Essay review of *Logical Consequence* by Gila Sher, Cambridge University Press 2022.
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Logical Consequence is part of CUP's Elements series, which aims to give advanced readers cutting-edge overviews of particular areas. Elements are tightly constrained in length, with an upper word limit of 30,000. The author, Gila Sher, has been writing and thinking about logical consequence for decades. In this Element, she does an excellent job of condensing her interesting and influential views into 86 pages. The book serves as both an introduction to the topic for those with a solid background in logic and philosophy as well as a more advanced overview of questions at the research frontier. It is a very welcome addition to the series.

This review has two parts. The first and shorter Part I is a précis of Sher's Element. In Part II, I compare Sher's views with those put forward in *One True Logic*, a monograph Owen Griffiths and I published in the same year as *Logical Consequence*. Although the two books are on the same topic, are written from a similar perspective, and advance similar conclusions, there are some key differences between them, which the comparison will bring out. Bare page references below are to *Logical Consequence*.

Part I: Précis

Logical consequence is arguably the most central concept of logic (p. 1). The theory of logical consequence has two branches: proof theory and semantics or model theory. The former systematises the common understanding of logical consequence as proof, the latter its common understanding in terms of truth (p. 2). The Element is concerned with the latter notion, whose study took off following Tarski's famous 1936 work (p. 3). There are many different types of consequence, for example material consequence (if the premises of an argument are true so must its conclusion be) or nomic consequence. Logical consequence is stronger than either of these (p. 4). The Element focuses on what Sher calls predicate logic (see Part II of this review for what this is).

A discussion of Tarski's approach to logical consequence in the 1930s follows (pp. 4–7). Gödel's First Incompleteness Theorem 'shows that the proof-theoretic concept of LC is significantly narrower than the concept informally used by mathematicians and others' (p. 8), which justifies the need for a non-proof-theoretic, semantic, definition of logical consequence. (All instances of 'LC' in quotations from the Element stand for 'logical consequence'.) Tarski's account is based on two fundamental features of logical consequence (symbolised as '⊨' below): *necessity* and *formality*, which may be cashed out as follows in terms of the truth predicate 'T' (p. 9).

> *Necessity:* An adequate definition of LC renders LCs *necessary*, that is, if $\Gamma \vDash S$, then *necessarily*, if all of the sentences in Γ are true, S is true. In symbols: $\Gamma \vDash S \supset$ $\mathbf{Nec}[T(\Gamma) \supset T(S)]$.
>
> *Formality:* An adequate definition of LC renders LCs *formal*, that is, if $\Gamma \vDash S$, then *formally*, if all of the sentences in Γ are true, S is true. In symbols: $\Gamma \vDash S \supset$ $\mathbf{For}[T(\Gamma) \supset T(S)]$.

Logical consequence is invariant under uniform replacement of non-logical constants by constants of the same syntactic type denoting different objects (p. 9), but a substitutional account of logical consequence would be inadequate (p. 10). This is for the familiar reason that the language may be impoverished, as well as the fact, not mentioned by Tarski, that it takes into account only sentences' truth in the actual world. Over the next ten pages (pp. 12 – 21), Sher gives a careful technical definition of logical consequence derived from Tarski's work for systems with linguistic expressions and objectual correlates of levels 0, 1 and 2 (respectively: individuals, their properties, properties of their properties). This standard, model-theoretic, account, avoids the problems faced by the substitutional account (pp. 21–22). Following Tarski, Sher remarks that this account relies on a demarcation of the logical constants (pp. 23–25).

To solve this problem, Sher recommends a methodological approach she calls *foundational holism* (pp. 27–29), developed in much greater detail in her 2016 book. The requirement of necessity stems from humans' interest in truth and knowledge, and thus in a method of inference that transmits truth with an especially strong force (p. 30). Sher then introduces the notion of invariance, over models containing actual or counterfactual individuals. She gives a precise account that Part II of this review will illustrate with examples. The first-level unary property *being human* (denoted by the predicate 'is human') is not invariant, since over some domains, some permutations of the domain disturb its extension, whereas the first-level binary property *being identical to* (denoted by 'is identical to') is invariant. Similarly, the second-level unary property *being a geological property* is not invariant. Sher mentions that the invariantist idea can be found in Kant, Frege and Mostowski (pp. 34–35), and in more developed form in Lindström and the later Tarski. The precise version of invariantism advocated here is isomorphism invariance, i.e. invariance with respect to bijections from one structure to another, structures being based on domains of actual or counterfactual individuals. This criterion of logical constancy has come to be known as the Tarski–Sher thesis, and we will return to it in Part II below. The account may be extended to sentential connectives by considering domains of atomic states of affairs (p. 39).

Sher relates the necessity and formality criteria to isomorphism invariance by means of five theses (pp. 41–45), quoted below.

> Thesis 1: Isomorphism-invariance is formality in the sense of strong structurality.
>
> Thesis 2: The formality of logical properties implies the formality of laws governing/describing them.
>
> Thesis 3: The formality of laws/principles governing/describing logical properties implies their necessity; indeed, it implies that theirs is an especially strong type of necessity.
>
> Thesis 4: LCs are based on objectual laws that are formal (in the invariantist sense) and as such have an especially strong modal force.
>
> Thesis 5: Consequences satisfying the semantic definition of LC are formally, hence maximally, necessary. Their formal/maximal necessity is due to the fact that they are based on formal-and-necessary laws. These laws connect the formal structures (formal skeletons of the situations) delineated by their premises and conclusions in all models, that is, in all (representations of) formally possible situations (*vis-à-vis* a given language).

The upshot is that the formality and necessity requirements on logical consequence have been met (p. 45).

Logical inference can be thought of as an especially powerful method of inference that links the world (i.e. reality), truth and logic. The following biconditionals articulate these links (p. 48):

> [Logic:] S_1, S_2, … *logically imply* S
>
> iff
>
> [Truth:] the correspondence-truth of S_1, S_2 … *guarantees, with an especially strong modal force*, the correspondence-truth of S
>
> iff
>
> [Reality:] the situations $\mathfrak{C}_1$,$\mathfrak{C}_2$, which do/would make the sentences S_1, S_2, … correspondence-true and whose formal structures correspond to the logical structures of these sentences, *formally necessitate* the situation $\mathfrak{C}$, which does/would make the sentence S correspondence-true and whose formal structure corresponds to the logical structure of S.

The fact that logical consequence is grounded in the formal features of reality explains certain of its features, for example generality and topic-neutrality (p. 49). Logic is epistemically normative because its highly necessary and widely applicable laws govern the world, so conforming to it aids and abets our attempts to know that world (p. 51). Because logic is more general than any other discipline, such as physics, say, physical laws have to abide by the laws of logic, but not the other way around, which makes logic in this sense more normative than any other discipline (p. 52). But this does not mean logical considerations always take priority over any others (p. 53).

Sher distinguishes her account of logical consequence from a mathematical precisification, of the sort one might give by using a background theory of models such as ZFC (p. 56). Her account may also be hospitable to non-bivalent approaches (e.g. Dummett's) if it turns out that reality's formal structure is not bivalent (p. 57). Mathematics and logic are distinct because first-level mathematical properties (e.g. *being identical to the number 1* or *being even*) are not invariant (hence not logical), although they are correlated with higher-level properties which are invariant and hence logical (p. 59). For example, cardinality quantifiers (such as 'there are exactly κ') are logical.

Sher then runs through some metalogical theorems (pp. 60–62), as well as some confusions regarding Tarski's 1966 talk on the logical constants (pp. 63–64), which gave the Tarski–Sher's thesis the first of its hyphenated names. She then responds to two criticisms by John Etchmendy in his 1990 book. The first attributes an elementary error involving the modal operator 'It is necessary that' (formalised as '**Nec**' above) to Tarski. The second is Etchemendy's famous argument that the model-theoretic conception offers neither a satisfactory representational nor an interpretational semantics. Sher's response here is that the model-theoretic conception involves a subtler blending of language and world than either the interpretational or representational semantics allows for. This means that Etchemendy's criticism fails (p. 69).

Another problem with the model-theoretic conception is that its models are sets, but the universe of sets itself is not a set. This leaves it open that a model-theoretic logical truth may not be a logical truth *simpliciter* (p. 70). Sher's response is threefold (pp. 70–72). First, she sees the criticism as levelled at a particular mathematical precisification of her account of logical consequence rather than the account itself. Other precisifications, perhaps not in terms of sets, may fare better. Second, reflection principles may come to the rescue: in first-order ZFC, for example, any property true of the universe is true in some set within it. Third, there may not be proper classes: perhaps proper-class talk is just a 'façon de parler' (manner of speaking). If one lesson of Russell's Paradox is that proper classes do not exist, there is no need for the model-theoretic apparatus to represent them.

We turn finally to criticisms of the specifically isomorphism-invariance criterion. Sher responds to Quine's insistence that logic is first-order logic (pp. 73 –74) and to Feferman's three arguments against isomorphism invariance (pp. 74–80), and throws in a criticism of Feferman's own approach to boot (p. 81).[1] As Feferman's criticisms have been discussed in Sher's previous (2016) book, and at great length in my own co-authored *One True Logic* (Part III) and earlier work (Griffiths & Paseau 2016), I shall not say more about them or Sher's responses here. Next, on pp. 82 –84, Sher tackles what in Part II of this review I'll call the Intensionality Problem and will discuss there. She then briefly mentions the undergeneration criticism before wrapping things up (pp. 85–86).

Part II: Differences

I now run through some differences between Sher's approach in *Logical Consequence* and that taken in *One True Logic*. These differences are not exhaustive, but to my mind they are the principal ones. As I said earlier, the two books have much in common. Broadly speaking, Sher and I are on the same side: we defend very similar ideas, sometimes (but not always) in fairly similar ways. To those far from this research area, or unsympathetic to our positions, focusing on the differences between us might seem like splitting hairs. But the differences are real, and the topic is a fundamental one in logic. This part of the review is therefore addressed to anyone who finds this sort of comparative exercise of interest and/or anyone who has read the précis and would like a more critical perspective on Sher's book.

Below, I outline five points of difference. The first three have more to do with emphasis than substance; the fourth and fifth are more substantive.

A. *Monism*

Logical monists claim that there is one true logic, logical pluralists that there are many. Beall & Restall (2006) and Shapiro (2014) are recent book-length defences of the latter; *One True Logic* is, in part, a defence of the former. We may define logical pluralism more precisely as the claim that at least two logics provide extensionally different but equally acceptable accounts of consequence between meaningful statements. Logical monism, in contrast, claims that a single logic provides this account. (I shall refer to these two rival positions as 'monism' and 'pluralism' respectively.)

Sher is quite clearly a monist. Which is not to say that she denies that there are other types of consequence than logical; indeed, she highlights the fact that there are many legitimate types and levels of consequence (see pages 44, 52, 60, 85 for exampl). Physical consequence—the implication that arises from physical necessity—is manifestly weaker than logical

[1] See Quine (1970) and Feferman (1999, 2010).

consequence. The type of consequence studied in modal logic—the logic of metaphysical necessity—is stronger than physical consequence, although weaker than logical consequence.[2] However, the existence of these other, non-logical, types of consequence does not threaten *logical* monism. Physical, modal, etc. and formal consequence all operate at different levels and mesh together. Sher is a logical monist because she thinks there is only one logic that gets the formal structure of reality right.

One point of difference between Sher's approach and *One True Logic*'s approach is that we try and offer a dialectically persuasive argument for monism, as our book's title suggests. Part I of *One True Logic* is devoted to that task. Sher's approach is different. Although she is not explicit about it, her monism seems to follow from two assumptions: (i) logic's job is to reflect the formal structure of reality, and (ii) reality has a unique and definite formal structure. Although the first assumption is clearly articulated in *Logical Consequence*, the second is implicit. The book and her work more generally could be considered an indirect argument for monism of the 'Ye shall know them by their fruits' sort: by going along with her assumptions and appreciating the power of her approach, you will be led to monism. Although this is a potentially strong argument for monism, I believe that one can offer more direct arguments, of the sort canvassed in *One True Logic*.

To see how ingrained Sher's monistic convictions are, we need look no further than the book's second paragraph, on page 1. Although not flagged as an argument for monism, it guides the reader down a monist path right from the start. Sher writes:

> Given a collection of sentences $\Gamma = \{S_1, S_2, \ldots, S_n, \ldots\}$, $n \geq 0$, and a sentence, S, either S follows logically from Γ— S is a LC [logical consequence] of Γ — or not. (p. 1)

This single sentence almost imperceptibly steers us towards monism. Expand it a little and it turns into a single-premise argument:

> *The Quick Argument for Monism*
> Each argument of English is either valid or invalid. Therefore, the correct logic is the one and only logic that captures the validity of all and only the valid arguments.

Is the Quick Argument sound? One could object to its premise on the basis that it is an instance of the Law of Excluded Middle (LEM). Anyone who has doubts about the LEM (in this sort of context, at least)—an intuitionist, perhaps—will not be convinced by it. Setting that aside, the main reason that the Quick Argument does not succeed is that it seems to presuppose monism about logic in its unrelativised conception of validity. At least some brands of pluralism hold that we should never speak of validity *simpliciter* but only validity in some logic or other.[3] From this perspective, the premise of the Quick Argument should

[2] There is a slight interpretative wrinkle here, in that on page 60, Sher says that it is not clear whether there is a unique type of metaphysical possibility/necessity and hence a unique type of metaphysical consequence. She clarifies that the type of impossibility involved in denying that any object can be both all-red and all-blue is weaker than formal (i.e. logical) possibility. It would be more accurate, then, to say that for Sher at least *one* important type of metaphysical consequence is weaker than logical consequence, and that she leaves it open that there may be other types of metaphysical consequence.
[3] Shapiro's pluralism, for example, is explicitly relativist.

therefore be: ‘Each argument of English is either valid$_L$ or invalid$_L$’. For the pluralist, there will be more than one specification of logic *L*, so the conclusion, as the pluralist sees it, is that exactly one logic captures the validity$_L$ of all and only the valid$_L$ arguments. This conclusion is perfectly compatible with pluralism, so long as there are different logics *L*. Logic L_1 captures validity$_{L1}$, logic L_2 captures validity$_{L2}$, and so on. The Quick Argument therefore seems to beg the question against the pluralist.

I do not, of course, mean to imply that Sher explicitly intended her compressed version of the Quick Argument on page 1 to constitute a knock-down argument for monism. My point is that monism is implicit in this passage, as elsewhere in the book. Sher does explicitly discuss the monism versus pluralist debate in one paragraph, where she allows for different subject matters possibly requiring different logics.

> It is also possible in principle that different parts of the world (e.g., its macroscopic and microscopic parts) differ in their formal structure, hence require different logics. (p. 58)

This is what Griffiths and I call horses-for-courses monism (2022, pp. 33–35). Different sub-logics are sovereign in their respective spheres, and the one true logic is their union. As long as no single argument is deemed valid by one correct logic and invalid by another logic, there is no clash and we have monism, albeit a disjunctive form of monism.

In sum, Sher is a monist. The reason seems to be that the world has a single and definite formal structure, even if some parts of it differ in their formal structure, just as a statue has a unique and definite shape, even if parts of it have different shapes. The correct logic is the one and only logic that gets this overall structure right. To be clear, the fact that she does not articulate and defend her monism more explicitly is no criticism of her Element, which cannot do everything in its brief compass.

B. *The correct logic*

The obvious question for any monist is: what is the correct one true logic? Neither Sher nor Griffiths and I advance a complete answer to that question. Both parties focus on what Sher calls ‘*predicate, formal*, or *mathematical* logic, which is the main modern successor of Aristotelian logic and is widely considered a core logic’ (p. 4, original italics). She clarifies that, for her, this excludes the likes of modal and relevance logic. Sher’s Element characterises the logical constants (see point of difference D below) but does not focus much on what exactly the correct predicate logic is. We are told briefly that it goes beyond first-order logic and includes ‘generalized first-order logics, (full) second-order logic, and other logics with lcs [logical constants] satisfying the isomorphism-invariance criterion’ (p. 55; the original is italicised).

Part II of *One True Logic*, in contrast, focuses exclusively on this question. Our claim is that the one true logic is highly, indeed in a sense maximally, infinitary in (what Sher calls) its predicate portion. We called this doctrine, with a touch of levity, the *L∞G∞S Hypothesis* (the first word is pronounced ‘Logos’). More precisely, let $FTT_{\infty\infty}$ be the logic of all finite types that (a) allows conjunction and disjunction over κ formulas, for any cardinal κ (finite or infinite); and (b) allows quantification over κ variables, for any cardinal κ (finite or infinite). Then according to the L∞G∞S Hypothesis, the intersection of the one true logic with the logic $FTT_{\infty\infty}$ also allows conjunction and disjunction over κ formulas and quantification over κ variables, for any cardinal κ. In particular, assuming first-order logic is at least a sublogic of the one true logic, the one true logic must contain the highly infinitary logic known as $L_{\infty\infty}$.

Our arguments for the *L∞G∞S Hypothesis* in *One True Logic* were twofold. The 'top-down' argument is very much of a piece with the sort of arguments Sher gives, and indeed is inspired by previous work of hers. However, our top-down argument is different from Sher's in that it does not rely on an account of the logical constants. Instead, it relies on a characterisation of logical extensions as isomorphism-invariant, and on the one true logic containing a connective for every logical operation. The main difference between her approach and ours is that we supplement the 'top-down' argument with 'bottom-up' ones. These are arguments based on relatively light theoretical commitments: although they do presuppose some facts about logical consequence and logical constants, they do not appeal to anything like a full-blown criterion of logicality. Chapter 5 of *One True Logic* contains the bottom-up arguments and Chapter 6 the top-down one.

Both Sher and we agree that the logic one uses to reason about some specific domain may, and typically will, be a small subset of the one true logic. The parallel is with logicians who regard first-order logic as the ultimate logic yet employ propositional logic or monadic first-order logic for some particular application. To say that the one true logic is *L* does not imply that we must always harness the full strength of *L* when exploring the implicational network of a language fragment. By way of analogy, we do not demand of engineers that they utilise the complete micro- and macro-physical theory of everything whenever they construct a bridge. This may seem an obvious point, but I emphasise it as it is not universally appreciated.

In sum, Sher focuses more on the logical constants and less on what the one true predicate logic looks like. It would be interesting to have more detail on the contours of the one true predicate logic as she sees it.

C. *Formalisation*

Sher takes logical consequence to be a relation between linguistic entities (p. 7). Griffiths and I agree.[4] However, to apply logic to sentences of a natural language such as English, we must be told what their logical form is. And logical form is an incurably logic-relative notion: different English sentences have different forms in different logics. To take a very simple example, the sentence 'Ann is friendly' has the form *p* in propositional logic, whereas its form in first-order logic is *Fa*.

What role does logical form play in an account of logical consequence? In *One True Logic,* we put things the following way. Call mappings from something like English sentences to sentences of a formal language *formalisations*. There are some constraints on formalisations, which we try to sketch in our book and which I have tried to say more about elsewhere.[5] With this notion in hand, we can say that *A* logically implies *B* just when *A*'s formalisation-in-*L* implies-according-to-*L* *B*'s formalisation-in-*L*, where *L* is the one true logic.[6]

[4] There may be a difference between Sher's approach, which is based on sentences, and ours, which is based on what we call statements (2022, pp. xxv–xxvi), of a language that is an extension of cleaned-up English (2022, pp. xxi–xxiv).

[5] See my (2019) and its sequel (2021).

[6] Since a sentence may have several formalisations, a further proviso is required to the effect that picking one does not matter. I'm skimming over details here, as elsewhere in this review.

Sher is a lot less explicit about this, but I think she would agree. On page 20 of *Logical Consequence,* she talks about abstracting from the meanings or contents of non-logical constants and treating them as schematic variables. Sher's abstraction is, in our terminology, formalisation. What we do not get in her work is a fuller account of what formalisation/abstraction involves. Since this notion plays such an important role in any overall account of logical consequence, more details would be welcome.

D. *Logical constants*

The first three points mainly bring out differences in emphasis or approach between *One True Logic* and *Logical Consequence*. Aside from the fact that Sher's monism is rooted in her formal-features account of logic whereas *One True Logic*'s is not, the points covered do not bespeak any major differences in view. The fourth and present point reflects a more substantive difference.

As mentioned in Part I of this review, the post-Tarskian understanding of logical consequence relies on a division between logical expressions, such as 'and' or 'is self-identical', and non-logical ones, such as 'is a tree' or 'Anna'. The former are formalised as expressions (e.g. '$\wedge$' and '=') whose interpretation is held fixed across models, whereas the latter are formalised as expressions (e.g. 'F' or 'a') whose interpretation varies across models. The problem of the logical constants is to offer a principled account of which is which. In the absence of such an account, we cannot be said to have a complete theory of logical consequence.

What, then, are logical constants? On the isomorphism-invariance approach Sher and I both favour, a necessary condition for an expression *e* to be a logical constant is that it denotes an isomorphism-invariant extension. This condition is not usually taken to be sufficient because expressions that are extensionally equivalent may differ in their logical status. For example, 'is non-self-identical' is logical, whereas 'is a twentieth-century American pope' is not; yet both expressions have the same empty extension, which is isomorphism-invariant. We may call this the *Intensionality Problem.*

To solve the problem, it is natural to suppose that we must find something that distinguishes logical expressions denoting isomorphism-invariant extensions from non-logical ones. One type of approach would be to claim that, to be logical, an expression's extension must be isomorphism invariant on every domain of a certain expanded type. In particular, one could consider possible as well as actual domains. Doing so would drive a wedge between 'is non-self-identical' and 'is a twentieth-century American pope': the former has empty extension in every possible domain, whereas in some possible domains the latter has non-empty extension. This is a modal response, because the range of domains includes both possible and actual ones.

The modal response doesn't work, however. The problem can be revived by considering expressions such as 'is a male widow' or 'is a regular heptahedron', not usually thought to be logical.[7] These expressions are necessarily empty in virtue of meaning but not in virtue of

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Evidently, the account should also be generalised to premise sets containing more than a single sentence. We return to this in point E below.

[7] These examples of expressions whose extensions are empty of metaphysical but not logical necessity are owed to Gómez-Torrente (2002). However, 'male widow' does not clearly belong to this category, because it is metaphysically possible for there to be a certain sort of

form; in first-order logic, for instance, they would be formalised as ‘$Mx \wedge Wx$’ and ‘$Rx \wedge Hx$’ respectively. Since an expression such as ‘is a male widow’ necessarily has an empty extension, allowing metaphysically possible domains would not distinguish its extension from that of ‘is non-self-identical’.

If the modal response—invoking possible domains—is to result in extensional adequacy, it seems that the only way to distinguish being non-self-identical from being a male widow is to allow logically possible domains. The problem now, however, is that logical constancy rests on the notion of logical possibility. But to analyse logical constants in terms of logical possibility would make the overall account of logical consequence circular, since in the Tarskian tradition logical consequence is analysed in terms of the logical constants. To see the circularity, note that logical consequence and logical possibility are interdefinable with a minimum of machinery: a sentence *A* is logically possible if and only if it’s not the case that every sentence is a logical consequence of *A*; and a sentence *C* is a logical consequence of some set of sentences *S* if and only if it’s not logically possible for all the sentences in *S* to be true and *C* to be false. To avoid circularity, we cannot analyse logical constancy in terms of logical possibility or logically possible domains.

How does Sher deal with the logical constants? She offers the following analysis of logical constancy (pp. 38–9; original italics removed):

> (Predicative) constant *C* is logical iff
>
> (a) *C* denotes a property, *P*, of the same level and arity as *C*, and
>
> (b) The denotation of *C* is defined in advance for all domains of actual-counterfactual individuals, hence for all models, and
>
> (c) *C* is a rigid designator; its denotation is defined by an extensional function and is identified with its extension, and
>
> (d) The denotation, P^{C}, of *C* is invariant under all isomorphisms (of structures for P^{C})…

Condition (a) is a background assumption and condition (d) is the isomorphism-invariance constraint. Sher avoids the Intensionality Problem principally by means of clause (b), because the models she operates with are all formally possible ones. This is made particularly clear in this passage:

> What is the scope of actual-counterfactual individuals in contexts concerning logic? Their scope is very broad. It includes individuals that are physically and even metaphysically impossible. More precisely, it includes all and only *formally possible* individuals. To be formally possible, an individual cannot be formally – hence, logically – contradictory or impossible, but it can be physically or metaphysically contradictory or impossible. (p. 43)

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intersex person who is both male and female, and who loses their spouse. Such a person would arguably be both male and a widow. I have kept ‘male widow’, because it is the example Sher uses (pp. 82–84) and the one other contributors to the literature do too (e.g. MacFarlane 2015), following Gómez-Torrente.

It is precisely this feature that affords her an answer to the Intensionality Problem, as she tells us herself:

> The impossibility of being a male widow (like the impossibility of being both all-red and blue, see Section 3.4) is *not* a *formal* impossibility. Therefore, there are structures in which some formally possible individuals are male widows, and "is-a-male-widow" is nonempty in such domains, hence cannot be identified with "is-not-self-identical." In short, "is-a-male-widow" is not invariant under all isomorphisms of structures (with domains of formally possible individuals). (p. 84; emphasis original)

This approach apparently succeeds in achieving extensional adequacy. But it does so by invoking the notion of formal possibility. And although Sher says several things about formal possibility, the worry is that she has not said enough to fend off the circularity objection.

Why so? For Sher, formal possibility is the worldly counterpart of the notion of logical possibility, so in this respect the two certainly differ. She also explains that male widows, all-red-and-all-blue objects, and the like are formally possible, and that formal possibility is stronger than metaphysical possibility (as explained under heading A above). This means that her account of logical consequence is not starkly circular in the way that an analysis of logical constancy that employs the notion of a logically possible domain would be. But the worry is that it remains circular, because we have not been told enough about formal possibility beyond what we already knew about logical possibility, recast in a worldly rather than linguistic key and replacing any occurrence of the word 'logical 'with the word 'formal'. Of course, Sher combines condition (b) above with others, notably rigid designation and isomorphism invariance, respectively conditions (c) and (d) above, which makes her account of logical constancy interesting and substantial. But that does not absolve condition (b), and hence the overall account, of ultimate circularity. This is why, to my mind, Sher has not solved the Intensionality Problem. In fairness, I should add that it's clear from the book and from follow-up discussion that Sher disagrees with this verdict.

In *One True Logic*, our approach is different. We decline the difficult task of analysing logical constancy, and rely instead on the analysis of logical extension. That allows us to argue that the one true logic is highly infinitary. We have tackled the problem of logical constancy, not needed for the argument in *One True Logic*, in subsequent work.[8]

E. *Formal structure*
The fifth and final point of difference is the most fundamental one and underlies all the substantial disagreements mentioned so far. Unlike Griffiths and me in *One True Logic*, Sher takes the ground of logical consequence to be the formal features of reality. I indicated under heading A above that this is what leads her to implicitly assume monism; and under D, that on its basis she takes herself to have solved the problem of logical constants.

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[8] Griffiths and Paseau (2024) proposes an account of logical constants that aims to be both extensionally adequate and non-circular. A further difference between Sher's approach and ours in *One True Logic* is that Sher accounts for the logicality of sentential connectives in terms of atomic states-of-affairs. We prefer to avoid commitment to states-of-affairs in a treatment of logicality (they may be needed for other reasons) and do things in terms of truth-values: see Chapter 6 of *One True Logic*.

As Sher sees it, the formal features of reality are regulated by formal laws with an especially strong modal force. Although she does not use the word, it would be accurate, I think, to say that for her the formal features of reality are the *truthmakers* of logical truths.

> …LC is grounded in formal laws governing the world, laws that due to their formality have an especially strong modal force, and the strong modal force of LC is due to the strong modal force of these laws. (p. 49; italics in the original removed)

The key question is how these formal features succeed in underpinning the model-theoretic account of logical consequence. We may succinctly and schematically express Sher's account as follows (see the passage on p. 48 quoted in Part I of this review):

> *Formal-Features Account (FFA)*
> A logically implies $B =_{exp}$ the situation $\mathfrak{C}$ that makes the sentence A correspondence-true formally necessitates the situation $\mathfrak{C}*$ that makes the sentence B correspondence-true.

Here, A and B are sentences and '$=_{exp}$' means that the left-hand-side is explained in terms of the right-hand-side. Compare this to the model-theoretic account encountered earlier and which may be succinctly expressed as follows.

> *Model-Theoretic Account (MTA)*
> A implies $B =_{exp}$ any model of the formalisation of A is a model of the formalisation of B

For both accounts, the many-premise version generalises the single-premise one given in the obvious way. In the case of the model-theoretic account, the monist will take the formalisations of A and B to be those in the single correct logic, and models to be appropriate ones for that same logic. Of course, both equations hide a mass of complexity behind them, but they will do for our purposes here.[9]

How do the formal-features account and the model-theoretic account relate to one another? Sher takes the latter to be a mathematical precisification of the former. Not one she is necessarily committed to, but one way this precisification could go. In evaluating her philosophical definition—namely the formal-features account—she urges that

> …it is important to distinguish between the definition itself [FFA] and its mathematical precisifications. When the main goal is to investigate the *mathematical* properties of logic, it is reasonable to identify logic with a certain mathematical precisification. But in a *philosophical* study of logic this distorts our understanding, leading us to attribute weaknesses or peculiarities of the mathematical background theory to logic itself. (p. 71)

So let us examine how the formal-features account might underpin the model-theoretic one – that is to say, how the formal structure of reality underpins the more mathematical account of logical consequence in terms of models.

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[9] Griffiths and Paseau (2025) has more on the model-theoretic account of validity, as well as its rival proof-theoretic account.

Suppose for a moment that 'reality' is the (proper-class-size) universe of sets; models are sets, as standard; and the correct logic in which to cast set theory is first-order logic. In that case, we know by the Reflection Theorem (owed to Lévy and Montague) that any formula true in the universe must be true in some model: if ZFC proves that φ then ZFC proves that φ-in-some-model-*M*. This obviously generalises to any finite $\varphi_1, \ldots, \varphi_n$ (since φ can be taken as their conjunction), but it does not generalise to infinitely many $\varphi_1, \ldots, \varphi_n, \ldots$.[10]

In short, one way to connect the formal-features account and the model-theoretic one is to see the latter as a consequence of the former. The hope is that we can find some sort of Reflection principle that combines with the former to imply the latter. Schematically:

Formal-Features Account of Logical Consequence (FFA)

+

Reflection Principle

$\Downarrow$

Model-Theoretic Account of Logical Consequence (MTA)

Sher doesn't put things quite this explicitly, but hints at this picture in response to the objection that reality is not a model (see the summary of pp. 70 –72 above).

Deriving the model-theoretic account from the formal-features one in this sort of way presents quite a daunting technical challenge. The first-order Reflection Theorem is of no immediate help in this context, for several reasons. One is that reality for Sher consists not just of actual individuals, but also all counterfactual ones, so reality for her cannot be equated with the universe of sets. The second is that logical consequence is a relation that holds between premise sets of arbitrary size and conclusions, and the first-order Reflection Theorem is, as we have seen, limited to finite premise sets. Third, Sher like me thinks that the one true predicate logic vastly outstrips first-order logic, so presumably the correct model theory will be cast in this very powerful logic. But the most obvious generalisation of the Reflection Theorem to third-order languages is known to be false. Far from there being a guarantee that Reflection will hold for set theory cast in a very powerful logic, the prospect has an air of falsity to it.

As mentioned, Sher is careful to distinguish her philosophical account of logical consequence from its mathematical precisifications (see pp. 56 –57 and p. 71). In that, she is surely right: there are many ways to make mathematically precise her fundamental idea. But to my mind, the burden of proof lies more with her than with those who do not embrace her formal-features account of logical consequence. Since, as I would put it, Sher's aim is to provide a metaphysical foundation for logic, it behoves her to show that her proposed foundation and standard logical practice are compatible. So she must find some way of showing how the standard model-theoretic account (MTA) flows from the formal-features account (FFA). Or, if she rejects this standard account in favour of some alternative mathematical precisification (MTA* say), she must (a) motivate this alternative MTA* as a better precisification than the standard model-theoretic one; and (b) show how MTA* follows from the formal-features account.

[10] The theorem is a standard and elementary one; consult pretty much any textbook on axiomatic set theory for the proof.

What is the alternative to the formal-features account explanation of logical consequence? The main alternative, I suggest, is to stop at the standard model-theoretic account of logical consequence, i.e. MTA. On this approach, there is no need to seek some ultimate metaphysical grounding for the model-theoretic account; that account itself is the end-point. From Sher's perspective, this sort of explanation doesn't go far enough: it stops at the mathematical precisification before we get to the underlying metaphysical basis (the formal structure of reality), which it precisifies. She would presumably see it as explanatorily shallow—or at least missing the depth which her formal-features account affords by adding a further explanatory layer to (something like) the model-theoretic account.[11] From the rival perspective that stops at the model-theoretic account, that purported explanatory depth is illusory; explanation has to stop somewhere, and the model-theoretic account is a better stopping-point than the formal-features account. One clear benefit is that we avoid the thorny question of how to derive MTA (or MTA*) from FFA.

One of the main residual questions for the rival approach to Sher's is what the logical constants are. The question is a compulsory one because the account avails itself of a distinction between vocabulary items whose interpretation is held fixed (the logical ones) and those whose interpretation varies (the non-logical ones). Although this is not the place to develop an answer, I believe there are grounds for optimism that one can do so.[12]

Let me end by praising *Logical Consequence* once more. It is quite a challenge to condense so many ideas and present them so clearly in such a relatively small number of pages. And the differences between Sher's views and my own should not obscure the great many common points in how we approach the topic and where we end up.[13]

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[11] I say 'something like' because, as indicated, the mathematical precisification of the formal-features account is, for Sher, up for grabs.

[12] See the aforementioned Griffiths and Paseau (2024).

[13] I would like to thank Gila Sher and Tibo Rushbrooke for generous discussion of earlier drafts of this review.

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