Philosophical and mathematical reflection on Riemann's hypothesis. I Reframing in Hilbert arithmetic

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Abstract. What should be the "physical interpretation" of Riemann's hypothesis? Can its eventual physical interpretation pioneer a pathway for the proper mathematical proof? Answers to both questions are researched in the framework of ontomathematics inherently involving the unity of physics, mathematics, and philosophy. After that viewpoint, a philosophical method for reinterpreting most fundamental mathematical problems (in particular, the seven "Millennium Problems" of CMI) is suggested. Loosely speaking, it consists in determining the ontomathematical "forest" in which the "tree" of a certain very essential mathematical problem is situated, after which the shortest silogism eventually needing a relevant "Gestalt change" appears to be natural and almost obvious, furthermore rather elementarily provable. One even notices that many (if not all) most fundamental problems of contemporary mathematics appeal to the same "Gestalt change" needing the "Cartesian glasses" to be "put off" and Modernity and its episteme to be abandoned. As for mathematics itself, one can conjecture that many or all of the most fundamental problems are (or at least, are linkable to) Gödel's insoluble statements. Ontomathematics suggests a general framework (also) for resolving many essential mathematical problems by breaking the Cartesian prejudice established by Modernity: then, Riemann's hypothesis can be reformulated in terms of the qubit Hilbert space so that the "zeta function" belongs to it. If one manages to demonstrate this, Riemann's hypothesis is rather easily provable since it refers to the fundamental reducibility of any qubit to a single bit "after measurement". From the newly introduced viewpoint, the zeta function is "physically" continued also at its single pole therefore tracing its interpretation by means of the Noether (1918) first theorem. Its application for proving Riemann's hypothesis is sketched in order to be elaborated in the next, second part of the study.
Keywords: bit and qubit, equation, Gödel mathematics versus Hilbert mathematics, Hilbert arithmetic, Noether's symmetry and conservation theorem, nonstandard bijection, ontomathematics, quantum information, Riemann's hypothesis, Riemann's zeta function.

I RIEMANN'S HYPOTHESIS: FROM RIEMANN TO A CMI "MILLENNIUM PROBLEM"

Riemann's Hypothesis (RH) stands as one of the most profound and enduring problems in mathematics. First formulated by Bernhard Riemann in his seminal 1859 paper, the conjecture is suggested in relation to the distribution of prime numbers, but it refers verbatim to the "nontrivial zeros" of the Riemann zeta function. Despite extensive efforts, RH remains unresolved, making it one of the seven Clay Mathematics Institute's Millennium Prize Problems, offering a $1 million reward for its proof[1]. This historical introduction traces the development of RH, its foundational results, and key conjectures that have emerged over time[2].

[1] Clay Mathematics Institute (2000).
[2] Oort, Schappacher (2016).

Bernhard Riemann involved his famous hypothesis in his only number theory paper, “Über die Anzahl der Primzahlen unter einer gegebenen Grösse” [“On the Number of Primes Less Than a Given Magnitude”] (Riemann 1859). The paper extended Euler’s work on the zeta function and formulated the fundamental connection between prime numbers and the complex analysis of $\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$, for $Re(s) > 1$. Riemann built on earlier work by Leonhard Euler, who in the 1730s linked the zeta function to primes through the Euler product. Euler’s insights, however, were confined to real values of the variable. Riemann’s breakthrough lay in treating it to be complex. Euler demonstrated that this function could be expressed as an infinite product over primes (a result now known as the Euler product formula): $\zeta(s) = \prod_{primes} (1 - \frac{1}{p^s})^{-1}$. This connection between the zeta function and prime numbers laid the groundwork for future explorations into their interplay. Fast forward to the mid-19th century, when Bernhard Riemann extended Euler's ideas to the realm of complex numbers, analytically continuing $\zeta(s)$ beyond its domain of convergence everywhere on the complex plane where it is defined except its (single) simple pole (which is at $s = 1$). He also derived a functional equation relating ζ(s) and ζ(1−s), which revealed symmetry about the critical line Re(s)=$\frac{1}{2}$:

$\pi^{-s/2}\Gamma(\frac{s}{2})\zeta(s) = \pi^{-\frac{(1-s)}{2}}\Gamma(\frac{1-s}{2})\zeta(1 - s)$ - gamma form of the functional equation:

$\zeta(s) = 2^s\pi^{s-1}sin(\frac{\pi s}{2})\Gamma(1 - s)\zeta(s - 1)$ - Riemann's standard functional equation.

The key sinusoidal multiplier in this equation is: $sin(\frac{\pi s}{2})$. This multiplier is responsible for producing all trivial zeros of the Riemann zeta function. The trivial zeros occur at: $s = - 2, - 4, - 6, - 8$, ..., i.e. all even negative integers, which are precisely the roots of the sinusoidal multiplier. Thus, the trivial zeros of ζ(s) come directly from the vanishing of this sine function in the functional equation.

Riemann then focused on the non-trivial zeros of ζ(s), i.e., those not located at negative even integers due to the sinusoidal multiplier in the above functional equation and called “trivial zeros”[3] . The nontrivial zeros are crucial because they encode information about the distribution of prime numbers through the explicit formula he developed. Specifically, Riemann conjectured that all non-trivial zeros lie on the critical line Re(s)=$\frac{1}{2}$. This assertion became the cornerstone of what is known as “Riemann’s hypothesis”, one of the most central unsolved problems in mathematics (Edwards 1974).

RH stands as one of the most profound unsolved problems in mathematics, bridging number theory, complex analysis, and mathematical physics. Proposed by Bernhard Riemann in

[3] Riemann’s functional equation and the trivial zeros implied by it enters the ground of the present study as a necessary premise. The equation is very well investigated: e.g., Li 2011; Chaudhry, Al-Baiyat, Al-Humaidi 2010; Culp-Ressler, Flood, Heath, Pribitkin 2000; Knopp 1994; Rooney 1994.

1859, it concerns the distribution of prime numbers and the properties of the Riemann zeta function, ζ(s). This hypothesis has captivated mathematicians for over a century and a half, inspiring countless investigations, partial results, and speculative connections to other fields.

The story begins with Leonhard Euler's work on a few infinite series in the 18th century. In his seminal paper "Variae observationes circa series infinitas" (Euler, 1737), Euler introduced the zeta function for real arguments greater than 1 and demonstrated that this function could be expressed as an infinite product over primes.

Following Riemann's death in 1866, interest in his hypothesis waned until the late 19th century, when mathematicians began systematically studying the zeta function. Edmund Landau provided foundational contributions to the theory of the zeta function, proving important bounds on its growth (Landau, 1909). Around the same time, Jacques Hadamard and Charles Jean de la Vallée-Poussin independently proved the Prime Number Theorem (Hadamard, 1896; de la Vallée-Poussin, 1896), showing that the number of primes less than x, denoted π(x), asymptotically satisfies: $\pi(x) \sim \frac{x}{log(x)}$. Their proofs relied on showing that $\zeta(s)$ has no zeros for $Re(s) = 1$, thus strengthening the connection between RH and prime number distribution (Hadamard, 1896; de la Vallée-Poussin, 1896).

The first numerical verification of RH was carried out by Gram (1903), followed by extensive calculations by Littlewood and Hardy (1918), and Siegel (1932). Alan Turing later devised computational methods to verify zeros on the critical line (Turing, 1953). As of recent years, billions of nontrivial zeros have been computed, all of which lie on $Re(s) = \frac{1}{2}$, lending strong empirical support to RH (Odlyzko, 1987).

Despite these advances, no general proof (or disproof) of the RH has emerged. However, several significant results have illuminated its structure and implications: (1) critical strip boundaries; (2) density estimates; (3) explicit formulae; (4) generalizations. It was shown that all non-trivial zeros must lie within the critical strip $0 < Re(s) < 1$ (de la Vallée-Poussin, 1896). This eliminates the possibility of zeros outside this region. Norman Levinson made substantial progress by proving that at least one-third of the non-trivial zeros lie on the critical line (Levinson, 1974). Subsequent refinements increased this proportion to approximately two-thirds (Conrey, 1989). Riemann's explicit formula connects the zeros of ζ(s) to fluctuations in the distribution of primes. This relationship provides deep insights into both areas but remains difficult to fully exploit due to the complexity of the zeta function. The RH has inspired numerous generalizations, such as the Extended Riemann Hypothesis (ERH) for Dirichlet L-functions and the generalized Riemann hypothesis (GRH) for Dedekind zeta functions. While stronger, these conjectures share the same core challenge: proving that relevant zeros align along specific lines. RH has inspired numerous other generalizations and equivalent formulations. Some notable ones include: (1) Selberg's Trace Formula; (2) Weil's Criterion; Langlands Program. Atle Selberg (1956) developed an analogue of the Riemann hypothesis for automorphic L-functions, laying the foundation for modern analytic number theory (Selberg, 1956). André Weil (1948) proved an equivalent formulation of RH for function fields, deepening the connection between RH and algebraic geometry (Weil 1948)[4]. Robert Langlands extended RH's implications by connecting zeta functions to automorphic forms, a framework central to modern number theory (Langlands, 1970).

David Hilbert (1900) listed RH as one of his 23 unsolved problems, underscoring its importance. Later, George Pólya and others suggested a spectral interpretation of RH, hypothesizing that the zeros $\zeta(s)$ of might correspond to eigenvalues of a yet unknown Hermitian operator (Pólya, 1926). This perspective has led to deep connections between RH and random matrix theory, particularly through the work of Montgomery (1973), who found links between the pairwise spacing of zeros and eigenvalues of random Hermitian matrices (Montgomery, 1973).

The RH transcends pure number theory, finding surprising links to diverse disciplines: (1) quantum mechanics; (2) random matrix theory; (3) algebraic geometry. Michael Berry and Jonathan Keating (1999) proposed a tantalizing connection between the RH and quantum chaos. They suggested that the distribution of zeta zeros might correspond to energy levels in certain quantum systems governed by chaotic dynamics. Andrew Odlyzko's numerical studies revealed striking similarities between the spacing of zeta zeros and eigenvalue distributions from random matrix ensembles (Odlyzko, 1987). This connection hints at underlying symmetries yet to be fully understood. The Weil Conjectures, proven by Pierre Deligne, established analogues of the RH for zeta functions associated with algebraic varieties over finite fields (Deligne, 1974). Although distinct from the classical RH, these results demonstrate broader applicability of similar principles.

Contemporary research continues to explore new avenues toward resolving the RH. Some prominent strategies include: (1) analytic methods; (2) computational verification; (3) interdisciplinary insights. Those include correspondingly refining estimates for ζ(s) and related functions to constrain zero locations: extending numerical checks to higher ranges of imaginary parts of zeros; leveraging tools from physics, probability, and computer science to uncover hidden structures.

However, fundamental obstacles persist. The intricate nature of the zeta function, combined with its apparent resistance to traditional techniques, makes the problem exceptionally challenging. Moreover, any proof would likely require novel concepts beyond current frameworks. If proven true, the RH would revolutionize our understanding of prime numbers and their distribution. Applications range from cryptography to algorithm design, as many modern encryption schemes depend on assumptions tied to prime factorization. Beyond practical benefits, the RH embodies humanity's quest for order amidst apparent randomness - a testament to the enduring allure of mathematics.

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[4] Weil Conjectures (1949): André Weil proved RH for function fields (curves over finite fields), linking zeta zeros to eigenvalues of Frobenius maps. His work laid the groundwork for étale cohomology. Grand Riemann Hypothesis: Extends RH to all global L-functions, including Dirichlet L-functions and modular forms. Deligne's Proof (1974): Using Weil's framework, Deligne validated the Weil conjectures in full, earning a Fields Medal.

Riemann's hypothesis remains one of the greatest open problems in mathematics, with deep implications across number theory, complex analysis, and mathematical physics. Despite extensive numerical verification and partial results, a formal proof or counterexample continues to elude mathematicians. Future advancements in analytic number theory, random matrix theory, and quantum chaos may hold the key to resolving this century-old mystery.

II TWO ALTERNATIVE VIEPOINTS TO "MATHEMATICAL PROOF": FROM MATHEMATHICS AND (OR) FROM PHILOSOPHY

Historically, the proofs of mathematical theorems, being reducible to a few initial statements (eventually, axioms), is always a silogism. Only in the end of the 19th century, and especially during the 20th century and mathematical logic, they have been granted to be necessarily representable as a calculation in Boolean algebra, furthermore obeying Gentzen's cut theorem[5] in the case of being of any "infinite" length.

Though, the cut theorem means some "infinite" silogism in general, it can be otherwise seen: (at least) conjecturing the option for any silogism to be "cut" to a minimal "shortest" length therefore forcing an analogy to the principle of least action in physics, however remaining groundless, "superficial", not more than a metaphor after the Cartesian "abyss" between the "material" physics on the "bodily shore" and the "ideal" logic and mathematics on the opposite "mental shore" by and during Modernity.

However, if one dare put off the "Cartesian glasses" (as ontomathematics, discussed in detail in other papers, e.g., Penchev 2024 April 16; 2023 November 2, does), the unity of Gentzen's cut theorem and the principle of least action can be reflected, reasoned, and mathematically proved in the final analysis: and here is how though only cursorily outlined since it is more or less explicitly meant in other papers (Penchev 2022 August 3):

So, the cut theorem and its eventual corollary about the "shortest silogism hypothesis" should refer to Hilbert arithmetic in a *narrow* sense, and accordingly, to the principle of least action, to Hilbert arithmetic in a *wide* sense, being equivalent to the qubit Hilbert space unlike the former resulting particularly into Boolean algebra isomorphic to classical propositional logic or to the bit calculation space of any Tirung machine. The class of equivalence of all values of any qubit (i.e., an "empty qubit") is in turn isomorphic to a bit of classical information. Once any bit is defined to be isomorphic to an empty qubit, it is practically tautological to be said that the

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[5] Cut-elimination in a context relevant to the present context: e.g., Mancosu, Galvan, Zach 2021; Baaz, Leitsch 2014; 2011; 2006; 2000; Ciabattoni, Galatos, Terui 2012; Dowek, Hermant 2012; Mints 2012; 2008; Tiu, Momigliano 2012; Aral 2011; 1998; Beckmann, Buss 2011; Kamid, Wansing 2011; Leitsch, Baaz 2011; Galatos, Ono 2010; Pattinson, Schröder 2010; Brünnler, Studer 2009; Kikuchi 2008; Rasga 2007; Terui 2007; Borisavljević 2006; 1999; Brünnler 2006; Ciabattoni 2006; Zamansky,. Avron 2006; Zhang 2006; 1991; Alberucci, Jäger 2005; Belardinelli, Jipsen, Ono 2004; Cerrit, Kesner 2004; Bakel 2003; Gutiérrez, Ruiz 2003; Okada 2002; 1999; McDowel, Miller 2000; Avigad 2001; Elbl 2001; Gilmore 2001; Tanaka 2001; Pfenning 2000; Došen 1999; Gelder, Okushi 1999; Dyckhoff, Pinto 1998; Faggian, Sambin 1998; Gabbay, Olivetti 1998; Carbone 1997; Kashima, Shimura 1994; Markett 1994; Vauzeilles 1993; Hudelmaier 1992; Ungar 1992; Gordeev 1987; Szabo 1987; Yasuhara 1982; Mönting 1981; Pottinger 1977; Wessels 1977; Zucker 1974; 1974a; Bowen 1972; 1972a.

ultimate statistical collection of an “empty qubit”, i.e., all possible values of a qubit, would result for both options of a bit (e.g., notated as “true” and “false” or as “1” and “0”, etc.) to be equally probable: this means that the probability of each of them is necessarily ½ (after many enough measurements). The above quite short and obvious consideration (being almost trivial) implies the too sophisticated hypothesis of Riemann if one would manage to show that Riemann's “zeta function”[6] can be represented as a qubit wave function, and then, that its nontrivial “zeros” correspond to the projection of the qubits at issue into bits of information. What is essential for the present section is the conjecture that Riemann’s hypothesis is eventually inferable of the relation of Hilbert arithmetic in both narrow and wide senses:

Granting the alleged “short silogism conjecture”, one may assign it to the mathematical proof from a “philosophical viewpoint” opposing it to the proper mathematical one meaning all accomplished already proofs or eventually accomplishable ever in the future regardless of their “length” (depending also on the chosen tuple of axioms).

If a finite silogism (whether proving or rejecting any mathematical statement) is known, the option of its abbreviation can be important only technically and practically, e.g., implying new applications, rather than fundamentally, once the corresponding theorem (following from a certain list of axioms) has been somehow and anyway already proved. However, the case of a silogism being infinite is worth to be considered philosophically, especially ontomathematically and furthermore, in the context of Gödel incompleteness theorems (1931):

So, Gentzen’s cut theorem implies the reducibility of any infinite silogism to a finite one, but Gödel’s incompleteness theorem (“Satz VI”) implies for any infinite silogism not to be inferable from any finite syllogism as a fundamental restriction: whether (either) for contradiction or for incompleteness. If one grants that the two theorems are consistent to each other (even more so that they are really consistent), they involve an asymmetry of finiteness (more precisely, the arithmetic finiteness) to “infinity” (the proper set-theoretical infinity”). The former is accessible from the latter for Gentzen’s theorem, but not vice versa: for Gödel’s theorem.

As for the same observation applied to the relation to Hilbert arithmetic in both senses (narrow and wide), it implies their derivative asymmetry. Furthermore, it is definitive since each unit of in the former is an empty qubit, i.e., a class of equivalence. Thus, a unit in a narrow arithmetic sense can be unambiguously inferred from all possible values, but vice versa: not at all.

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[6] It is widely discussed, but the present context essentially restricts the relevant references: e.g., Heymann 2024; Arvashan 2021; Heath-Brown 2020; Bellissard 2017; Bender, Brody, Müller 2017; Ford. Zaharescu 2016; Fukushima 2016; Hassani 2014; Masser 2011; Meyrath 2011; Laurinčikas 2010; 1989; 1987 1987a; 1985; 1985a; Frankenhuijsen 2008; 2005; Ki 2008; Montgomery, Vaughan 2007; Ding, Feng 2006; Goldston, Montgomery, Soundararajan 2005; Liao, Yang 2003; Burnol 2002; Hall 2002; Keating, Snaith 2000; Musès 2000; Connes 1999; Bhaduri, Khare, Reimann, Tomusiak 1997; Lang 1994; Butzer, Hauss 1992; Karatsuba, Voronin, Koblitz 1992; Groenewald 1991; Julia 1990; Anderson 1986; 1983; Bump, Ng 1986; Ivić 1985; Apostol, Vu 1984; Ghosh 1983; Mueller 1982; Davenport 1980; 1967; Brent 1979; Apostol 1976; Levinson 1973; 1969; Montgomery 1979; Hardy, Littlewood 1921; 1921a; 1918; 1914; Backlund 1918.

Particularly, many enough physicists, including Einstein himself, had and have not managed to penetrate into the special consistency of that asymmetry implying that quantum mechanics is really complete (in the exact meaning of the theorems of absence of hidden variables in quantum mechanics: Kochen, Specker 1967; Neuman 1932), on the one hand; but nonetheless, simultaneously probabilistic (being obvious after Max Born's interpretation), on the other hand.

In fact, those completeness and probabilisticality of quantum mechanics repeat the special consistency of Gentezen's theorem and Gödel's theorem to be true simultaneously. Indeed, quantum mechanics is complete in the "straight" direction, after which no hidden variable to determine additionally the measured quantum state (being definitively finite) just as any infinite syllogism can be unambiguously "cut" in Gentzen's manner. Nonetheless, the collection of all possible measurements of a certain quantum state can be only a statistical mapping (thus a surrection rather than a bijection) of the coherent quantum state being measured, implying for the statistical collection to be fundamentally incomplete just as any arithmetically enumerated set after Gödel being finite is incomplete (if not contradictory) to any actual infinite set.

As for the proper subjects of the present study and section in it, the eventually borrowed structure for completeness and incompleteness to be asymmetric to each other, corresponds to the qubit - bit asymmetry and the asymmetry of the philosophical - mathematical viewpoints to the mathematical proof accordingly. Then, and interpreting the borrowed structure, one can say that the reduction of a qubit to a corresponding bit does not need any hidden variables (thus being unambiguous and deterministic), but nonetheless, the statistical collection of all possible reduction of a qubit to a bit is fundamentally probabilistic (and only in that sense, "incomplete": a viewpoint involved by Einstein, Posolsky, and Rosen rigorously enough in their notorious article as for quantum mechanics itself). That observation is relevant to the interpretation of Riemann's hypothesis[7] in terms of Hilbert arithmetic isomorphically after applying the afore-introduced structure, the logical viewpoint to the mathematical proof implies the philosophical viewpoint to it, but not vice versa (as if, but only seemingly and ostensibly reasoning common sense's judgement on philosophy: to be simultaneously true and "useless").

Now, the above considerations will be applied to a crucial particular case after granting the beginning and the end of a syllogism to be linked by an infinite sequences of elementary

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[7] The present context restricts the huge volume of papers about Riemann's hypothesis: Kapustin 2019; Ovchinnikov 2019; Moxley 2018; Omar, Bouanani 2017; 2010; Oort, Schappacher 2016; Yang 2016; Mazur, Stein 2015; Bui, Lester, Milinovich 2014; Yakubovich 2014; Carneiro, Chandee, Milinovich 2013; Vericat 2013; Gonek 2012; Sondow 2012; Sekatskii, Beltraminelli, Merlini 2012; Caveney, Nicolas, Sondow 2011; Masser 2011; Schumayer, Hutchinson 2011; Schumayer, Hutchinson 2011; Planat, Solé, Omar 2010; Sondow, Dumitrescu 2010; Suzuki 2009; Borwein, Choi, Rooney, Weirathmueller 2008; Chahal, Osserman 2008; Haight 2008; Wójtowicz 2007; Granville 2007; Bagchi 2006; Booker 2006; Voros 2006; Wan, Haessig 2004; Bombieri 2000; Bombieri, Lagarias 1999; Sheats 1998; Rudnick, Sarnak 1996; Bender, Patashnik, Rumsey 1994; Wolf 1994; Baezduarte 1993; Mikolás, Sato 1992; Spector 1990; Roesler 1987; 1986; Bercovici, Foias 1984; Sarkaria 1984; Randol 1978; Miller 1976; Segal 1975; Moreno 1974; Weil 1952; Wintner 1944; 1943; Siegel 1932; etc.

syllogisms (therefore constituting a well-ordered actually infinite set), so that they are to be situated (idempotentently) of the two opposite “shores” of the Cartesian “abyss” thus joining mathematically (and therefore, necessarily) a physically measurable conclusion (or idempotently, tuple of premises) with a list of only mathematical conditions (or idempotently, theorem). Once that construction has been imagined (obviously, being absolutely consistent), it generates the following troubles about Gödel’s incompleteness theorem and Gentzen’s “cut rule”, on the one hand, and the Cartesian “abyss”, adopted by Modernity, on the other hand:

The “abyss” at issue is postulated to be absolute, including its interpretation in physics strictly forbidden, e.g., as the notorious “creatio ex nihilo”, i.e. any violation of energy (or mass) conservation as well as in relation to the recently discovered “dark mass” and “dark energy” (otherwise being easily explicable by the permanent violations of mass and energy conservation, correspondingly), even crucially prevailing in the universe over the particular case of their conservation meant alone by classical physics and science. The mythical “Big Bang” has been also introduced just to “conserve energy conservation”. One can immediately notice that Gödel’s incompleteness paper is consistent with the same viewpoint heralded by Modernity once the arithmetic finiteness has been identified with “mind” (“subject”): and the set-theoretical “infinity”, with “body” (“object”).

However, if one admits that the “abyss” between them can be equivalently substituted by an infinite series of intermediate conclusions (for example, as projective geometry postulates for any two parallel straight lines in Euclidean space to cross each other at the “infinity point”), and applying the Gentzen cut rule, it can be always reduced to a finite proof. The last construction cancels in fact the Cartesian abyss since it states (rather figuratively said) that a finite bridge can always connect any two points, each of which belonging to one of the two opposite “shores” (therefore particularly allowing for “creating energy from nothing” as well as its “annihilation” reversely to nothing).

However, the “bridge” metaphor is partly misleading since any real bridge connects equally well in both directions, unlike the case meant by the metaphor, after which it is only “semipermeable”: so that Gentzen’s theorem stating its “permeability” in the one direction (from infinity to finiteness) can be consistent with the converse impermeability established by Gödel’s theorem. Furthermore, the former refers to a series of elementary conclusions (thus being inherently well-ordered without needing the axiom of choice) unlike the latter since the arithmetic enumeration is idempotentently reversible (though that argument is rather psychological than formal). Translated about energy conservation in physics, that “Solomonic” semi-pemeability would mean that energy can only vanish rather than be created (thus contradicting the standard understanding of energy able to both increase and decrease). However, that newly introduced "one-directional" energy would be quite consistent with its conjugated counterpart of time being inherently and oppositely (only increasing) “one-directional”, “one-way”.

However, one can reflect on the just sketched troubles quite differently by distinguishing the following two alternative cases. The one consists in the standard interpretation including that of Gödel himself, that his "incompleteness statement" (namely, "Satz VI") is really a theorem following from arithmetic, set theory, and propositional logic. The other one (being quite nonstandard, but not inconsistent) adopts it to be an axiom usually unarticulated whether in mathematics or in meta-mathematics therefore rejecting for the "bridge" over the Cartesian "abyss" to be "semipermeteable" (which is an absolutely relevant solution as it is above demonstrated in detail). If the "bridge" at issue is granted in advance to be only "two-directional", "two-way" (i.e. permeatable in both directions: from finiteness to infinity as well as vice versa), the Gödel (1931) incompleteness theorem postulates for it not to exist: on the contrary, the cut rule adopts it. Of course, the former option coinciding with the contemporary standard mathematics is very well investigated, and particularly, the seven CMI "Problems of the Millennium" mean in default the same framework in which Gödel insoluble statements hold.

Then, an admissible conjecture (advocated also in more detail below, in the last section of *Part II*) is that the seven problems at issue are insoluble statements (or depending on those, linkable to those, etc.), which is the real reason not to be resolved until now by the huge army of professional mathematicians de-puzzling them for centuries. As for Riemann's hypothesis also belonging to the seven ones (if it can be connected to the reducibility of a qubit to a bit, in detail in *Sections III* and *IV* below), it would occupy a privileged position among them, being a meta-position linkable to the interpretation of the incompleteness theorem: either an axiom or a theorem.

Then, following a mathematical behavior as after Lobachevsky, one can start cautiously investigating "non-Gödelian mathematics" searching for eventual contradictions therefore conforming by "reductio ad absurdum" that the incompleteness theorem is really a theorem rather than an axiom. Alas, those contradictions cannot be revealed. The newly discovered world is quite extraordinary, but nonetheless, internally absolutely consistent just as in the original pattern of Lobachevsky. Of course, it contradicts common sense's prejudice drastically, but it is rather an advantage than a disadvantage (since Copernicus's age) for any scientific hypothesis (even articulated by Niels Bohr's famous and widely coined reply: "Your idea is crazy, but not crazy enough to be true"). However, the here challenged prejudice is much more fundamental than those being overcome by Copernicus or Lobachevsky accordingly:

What is at stake now is the most fundamental principle in the foundations of Modernity (including the social order and organization, for example) rather than only those of mathematics, physics or even modern experimental and theoretical science in general. It can be relevantly represented by the Cartesian dogma about no smooth way between the two "shores" of its "abyss" and already discussed above.

Gentzen's cut rule obviously violates it though partly and only for mathematics demonstrating at least the semi-permiteability of that forbidden smooth transition. On the contrary, Gödel's incompleteness theorem is absolutely consistent with it, in fact reproducing it in mathematics by situating it between arithmetic and set theory. Only correlating both theorems to each other, their eventual (or alleged) mutual consistency forces to distinguish the two opposite directions of the continuous "bridge" between finiteness and infinity.

However, the problem of their mutual consistency allows for another solution, after which the extraordinary semi-permeteability between finiteness and infinity is rejected, therefore restoring the usual two-directionality or symmetry between increasing and decreasing any mathematical quantity. Due to the same circumstance, non-Gödelian mathematics is to be admitted after his notorious theorem have been in advance granted to be an axiom (thus replaceable by its negation just as the Fifth postulate of Euclid after Lobachevsky) rather than a theorem. One can gradually build a new mathematical universe, absolutely internally consistent, but shockingly different from common sense's prejudice implying a series of "ridiculous corollaries" (such as that the "Big Bang" is a myth; humankind can master "creatio ex nigilo"; nature is not "blind"; people can "negotiate" with nature utilizing the language of mathematics, not as a metaphor, etc. and etc.).

As for the most difficult and fundamental mathematical problems (as those seven offered by CMI), and particularly Riemann's hypotheses, Hilbert arithmetic in the foundations of Hilbert mathematics (being a representative, possibly the simplest one belonging to the class of non-Gödelian mathematics) supplies a general method for resolving them if the conjecture that all of them are linkable to Gödel insoluble statements is reasonable and true. As for Riemann's hypothesis itself, it occupies a privileged position being an immediate corollary from the formulation of the universal method in question, furthermore regulating the necessary relation between quantum information and classical information in Hilbert arithmetic in both wide sense and narrow senses accordingly: so that the class of equivalence of all possible values of a qubit (i.e., an "empty" qubit or a unit of Hilbert arithmetic in a wide sense) is granted to be a bit of classical information and a usual unit of Hilbert arithmetic in a narrow sense, and further reducible to a usual unit of Peano arithmetic.

One can follow the pattern borrowed from the application of non-Euclidean geometries (properly, that referring to pseudo-Riemmanian space) to general relativity (a physical rather than mathematical theory of gravitation in common opinion), therefore asking for the "real mathematics" (i.e. Gödelian or not) of the being itself rather than only as of the universe just as astronomers and cosmologists nowadays question about the "real geometry of the universe". One can enumerate a series of observable and measurable effects from the hypothesis that the real mathematics of our "being" at all is not Gödelian, and then check what the case is: as for us and our being. Among that very long list of eventual confirmations (respectively rejections) of which mathematics (either Gödel mathematics or Hilbert mathematics) the being obeys, one may (quite cursorily) enumerate the following:

Classical quantum mechanics corresponds to Gödel mathematics after complying with the following list: energy conservation, unitarity, Hermitian operators for physical quantities, the special quantity of time without a relevant Hermitian operator, the Standard model, etc. The hypothesis of the "Big Bang", respectively the absolute prohibition of any "creation from nothing" after it would contradict Hilbert mathematics. The contemporary "solid" organization of society consisting of a numerous amount of the "microcrystals" of various hierarchies contradicting each other satisfies rather the Cartesian dogma of the "abyss" so that the society is situated on its “material shore” (including the *institution* of science itself), but science *by itself* is on the opposite, “ideal shore” thus not being able to influence directly on the society but only by the mediation of human decisions. That and any other confession of the Cartesian dogma mean implicitly, but necessarily, just Gödel mathematics. The fundamental controversies are much more (as well as discussed in other papers: e.g., *Penchev 2024 April 16; 2023 May 3*), but even only the mentioned a few ones are sufficient to outline the size of eventual changes relevant to the adoption of Hilbert mathematics, advocated in the present paper as for the essential, but only mathematical and this particular problem of Riemann’s hypothesis.

III IS MATHEMATICS REALLY DIFFICULT? IS MATHEMATICS, FIRST OF ALl, SILOGISMS & CALCULTATIONS OR THINKING?

Once the option of Hilbert mathematics has been at all admitted for consideration, though in the particular context of Riemann’s hypothesis, an unusual sequence within its framework (furthermore not mentioned yet in other papers) can be discussed. Mathematics is so difficult for almost all people, needing too rare human capabilities featuring all mathematicians only it is identified with a very extended calculation: a particular case of which is one, usually too long syllogism being “normal” for the mathematical proof in general, along with all mathematical notations and rigorous, also too artificial definitions, so that both transform any professional mathematical paper in a text written in an absolutely inaccessible language (figuratively speaking, “in Chinese” rather than in English).

Even much more, that common understanding of mathematics, namely, to be accessible only “for chosen people” (who are professional mathematicians) corresponds to the location of mathematics in the episteme of Modernity: on the “ideal shore” thus being absolutely separated from physics (on the opposite, “material shore”), or philosophy inherently endeavoring to unified viewpoints to the world. However, that sophistication of mathematics, distinguishing it from physics and philosophy, features it only during late Modernity (for example, starting from the second half of the 19th centuries until nowadays). Even in early Modernity, in Descartes, Leibniz, and Newton’s age, it had been closely linked to philosophy and physics.

Speaking loosely, the human mind very easily recognizes images, especially those to which had been trained in childhood (or eventually later), but calculates very difficulty in general, especially in a comparison with a contemporary superpowerful computer, but even to humans’ personal smartphones, for example. On the contrary, a contemporary computer is a Turing machine and calculates quite successfully, being supplied by the relevant software program, but only the recent achievements in AI allow the recognition of images or language patterns in a degree comparable with that of normal humans.

If one grants the deductive and axiomatic method in mathematics to be omnipresent and universal for it, the proper creative stage should be restricted to the choice of a tuple of consistent axioms such as those of geometry suggested by Euclid and ultimately perfected by Hilbert. Then, the proofs of all inferable theorems need only logical calculations (what all mathematical proofs are), and consequently they might be accomplished by Turing machine computers only provided by relevant software. So, one can suggest that the rather rare human mathematical capability is already redundant since it can be replaced by much more powerful tools such as the contemporary supercomputers (eventually trained within LLM) just as the human physical strength had been substituted a long time ago by much more powerful machines only controlled by people.

However, real mathematical creativity refers much more to the recognition of abstract “images”, patterns and models (what all mathematical structures are) than to calculations including logical ones. One can involve Gentzen’s cut rule more or less figuratively to describe what a professional mathematician really does. Initially, he or she manages to recognize such an abstract “image”, however that recognition is an “infinite” (logical) calculation and thus irrepresentable as a proof needing not more than a few decades of pages to be exhaustively exhibited. However, the cut rule guarantees that the “infinite” calculation at issue, being due to the human capability for recognizing both abstract and real “images”, can be reduced to a finite syllogism, even to a few pages almost always, so that other mathematicians can repeat and check it. On the contrary, any Turing machine encounters an unavoidable obstacle because of the Gödel incompleteness theorem which does not allow the recognition of any “image” whether abstract or not to be achieved as a finite calculation.

So, the aforementioned suggestion that contemporary computers might replace human mathematicians being "intellectually” much more powerful just as, e.g., excavators are *physically* much more powerful than human diggers, is not relevant. The reason is that the mathematicians do not “dig intellectually”, but: (1) penetrate suddenly “somehow” though after sustaining efforts (fruitless in the most cases), but anyway resulting in an “insight”: an abstract “image”; (2) also creatively, though being guaranteed by the “cut rule” in general, they are able to deliver a finite syllogism instead of the initial “infinite” insight so that a proof usually no longer than a few pages is already accessible to many people, in fact, specially and very trained such as professional mathematicians. Only to the somehow “stolen from the Gods” written proof, the deductive and axiomatic method would be relevant, but it is thoroughly inappropriate for describing the “theft” itself. Brouwer’s intuitionism (for example) by his concepts of “creative subject” and “bar induction”, etc. fits much more to that objective.

However, meaning just sketched mental “image” how a professional mathematician really creates by "insights", then “cutting” them into finite proofs, thus absolutely in the framework of the standard (i.e. Gödel) mathematics, one can attempt to represent it otherwise, but also absolutely rigorously: within Hilbert mathematics, more exactly, in terms of Hilbert arithmetic in both narrow and wide sense as well as their relation (which is crucial for Riemann’s hypothesis, in particular, and thus for the context of the present study).

The essential difference consists in the circumstance that the “semi-permeability” between finiteness and infinity in the standard mathematics is replaced by their absolute permeability in both directions. That difference can be very instructively visualized by the real debate about Wiles’s proof of Fetmat’s last theorem by the modularity theorem (i e. the Taniyama - Shimura - Weil hypothesis, before Wiles’s proof): whether it is thoroughly within the standard mathematics or not if it needs inaccessible cardinal numbers, rather countable (more exactly specified to be subcountable) inaccessible cardinal numbers[8].

One can rather elementarily demonstrate that Fermat's last theorem is a Gödel insoluble statement by the mediation of Yablo's paradox (in detail, e.g., in: *Penchev 2021 March 9*). Andrew Wiles himself and many other mathematicians state that it is thoroughly within the standard mathematics referring to tenets, which can be represented in the present context as relating to Gentzen's cut rule since it "jumps from infinity into finiteness" directly, therefore avoiding the gradual descent in the "ladder from infinity to finiteness" and thus, all inaccessible subcountable cardinals corresponding to that descent.

On the contrary, the adherents of the opposite opinion refer to the fact that Fermat's last theorem is an arithmetic statement so that it cannot be generalized to its counterpart which is properly proved as a corollary from the modularity theorem otherwise than utilizing those (subcountable) cardinal numbers out of the standard mathematics. So, Wiles's proof either only shifts the troubles of its proof into the problem whether both versions of Fetmat's last proof may or may not be identified, on the one hand, or, on the other hand, if his proof is granted to be correct, it has implicitly and secretly involved those (subcountable) cardinal numbers in advance therefore going out of the standard mathematics. As for me personally, I belong to the latter team of mathematicians or philosophers due to the the following argument:

In fact, Andrew Wiles's approach for proving Fermat's last theorem can be generalized and then utilized to resolve all Gödel insoluble statements: namely, by substituting it with the "same statement" in another axiomatic system. That substitution is incorrect or at least disputable, and it can be visualized by the following, obviously wrong "conclusion":

One considers the "theorem" that the sum of the angles of any triangle is NOT equal to 2π [rad] in Euclidean geometry after its substitution with the "same statement" about non-Euclidean geometry in general, after which it is claimed to be also proved as for the particular case of Euclidean geometry.

Obviously, and analogically, Fermat's last theorem is an arithmetic statement relating only to the particular case of finite sets if it is meant from the viewpoint of set theory. If one starts from Fermat's last theorem, the modularity theorem cannot be reached from it otherwise than crossing all inaccessible (subcountable) cardinal numbers, loosely speaking, situated between finiteness and infinity. Andrew Wiles bypassed that inaccessibility by considering it only in the opposite direction, in which the cut rule is already applicable and legitimate. However, an unarticulated postulation implying that the generalization of Fermat's last theorem

[8] All inaccessible subcountable cardinal numbers are: (1) crucially important for ontomathematics and thus, for the approach of the present study; (2) out of the standard mathematics; (3) intensively elaborated in category theory, especially by and after Grothendieck: e.g., Ernst 2020; Linnebo 2017; Feferman 2013; 1969; Hamkins 2012; Baez 2010; Caramello 2010; Shulman 2010; Heunen, Landsman, Spitters 2009; Lurie 2009; Kashiwara, Schapira 2006; Jackson 2004; Hellman 2003; Kanamori 2003; Johnstone 2002; Mac Lane 1998; Awodey 1996; Kontsevich 1995; Lawvere 1994; Grothendieck 1986; 1957; Adámek, Herrlich, Strecker 1990; Drake 1974; Artin, Grothendieck, Verdier 1972; Bourbaki 1972; Tarski 1938.

from finite sets to infinite sets is THE SAME as what is the corollary from the modularity theorem.

One can easily notice that the application of the implicit axiom at issue is able to "resolve" any Gödel insoluble statement just by virtue of the circumstance that the axiom in question and the Gödel incompleteness theorem ("Satz VI" in 1931's paper) are the two dichotomic versions of the same axiom just as Euclid's "Fifth postulate" and its negation.

So, Andrew's Wiles proof is not correct, on the one hand, but that incorrectness is due to an unarticulated distinction, after which the Gödel incompleteness "theorem" is really a theorem once mathematics is identified with "Gödel mathematics" following the general prejudice of Modernity about the Cartesian abyss separating mathematics from reality.

Thus, Andrew Wiles can in fact and really replace Fermat's last theorem formulated in the standard mathematics (sharing the "incompleteness theorem") with the "same" theorem in the alternative axiomatics therefore postulating the negation of the "incompleteness theorem" (for which the notation of "Hilbert mathematics" is introduced and reasoned in other papers: *Penchev 2024 April 16; 2023 May 3*; *2023 July 16; 2023 January 3*; *2022 October 21*).

So, Andrew Wiles's proof and Gödel incompleteness theorems share the same absence of the articulated distinction but from opposite viewpoints: the former belongs to Hilbert mathematics (and more exactly said, to Hilbert arithmetic), and the latter, to Gödel mathematics, and thus, to standard relation of arithmetic to set theory obeying the Gödel dichotomy.

That elucidation in full detail is necessary since Riemann' s hypothesis is analogically a theorem in Hilbert arithmetic (thus in Hilbert mathematics) rather than in Gödel mathematics after encountering (though rather unexpectedly) with the "teleportation theorem" as follows:

Literally, the teleportation theorem states that quantum information though transmitted instantaneously needs additionally two opposition (wrongly reckoned to be "two bits") or a bit of classical information for being completely restored by its receiver. However, it will be now reinterpreted by means of the Gentzen / Gödel semi-permetability of the transition between finiteness and infinity. So, a qubit is reducible to a bit without any "hidden variables" (i.e. in "Gentzen's direction"), but not vice versa (i.e. in "Godel's direction"). So, the reinterpreted "teleportation theorem" means that a bit of classical information is necessary to complement it additionally if the transfer of quantum information is granted to be symmetrically two-directional just as that of classical information: in other words, if its transfer from "Alice" to "Bob" implies necessarily its converse transferability: from "Bob" to "Alice". Thus, a qubit is reduced to certain bit as for Alice rather than for Bob: so she has to message that certain bit by means of a "classical channel" since it is impossible to be transmitted by any "quantum channel" including that by which Alice's qubit has been instantaneously transmitted without "distortions" and "disturbances".

As for Riemann's hypothesis properly, one encounters Wiles's problem in relation to Fermat's last theorem: the further shown proof refers to Hilbert arithmetic and thus to Hilbert mathematics rather than to Gödel mathematics. Following the just sketched pattern borrowed from the teleportation theorem, a bit of classical information misses fundamentally for the same proof to be valid in the standard mathematics. In fact, the "same bit" is not available in Wiles's proof as well, and that absence is fundamental and irreparable. That bit is absolutely certain: it is that of the two options of "Hilbert mathematics" versus "Gödel mathematics" in both cases. It can be explicated as for the intended proof of Riemann's hypothesis in the following way:

The fundamental mathematical "relation of equivalence" is reflexive in definition so that $"A = B" \Rightarrow "B = A"$. However, it is not true (though loosely speaking) if "A" is infinite, but "B" is finite and the "cut rule" is investigated as a suggested relation of equivalence, namely logical equivalence. Then, $"A = B"$ (after Gentzen's cut rule theorem), but nonetheless $"\neg(B = A)"$ (after Gödel's incompleteness theorem). In the case granted to be relevant to Riemann's hypothesis, the just outlined asymmetry means that a qubit implies a bit, but not vice versa, and the asymmetry at issue is established also by virtue of the "teleportation theorem". Meaning the forthcoming consideration (including what is forthcoming in the "Part II" of the paper), the following paradoxical corollary is implied. Riemann's "zeta function"[9] implies that all its "nontrivial zeros" share the property Re{0}=½. So that: it is a relation of equivalence about two infinite sets, which is a particular case of the class of all bijections between two equally powerful sets where the bijection is the identical mapping. Consequently and only as for infinite sets (by virtue of being bijectively mappable into true subsets applied to the identical bijection), the statement reflexive to the aforementioned statement about the zeros of the "zeta function" is not true in the standard mathematics, in which Gödel's incompleteness theorem holds, and then, Riemann's hypothesis is the serial Gödel insoluble statement, in particular being due to the extraordinary observation that the identity (i.e. not only being equally powerful) of an infinite set with its true subset (even including finite) is both admissible and non-reflexive:

On the contrary, if one would like to restore the usual reflexivity of the equivalence or identity (borrowed from finite sets) to infinite sets as well, the inaccessible subcountable cardinal numbers are unavoidable to be involved, though eventually (and rather) implicitly. So, the framework of the standard mathematics turns out to be necessarily abandoned. Wiles's proof of Fermat's last theorem has abandoned it though implicitly and even rejecting to have done it.

The present paper does it, on the contrary, explicitly, even expressly emphasizing it after demonstrating that Riemann's hypothesis is an insoluble statement including by virtue of the "teleportation theorem" implying that at least a "zero" is to be added following the "converse direction" (loosely speaking, that "from finiteness to infinity") therefore rejecting Riemann's hypothesis as in relation to the strict framework of the standard mathematics. So, the efforts for

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[9] For example, Ovchinnikov 2019; Rodgers 2018; Ford, Luca, Moree 2014; Choi, Chung, Kim 2012; Rządkowski 2012; Sekatskii, Beltraminelli, Merlini 2012; Schumayer, Hutchinson 2011a; Sondow, Dumitrescu 2010; Voros 2009; 2006a; 2000; Chahal, Osserman 2008; Ng 2008; Pegg 2004; Šležəvičien, Steuding 2004; Conrey 2003; Corfield 2003; Derbyshire 2003; Elizalde, Moretti, Zerbini 2003; Ye 1999; Connes 1998; Csordas, Norfolk, Varga 1994; 1991; 1986; Elizalde 1994; Rubinstein, Sarnak 1994; Sankaranarayanan, Srinivas 1993; Odlyzko 1992; 1987; n.d.; Csordas, Varga 1990; Cobeli, Vâjâitu, Zaharescu 1989; Patterson 1988; Srivastava 1988; Goldston 1987; Woodcock 1987;Dieudonné 1981; Ribenboim 1979; Stechkin 1970; Selberg 1956; 1946; Riemann 1859;

proving Riemann's hypothesis in that framework are meaningless since it is a serial Gödel insoluble statement.

Instead of that unachievable objective (in fact, even ridiculous. nonsense, and absurd just as "perpetuum mobile" in the framework of Carnot's thermodynamics), it is researched and proved in Hibbert arithmetic showing that it is a corollary from the way in which its two "senses", "narrow" and "wide", are *reflexively* related.

Simultaneously, the example of the present consideration being inherently philosophical rather than technical and mathematical (obvious even for the absence of the usual "abracadabra" of numerous artificial notations) serves for elucidating the proper subject of the present section: mathematics is not difficult if one thinks rather than calculates since thinking is much more relevant to humans than calculations, even more so that all computers nowadays make those incomparably better and faster than any mathematician.

Indeed, one might figuratively represent and oppose the ostensibly "mathematical" approach, i.e. reducible to some finite calculation, *versus* "philosophical thinking" as follows. The former would be properly local just like a human moving within a forest and thus able to see only separate trees, so that any of them reveals the horizon of a finite number of neighboring trees, but never the forest itself, even reaching the end of the forest, so that there are no more trees "in front", only "in back". One might see the forest as a whole "from above, from the air", at a sufficient height, or on a map (for example, by means of an app and the personal smartphone).

Obviously, the exhaustive description even of all the trees of the forest is irrelevant for finding the shortest pathway between two trees, respectively the second one starting from a given one. Indeed: if the latter is well enough described, one might eventually reach it merely wandering through the woods or combing looking for a particular item with enough searchers spread out close together. Though the probability for finding that tree or any item is much greater in the latter case, it is too small in comparison with searching "from the air", e.g., by a helicopter or knowing the location of the tree on the map thus needing only the best route to it (or at least a possible way, though not the most optimal one).

One more feature of the metaphor is relevant. The viewpoint "from the air" or "on the map" means a *new dimension*, which facilitates crucially the research whether being a new "physical" dimension or a "mental and abstract" one as in the case of a map. The "quantum leap" in a new dimension is what allows for the essential, including exponentially shortening of the search time. The suggested metaphor can be applied for distinguishing the proper mathematical and philosophical viewpoints to the class of all proofs (including incorrect ones). In fact, all great mathematicians have tended to the "philosophical approach" for proving theorems since their "insights" correspond to a new, but mental "dimension", so that they have been able to observe the joint context including the premises and conclusion simultaneously in the figured image as a whole, and then, to behold and discern a finite syllogism for linking them, therefore making a relevant cut-eliminitation, eventually resulting in a correct proof traceable and confirmable by other mathematicians. Obviously, the probability to be found by "combing methodically the forest" (not speaking of "wandering randomly") is insignificant in comparison with the sudden "insight", however unpredictable and uncontrollable, unfortunately. Of course, reliable methods, whether philosophical or not, for insight would be amazing therefore exponentially accelerating human cognition.

One is forced to recognize that any general and reliable "methods for insights" are forbidden from the episteme of Modernity just as any methods allowing for bridging the opposite shores of the Cartesian "abyss". The single and unique arbiter (excluding "God Himself") able to bridge them is humankind in general or corresponding human professionals deciding at their own discretion about any problem touching entities of both shores simultaneously. Those fundamental human decisions (mathematically, belonging to the class of choices) correlate with human hierarchies (mathematically, belonging to the class of well-orderings). Those "methods of insight" would wave the monopoly of human decisions of how to bridge the two "shores" of the (non-existent really) "abyss" alleged by Cartesianism to establish humankind's absolute domination all over the rest nature as "God's only vicegerent on earth".

Furthermore, one can consider the mathematical insight for discerning an eventual finite proof (such as those of famous mathematical proofs) if the two "shores" of the seeming "abyss" would be necessarily linked by any mathematical statement being discussed. The conjecture more or less reasoned and justified in other papers (e.g. Penchev 2023 July 16) identifies the following classes: the one is all Gödel insoluble statements; the other is that of all *mathematical* (expressly emphasizing, just "mathematical") statements questioning or implicitly touching the eventual "bridge" between the shores at issue. Thus, a corollary from it states that the seven "Millennium Problems" offered by CMI, and consequently Riemann's hypothesis in particular, are those insoluble statements necessarily existing in the standard mathematics thus appealing to overcome both Modernity and its cognitive "episteme": to be whether destroyed or replaced by their (preferably, conservative) generalizations.

As for Riemann"s hypothesis itself, it should occupy a privileged position among them since its correctness might be inferred from the just above formulated identification of those two classes thus depending directly on that meta-statement both philosophical and mathematical (and even physical, thus implying experimentally testable conclusions in the area of quantum information, originating immediately from Riemann's hypothesis), Indeed, let one consider afore-suggested case for whatever mathematical conjecture needing or implying a bridge between the two shores thoroughly within mathematics itself. Being divided by an abyss, one can admit two independent suggestions as well as postulate their identification:

The one states that the "logical distance" (i.e., the minimal cardinal number of the set of mediating implications) is infinite. The other would suggest that the concept of "logical distance" cannot be at all defined in the discussed case. In fact, one may involve the situation about Euclid's "fifth postulate" (to be even isomorphic), furthermore investigated in full detail a long time ago. For any two parallel lines in Euclidean space one may postulate: (1) they cross each other in a newly introduced "infinite point" (as, e.g., projective geometry adopts); they do not cross each other (what is the literal position of geometry bracketing the following two options):

(2.1) they do not cross each other in any finite distance; (2.2) they do not cross each other at all, including any newly introduced infinite point; (3) one can consistently identify (1) and (2.1) granting that projective geometry generalizes Euclidean geometry.

Isomorphically applying the outlined structure: (1) there exists an infinite syllogism able to link any two statements, each of which belongs to the one of the two opposite shores and then "cut-eliminating" it, this implies a bridge between the shores buildable just between the two statements at issue; (2.1) that bridge buildable in Gentzen's manner is impossible so that the interpretation of the above option (3) would be prevented if one has in advance granted the cut-elimination, even more so due to the Gödel incompleteness theorem (since it may be identified with "2.1"). However, meaning the eventual "semi-permeability" of the bridge between finiteness and infinity (allowing for "Gentzen" transitions from the latter to the former, but not vice versa, after Gödel), the option (3) can be conserved even keeping the cut-elimination theorem. In fact, that is the commonly accepted solution of the standard mathematics thus conserving for it to be absolutely consistent involving the extraordinary semi-permeability between finiteness and infinity because of the introduction of "actual infinity" by set theory (more exactly said, since "actual infinity" is not consistent with the axiom of induction in arithmetic).

Indeed, one can easily demonstrate that the semi-permeability at issue is a corollary only from the simultaneous application of the axiom of induction and axiom of infinity borrowed from the Peano and ZFC lists of axioms according to arithmetic and set theory correspondingly, and granting (as usual) for both to be first order logics to the same "zero order" logic of Aristotelian propositional logic. Once the axiom of induction has been in advance admitted (thus, in the one, "impermeable" direction, that "from finiteness to infinity"), it implies that any syllogism is finite[10], and the set of all Gödel insoluble statements is not only nonempty, but infinite. Speaking loosely, any statement belonging to the set at issue needs an "infinite" syllogism, thus being inaccessible once the axiom of induction holds.

However, in the converse, "permeable" direction "from infinity to finiteness", thus granting the axiom of infinity rather than that of induction being inconsistent with the former, the cut-elimination theorem is correct and provable needing in the final analysis the obvious statement that any infinite set (thus and particularly, the elements of which are statements belonging to the same syllogism) contains finite subsets. One "temptation" consists in the replacement of an arithmetic statement by its set-theoretical counterpart ostensibly, but only seemingly and in fact wrongly.

The logical mistake is isomorphic to proving a statement formulated in Euclidean geometry by its ostensibly equivalent replacement with its counterpart in non-Euclidean geometry as for all theorems for which the fifth postulate is a necessary premise. The mismatch for them is obvious, e.g., as about the sum of the angles of any triangle. Regardless of the fact

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[10] Following the axiom of induction, any natural number is finite since: "1" is finite; if "n" is finite, "n+1" is finite; consequently, any natural number is finite according to the axiom of induction. Then, any syllogism consisting of any natural number of steps is finite as well.

that all the rest theorems are identical, the counterparts, each belonging to either, also cannot be granted to be the same due to the difference in their corresponding contexts.

If one moves any material macroscopic “thing” from a room into another, that “thing” is granted to be the same and independent of the mismatching contexts of two different rooms. Classical science defines all entities being things absolutely rather than relatively to their contexts. Nonetheless, any quantum entity in a certain state and described by a corresponding wave function is relative to the context of the whole quantum system to which it belongs due to the phenomena of entanglement even in the case of zero entanglement. Thus, it cannot be granted to be the same (or indistinguishable) being moved in another quantum system,

An amazing illustration is the debate about Andrew Wiles’s proof of Fermat’s last theorem since the former infers the latter as a corollary from the modularity theorem. The issue consists in the suggestion that Fermat’s original statement and its counterpart deduced immediately from the modularity theorem are not the same though being formulated identically to the their inconsistent contexts: Fermat’s last theorem is an arithmetic statement thus needing the axiom of induction versus its set-theoretical counterpart inferred from the modularity theorem and thus needing the axiom of infinity in the final analysis.

One can even suggest the exact spot in Wiles’s proof[11] itself where and when in its real history, their replacement has been accomplished: Andrew Wiles had retired and isolated for about ten years to prove the Tanyama - Shimura - Weyl conjecture linking the discrete modular forms and smooth elliptic curves from which Fermat’s last theorem follows necessary as a corollary. After he had finished the proof at least as a detailed plan, he presented it to his colleague Nick Katz, a mathematician at Princeton, who would review the proof and found the crucial gap, (at a “private seminar” during a few months), who approved it, and then sent it for reviewing and publishing into the authoritative mathematical journal “Annals of mathematics”, where Wiles would send his paper, which would be published there in 1995. Since the proof was too long and encompassed a huge number of mathematical areas, it was divided into relatively standalone parts and distributed between leading mathematicians in the relevant fields. Nick Katz was one of them and took a small part of about 10-20 pages, in which he found a crucial mistake, due to which, the text was returned to Andrew Wiles for revision and correction, who managed to do this for a year or two. The corrected proof was accepted by the team of reviewers after that (Singh 1997: 265-304) .

Meaning the present context about the “semi-permeability between finiteness and infinity”, one may admit that the correction at issue substituted the original, proper arithmetic statement of Fermat with its identically formulated counterpart, now in the context of set theory inconsistent to arithmetic after Gödel (1931). The replacement remained implicit, imperceptible, and unarticulated due to the change of the context, in which a statement has been formulated literally in the same way in both cases. In other words, one is to distinguish: (1) Fermat’s last

[11] Here are concentrated references in the present study relating to Wiles’s proof of Fermat’s last theorem: Wiles 1995; Singh 1997; 1997a; McLarty 2010; Darmon 1999; Cornell, Silverman, Stevens 1997; Resnik 1997; Gouvêa 1997; Ribet 1995; Mazur 1994

theorem in the complete context of the whole proof; (2) Fermat's last theorem in that part, checked by Nick Katz, and only repaired. Fermat's last theorem is not the same as an arithmetic statement (as, in fact, Fermat formulated it) and as a corollary from modularity theorem. The replacement of the two literally identically worded statements was hidden by the implicit substitution of the arithmetic context by its set-theoretical counterpart being inconsistent to the former.

One more illustration though imagined might be a counterfactual situation, after which one of the numerous inventors of "perpetuum mobile" would suggest to the French academy a too complicated mechanism: so sophisticated for encompassing mutual transformations of different forms of energy that its verification would be distributed between different scientists, each of whom will be a leading specialist in one of the scientific fields corresponding to all energy transformations within the checked mechanism supposedly demonstrating at least one violation of the principle of Carnot's thermodynamics. One of them might find a backdoor, through which the environmental energy would flow into the perpetual motion machine, in that part of the mechanism which should be carefully considered by him. Then, the mechanism would be returned to its inventor for removing that backdoor. For a year or two of effort, the backdoor might be indeed closed, and the French academy would approve that embodiment of "perpetuum mobile".

However, we know that it is impossible, and the closing of the backdoor in question had somehow, but necessarily opened other backdoors for environmental energy inflow, most probably in the relation of the repaired part to some others of all the rest. We would even assume a possible collusion for defrauding the French Academy. The replacement of the context as the basis of intentional or unintentional fraud is obvious: the context of the perpetual engine as a whole is substituted by that of the part returned for repairing, and the fundamental law of energy conservation assists its necessary existence to be suspected or even categorically stated including without or before its explicit demonstration. Energy conservation is what allows us to state that closing the initial backdoor has in turn opened at least a new one for inflowing energy from its environment even without being able to explicitly demonstrate the newly opened backdoor.

Analogically, one can show rather elementarily that Fermat's last theorem is a Gödel insoluble statement in the standard mathematics by the mediation of Yablo's paradox (Penchev 2021 March 9). Nonetheless, its set-theoretical counterpart is provable in the same framework relying on cut-elimination in the final account. However, the substitution of the former by the latter, though being incorrect, may be involved quietly and by stealth, rather implicitly only by means of the change of their linked contexts (contradicting each other being meant simultaneously). Nonetheless, one should notice that the nonlegitime contextual change is equivalent to a relevant Gestalt change accompanying all crucial scientific discoveries resulting in "scientific revolutions" (e.g., after Kuhn).

So, what has been done silently realizing it to be ostensibly "criminal" thus needing to remain hidden and secret, can be simultaneously done absolutely publicly since the contextual and Gestalt changes are equivalent to each other and even might be bijectively mapped, however therefore appealing to a new scientific revolution. As for mathematics itself, it would not be especially revolutionary in its aspect to the proof of Fermat's last theorem, only postulating the transition between finiteness and infinity to be "two-way" instead of its "semi-permeability" in the standard mathematics, after which the secret replacement of the context can be openly heralded to be a Gestalt change thus eventually implying a scientific revolution though seeming not properly revolutionary from the internal viewpoint of mathematics itself.

However, the change at issue, touching only the relation between finiteness and infinity implies grandiose sequences as for the localization of mathematics in the contemporary organization of cognition fundamentally transforming the latter and even in relation to socially ordered hierarchies inherently linking social and scientific revolutions (in much more detail investigated in other papers: e.g., *Penchev 2025 February 13; 2024 October 2; 2024 August 5; 2023 December 6*).

The present section of the study will pay sharply and discernibly attention to a quite particular aspect of that fundamental Gestalt change, namely the comparison of the usual viewpoint to any mathematical proof (to be an extended syllogism and thus always a finite calculation though both mathematical and logical), on the one hand, with the newly offered philosophical viewpoint, on the other hand, both being especially clearly distinguishable as for problems being insoluble in the standard mathematics after Gödel (1931). Due to the cut-elimination "semi-permeability", the former implies the latter, but not vice versa, as for the standard mathematic, after which in particular, Wiles's proof is "criminal" therefore needing the semilegal contextual change about Fermat's last theorem also traceable in the history of the publication as sketched above. In fact, it is not really "criminal", but appealing to a civilization (rather than only scientific or social) revolution replacing Modernity by Postmodernity (but in an absolute rigorous, ontomathematical meaning): an objective, which Andrew Wiles, realizing himself as a (modern) mathematician in a proper and narrow sense (thus not as a philosopher like Descartes, Leibniz, or Newton), would not even suspect, therefore needing the semilegal change of contexts at issue regardless of becoming aware of the replacement or not.

However, after ontomathematics. newly introduced, not only Wiles's proof is legitimate and correct, but also the philosophical viewpoint to the mathematical proofs becomes very important practically: and here is why and how. From the viewpoint of the standard mathematics in turn originating and thus consistent with Modernity, the mathematical proof implies the option for it to be philosophically reflected, but not vice versa: there does not exist any "philosophical method" for theorems to be really proved in its framework. At best, philosophical considerations can serve as heuristic and guiding, but anyway ambiguous enough and contradictory in fact. So, they would replace the real mathematical proofs with speculations, ones' opinions and interests, as well as with their abstract, but ideological reflections. So, philosophy should be kept away from mathematics and all the rigorous science sharply distinguishing the latter from "art and humanities" (where philosophy has been exiled).

However, if the mutual "permeability" between finiteness and infinity has been in advance postulated thus suspending Gödel mathematics and substituting it by Hilbert mathematics, there exists necessarily some unambiguous pathway from philosophical reflections to an absolutely rigorous and correct proof for any mathematical theorem, in principle. i.e., *ontomathematically*. This means, a counterpart of the "converse cut-elimination" links philosophical reflections as premises and a theorem, to which they are relevant, as a conclusion, or in other words, infinity is able to be unambiguously restored from finiteness by means of only doubling the latter, (which Hilbert arithmetic realizes in particular involving two dual Peano arithmetics). The study's objective is just to trace and also reflect the syllogism starting from philosophical considerations (as those in the article) to the eventual proof of Riemann's hypothesis itself as a conclusion from them, thus more and less restoring the syncretic unity of mathematics, physics, and philosophy in Modernity's dawn, for example, revealable after Descartes, Leibniz or Newton if one dare interpret them in a proper *postmodern* manner.

Following that objective, mathematics is not an extended calculation (including or mainly logical) any more: even more so that any contemporary computer (including the readers' personal smartphones), always being a "Turing machine", is able to do that incomparably better and faster than any human (nonetheless, nobody thinks of the own smartphone to be really "smarter", i.e. more reasonable than its human possessor, or even merely "smart" in a proper sense).

So, mathematics is neither "difficult" any more since humans' minds and brains are rather inappropriate for calculations, nor their thinking is (reducible to) calculations. However, mathematics continues to be difficult in another sense, e.g., after Heidegger's "we do not yet think" interpretable as the inability of proper philosophical thought and consisting, roughly speaking, in the vision of abstract images: "philosophical insight" should substitute the extended (logical or not) calculations.

In fact, all great modern mathematicians, though always representing the final result as a finite proof, i.e. as a logical calculation, think just philosophically, as a necessary condition for their rare enough really mathematical capabilities. In other words, they are unavoidably "mental visioners" able to see, permanently to watch and observe the same abstraction, thus being inherently infinite as any other image "reflecting reality" or figured. After being mathematically educated, each of them can more or less successfully represent and describe it as a finite syllogism, therefore "bracketing" it for the purpose of communication (first of all, with other mathematicians). His or her colleagues after reading the proof, in fact, always create an own mental image starting from the shared syllogism similar to a two-dimensional hologram which restores a threedimensional image. Then, the colleagues understand the offered proof as well as they might observe that the "holographic" image is inconsistent due to some hidden logical contradiction within the syllogism, i.e. the proof is incorrect, wrong, false: however regardless of being correct or not, other mathematicians have to *understand* it before that, i.e. to restore "holographically" a mental image more or less similar to that observed by the author of the proof.

IV DOES PHILOSOPHY ALLOWS FOR ONE TO STOP CALCULATING IN ORDER TO START THINKING MATHEMATICALLY?

One is to put now the philosophical "problem of understanding", for example, following Heidegger and Gadamer's hermeneutics in its more or less practical meaning for distinguishing all humans' thinking from the Turung machine calculations regardless of how much (or rather "how many) extended they would be, including in relation to mathematical thinking being irreducible to those calculations. As for the just suggested "holographic metaphor" about the understanding of any mathematical proof being formally always a finite syllogism, i.e. a logical calculation, it is "dual" (or "complementary") to the (preceding "cut elimination" of the author's mind) "infinite" mental image, which has resulted into the written down proof, now being read and understood by other mathematicians, thus restoring more or less adequately the initial "hologramna" in the "author's head".

So, if cut-elemination recognized to be absolutely legitimate including within the standard mathematics collapses "from infinity to finiteness", the mathematicians' understanding (eventually generalizable by philosophical hermeneutics) runs in the opposite direction, i.e. from finiteness to infinity thus remaining strictly forbidden in the same framework for being self-contradictory (once it endeavors to overcome the incompleteness of arithmetic to set theory after Gödel). However, postulating for the "barrier" between finiteness and infinity to be two-way permeable in Hilbert arithmetic, the understanding of human thinking in relation to any Turing machine calculations, including the mathematicians' understanding of the proof at issue, is already thoroughly within mathematics and thus formally reflectable.

The problem of the just formal representation of understanding (initially borrowed from all humans' thinking) is crucial for "AI", which (or maybe "who" in the future) should learn to understand, including just a mathematician is able to "understand" the finite syllogism of the offered proof. The standard mathematics obeying the Gödel dichotomy (about the relation between arithmetic and set theory and consisting into the dilemma "either incompleteness or contradiction") in fact follows Cartesianism and its non-reflectable "mind - body" abyss establishing for humankind to be the only "Heron", "Erodius" able to link by the vehicle of the choice and decision between its two shores therefore establishing to be the privileged arbiter alone. Thus and in fact, the "mind - problem" is tabooed in Modernity, however being crucial for AI (needing Postmodernity in an absolutely rigorous meaning in order to appear or to be created). Thus Hilbert arithmetic (being inherently inconsistent with Modernity), therefore, is what is able to supply the key to AI after involving mathematicians' creative thinking within mathematics itself (i.e. already "Hilbert mathematics" being an alternative of the standard mathematics).

As an illustration, Wiles's proof is really accomplished within it since Fermat's last theorem is an insoluble statement in the strict framework of the standard mathematics, though its author himself is adherent to the latter and tried to hide the former. The intended in the present study proof of Riemann's hypothesis is also impossible in the standard mathematics, analogically being an insoluble statement within it. However unlike Wiles's proof only touches that fundamental problem, the eventual proof of Riemann's hypothesis would embodied it being a corollary from its formal and internally mathematical representation of the symmetrical (i.e.

“two-way permeable”) of finiteness (e.g., a bit of information) and infinity (e.g., a qubit of quantum information) since the symmetrical or (“mutually, two-way possible”) transition between “bit” and “qubit” is crucial for the proof of Riemann’s hypothesis.

However, the proper interest of present section is in the given particular trouble of how formally and mathematically the recipients of a mathematical proof, once they are really able to understand it, restore “holographically” the author’s initial mental image starting from its finite record as a syllogisms (meaning as a background also how to describe “understanding” as (or like) that of Heidegger and Gadamer’s philosophical hermeneutics furthermore being inherent and definitive for human thinking) and then, how to learn AI to understand following the successful pattern of human understanding.

Or in other words, how a qubit can be restored from a bit, to which the former has been collapsed, e.g. after measurement. Quantum mechanics, especially that is not within the “Procrustian bed” of classical quantum mechanics (culminated in the Standard model) resolves the same problem since it proves that “no hidden variables” in it and thus the eventual collection of all possible measurements of any quantum entity coincide with reality, or speaking in a Kantian manner, with the quantum entity “by itself" (“an Sich”). What is to be abandoned in classical quantum mechanics is just the analogical “semi-permeability” from the coherent and thus nonlocal state “by itself” “a priori”, i.e. before measurement, to the statistical and thus local collection of results “after measurement" or in the Kantian manner “for us”, “a posteriori”. Thus, classical quantum mechanics is univocal with the standard mathematics originating from the episteme required by Modernity since they share the same “semi-permeability”.

“Unfortunately”, epistemology (including that after Kant’s “Copernican revolution”) is not satisfied by that “semi-permeability” and postulates the identity of the quantum entities “by themselves” and “for us” (eventually, after a newly introduced by him “transcendental method”). As for the particular problem about the “bit-qibit” relation, this is to establish for it to be symmetric, and the corresponding transition, two-way. That “tension” between the two alternative options, either “one way” or “two-way”, about the transition between finiteness and infinity will be now interpreted in relation to the special problem for the “understanding” of any mathematical proof being always a finite syllogism, in order to be represented formally and mathematically though being a hidden and sudden “insight” by itself, a Gestalt change, inaccessible to be reflected by the mathematician's conscious experience whether really or ostensibly.

What is given as the initial conditions of the problem are both transitions: (1) the former is within the author’s mind who has seen, watched, understood a mental image in which the premises and conclusion of the theorem are situated in the same “Gestalt” in a consistent way. However, the trouble is how the mathematician’s insight being always one’s subjective experience to be communicated and then, checked and utilized by other mathematicians. So, the author is forced to “collapse” the insight, i.e. an infinite mental image to a finite syllogism, fortunately, thoroughly writtenable down as a tuple of information on some material carrier such as paper, computer memory, online accessible resources, etc. Only then, (2) one might start reading and understanding it therefore restoring the author's or authors' initial Gestalt, their "mental images", however being meanwhile cut-eliminated into the written down syllogism for being communicated. Those colleagues reading the same text, nonetheless, will understand it by means of a different, personal, subjective, unique and thus untransferable Gestalt, mental image featuring the human capability of understanding, however remaining inaccessible to any Turing machine computer and thus being fundamentally irreproducible by it.

If the problem searching for its solution has been in advance formalized as just above, it can be already represented formally and logically in terms of the transition between the extensional and intentional description of the same set whether finite or infinite as follows. One is to grant the set of all possible subjective Gestalts or mental images, speaking metaphorically the "forest" (more exactly different subsets of all "trees" of that "forest") necessarily including the "trees" of the premises and conclusion of the theorem at issue. Then, the author, somehow managed to see any relevant part of that "forest" (this means at least one element of the afore-defined set), wishes to describe the set itself as the class of equivalence of all its elements, i.e. all possible subjective understandings, however having only a single sample (at best, a few ones) in the own head. Following the very well established mathematical tradition, what is sufficient is to be only described and written down the necessary logical pathway able to link the premises and conclusion by a consistent syllogism regardless of not being the optimal one initially, but prefectable in the future. Recording that consistent syllogism, the author suggests therefore the relevant characteristic property (consequently, finite) unambiguously describing a set of possible Gestalts, mental images, parts of the "forest", i.e. "understandings of the intended theorem": and what is crucial, fortunately, already absolutely communicable being finite unlike the author's initial, quite subjective mental image.

Obviously, the circumstance for that procedure collapsing the initial mental image to be always accomplishable is guaranteed by the cut-elimination theorem. However, in the present context, it is seen otherwise: as converting (at least) one sample belonging to the extension of the set of all possible understandings (namely, that insight somehow occurred in the author's head) to the relevant characteristic property of the set of many possible understandings, i.e. by its intension, as a consistent syllogism able to link the premises and conclusion. Indeed, any syllogism borrowed from traditional logic can be equally represented and transformed, if need be, by Boolean algebra, and thus, as a logical property explicitly, which can be in turn interpreted to be a set-theoretical characteristic property, as in the investigated case, for example.

All mathematicians who are readers are to accomplish the inverse mental operation: having the characteristic property of the set of understandings, i.e. the syllogism of the proof, each of them needs to reveal at least one "subjective" element of its extension, i.e. to *understand* the proof. So, they move "from intensionality to extensionality" in order to understand, and that latter direction can be chosen for the formal and mathematical definition of "understanding" by simultaneously uniting the transition from (arithmetic) finiteness to (set-theoretical) infinity with the (dual) transition from (logical) intension to (set-theoretical) extension.

Obviously, the standard mathematics after Gödel (1931) prohibits the former transition, but not the latter, for example, again after Gödel (1930), therefore appealing rather paradoxically to be unified remaining impossible in the strict framework of the standard mathematics absolutely distinguishing (Peano) arithmetic and (classical) propositional logic. Thus, "understanding" also cannot be formally defined within the standard mathematics therefore tabooing an almost "sacral" capability which features uniquely "human thinking", consequently inaccessible for all Turing machine computers.

However, one should question before that: whether the Turing machine is able at all to accomplish the inverse, "cur-elimination" transition, now interpreted as that from extension to intension. According to the "forest" metaphor, whether it might formalize the context of a relevant part of the "forest" into a correct syllogism able to link the premises and conclusion of the theorem being proved; i.e., if the input data are a fussy and unclear context, it will be able to reduce them (or it) to an ambiguous characteristic property, also traditionally representable as a syllogism. Any creative professional mathematician manages sometimes to do it resulting in a correct proof following the general pathway guaranteed by the cut-elimination theorem, and thus thoroughly within the standard mathematics. The problem is whether (loosely speaking) some relevant software might at all exist so that those input data forcing a Turing machine computer to "get stuck", that software program would be able to "halt" it therefore producing an ultimate finite result, however being relevant to the input "infinite" data. Supposedly, the just formulated problem is not investigated enough.

Regardless of the eventual solution of the above problem, the mathematicians who read, understand and check the proof are to accomplish the converse transition. Again by the "forest" metaphor, they have to imagine the "forest" as a whole (or a relevant part of it) being supplied only by the "road" (i.e. by the syllogism as a "logical pathway") between the "trees" of the premises and conclusion. So, meaning those considerations, one can conjecture that the elucidation of the two-way transition, both from extension to intension and vice versa (in turn being in detail researched by contemporary logic including mathematical logic) is the essence and ground of the formal and mathematical concept of "understanding"; as well as particularly to the present context of Riemann's hypothesis: for "bit" to be interpreted as the class of equivalence of all "intensions" of "qubit", and respectively vice versa, "qubit", as the extension of "bit".

A preliminary notice is necessary as for the formal and mathematical definition of "interpretation" being inherently linkable to the also formal and mathematical "understanding" as above: even more so that contemporary philosophical hermeneutics connects them closely. Interpretation is the transition from a universal zero-order logical expression (being syntactically correct in definition) and the same, however in a certain first-order logic, i.e., a "mathematical theory": thus, the specific tuple of axioms of the theory at issue is added to the universal list of axioms of propositional logic during the so-defined interpretation. Then, "interpretation" in a wide (or "wider") sense would add more consistent axioms therefore constituting a more specific structure starting from a more general one. Furthermore, one might introduce the concept of "deinterpretation" meaning the inverse operation, i.e. removing axioms conserving consistency.

Anyway, both interpretation and deinterpretation being inherently intensional admit their extensional counterparts therefore decreasing or increasing a certain class of equivalency. Understanding can be also seen as a kind of generalized interpretation and deinterpretation able to transcend in both directions accordingly through the barrier between extension and extension eventually identifying it (being logical) with its counterpart between finiteness and infinity (being set-theoretical): even more so that that propositional logic and set theory share the same structure of Boolean algebra so that the former is to be intentionally seen versus the latter, which in turn is seen extensionally therefore simultaneously observing "understanding" to reveal in both directions the equivalent dual counterpart: this means the intensional ("cut-elimination") twin of an extensional definition as well as vice versa.

The Turing machine is fundamentally unable to transcend between that barrier since transcending it would be equivalent for the Turing machine to resolve the halting problem. Consequently, it is inherently deprived of the human capability of "understanding" gifted to humankind by nature (or "by God Himself"). Nonetheless if one considers a pair of Turing machines furthermore idempotently dual to each other and supplied by the option to communicate (though initially, in a loose and intuitive sense) regardless of their duality, they (but only together) would be capable of understanding, working jointly, i.e. (somehow) communicating to each other.

Now, one might stare at what should mean for two Turing machines initially defined to be absolutely independent of each other (after being "dual") to communicate, i.e. to exchange information furthermore adopting it. Speaking loosely, the mutual acceptance of information means for certain segments of their types to be shared, regardless of which of them has calculated it (being the sender of information, and its twin, the recipient of the communicated information) mutually restricting each other, the degrees of freedom for the forthcoming computations, so they can be equivalently interpreted to be "entangled" after exchanging information.

In other words, two Turing machines communicating to each other (as this is rigorously defined above) constitutes a "quantum computer", and two bits "communicating" to each other (furthermore definable exactly after following the same pattern of what communication should mean) constitutes a qubit, in turn linkable to Riemann's hypothesis though here inferred from the formal approach to understanding, and starting (in the present section) from the problem of how the "mathematical proof" to be reflected philosophically: using again the "forest" metaphor, how a mathematical proof would be alternatively accomplished by its context, from the "forest", in which it is situated, rather than by the explicit pathway connecting the premises and conclusion by a relevant, more or less sophisticated syllogism as usual, not needing at all or as if "bracketing" its understanding. In fact, the understanding at issue is always done by human mathematicians, however remaining absolutely non-reflectable in the standard mathematics in turn originating from the Cartesian episteme of Modernity.

Then, the conclusion of the present section is that "philosophy does allow for one to stop calculating in order to start thinking mathematically" however only as far as the rigid framework of the standard mathematics has been in advance broken. The same option is formally representable by the relation of "bit" and "qubit", in turn able to underlie the proof of Riemann's hypothesis in Hilbert arithmetic since the "bit - qubit" relation is what unifies its "narrow sense" and "wide sense".

V MATHEMATICAL PROOFS IN ONTOMATHEMATICS: FROM HILBERT ARITHMETIC TO HILBERT MATHEMATICS

As this is explicitly demonstrated above, the mathematical proof in the standard mathematics is necessarily restricted to the resultative syllogism connecting the premises and conclusion of any theorem. It needs (human) mathematicians able to understand, but the concept of understanding fundamentally cannot be formally represented. Indeed, the cut-elimination theorem supplies the existence of a finite syllogism, but not the constructive way for the relevant syllogism to be revealed starting from the initial understanding somehow realized by the author, thus also needing the creative capabilities for reducing the subjective mental image (which the author's understanding is) to the ultimate syllogism not being unambiguously implied and thus predetermined by the former.

As for the understanding by itself and real subjective understandings of the author and readers of the proof, they need human mathematicians and their thinking only able to transfer over the Cartesian "abyss" established by Modernity also to mathematics. The standard mathematics (as mathematics is reduced during and by Modernity, or Gödel mathematics) is situated only on the mental shore of "mind", where only the ultimate syllogism can be thoroughly located and correspondingly the mathematical proof is necessarily identified with the syllogism needing mathematicians gifted as all humans by capability of understanding, by which are to "steal" the proof at issue "from the Gods" (as in the myth of Prometheus).

The reason for that identification originates from the location of mathematics in Modernity to be absolutely divided and opposed to "body", physics, the material world, etc., all of which are situated on the other shore. If any connection or communication between the two shores is anyway necessary, they can be accomplished only by people, by virtue of an alleged monopoly for their ostensible domination over nature, which in turn is granted to be "blind" and as if exiled thoroughly on the opposite, material shore. Thus the mathematical proof in particular needs human mathematicians due to the whole social and scientific organization during and within Modernity, who are anyway rather paradoxically doomed to be unable to "understand the understanding itself", even only to reflect the misunderstanding at issue:

In particular, the syllogism of any given mathematical proof is somehow supplied by mathematicians due to their creativity but not being reflectable by themselves. The reason for that consists in the necessity for what is properly "understanding by itself" to link the two shores thus turning out to be situated within the absolutely inaccessible abyss which divides them unsurmountably. So, the authors manage somehow to transfer themselves to the opposite shore, after which they return into the initial shore of mathematics however already supplied with the “stolen” finite syllogism, triumphally heralding their success to all other mathematicians or merely curious citizens eagerly awaiting them on this shore by means of the journal article publishing the text of the cherished proof.

Provided by it, other mathematicians drive off to an allegorical “understanding travel” whether confirming or rejecting the proof at issue. Unfortunately, they are also incapable of reflecting and describing their understanding since they have miraculously understood the proof with sudden illumination and insight. Even if some mathematicians among both authors and readers would manage to realize consciously the way for understanding in detail, therefore suggesting a more or less formal method of understanding, they will collide the taboo of Modernity veiled all topics about the reflection on crossing the Cartesian abyss. However, the present paper does not confess, recognize or obey the taboo in question; even much more, it endeavors to assist the efforts about the ridiculous prohibition to be destroyed and suspended in the forthcoming future of Postmodernity.

One might rather instructively and productively complement the “abyss” metaphor by the “forest” one. Even more, the forest metaphor can be involved in two ways: (1) an “abyss” to divide two or more “trees” alleged to be linkable by a given problem, i.e. situating them on its opposite “shores”; (2) by the “abyss” dividing any “trees” by themselves and the “forest” as such. The latter version is more general since it includes the former as a particular case including also the case for the trees not to be divided by an “abyss” meant to be a metaphor of Cartesianism itself. Considering now both above options of the latter, they can be further interpreted as follows: (2.2) finite syllogism or calculation might be the relevant pathway able to connect “trees” not being cut by an “abyss”: infinite after have “cut” by an “abyss”.

On the contrary, Hilbert mathematics (already introduced and described in detail in other papers: *Penchev 2024 April 16*; *2023 July 16; 2023 May 16; 2023 January 3; 2022 October 2*) contents "understanding" (mathematically formalized) inherently and in definition. In particular, the underlain by it “Hilbert mathematics” does not contain any insoluble statements: unlike the standard mathematics since the mathematical proof (as a “finite syllogism”) and its understanding (as an “infinite syllogism”) are equivalent (rather than only the latter to imply the former by virtue of the cut-elimination theorem as in the standard mathematics). One may use the metaphor (or eventually, the interpretation including in a rigorous sense) of Riemann's spherical geometry being an alternative of Euclidean geometry so that any lines cross each other after an arbitrary, but always finite distance thus not allowing them to be ever parallel. The underlying idea suggests the length of the proving syllogism and the distance after which two certain lines will cross each other to be identified.

The “qubit-bit” two-way relation suggested to imply Riemann’s hypothesis in Hilbert mathematics (but not being relevant in the standard mathematics) connects any (finite) syllogism and its (infinite) understanding, on the one hand, and the “narrow” and “wide” senses of Hilbert arithmetic, on other hand. Indeed, its narrow sense also representable by two dual, anti-isometric Peano arithmetics originates from its wide sense (in turn being isomorphic to the qubit Hilbert space) under the additional condition for the identification of the n^{th} arithmetic unit with the nth "empty qubit" of the qubit Hilbert space where "empty qubit" means the class of equivalence of all possible values, which might be, e.g., "recorded" in the corresponding nth "empty qubit".

However, and furthermore, Hilbert arithmetic postulates that its two senses are equivalent to each other rather than only the narrow sense to be derivative from the wide one. Thus, any arithmetic unit is interpretable as an "empty qubit", or in other words, as a sell of the quantum Turing machine tape, in which one among an infinite set of "digits" (rather than only two as the proper Turing machine) are recordable, and then, as a certain qubit, belonging to a certain qubit wave function and thus, to a certain quantum state. That fundamental postulate implies the verification of Riemann's hypothesis (in detail, further): thus, one might allege that such a kind of solution is a "solution" (in quotation marks, i.e. a "fraud" properly) as far as its solution has been in advance postulated, after which it is trivially deduced "from itself" by what "identity" is.

However, the same approach implies for Riemann's hypothesis to be an insoluble statement in the strict framework in the standard mathematics, hence, not to be a theorem, or to be rejected in the final analysis. Nonetheless that negative solution is not mathematically especially meaningful since it closes rather than reveals new horizons for mathematics. Fortunately, that closure and depriving of horizons refers only to the standard mathematic and can be conversely interpreted as for the latter not to be the "real mathematics" of our world (only generalizing the pattern borrowed from the statement that Euclidean space is not the "real geometry" of our world after general relativity).

Furthermore, Wiles's proof can be considered to be a precedent for suspending the standard mathematics since one can rather elementarily demonstrate that Fermat's last theorem is an insoluble statement by means of Yablo's paradox (in detail: *Penchev 2021 March 9*). This literally implies that Fermat's last theorem is false in the framework in question. Nonetheless, one can modify implicitly the standard mathematics (in fact complementing it by inaccessible subcountable cardinal numbers) so that Fermat's last theorem is true within it. The fact that the same statement can be a true theorem after one tuple of axioms, but false after an alternative one is trivial, and can be immediately realized by pairs of the same statements in both Euclidean and non-Euclidean geometries.

Even history can be counterfactually reinterpreted if Euclid had formulated the Fifth Postulate as a hypothesis rather than as an axiom (neglecting his distinction between "axioms" and "postulates"). Then, Lobachevsky would prove that the Fifth "hypothesis" is false merely being an axiom rather than a problem. However, Lobachevski's geometry is much more valuable for mathematics than the simple rejection of Euclid's "Fifth Postulate hypothesis" (counterfactually suggested). Quite verbatim, Wiles elaborated an absolutely new non-Peano arithmetic, in which Fermat's last theorem was and is a true theorem, however he lives in our epoch, in which any revolutionary scientific discoveries, even more so that they push to social revolutions, are not already welcome Accordingly, he was forced (whether consciously or unconsciously) to represent his finding in a "socially acceptable", i.e. conservative way as if proving at last the hypothesis at issue conjectured about four centuries ago. Also counterfactually, one can admit that if Wiles would live in the 19th century, he would represent his "proof" as the discovery of a new non-Peano arithmetic: being absolutely consistent with set theory and thus not allowing for any insoluble statements as after Gödel (1931).

So, Wiles's proof or "proof" is referred to be a precedent only partly: its social conservatism (for which he is a "sir" in particular) is thoroughly rejected, hence, not especially difficultly, in favor in challenging (not only scientific) revolutionarism in the stylish manner of the 19th century therefore heralding in full compliance that "the king is nude" (in more detail: *Penchev 2024 August 5; 2023 December 6*). So, the precedent is followed only in the part replacing the hypothesis (which is originally Fermat's last theorem, but now Riemann's one is at stake) from the context, in which it is an insoluble statement, to that one, being absolutely newly introduced, in which it will be really a true theorem. However, the hypocritical socially wishful pretense (as if Riemann's hypothesis was proved in the strict framework of the standard mathematics) is abandoned since it is an insoluble statement within it, What will be proved is its identical counterpart in that relevant context, in which is really provable, by the way, but not surprisingly in the same sense in which Wiles's proof is relevant because the Gödel incompleteness (or inconsistency) of arithmetic to set theory is the same obstacle in both cases.

However (and it is expressly emphasized above), Riemann's hypothesis is to be related (as a direct corollary) to what any proof (including that of itself) is in Hilbert mathematics unlike in Gödel mathematics (being identical with the standard mathematics). For example, the ultimate syllogism of the proof and that mathematician who writes it down are divided on the opposite shores of the omnipresent during Modernity Cartesian "abyss" "omnipresent" only within it, though). So, the fact that some mathematician is able to write down a proof as an extended syllogism is trivial, but the converse suggestion (jokingly said, that some proof is able to generate a mathematician: being "God's privilege" alone) is ridiculous and nonsense. Nonetheless, that is the case, but in Hilbert mathematics, and if one would wish to challenge common sense drastically enough (a usual entertainment of science at least since Copernicus's age and including "Schrödinger's cat").

VI TOWARD "PART II" OF THE STUDY: RIEMANN'S HYPOTHESIS UNDERSTOOD BY ONTOMATHEMATICS

As this is elucidated in more detail in the introductory *Section I*, Riemann's hypothesis means only those "zeros" of Riemann's zeta function (also called "non-trivial zeros") which do not follow immediately from Riemann's equation (also called "trivial zeros"). Speaking loosely, all trivial zeros are due to its "wavy appearance" (in turn for the imaginary part of its variable resulting into trigonometric functions with that "wavy appearance") because of which it regularly crosses the zero plane just in the trivial zeros, and Riemann's equation is what explicitly demonstrates that the sinusoidal multiplier implies them.

Following the above pattern of thought, one could easily conjecture that, respectively, all nontrivial zeros[12] should be for the real part of the complex variable. Then, one needs a relevant

[12] The present study emphasizes the link between the regularity of all trivial zeros and the "conservation" of all nontrivial zeros to share the line "Re=½" conjectured by Riemann's hypothesis. The properties and

interpretation of that real part for which Riemann's zeta function is to be seen by the mediation of some suitable (qubit) wave function, for example, since the latter is the characteristic function of a certain probabilistic distribution, and its real part possesses a clear probabilistic meaning, by which and reflecting back the real part of the zeta function itself can be probabilistically interpreted as well and respectively, its nontrivial zeros in turn.

Furthermore, if one considers any bit as an elementary function granting for its two alternatives to fluctuate only under the condition their sum to be a unit, it would be a bijective counterpart of a certain qubit where the two alternatives of a bit are additionally granted to be (idempotently exchangeable) two mutually orthogonal subspaces of the separable complex Hilbert space, e.g., two consecutive, "n^{th}" and "$(n+1)^{th}$" axes of it (as usual to be a qubit defined): i.e., "$e^{i\omega n}$" and "$e^{i\omega(n+1)}$".

If one uses the representation of a qubit as a unit ball (in Euclidean space), within which a point is chosen ("a certain value is recorded"), that would correspond to the projection of the unit ball and a certain point within it onto its "great circle". Obviously, that projection being a single one is not equivalent to a qubit, but a pair of projections onto two orthogonal great circles is already absolutely equivalent. In fact, a bit is two orthogonal (or "complementary") oppositions regardless of the commonly accepted, but wrong prejudice to be a single one (in much more detail in: *Penchev 2021 July 8*). The following illustration would be here enough:

A Turing machine type cell is what will be considered: the obvious opposition is that between the two states recordable on it and usually notated as "0" and "1". However, another implicit opposition consists in the preliminary dilemma whether either of them is recorded or not. Then, if both oppositions are generalized to be "bit functions" (introducible as above), they would constitute just a qubit. The former opposition can be called to be "explicit", and attributed to the real part of the complex variable, and the latter, "implicit", attachable to the imaginary part accordingly: both rather provisionally, only for the distinction, since they are formally exchangeable but needing a compensating rotation equivalent to the exchange of the abscissa and the ordinate of the complex plane.

Now, one might attempt to reason initially the trivial zeros of Riemann's zeta function by the afore-involved "qubit pattern" leaving the rigorous proof for *Part II.* For example, one can figure a sinusoidal variable changing the "implicit" probability for either "explicit" alternative to be at all recorded. When the probability for recording either of both would turn out to be zero, that circumstance would result in a corresponding "trivial zero" if that sinusoidal variable and the sinusoidal multiplier in Riemann's factorization would be identified.

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interrelations (including oppositions) are studied in many papers (e.g., Heymann 2024; Salah, Rehman, Al-Buwaiqi 2022; Heath-Brown 2020; Ovchinnikov 2019; Hassani 2014; Choi, Chung, Kim 2012; Bui 2011; Sierra, Rodríguez-Laguna 2011; Voros 2009; 2000; Ki 2008; Ng 2008; Sierra 2008; Pegg 2004; Sierra, Townsend 2004; Šleževičien, Steuding 2004; Hall 2002; Conrey, Iwaniec 2000; Connes 1999; Csordas, Wayne, Varga 1994; Conrey 1989; Anderson 1983; Mueller 1982; Levinson 1974; 1969; Montgomery 1973; Stechkin 1970; Davenport 1967; Hardy, Littlewood 1921; 1921a; Backlund 1918; Hadamard 1896; Riemann 1859.

Well, if one rotates the complex plane for exchanging the two "bit variables", therefore transforming the implicit into the explicit one, it would behave in the same way, anyway questioning the correctness of the identification of the implicit explicit ones. In other words, one might suggest that the former explicit one, now transformed into implicit, conducts itself quite otherwise than before (when it was "explicite"). Nonetheless, the involved "qubit" model itself does not support that suspicion just above since the exchange is obviously invariant to the introduced rotation not being able to distinguish which is explicit from which is implicit.

However, the nontrivial zeros unlike the trivial ones are not regular, eventually absolutely randomly distributed though all of them situated only on the line "$y(s) = Re(s) = 1/2$" if Riemann's hypothesis would be proved. That observation seems to reject (at least, at first glance and literally) the assumption that the zeta function would allow for that "qubit" model. In fact, the failure is due to the following mismatch: the two bit variables granted to be equivalent to a qubit variable are complementary by themselves, but that circumstance is not conserved if they have been jointly considered by the "qubit" model.

Sharpening, the question can be reformulated: does the exchange between the explicit and implicit bit variables transform the regular behavior of the explicit one into fundamentally random as for the implicit one? Can those two ones be interpreted to be complementary to each other in the narrow proper sense of quantum mechanics including? How should one understand complementarity in terms of qubit models being invariant to the rotation inherently canceling the same complementarity at issue.

One possible reason, respectively, a way out is suggested by Hilbert arithmetic, more or less successfully attempting to reconcile both qubit representation and complementarity after distributing them into different features, respectively, substructures: so, the qubit model is concentrated explicitly only in Hilbert arithmetic in a wide sense, and complementarity, in duality, respectively, by doubling dually any unit or qubit by its "twin", i.e., dual counterpart so that the relation of any (mutually and idempotently) dual pair is that of complementarity. That approach in particular does not allow the above confusing interpretation since the two bit variables are either consistent in qubit or, however, complementary as for any dual counterparts.

However, and simultaneously, the qubit model (anyway applied literally, implying for the two bit variables to be consistent rather than complementary to each other) is crucial for the eventual resolution of Riemann's hypothesis in Hilbert arithmetic. For avoiding the just mentioned direct contradiction, one can bypass it by introducing the bijection of a qubit in which its two bit variables are consistent to each other onto the same qubit, but in which they are complementary to each other. Riemann's hypothesis is properly formulated as for the latter members of the bijection at issue though it proves their former counterparts and thus the bijection itself since it is able to legitimate the transition between them and substitute them consistently if need be.

An additional preliminary notice is necessary to elucidate what is exactly changed after their mutual replacement, or in other words: how can be complementarity equivalently represented including it in a model in which the alternatives being initially granted to be mutually complementary to each other, now are consistent, i.e. simultaneously representable. Which property or relation is sufficient to be added to the “wrongly” represented complementarity as consistency so that it will be truly corrected (of course, if that property or relation would at all exist). Meaning all “plots” in the literature of quantum mechanics, one is instantly able to guess "randomness" regardless of whether it is interpreted to be a relation or a property.

Reflecting philosophically the bijection between two alternative interpretations of the same qubit after involving that “randomness” being fundamental for quantum mechanics, one can now see it as a reformulation of what the theorems about the absence of hidden variables (Kochen, Specker 1967; Neumann 1932) mean: thus granting them as a rigorous proof of the correctness of the conjectured substitution. Indeed, the bijection at issues states that a qubit (for which one can use Einstein’s pejorative metaphor of “God playing dice”) is “rolled dice”, i.e. meaning to be absolutely random, on the one hand (or, on the other side of the bijection), but simultaneously, “divine” and therefore infinitely regular (only seeming to humans to be absolutely random by virtue of their inherent “finitude” or physically, rather locality).

In other words, the just newly introduced bijection is only another proper mathematical expression of a traditional “trope” in the “genre” and discourse of quantum mechanics: though initially involved by Einstein satirically, afterwards it was proved absolutely seriously and mathematically, which allows now for it to be granted “ready-for-use” for proving Riemann’s hypothesis just in Hilbert arithmetic (respectively in Hilbert mathematics) and only in it, being inadmissible in the standard (“Gödel”) mathematics since it is also equivalent to the correctness of the transition from finiteness to infinity (or respectively, from arithmetic to set theory, after Gödel). Thus Riemann's hypothesis will be proved just only in Hilbert arithmetic and mathematics crucially depending on the bijection in question (including on the obviously philosophical discourse for a proof claiming to be inherently mathematical) therefore remaining still insoluble in the standard mathematics: Even more: the eventual necessity of that bijection for the proof of Riemann’s hypothesis would imply that it is improvable in the strict framework of the standard mathematics.

Until now, that bijection was utilized in relation to Riemann’s hypothesis only in order to justify and reconcile the elementary regularity of the “trivial zeros” of Riemann’s zeta function with the absolute random distribution of its “nontrivial zeros”. However, it is much more powerful tool thus allowing at the same time for confirming Riemann’s hypothesis by deducing it directly from that elementary regularity of all trivial zeros originating from the regular crossing the abscissa by the corresponding sinusoidal function by virtue of Riemann’s factorization of his zeta function. Indeed, one can consider all trivial zeros as an additive semigroup and then, applying conversely (since that is allowed in Hilbert arithmetic) whether the well-ordering “theorem” or the axiom of choice, one can transform it into a Lie group and refer to Emmy Noether’s first theorem including by virtue of that bijection, the physical interpretation of it and that of a qubit, by the mediation of which her theorem can be involved as far as it is formulated to physical quantities. Resultively, all nontrivial zeros obey necessarily the conservation of “

$Re\{"0"_i\} = \frac{1}{2};\ i = 1, 2, 3, \ldots$": even more so since the conservation constant (namely, "½") is reciprocal to the "step" (namely, "2") by which the trivial zeros follow one after one. So, Riemann's hypothesis would be proved, needing "a few pages" (figuratively said) for writing down the proof but only in Hilbert mathematics due to the crucial mediation of Hilbert arithmetic. Thus, the Gestalt change (from the standard mathematics into Hilbert one) is sufficient to transform a "Millennium Problem" (according to CMI) into a theorem.

The proper mathematical argumentation and tenets will be discussed in the next, second part of the paper. A few rather philosophical or methodological reflections are what follow here. As an immediate objective, the present investigation tries to prove Riemann's hypothesis, but by both Gestalt change from the standard ("Gödel") mathematics to the newly introduced Hilbert mathematics also interpretable as "ontomathematics", in which mathematics, physics and philosophy are merged, even rather indistinguishably, on the one hand, and by a relevant syllogism, but already in the just involved innovative framework, in which many necessary syntheses are consistent and admissible being absolutely prohibited in the previous, former one. Furthermore those forbidden logical and mathematical steps are proved to be necessary therefore providing them by a relevant proof, which results into an insoluble statement if one is restricted to remain in the standard mathematics, which, first of all and crucially, does not allow for the continuous, thus "two-way" transition between finiteness and infinity by the newly-introduced "ladder" of inaccessible subcountable cardinal numbers, through which the transformation between finite logical well-orderings, i.e. syllogisms and "infinite" mental images is easy, free, and permanent. So, one might coin the metaphor for "corpus callosum" (between the left logical "hemisphere" producing inherently finite syllogisms and the right intuitive "hemisphere" in turn generating "infinite" mental images: at least as a "scientific myth") for the two-way ladder closely linking finiteness and infinity by all inaccessible subcountable cardinal numbers.

One can introduce the term of "nonstandard bijection" as for that two-way mapping between finiteness and infinity symbolized by the discussed "bit - qubit" unambiguous relation in turn implying the proof of Riemann's hypothesis as sufficiently reasoned only by virtue of the same Gestalt change introducing "nonstandard bijection". "Standard bijection" is impossible for sets of different cardinality whether finite or infinite unlike "nonstandard" one. The term of "*nonstandard* bijection" is chosen for the concept of "nonstandard model" after the Löwenheim-Skolem theorem and Robinson's "nonstandard analysis" applying it to the main approach of classical physics for building mathematical models and particularly crucial for the bridge between all trivial and all nontrivial zeros of Riemann's zeta function via Noether's first theorem (1918) therefore utilizing both directions of "nonstandard bijection".

It is to be unambiguously defined as the mapping able to link the initial set and the set of low cardinality corresponding to the model of a certain structure rooted in the former. One can immediately see that the mapping at issue cannot be any standard bijection due to the different cardinality of the related sets. That is "on the one hand", but nonetheless, or "on the other hand", that should be a bijection in a "nonstandard sense" due the definitive adequacy of "model" and any given model of whatever. So, the concept of "nonstandard bijection" captures both contradictory conditions in a consistent way, or loosely and paradoxically speaking, it is both bijective and non-bijective, but both are in different, or rather complementary relations, therefore preventing the direct logical contradiction; namely, it is non-bijective "locally" or "inside", but bijective "globally" or "outside". Figuratively speaking, "nonstand bijection" means to map two "forests" of different number of "trees": both being a forest, they can be bijectively mapped only globally (one "forest" - another "forest"), but not locally, "tree by tree" since the number of trees is different.

The extraordinarity of "nonstandard bijection" stands out especially after its two-way utilization as the proof of Riemann's hypothesis needs; and that "exoticism" can be illustrated by the two-way nonstandard bijection of two "forests" with different numbers of "trees". So, one maps bijectively, but nonstandardly, the former forest onto the latter "forest" (in the "straight direction"), and then, the mapped representation is in turn again mapped back onto the former "forest" (in the "converse direction"). As for "standard bijection", the initial forest and its two-way mapped image are identical, and this statement is trivial, but wrong as for "nonstandard bijection" visible after investigating its local behavior, i.e. in relation to the trees of both forests, meaning that their numbers are different. Furthermore, the concept of "nonstandard model" itself implies that the number of trees just of the latter forest is less than that of the former.

Let, now, one be following the local behavior of nonstandard bijection after two-way applying. A greater number of trees is mapped onto a smaller number, which implies for that mapping, if it is standardly interpreted, to be a surjection; as for the converse mapping, it is even impossible to be a standard function, but only a generalized one, called also "distribution", thus inherently-interpretable as a probabilistic distribution in the virtue of the following observation. The standard functions map unambiguously (a given one though possibly not a single element of the former set) onto just a certain element of the latter. On the contrary, a "distribution" maps "one - many", i.e. onto many elements, involving a probabilistic distribution due to the different probabilities for each eventual image to the really chosen image so that the image is ambiguous, but the probabilistic distribution of all possible images is unambiguous.

Summarizing, a two-way applied "nonstandard bijection" behaves quite differently than a standard one idemponently and uniformly exchanging the former and latter sets by virtue of the same cardinality. It maps divergently or randomly, therefore melting or blurring the former set being mapped onto itself once applied "two-way". So, i.e. the globally observed "nonstandard bijection", first applied "one-way" (i.e. in the "straight direction"), decreases the cardinal number (from an uncountable to a countable set in the case), and then "restores" the initial cardinal number (i.e. now decreases it reversely), at the same time randomly mapping itself onto its subset (of the next lower cardinality).

One can consider how that two-way applied nonstandard bijection would be interpretable as for Riemann's hypothesis to be proved via Noether's first theorem (1918). To trace the consecutive transformations, one should interpret the complex plane, on which Riemann's zeta function is defined, physically so that the abscissa is "time", and the ordinate is "energy" (and it will be notated to be "Noether complex plane"), meaning that the only condition for the physical dimensions of both “axes” is the physical dimension of their product to be “action”-dimensional for her theorem to be relevant.

Now, one maps both groups of all trivial and nontrivial[13] zeros on the usual complex plane initially simultaneously granting Riemann's hypothesis to be true. Then, the physically dimensionless complex plane is “interpreted” to be the just newly introduced “Noether complex plane”. That construction, though being obvious and thoroughly admissible as an illustration of one’s thought, is, in fact, categorically forbidden in both mathematics and physics during Modernity obeying Cartesianism’s dogma for strictly dividing on the two opposite “shores” accordingly. Any statement referring to their unity by itself can be only a visualization of one’s thought (e.g. mine in the case), but it is to be represented as a correct finite (though arbitrarily long) syllogism in order to be legalized, for example, by publishing in some scientific journal (since nobody would accept that illustration seriously to be discussed for publishing).

The present study breaks that dogma, but in a rigorous and logically correct way: it rather postulates “Hilbert arithmetic” and “Hilbert mathematics” as a consistent alternative of the standard mathematics and its foundations, intending to prove Riemann’s hypothesis simply enough in the final analysis. The close link between physics and mathematics maps bijectively (*nonstandardly* bijectively) Hilbert arithmetic in both narrow and wide senses as well as their elements, bits and qubits, thus finally implying Riemann’s hypothesis itself. Furthermore, that ontomathematical consideration is necessary for its proof, i.e., as for its counterpart in the standard mathematics, it remains insoluble, critically needing the unity of physics and mathematics.

In particular, and thus absolutely consistently with the onthomathematical approach, the transformation of the complex plane into the just introduced “Noether complex plane” is involved for the Noether (1918) first theorem to be utilized in relation to the nontrivial zeros of Riemann’s zeta function, which turn out to be randomly disseminated onto the line $Re = \frac{1}{2}$ (as for the complex plane), and thus, supposedly for the Noether complex plane before that, implying a nonstandard bijection due to the internal structure of the theorem, as if transmitting through a set of a less cardinal number. So, the internal structure of the theorem should involve the two-way applied nonstandard bijection between Hilbert arithmetic in a wide sense (possessing uncountable cardinal number) and Hilbert arithmetic in a narrow sense (with a countable cardinal number), on the one hand. As for the latter, and thus, on the other hand, an internal standard bijection between the two dual Peano arithmetics is involved (within each pair of dual natural numbers of the same ordinal number). The same bijection is represented as the definitive trivial bijection (the $\pi/2$ rotation) exchanging the axes of the (Noether or usual) complex plain. So, duality is represented (as usual or at least, very often) by means of the complex plane. That circumstance can refer to the uniform reflection of the Gauss - Lucas theorem (Gauss 1799; Lucas 1879) and the Kochen - Specker (1967) theorem, however meaning

[13] The nontrivial zeros are represented “in module” since that is a complete information for any pair of conjugate nontrivial zeros.

that the uniform reflection at issue originates from ontomathematics, but being stigmatized by Modernity due to the Cartesian abyss established by it.

The Noether (1918) first theorem is not formulated in terms of Hilbert arithmetic in a narrow sense (even not in those in Hilbert arithmetic in a wide sense, but its physical sense directly corresponds to the framework of the latter), so that the former should be deduced. What is given as for the presupposed context of its eventual application to Riemann's hypothesis consists in the reinterpretation of the complex plane, on which the zeta function is defined and which contains its values, both to be, first, the Noether complex plane, and, then, its discrete equivalents after the inherent internal bijection between its two "senses", which in turn can be divided into two substages: the one is a standard bijection, but the other is a nonstandard bijection (as above):

The former, standard bijection is trivial: it maps the serial number of each, "n", in just the same natural number, "N": "$n \leftrightarrow N$". The lather, nonstandard one is defined as a class of equivalence, by mapping of the uncountable (according to the conventional definition) set of all values of each qubit into a single one, the "empty" (figuratively said) qubit; in other words, equivalent to all standard operations of human thinking transforming a class of any objects (whatever they be) into a single mathematical object (for example, a collection consisting of any "n" entities into the natural number "N"). So, the uncountable set of all values of qubits of the qubit Hilbert space is transformed into the countable set of all natural numbers. One can immediately notice that the nonstandard bijection is a necessary condition for the ontomathematical unity of physics and mathematics so that if the latter is presupposed (as in the present study), it in turn implies the nonstandard bijection and thus justifies its utilization.

Furthermore, the sense of the discussed Noether theorem (by itself linking physics and mathematics) is obvious in terms of Hilber mathematics originating directly from the trivial conventional identification of the dual Peano arithmetic and the real line implied by it as represented by the ordinate of the complex plane (according to the tradition since the exchange of the abscissa and the ordinate is idempotent). That idea can be also interpreted to transfer the equivalence of the Lagrangian and Hamiltonian approaches from mechanics to mathematics, even to its ontomathematical foundations:

Then, the Noether theorem at issue refers to the fundamental mathematical equivalence of a class of equivalence and that entity, into which it results (sometimes notated as the equivalence of "extension" and "intension": for example, the same set whether represented by means of its elements or by its characteristic property): by the way, meant by the nonstandard bijection and embedded into the nonstandard bijection of Hilbert arithmetic in both wide and narrow sense. "Symmetry", including in the exact particular meaning of the Noether first theorem, is to be related to the former (i.e. to the "class of equivalency" since all elements symmetrical to each other are equated just by virtue of the symmetry), and "conservation", accordingly, to the latter (since it substitutes all the symmetry and thus all symmetrical elements with the single conserved value).

So, the idea of the Noether theorem to be applied to Riemann's hypothesis infers the line "$Re = \frac{1}{2}$" conjectured to be shared by all nontrivial zeros as the "conservation" (in the just deduced new interpretation) correlative to the obvious and trivial semigroup of all trivial zeros after applying the nonstandard bijection able to explain the contrast of the latter regularity to the former randomness. The present approach considers the two groups of zeros as dual in a mathematical sense or as complementary in a sense relatable to quantum mechanics, corresponding to the representation of duality by the complex plane as that is explained above or as to conjugate physical quantities just as that Noether's theorem needs. However, is it admissible and provable in the rigorous meaning required by any syllogism?

What can be demonstrated is that Riemann's zeta function can be more or less conventionally interpreted as a wave function, which mathematically means, as a square-integrable function additionally defining it to be square-integrable also at its pole ($s = 1$) which is (standardly or disputably) out of its domain not being analytical in it, but anyway remaining square-integrable in any neighborhood of it (whether an infinitesimal neighborhood or not). So, whether the zeta function is square-integrable at its pole or not, this can be consistently postulated in both alternative versions.

Ontomathematicics postulates naturally it to be the square-integrable at the pole and thus, a wave function due the following considerations which are not necessary, but only consistent. So, the detailed sketch of the eventual proof of Riemann's hypothesis in the next "Part II" (complying with the newly introduced framework of Hilbert arithmetic, and thus, Hilbert mathematics) relies crucially on interpreting the zeta function to be a wave function (i.e. square integrable) all over the complex plane conventionally: including its pole ($s = 1$). A few more or less intuitive tenets are able partly to reason that assumption, but they are not sufficient (in the strict logical sense) thus needing human free will's decision between two options neither of which is logically necessary (including the pair of both is not logically necessary as well). However, the traditional one follows the fundamental worldview of Modernity, in particular, in relation to the Cartesian "abyss" gapping mathematics from reality on two opposite "shores", thus needing (for example, for "tolerating diversity and maintaining balance") humankind to stare at its alternative in full detail:

Only not being definable at its pole directly or after continuing analytically, it is anyway square-integrable in all neighborhoods (whether infinitesimal or finite) of the pole. Furthermore, it is bounded almost everywhere on the complex plane. The very conception of analytical continuation can be in turn "analytically" (though rather "physically" in a proper sense) continued after defining it unambiguously to possess an "infinite value" simultaneously conserving at it the universal square-integrability (or in other words, conventionally continuing that property including at "$s = 1$") by a function only "looking like" the Dirac delta function but different from the latter due to its square-integrability (rather than integrability) at the pole.

Then, that continued at the pole zeta function can be immediately interpreted physically, independently of being inherently mathematical, but not as a model of whatever physical phenomena might be, but itself "physically continued" in the afore-defined meaning in a sense extending further (i.e. physically) the concept of analytical continuation. One can state that the "physical continuation" of the zeta function is an unambiguously[14] determined wave function and thus a quantum state (however, thus including a singularity at its pole, also interpretable to link physics and mathematics by a bridge over the Cartesian "abyss"). One can suggest a further generalization of the above case solution by introducing the concept of physical continuation of an analytic function under the additional condition of being square integrable in order to be interpretable as a wave function and a quantum state accordingly (additionally discussing its singularity).

As for the physically continued zeta function, it is necessary to reason and mathematically justify the Noether (1918) first theorem to be consistently applied for the proof of Riemann's hypothesis.This implies for all trivial zeros to be interpreted as discrete semigroup after the nonstandard bijection of the corresponding Lie group in the sense of the cited theorem (e.g. that of time translations as for energy conservation and being conjugate to all nontrivial zeros according to the intended design), in turn now interpreted to address the conserving quantity in the same context.

So, the more or less conventionally postulated condition for the zeta function to be "physically continuable" is furthermore sufficient (in the exact meaning of "sufficient condition") for involving Noether's theorem in turn being crucially important for proving Riemann's hypothesis in Hilbert mathematics by a reason turning out to be quite natural but in the newly introduced context rather than within the original, traditionally found one of complex analysis or number theory. It consists in a physically interpreted connection of the groups of trivial and nontrivial zeros, otherwise also researched in detail with results concentrated in a few "constellations"[15]: (1) "functional equation" (after which the trivial zeros serve as a boundary that constrains the distribution of nontrivial zeros); (2) "Weil's explicit formula" (so that the trivial and nontrivial zeros are Fourier duals, balancing spectral contributions); (3) "spectral theory" (interpreting the trivial zeros to act as "stabilizers" in the nontrivial energy level structure); (4) "p-adic analysis" (where both types of zeros appear in common arithmetic interpolations); (5) "logarithmic derivative": (for the trivial zeros to regulate the singularities of the nontrivial via cancellation mechanisms).

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[14] The statement about the unambiguously determined wave function, which is the counterpart of Riemann's zeta function is reasoned and proved in *Part II* (*Section II*) of the paper.

[15] Eventual connections between trivial and nontrivial zeros are researched in many papers (e.g. Heymann 2024; Salah, Rehman, Al-Buwaiq 2022; Heath-Brown 2020; Rodgers 2018; Ford, Zaharescu 2016; Fukushima 2016; Ford, Luca, Moree 2014;Hassani 2014; Bender, Orszag 2013; Conrey 2010; 2003; Montgomery, Vaughan 2007; Goldston, Montgomery, Soundararajan 2005; Derbyshire 2004; Pegg 2004; Havil 2003; Iwaniec, Kowalski 2002; Bombieri 2000; Voros 2000; Keating, Snaith 2000; Berry, Keating 1999;Conrey, Iwaniec 2000; Patterson 1995; Lang 1994; Rubinstein, Sarnak 1994;Karatsuba, Voronin 1992; Odlyzko 1992; 1987; n.d.; Ivić 1985; Gelbart 1984; Davenport 1980; Brent 1979;Zagier 1977; Apostol 1976; Edwards 1974; Levinson 1974; Montgomery 1973; Iwasawa 1972; Davenport 1967; Selberg 1956; 1946; Weil 1952; Titchmarsh 1951; Wiener 1951; Ingham 1932; Backlund 1918; Hardy, Littlewood 1921a; 1914; Hadamard 1896; Riemann 1859). However, that in the present study was not found among them, even implicitly or by hinting.

Among the enumerated directions for researching eventual hidden and mediated correlations of the two groups of zeros, only (2) might be connected (though rather loosely) to the approach advocated by the present study. The reason is many times emphasized above: Riemann"s hypothesis formulated in terms of complex analysis and linked by Riemann himself to number theory is situated strictly within "pure mathematics" and gapped from physics hardly visible on the too remote opposite "shore".

Anyway, the Hilbert-Polya conjecture[16] positing that the nontrivial zeros of the Riemann zeta function correspond to the eigenvalues of a self-adjoint operator (furthermore involving a spectral interpretation of these zeros), at least implicitly links it to a physical quantity as far as quantum mechanics interprets the latter as the former. For example, Connes (1999) or Selberg (1956) suggest that while the Hilbert–Pólya conjecture directly concerns nontrivial zeros, there are theoretical frameworks in which both trivial and nontrivial zeros are interconnected through spectral and geometric interpretations. Though following the ideas of the Hilbert-Polya conjecture, an implicit ontomathematical horizon for investigations appears, it remains inaccessible since those would need to overcome the "abyss" dividing mathematics from physics established by Modernity (forcing them to be situated on the opposite "shores").

The problem for Riemann's hypothesis to be interpreted both physically and mathematically in an indistinguishable way thus explicitly rejecting those dogmas can be accomplished only by introducing Hilbert arithmetic in the unity of its narrow and wide senses (what the present study does properly). Nonetheless, one can also extrapolate the available physical interpretations of Riemann's hypothesis as those by the mediation of the Hilbert - Polya conjecture[17] into thought experiments of how both might be confirmed or rejected empirically[18].

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[16] References include the original papers of Hilbert (1912; 1902; 1900; 1897) and Polya (1982; 1954; 1927; 1927a; 1926; 1926a; 1921; 1919; 1914; Pólya, Szegő 1925) as well as many papers discussing sometimes including physical interpretation of the hypothesis (e.g., Yakaboylu 2023; Bellissard 2017; Bender, Brody, Müller 2017; Broughan 2017; Andrade 2013; Sierra Rodríguez-Laguna 2011; Mehta 2004; Conrey 2003; Iwaniec, Kowalski 2002; Bombieri 2000; Sarnak 2000; Andrews, Askey, Roy 1999; Berry, Keating 1999; Connes 1999; 1998; Elizalde 1994; Odlyzko 1987; Ivić 1985; Hejhal 1983; Edwards 1974; Montgomery 1973; Szegő 1972; Reid 1970; Selberg 1956; Titchmarsh 1951; Wigner 1951).

[17] For example, Yakaboylu 2023; Planat, Aschheim, Amara, Irwin 2019; Bender, Brody, Müller 2017; Bellissard 2017; Broughan 2017; Andrade 2013; Berry 2012;Sierra, Rodríguez-Laguna 2011; Schumayer, Hutchinson 2011; Sierra 2008; Sarnak 2000; Berry, Keating 1999; 1999a; Wigner 1951.

[18] The Hilbert–Pólya conjecture suggests that the nontrivial zeros of the Riemann zeta function correspond to the eigenvalues of a self-adjoint operator. While numerous theoretical models attempt to establish this link, an empirical test remains elusive. The following thought experiment proposes a possible framework for experimentally probing the conjecture using principles from quantum chaos and spectral analysis. Consider a quantum billiard system in which a single particle, such as an electron, is confined within a two-dimensional cavity with boundaries shaped to induce chaotic motion. A well-known example is the Sinai billiard or a stadium-shaped cavity, where classical trajectories exhibit chaotic behavior, leading to quantum energy levels that resemble spectra governed by Random Matrix Theory (RMT). One can utilize microwave cavity resonators, laser spectroscopy, or ultracold atoms to measure the energy eigenvalues of the quantum billiard system with high accuracy. The goal is to obtain the spacing distribution of these energy levels and compare them to the Gaussian Unitary Ensemble

(GUE), which is known to statistically model the distribution of nontrivial zeros of the Riemann zeta function. If the quantum billiard's eigenvalues match the GUE statistics, it suggests that the system's underlying operator shares properties with the conjectured Hilbert–Pólya operator, supporting the hypothesis that such a self-adjoint operator exists for the Riemann zeta function. If deviations arise, this could either point to limitations in the experiment or indicate that the conjecture requires refinements. A complementary approach involves simulating a synthetic quantum system using ultracold atoms trapped in an optical lattice. Design an artificial Hamiltonian in which the energy spectrum mirrors the expected eigenvalues of the conjectured operator. Measure the system's energy levels and analyze whether they match the nontrivial zeros' distribution. A successful match would provide a physical realization of the conjectured self-adjoint operator, reinforcing the spectral interpretation of the Riemann hypothesis. Given the overwhelming numerical evidence that the zeros of $\zeta(s)$ obey GUE statistics, one can believe these experiments would likely provide support for the Hilbert–Pólya conjecture. A direct experimental realization of the associated Hamiltonian would be groundbreaking, bridging number theory and physics in an unprecedented way. However, should deviations from GUE statistics occur, they may point to refinements needed in either the experimental framework or the conjecture itself.

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