Frege: Two Theses, Two Senses

CARLO PENCO
Department of Philosophy, University of Genova
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One particular topic in the literature on Frege's conception of sense relates to two apparently contradictory theses held by Frege: the isomorphism of thought and language on one hand and the expressibility of a thought by different sentences on the other. I will divide the paper into five sections. In (1) I introduce the problem of the tension in Frege's thought. In (2) I discuss the main attempts to resolve the conflict between Frege's two contradictory claims, showing what is wrong with some of them. In (3), I analyze where, in Frege's writings and discussions on sense identity, one can find grounds for two different conceptions of sense. In (4) I show how the two contradictory theses held by Frege are connected with different concerns, compelling Frege to a constant oscillation in terminology. In (5) I summarize two further reasons that prevented Frege from making the distinction between two conceptions of sense clear: (i) the antipsychologism problem and (ii) the overlap of traditions in German literature contemporary to Frege about the concept of value. I conclude with a hint for a reconstruction of the Fregean notion of 'thought' which resolves the contradiction between his two theses.

1. Introduction

From 1891 onwards, the concept of conceptual content defined in Frege's *Begriffsschrift* splits into three different notions: sense, reference and extension. However, Frege applied the tripartite distinction only to predicates; for names and sentences, he used only the distinction between sense and reference, identifying reference with extension. The concept of *thought* became constantly referred to as the *sense of a sentence.* The identification of the thought with the sense of a sentence hides a tension in Frege's conception of thought. This tension is manifested by his simultaneously holding two different claims, which at some point evidently clash. Frege held that a single thought can be expressed by different sentences, yet at the same time he held that a sentence is a picture of the thought it expresses. What's wrong with this position? Why can't we simply think of different sentences as different ways to picture one thought? An extreme interpretation of this idea - that a sentence is a picture of a thought - implies that there is a strict correspondence between the elements of the thought and the elements of the sentence. Such a view suggests that there must be one - and only one - analysis of a thought which is mirrored by a unique sentence; this is contrary to the idea that many different

sentences can express the same thought. Therefore, we reach here two contrasting views which can be expressed as follows:

[1] Structurally different sentences can express the same thought.
[2] The structure of a sentence (uniquely) reflects the structure of the thought it expresses.

Can the same notion of thought be used to support both claims? My answer is no. I will claim that Frege uses the term 'thought' or 'sense of a sentence' to answer different sets of problems (§ 4). To begin with, Frege was striving towards the clarification of sense as truth condition (as it has been developed by Wittgenstein and Carnap after him); at the same time, he had difficulty in isolating that notion clearly because a stronger conception of sense was required by the internal needs of his philosophy. The stronger conception of sense should take care of the epistemic aspects not explainable by the simpler conception of sense as truth condition. I will try to develop this intuitive and traditional line of interpretation1 against more sophisticated attempts to make Frege coherent at prices that I think are too high to pay.

2. Some Previous Explanations

A justification for Frege's simultaneously holding both theses [1] and [2] with a single concept of sense can be found in Dummett's claim that Frege uses two kinds of analysis2. The first aims at showing how a sentence can be decomposed in different ways, sorting *different* internal patterns (i.e. different choices regarding what counts as function and what as argument).3 The second is used to show that a sentence is the expression of a *unique* set of building blocks. This second thesis seems, however, to hold only for formalized languages4, while the first is also often applied to sentences in languages that have not yet been put in the frame of a formal semantic theory.5

Dummett's solution to the apparent contradiction between [1] and [2] can therefore be summarized as follows. Thesis [1] is needed to exp lain the way we reach new concepts through the discovery of different patterns in the same sentence. Once we have defined a

1 This more intuitive line has been hinted at by authors such as Shwayder (1976), Kitcher (1978), Hodes (1982), Currie (1982).
2 Basically these two kinds of analysis could be interpreted as function-argument analysis and whole-part analysis. We shall keep this distinction in the background. In general, see Dummett (1973), p.65; Dummett (1981), ch. 15. Briefly stated: while there are many *decompositions* of a sentence, there is only one *analysis* of it; and it is the analysis which is relevant to its sense. For this, to be faithful to Frege's intentions, there cannot be two sentences differing in essential structure but expressing the same sense (Dummett (1981), p.327). Dummett's claims have been challenged by Levine 2002. I will give here a different criticism of the consequences of Dummett's solution.
3 Frege has shown us the formation and fruitfulness of logical thinking: we may decompose a judgment (via extraction of functions) and pick a pattern common to many different sentences to arrive at a new concept. Once the pattern is picked up, we have to keep it as the common structure of different thoughts. The patterns discovered in logical proofs derive from our discernment of what is not immediately apparent, but was there to be discerned. See for instance Dummett (1981), p.277; Dummett (1991) pp.195-199.
4 The uniqueness of analysis of the sentence is relative to the semantic theory adopted; within that theory, the construction of each sentence must be unique, on pain of ambiguity (Dummett (1981), p.291)
5 For Frege, expressions such as <<there is at least one square root of 4>> or <<the concept square root of four is realized>> or <<the number 4 has the property that there is something of which it is the square>> are different sentences expressing the same thought. But when we decide to give a reconstruction in a quantificational language, the analysis will be unique: $\exists x \sqrt{4}x$ has one and only one analysis in constituents when considered as part of a formalized language that aims for a perspicuous representation of the pure thought.

logical language, where everything is explicit and the distinction in function and argument is clearly expressed, then for any sentence in our language there will correspond one -and only one- determinate thought, and consequently, thesis [2] firmly holds. We might call this last claim the thesis of logical isomorphism. It has a certain appeal with respect to the Fregean idea of revealing the real structure of the thought with logical notation. Even if basically correct, however, Dummett's solution has some evident disadvantages. On the one hand, it reduces the thesis of isomorphism of thought and language to a tautology, for if a given sentence is not considered isomorphic with the thought it expresses, then it is just a sentence which does not convey that thought in its most fundamental form. And in such circumstances, it is unclear how to decide which is *the* most fundamental form of the thought (see Beaney 1996, p. 242), unless we accept that (some) standard logical notation is exactly what is needed. On the other hand, even isomorphism intended in this way (as logical notation) is contradicted by other examples given by Frege: pairs of sentences like 'if A then B' and 'not (A and not B)' or 'A and B' and 'B and A' express the same thought. The same thought is therefore expressed in different ways in his logical formalism1. Beyond that, Dummett's solution has the disadvantage of making Frege too often wrong or guilty of changing his mind on basic points. I will give an example.

One of the main points of contrast among interpreters is what to do about the idea that each sentence in the following *Grundlagen* pairs expresses the same sense:

A1 *a* is parallel to *b.*

A2 The direction of *a* = the direction of *b.*

B1 There are as many *Fs* as *Gs.*

B2 The number of *Fs* is the same as the number of *Gs.*

To these we may add the corresponding Axiom V of *Grundgesetze*, which is composed by the pair B1/B2 in more technical jargon:

B1' For every *a (f(a) = g(a))*

B2' The value range of *f* = the value range of *g*

Dummett finds it very difficult to reconcile the idea that these pairs express the same sense with later ideas hold by Frege concerning the difference of senses2. Eventually Dummett claims that Frege was wrong in accepting the thesis that each sentence in the *Grundlagen* pairs expresses the same sense, and that he changed his mind, especially regarding Axiom V. According to Dummett, in the *Grundgesetze* Frege no longer accepted the idea that the two sides of Axiom V expressed the same sense. The rationale that Dummett gives for this

1 We may say, like Dummett (1981), p.332, that Frege believes that to grasp the sense of 'and' we have to recognize that the conjunction is commutative and idempotent. It is not clear, however, how much has to be recognized to grasp the sense of more complex formulas. This point regarding the problem of sense understanding will be developed later (§ 3, sec.5).
2 Dummett (1981), p.332

change is the transparency principle, which was clearly expressed by Frege in 1906 in an unpublished paper1.

Dummett labels as an 'aberration' the idea, maintained by Frege in a letter to Husserl, that two sentences express the same thought when we can derive a contradiction from the supposition that one is false and the other is true. Dummett correctly suggests that this criterion requires that two analytically equivalent sentences express the same thought, a view irreconcilable with Frege's ideas about sense as stated in other writings.2 This criterion does however hint at a definition of thought which is consistent with all of the passages by Frege in which Dummett claims that Frege was wrong or changed his mind. I therefore suggest that the appearance of this aberration may result from Dummett's too strongly emphasizing the notion of sense given in *Über Sinn und Bedeutung*. The feeling of aberration may disappear if we take into account a hidden tension between two different conceptions of sense that are present, even if not acknowledged, in Frege's work.

Dummett almost never considers the possibility of an inner tension in Frege's conception of thought; the only time he does so is when he criticizes Currie's attempt to discern two distinct theories or conceptions of sense in Frege's writings. Currie distinguishes between two notions in Frege: one definded by the information conveyed by a sentence and one used to justify the replacement of an intuitive notion by a precise one, as in the analytic definition of arithmetical notions used in the *Grundlagen* pairs. Dummett claims that the justification given by Currie is not plausible because it is hard to see how there could be any acceptable concept of sense according to which an imprecise expression could bear the very same sense of a precise one.3 However, a distinction between two theories similar to the one held by Currie can be sustained on other grounds, both theoretical and textual, and can clarify Frege's statements about the identity of sense in the *Grundlagen* pairs as well as help to solve the apparent contradictions in his writings. This is the project of my paper.

Before going into the details, I will review the arguments for a viable alternative to Dummett's solution. Some of the most interesting alternative solutions4 rely on a sharp divide between epistemological and non-epistemological points of view.

Currie 1985 maintains that thesis [1] is the fundamental thesis of Frege's theory insofar as it is an ontological thesis on the thought, arguing, i.e., that the thought is an independent object which can be expressed in different ways. The two sides of Axiom V are thus two ways of expressing the same thought, and no language would be ideal in which

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1 Dummett (1989), p.293. He is right, contrary to Sluga, in pointing out that Frege did not say in *Gg* that the two sides express the same sense, but neither did Frege deny it. See later at § 3, section 2.

2 Dummett (1981), p.325.

3 Dummett (1989), p.299. Currie's distinction is related to two different internal needs in Frege's philosophy of mathematics: (a) the problem of establishing identity of reference between numerical designators; (b) the problem of explaining the non-triviality of mathematical truths considered as analytical. Dummett probably agrees on the second point, while he disagrees on the first. The theory of strong sense which justifies (a) can be considered a method by which we can come to know that two expressions refer to the same object, thus ensuring conceptual continuity between ordinary arithmetic and his logical form, without invoking arithmetical proof, empirical procedures, or any direct intuition of the nature of numbers (Currie 1982, p.17).

4 Currie (1985), Garavaso (1991) and Bell (1997)

that principle could be not expressed (294). On the other hand, according to Currie, thesis [2] - and the idea that a thought is build on constituents which correspond, *by and large*, to the parts of the sentence - cannot be considered an ontological claim, but only an epistemological one. To *understand* a thought, we need to have some sentence whose structure expresses the structure of the thought.

The distinction between the epistemological and non-epistemological1 points of view in Frege's thought has an immediate merit. It helps in solving some of the typical problems about indirect contexts by establishing an intuitive difference between two ways of reporting beliefs. We should distinguish in general between thought expressed by a sentence and thought *as* expressed by a sentence; hence, we should distinguish between: 2

a) s believes the thought *as* expressed by p

b) s believes the thought expressed by p

Taken at face value, these two ways of reporting beliefs may be useful. We might say that if s believes the thought expressed by A B, she therefore believes the thought expressed by ¬ (A ¬ B). However, let us imagine that she is a beginner in a logic course: we cannot then say that, if she believes the thought *as* expressed by A B, she therefore believes the thought *as* expressed by ¬ (A ¬ B).

The main problem with this solution is that it drives us towards a very counter-intuitive interpretation of Frege. If further pursued without other restrictions, the distinction between epistemological and nonepistemological readings of thought brings about a conclusion that is both difficult to avoid and difficult to accept, i.e., that thoughts *per se*, being compatible with different analyses, do not possess a unique inner composition, and ultimately lack a definite structure. This conclusion is explicitly claimed in Garavaso 1991 and Bell 1996. Bell suggests that the best way to render Frege coherent is to make him unaware of the dilemma because the two claims pertain to different problems: [1] concerns the nature of thoughts and their relation to the language in which we express them, while [2] concerns the senses of the sentences and the nature of linguistic understanding.3 Bell's main argument is that Frege himself used two distinct models to account for the structure of thoughts: the function/argument model and the part/whole model. The part/whole model is fundamentally linked to thesis [2] and to the idea of sense and of hysomorphism between the structure of linguistic meaning and the structure of the sentences. The function/argument model, linked to thesis [1], implies that a thought can have different analyses in function and argument. Therefore - Bell claims - the function/argument analysis does *not* reveal the intrinsic structure of the thought. The price to be paid for establishing harmony between Frege's two apparently contradictory theses is that we are now compelled to deny that thoughts have a determinate intrinsic structure.

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1 The nonepistemological may be either ontological (Currie 1985) or logical (Garavaso 1991).
2 See Garavaso (1991), p.204; Beaney (1996), p.245.
3 Bell (1996) p.594

Bell's conclusion involves a split between conceptual content and linguistic meaning, where conceptual content is the unstructured thought and linguistic meaning is the structured sense of a linguistic expression. His solution is reminiscent of the one given in Wagner 1983 who, following Sluga, speaks of Fregean contents as unstructured bearers of truth values. Fregean contents are prior to propositions which are structures which the mind formulates by way of representing contents to itself. In fact, Bell's split must contend with at least three significant difficulties:

(i) an argument given by Frege and repeated by Dummett1 excludes the possibility of unstructured thoughts: properties and relations arise simultaneously with the first judgments in which they are ascribed to things. Therefore we cannot think of grasping a thought as grasping something without a structure. The content that we grasp is already articulated;

(ii) furthermore, Bell's assimilation of sense to linguistic meaning seems to run counter to the Fregean conception that senses are the ways in which to give the reference.This idea is what makes Frege's theory diverge from direct reference theories: senses are not 'Kaplan' characters, not something pertaining only to the form of expression types, but rather something like character *plus* context (as Frege suggests in speaking of indexicals);

(iii) ultimately, the conception of thought as unarticulated conceptual content does not fit the logical study of pure thought which was Frege's main concern. The idea of unarticulated conceptual content might perhaps be more applicable to a psychological study of mental processes, where thinking could be defined as the process of making a structure emerge from a previously unstructured (neural) network. If this is indeed a possible interpretation of unstructured thought, then we are here *very* far away from Frege's concern.

To summarize, I accept Dummett's criticism on thought as unstructured content. I also accept, however, contrary to Dummett, the idea that there is a hidden tension in Frege's conception of sense. I disagree with the specific solutions to this tension given by Currie and Bell. The first offers an interpretation too narrow for a sound conception of thought, and the second cuts too neatly between thought and sense, transforming thought into an unstructured element of the mind and sense into linguistic meaning2. In contrast with these claims I will suggest another way out, which is coherent with Frege's main tenets, even if it needs some revision of Frege's classification of sense and reference. I will begin with the textual evidence (part 3), and then I will try to show how Frege was partly unaware of two really contrasting uses of sense and thought found in his own work (part 4). I will then give some tentative conclusions.

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1 The argument is given in 'Booles rechnende Logik' (*NS*18-9) and expanded in Dummett (1989).

2 The difference between sense and linguistic meaning has been widely discussed by Burge (1990) which considers the reduction of senses to linguistic meanings one of the main mistakes in the interpretations of Frege. A simple example: the sense of 'today' is the same as the sense of 'yesterday' uttered in the context of the day after (Frege 1918, p.64). Still 'today' and 'yesterday' have different linguistic meanings.

3. Criteria for Identity

As Dummett 1981 suggests, we must scrutinize carefully the comparatively rare cases in which Frege positively asserts that distinct sentences express the same thought. These instances occur mostly in the scattered definitions of a criterion for identity, as well as in few other passages. Frege gave at least four different criteria for identity of thoughts:

(1) Criterion based on inferential potential (general substitutability).
(2) Criterion based on immediate recognition.
(3) Criterion based on logical equivalence.
(4) Criterion based on substitutability in indirect contexts.

(1) *Inferential potential (sense as inferential content).* This is the basic definition of sense-identity in the *Begriffsschrift* (§ 3), a (dynamic) definition that Frege never rejected. Even if two sentences in active and passive form have a slight difference in sense (*eine geringe Verschiedenheit des Sinnes*), they mostly give preponderant agreement in sense. Frege speaks of conceptual content as the part of the content which is the same in both. Two judgments have the same conceptual content if the conclusions that can be drawn from one when combined with certain others also always follow from the second when combined with the same judgments. What the two judgments have in common is their substitutability in inferences, or their inferential power, or the ability to preserve the same consequences. The idea of sense as inferential potential is the core of a definition of sense, or conceptual *content*, based on substitutability, as Brandom 1994 has abundantly shown.

Frege's definition of content, however, is given before the distinction is made between sense and reference and could be used for both1. Beaney (1996, ch.2.5) speaks of veritable content (reference or truth value) and logical content; the criterion could be clarified by saying that (a) two sentences have the same *veritable content* if they are materially equivalent and (b) two sentences have the same *logical content* if they are logically equivalent (provably materially equivalent). We might speak also of a third way to interpret criterion (1), i.e. (c) two sentences have the same *inferential content* if one can replace the other without rendering any correct proof incorrect.

If we take definition (b), we may consider it to be either a source for or a possible anticipation of sense as a truth condition. Marconi 1991 points out that if inferential potential is defined with respect to some logic L and L is complete with respect to some semantic S, then sense-as-inferential-potential is the same as sense-as-truth-conditions, where truth conditions are given in terms of S.

1 See also Picardi (1993).

There is, however, some problem here with the 'slight differences in sense'. Depending on which collateral judgments and peculiar interests we take into account, the slight difference in sense which Frege spoke about could become relevant for deduction. In that case, if we still wanted to identify conceptual content with logical equivalence, we should at least make logical equivalence relative to contexts of possible collateral judgments. Frege's 'small difference of sense' is typically considered in light of the reciprocal influence of speaker and listener. The relation between speakers is something Frege assumes not to be of concern to logic. However, if we enlarge the scope of logic from a representation of mathematical reasoning to a representation of common sense reasoning, we have to touch upon pragmatic problems of competence and communication (think of Gricean implicatures). We might therefore use the criterion of inferential potential to define a concept of equivalence more fine grained than that of logical equivalence. The general criterion stated in the *Begriffsschrift* should therefore be considered a programmatic one, which can have different levels of application and refinement.

(2) *Intuitive recognizability (transparency of sense)*. Criterion (2) is the following: two sentences which we understand express the same thought if we cannot understand a thought as expressed by one sentence without recognizing that it is the same as the thought expressed by another sentence.

This criterion is explicitly stated in *A Brief Survey of my Logical Doctrine* (1906, p.213) where Frege says that two sentences A and B have the same content or are equivalent when anyone who recognizes the content of A as true must straight away also recognize that of B as true, and conversely, anyone who recognizes the content of B must immediately recognize that of A. This criterion is based on the assumption that there is no difficulty in grasping the content of A or B. It is therefore a conditional principle, which could be expressed as: if somebody grasps the content of two expressions, she therefore recognizes whether they express the same thought. In this survey, Frege repeats his standard idea that the thought is the part of the content that is not affected by poetic aura (tone, coloring). This principle was reaffirmed in his latest writings (e.g. *Gedankengefüge*), where Frege speaks of the immediate recognition of sense identity concerning pairs of sentences like 'A' and 'not(notA)', or 'A and B' and 'B and A'. However, some ambiguity on Frege's side casts serious doubt on the reliability of this definition. In his paper *Die Verneinung*, Frege contradicts what is said in *Gedankengefüge* and treats 'A' and 'not(notA)' as sentences with different senses.1 Later, in *Logik in der Mathematik,* Frege seems to dismiss the principle (NS 227), saying that instead of analyzing a given

1 In *Gedankengefüge* (p. 444) Frege says that 'not(notB)' has the same sense as 'B', while in *Die Verneinung* he explicitly speaks of '*the two thoughts*, A and the negation of the negation of A'(p.157). If we have here *two* different thoughts (even if they necessarily have the same truth value), we have, by definitions, *two* different senses expressed by the two different sentences A and not(notA).

expression through a synonymous one, we can give a stipulation for using B instead of A. These oscillations on Frege's side cast some doubts on the reliability of this criterion.

However, even if not generally accepted in all stages of Frege's work, this criterion may explain Frege's claims for the identity of content in the *Grundlagen* pairs. In *Grundgesetze*, Frege warns that a dispute might break out over Axiom V because the two sides of the Axiom did not satisfy completely the requirement of intuitive recognisability. Dummett takes this remark as grounds for claiming that Frege no longer considered the two sides of the axiom to have the same sense or to express the same thought. But the fact that the two sides of axiom V do not have the same sense is not the problem here. They *should* have possessed the same sense *if* they worked.

(3) *Logical or analytical equivalence: towards sense as truth condition.* In paragraph 32 of *Grundgesetze*, entitled 'Every Sentence of *Begriffsschrift* Expresses a Thought', Frege says that (i) it is determined by stipulation under what conditions a sentence refers to the True; and (ii) the sense of a sentence, the thought, is the thought that these conditions are fulfilled. This passage has been perceived to be the forerunner to the concept of sense as truth condition developed by Wittgenstein and Carnap. Frege is referring to the stipulations of his primitive signs, and possibly to something analogous to meaning postulates.

If sense is intended as truth condition, sameness of sense is to be identified with logical equivalence. Actually Frege gave a definition of sameness of thought as logical equivalence in the first of two letters to Husserl in 1906. After stating that his notation did not represent mental processes, he defines thought as the content which is shared by equipollent sentences, disregarding the colouring or intonation (what elsewhere he calls tone, the heir to what in the *Begriffsschrift* are called 'slight differences in sense'). Then, after having hinted at a possible canonical form for a system of equipollent propositions, he poses the question whether A B and ¬ (A ¬ B) are equipollent. Frege displays the truth table of the conditional, then he shows that the *negation* of A B amounts to A ¬ B; therefore he concludes that the two sentences above are equipollent. Equipollence is here logical equivalence (identity of truth conditions). Given that equipollent sentences have been defined as sentences with the same thought, we may conclude that here Frege defines thought as truth condition, and that he uses logical equivalence as the criterion for identity of thoughts. Besides this, he demonstrates a general tendency to consider the transformation of logically equivalent sentences as sense preserving. In *Begründung meiner strengeren Grundsätze des Definierens* (1897), in a contraposition (such as the trasformation of A B into ¬ B ¬ A) 'the sense is ...hardly affected' because the transformed sentence 'gives no more and no less information than before' (NS 166). Still, we may note that the concept of information is relative to the speaker, and that from the

logical point of view, we have some information on the different complexity of the two formulas.1

In a second letter to Husserl in 1906, Frege presents sense identity as analytical equivalence (which we could express after Carnap as equivalence in all possible worlds depending on the stipulation of meaning postulates). As we have seen, Dummett thought this idea to be an 'aberration' in Frege's private correspondence. However, the clarity of the elucidations given by Frege prevents us from rapidly dismissing this criterion for logical equivalence or analytical equivalence which hints at a definition of sense that can be called 'semantic'.

The logical criterion for the identity of thoughts as truth conditions are possible explanations also for the remarks on inferential potential hinted at in the *Begriffsschrift* and for the remarks in *Gedankengefüge* on logically equivalent formulas with different logical operators. Analytical equivalence is also compatible with the concept of sameness of sense exhibited by the sentences of the *Grundlagen* pairs (our recognition of the identity of content in the *Grundlagen* pairs can be seen as recognition of their analytic equivalence, as a matter of stipulation).

(4) *Substitutability in indirect speech.* Identity of objects is given by truth-preserving substitutability of co-referring terms. An apparent counterexample to general truth-preserving substitutability is given by indirect speech. Frege exploits sense to explain indirect speech, with the consequence that he treats sense as that which is referred to in indirect speech. In this way, he preserves the general criterion for identity of objects (senses are objects which have to be identical in order to be substituted salva veritate in indirect speech). However, Frege gives no example of substitutability of identical senses in indirect speech, but only counterexamples to substitutability such as:

from: *s* believes that the morning star is a planet
you cannot derive: *s* believes that the evening star is a planet

The counterexamples bring us to the *intuitive criterion of difference of thoughts*: if it is possible to understand two sentences and coherently believe what one expresses while not believing what the other expresses, then those sentences express different senses (different thoughts). The main novelty of this intuitive criterion is that it links expressly the definition of sense to the cognitive capacities of the speakers: therefore, this notion of sense can be called cognitive or epistemic.2

1 Actually, the passage quoted above is related to a discussion on the difference between Frege's view and Peano's. Frege shows that if we follow Peano, with admissible transformations given by contraposition, the referents of the letters may be modified with damage to logical clarity. The general point that there is a slight different in sense even in logically equivalent formulas is developed in Penco 2002. Here it is shown that Frege *could* have treated expressions like A B and ¬ B ¬ A as differing in sense, if he had used - instead of using a conception of absolute rationality in logic - a conception of limited rationality.

d 2 Calling it 'cognitive' might cause confusion because the conception of semantic sense also has a cognitive aspect (see later at § 4). The 'locus classicus' of the intuitive criterion of the difference of thoughts is Evans (1982, p.18) commenting on Frege 1892. The formulation has also been called 'rational cotenability'. See Sainsbury 1999 and a discussion on interpretations of rational cotenability in Penco 2002.

While never formulated by Frege, a criterion of identity of sense based on substitutability in indirect contexts could be formulated as the following: two sentences express the same sense if they are substitutable in indirect contexts *salva veritate*. Frege's discussion of indirect speech results from the epistemological concerns for which he wanted the notion of sense, i.e., the fruitfulness of mathematical truths. The example of Hesperus and Phosphorus is a shorthand way to show that numerical equations are not boring examples of the principle of identity; on the contrary numerical equations give new knowledge. At the same time, the principle of substitutivity in indirect speech could be interpreted as a case against the attribution of sense identity of the *Grundlagen* pairs or of pairs of logically equivalent formal sentences. This is clear if the sentences are complex enough that a speaker who can be said to understand both may miss their logical equivalence. The criterion of substitutability in indirect speech is therefore certainly more fine-grained than the other criteria hinted at in Frege's writings.

(5) *Relationships Among the Criteria*

Taken together, the four criteria discussed above are not consistent, and each of them has its own difficulties. We have seen that the first criterion is vague enough to give rise to different interpretations. I will describe in more detail some of the difficulties with the other criteria.

The second criterion of *immediate recognizability* does not conflict *per se* with the conception of thought as truth condition if we take into account the restriction given in (2) that we have no difficulty in grasping the senses. In fact, we may recognize immediately two sentences as logically equivalent *if* we grasp their respective truth conditions. This interpretation also fits the examples given by Frege when he speaks about the composition of thoughts. Why do we immediately grasp the identity of the thoughts 'A B' and 'B A' or 'A B' and '¬ A B'? An answer could be that it is easy for us to grasp their truth conditions, therefore it is easy to see that the two sentences express the same truth condition and therefore the same sense. This step does not apply in the case of empirical propositions, where it is not immediately recognizable that they express the same truth condition ('Hesperus is a planet' and 'Phosphorus is a planet').

However, in speaking of sense as linked to *immediate recognition* (2), we are always in the empirical realm: people often do not know, are unaware of, or find it difficult to realize completely the sense of an expression. Frege himself refers often to this problem: even mathematicians have difficulty in acknowledging the sense of some complex mathematical operations or functions. This point poses a problem about transparency of senses.1 If we admit that we are often unable to grasp senses *fully*, how can we recognize

1 The contrast between claims about *easiness* and claims about *difficulty* of grasping a sense in Frege's writings has been discussed by Burge (1990) and criticized by Kemp (1996) (see also Stjernberg *forth*). For a discussion of problems about transparency with respect

sense identity? If it is so easy for a speaker to have difficulty in fully comprehending the sense of some expressions, how can we say that immediate recognizability is a suitable criterion for identity of sense? We could overcome this problem if we were to apply intuitive recognizability to ideal speakers and to consider sense in terms of that which is immediately evident to an ideal mind (to *the* mind, not to minds).1 This last solution fits together nicely with Frege's main views of psychological problems, and helps us to avoid the risk of giving too much emphasis to psychological questions. The solution of the ideal mind is compatible, even if not identical, with a normative attitude about sense identity: we might say that two sentences with the same sense have such a structure that *if* we recognize the first as true, then we *must* recognize as true also the second.2

Both ideal mind recognizability and normative recognizability have some disadvantages. Some of Frege's examples apparently rely on the *human* ability to make immediate substitutions, like in the cases of 'A and B' and 'B and A'. Here it seems that Frege wanted to make us think of sense recognition as linked to the most elementary patterns of substitutions *we* are able to make in a chain of reasoning. Besides, immediate recognizability is not a viable criterion unless ideal or normative recognition can be expressed by a computational procedure: we have to know which kinds of computation we should perform if we (or the ideal mind or machine) are to make the appropriate substitutions that result from immediate recognition. We are then back to the problem of the structure of substitutability.

The criterion of *substitutability in indirect speech* (4) seems to be a specification of the substitution strategy that lies at the heart of the general definition (1). It is, however, not easy to make criterion (4) match the other two remaining criteria. We cannot use (4) together with the conception of sense identity as logical equivalence (3): we would reach the unwanted conclusion of the speaker's logical omniscience - a problem which has caused so much trouble to the philosophical community after Frege. *If* we identify sense identity with logical equivalence *and*, at the same time, allow the substitution of logically equivalent sentences in indirect contexts, we fall into the standard argument: if s knows that 2+2=4 and 2+2=4 is logically equivalent to any other mathematical or logical truth, then s knows every mathematical or logical truth. Hence, we would have to claim that every speaker who knows or believes a logical or mathematical truth also knows or believes all logical or

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to *de re* sense, see Brandom (1994), ch.8. The topic of transparency also poses a problem with the treatment of Frege's analyticity presented by Boghossian (1996). Here we need the speaker's full awareness of senses in order to have analyticity.

1 Think of the way Frege in *Der Gedanke* (p.74) describes the task of logic and mathematics as the investigation of *the* mind; of *the* mind, not of minds. Stjernberg (*types.*) and Stuhlmann-Laeisz 1995 give a solution of this sort, using the notion of an ideal subject or mind which makes the logical criterion compatible and easily transformed into an (ideal) epistemic criterion: sentence A expresses the same thought as sentence B if and only if an ideal subject cannot believe that A is true and at the same time believe that B is false (and vice versa).

2 As Grassia (*ty pes.*) suggests, we have two different ways to interpret the Fregean principle of immediate recognizability as (1) two sentences are synonymous iff every competent speaker who recognizes one of them as true immediately recognizes the other as true too; (2) two sentences are synonymous iff they are so related that any speaker who recognizes one of them as true *must* recognize the other as true too. For Grassia it is the relation of analytic coextensionality that *binds* the speakers to recognize two sentences as synonymous.

mathematical truths. Might we conclude again that Frege's logical system represents Gods viewpoint?

An ideal mind or that of God does not suffice to solve this problem, because the criterion of *sameness of sense as logical equivalence* apparently clashes with another relevant point. From the point of view of Frege's logicism, true mathematical equations are logically equivalent. However true mathematical equations, even $2^2=4$ and $2+2=4$, have different senses or express different thoughts. The passages in which Frege says they have different senses are so abundant1 that we feel almost compelled to reject the logical equivalence criterion as generally acceptable.

We might then adhere more strictly to the criterion of substitutivity in indirect contexts. However, if we want to make *this* criterion more precise, we have more than one difficulty. Two sentences could be said to have the same sense if they have the same logical form and their subsentential parts have the same sense. Which is the criterion of sameness of sense of subsentential parts? Dummett suggests that there probably is no sharp criterion for sameness of sense short of the rather boring notion of intensional isomorphism.2 Whether boring or not, even intensional isomorphism has its flaws when propositional attitudes are concerned, as Mates' puzzle shows. Mates' puzzle could, however, be resolved by the application of a Fregean (conditional) Transparency Principle: *if* we grasp the senses of two expressions, we always recognize whether these expressions have the same sense. It is apparent that failures in sense recognition bring about failures in substitutability; but we may not be interested in these kinds of failures.3 Therefore, the criterion of substitutability in indirect speech could still be used as a viable criterion of sense identity (throught something like intensional isomorphism), given a presupposed notion of sense recognition (or the ability to grasp the sense). In this respect, the criterion of substitutability is similar to the criterion of immediate recognition: both depend on the concept of sense understanding.

Dummett claims that the problem of sense understanding is the main problem in Frege, and that we should set aside the problem of synonymy to concentrate on the problem of grasping the sense. If we concentrate on the problem of sense understanding, however, it seems really difficult to give an objective representation of the structure of thought, unless we do so in terms of one's ability to make the right substitutions. We end up running in a circle: to have a notion of grasping the sense, we need a notion of correct substitutions. But, as we have already seen, to have a viable notion of substitution, we need a notion of grasping the sense.

This circle could be broken in two different ways: the first is to follow a Wittgensteinian strategy, defining grasping the sense as the ability to follow rules, with no

1 Both in published works (as in Frege 1891, p.13, Frege 1893, § 2) and in letters to Peano and Russell.
2 Dummett (1991), p.341.
3 This idea could be developed using Kripke's hints about a Fregean treatment of Mates (see Kripke 1975, note 15).

need to have explicit acknowledgement either of intuition or of substitutivity of synonyms.1 The second is to give careful attention to the *degrees* and *contexts* of substitutability. We might think of the criterion of substitutability as a criterion that gives degrees of the sameness of sense, defined as substitutability in kinds of contexts. We should be content with a criterion that permits us to indefinitely specify more and more refined conceptions of sense-identity. In fact, perfect synonymy as substitutability in every context does not exist, and we have before us cases that range from ideal perfect substitutability and occasional substitutability. We might therefore conclude, as Beaney suggests, that we should consider as many notions of meaning as there are distinguishable types of contexts in which substitutions of linguistic expressions occur.2

In fact, even the assumption that senses are context-dependent entities does not permit us to avoid the need for distinguishing between the two main aspects of sense hinted at by Frege in his attempt to give criteria of sense identity: the semantic one and the epistemic one. Brandom 1986 suggested that, after Kripke's and Putnam's analyses, the most urgent problem is the investigation of the interrelation between these two kinds of senses. The investigations that have been pursued thus far, as developed in dual theories of meaning, have separated the two aspects of sense into two separate components, one dealing with the world and the other dealing with the mind. Probably, as Putnam said, no unique concept may account both for the task of fixing the reference and for that of giving a synonym. We are not bound, however, to have two separate concepts, heirs of the Cartesian divide,3 but might find within the tensions of Frege's conception of sense a unifying answer. To respond to Brandom's suggestion in a Fregean setting means to solve the problem of understanding the internal link between sense as truth condition and sense as epistemic or cognitive content. Once we have clarified the connection between these two levels of sense, we might develop further analyses that move towards formulating contextual restrictions to the application of the principle of compositionality.

4. Constant Oscillation between Different Concerns

1 I will not follow this line of thought here, just hint at it. The clash between immediate recognition and substitutivity deals with the contrast between intuition and proof. Immediate recognition of sameness of sense implies substitutivity, but also vice-versa: we have always to resort to some kind of pattern recognition to ascertain whether some substitution is allowed. Hilbert spoke of the intuition of symbols, and so did Wittgenstein in the *Tractatus.* In the *Remarks on the Foundation of Mathematics,* Wittgenstein discussed the surveyability of the proof and the problem of pattern recognition. Frege, in *Gedankengefüge* (p. 39), thinks of the criteria of immediate recognition as referring to the most elementary steps in a proof: that 'A and B' has the same sense as 'B and A' we may see without proof by merely being aware of the sense. In fact, what Frege says here implies that at least in most of the more complex cases we need a proof to check whether two sentences have the same sense. And a proof is made of individual steps of which each should be surveyable and immediately recognizable. However, if we take the conception of sense as truth condition and truth table methods as one example of such a proof, we cannot avoid the conclusion that all logical truths of propositional calculus have the same sense. And certainly this was not a welcome result of Frege's notion of pure thought, whose purpose was also to distinguish parts inside the realm of the True. The fact that Frege did not pursue coherently this analysis of sense and meaning probably helped to nourish Wittgensteins work on the problem of immediate recognition in the elementary steps of proofs. This problem, summed up by Brouwer's discussion on intuition at each step of a proof, eventually brought Wittgenstein to develop the argument of rule-following. For a discussion of the abandonment of pattern-recognition analysis in Wittgenstein, see Penco (1994).

2 Beaney (1996), p.258

3 See McDowell (1986).

Frege oscillates between different criteria of sense as a result of the different purposes and concerns for which he wants the notion of sense. However, speaking of semantic vs. cognitive (epistemic) content does not amount to saying that semantic content does not regard cognition. We have two different concerns regarding cognition, linked to two distinctions:

1) the distinction of cognitive vs. emotive (tone);

2) the distinction of cognitive value vs. semantic value.

If we want to clarify the two different notions of sense in Frege, we need to follow each of these concerns. Thesis [1] or the claim that structurally different sentences may express the same thought is consistent with the idea of sense as truth condition and with the criteria of identity of thought based on logical equivalence. Arguments supporting thesis [1] appear most often in the context of the distinction between sense and tone, and, in particular, of the idea that different sentences in natural language may have the same logical consequences. Formal languages can express in a single formula different equivalent sentences of natural language. To say that two expressions in natural and formal language have the same sense is different than saying that the sense is imprecisely contained in natural language expressions and fully expressed in formal languages. Both formulations, in natural and formal language, admit an interpretation in terms of sense identity as logical or analytical equivalence. Using an example by Beaney, we may say that no expert who knows that NaCl+H2O = Na+Cl+H2O will reject the truth of the natural language sentence <<salt dissolves in water>> and vice versa. Following the criterion fo analytical equivalence (3) we may say that these two expressions express the same thought.

The step from this point to the treatment of different expressions in formal language is given in Frege's first 1906 letter to Husserl where he says that we may find some canonical or normal formula which could give in a single expression a set of equivalent logical formulas. Frege speaks of the pragmatic utility of having different equivalent sentences of a formal language: a particular formulation may be more useful than another for helping people in deduction. Different expressions, both in natural and in formal language, may express the same thought. This thought has an inner structure, which may be well represented by what later has been called truth condition (Wittgenstein) or intension (Carnap). This assessment of sense identity as logical equivalence, far from being an aberration as Dummett suggested, is not only consistent with Frege's work, but also with the tradition following Frege.

Certainly Wittgenstein, who prided himself on being able to recite by heart entire sections of Frege's writings, didn't miss the passage in *Grundgesetze* par.32. Here Frege says that the sense of a sentence is the thought that the truth conditions are fulfilled and that to know the sense you have to know the truth conditions which have to be fulfilled. If Wittgenstein was the first to define explicitly sense as truth condition, Carnap, who followed Frege's lectures in Jena, developed the idea that cognitive meaning is truth

conditional meaning, or intension. Carnap was a pupil of Frege and his notes from the lectures held by Frege in Jena in 1910-1913 show the deep influence of Frege on his thought1. In *Meaning and Necessity*, Carnap eventually defines intension as 'cognitive meaning'. Even if the term could have been more suitable for defining intensional structures, we have to remark that the main debate at the time was still between cognitive vs. emotive, as Carnap mentions at the beginning of his book, a contrast which strongly resembles the contrast between sense and tone in Frege. The first development of Frege's ideas - the definition of sense as truth condition - was based on an aspect of his thought which was certainly well rooted in his writings; this aspect however has never been made completely explicit by him because of the contrast with another aspect, related to a more fine-grained definition of sense.

What then about thesis [2]? In the early period of logic and philosophy of language, ideas grew interconnectedly, so that it now requires time and caution to disentangle the contributions of different philosophers. It is tempting to think that Frege's tendency to reason in terms of the isomorphism between thought and language is a late influence from his former pupil Wittgenstein. Frege read the *Tractatus* during World War I, before writing the essay *Gedankengefüge* (1923), in which he states his major proposition on isomorphism.2 But Frege's strict correlation here between linguistic expression and sense once again matches very well with his former analyses of the difference between thoughts expressed by different mathematical propositions. Frege's building block theory of the composition of senses antedates his reading of the *Tractatus* and we may easily see that most of Frege's suggestions on thought-language isomorphism are scattered throughout his writings.

Thesis [2], or the claim that to any component in the sentence there is a correspondent component in the thought, is coherent with the idea of sense as epistemic value and with the stricter criterion of substitutability in indirect speech. Here, for instance, A B and ¬ (A ¬ B) may be considered to have different cognitive values. A speaker may believe that A B and disbelieve that ¬ (A ¬ B). Given the intuitive criterion of difference of thoughts, the two sentences have different senses.3 This is contrary to the idea that, being logically equivalent, the two sentences should express the same thought. However, discussions that leads towards the idea of thought equivalence as logical equivalence never encounter the problem of the cognitive value of identity statements. This last problem normally appears within discussions that clarify the distinction between sense and reference and justify the informativeness of assertions of identity. When Frege says that two sentences with the same truth-value may express *different* thoughts, i.e. different

1 See the notes taken by Carnap and edited by Gabriel (here:Frege 1996).
2 WB p.67. Frege read Wittgenstein's *Tractatus* while submitting his paper 'Der Gedanke' for publication (actually, he wrote to Wittgenstein that he couldn't get beyond the first definitions fo the book, but we may assume that at least he gave a look at it). His terminology itself is reminiscent of Wittgenstein's.
3 A further argument on limited rationality should be required for the rationality of simultaneous holding the two basic sentences. See on that Penco 2002.

senses, he most often raises the problem of changes in sense provoked by substitutions of co-referential elements in a sentence. When he speaks of *identity* of thoughts, he is almost always preoccupied with the other concerns relating to the difference of sense and tone we have discussed above. There are therefore two lines of preoccupation which lead to the contemplation of two different aspects of the sense of a sentence; these two lines of preoccupation are typically given in different contexts - even if in the same paper**.** They don't clash with one another explicitly; another reason for this is the still embryonic state of the characterization of sense (and therefore thought) as truth condition.

The differentiation between semantic and epistemic conceptions of sense does not need to be reduced to a difference of ontological vs. epistemological points of view. The epistemic conception of sense reveals an *intrinsic* aspect of thought, i.e. the different ways in which a thought can be expressed: it is essential to the thought that it be thinkable in different ways. We may devise canonical forms to express a class of logically equivalent expressions. Pragmatic considerations may press us to devise and use different formulations, but these pragmatic considerations rely on an intrinsic aspect of the structure of thought. A thought has *per se* different ways of being expressed, all of them relying on the basic structure expressed by the truth condition. A thought - as truth condition - has always to be taken together with the different procedures through which we arrive at determining the truth condition. These procedures may be considered to be the accessibility conditions of thoughts. From a Fregean point of view, we might say that these procedures do not depend on human beings, but are intrinsic to the thought.

Summarizing: the oscillation in Frege's theory between criteria of identity and difference of senses reflects an internal tension between two different conceptions of sense whose contrast Frege never became aware of; there were reasons, however, why he was unaware of the tension. First of all, he treated each of these two conceptions of sense in different contexts which responded to distinct concerns.1 I have tried to give here some suggestions for the ways in which we might see his discussion of sense as encapsulated in different contexts and concerns that prevented him from recognizing the possibility of contradictions that would have arisen if he had compared the different principles underlying his definitions of thought as sense of a sentence. I will now consider some further motivations for Frege's inability to resolve the tension caused by a possible contradiction in his claims and suggest a way in which his system could have developed a clear distinction to solve the claim.

5. Why Frege Didn't Get It Right: Hints Toward a Reconstruction of Fregean Thought

1 Picardi 1993 has insisted on the many different preoccupations that lie behind Fregean attempts to give a criterion for identity of thoughts. We might distinguish in this way different categories of synonymous pairs of sentences. Think for instance of synonymous sentences which (i) differ in tone, (ii) differ in grammatical construction, (iii) are paraphrases or elucidations (iv) are logical transformations. I follow here a more basic distinction, which is compatible with a more detailed analysis of these differences.

A criterion of identity between thoughts that is more fine-grained than logical equivalence is not easy to give. In his first 1906 letter to Husserl, after having defined logical equivalence (equipollence) as the criterion of identity of thoughts, Frege hints at the problem of a more restricted idea of equivalence: the problem of congruence between thoughts.1 But he says that it is not possible to draw a clear and recognizable line between merely equipollent and congruent propositions. Even sentences which appear congruent when presented in print can be pronounced with a different intonation and are not therefore equivalent in every respect [WB 67]. How may we interpret the idea of congruent sentences, sentences equivalent in every respect, or sentences which express *exactly* the same thought? Shall we look for something more fine-grained than logical equivalence? Or are logically equivalent expressions also congruent? Frege raises some doubts on the subject: 'this could be debated for a hundred years or more. At least I do not see what criterion would allow us to decide this question objectively'. Maybe this skepticism explains why he never stated any precise criterion of identity for his epistemic conception of sense, but suggested only an intuitive criterion of difference.

The problem of the definition of a relation of identity more fine grained than logical equivalence has been debated almost a hundred years. The distinctions between intension and intensional structure (Carnap), between equivalence and strong equivalence (Barwise and Perry), between content and character (Kaplan), between sense and thought (Perry), between structured propositions and modes of presentation (Schiffer), between truth conditional content and representational content (Kamp), between ingredient sense and assertoric content (Dummett) and between subjunctive intensions and epistemic intensions (Chalmers) can be interpreted as variations on this theme.2

Evans 1982, McDowell 1984, and Recanati 1993 have suggested that direct reference theories can be considered variants of the Fregean theory or at least that their difference is more superficial than substantive. A deeper continuity in the recent history of philosophy of language could be better understood by realizing how these attempts to characterize a multiplicity of levels of meaning are dealing with and explicilty expressing

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1 See Frege's first letter to Husserl in 1906 in WB, passim. In *Grundlagen* par.65, p.76 Frege, referring to the difference between *dasselbe* (the same) and *gleich* (equal), comments: *the same* may appear to express complete agreement, *equal* only agreement in this or that respect. Soon Frege leaves the question of the difference between the two terms unanswered, and relies on Universal Substitutability which contains all the laws of identity. This solution could be considered another clue as to why he chooses to give no answer to the problem of giving a criterion stricter than logical equivalence. Maybe this is also a reason why he never stated any precise criterion of *identity* for his epistemic conception of sense (congruence?), but suggested only an intuitive criterion of *difference*. For these reasons, it is difficult to present a clear Fregean idea
of substitutivity of synonyms, as required for Boghossians Fregean Analyticity.

2 A proliferation of distinctions of different levels of sense has also developed within direct reference theories - see Almog (1984), Corazza (1995), Hahn (1995), Braun (1996). Perry (1977) suggested, after Kaplan's distinction between character and content, making a break between 'sense' - which is entertained - and 'thought' - which is apprehended. Perry's distinction and some of his arguments are one source for what I have been discussing here. The conception of thought entertained by Perry looks too unfregean, however, to be proposed as a development of a Fregean theory of thought, and his theory of sense in that paper tends to reduce sense to linguistic meaning. Later (2001), Perry develops a different strategy, distinguishing three levels of analysis of content, which he finds implicit in *Begriffsschrift* and abolished in *Über Sinn un Bedeutung*. In *Über Sinn un Bedeutung*, we have Truth value at the level of *Bedeutung* and thought at the level of *Sinn*. What disappears from the scene is the subject matter, the situation represented, or how the things would be if the sentence were true. Actually Perry speaks of three kinds of propositions: the mode-of-designation proposition, the subject-matter proposition and the mode-of-presentation proposition. I will not follow his terminology here in order to stay within the Fregean terminology, up to a certain point.

Frege's hidden ambiguity on the notion of thought as sense of a sentence. We are still searching for a unifying theory that could clarify the connection between the semantic and the epistemic aspects of the Fregean notion of thought1.

Why didn't Frege realize that his conception of thought as sense of a sentence was oscillating between the concept of a general structure (common to many logically equivalent sentences) and the idea of a more fine-grained structure that made these sentences different from another? In addition to the arguments given in the previous paragraph, I would like to suggest two other reasons:

(i) *The antipsychologism problem.* Frege would have probably been reluctant to develop an explicit epistemic notion of sense because its psychologistic aspects might have collided with the anti-psychologistic tenets of his theory. In fact, in a proper definition of epistemic sense we should take into account the psychological limitations of a speaker's mind. As noted before Frege eventually wanted to give a logical theory of *the* mind, not a psychological theory of minds. Frege would not have liked a too strict connection between his conception of sense and the psychological processes of the speaker. For that reason, he maintained that grasping a sense is a 'mysterious' process in which the subjective comes in contact with the objective2. Furthermore, he claimed that a property of a thought will be called inessential if it consists in, or follows from, the fact that this thought is grasped by a thinker.3 Thoughts, although they can be grasped by the minds of single speakers, are completely autonomous of such mental processes; therefore, their definition must be independent of the different ways that speakers use in grasping them.

But is it really necessary to get involved in the working of actual mental processes in order to define epistemic senses? We might just speak, instead, of the different ways a thought can be expressed, or the different ways a mathematical proposition can be computed. This was how Frege used to explain the difference between sense and reference in his letters to Peano and Russell. There is a cognitive value in the fact that *different* computational structures such as 7=7 and 52 211-4/753 = 7 give the same Truth Value. They not only give the same truth value, but - assuming the logicistic reduction of arithmetic to logic - they are also logically equivalent, that is they have the same truth condition (we would say now that they are true in all possible worlds). What then are these computational structures? Certainly not mental processes, even if they may be reproduced by mental processes.

The cognitive or epistemic sense as represented or given by computational procedures characterizes the difference between logically equivalent expressions like the ones given above. But this approach can be easily extended to other kinds of sentences, and we may say that the procedure to compute A B is different from the procedure to

1 A discussion of the search for a unified theory (and a criticism of the attempt made by Chalmers) is given in Marconi, *forth*.
2 As in a famous passage in *Logik*, in NS, p.157:'und vielleicht is dieser Vorgang der geheimnisvollte von alle.'
3 *Der Gedanke*, p.76

compute A B; we have here a clear example of the same semantic sense (the same thought expressed by the two sentences) and different epistemic sense (the two ways to arrive at that same thought). We may conclude that a distinction between thought and thinking process may be represented in an objective way. The thinking processes may be considered processes of an ideal mind and therefore free from the Fregean criticism of psychologism; they are not necessarily derived from subjective processes, but are abstract procedures, even if some of them may have psycholological plausibility. The apparent ambiguity in Frege's conception of thought as the sense of a sentence can be explained away if we distinguish thought as semantic sense and thinking procedures as epistemic sense.

(ii) *The choice of truth value as Bedeutung of sentences*. The definition of the *Bedeutung* of a sentence as its truth value, although an important step in semantics, leaves too much conceptual space open for the notion of sense. This does not happen in respect to predicates. In a letter to Husserl written in 1891, Frege uses a tripartite distinction of *Sinn*, *Bedeutung* and *Extension* with respect to predicates. Why did he decide to identify *Bedeutung* and *Extension* when speaking of sentences? An interesting answer is suggested in Gabriel 1984. Gabriel analyses the ambiguity of the conception of value in the Fregean writings, starting from the source of the term, which comes from both mathematics and value theory in nineteenth-century philosophy. In overlapping these two traditions, Frege did not make a proper distinction between *Bedeutung* as extension (truth value) and *Bedeutung* as meaning (significance). Intended as (mathematical) value of a characteristic function, the *Bedeutung* can be easily interpreted as extension, as the *objects* 1 or 0, or as the True or the False. Frege's schema of sense and reference for names is therefore easily applied to sentences: sentences are names of objects of a peculiar kind, truth values. But *Bedeutung* as the value of significance should be accounted for in a more complex way, as many commentators remark. As Beaney 1994 suggests (p.233), in restricting the *Bedeutung* of sentences to truth value, the notion of *Sinn* was given too much work to do. If Frege had also applied the distinction between *Bedeutung* and *Extension* to sentences, he could have given a better account of his theory of sense. One of Frege's main motivations for his distinction between reference and extension for predicates was the development of science. We may wait to give an extension to predicates when we are not yet sure that *there is* an extension; still we need to refer to concepts, to have concepts as referents of our predicate terms, in developing a hypothesis. Why not apply the same strategy to sentences? We may use sentences in science in a hypothetical way. We may not know whether they are true or false, still we need to *refer* to the thought expressed in them, i.e. to their truth condition. On the one hand we may consider truth conditions to be *references* (*Bedeutungen*) of sentences and at the same time keep the idea that truth values are the *extensions* of sentences. On the other hand we have different ways to access thoughts, i.e. different ways to verify which truth condition they express. These different ways can cover the epistemic aspect of Fregean *senses*. In such a proposal, extending Frege's three-

level analysis to sentences, we would have a way to make clear the difference between semantic content, the *truth conditional content* or what we refer to when speaking of an objective thought, and the *epistemic content* or the way in which we verify or arrive at defining the semantic content (when we are able to do that).

(iii) *a Fregean solution to Frege's two conception of sense.* It would seems not completely alien to Frege's ideas to generalize the tripartite division of sense, reference and extension made for predicates to other kinds of linguistic expressions. Pursuing a distinction of this kind for sentences and singular terms, we could find a Fregean solution to the problems discussed above. It would be very different from the original Fregean solution, still it would save some relevant aspects of his idea of the objectivity of thought. Let us keep our attention on sentences. If truth-values are the extensions and truth conditions the referents of sentences, (epistemic) senses could be represented by the different procedures through which each formula is given a truth condition. Even formulae expressing logical truths, which - being always true - have the same truth conditions, should have different (epistemic) senses (the differences resulting from the complexity of the formulae).

All this is apparently very sketchy and points towards a rational reconstruction of some of Frege's ideas that can resolve the contradictory tensions in Frege's logic analysed above. We shall give here a hint towards how such reconstruction might be achieved, a topic which deserves more extensive discussion. Accepting, for the sake of simplicity, the modern terminology for intension as functions from possible words to extensions, we might sketch the development of the tripartite Fregean schema in the following way:

epistemic sense	procedures associated with the intension
semantic sense	intension (function from p.w. to truth values)
extension	extension (truth value)

Here we compare Frege's terminology with some of the corresponding concepts in our contemporary terminology. It is worth noting that the idea that intension corresponds to the Fregean reference and not to the Fregean sense (a suggestion given by Carnap) is taken explicitly by Cresswell 19851. Abandoning Fregean terminology, we may rely on the distinction between intension (function from possible worlds to extensions) and procedures attached to the intension. The different procedures that compute the function can be

1 Developing his ideas on possible world semantics, Cresswell 2002 takes it for granted the idea of unstructured proposition (set of possible worlds) and the idea that 'structured meanings are what they are only because the unstructured propositions ...are what they are'. The idea might also be reversed saying that unstructured propositions are what they are only insofar they are accessible by structured meanings. But the debate is ambiguous; the distinction between unstructured and structured propositions is compared to the hypotetical distinction between 'coarse grained numbers' such as 12 and 'fine grained numbers' such as <+,5,7>. We may accept that numbers are objects named by numerals like propositions are objects named by sentences. However numbers are given only in a system in which it is essential (or internal property) of each number to be computed in different ways inside the system. Numbers do not stay alone in the series of numbers but belong to a structure of calculation. Without that, they would be unconceivable.

considered a representation (and explication) of what Frege used to call 'sense' in its epistemic aspect.

It might be asked whether this attempt is still a 'Fregean' conception of thought. Certainly, it does not follow the *Basic Criteria for a Fregean Representation of Thought* given by Bell 1987. In fact, Bell's criteria have to change if we maintain a conception of thought that allows a thought to be considered together with its objective ways of being thinkable. Our criteria will therefore be different from the classical ones insofar as we need to distinguish semantic sense (Thought) and epistemic sense (Thinking). We might propose some criteria along the following lines:

1) Thought as the referent of a sentence (the state of affairs represented or the truth condition) may be given by a formula in a canonical form (think, for example, to the disjunctive normal form in propositional logic). Grasping a thought is to understand the truth condition represented by the sentence. Certainly this is a major change from the Fregean setting, but the separation between reference (truth condition) and extension (truth-value) of sentences justifies it. This change leaves open a space for the sense of a sentence in the following way:

2) The canonical expression of a thought corresponds to a class of equivalent expressions. The different expressions represent either different ways in which the same truth condition is given or different associated procedures. They give our epistemic accessibility to the Thought, to the different senses in which a Thought is presented.

3) These different ways of getting at a unique structure must be taken as intrinsic to Thought, and not dependent upon our psychology. When expressed in a language, however, they represent *our* explicit representation of the working of the Thought.

4) The semantic and epistemic aspects of Thought must comply with compositionality. As far as truth condition is concerned, this is already clear in model theoretic tradition. As far as senses as procedures are concerned, we rely on the wide discussion in A.I. about compositionality of procedures1.

With the development of intelligent systems, we have to better work out a computational theory of thought (or, as we could put it after Frege, of a computational theory of 'the' mind). From the point of view of a computational theory of thought, we may say that truth conditions and the way to get *at* them are both part of the objective structure of the Thought. The thought is not an unstructured content, but something that intrinsically comes with its own different ways of thinking it. Thoughts, or thinkables, are to be studied at different levels of complexity: intensions are the most unifying structures, representing the basic semantic stuff; procedures attached to intensions are the ways to make intensions realized in different ways. Frege's tensions could be overcome by defining Thought as a multi-level structure: both as intension and as procedure, i.e., both as

1 Since Winograd 1972.

semantic and objective truth condition and, procedurally, as objective ways to determine truth conditions - so that a given intension (truth condition) may be associated with different procedures. Returning to a kind of example previously discussed, we might assert that a speaker who knows A B is able to apply a different computational procedure than one who knows A v B. Whoever knows that the two formulas are equivalent has a new knowledge: two different procedures bring about the same truth condition.

Thus, we need to give a representation not only of what the thought is, but also of how it is given, its mode of presentation (the expression of our computational capacities). It is a property of the thought to be thinkable in different ways; we cannot separate into two realms an unstructured thought and a structured linguistic meaning: the thought has its own general structure (intensions) which is the source of all of its possible objective articulations, of different kinds of (context dependent) procedures. The two apparently contradictory theses in Frege are ultimately resolved through a careful assessment of two inextricably connected aspects of our thoughts.

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