*” (*ibid.*, 19): an analysis specific to loci, *i.e.* an analytic topology. Algebra represents quantities with appropriate symbols and operations: other symbols and operations can calculate forms, angles, orientations, movements, in their qualitative aspects too.

The most important use of this analysis, anyway, is to help in geometrical reasoning: “on trouve ainsi par une espece de calcul”, the same words used to describe the advantages of infinitesimal analysis, “tous ce que la geometrie enseigne jusqu'aux elemens d'une maniere analytique et determinée” (*ibid.*, 26). By this calculus it is possible to determine analytically everything that belongs to geometry, up to its most fundamental elements.

As an obvious example of immaterial cognitive technology, this new *analysis situs* is, of course, an art of characters, and an art of invention: “Cette caracteristique”, adds Leibniz, will even express in symbols all mechanical structures and will help us to find new geometrical constructions, “à trouver de belles constructions”, since it contains at one time both the procedures of calculus and of construction” (GM 2, 30-31).

VIII An engine for your thoughts

"Quod omnium maxime quaero est Machina, quae pro nobis faciat operationes analyticas, quemadmodum Arithmetica a me reperta facit numericas" (Leibniz A, VI, 3, 412). What I most desire, Leibniz writes already in 1674, is a machine that performs analytical operations, just as the calculating machine he invented carries out the arithmetic ones. This idea of an analytical engine is hindered, one may say, by the inadequacy of its programming language, since "the universal analysis depends on the development of a universal character" (Leibniz A, VI, 3, 413). Meanwhile, for the use of complex reasonings, it is acceptable to surrogate the required special-purpose characters with generic characters, such as the letters used in geometry[5]. But in general the signs we presently use to compose analytical formulas, adds Leibniz, can't suitably express the mental operations involved in their treatment, by means of simple analytical procedures as transpositions or linear transformations. Anyway, it is not an impossible task, since "omnes cogitationes non sunt nisi simplices complicationes idearum" (*ibid.*): thoughts derive in the ultimate analysis from simple components, simply combined, as words are composed by simple letters, and the complex apparatus of thoughts needs only to be brought back to such simplicity.

But in reality our thoughts aren't so transparent: even if we were able to perform thorough analyses of the concepts we use, we would not *ipso facto* be aware of its results at any moment of our thinking processes: "when a notion is very composite, we can't think of all its ingredients together, as with an intuitive notion" (Leibniz GM, IV, 610). Leibniz discusses such issues in his *Meditationes de cognitione, veritate et ideis*, a short essay published in 1684 and dealing mainly with the classification of ideas into clear, distinct, obscure, adequate etc. We have a distinct notion of something, Leibniz affirms, if our knowledge contains enough marks to discern it from all similar objects. But "in most cases, in particular when a very complex Analysis is required, we can't represent intuitively the whole nature of the object, and we use signs instead" (*ibid.*).

This sort of reasoning, says Leibniz, can be called "blind reasoning, or also symbolic reasoning, as we make use of in Algebra and Arithmetic, and indeed in every moment" (*ibid.*). Symbols like those of analysis, are the true instruments of thought: in particular, they are for human thought a sort of indispensable blind-flying instruments—under conditions where normal thought is "blind-thought". "Et huius generis *cogitationes*", in Leibniz's words, "soleo vocare *caecas*, quibus nihil apud homines frequentius aut necessarium magis" (Leibniz A, VI, 2, 481). This is the most intimate kernel and the real operational mode of human thought: that it operates mostly by means of symbols, that is to say it operates in the same way as algebraic algorithms, or analytical algorithms do—those of the "literal" or "specious" analysis. That's why this last one is so successful, and useful, and sure, in matters so difficult and general as reasoning and problem solving: "Hinc Symbolica illa recentiorum analysis (...) tanti est ad celeriter et secure ratiocinandum usus" (*ibid.*).

The *cogitatio caeca* or *symbolica* finally is, according to Leibniz, in itself the best human instrument for problem solving, that is to say for the augmentation of "both knowledge, and happyness" (*ibid.*)—and mathematical analysis mirrors it. Not bad, in the end.

Endnotes

[1] In the *De modo perveniendi as veram corporum analysin* of 1677: "Duplex est resolutio: una corporum in varias qualitates per phenomena seu experimenta, altera, qualitatem sensibilium in causas sive rationes per ratiocinationem" (Leibniz GP, VII, 268). If we combine such analyses with experiments, adds Leibniz, we'll easily determine the causes of any quality found in any physical subject.

[2] For instance, a quite conformable statement can be read in Newton's *Optics*: "The Synthesis consists in assuming the Causes discover'd, and establish'd as Principles, and by them explaining the Phænomena proceeding from them" (Newton 1721, 380).

[3] It must be observed that the instruments intended for the combinatory are mostly traditional, static and trite; the instruments for analysis powerfully embody innovation.

[4] And in the *Elementa nova matheseos universalis* (written between 1684 and 1687): "Tradetur et Synthesis et Analysis, sive tam Combinatoria, quam Algebra." (Leibniz VE, 987).

[5] In this way, if the specific knowledge that enters in a logical calculation is already set up, it will be easier to coordinate this particular specimen of the art to the general frame of the universal characteristic.

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