The Semantics and Pragmatics of Argumentation

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13.0 Questions and Answers

(1) Why do you think both linguists and philosophers find the semantics and pragmatics of argumentation interesting? Arguments have been the object of philosophical interest for a long time. Logicians and philosophers have studied the formal properties of arguments at least since Aristotle and have long discussed the logical sense of arguments as sets of premises and conclusions (Hamblin (1970), Walton (1990), Parsons (1996), Rumfitt (2015)). The structure of arguments has been investigated by epistemologists (e.g., Pollock (1987), Pollock (1991a), Pollock (1991b), Pollock (2010)), and has given rise to formal argumentation theory, which has developed into a branch of computer science in its own right (e.g., Dung (1995), Wan et al. (2009), Prakken (2010))). Philosophers of mind have contemplated the nature of reasoning and inference as mental acts and theorize about the relation between those mental acts and doxastic states, such as beliefs and credences (e.g., Longino (1978), Broome (2013), Neta (2013), Boghossian (2014))). By contrast, comparatively less attention has been paid to arguments as a distinctive kind of *discourse*, with its own semantics and pragmatics. Most work on speech act theory fails to discuss arguments as a kind of speech act (cf. Austin (1975), Searle (1969), Searle and Vanderveken (1985)). Even recent discussions of speech acts tend to focus primarily on assertions, orders, imperatives, and interrogatives (cf. Murray and Starr (2018), Murray and Starr (2020), Fogal et al. (2018)). Though arguments have not been widely studied *qua* linguistic constructions, they are central to linguistic theory and to philosophy (Dutilh Novaes (2021)). Just like we use language for exchanging information, for raising questions, for issuing orders, for making suppositions, etc., we also use language to give arguments, as when we argue on behalf of a certain conclusion and when we share our reasonings. Indeed, giving arguments is one of philosophers' favorite speech acts; and it is quite remarkably widespread outside the philosophy classroom.

(2) What recent developments in linguistics and philosophy do you think are most exciting in thinking about the semantics and pragmatics of argumentation? Recent developments in linguistics provide ample new resources for providing a semantics and pragmatics argumentation. We make arguments through constructions of the form:

(1)   a. $P_1, \ldots, P_n$. Therefore/thus/hence/so C;
         b.   Suppose $P_1, \ldots, P_n$. Then C.

These constructions are sets of sentences — or *discourses*. It is therefore natural to study these constructions by looking at semantic approaches that take discourses rather than sentences to be the main unit of semantic analysis. Because of this, dynamic approaches to the semantics of arguments will be at the center of my discussion. In particular, I will discuss the resources that discourse coherence approaches (Hobbs 1985; Asher 1993; Asher and Lascarides 2003; Kehler 2002) as well as dynamic semantic approaches to the study of language (Veltman 1985, 1996; Beaver 2001; Kaufmann 2000; Brasoveanu 2007; Gillies 2009; Murray 2014; Willer 2013; Starr 2014a,b; Pavese 2017, 2021; Kocurek and Pavese 2021) have to understand the semantics and dynamics of arguments.

(3) What do you consider to be the key ingredients in adequately analyzing the semantics and pragmatics of argumentation? Speech acts tend to be conventionally associated with certain linguistic features. For example, assertions are associated with the declarative mood of sentences; suppositions with the subjective mood, orders with the imperatival mood, questions with interrogative features, etc. Like other speech acts, giving an argument is conventionally associated with certain grammatical constructions of the form as (1-a) and (1-b) above. In order to study the speech act of giving an argument, I will therefore look at the semantics and pragmatics of words such as 'therefore', 'thus', 'so', 'hence', and 'then' — argument connectives, as Beaver (2001, 209) calls them — which are used in natural languages to signal the presence of arguments and to express relations between premises and conclusions. These argument connectives exhibit a distinctive *anaphoric behavior*. Their anaphoric component enables arguments to make use of multiple bodies of information at once. They often consist of multiple suppositions (as in proof by cases), suppositions within suppositions (as in conditional proofs), and so on. As we will see, in order to model these anaphoric relations, discourses have to be thought not simply as a sequences of sentences, but as sequences of labeled sentences — which can track different information states as different sets of premises and suppositions. It also requires thinking of contexts as more structured as usually required in dynamic semantics — not simply as information states or sets of possible worlds, but as having a distinctive *layered* (indeed, tree-like) structure (Kocurek and Pavese (2021)).

(4) What do you consider to be the outstanding questions pertaining to the semantics and pragmatics of argumentation? Here are a few outstanding questions pertaining the semantics and pragmatics of argumentations: what does the

speech act of arguing and making an argument amount to? In particular, how does it affect the context set? What relations do argument connectives express (if any) between premises and conclusions? In virtue of what mechanisms (i.e., presupposition, implicature, etc.) do they get to express those relations? How does the semantics of these words compare to their counterparts in formal languages? How are we to think of the syntax of argumentative discourses and how are we to model contexts in order to model the dynamics of argumentative discourses? Can a unified semantics of argument connectives be provided across their deductive, practical, causal, and inductive usages? How are we to think of the syntax of argumentative discourses and how are we to model contexts in order to model the dynamics of argumentative discourses? What do argument connectives such as 'therefore' contribute to the arguments where it occurs? What is the nature of the support relation tested by argument connectives? How are we to model the subtle differences between argument connectives — between 'therefore', 'then', 'so', 'thus', and 'hence'? What makes a discourse an argument, rather than an *explanation*? How are we to characterize the distinctive utterance force of arguments versus explanations? Are there such things as zero-premises arguments in natural languages? How do deductive arguments in natural language differ, if at all, from proofs in natural deduction systems — such as Fitch's proofs?

13.1 Introduction

This chapter overviews recent work on the semantics and pragmatics of arguments. In natural languages, arguments are conventionally associated with particular grammatical constructions, such as:

(2)   a. $P_1, \ldots, P_n$. Therefore, C;
      b.   Suppose $P_1, \ldots, P_n$. Then, C.

These constructions involve argument words such as 'therefore','thus', 'so, 'hence' and 'then' — *entailment words* (cf. Brasoveanu (2007)) or, as I will call them, following Beaver 2001, 209, *argument connectives* — which are used in natural languages to signal the presence of arguments. It is, therefore, natural to study the speech act of giving an argument by looking at semantics and pragmatics of argument connectives.[1]

The first six sections of this chapter look at the semantics of argument connectives. Because arguments typically stretch through discourse, and argument connectives are kinds of discourse connectives, it is natural to start with semantic approaches that take discourses rather than sentences to be the main unit of semantic analysis. Recent developments in linguistics provide ample new resources for a semantics of argumentation. In particular, I will discuss the resources that discourse coherence approaches as well as dynamic approaches to the study of language have to understand the semantics of argument connectives. §2 compares argument connectives in English to their formal counterparts in proof theory. §3 explores thinking of argument connectives as expressing *discourse coherence relations* (e.g., Asher 1993; Asher and Lascarides 2003; Bras et al. 2001a,b; Le Draoulec and Bras 2007; Bras et al. 2009; Jasinskaja and Karagjosova 2020). §4 discusses Grice's view according to which argument connectives come with an associated conventional implicature and compares it to the competing analysis on

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[1]Even recent discussions of speech acts tend to focus primarily on assertions, orders, imperatives, and interrogatives (cf. Fogal et al. (2018)). Some discussion of argumentation can be found in van Eemeren and Grootendorst 1982, 2004, who investigate arguments and argumentation, but primarily as a tool to overcome dialectical conflict and in Mercier and Sperber (2011) who use arguments and argumentation theory for a philosophical theory of reasoning, and in Koralus and Mascarenhas (2013) who draw an interesting parallel between reasoning as a psychological process and arguments in natural languages and highlight the question-sensitivity of both. There is some discussion of argument connectives such as 'therefore' in *discourse coherence theory* (e.g., Hobbs 1985; Asher 1993; Asher and Lascarides 2003; Asher and Gillies 2003; Kehler 2002; Stojnić 2022), though these discussions fall well short of giving a systematic semantics for 'therefore' in all of its uses.

which 'therefore' is a presupposition trigger (Pavese 2017; Stokke 2017; Pavese 2021). §5 discusses Brasoveanu (2007)'s proposal that semantically 'therefore' works as a modal, akin to epistemic 'must'. §6 examines dynamic analyses of argument connectives (Pavese 2017; Kocurek and Pavese 2021), with an eye to highlight the scope and the advantages of these sorts of analyses. The final section (§7) looks at the pragmatics of argument connectives and at the difference between arguments and explanations. §8 concludes.

13.2 Arguments in logic and in natural languages

Consider **Argument Schema**, with the horizontal line taking a list of premises and a conclusion into an argument:

Argument Schema

$$\frac{\phi_1, \ldots, \phi_n}{\psi}$$

Now, compare **Argument Schema** to the following arguments in English:

(3)   a.   There is no on-going epidemic crisis. Therefore, there is no need for vaccines.

        b.   It is raining. Therefore, the streets are wet.

        c.   I am smelling gas in the kitchen. Therefore, there is a gas leak.

        d.   This substance turns litmus paper red. Therefore, this substance is an acid.

These arguments all have the form "Φ, Therefore ψ" where Φ is the ordered set of premises $\phi_1, \ldots, \phi_n$ and ψ is the conclusion. Because of the syntactic resemblance of **Argument Schema** and (3-a)-(3-d), it is tempting to think of 'therefore' and other argument connectives such as 'thus', 'so, 'hence' and 'then' as having the same meaning as the horizontal line (e.g., Rumfitt 2015, 53).

However, **Argument Schema** is not perfectly translated by the construction "Φ. Therefore/Thus/Hence/Then ψ"; nor is the horizontal line perfectly translated by the argument connectives available in English. First of all, the horizontal line does not require *premises*, for it tolerates conclusions without premises, as in the case of theorems:

Theorem

$$\overline{\psi \vee \neg\psi}$$

By contrast, 'therefore', 'thus', 'so', 'hence', 'then', etc. do require explicit premises:[2]

(4)   a. ??Therefore/hence, we should leave (looking at one's partner's uncomfortable face).

         b. ??Therefore/hence, the streets are wet (looking at the rain pouring outside).

         c. ??Therefore/hence, either it is raining or it is not raining.

A plausible explanation for this contrast is that 'therefore', 'thus', 'so', 'hence', and 'then' differ from the horizontal line in that they contain an anaphoric element — (cf. Brasoveanu 2007, 296; Kocurek and Pavese (2021)). Like anaphors, argument connectives require not just an antecedent but its *explicit* occurrence.[3]

That is the first difference between 'therefore' and the horizontal line. Here is a second difference (cf. Pavese 2017, 95-6; Pavese (2021)). In **Argument Schema**, the premises can be supposed, rather than asserted. By contrast, 'therefore' (and 'hence', 'thus', 'so') is not always allowed in the context of a supposition:

(6)   a.   It is raining. Therefore/so/hence, the streets are wet.

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[2]As Pauline Jacobson has pointed out to me (p.c.), the use of 'so' strikingly differs from the use of 'therefore' in this regard, in that 'so' can also be used without premises, as in "So, you have arrived!". On the other hand, 'so' can also be used anaphorically, in non-argumentative use, as when we say 'I think so'. See Needham (2012) for a discussion of theses uses of 'so' and Krifka (2013), Elswyk (2019) for a more general discussion of propositional anaphora. Hence, 'so' seems to have a *deictic* use as well as an anaphoric use. By contrast, 'therefore' seems to privilege an anaphoric use. (However, see Neta 2013, 399–406 for the claim that 'therefore' is a deictic expression.) For a more careful comparison of the subtle differences between argument connectives, see Kocurek and Pavese (2021).

[3]There is not to say that premise-less arguments cannot be made in natural languages. Natural languages seem to resort to other devices to express premise-less arguments, —i.e., locutions such as 'by logic'. Cf. Pavese (2021) for a discussion of these issues. Moreover, not every argument connective attaches to conclusions in the same way 'therefore' and 'so' do. For example, 'since' is an argument connective in (5):

(5)   Since it is raining, streets will be wet.

But here it attaches to 'it is raining' which is intuitively the premise of the argument.

    b. ??Suppose it is raining; therefore/so/hence the streets are wet.

    c.   If it is raining, therefore/so/hence the streets are wet.

    d.???If Mary is English, therefore/so/hence she is brave.

    e.???Suppose Mark is an Englishman. Therefore/so/hence, he is brave.

Under supposition, connectives like 'then' are much preferred to 'therefore':

(7)   a.   Suppose Φ; then, ψ.

        b.   Suppose it is raining. Then, the streets are wet.

        c.   If it is raining, then the streets are wet.

        d.   If Mary is English, then she is brave.

        e.   Suppose Mark is an Englishman. Then, he is brave.

For this reason, Pavese (2017) speculates that the slight infelicity of (6-b) may indicate that 'therefore' is more similar to the square — i.e., '$\square$' — that ends proofs than to the horizontal line in **Argument Schema**:

 Proof of Theorem

     Theorem . . . $\square$

Just like '$\square$', 'therefore' would require its premises having been discharged and not conditionally dependent on other premises.

    However, the data is more complex than Pavese (2017) recognizes and should be assessed with caution. 'Therefore' can be licensed in the context of supposition. For example, consider:

(8)   a.   If it were raining, the streets would, therefore, be wet.

        b.   Suppose it were raining; the streets would, therefore, be wet.

        c.   If Mary were English, she would, therefore, be brave.

        d.   Suppose Mark were anEnglishman. He would, therefore, be brave.

'Therefore' is licensed in this construction, where the mood of the linguistic environment is subjunctive. In this respect, 'therefore', 'thus', 'so', and 'hence' differ from 'then', for 'then' is permitted within the scope of a supposition whether or not the mood is indicative:[4]

(9)   a.   Suppose it were raining. Then, the streets would be wet.

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[4]Indeed, in these and other respects, 'then' and 'therefore' seem to be in complementary distribution. See Kocurek and Pavese (2021) for more discussion of this point.

b. If it were raining, then the streets would be wet.
c. If Mary were English, then she would brave.
d. Suppose Mark were an Englishman. Then, he would be brave.

Moreover, 'therefore' is at least tolerated with so-called 'advertising conditionals' — interrogatives that play a role in discourse similar to that of antecedents of conditionals:

(10) a. Single? (Then) You have not visited Match.com. (Starr 2014a, 4)
      b. Single? Therefore, you have not visited Match.com.
      c. Still looking for a good pizzeria? Therefore you have not tried Franco's yet.

This suggests that at least under certain conditions, 'therefore' can appear in suppositional contexts (cf. Pavese (2021)).

Another respect under which argument connectives in English differ from the horizontal line in **Argument Schema** is that while their premises have to be declarative, their conclusion does not need to be.[5] Several philosophers have observed that imperatives can appear as conclusions of arguments (e.g., Parsons 2011, 2013; Charlow 2014; Starr 2020):

(11) If May arrives late tonight, you should go to the store. As a matter of fact, Mary is arriving late. Therefore, go to the store!

In addition to allowing imperative conclusions, argument connectives can also have *interrogative* conclusions:

(12) The doctor and the lawyer were the two main and only suspects. But then the detective found a stethoscope near the location of the murder. Therefore, who is the chief suspect now?

The final important observation is that argument connectives in English differ from the horizontal line in that they can also appear in non-deductive arguments, both in inductive arguments such as (13-a)-(13-c), in abductive arguments such as (13-c)(13-d), in causal arguments as in (14-a)-(14-d), as well as practical arguments, such as (14-e):

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[5]I will be assuming throughout that arguments cannot have imperatives or interrogatives as premises but even here the data is rather subtle. See Kocurek and Pavese (2021) for some discussion.

(13) a. It happened, therefore it can happen again: this is the core of what we have to say. It can happen, and it can happen everywhere. (from Primo Levi The Drowned and the Saved, Vintage; New York, 1989. pg. 199). [INDUCTIVE ARGUMENT]

b. Almost every raven is black, and the animal that we are about to observe is a raven. Therefore, it will be black too. [INDUCTIVE ARGUMENT]

c. Mark owns a Bentley. Therefore, he must be rich (Douven et al. 2013) [ABDUCTIVE ARGUMENT]

d. The victim has been killed with a screwdriver. Therefore, it must have been the carpenter. [ABDUCTIVE ARGUMENT]

(14) a. John pushed Max. Therefore, Max fell. [CAUSAL ARGUMENT]

b. John was desperate for financial reasons. Therefore, he killed himself. [CAUSAL ARGUMENT]

c. Mary qualified for the exam. Therefore, she could enroll. [CAUSAL ARGUMENT]

d. Reviewers are usually people who would have been poets, historians, biographers, etc., if they could; they have tried their talents at one or the other, and have failed; therefore they turn into critics. (Samuel Taylor Coleridge, Lectures on Shakespeare and Milton) [CAUSAL ARGUMENT]

e. We cannot put the face of a person on a stamp unless said person is deceased. My suggestion, therefore, is that you drop dead (attributed to J. Edward Day; letter, never mailed, to a petitioner who wanted himself portrayed on a postage stamp). (Brasoveanu 2007, 279) [PRACTICAL ARGUMENT]

To sum up, there are at least four dimensions along which argument connectives differ from the horizontal line in deductive logic. First, they differ in that they have an *anaphoric component*; second, they are *mood-sensitive*, in that whether they allow embedding under supposition and sub-arguments might depend on the mood of the linguistic environment. Thirdly, argument connectives can allow for *non-declarative conclusions* and, fourthly, they can occur with logical, causal and practical flavors, as well as in inductive and abductive arguments.

13.3 Argument Connectives within Discourse Coherence Theory

Giving an argument is a speech act that stretches through a discourse — i.e., from its premises to its conclusion. It is therefore natural to start an analysis of arguments by looking at the resources provided by *discourse coherence analysis* — an approach to the study of language and communication that aims at interpreting discourses by uncovering coherence relations between their segments (Asher 1993; Asher and Lascarides 2003). The crucial question behind a coherence discourse theoretic approach to the meaning of argument connectives is, then, *what kind* of coherence relation they express. The most notable discourse relations studied by discourse coherence theorists are NARRATION, ELABORATION, BACKGROUND, CONTINUATION, RESULT, CONTRAST, and EXPLANATION.

Although this literature has focused much more on *temporal* discourse connectives than on argument connectives, the general tendency in this literature is to assimilate the meaning of 'therefore' to the meaning of 'then' in its temporal uses and to its French counterpart 'alors' (cf. Bras et al. 2001a,b, 2009). According to the prevailing analysis, 'therefore' would then introduce RESULT (Hobbs 1985; Asher 1993; Asher and Lascarides 2003; Asher and Gillies 2003; Kehler 2002).[6] If the relation of RESULT is a causal relation: if it holds between two constituents, then the former causes the latter.

While this account captures well causal uses of 'therefore' as in (14-a)-(14-c), not every use of 'therefore' is plausibly causal in this fashion. For example, in the following arguments, the truth of the premises does not cause the truth of the conclusion:[7]

(15) a. All the girls have arrived. Therefore, Mary has also arrived.
      b. Mary has arrived. Therefore, somebody has arrived.
      c. 2 is even. Therefore either 2 is even or 3 is.

In order to extend their discourse coherence analysis to uses of 'therefore' that are recalcitrant to the causal analysis, Bras et al. (2009) proposes we appeal to INFERENTIAL RESULT — i.e., a relation holding between two events or proposi-

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[6]I am grateful to Nick Asher for correspondence here.

[7]For example, (15-b) violates counterfactual dependence that is plausibly necessary for a causal relation, for if Mary had not have arrived, somebody might still have arrived. Or consider a mathematical inference, such as (15-c), for which the counterfactual "If 2 were not even, it would be false that either 2 is even or 3 is" is a useless counterpossible.

tions just in case the latter is a logical consequence of the former (K_α indicates a constituent's way of describing an event α and the arrow stands for the material conditional):

INFERENTIAL RESULT (α, β) iff $\Box(K_\alpha \rightarrow K_\beta)$.

However, not every non-narrative use of argument connectives can be analyzed in terms of INFERENTIAL RESULT. For example, consider the use of 'therefore' in inductive, abductive, or practical arguments, as in (13-c)-(14-e). None of these arguments plausibly express INFERENTIAL RESULT. Even if we restrict INFERENTIAL RESULT to the deductive uses of argument connectives, the problem remains that this approach would result in a rather disunified theory of the meaning of argument connectives. We are told that sometimes discourses involving 'therefore' express the causal relation of RESULT, sometimes they express a different discourse relation altogether — i.e., INFERENTIAL RESULT or classical entailment in deductive uses, and maybe some other discourse relations in practical and inductive uses.

Here is a unifying proposal, one that preserves the discourse coherence theorists' important insight that 'therefore' is a discourse connector expressing some or other discourse relation. Suppose we understand RESULT in terms of a *restricted* notion of entailment. For example, we might understand RESULT in terms of nomological entailment — entailment given the laws of nature — or default entailment, as in Asher and Morreau (1990) and Morreau (1992). (cf. also, Meyer and van der Hoek 1993; Weydert 1995; Veltman 1996). Quite independently of the consideration of argument connectives, Altshuler (2016) has proposed that we understand RESULT in terms of *enthymematic nomological entailment*.[8] ϕ enthymematically entails the proposition ψ, if and only if there is a nonempty set of propositions Φ such that $\Phi \cup \{\phi\}$ logically entails ψ. For example, consider again (14-a). While John's having pushed Max does not entail that Max fell, Altshuler 2016, 70-1 proposes John's having pushed Max might enthymematically entail that Max fell, for John's having pushed Max in conjunction with an appropriate set of background propositions might entail that Max fell.[9]

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[8]See also Kehler (2002) (section 3.1).

[9]When we interpret (14-a), we might assume that in normal circumstances, if one is pushed sufficiently strongly, then one will fall and that Josh must have pushed Max sufficiently strongly. As Altshuler (2016) observes, these background propositions may come from a wide variety of sources, from shared knowledge or from the discourse itself. In the case of RESULT, Altshuler proposes that we might understand the relation between two constituents as a form of entailment

Following and extending this proposal, we might then take argument connectives in their deductive uses to express a non-restricted form of entailment — i.e., classical (or relevantist) entailment; by contrast, in their causal uses, they express *nomological entailment* and in their practical uses *practical entailment* — entailment given the prudential/practical/moral laws. Inductive uses might be understood in terms of a restricted form of entailment as well, where the restriction comes from the general principle of uniformity of nature or a specific version thereof (cf. Kocurek and Pavese (2021) for this unifying idea). On this proposal, every use of argument connectives expresses some more or less general relation of entailment. We thereby reach unification across uses of argument connectives while preserving the differences.

In conclusion, discourse coherence theory provides us with the resources to study the semantics and pragmatics of arguments from the correct methodological standpoint: because arguments are discourses, this approach analyzes argument connectives as discourse connectors and thus as expressing discourse relations. From our discussion, however, it emerges that argument connectives appear with a variety of different flavors (narrative, causal, inferential, etc.), and so the question arises of what unified discourse relation they express. In order to capture what is common to all of these uses, it seems promising to think of the relevant discourse relations in terms of more or less restricted relations of entailment.

13.4 Conventional implicature or presupposition?

In "Logic and Conversation", Grice 1975, 44–45 uses the case of 'therefore' to illustrate the notion of a *conventional implicature*. Grice observes that in an argument such as (16-a) and in a sentence such as (16-b), 'therefore' contributes the content that the premise entails the conclusion — in other words, it contributes *Target Content*:

(16) a. Jill is English. Therefore, she is brave. (*'therefore'-argument*)
   b. Jill is English and she is, therefore, brave. (*'therefore'-sentence*)
   c. Jill is English and she is brave.
   d. Her being brave follows from her being English. (*Target Content*)

—i.e., *nomological entailment*. This discourse relation between a constituent σ_1 and a constituent σ_2 holds just in case σ_1 entails σ_2, together with the relevant laws L as well as the other relevant background propositions.

Grice points out that in an argument such as (16-a) or in a sentence such as (16-b), *Target Content* is communicated without being asserted, for by saying (16-b), one commits to *Target Content*'s being true but whether *Target Content* is true does not contribute to what is said by (16-b). Grice took this to indicate that *Target Content* is only conventionally implicated by 'therefore', for he further thought that (16-b) would not be false if *Target Content* were false. It is customary for linguists and philosophers to follow Grice here. For example, Potts 2007, 2 tells us that the content associated with 'therefore' is a relatively uncontroversial example of a conventional implicature (see also Neta (2013) and Wayne 2014, section 2). Whether the conventional implicature analysis of 'therefore' best models the behavior of 'therefore' is, however, questionable. Some have argued that several considerations suggest that the explanatory category of *presuppositions*, rather than that of conventional implicatures, might actually better capture the status of the sort of content that is conveyed by argument connectives (see Pavese (2017), Stokke (2017), Pavese (2021)).

The first kind of evidence for this claim is that 'therefore' satisfies the usual tests for presupposition triggers: *Projectability* and *Not-At-Issuedness*. Start with *Projectability*. Like standard presupposition triggers, *Target Content* projects out of embeddings — i.e., out of negation (17-a), out of questions (17-b), in the antecedents of conditionals (17-c), out of possibility modals (17-d) and out of evidential modal and probability adverbs (17-e), as can be seen from the fact that all of the following sentences still convey that Mary's braveness follows from her being English:

(17)   a.   It is not the case that Mary is English and, therefore, brave. (*Negation*)

          b.   Is Mary English and, therefore, brave? (*Question*)

          c.   If Mary is English and, therefore, brave, she will act as such. (*Antecedent of a conditional*)

          d.   It might be that Mary is English and, therefore, brave. (*Possibility Modal*)

          e.   Presumably Mary is English and therefore brave. (*Evidential modal, probability adverb*)

Some speakers also hear a non-projective reading for *Negation* (17-a). On this projective reading, we are not simply denying that Mary is English. We are denying that her braveness follows from her being English. However, the claim that 'therefore' works as a presupposition trigger in (17-a) is compatible with

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(17-a) also having a non-projective reading. For example, consider (18):

(18)   The tarts were not stolen by the knave: there is no knave.

Clearly, the definite article in 'the knave' must have a non-projective reading in "The tarts were not stolen by the knave," for else (18) would have to be infelicitous. Presumably, whatever explains the non-projective reading in (18) can explain the non-projective reading in (17-a) (cf. Abrusán (2016), Abrusán (2022)). The standard explanations for non-projective readings under negation are available here: maybe we are dealing with two different kinds of negation (metalinguistic negation *versus* negation *simpliciter* (cf. Horn (1972), Horn (1985)); or we might be dealing with an example of local accommodation (cf. Heim (1983)); or we might appeal to Bochvar (1939)'s A operator (cf. Beaver (1985), Beaver and Krahmer (2001)).

Hence, *Target Content* is projectable to the extent to which presuppositions are usually taken to be projectable. Moreover, *Target Content* satisfies the second standard set of tests for spotting presupposition triggers — i.e., the *not-at-issuedness* tests. *Target Content* also cannot be directly challenged — i.e., (19-a) and (19-b) — in striking contrast to when it is instead made explicit — i.e., (19-c)-(19-d):

(19)   a.   Jill is English and, therefore, she is brave.
                 *That is false/That is not true.
         b.   Jill is English. Therefore, she is brave.
                 *That is false/That is not true.
         c.   Jill is English and from that it follows that she is brave.
                 That is false/That is not true.
         d.   Jill is English. It follows from that that she is brave.
                 That is false/That is not true.
         e.   Jill is English and, therefore, she is brave. Hey, wait a minute! Not all English people are brave!
         f.   Jill is English. Therefore, she is brave. What? Not all English people are brave!

While the *Target Content* cannot be directly challenged, it can be *indirectly* challenged, by taking some distance from the utterance, as evidenced by (19-e) and (19-f), through locutions such as 'wait a minute' and 'what?'. Note that this phenomenon is not just observable for inferential uses of 'therefore'. The same pattern is observable for narrative uses of 'therefore' too:

(20) a. John was desperate for financial reasons. Therefore, he killed himself.
       b. *That is false/*That is not true. He did not kill himself for financial reasons.
       c.   Wait a moment!!! He did not kill himself for financial reasons.
       d. What?? He did not kill himself for financial reasons.

That suggests that whether the relation expressed by 'therefore' is classical entailment (in inferential uses of 'therefore') or some restricted notion of entailment (as in narrative uses of 'therefore'), such relation is backgrounded in the way presuppositions are.

Like presuppositions, *Target Content* also cannot be canceled when unembedded, on pain of Moorean paradoxicality:

(21) a. ??Jill is English. Therefore, she is brave. But her braveness does not follow from her being English.
       b. ??Jill is English. Therefore, she is brave. But I do not believe/know that her being brave follows from her being English.

And like other strong presupposition triggers, which cannot felicitously follow retraction (cf, Pearson (2010)), 'therefore' cannot follow retraction either, as evidenced by (22-a) and (22-b)

(22) a. ??Well, I do not know if her braveness follows from her being English. But Mary is English. And therefore, she is brave.
       b. ??Well, I do not know if her being from the North follows from her being progressive. But Mary is a progressive. And therefore, she is from the North.

Finally, just like presuppositions issued by strong presupposition triggers *Target Content* cannot even be suspended, as evidenced by (23-c) (Abrusán (2016), Abrusán (2022)):

(23) ??I have no idea whether all English people are brave. ?? But if Mary is English and therefore brave, she will act as such.

Do these tests suffice to show that 'therefore' is a presupposition trigger? Now, the boundaries between conventional implicatures and presuppositions are notoriously hard to draw. And many supposed examples of conventional implicatures also satisfy many of the aforementioned tests. However, there are some additional

considerations that suggest that the presuppositional analysis is more explanatory of the behavior of argument connectives. Conventional implicatures project *even more massively* than presuppositions (Potts 2015, 31). For example, additive articles such as 'too' and 'also' project out of standard plugs such as attitude reports (cf. Karttunen (1973)). By contrast, the presupposition associated with 'therefore' can be plugged by belief reports:

(24)   George believes that Mary is English and, therefore, brave. (*Belief operator*)

Also under epistemic modals and negation, not-projective readings are sometimes available for 'therefore' (cf. Pavese (2021), Kocurek and Pavese (2021) for discussion).

Moreover, it seems a necessary condition for presuppositions that a sentence s presupposes p only if s does not warrant an inference to p when s is in an entailment-canceling environment and when p is locally entailed (cf Mandelkern (2016)). This condition is satisfied also by discourses featuring 'therefore'. For example, the following conditionals do not entail *Target Content*:

(25)   a.   If being brave follows from being English, Mary is English and, therefore, brave.
         b.   If liking the Steelers follows from being from Pittsburgh, then Mary likes the Steelers and, therefore, she is from Pittsburgh.

In conclusion, the presuppositional analysis seems to capture the projective behavior associated with 'therefore' better than the conventional implicature analysis.[10] I take it, however, that the real interesting question — and the one I will focus on going forward — is not how to label 'therefore' (whether as a presuppositional trigger or as a conventional implicature trigger) but rather how best to formally model its projective and non-projective behavior. It is to this question which I turn next.

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[10]Vaassen and Sandgren (2021) argue that 'therefore' is not a presupposition trigger on the grounds that non-projective readings are available for 'therefore' under epistemic modals, negation, and interrogatives. But the mere availability of non-projective readings is only evidence against the conventional implicature analysis and is compatible with 'therefore''s being a presupposition trigger, since its being a presupposition trigger does *not* entail that its content never projects (cf., e.g., Karttunen (1974)). See both Pavese (2021) and Kocurek and Pavese (2021) for discussion. As observed by Kocurek and Pavese (2021), a dynamic semantics for 'therefore' as a presuppositional trigger can capture both projective and non-projective readings.

13.5 'Therefore' as a Modal

Another important observation about the meaning of 'therefore' is that it closely resembles that of necessity modals. For example, (26) is very close in meaning to the modalized conditional (27):

(26)   a.   Sarah saw a puppy. Therefore, she petted it.
           b.   If Sarah saw a puppy, she (obviously/necessarily/must have) petted it.

provided that we add to (26-b) the premise (27):

(27)   Sarah saw a puppy.

Moreover, as we have seen in (13-a)–(14-e), 'therefore' comes in different flavors (logical, causal, practical, inductive, abductive). So in this respect too it resembles modals (cf. Kratzer 1977, 2002). On these bases, following Kratzer's analysis of modals, Brasoveanu (2007) proposes we understand different flavors of 'therefore' as resulting from a restriction of the corresponding 'modal base'. A modal base is a variable function from a world to a set of propositions, modeling the nature of the contextual assumptions — whether causal, practical, or epistemic. Its intersection returns the set of possible words in which all the propositions in the modal base are true. The logical consequence flavor of 'therefore' derives from an empty modal base, whose intersection is the universe. This formally captures the fact that logical consequence is the unrestricted flavor of 'therefore'.

    This approach captures both the similarity between 'therefore' and 'must' and several possible flavors with which 'therefore' is used. However, it is unclear that this approach resorting to modal bases can effectively model inductive and abductive uses of 'therefore'. Inductive arguments are notoriously non-monotonic. For example, consider:

(28)   a.   The sun has risen every day in the past. Therefore, the sun will rise again tomorrow.
           b.   The sun has risen every day in the past. And today is the end of the world. ??Therefore, the sun will rise again tomorrow.

If we apply the modal base approach to (28-a), we get that in any context where (28-a) is felicitious, (28-b) should be, too. For suppose in our current state s, when we update s with the premises in (28-a), each world in the resulting state s' is assigned by the modal base a set of propositions whose intersection supports

the conclusion. Let s'' be the result of updating s with the premises in (28-b). Since every world in s'' is a world in s', when we apply the modal base to a world in s'', it also supports the conclusion. One way Brasoveanu's approach could be extended to model the non-monotonicity of inductive arguments is by appeal to some context-shift. But it is difficult to see how the sort of context-shifts needed could be motivated. This observation does not undermine the important similarity between 'therefore' and 'must' observed by Brasoveanu (2007), for 'must' seems to be amenable to inductive uses too, as in:

(29)   All swans observed so far have been white. The next must be white too.

However, it does raise the issue of how to model inductive and abductive uses of both 'therefore' and modals. (For promising work in this respect, see Del Pinal (2021)).

13. 6 Dynamic Treatments of Argument Connectives

13.6.1 A Simple Semantics

So far, we have observed that argument connectives appear to behave as presupposition triggers and that they also resembles modals. Any semantic analysis ought to capture these two sets of data. Pavese (2017) suggests that dynamic semantics offers the tools to develop an analysis that meets this desiderata. Kocurek and Pavese (2021) improve on Pavese (2017)'s analysis and develop this proposal in some detail. Here, I review some of the most important aspects of these dynamic analyses.

In dynamic semantics, a test is an expression whose role is to check that the context satisfies certain constraints, as Veltman (1996)'s 'might' or von Fintel and Gillies (2007)'s 'must'. These expressions check that the context supports their prejacent: so "It might be raining" checks that the context supports the sentence that it is raining.

Define an INFORMATION STATE as a set $s \subseteq W$ of worlds. We define the update effect of a sentence on an information state recursively, as follows:

$$s[p] = \{w \in s \mid w(p) = 1\}$$
$$s[\neg\phi] = s - s[\phi]$$
$$s[\phi \wedge \psi] = s[\phi][\psi]$$

$$s[\phi \vee \psi] = s[\phi] \cup s[\psi]$$
$$s[\Box\phi] = \{w \in s \mid s[\phi] = s\}$$
$$s[\Diamond\phi] = \{w \in s \mid s[\phi] \neq \varnothing\}$$
$$s[\phi \rightarrow \psi] = \{w \in s \mid s[\phi][\psi] = s[\phi]\}.$$
$$s[\therefore \phi] = \begin{cases} s & \text{if } s[\phi] = s \\ \text{undefined} & \text{otherwise} \end{cases}$$

In the above definition, $\Box$, $\Diamond$, $\rightarrow$ $\therefore$ are all tests. $\Diamond$ (corresponding to Veltman (1996)'s 'might') tests whether the context is compatible with its prejacent; if not, it returns the empty set. $\Box$ (corresponding to von Fintel and Gillies (2007) and von Fintel and Gillies (2010)'s 'must') tests that the context supports its prejacent — i.e., that $s[\phi]=s$. If not, it returns the empty set. Notice that $\therefore$ (corresponding to our 'therefore') is similar to '$\Box$' — like '$\Box$' it checks that the current context (augmented with '$\therefore$'s antecedents) supports the conclusion. $\therefore$ also closely resembles $\rightarrow$ (corresponding to Veltman (1985)'s conditional): the latter tests whether the context augmented with the antecedent supports the consequent; '$\therefore$' tests whether the context augmented with the premises support the conclusion. One respect in which discourses containing 'therefore' differ from Veltman (1985)'s conditional is that Veltman (1985) conditionals return the initial context after the test. But intuitively, an argument updates the context with the premises. For example, an argument with assertoric premises P after the checking must return the context updated with P. To see why this must be so, consider:

(30)   Paolo is from Turin$_i$. Therefore$_i$ he is from Piedmont$_j$. And, therefore$_j$ he is from Italy.

If in (30), 'therefore$_i$ he is from Piedmont$_j$ ' returned the context antecedent to the update with 'Paolo is in Turin$_i$', the output context might not support the proposition that Paolo is from Italy. So we cannot explain why (30) is a good argument. This observation motivates taking the entry for $\therefore$ to model this feature of 'therefore': $\therefore$ takes the current context (already updated with its antecedents) and returns that context if the test is positive. This explains why successive 'therefore' can test the context so updated with the earlier premises (see Kocurek and Pavese (2021) for a proposal on which the conditional test also returns the context updated with the antecedents, motivated by the need to model modal subordination under conditionals).

These entries allow to capture the similarities between necessity modals such as 'must' and 'necessarily' and 'therefore' that we have observed in the previous

section. On this proposal, one notable difference between 'therefore' and 'must' that is relevant for our purposes is that if the test fails, the former returns an undefined value rather than the empty set. This feature is needed to account for the different projective behavior of 'therefore', 'must' and the conditional. Conditionals and 'must' are not plausibly presupposition triggers. 'Must'-sentences, and in general sentences containing modals, do not need to presuppose that the context supports their prejacent. Consider:

(31)   a.   It is not the case that Mark is a progressive and must be from the North.
       b.   Is Mark a progressive and must be from the North?
       c.   If Mark is a progressive and must be from the North, he will not vote for Trump.
       d.   It might be that Mark is a progressive and must be from the North.

None of these convey that Mark's being from the North follows in any way from him being a progressive. Conditionals also do not project out when embedded in antecedent:

(32)   If Jen gets angry if irritated, you should not mock her.

(32) does not presuppose that Jen will get angry follows from her being irritated. 'Therefore' seems to differ from other tests such as conditionals and 'must' in that the checking is done by the presupposition triggered by 'therefore.'

'Therefore'-discourses are infelicitous if the checking is not positive, like in the case of 'must'-sentences and Veltman (1985)'s conditional. But in the case of 'therefore', the infelicity is due to *presupposition failure*. Because of its behavior as a presupposition trigger, it is more accurate to give 'therefore' a semantic entry similar to the one that Beaver 2001, 156–162 assigns to the presuppositional operator 'δ':

$$s[\delta\phi] = \begin{cases} s & \text{if } s[\phi] = s \\ \text{undefined} & \text{otherwise} \end{cases}$$

Compare $\square$ on one hand and δ and $\therefore$ on the other. They only differ in that the former returns the empty set if the context does not support ϕ, whereas the latter returns an undefined value. The difference between these two 'fail' values — undefinedness *versus* the empty set — is important. A semantic entry that returns the empty set receives a non-fail value under negation. But in order to account

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for the projection of the presupposition from a sentence containing 'therefore' to its negation, the negation of that sentence must also receive a fail value if the sentence does. Choosing 'undefined', rather than the empty set, gives the desired result here — i.e., that the negation of the sentence containing 'therefore' will also be undefined.

This analysis can be illustrated with the following example. Consider:

(33)   It's not the case that Mark is progressive and, therefore, from the North.

$$\neg(p \wedge \therefore n)$$

Compositionally, we get that the meaning of (33) is the following function:

$$
\begin{aligned}
s[\neg(p \wedge \therefore n)] &= s - s[p \wedge \therefore n] \\
&= s - s[p][\therefore n] \\
&= \begin{cases} s - s[p] & \text{if } s[p][n] = s[p] \\ \text{undefined} & \text{otherwise} \end{cases}
\end{aligned}
$$

13.6.2 Refining the Analysis: Supposition, Parenthetical, and Subarguments

While this analysis might be a good starting point, it is oversimplified in several ways. One way in which it is oversimplified is that it says nothing about how to model arguments that have not premises but other arguments as antecedents, such as conditional proofs:

(34)   Suppose Paolo is from Turin, Then he is from Piedmont. Therefore, if Paolo is from Turin he is from Piedmont.

Moreover, argumentative discourses seem to have a layered structure: suppositions introduce new states of information, at a different level from categorical states of information, and suppositions can be embedded to add further levels. For example, consider:

(35)   Paolo is either from Turin or from Madrid. Suppose$_1$, on the one hand, that he is from Turin. Then$_1$ either he did his PhD there or he did it in the US. Suppose$_{1.1}$ he did his PhD in Turin. Then$_{1.1}$, he studied Umberto Eco's work. Suppose$_{1.2}$ instead he did his PhD in the US. Then$_{1.2}$ he

studied linguistics. Therefore$_1$, he either did continental philosophy or philosophy of language. Now on the other hand, suppose$_2$ he is from Madrid. Then$_2$ he definitely did his PhD in the US. Therefore$_2$, he studied linguistics. Either way, therefore, he did either continental philosophy or philosophy of language.

As the indexes indicate, in (35), supposition$_1$ introduces a new layer, over and above the categorical context where 'Paolo is either from Turin or from Madrid'. Moreover, suppositions can be embedded one after the other (as supposition 1 and supposition 1.1) or might be independent (as supposition 1 and supposition 2). 'therefore' and 'then' might test the context introduced by the most recent premises or suppositions (as 'then$_2$' and 'therefore$_2$') or refer back to suppositions introduced earlier (as 'therefore$_1$'). Finally, after a supposition, parentheticals can be used to add information to the categorical level and to every level above. For example, consider:

(36)   Suppose Mary went to the grocery store this morning. [Have you been? It's a great store with great fruit.] She bought some fruit. Therefore, she can make a fruit salad.

To model the discourse in (36), we need to be able to exit the suppositional context, update the categorical context, and then return back to that suppositional context. In (36), however, the information added by the parenthetical to the categorical content seems to percolate up to the suppositional context too. Ideally, a theory of argumentative discourse ought to be able to account for these complexities. It seems that in order to model discourses such as (36), we need to refine Pavese (2017)'s analysis in some important ways.

Kocurek and Pavese (2021) propose we can model these data by adding structure both to the syntax of discourses as well as to the contexts used to interpret them. In order to capture the syntax of argumentative discourses such as the above, they propose we take discourses not just as sequences of sentences but rather as sequences of *labeled sentences*. A labeled sentence is a pair of the form $\langle n, \phi \rangle$, which we write as $n \colon \phi$ for short (Throughout, we use $\varnothing$ to stand for the empty tuple $\langle \rangle$). So parts of discourses are labeled sentences. Here, n is a label, which is a sequence of numbers (where, for shorthand, we write $\langle n_1, \ldots, n_k \rangle$ as $n_1.n_2.\ldots.n_k$) that represents which suppositions are active, and ϕ is a sentence. Labels enable to keep track of which suppositions are active when and to model the function of parentheticals of going back to the categorical contexts. So for example, the following is a representation of (36) with labeled sentences (where m

= 'Mary went to the grocery this morning'; g = 'Have you been? It's a great store with great fruit'; b = 'She bought some fruit'; f = 'She can make a fruit salad').

$$1\colon m, \quad \varnothing\colon g, \quad 1\colon b, \quad 1\colon \therefore f$$

The second move is to distinguish between the meaning of a sentence and the meaning of a part of a discourse — or labeled sentence. The meaning of a sentence is simply its update effect on information states — i.e., a function from information states to information states, as outlined in §6.1. This semantics would suffice if argumentative discourse did not have the layered structure we have seen it does have and if argument connectives did not license different anaphoric relations towards their antecedents. This further information is captured by parts of discourses or labeled sentences. So, in order to capture suppositional reasoning as well as the anaphoric relations that argument connectives establish in discourse, we ought to interpret labeled sentences as well. While the meaning of sentences is a function from information states to information states, the meaning of parts of discourses is its update effects on a context. Instead of modeling contexts as information states, Kocurek and Pavese (2021) model contexts rather as *labeled trees* — i.e., a tree where each node is an information state which is given its own label. Labeled trees contain much more structure than simple information states. They also contain more structure than stacks of information states of the sort proposed by Kaufmann (2000) to model suppositional reasoning. Labeled trees differ from stacks of information states in that (1) they allow non-linear branching, so that independent suppositions can be modeled at the same "level" as well as at different levels and (2) can model anaphoric relations, which will allow us to temporarily exit a suppositional context and later to return to that context. This also allows us to capture the distinctive ability of 'therefore' to be anaphoric on different suppositional contexts. A CONTEXT is a partial function $c\colon \mathbb{N}^{<\omega} \to \wp W$ from labels (i.e., sequences of numbers) to information states, where:

- $\varnothing \in \mathrm{dom}(c)$ (i.e., the categorical state is always defined);

- if $\langle n_1, \ldots, n_{k+1} \rangle \in \mathrm{dom}(c)$, then $\langle n_1, \ldots, n_k \rangle \in \mathrm{dom}(c)$ (i.e., a subsuppositional state is defined only when its parent suppositional state is defined).

The value of a context applied to the empty sequence is the CATEGORICAL STATE, denoted by $c_\varnothing$. The value of a context applied to a non-empty sequence is a SUPPOSITIONAL STATE. So for example, $n\colon \phi$ will tell us to update c_n with ϕ.

However, when we introduce a new supposition in a discourse, we don't simply update the current information state with that supposition (suppositions are not just assertions). Rather, we *create* a new information state updated with that supposition so that subsequent updates concern this new state as opposed to (say) the categorical state. The new supposition effectively copies the information state of its parent and then updates that state with the supposition.

Formalizing, where $n = \langle n_1, \ldots, n_{k+1} \rangle$ is a label, let $n^- = \langle n_1, \ldots, n_k \rangle$ ($\varnothing^-$ is undefined). This will allow us to keep track of which information state gets copied when a new supposition is introduced. For labels n and k, we write $n \sqsubseteq k$ just in case n is an initial segment of k and $n \sqsubset k$ just in case n is a proper initial segment of k (i.e., k is "above" n in the labeled tree). Where c is a context, let $c \uparrow_n \phi$ be the result of replacing c_k with $c_k[\phi]$ for each $k \in \text{dom}(c)$ such that $k \sqsupseteq n$ (i.e., $c \uparrow_n \phi$ updates c_n and all information states "above" c_n in the tree with ϕ). Finally, where s is an information state, let $c[n \mapsto s]$ be just like c except that $c_n = s$:

$$c[n \colon \phi] = \begin{cases} c \uparrow_n \phi & \text{if } c_n \text{ is defined} \\ c[n \mapsto c_{n^-}[\phi]] & \text{if } c_n \text{ is not defined but } c_{n^-} \text{ is defined} \\ \text{undefined} & \text{otherwise} \end{cases}$$

Unpacking this semantic clause: If c_n is defined, we update c_n and all subsequent states above it with ϕ. If $n = \varnothing$ (the categorical state), then every state that's currently defined is updated with ϕ. If $n = \langle n_1, \ldots, n_k \rangle$, then we only update states assigned to a label that starts with $n_1, \ldots, n_k$. If c_n is undefined, that means we're creating a new suppositional state:

- First, find the state whose label is right below n (so, e.g., if $n = \langle 1 \rangle$, then the label right below n is $\langle \rangle$, i.e., the label of the categorical state).

- Next, copy the state with that label and assign n to that state. Finally, update that copied state with ϕ.

This semantics for parts of discourses can be illustrated by considering two examples. Under a plausible interpretation, the following discourse is represented as the following sequence of labeled sentences:

(37)　　Either it is raining or not. Suppose it's raining. Then better to take the umbrella. Suppose it is not raining. Then, taking the umbrella will do no harm. Therefore, you should take the umbrella.

$$\varnothing\colon (r \vee \neg r), \quad 1\colon r, \quad 1\colon \therefore u, \quad 2\colon \neg r, \quad 2\colon \therefore u, \quad \varnothing\colon \therefore u$$

The dynamics of this discourse can be summarized as follows: First, we update the categorical state s with the trivial disjunction $r \vee \neg r$ (so no change). Next, $1\colon r$ requires setting $c_1 = s[r]$. Then $1\colon \therefore u$ tests $s[r][u] = s[r]$. If it passes, it returns $s[r]$ as c_1. Otherwise, the context is undefined. Assuming $s[r]$ passes the test, $2\colon \neg r$ requires defining a new information state $c_2 = s[\neg r]$. Then $2\colon \therefore u$ tests $s[\neg r][u] = s[\neg r]$. If it passes, it returns $s[\neg r]$ as c_2. Otherwise, the context is undefined. Assuming $s[\neg r]$ passes the test, $\therefore u$ tests $s[u] = s$. Since $s[r]$ and $s[\neg r]$ have passed this test, s will, too. Or consider the following example with a parenthetical:

(38)   Suppose Mary went to the grocery store this morning. [Have you been? It's a great store.] Then she bought some fruit. Therefore, she can make a fruit salad.

This is represented as:

$$1\colon m, \quad \varnothing\colon g, \quad 1\colon \therefore b, \quad 1\colon \therefore f$$

First, we introduce a suppositional context c_1 by copying s and updating it with $s[m]$. Next, $\varnothing\colon g$ updates both the categorical context s and the suppositional context $s[m]$ with g. Then $1\colon \therefore b$ tests $s[m][g][b] = s[m][g]$. If it passes, it returns $s[m][g]$ as c_1. Otherwise, the context crashes. Likewise for $1\colon \therefore f$.

13.6.3 Further Issues

The semantics for argumentative discourses outlined above can be extended to model modal subordination effects (see Kocurek and Pavese (2021)) as well as subjective arguments, though I do not have space to discuss these extensions. Let me conclude this discussion of the semantics of arguments by looking at some further open issues.

The dynamic analysis of argument connectives presented in the previous two sections takes argument connectives to be 'presuppositional' tests. On this analysis, a categorical argument is a matter of first asserting the premises and then drawing a conclusion from the premises, by presupposing that the conclusion follows from the premises. It might therefore seem as if arguments can never be informative. However, this conclusion is not correct, for presuppositions *can* be

informative. Suppose it is not known in the context that Pittsburgh is in Pennsylvania. The presupposition triggered by (39) is most likely to be accommodated in this context and this accommodation will result in restricting the context set, by ruling out possibilities where Pittsburgh is located in a state other than Pennsylvania:

(39)   John is in Pittsburgh. Therefore, John is in Pennsylvania.

Hence, although the presupposition associated with 'therefore' generally works as a test checking that the context satisfies certain constraints, just like other kinds of presuppositions, it can sometimes be informative (cf. Pavese (2021) for discussion of these issues and how they relate to the problem of deduction and Kocurek and Pavese (2021) for yet a different way to account for informative uses of 'therefore').

Arguments such as (39) sound weird to common speakers and so do arguments such as the following:

(40)   a.   Paris is in France. Therefore, either it is raining in Ecuador now or it is not.
         b.   Paris is in France. Therefore, if today is Wednesday then today is Wednesday.
         c.   Paris is in France. Therefore, if today is Wednesday, then Paris is in France.

Because they are all classically valid, and also sound, the current semantics cannot predict their infelicity. One might blame it on the pragmatics and allege that their weirdness has to do with their conclusions not being relevant to the premises. An alternative thought is, nonetheless, worth exploring. Notoriously, the weirdness of these patterns of inferences has motivated relevance logic (MacColl (1908); Belnap (1960); Anderson et al. (2017)). Argument connectives might test for relevantist, rather than classical, support.

As we have seen in §2, arguments can have non-declarative conclusions too. These kinds of arguments suggest that drawing a conclusion from certain premises can be a matter of checking that the context supports the conclusion even if the conclusion is not declarative.[11] Start with arguments with imperative conclusions,

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[11]It might be helpful to draw again a comparison with epistemic modals like 'must' and 'might'. Although not every use of these epistemic modals in the scope of questions is always felicitous (cf. Dorr and Hawthorne (2013)), many have observed that some uses of these modals are acceptable in questions. For example, Papafragou 2006, 1692 observes that the following

as in "Ψ; therefore, φ!". If imperatives express propositions, as on a propositional-ist semantics of imperatives (e.g., Lewis (1972); Aloni (2007); Schwager (2006)), modeling arguments with imperatival conclusions just amounts to testing that the context augmented with the premises supports the proposition expressed by the imperative. On an expressivist semantics for imperatives, instead, things are not so simple and modeling imperatival conclusions requiring thinking of information states as having more structure than just sets of possible worlds. For example, on a Starr (2020)'s preference semantics, context ought to be modeled as involving a set of preferences. On this semantics, testing for support of an imperative by the context amounts to testing that the preferences expressed by the imperatives are already in the context. Finally, consider how to model uses of 'therefore' that embed interrogatives, such as (12). Kocurek and Pavese (2021) propose we piggyback on recent dynamic theories, which take the change effect potential of interrogatives to be that of raising issues. Following Groenendijk et al. (2003) and Aloni et al. (2007), we can model this idea by thinking of an information state not as a set of possible worlds, but rather as a partition on possible worlds — i.e., as a set of mutually disjoint but jointly exhaustive sets, or *cells*. An interrogative

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exchange is felicitous:

(41)   a.   If it might rain tomorrow, people should take their umbrella.
        b.   But may it rain tomorrow?

Along similar lines, Hacquard and Wellwood 2012, 7 observe that the following interrogatives also have a distinctively epistemic interpretation:

(42)   a.   With the owners and the players on opposite sides philosophically and economi-
              cally, what might they talk about at the next bargaining session?
        b.   Might he be blackballed by all institutions of higher learning?

In this respect, then, 'therefore,' 'hence,' and 'so' resemble standard tests. There is an important difference between 'must' and 'might', on one hand, and 'therefore', 'hence', 'so', on the other. As we have seen, argument connectives can also tolerate imperative conclusions, whereas nei-ther 'might' nor 'must' can occur in imperatives (although the reason for this infelicity might be syntactic):

(43)   a.   ??Might go to the store!
        b.   ??Must go to the store!

   As Julien Schlöder pointed out to me (p.c.), "Maybe go to the store" is instead perfectly fine. See Incurvati and Schlöder (2019) for a helpful discussion of the differences between 'might', on one hand, and 'maybe' and 'perhaps' on the other. This sentence does have an acceptable reading, on which 'must' receives a deontic interpretation.

might refine the partition by dividing current cells into smaller subsets. So effectively, when using 'therefore' with an interrogative conclusion, we are testing that adding $?\phi$ would not further refine the partition.

13.7 The Pragmatics of Arguments

So much for the semantics of arguments. Onto the pragmatics. How are we to model the speech act of giving an argument? To begin, compare the following two discourses:

(44)   a.   It is raining. I conclude that the streets are wet.
        b.   It is raining. Therefore, the streets are wet.

Prima facie, these two discourses are equivalent. The locution "I conclude that..." seems to mark the speech act of concluding. It is tempting, then, to assimilate the meaning of 'therefore' to the meaning of 'I conclude that...'. On this analysis, argument connectives such as 'therefore' work as a *speech act modifier* — taking pairs of sentence types, into a distinctive kind of speech act — i.e., the speech act of giving an argument for a certain conclusion.[12]

One issue with this analysis is that argument connectives are not always used to make arguments. Consider again (45-a)-(45-d) from §2:

(45)   a.   John pushed Max. Therefore, Max fell.
        b.   John was desperate for financial reasons. Therefore, he killed himself.
        c.   Mary qualified for the exam. Therefore, she enrolled.
        d.   Max passed his A-levels. Therefore, he could go to the university.

While superficially, these discourses have the *same form* of an argument, they can be used to make other speech acts too. For example, one may utter, say, (45-a) without arguing for the conclusion that Max fell. In fact, the most common use of (45-a) is simply to explain what happened when John pushed Max (suppose (45-a) is used in the process of reporting what happened yesterday). In this use, the discourse does not necessarily have argumentative force. Rather, it uses 'therefore' *narratively* or *explanatorily*. Similarly for (45-b). Arguments and explanations

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[12]For example, some take epistemic modals such as "might" to be speech act modifiers in that they 'modulate' assertoric force. See for example, Westmoreland (1998) and Yalcin 2005, 251. Others argue that intonation is a speech act modifier. See Heim et al. (2016).

are different kinds of speech acts. That can be seen simply by observing that while an explanation might presuppose the truth of its explanandum, an argument cannot presuppose the truth of its conclusion, on pain of being question-begging. For example, one might use (45-a) in the course of an explanation of how Max fell, in a context where it is already common ground that Max fell. As used in this explanation, (45-a) is not the same as an argument.

It is also tempting to think that the causal uses are explanatory and not argumentative whereas the logical uses are argumentative but not explanatory. However, this cannot be correct, as there are causal and *yet* argumentative uses of 'therefore'. For example, consider **TRIAL**:

TRIAL   In a trial where John is accused of murdering his wife, the prosecutor argues for his conviction, as follows:

(46)   John was financially desperate, ruthless, and knew about his wife's savings. Therefore, he killed his wife to get her money.

The discourse (46) in **TRIAL** can undeniably be used in an argument — for example, an argument aiming to convince the jury of the fact that John has killed his wife. And yet the relation expressed by this use of 'therefore' is causal, if anything is.

There are also deductive uses of 'therefore' in *explanations*. For example, consider the following (Hempel (1962), Railton (1978)):

1 Whenever knees impact tables on which an inkwell sits and further conditions K are met (where K specifies that the impact is sufficiently forceful, etc.), the inkwell will tip over. (Reference to K is necessary since the impact of knees on table with inkwells does not always result in tipping.)

2 My knee impacted a table on which an inkwell sits and further conditions K are met.

Explanandum   Therefore, the inkwell tipped over.

In this explanation of why the inkwell tipped over, that the inkwell tipped over deductively follows from the premises. In this sense, there are logical uses of 'therefore' in explanations too.

The conclusion is that the distinction between argumentative uses of 'therefore' and explanatory uses of 'therefore' cuts across the distinction between causal

and logical meaning of 'therefore'. How are we to capture this distinction between argumentative uses of 'therefore' and explanatory uses of 'therefore'? This distinction might have to be captured not at the level of the semantics of arguments but rather at the level of the *pragmatics* of arguments. Chierchia and McConnell-Ginet (2000) have introduced an important distinction (then defended and elaborated by Murray and Starr (2018) and Murray and Starr (2020)) between CONVENTIONAL FORCE and UTTERANCE FORCE. The CONVENTIONAL FORCE of a sentence type consists in the distinctive ways different sentence types are used to change the context — e.g., declaratives are used to change the common ground, by adding a proposition to the common ground (Stalnaker (1978)); interrogatives affect the questions under discussion (e.g., Groenendijk and Stokhof (1982), Roberts (1996)) and imperatives the to do list (e.g., Portner (2004), Portner (2007), Starr (2020), Roberts (1996)). UTTERANCE FORCE, by contrast, consists in the distinctive ways *utterance types* change the context. This is the *total force* of an utterance, while the conventional force is the way a sentence's meaning constrains utterance force. Crucially, as Murray and Starr (2020) argue, conventional force underdetermines utterance force. For example, assertions are conventionally associated with declarative sentences. However, declarative sentences can also be used to make conjectures, to lie, to pretend, etc. So, while the conventional force of a speech act is conventionalized and can be modeled by looking at its invariant conversational effects on a public scoreboard, the utterance force of a speech act might vary depending on the effects of the speech act on the private mental states of the participants to the conversations as well as on the mental state of the utterer.

Suppose we apply this distinction between conventional force and utterance force to the case of argument connectives and discourses that feature them. The proposal then is that across all of its uses — causal, explanatory, as well as practical, inductive, deductive — argument connectives have the same conventional force. As we have seen, following Kocurek and Pavese (2021), the core meaning of argument connectives might be dynamic across the board: all uses of 'therefore' express that the premises in the context (logically, causally, nomologically, probabilistically) support the conclusion. However, in addition to argument connectives' having this dynamic meaning, uses of discourses with argument connectives come with a distinctive utterance force — in some cases with the force of an argument, in others with the force of an explanation. If that is correct, then the distinctive force of arguing *versus* explaining can be recovered at the level of argument connectives' utterance force.

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13.8 Conclusions

This chapter has overviewed recent studies on the semantics and pragmatics of arguments. From this discussion several issues emerge for further research. These include: How are we to think of the syntax of argumentative discourses and how are we to model contexts in order to model the dynamics of argumentative discourses? What consequences does the presuppositional nature of 'therefore' have on how to think of arguments? What is the nature of the support relation tested by argument connectives? How do we define entailment for arguments understood as sequences of labeled sentences? What makes a discourse an argument, rather than an explanation? At which level of linguistic analysis lies the difference between arguments and explanations? How are we to characterize the utterance force distinctive of arguments? Are there such things as zero-premises arguments in natural languages? How do deductive arguments in natural language differ, if at all, from proofs in natural deduction systems — such as Fitch's proofs? Although many of the issues pertaining the semantics and pragmatics of argumentation are left open for further research, I hope to have made a plausible case that they deserve attention since foundational questions concerning the nature of context and discourse, as well as their dynamics, turn on them.

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