

# THE SYMMETRIES OF QUANTUM AND CLASSICAL INFORMATION. THE RESURRECTED “ETHER” OF QUANTUM INFORMATION

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*Abstract:* The paper considers the symmetries of a bit of information corresponding to one, two or three qubits of quantum information and identifiable as the three basic symmetries of the Standard model,  $U(1)$ ,  $SU(2)$ , and  $SU(3)$  accordingly. They refer to “empty qubits” (or the free variable of quantum information), i.e. those in which no point is chosen (recorded). The choice of a certain point, i.e. the certain coefficients of superposition ( $\alpha, \beta \in \mathbb{C}: |\alpha|^2 + |\beta|^2 = 1$ ) in any qubit ( $\alpha|0\rangle + \beta|1\rangle$ ), violates those symmetries. It can be represented furthermore as the choice of a privileged reference frame (e.g. that of the Big Bang), which can be described exhaustively by means of 16 numbers (4 for position, 4 for velocity, and 8 for acceleration) independently of time, but in space-time continuum, and still one, 17<sup>th</sup> number is necessary for the mass of rest of the observer in it. The same 17 numbers describing exhaustively a privileged reference frame thus granted to be “zero”, respectively a certain violation of all the three symmetries of the Standard model or the “record” in a qubit in general, can be represented as 17 elementary wave functions (or classes of wave functions) after the bijection of natural and transfinite natural (ordinal) numbers in Hilbert arithmetic and further identified as those corresponding to the 17 elementary particles of the Standard model. Two generalizations of the relevant concepts of general relativity are introduced: (1) “discrete reference frame” to the class of all arbitrarily accelerated reference frame constituting a smooth manifold; (2) a still more general principle of relativity to the general principle of relativity, and meaning the conservation of quantum information as to all discrete reference frames as to the smooth manifold of all reference frames of general relativity. Then, the bijective transition from an accelerated reference frame to the 17 elementary wave functions of the Standard model can be interpreted by the still more general principle of relativity as the equivalent redescription of a privileged reference frame: smooth into a discrete one. The conservation of quantum information related to the generalization of the concept of reference frame can be interpreted as restoring the concept of the ether, an absolutely immovable medium and reference frame in Newtonian mechanics, to which the relative motion can be interpreted as an absolute one, or logically: the relations, as properties. The new ether is to consist of qubits (or quantum information). One can track the conceptual pathway of the “ether” from Newtonian mechanics via special relativity, via general relativity, via quantum mechanics to the theory of quantum information (or “quantum mechanics and information”). The identification of entanglement and gravity can be considered also as a ‘byproduct’ implied by the transition from the smooth “ether of special and general relativity” to the “flat” ether of quantum mechanics and information. The qubit ether is out of the “temporal screen” in general and is depicted on it as both matter and energy, both dark and visible.

*Key words:* confinement, discrete reference frame, still more general principle of relativity, quantum-information conservation, symmetries of information, the ether

## I INTRODUCTION

If quantum information is conserved, and the first Noether (1918) theorem is applicable, a Lie group of action corresponds. If that is the case, that Lie group is to be that of the Hermitian operators of action in the separable complex Hilbert space.

However, the statement that quantum information is conserved is ambiguous since it can refer to a few different meanings about what is conserved, e.g.:

(1) This is the qubit Hilbert space itself (being equivalent to the separable complex Hilbert space<sup>1</sup>) following from the conservation of all “empty qubits”: i.e an abstract superposition of two neighbouring axes of the separable complex Hilbert space, in which both norming complex coefficients  $\alpha, \beta$  ( $\alpha, \beta: |\alpha|^2 + |\beta|^2 = 1$ ) are uncertain yet. The qubit Hilbert space can be interpreted also as the class of equivalence of all wave functions.

(2) This refers to a certain wave function being relevant to a quantum state, which would mean that the wave function and quantum state always correspond to each other, and that quantum state does not possess any more exhaustive description than wave wave function supplies. Any wave function is a certain value assignable to the free variable of quantum information (as the qubit Hilbert space can be interpreted as well).

(3) This is the set of all wave functions rather than the class of their equivalence as in (1) of the same enumeration. In other words, one is to distinguish the qubit Hilbert space as a set of elements (which are wave functions as to the case) from it as an abstract mathematical structure (respectively, the “free variable of quantum information”): right as to the former case.

Whatever among the enumerated meanings (or the combinations of them) would be signified, the eventual conservation of quantum information is undergone to another ambiguity due to the applicability of both first and second<sup>2</sup> Noether (1918) theorems. The investigation and resolution of that ambiguity need the distinction and then mapping (as a *bijection*) of the usual quantity of time and the physical correlate of transcendental time as what the qubit Hilbert space

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<sup>1</sup> The equivalence is valid if one accepts that (1) the two “axes” of any qubit are commutative, and (2), the qubit Hilbert space is defined as the class of all combinations of two axes of the separable complex Hilbert space. Here and further, “class” and “set” will be distinguished correspondingly as: (1) all entities (including elements of a set) satisfying a logical condition and (2) all entities which can be enumerated unambiguously by a certain bijection on the set of all natural numbers after the axiom of choice. Very often, the logical condition of (1) and the bijection of (2) determines themselves mutually, and then “class” and “set” are synonyms. “Class” is preferable if the elements of the corresponding set are too much (or too many after the axiom of choice) therefore going out from the “classical” set theory into the inaccessible cardinals whether uncountable or countable (accordingly).

<sup>2</sup> The second Noether theorem means the usual quantity of time as a single parameter as a single parametric Lie group and a single corresponding differential equation, the only existence of which the theorem states. The first Noether theorem means what is a parametric group in the second theorem is a direct (physically dimensionless) correlate of action and can generate in turn relevant parametric groups though in a generalized sense since the parameter at issue is bijectively mapped in the real axis. In fact formally, any physical quantity (as time in the case) is bijectively mapped in the real axis, but generalization means any nontrivial bijection (for example valid under additional physical or mathematical conditions).

is discussed below as in other papers (Penchev 2020 October 20; 2020 August 25). The construction of that bijection in detail suggests the following stages:

(1) The usual quantity of time is to be discretized and represented as the well-ordering of successive natural numbers starting from “1” in virtue of the axiom of choice, after which the continuum of all real numbers can be mapped bijectively in the set of all natural numbers.

(2) The series of natural numbers is substituted by a series of empty bits of information and enumerated by the replaced series:  $1 \leftrightarrow B1$ ;  $2 \leftrightarrow B2$ ;  $3 \leftrightarrow B3$ ; ... ;  $n \leftrightarrow Bn$ ; ...

One establishes the following notations about the structure of a bit of information: the chosen state of a bit,  $B_c$ ; the unchosen state of a bit,  $B_{uc}$ ; the state of a bit before choice (an empty qubit),  $B$ ; a certain bit enumerated as  $n^{\text{th}}$ :

$$Bn \rightarrow \left\{ B_c n, B_{uc} n \right\}$$

(3) The bijection of the set of all natural numbers, “{N}” into all natural numbers, “N<sup>+</sup>” exists in virtue of the axiom of choice. The former satisfies the axioms of set theory (e.g. “ZFC”), and the latter, those of Peano arithmetic. One defines “N<sup>-</sup>” by the modification of Peano axioms as follows: the first element of “N<sup>-</sup>” is “ $\omega$ ”, a certain well-ordering corresponding to the ordinal number of “{N}” meaning namely only the well-ordering of “N<sup>-</sup>” (among all well-orderings relevant to the ordinal number of “{N}”) constructed by the modified function successor,  $n_{next} = n - 1$  so that “N<sup>+</sup>” and “N<sup>-</sup>” consists of all elements of “{N}” and exists an isomorphism of “N<sup>+</sup>” and “N<sup>-</sup>” after the modification (i.e. the interpretation of  $n_{next} = n - 1$  for  $n_{next} = n + 1$ , and as the first element, “ $\omega$ ”, for the first element as “1”). The notation “N<sup>0</sup>” means the set without any well-ordering, which can be defined as the set “{N}” in set theory without the axiom of choice as the set “{N}” to which the axiom of choice can be applied, but it is not applied actually yet. In other words, “N<sup>0</sup>” means the set “{N}” “before choice” being analogical to a “quantum coherent state before measurement”<sup>3</sup>.

(4) A special operation “localize”  $\Lambda(S, a)$  defined on all non-empty sets maps a certain set “S” “within” any single element of it, “a”. In other words, that operation of localization replaces a certain element “a” by the pair “a, S”, therefore being self-predicative definitively.

The operation formalizes the concept of the invariance of global and local spaces (formally vector spaces) in quantum mechanics and the Standard model by which any operator acting on the global space is “localized” as acting on the local space associable with a single “point”

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<sup>3</sup> The triple, “N<sup>0</sup>, “N<sup>+</sup>, and “N<sup>-</sup>” formalizes the concepts of “coherent state”, “measurement”, and “conjuncted measurement” (i.e. the measurement of any quantity, the operator of which does not commute with the operator of the quantity by the “measurement” at issue) in quantum mechanics correspondingly.

(vector) of it<sup>4</sup>. Thus, one can construct a “quantum field” mapping the vector space at issue into a class of operators acting on it.

(5) The operation of localization is applied to the of set of all natural numbers, “{N}”, and to all natural numbers, “n”, that is:  $\forall n \in \{N\}, \exists \Lambda_n(n, \{N\})$  where  $\Lambda_n$  are granted to be different in general. Then, one defines a triple relevant to  $\forall n \in \{N\}, \exists \Lambda_n(n, \{N\})$ , and namely replacing “{N}”: (5.1)  $\forall n \in \{N\}, \exists \Lambda_n^0(n, N^0)$ , (5.2)  $\forall n \in \{N\}, \exists \Lambda_n^+(n, N^+)$ , and  $\forall n \in \{N\}, \exists \Lambda_n^-(n, N^-)$ . The triple is determined unambiguously due to the tree bijections (valid in virtue of the axiom of choice):  $\{N\} \leftrightarrow N^0$ ,  $\{N\} \leftrightarrow N^+$ ,  $\{N\} \leftrightarrow N^-$  correspondingly.

(6) One interprets (5.1), (5.2), and (5.3) as a bit of information, and following the notations in (2) as to the set of all natural numbers, as to any certain natural number, namely:

$$(6.1) B =_{def} \Lambda_n^0(n, N^0), \forall n \in \{N\} \text{ or } (6.1)' Bn =_{def} \Lambda_n^0(n, N^0), n = n_{const};$$

$$(6.2) B_c =_{def} \Lambda_n^+(n, N^+), \forall n \in \{N\} \text{ or } (6.2)' Bn_c =_{def} \Lambda_n^+(n, N^+), n = n_{const};$$

$$(6.3) B_{uc} =_{def} \Lambda_n^-(n, N^-), \forall n \in \{N\} \text{ or } (6.3)' Bn_{uc} =_{def} \Lambda_n^-(n, N^-), n = n_{const}$$

(7) The interpretation in (6), i.e. both (6.1), (6.2), 6(3), on the one hand, and (6.1)', (6.2)', 6(3)', on the other hand, define a qubit of quantum information due to the equivalent definition of “qubit” (Penchev 2020 July 10) as the choice of an element belonging to an infinite set. The former triple defines a qubit as the qubit Hilbert space, and the later, as a single qubit to the infinite series of qubits relevant to the qubit Hilbert space. No contradiction arises since a countable set consisting of countable sets  $\{S\}$  is countable also in relation to the elements of all countable sets  $\{S\}$ .

The advantage of that absolutely formal definition of “qubit” consists in the use only of terms of set theory and arithmetic rather than those of quantum mechanics and information. This is necessary if one need unify the foundations of mathematics (as arithmetic and set theory) and those of quantum mechanics and information: an idea seeming to be even ridiculous at first glance or from the viewpoint of the scientific common sense since mathematics and quantum mechanics (physics) are not only different sciences, but belonging to disjunctive classes: postulative science versus empirical and experimental science. Furthermore, mathematics seems to be much more fundamental than quantum mechanics. So, the perception of the present paper needs a relevant “Gestalt change” justifiable by the philosophy and history of quantum information (Penchev 2009).

However, the bijection of the triple of Peano arithmetics and the qubit Hilbert space sharing the structure of a bit of information (Penchev 2020 August 25) can serve as a sufficient

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<sup>4</sup> The conception of “local and global space” only interprets “spacely” (i.e. in terms of Euclidean space) the “non-Euclidean-space” base of quantum mechanics of two idempotently dual separable complex Hilbert space. Here and further “separable” is added to the “complex Hilbert space” for the extended term of both to be valid to classes avoiding the axiom of choice standardly involving by set theory rather than by propositional logic.

justification of merging the foundations of mathematics and quantum mechanics. If one redefines each of the two twin and oppositely directed Peano arithmetics in relation to the dual counterpart and replacing its first element and function successor with the proper ones, the latter counterpart would represent a transfinite Peano arithmetic of transfinite ordinals (which can be redefined in turn as bijectively corresponding “transfinite natural numbers”) in relation to the former counterpart.

Furthermore meaning the tracked bijection of the “bit triple” of Peano arithmetics and the qubit Hilbert space, there exists a bijection (involving the axiom of choice explicitly) of all wave functions (i.e. all elements of the qubit Hilbert space) and the two dual transfinite complements of the two dual finite Peano arithmetics. Then, mathematics considers the finite “bit-like”<sup>5</sup> triple of Peano arithmetics, and quantum mechanics considers the transfinite “bit-like” triple of Peano arithmetics: the qubit Hilbert space is isomorphic.

The following metaphor seems to be relevant: quantum mechanics (underlying all physics) is the “arithmetic of infinity” in opposition to the usual Peano arithmetic of the finite. The concept of “Hilbert arithmetic” means the bitlike triple of Peano arithmetics and it is definitively isomorphic to the qubit Hilbert space.

The philosophical reflection of Hilbert arithmetic is relevant to a quantum neo-Pythagoreanism establishing particularly the completeness of mathematics being Pythagorean metaphysics and including the world within its scope. That kind of complete mathematics and not less complete physics isomorphic to mathematics (once it has been based in a way similar to the foundations quantum mechanics) can be called and defined rigorously as “Hilbert mathematics” in opposition to “Gödel mathematics” (the latter is excluding the world from its immanent scope)<sup>6</sup>.

## II TRANSCENDENTAL TIME

The above “interpretative bijection” of the real axis (relevant as to the quantity of reversible time as to any other physical quantity) as the qubit Hilbert space (sharing the same kind of double relevance) is able to elucidate simultaneously the following two statements: (1) Pauli’s famous phrase about “time being only a number”, but not an operator in quantum mechanics unlike all other physical quantities and equivalent to the “conservation of energy conservation” in it; (2) quantum information is able to generalize the quantity of time of classical physics as to any temporal process rather than only to quantum mechanics as the formalism of the separable complex Hilbert space is interpreted usually by the “scientific common sense”.

The former (Pauli) statement distinguishes the corresponding former member (the real axis of time) of the built “interpretative bijection” from the latter one. The latter (and advocated by the present paper) statement refers to the same bijection as a sufficient justification for the qubit

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<sup>5</sup> The term “bit-like” (or bitlike) means a structure isomorphic to a bit of information: a qubit represents one Peano arithmetic as a state of bit in the case and just as the qubit Hilbert space or any certain number-sake pairs of its qubits.

<sup>6</sup> The conception of Hilbert mathematics versus Gödel mathematics has been formulated initially in: Penchev 2010.

Hilbert space to be considerable as a generalization of time and thus applicable in principle to any temporal process studied by any science.

Particularly, the qubit Hilbert space can be interpreted as a special, physically dimensionless quantity of quantum information being a correlate to the Lie group of action in the first Noether (1918) theorem. That group of action is naturally to be that of the corresponding Hermitian operators<sup>7</sup> acting on the separable complex Hilbert space and redefinable unambiguously to the qubit one.

Reversible time can be introduced as a formal parameter of that group, and implying the particular case of energy conservation once that time has been granted as a real physical quantity (as classical physics and the “classical” quantum mechanics<sup>8</sup> do). Nonetheless, the Lie group of Hermitian action operators is able not to be associated with time, therefore suspending energy conservation in general and establishing the conservation of quantum information as its immediate correlate in the first Noether (1918) theorem and generalizing energy conservation<sup>9</sup> (being valid only on the “screen of time”, i.e. only after postulating the associable formal parameter of the group as a real physical quantity).

The conservation of quantum information in relation to the first Noether (1918) theorem (and unlike to the second one, which will be discussed a little further) is to be interpreted as the conservation of the free variable of quantum information, i.e. the qubit Hilbert space itself as relevant and thus valid to any physical process (rather than only to temporal ones). That statement seems to be obvious, even trivial<sup>10</sup> as far as “physical process” can be defined by the measurable physical action produced by it, and the quantity of action (as any other quantity in quantum mechanics) needs the separable complex Hilbert space (equivalent to the qubit one) as a necessary condition to be at all definable.

However, the sense of the so interpreted conservation of quantum information consists in its relation to its interpretation by means of the second Noether (1918) theorem for the particular case of a single-parametric group and which is forthcoming now:

The former case (applying the first theorem) can be interpreted as the conservation of the parameter of that group, namely the free variable of quantum information, what the qubit Hilbert space is. That parameter is mapped as into a certain wave function as into all wave functions

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<sup>7</sup> One should notice that the Lie group of Hermitian operators is isomorphic to all physical quantities definable in quantum mechanics and including that of action: it can also be investigated as a paradigmatic and justifying pattern both philosophically and physically & mathematically.

<sup>8</sup> The term “classical quantum mechanics” means it without the theory of quantum information, i.e. without the domain of entanglement and its phenomena.

<sup>9</sup> Philosophical as well as historical and scientific dimensions of the transition to that generalization are discussed in: *Penchev 2020 October 5*.

<sup>10</sup> The statement is trivial only physically rather than philosophically or philosophically and mathematically. It establishes that the qubit Hilbert space, respectively the free variable of quantum information is the ultimate structure, respectively variable in virtue of the completeness provable internally, i.e. purely mathematically only within the structure at issue. In other words, quantum information can be interpreted as the ultimate most fundamental substance of the physical world rather than as its universal substance according to the contemporary stage of physical cognition (but presumably replaceable by some still more general one in future).

(in virtue of the axiom of choice) and both considered as two different one-parametric groups, furthermore sharing the same parameter, are isomorphic to each other, on the one hand, and to the group of the parameter, on the other hand.

Then, as any single wave function as all wave functions can be interpreted by the second theorem stating the existence of an implicit function given by the differential equation (being one single for one single-parametric group, what is the case) of the functional (called also “variative”) derivative<sup>11</sup> of the wave function and the physical quantity of action. The physical sense is that the functional derivative of any wave function (being another wave function) causes a certain physical action not less than the minimally possible physical action equal to the Planck constant. So, the application of the second theorem justifies the fundamental conjecture that an unambiguous link of the Planck constant (possessing the physical dimension of action) and the unit of information (being physically dimensionless) exists necessarily.

Further, that physical sense for the second theorem to be applied can be distinguished and elucidated as to both cases: either a certain wave function or all wave functions (to which the second theorem states the existence of the differential equation at issue). A certain wave function is to be referred to the functional itself, and the case of all wave functions, to the variable of that functional, therefore being idempotent to each other in the case of the separable complex Hilbert space. In other words, the second theorem turns out to be applicable to both dual Hilbert spaces simultaneously, but in the same sense.

A smooth metric of the separable complex Hilbert space (what is necessary for the application of the second theorem) can be defined in different ways, some of which may be equivalent to each other. Being the most intuitive and sufficient for the present objectivity, one can utilize the smooth metric of phase shift meaning infinitesimal phase variations, i.e.  $\frac{d\Psi}{d\varphi}$  where the phase vector, “ $\{\varphi_n\}_{n=1,\dots,\infty}$ ”, consists of the separate phase shifts for each axis, “ $n$ ”, of the separable complex Hilbert space:  $e^{i(n\omega + \varphi_n)}$ .

Then, phase variation is interpretable as “entanglement variation” and therefore generating physical action in virtue of the application of the second theorem in relation to a wave function (all wave functions) as a one-parametric group of the single parameter of  $N^+$  ( $N^-$ , respectively, for the dual and idempotent case of all wave functions).

Though  $N^+$  ( $N^-$ ) is not continuous, its parameter in turn is bijectively<sup>12</sup> the smooth reversible time (undergone to the axiom of choice), and the latter can be considered as the parameter in order to satisfy the conditions of the second theorem literally.

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<sup>11</sup> Functional (or variative) derivative of a functional (what a wave function is in relation to a mapping of the space dual to which it belongs) is defined by the infinitesimal change of the value of the functional to the corresponding infinitesimal change of the function (variable of the functional) in the metric of the function space (which the separable complex Hilbert space is in the case). The metric is meant to be determined explicitly since a function space usually admits more than one metrics.

<sup>12</sup> The bijection is due to the “joint triple relativity” considered above.

The conclusion is that the second theorem implies for the functional derivative meant by it to be interpretable physically as entanglement and then equitable to physical action, therefore simultaneously equating (but eventually not identifying) the units of information (whether qubit or bit) and the natural unit of action, what the Planck constant is.

As far as the subject of the application whether of the second theorem (as to the “values”) or the first theorem (as to the “free variable”) is the same, quantum information, one may identify their statements: namely, the conservation of quantum information (according to the first theorem) can be considered as equivalent to the physical action of entanglement, or quantum information (according to the second theorem).

So, the physical sense of a law more general than energy (energy-momenta) conservation, namely the conservation of quantum information<sup>13</sup> can be established and elucidated as managing all physical phenomena out of time (i.e. entanglement).

One can notice that the mathematical “resource” of entanglement are those mappings which are not functions, i.e. many-to-many (or even rather “much-to-much”) mappings, the most general case of mappings, after which the shared values (or rather “interval”) of two or more of those mappings implies for them to be entangled. As far as classical physics<sup>14</sup> remains thoroughly within the mappings-functions, entanglement is fundamentally inaccessible in its framework.

The reason for classical physics to privilege the functions is due to the “temporal screen” equivalent to their physical application: indeed, any function interpretable physically admits the parameter (variable) of time unlike all the much-to-much mappings managed to steal by the “backdoor” of quantum mechanics into physics and therefore rejecting the universality of time.

However, the universality of time is unconditionally true for any human experience. The situation seems to be paradoxical: our cognition, though in the framework of the experimental science, what quantum mechanics is, turns out to be wider (even rather “much more wider”) than our empirical experience: registrable and restrictable to the macroscopic “apparatus”, as to quantum mechanics.

The same observation implies radical physical and philosophical conclusions also in relation to time: it should be considered as an emergent macroscopic property and inherited in our science from human experience developed initially in that macroscopic cognitive niche, but “left

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<sup>13</sup> Quantum information conservation is tracked from the foundations of quantum mechanics in detail in a previous paper (Penchev 2020 October 5).

<sup>14</sup> The “prejudice of function” shared by classical physics is so steady that even quantum mechanics is formulated in terms of functions, namely wave functions. Thus, a wave function means unambiguously a probability density distribution of a quantum state unlike any function in classical physics referring to *real* values rather than only *possible* ones. Anyway, “function” is saved acting to a fundamental different class of variables: probability density distributions. That approach corresponding to the “function tradition” in physics is isomorphic to introducing “much-to-much” mappings reduced to two dual and complimentary “much-to-one” functions distributed alternatively either in the one dual Hilbert space or in the other one. However, entanglement violates even that way-out for saving the paradigm of function”, demonstrating that “much-to-much” mappings imply a continuum of functions besides the two “much-to-one” functions orthogonal or complementary to each other, to which the “classical” quantum mechanics reduces it.

already the cradle” (paraphrasing the famous metaphor of Tsiolkovsky) by quantum mechanics and especially, by the theory of quantum information.

Intermediately summarizing, what corresponds philosophically to quantum information, a physical and mathematical concept, is the totality. One can demonstrate that quantum information is conserved, and even that it is the ultimate conservation law also in mathematics (as consistently complete) rather than only in physics (where generalizes energy, respectively energy-momentum, conservation). Quantum information conservation is ultimate since it originates from the “conservation of the totality”, which is its definitive property: to be itself by itself, i.e. self-identical and *conserving* itself.

In other words, what one means after quantum information conservation is the postulation of the totality in the final analysis, an achievement of classical German philosophy from a few centuries ago. The philosophical category of Time (with capital letter) as well as the empirical and physical concept and quantity of time is figuratively only a “screen” situated *within* the totality, originating emergently from the macroscopic level of human experience, but far not exhausting what the totality is: there exists a much more huge domain of the totality being out of the temporal screen, to which physics touches precariously and “darkly” only recently by “dark matter” and “dark energy”.

Furthermore, quantum information conservation implies all phenomena of entanglement: corroborated experimentally very well (that is: right on the screen of time). Being out of the temporal screen, they violate energy conservation in general, or in other words, they represent the violations of that symmetry to which energy conservation correlates, but within the unconditional framework of the totality, respectively quantum information conservation.

### III THE “TEMPORAL SCREEN” IN DETAIL

The next deductive step is to describe the temporal screen and energy conservation in the same terms of quantum information as entanglement; that is: entanglement as the violation of those symmetries able to represent supposedly the temporal screen and energy conservation exhaustively. The hope is that this step will assist in building a paradigm for translating between the “languages” of “external reference frames” and quantum states (being the well-known and utilized “language” of quantum mechanics or quantum information).

A qubit can be visualized as a usual ball in the usual 3D space of our experience with the following properties: to be unit (i.e. its radius is a unit); its orientation is constant (i.e. a certain, possibly orthogonal coordinate system is chosen and permanent); to be “empty” (i.e. no point is chosen within it initially); and last, only one point (being arbitrary, including the surface) can be chosen (“recorded”) in it. The justification of that visualization is its isomorphism with the standard definition of a qubit in quantum mechanics as the normed superposition of two orthogonal subspaces (non necessarily convex to each other) of the separable complex Hilbert space.

Then all that Hilbert space can be represented as an infinite series of balls (not necessarily convex to each other as its isomorphism with Minkowski space can demonstrate: *Penchev 2020*

July 14), but equivalent idempotently to a single ball in the dual space (also in virtue of the axiom of choice).

As far as the pseudo-Riemannian space of general relativity is homeomorphic to Minkowski space, and the latter is isomorphic to the separable complex Hilbert space of quantum mechanics, in turn isomorphic to the qubit Hilbert space, pseudo-Riemannian space is homeomorphic to the qubit space. Just the last conclusion is the physical and mathematical objectivity of the suggested visualization; or coining a metaphor: *gravitation* (as it is described by general relativity) is *deformed information* (as quantum mechanics is reformulated in terms of quantum information). Then, entanglement being the class of all homeomorphisms of the qubit Hilbert space is to coincide with gravity defined according to general relativity and being the class of all homeomorphisms of Minkowski space (as pseudo-Riemannian space can be defined as well); or figuratively: *deformed information is both gravitation and entanglement* (after exchanging the observer's reference frame between internal or external one correspondingly).

So, energy conservation (or respectively, the screen of time) can be described in the framework of quantum information conservation as a fundamental symmetry of the qubit Hilbert space visualized as above: as three qubits,  $Q_B, Q_{B_c}, Q_{B_{uc}}$ , furthermore each of them isomorphic

to an empty unit ball, and in the meta-framework of a bit of information:  $B =_{def} Q_B; B_c =_{def} Q_{B_c}; B_{uc} =_{def} Q_{B_{uc}}$ .

Thus, three different symmetries valid simultaneously are able to describe that “bit-like triple of qubits”: (1) the symmetry of each qubit separately (being a symmetry of a ball and thus isomorphic to “ $U(1)$ ”); (2) the symmetry of any pair of two qubits (being a symmetry of two unit balls and isomorphic to “ $SU(2)$ ”); and last, (3) the single symmetry of the triple of qubits (and being a symmetry of three unit balls is isomorphic to “ $SU(3)$ ”). Obviously, no other symmetries are relevant to the triple of empty qubits and thus, the composite symmetry “ $U(1) \otimes SU(2) \otimes SU(3)$ ” (where “ $\otimes$ ” means ‘tensor product’) is able to describe exhaustively the screen of time (respectively, the conservation of energy or energy-momentum) in terms of quantum information (respectively, its conservation).

In fact, the same composite symmetry, and thus the “temporal screen”, is postulated in the Standard model without any reason to be suggested to justify it. On the contrary, the phenomena of entanglement or the theory of quantum information go out of that screen, therefore admitting and depicting a physical world both alternative and generalizing what is “painted” by the Standard model.

If the composite symmetry holds, a conservative system corresponds therefore not experiencing any physical impact. The translation of “ $U(1) \otimes SU(2) \otimes SU(3)$ ” into the language of relative reference frames is: “a reference frame at rest to the observer's”. The sense of that translation (inferred and described above) consists in the exchange between an internal (continuous or smooth) reference frame and an external (discrete or quantum) reference frame. Then, the temporal screen (including the Standard model), postulated a discrete frame corresponding to a continuous reference frame at rest to the observer's and consequently, the

external reference frame established by the Standard model or the temporal screen, is to be defined or identified as “being at rest” (after the translation into “Quantum”, i.e. into quantum language).

On the contrary, any shift (motion) to the observer’s quantum reference frame generates entanglement in virtue of determining implicitly a second quantum system (respectively, a discrete reference frame being at rest to itself as the former one). The case of shifting only the position of the latter (i.e. with zero speed and acceleration) might seem most problematic (to be linked to entanglement) since two orthogonal qubit spaces for each of both are relevant to describe their system therefore non-entangled. However, one can discuss the case otherwise:

The system is a state of decoherence: but only now and admissibly, preceded by a series of less and less entangled states corresponding to a smooth motion of the internal counterpart of the discrete reference at issue, from the position identical to that of the former discrete reference frame to its present one.

As to the case of any nonzero speed or acceleration, the “Gedankenexperiment” of the triple article (Einstein, Podolsky, Rosen 1935) can be utilized to map unambiguously an entangled state of two quantum systems situated within the local spaces of two points associable with two reference frames of the global smooth space moving to each other.

Their thought experiment offers for two particles prepared in advance to be in an entangled state to be proved that they interact to each other determining their states mutually and unambiguously, including physical quantities incommensurable to each other in accordance with quantum uncertainty therefore violating it. As any classical physical interaction determines mutually the states of the interacting entities, entanglement after the EPR thought experiment is similar to a new physical interaction, however acting instantly and thus exceeding the fundamental limitation (constant) of the light speed in a vacuum and thus being out of special and general relativity: Einstein coined the metaphor of a “spooky action at a distance” to emphasize the contradiction.

Now, a huge number of experiments confirm many phenomena of entanglement therefore admitting to be changed the Gestalt of the thought experiment since it is corroborated sufficiently. In fact, EPR can supply unambiguous rules for “translating” between the smooth and temporal “language” of relativity and the quantum and atemporal “language” of entanglement since it describes the same phenomenon in both “languages” simultaneously and offers a “Rosetta stone” how the “unknown language” of entanglement to be interpreted into the “known language” of relativity and reference frames.

The description in terms of relativity is: two reference frames at an arbitrary distance and moving relatively to each other with a speed less than that of light in a vacuum. So, the pseudo-Riemannian space of general relativity (or particularly after zero acceleration, the Minkowski space of special relativity) is the “cognitive medium” relevant to the description. It is temporal and obeys the postulate of not exceeding the light speed in a vacuum.

The description in terms of quantum mechanics is: two separable complex Hilbert spaces each of which containing the wave function describing the state of either the one or the other are

entangled. Thus, at least a bit-like triple<sup>15</sup> number-sake axes are mutually rotated to each other therefore rejecting the orthogonality and independence of the three corresponding separable complex Hilbert spaces as well. That bit-like triple consists of an axis (all axes) of the Hilbert space associated with: (1) the system of both particles; (2) the one of them; (3) the other one.

The relevant cognitive medium consists of the separable complex Hilbert space, however generalized by the concept of deformation between its dual spaces being arbitrarily rotated to each other. That is: there exists a nonzero vector of rotation,  $\{Q_i\}_{i=1}^{\infty}$ , a series of qubits,  $Q_i$ : consequently a wave function, which can be even identified with the wave function of the system of both particles<sup>16</sup> situated as a “point” in the relevant qubit Hilbert space, but furthermore determining it.

That vector of rotation  $\{Q_i\}_{i=1}^{\infty}$  can be obtained from an entanglement tensor of an arbitrary or infinite dimension  $\{Q_{i,j,k,l \dots (\infty)}^{i,j,k,l \dots (\infty)}\}_{i=1}^{\infty}$  where the low indexes are covariant, and the upper ones, contravariant. Then,  $Q_{i,j,k,l \dots (\infty)}^{i,j,k,l \dots (\infty)}$  means the rotation which represents the entanglement of the axes  $i, j, k, l \dots (\infty)$  of both entangled systems. The justification of the equivalence  $\{Q_i\}_{i=1}^{\infty} = \{Q_{i,j,k,l \dots (\infty)}^{i,j,k,l \dots (\infty)}\}_{i=1}^{\infty}$  relies on the axiom of choice and the unitarity of the complex Hilbert space in the final analysis, but will also be inferred in detail below.

If the case is more than two entangles system, it can be reduced to that of two ones by a smooth field of entanglement following the paradigm of general relativity to resolve an analogical problem representing the gravitational interaction of many than two objects as the changes and corresponding derivatives in the same point of the field. The continual tensor field of entanglement can be again enumerated in virtue of the axiom of choice granted already in the case of two.

On the contrary, the latter description of the EPR experiment by Hilbert spaces in a few paragraphs above neither is temporal nor obeys the postulate of non-exceeding the light speed in a vacuum. Both properties are meant by the metaphor for entanglement to be “instant”.

The EPR experiment (as a “Rosetta stone”) allows both descriptions to be linked unambiguously to each other and consequently, all the nontemporal phenomena of entanglement to be projected on the standard temporal screen of physics used by special and general relativity accordingly. This projection is a function, but not a bijection: the mapping depicts pairs of vectors into their relative rotation (possibly being the same for absolutely different pairs of

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<sup>15</sup> The EPR experiment means the maximally possible degree of entanglement where all axes are mutually rotated, but this is not a necessary condition.

<sup>16</sup> That option to identify the mutual rotation with the wave function of the system by means of the qubit Hilbert space allows for quantum information conservation to be formulated: this is the main reason to be introduced.

vectors). The relative rotation is also a vector of the same dimensionality rather than their scalar product.

The identification of the composite symmetry  $\{[U(1)] \otimes [SU(2)] \otimes [SU(3)]\}$  with a “discrete” reference frame at rest to the observer (respectively, to the observer’s reference frame) was inferred above loosely. However, sufficiently rigorously, it should be postulated being rather obvious: the reference frame at rest and the composite symmetry are the simplest ones, both, and equating them seems to be the most natural suggestion. In fact, any other and arbitrarily accelerated reference frame can be granted to be the zero one (in virtue of additional considerations, e.g. as to the reference frame of the Big Bang), to which any other is to be determined absolutely.

#### IV THE “ETHER”: A PRIVILEGED DISCRETE FRAME AS THE STANDARD MODEL

The hypothesis for any reference frame different from the observer’s and being furthermore “zero” naturally, restores that of “ether”, which was rejected by special relativity, but being consistent to general relativity according to Einstein’s word himself (Einstein 1920).

The “Big Bang” conception restores the privilege of just one certain reference frame: its proper, and thus mediately, that of a generalized “ether” definable as the choice of any other reference frame different from the observer’s to be “zero” in virtue of any additional consideration: i.e. that of the Big Bang as to the case.

A conjecture advocated further is that the Standard model is not other than the exhaustive description of the Big Bang reference frame, however in the discrete terms of quantum mechanics rather than to the smooth ones of general relativity. The correspondence is the following:

The four “electromagnetic” members of the Standard table of elementary particles and relevant to the  $U(1)$  symmetry refers to the four coordinates of the privileged position; the next four “weak” members supplied by the  $SU(2)$  symmetry, to the privileged speed. Since the representation is quantum or discrete, it is to correspond to Hamiltonian mechanics, i.e. position and speed are independent variables, and acceleration is to be still one pair of position and speed and independent of the former pair:

Thus, the eight “strong” members and referable to  $SU(3)$  correspond to the latter pair in accordance with Hamiltonian independence<sup>17</sup>.

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<sup>17</sup> The Hamiltonian definitive independence of time implies for acceleration to be a second pair of position and velocity changed jumpily. The fundamental Newton principle linking proportionally the acting force “F” and the acquired acceleration “a” due to it (“F=ma”) implies a bijection of the external reference frame (relevant to “F”) and the internal reference frame (in which “a” is meant as a second pair of position and speed if the consideration is to be consistent with the external reference frame being discrete). If one needs to add more forces (for example acting implicitly unlike the explicitly declared and investigated one, this can be expressed by varying the component of positions and velocity into the *inseparable* second pair due to acceleration (respectively, force field acting there). This is meant by

Thus, the sixteen Standard elementary particles (i.e. without the Higgs boson) can be interpreted as sixteen positions necessary and sufficient to define a certain and arbitrarily accelerated reference frame being interpretable as positions after  $U(1)$  or as eight speeds after  $SU(2)$ , or as four accelerations after  $SU(3)$ . That definition does not involve time, the absence of which is relevant both to quantum mechanics (meant as discrete reference frames, includingly) and to the Big Bang “where” (or “when”) there does not exist time yet.

The Higgs boson itself associates a certain mass at rest to that privileged reference frame following the Higgs mechanism of “spontaneous violation of symmetry”. From the “temporal viewpoint”, that violation is to be related to the time symmetry “forwards” and “backwards”.

Mass at rest in general can be interpreted as that asymmetry or respectively, as a compensating parameter able to restore the initial time symmetry of the nothing or “ex nihilo” (Penchev 2013; Penchev 2019). The Higgs mechanism interprets the same, excluding time:

If one considers the partial composite electroweak symmetry  $\{[U(1)] \otimes [SU(2)]\}$ , its decomposition to the two independent symmetries,  $[U(1)]$  and  $[SU(2)]$ , implies a nonzero violation of symmetry due to the “entanglement” of them:  $[U(1)] \otimes [SU(2)] - \{[U(1)] \otimes [SU(2)]\} \neq 0$ ; that is: their composite symmetry is different from their tensor product<sup>18</sup>.

One can interpret them as symmetries of two balls either separately,  $2 \times [U(1)]$ , or together,  $[SU(2)]$ . Then, the spontaneous violation of symmetry can be visualized as any nonzero intersection of the two balls, which differs from that of a tensor product where the intersection at issue is zero. The relation of  $[U(1)]$  to  $[SU(2)]$  is different in each case. The physical quantities determined by the Higgs boson choose a certain interaction following the same visualization.

Since  $SU(3)$  is isomorphic to  $\{[U(1)] \otimes [SU(2)]\}$  (e.g. being representable as  $\{[SU'(3)] \otimes [SU''(3)]\}$ ), the Higgs mechanism is to be only repeated in relation to it. Furthermore, the identical repetition of it as to  $SU(3)$  is also obvious after the interpretation in terms of acceleration and doubling both tetrads of the electroweak interaction. The intersection of the visualizing balls remains the same<sup>19</sup> whether to  $\{[U(1)] \otimes [SU(2)]\}$  or to  $\{[SU'(3)] \otimes [SU''(3)]\}$ : in other words, mass at rest is independent of acceleration, interpreting physically.

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confinement in strong interaction: it will be discussed in detail in Section 7 as to the changed viewpoint in the present paper.

<sup>18</sup> Entanglement is defined analogically, however, in relation to wave functions rather than to symmetries. Following the present context, the same difference would consist only in the distinction between values of quantum information (wave functions), on the one hand, and the free variable of quantum information (the qubit Hilbert space), on the other hand. In other words, the Higgs mechanism (as “entanglement”), referring to the free variable, is universal and the same to any certain wave function.

<sup>19</sup> Also due to the interpretation of the Higgs mechanism as entanglement in relation to the free variable of quantum information (as in the previous footnote): then, that same universal entanglement of the free variable should be shared by both electroweak two tetrads and strong two tetrads of the Standard fundamental particles.

The eventual reference frame of the Big Bang (furthermore associable with a certain mass at rest) as equivalent to the Standard model in a sense implies an additional conceptual shift, at that hidden or implicit. While the Big Bang thought as a certain reference accelerated frame possessing a nonzero mass at rest means it to be actual and real, as all smooth or continuous reference frames investigated by general relativity, the transition to the generalization of its discrete reference and interpreted as the Standard model implies for the Big Bang to be understood as any quantum phenomenon being invariant to the transition from reversible to irreversible time or from the state out of the temporal screen (quantum information) to the state projected on it as matter and energy including dark ones:

Anything being real and actual (e.g. as the Big Bang only in the framework of general relativity and classical physics) is meant to happen only within the temporal screen therefore implying energy conservation. However, the Big Bang by itself is an incredibly huge violation of energy conservation, after which the energy of the future universe appears “ex nihilo”. This is an obvious nonsense demonstrating that the extrapolation to the Big Bang to be a real phenomenon occurring only within the temporal screen as all others studied by relativity and classical physics is self-contradictory and thus false:

Regardless of its mega-size, only the quantum description is consistent and relevant to it being a colossal quantum leap: even more, all the following history of the universe (in which humankind takes part as well) is the complimentary, smooth description of that Bih Quantum Leap, and at that, only partial, being still unfinished yet. Accepting that viewpoint, the Big Bang cannot happen in a certain and singular moment of time ( $t = 0$ ) but it is to describe how time arises in any moment of time: or in other words, how the fundamentally atemporal quantum information generates time in any moment of time therefore projecting itself on the temporal screen. The Big Bang concentrated only in the initial moment of time ( $t = 0$ ) is rather mythical and generated by our cognitive prejudice that time and thus, the temporal screen is a necessary condition for any physical process. Even on the contrary, that prejudice blames any cognition claiming to study nontemporal physical phenomena to be anti-scientific, mystical and similar to religion in the final analysis.

However, the occurrence of time itself cannot happen in time without generating a logical vicious circle in turn inconsistent to any science. So, what is to be sacrificed is the prejudice of the Big Bang as a real and actual event similar to all others studied by special or general relativity or classical physics and regardless of its immense dimensions.

Instead of that, it should be understood as the permanent quantum process of decoherence taking place anywhere and whenever. Indeed, the magnitudes of all physical quantities associated with decoherence are quantum, but if they are integrated all over space-time, the colossal size of the Big Bang is to be equivalent though concentrated into a single point of space-time:

If the integration is restricted only to the quantity of time, i.e. all time acquired thanks to decoherence all over the universe, one obtains the alleged age of the universe: a little less than fifteen billion years according to the contemporary measurements and calculations. If one asks anyway what had existed *before* that period of time, the correct answer is not “nothing”, but

quantum information: being definitively nontemporal so that the relation itself meant by “before” is not quite relevant.

The consideration of the Standard table of the seventeen most fundamental particles as seventeen single numbers necessary to be determined a certain reference frame without involving time, or in other words, discretely or quantumly might generate decisive objections and criticism:

It identifies illegally and unfoundedly physical entities such as the fundamental elementary particles of the Standard model with mathematical entities such as mathematical entities, numbers (whether real or rational or integers or natural numbers, always in the final analysis), at that without considering the latter (being too simple and underferiated) as models of the former. The correspondence of the seventeen particles and the seventeen numbers is only an artificial analogy in virtue of which any sets consisting of the same number of elements might be identified not less unfoundedly.

The reason for the identification is the following. In fact, any elementary particles including the seventeen fundamental ones at issue can be defined in terms of the equivalent language of wave functions due to the wave-particle duality as classes of equivalence of wave functions, which in turn can be identified unambiguously with certain wave functions (possibly, finitely-dimensional, i.e. belonging as “points” to finitely-dimensional Hilbert spaces).

One can identify all wave functions with all transfinite ordinal numbers less than the least infinite ordinal number, then, the latter, with the transfinite natural numbers (in definition); in turn them, with the dual Peano arithmetic. Thus, all the physical world represented by all possible quantum states as all possible wave functions is isomorphic to all the mathematical world represented by the two dual Peano arithmetics. The constructed bijection of quantum states and natural numbers (Penchev 2020 August 25) is what is necessary to give ground for the problematic identification of seventeen numbers with seventeen particles.

A philosophical reflection on it would show that the bijection violates the cognitive prejudice for the physical world to be sharply distinguished from the mathematical one being a bridge between them excluding any clear boundary. That distinction in turn relies on Descartes’s dualism underlying the cognitive episteme of Modernity. Consequently and in the final analysis, the concept of discrete reference frame as a further generalization of Einstein’s general relativity (“covariance”), in fact referable only to continuous reference frames, goes out of the Modern episteme, or speaking loosely, this is an “impossible knowledge” being inconsistent to all the corpus of modern cognition therefore needing a “scientific revolution”.

Furthermore, that identification of wave functions and natural numbers implies the axiom of choice as a necessary condition. Thus, the identification remains uncertain constructively, or existing only purely and mathematically: as a result, any repetition of creating the bijection again would suggest a new, fundamentally random and different bijection in general. In other words, only a class of different bijections can be chosen in virtue of the axiom of choice, among which still one wave function would be necessary to determine the bijection unambiguously.

That additional wave function can be suggested (in physical considerations) to be the same as that due to the Higgs mechanism or the “initial entanglement of the free variable of quantum

information” and discussed above. Then, it, by a scalar product with any wave function, would define a functional of all wave functions (or all quantum states) into all natural numbers therefore determining ultimately and unambiguously a certain natural number to a certain fundamental elementary particle of the Standard model.

On the other hand, that functional can be bijective to the functional which the dual counterpart of Hilbert space represents in relation to the counter itself (whatever of both be) also explainable physically as the Higgs mechanism or the “initial entanglement”. Indeed, if the functional definitive for the dual Hilbert space is granted to be fundamentally symmetric and equivalent to the tensor product  $U(1) \otimes SU(2) \otimes SU(3)$ , the other functional bijective to the former would correspond to the Higgs violation of the symmetry by the composite symmetry  $\{U(1) \otimes SU(2) \otimes SU(3)\}$  of the Standard model.

The consideration allows for the wave function of the Higgs boson to be identified as the unknown wave function determining ultimately the latter functional by a scalar product. One can notice an ostensible asymmetry or even, inconsistency between the odd and even tetrads of the Standard model:

The scalar product of Higgs boson wave function with the wave functions of the second tetrad means physically the quantities of momenta and energy, and with those of the fourth tetrad, forces and work. However, the corresponding scalar products with the wave functions of the odd tetrads possess no traditional physical interpretations.

This can be explained by the predominant Lagrangian representations in mechanics especially historically and initially: it suggests dynamics involving mass at rest to be related to the quantities derivatives such as speeds and accelerations, rather than to the quantities not derivatives such as positions. On the contrary, the alternative Hamiltonian representation being furthermore equivalent to the Lagrangian one treats them as equivalent and symmetric. So, the scalar product of them with the Higgs boson wave function is not less meaningful physically though the tradition in physics has not established any denotements of them.

Furthermore, one can doubt about the physical meaning of the quantities of the fourth column corresponding to the fourth column of the fundamental elementary particles of the Standard model and being relevant to time such as the “time derivative of time”, i.e. the “speed of time” or the “second time derivative of time”, i.e. the “acceleration of time”. They correspond to the approach of general relativity for interpreting time in similarity to space distances or coordinates, in a space-like way, after which time speed and acceleration correspond to the “curve of time” in pseudo-Riemannian space. The natural correlate of “time speed” in energy-momentum tensor is just energy, and then, “time acceleration” corresponds to the quantity of work (due to  $x = ct$  and  $x^2 = c^2 t^2$  accordingly and valid in relativity).

The problem about the privileged reference frame, which general relativity brings with itself in quantum mechanics, implies real or seeming paradoxes. The emergence of both relativity and quantum mechanics was accompanied with ostensible contradictions, inconsistencies or “incompleteness”, in fact, only for the sharp opposition to the prejudice of common sense including the scientific common sense granting particular principles as universal and inviolable

in the new areas of cognition not less in the classical and traditional ones. Sometimes, the new scientific approaches and ideas only relevant to relativity or quantum mechanics seemed or were claimed to be even anti-scientific.

The generalizing introduction of external reference frames implying the reinterpretation of quantum mechanics and information into terms of quantum mechanics collides their foundations, the inconsistency of which is a fundamental obstacle for “quantum gravity”. The consideration of the “Big Bang” (valid in virtue of the extrapolation of the present description in terms of general relativity into the past) as only virtual or as a projectional integrating of all decoherence all over the universe “backwards” can be considered as such an ostensible contradiction in fact only to the prejudice.

## V THE INVARIANCE OF THE FUNDAMENTAL PHYSICAL LAWS AND THE NEW GENERALIZATIONS OF THE “ETHER”

However, the problem about the privileged reference frame challenges even the core of all physical knowledge until now: the invariance of the physical laws to all reference frames. There exist physical invariances valid in general relativity and in relation to any continuous and arbitrarily accelerated reference frame, but invalid to the unification of it with quantum mechanics or in relation to the newly introduced discrete reference frames:

Now, the conservation of quantum information is universally valid to both continuous and discrete reference frames, but energy conservation not. Furthermore, energy conservation implies the “temporal screen” to any phenomena claiming to be physical until now. On the contrary, the conservation of quantum information allows for entanglement to be out of the “screen of time” therefore admitting the existence of physical laws valid to all “projected on the wall of Plato’s cave”, i.e. to the “shadows”, but not to what is projected and only visible as shadows.

Transiting between the reference frames associated with the “walls of Plato’s caves” situated anywhere in the universe, one would notice that the projections on the wall(s) depend on the coordinates of its cave in the universe: in other words, the shadows of different caves will obey different physical laws being only projections, but the invariant physical laws will relate only to what is projected, i.e. properly only to quantum information implying a mapping of the coordinates of the cave in the universe into the particular physical laws valid to the corresponding wall, or in other words, a “wall field” or “cave field”, meaning the collection of certain particular physical laws established experimentally and thus valid only to the wall at issue, but not as to any other cave or wall arbitrarily distant in the universe.

The problem turns out to be gigantic if one had understood that the Standard model is (was) to belong to that collection of partial physical laws valid only on the “wall of the terrestrial or solar cave” but not at any point sufficiently distant from Earth or the Sun.

The problem is closely linked to the opposite solutions about the existence of the privileged reference frame offered by general relativity or quantum mechanics. Traditionally and historically, this is the “problem of the ether” (a hypothetical physical medium being in the state

of absolute rest to anything else), respectively a reference frame at absolute rest being associable with the ether.

The fundamental constant of light speed in a vacuum contradicts the hypothesis of the ether if it is interpreted as a certain privileged reference frame in the usual physical Euclidean 3D space. As far as Einstein's theory of special relativity postulates that constant, it contradicts the hypothesis so interpreted in turn.

However, the ether can be generalized relatively in a way to follow from special relativity rather than only to be consistent to it. For example, if the ether is a certain privileged reference frame, it can be that one linked to the light cone in Minkowski space. This implies the "observer" to be also generalized with zero mass at rest (what no human being might be) to be able to be situated in the "light reference frame", furthermore featuring by zero space-time "size" ( $r^2 = x^2 + y^2 + z^2 - c^2 t^2 = 0$ ) as the electromagnetic radiation itself.

Then, distance ( $r$ ), velocity ( $dr/dt$ ), and acceleration ( $dr^2/d^2t$ ) will be invariant to that redefined "ether" valid in Minkowski space and thus being the absolute distance, velocity, and acceleration. If one defines mass at rest as a "spontaneous violation of symmetry" (like the Higgs mechanism); that is the derivative of mass at rest associable with any point within the light cone to the space-time distance ( $dm/dr$ ), and more precisely, as the following integral

$(m(r) = \int_0^r \frac{dm}{dr} dr = \frac{m}{r})$ , mass at rest would be invariant to the "ether of light" and thus "absolute" as well. If  $r$  changes relatively small ( $\Delta r/r \approx 0$ ), one would restore the usual suggestion of mass at rest to be constant as far as  $r \approx ct$  in relation to the entities studied by classical mechanics.

One can state that the concept of the "ether" has been conserved in special relativity by the generalization of the ether to Minkowski space, linking it to the light cone, and redefining mass at rest relevantly. The description to that new ether is invariant and "absolute" in that sense in the framework of special relativity. One dare state that the "true physical laws" are those in relation to the "light ether" and the conceptual change to Newtonian mechanics consist only in the relevant redefinition of what the ether is:

The physical quantities of Newtonian mechanics are borrowed from human experience sharing the same implicit assumptions. When our cognition turns out to be far from the area in which those assumptions are valid, it needs new relevant physical quantities (such as space-time distance, mass at rest) definable by the traditional ones originating from human experience, after which the description by the new quantities can be true or "absolute".

The innovation from general to special relativity can be visualized as only the deformation of the light cone due to the gravitational field. Then, space-time distance turns out to depend on quantities relevant to the light cone deformation, but the conceptual paradigm of the newly introduced ether can be conserved only generalizing (or arbitrarily deforming) Minkowski space to pseudo-Riemannian space.

However, Einstein (1920) showed that the classical concept of the ether can be restored in general relativity in virtue of the conservation of energy-momentum (but not energy conservation and momentum conservation separately as in classical mechanics and special relativity) in the final analysis: it is consistent to general relativity (unlike special relativity), but does not follow from it necessarily. Any reference in which energy-momentum conservation is valid can be considered as the same and the absolute one and relevant to the classical ether: in fact, it can be reformulated exhaustively only in terms of energy-momentum conservation as that immovable medium to which the description in terms of energy-momentum conservation is relevant.

In other words, if one follows the deformed space-time distance, the ether of the deformed light cone can be introduced in general relativity, but if the deformed energy-momentum is what is followed, the Newtonian ether can be restored in virtue of energy-momentum conservation (analogical to the invariance of space-time distance in special relativity and allowing for the light ether to be introduced consistently in it). Thus, general relativity introduces implicitly a new invariance of the description (or respectively symmetry) allowing for the identification of the Newtonian and “light” ethers which can be interpreted as complementary to each other in the sense and even rigorous meaning of quantum mechanics.

The newly restored Newtonian ether in general relativity is anyway a different or generalized one in a few features: (1) it means the class of all pairs of reference frames accelerated arbitrarily to each other and smooth trajectories for mutual transformations; (2) one certain reference frame among that class can be chosen conventionally or according to additional and external considerations (e.g. that of the Big Bang) to be the “zero one”, to which all relative physical quantities are transformed into conditionally “absolute quantities” (i.e. under the convention for the chosen reference to be the zero one and able to transform relations into properties); (3) the class of all pairs of reference frames in (1) is isomorphic to the class of all pairs of their corresponding quantities to that of the “ether” (the zero reference frame) in (2).

One may say, that what is the “ether” (respectively the privileged reference frame) has been changed or generalized, but that generalization has been interpreted as the absolute rejection of the ether, rather unusually in science, especially in physics. In fact, general relativity (also special relativity, quantum mechanics, and the Standard model, as will be articulated in the next few paragraphs) shares the new privilege of the whole, system, even the “totality” heralded yet by German transcendentalism as the only possible resolution of the Cartesian gap between the mental and spiritual, or the bodily and material:

What is the absolute in general relativity is the *class of all* arbitrarily accelerated reference frames rather than a certain one among them though the latter can be chosen conventionally or according to any additional consideration as representing that “class of all”, i.e. the wholeness, system, or even the totality. One may say, furthermore, that the privileged reference frame of the ether has been shifted cognitively from being “inside” to being “outside”, i.e. from one certain element to the class of all.

What corresponds unambiguously to that only cognitive shift of thought is physically the “reference frame of light” (though not “flat” as in special relativity, but arbitrarily “curved”

already): being situated on the exact boundary of all the physical (definitively in special and general relativity), it is able to represent the physical wholeness or “the totality”. Thus, one may say that the classical Newtonian ether has been splitted or doubled into two equal halves, which can be considered even to be the same (therefore sharing the formal structure of a bit of information).

(1) If the space-time viewpoint is what is to be kept, what is conserved is space-time distance, and the privileged reference frame of the ether turns out to be that of the “deformed” light cone in pseudo-Riemannian space. (2) If the disjunctivity of space versus time is what is to be kept, what is conserved is energy-momentum, and the privileged reference frame turns out to be that of the “class of all”, which is the class of all arbitrarily accelerated reference frames as to general relativity. At last (3), both (1) and (2) mean the same, or in other words, they are “relative” to each other (in the sense of relativity).

Then, the idea of the ether can be restored in general relativity, as Einstein elucidated yet in 1920, as consistent, but not necessary. Indeed, the eventual choice of a certain element among the class is consistent to the class, but not necessary: the class is absolutely determined without the choice of a certain “privileged” element conventionally or in virtue of other considerations.

However, meaning Einstein’s course of thought in relation to general relativity, it can be repeated literally in relation to special relativity, because the latter is only a particular case of the former for the case of a certain acceleration, zero. Indeed, if one considers the case of the nondeformed (i.e. “flat”) light cone of Minkowski space (rather than the deformed one of pseudo-Riemannian space), it can be identified as the “ether” and a privileged reference frame to be attached to it. If the Newtonian disjunctivity of space and time be kept, then, the class defined by energy-momentum conservation is disintegrated into two independent conservations: the one of energy, and the other of momentum and just as in classical physics. At last, both generalized ethers can be granted to be the same as above, in relation to general relativity.

Further, meaning the smooth sequence of the projections of pseudo-Riemannian space on Euclidean space parametrized by time, on which the axiom of choice acts, as an enumerable series of deformed Euclidean balls, the result is homeomorphic to the separable complex Hilbert space of quantum mechanics, furthermore distinguishing two cases as in the previous paragraph:

(1) The separable complex Hilbert space of the *classical*<sup>20</sup> quantum mechanics can be interpreted as homeomorphic to Minkowski space under the conditions of the axiom of choice implying the identification of the well-ordering of the latter (physically parametrized by time) with the absence of well-ordering in the former (after the unitary identification of vectors

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<sup>20</sup> The “classical” quantum mechanics can refer to a single quantum system needing correspondingly a single separable complex Hilbert space or can refer to many quantum systems being *unentangled* necessarily so that their joint complete system refers to a tensor product of their separable complex Hilbert space. Speaking loosely, the classical quantum mechanics researches the “flat” case however homeomorphic to the “deformed” one relevant to entanglement and investigated by the theory of quantum information naturally unifiable with the classical quantum mechanics into the contemporary quantum mechanics and information.

consisting of well-ordered components and functions consisting of the same components however commuting to each other).

(2) The separable complex Hilbert space of the theory of quantum information being the “flat” equivalent of entanglement therefore “flattens” isomorphically any homeomorphically deformed equivalent of the separable complex Hilbert in virtue of its infinite dimensionality allowing for any arbitrarily adding of complementing dimensions (i.e. able to represent any curved complex Hilbert space as the intended “flat” separable complex Hilbert adding to it arbitrarily many new dimensions (a set whether finite or infinite). In fact, the case (2) is homeomorphic to the pseudo-Riemannian space of general relativity.

The conclusion relevant to the kind of generalization is: the transition from special and general relativity to quantum mechanics and information conserves isomorphically any homeomorphic deformation in both, but distinguishes the latter from the former by the availability or not of the temporal screen (being mathematically equivalent to the availability or not of well-ordering after the use or not of the axiom of choice).

Meaning a relevant visualization by series of Euclidean balls, the generalization allows for the balls to be *exchanged* in any thinkable way in the separable complex Hilbert space, but not, in pseudo-Riemannian or Minkowski space, to the physical interpretations of which the “temporal screen” of well-ordering is a necessary condition. As to all homeomorphic deformations, the “balls” whether in the former or in the latter are equivalent including with respect to the peculiarity distinguishing them: the infinite dimensionality of the separable complex Hilbert space allows for any deformation in it to be flattened; a feature, which pseudo-Riemannian space is not able to share being finitely dimensional (four, or 3 convex + 1 nonconvex dimensions).

Those considerations suggest how the concept of the ether as well as its generalization to relativity can be translated in notions of quantum mechanics and information. Indeed, the qubit Hilbert space is to be related to Minkowski space, in which all “balls” (which implies all “axes”, since any ball is a pair of two “axes”) can be randomly mixed. One may say that the representation by Hilbert space in quantum mechanics is invariant to any course of time, respectively to any well-ordering of all moments of time, which in turn is equivalent to the reversible time of any coherent state.

The mapping between the “coherent” qubit Hilbert space and the ordered Minkowski space (which is even well-ordered under the condition of starting at a certain time) can be granted to be a bijection being divided into two sub-bijections under two different conditions: (1) the axiom of choice; (2) the normalized homeomorphism of a ball of any radius relevant to Minkowski space into a unit ball related to a qubit of Hilbert space.

Further, one can track what happens with the light cone of Minkowski space (meaning implicitly or as a target the generalized “luminous ether” of special relativity associated with it) after the above bijection of two phases: it is discretized (as quantization does), normalized, and randomly mixed. One might express this pictorially as to the Hilbert space of quantum mechanics by the figure of the “eternal ether” where “eternal” would refer to the physical and

mathematical independence of time due to the invariance to the class of all “orderings of time” (that is: all orderings of “balls” or “axes”).

The way for the arbitrarily “curved” ether of general relativity (because of the arbitrary deformation of the light cone inherited from special relativity as “flat” on the latter) to be transformed into the “eternal ether” (remaining “flat” or not needing to be “curved”) includes still two bijections under two new conditions and situated before (1) and after (2) correspondingly: (0) the normalizing homomorphism of an arbitrarily deformed light cone into a the standard “flat” light cone of Minkowski space or special relativity; (3) the unambiguous distinction of all normalizing homeomorphisms utilized in (0) by adding additional specifying and complementing qubits (also in the “flat” qubit Hilbert space) due to the arbitrary extensibility of the qubit Hilbert space able to add new and new dimensions, but remaining the same.

In other words, (0) is a surjection properly, and (3) is not a function, and consequently their composition  $(0) \circ (3) \equiv (0) \otimes (3)$  is a mapping of a too generalized kind: “much-many”. Nonetheless, it can be normalized to a usual bijection (i.e. “one-to-one”) by the condition of the axiom of choice, which is also used for (1), and still one condition consisting in adding additional specifying qubits in a way to distinguish unambiguously all surjections in (0).

An intuitive enough method for the constructive determination of all those complementing qubits is (e.g.) the following. Any homeomorphism of a certain light cone (deformed in an arbitrarily way) into the flat light cone can be distinguished from any other in the framework of the surjective class of those homeomorphisms as a well-ordered sequence of deformations of the unit ball (where the “unit”: “1” =  $ct_0$ ) in the course of time. That sequence can be discretized in virtue of the axiom of choice as an enumerated series of balls, then in turn transformed homeomorphically into an equivalent series of qubits which can be simply added as corresponding further dimensions to the qubits of the initial wave function.

So, a composite mapping is defined by  $(0) \circ (1) \circ (2) \circ (3)$  so that it is a bijection transforming any state of gravitational space-time field into a quantum state bijectively. Furthermore, (3) can be interpreted as (4): an entangled state of two separable complex Hilbert spaces which can be determined unambiguously under specifying conditions. Thus, still one and further bijection,  $(3) \circ (4)$  implying  $(0) \circ (4)$  due to the composite mapping  $(0) \circ (1) \circ (2) \circ (3)$ .

The physical meaning of  $(0) \circ (4)$  is “Gravity and entanglement are the same” speaking loosely, or “Entanglement being nontemporal is mapped as gravity on the “temporal screen” where it is fundamentally indistinguishable from the gravitational field caused by energy and matter in accordance with the Einstein field equation and “Mach’s principle” as it is formulated by Einstein (1918)”. The flat, but infinitely dimensional qubit Hilbert space of quantum mechanics able to represent both entangled and unentangled quantum systems turns out to be able to correspond bijectively to the curved and nonconvex, but finitely dimensional pseudo-Riemannian space, by which gravitational field is described in general relativity. Both

general relativity and quantum information mean the same utilizing, however, different languages.

The conception of the ether (respectively, that of the privileged reference frame linked to it) can be generalized further to both entangled and unentangled quantum reference frames bridging general relativity to quantum mechanics and information. That kind of generalized ether introduces a new definitive feature: unlike the well-ordered, though eventually “curve” ether of relativity, that of quantum mechanics is “eternal”, i.e. even any ordering misses rather than only a certain well-ordering available (as the former case); at that, it is always “flat” since any curving can be “flatten” by adding an additional number (infinite in general) of dimensions (axes) to the initial and also “flat” separable complex Hilbert space.

Thus, the ether of quantum mechanics and information, and to which the correspond privileged reference frame is ascribed, is to be the separable complex Hilbert space itself, or in other words, the free variable of quantum information: the class of all possible wave functions, each of which is relevant to a certain quantum state therefore implying an entity to which the state refers. The *substance* of quantum information itself, more fundamental<sup>21</sup> than energy and matter (which are always only a temporal projections needing time as a necessary condition which quantum mechanics does not obey) can be identified as that quantum ether as well as a certain quantum state: e.g. that possessing the zero wave function or that able to represent exhaustively and equivalently all the Standard model, or any other in the researcher's choice. Then, the existence as a material entity as a mental idea can be identified to be different from that zero wave function or even, to include it so that “nothing” to be existent in the latter case.

The fact that quantum information is always “flat” or can be equivalently “flatten” implies particularly the “conservation of energy conservation” in quantum mechanics and information (thus in the Standard model), but unlike general relativity, in which time is “spacelike”, and energy, “momentum-like”, and can be transformed in it. Therefore, neither energy nor momentum are conserved separately in general. The physical mediation between the Standard model with energy conservation and general relativity without that is carried out through the phenomena of entanglement (on the side of quantum mechanics) and dark matter & energy (on the other side of general relativity) by the conception of quantum information.

## VI THE VIEWPOINT OF THE “ETHER” TO PHYSICS

One can track sequently the generalization of the “ether” starting from Newtonian mechanics via special relativity, via general relativity, via quantum information, via the Standard model to

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<sup>21</sup> Even more: quantum information is not only more general, but the most general, i.e the ultimate substance in the following sense. It allows for a mathematical proof of completeness as what the theorems of the absence of hidden variables can be considered. In virtue of it, the completeness of quantum mechanics and that of mathematics can be identified as the same and embedded in Hilbert arithmetic being both a generalized arithmetic in the foundations of mathematics and the separable complex Hilbert space in the foundations of quantum mechanics and information (Penchev 2020 August 25). However, this does not imply that other ultimate substances cannot exist, but only that quantum information is one among them, eventually as the simplest one in a sense.

the ultimate end point of quantum mechanics and information summarizing the previous discussion as follows:

Initially, the ether is understood as an absolute immovable medium, to the reference frame of which any *relative* motion (i.e. quantitative *relation* of two reference frames in terms of the later theory of *relativity*) coincides with the *absolute* motion (i.e. a property) attributable to any physical body in Newtonian mechanics. Thus, the ether can be defined as a concept able to transform the physical properties which are relations in those which are properties:

This can be translated into a more contemporary, mathematical language as a hypothetical medium to which quantities being noninvariant in a theory (i.e. relations) to become invariant (i.e. properties), or respectively to a reference frame or an observer linked to that medium. That interpretation of the ether as a certain class of invariance can be continued further by the Noether (1918) theorems of conservation, and here is how:

Invariance can be interpreted in two ways simultaneously: as a symmetry or as a constant, “conserved” quantity. They are complementary to each other in a more generalized sense however originating and consistent with that of quantum mechanics: the symmetry means a class as the elements of a set (equivalent to that class); and the conserved quantity, the same class as a whole, i.e. as the set at issue. So, quantum complementarity is transformed is a more general concept of complementarity, namely that of a set and its element being situated in different hierarchical levels (e.g. in Russell’s or later type theory) furthermore relevant and consistent to quantum complementarity after the latter is formalized by means of dual Hilbert spaces, each of which is idempotently a functional of its counterpart consisting of a set of functions.

Thus, the ether conceptualized as above means a certain invariance relevant to the discussed theory, the corresponding symmetry and conservation; at that, the later (conservation) is substantialized; i.e. it is postulated to be a universal, “omnipresent” substance available as a certain amount in any physical entity<sup>22</sup>: e.g. matter, energy, or quantum information (as this paper advocates).

Then, the ether of special relativity would be related to the Lorentz invariance of space-time distance or energy-momentum, both being “flat” and non-transferable into each other (unlike general relativity) as well as to the medium and substance of light (being energy in the final analysis and available in any physical body according Einstein’s  $E = mc^2$ ), and to the symmetry of the class of all inertial reference frames. Then, the complete theory of special relativity can be understood as a new theory of the ether understood innovatively as an invariance (Lorentz’s), a medium-substance (light) penetrating anything as energy (“ $E = mc^2$ ”), and a symmetry (all inertial reference frames).

General relativity is not so revolutionary in the perfection of the concept of the ether as special relativity to Newtonian mechanics: the ether of general relativity is only “curved” (due to gravitational field) to that of special relativity. The corollaries from that gravitational distortion are: the Einstein field equation for the Lorentz symmetry; the curved energy-momentum

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<sup>22</sup> A visualizing metaphor might be that money is the “ether of the economy”.

possesses both non-zero energy and momentum projections simultaneously, and the curved space-time, both non-zero space and time projection, which can be interpreted as the mutual transformability of energy and momentum or that of space and time. The medium and substance of light is kept, but deformed by the gravitational field, and what is conserved is energy-momentum (but neither energy nor momentum separately in general). The symmetry of all reference frames is generalized to that of all arbitrarily accelerated frames smoothly transformable into each other.

If one grants that generalized conception of the “ether” as to general relativity, there appears a discrepancy between the logical and historical next steps of generalizations: they are the theory of quantum information (entanglement) and quantum mechanics correspondingly. In fact following a rigorously linear pathway of generalization, what would be to be the next after general relativity is the theory of quantum information. However historically, this did not happen (except perhaps as an implicit idea in the triple article of Einstein, Podolsky, and Rosen, 1935, however explicitly directed to challenge the “completeness of quantum mechanics”). Instead of that, quantum mechanics was established parallelly and absolutely independently of general relativity therefore generating a gap between them as well as all troubles about “quantum gravity” called to fill it, but remaining inconsistent until now; and quantum mechanics and general relativity, unconnected.

The present paper will not track that real course of history, but another, hypothetical and logical one so that the immediate generalizing counterpart as to the concept of the “ether” of general relativity will be quantum information, and the “flat” quantum mechanics will be represented as a further step after quantum information, but cyclically returning to special relativity closing a logical circle of successive generalizations<sup>23</sup>.

The necessary step from general relativity to quantum information (by the by, and analogical to that from special relativity to the classical quantum mechanics, which is “naively flat” and unlike the “sophisticatedly flat” quantum information) violates a fundamental “taboo” of experimental science not only physical: the postulate of general temporality stating that no natural process can occur out of time.

So, the generalization from the “curved”, but temporal ether of general relativity to that of quantum information allows for all moments of time to be “stirred randomly” in a way all orderings of them to be equivalent, and that kind of physical phenomena not to depend on time, unavoidably violating energy conservation in general, furthermore. Just, that arbitrary temporal mix allows for the next step from quantum information to the flat quantum information “sophisticatedly”: i.e. by the statement that a finite trajectory successive temporarily (as in general relativity) can be considered always as a “flat leap” from the initial to the final point, but consisting of an infinite number of components (“harmonics” of wave function or “axes” of the separable complex Hilbert space).

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<sup>23</sup> This logical circle is not “vicious”, but hermeneutical and generates the unification of special and general relativity, on the one hand, and quantum mechanics and information, on the other hand.

The “curved” light ether of general relativity is to be generalized to the “stirred”, or “eternal” ether of quantum information which is not a “light ether” already furthermore, but rather a quantum-information ether properly. The “curvature” can be kept initially as the tensor of mutual rotation of each “namesake” (or “numbersake”) pair of axes of the Hilbert spaces referring to two entangled quantum entities.

However, the term of “namesake” (“numbersake”) loses its meaning in virtue of the unitarity of Hilbert space though being relevant to the temporal pseudo-Riemannian space of general relativity. Just that property of unitarity allows quantum information to be always flattened to the “naively flat” Hilbert space of the “classical” quantum mechanics. One can figure metaphorically an imaginary “process of flattening” starting from the explicitly “curved” pseudo-Riemannian space through the intermediate stage of the qubit Hilbert space, “yet potentially curved” for its correspondence to the former, to the ultimate, unitarily “stirred” Hilbert space able to be explicitly flat because of that.

The way to be acquired the quantum information ether from that of general relativity may be visualized very well by the representation of a qubit as a ball in Euclidean space, within which (including its surface) a point is chosen. The picture of the ether consisting of those balls “theory by theory” seems as follows:

1. The “balls” have not appeared in Newtonian mechanics yet. In fact, the balls correspond to waves. Mechanics referred only to particles, and this is the reason for the “balls” to be missing then. Even optics was corpuscular due to the power of mechanics and its particle viewpoints. Anyway the undulatory theory of light also was available and competitive to the corpuscular one.

2. The balls were present implicitly in Einstein’s paper (1905) of special relativity: they became visible in Minkowski space, which is a spherical wave uniformly expanding in space in the course of time. Properly, Minkowski space consists of four identical copies of this expanding wave, each of which is determined as to the physical interpretation by two bits of the options:

- (1) either the past or the future (2) either sub-luminal or superluminal velocity. Thus, the balls, both well-ordered and “flat” (in the sense of rigorously spherical, i.e. not curved in any way and which follows from Minkowski space), entered in a fundamental physical theory. The “ether” as far as it would be to be generalized consists of consecutive light spheres (the surfaces of the balls) of different magnitudes (radiuses). Even wave-particle duality is available implicitly by duality of reference frames and the “light ether of balls”: the “light ethers of balls” can be interpreted as equivalent or “complimentary” to the subclass of inertial reference frames.

3. The same balls of special relativity are only homeomorphically deformed in general relativity as well as the light ether: thus, the equivalent or “complimentary” class consists of the smooth manifold of all reference frames.

4. The transition from the light ether of special and general relativity to the quantum-information ether of quantum mechanics and information is the innovative contribution and needs special attention. The essential problem is how an infinite continual set of deformed “gravitational balls” might be transformed into another infinite set of qubits, at that, countable standardly in a relevant way claiming to be equivalent. Four aspects of the problem follows after

the conceptual consideration is restricted to a single ball, therefore being able to be repeated as to an arbitrarily set of balls:

4.1. The arbitrarily deformed ball of general relativity (being a smooth manifold) consists of flat metrical neighborhoods about each point of the ball. Involving the idea of local and global space borrowed from the Standard model, one can identify the infinitesimally small “local space” of the neighborhood about each point with the global space, in which the curved ball at issue is situated. By applying the axiom of choice, the continual manifold of the curved ball is decomposed to a countable set of balls, each of which is assigned to a certain “flat” ball bijectively. Then, that set of flat balls is only interpreted as qubits.

4.2. Not worse, the same arbitrarily deformed ball can be represented in general relativity as two flat global balls, correspondingly contravariant and covariant, such that a tensor of local deformation, relevant to each point is mapped bijectively to any pair of corresponding points of both balls. Abandoning the many-dimensional order of the local tensors due the unitarity of the qubit Hilbert space (inherited from that of the separable complex Hilbert space), the components of any tensor can be represented by a well-ordered vector or as a function, in which its additive members are commutative (i.e. unordered).

4.3. Then 4.1 and 4.2 together imply by the mediation of the curved ball of general relativity that the function of entanglement being a non-Hermitian operator properly is equivalent to Hermitian operator after “flattening” the former according to rules isomorphically to the flattening of any local tensor to a corresponding vector or function if “order does not matter” (as the nontemporal generalization claims), and the the axiom of choice, before that, has transformed the smooth manifolds relevant to general relativity into the countable sets of qubits (respectively, of “axes” of the separable complex Hilbert space).

The physical sense of that “general flattening”, which quantum mechanics is always to implement, is the following. Any set (including infinite) of entangled subsystems can be considered as a single system of them, to the Hilbert space of which that of any of those entangled subsystems is not orthogonal in general. The Hilbert space of the system is “flat” definitively, and it exists necessarily if being interpreted as that of the universe. Its corresponding curvature (returning to the pseudo-Riemannian space of general relativity as to the universe) is zero: a statement coinciding with the experimental data strictly:

The global curvature of the universe can be granted to be really zero (Ries 1984; Pierpaoli 2000; Masi et al. 2002; Vardanian, Trotta, Silk 2009; Ahmed, Bamba, Salama 2020; Efstathiou, Gratton 2020). However, the reason for that is not that suggested by the “physical common sense”, namely that matter and energy (including dark ones) are “infinitely sparse”, so the curvature caused by them is infinitesimally small. It is false. Perhaps, as the visible mass and energy as dark ones are only relative and making sense only to the relation of a subsystem to another (including to the system of the universe as a whole). So, the global curvature of the universe itself turns out to be definitively zero, because this is the curvature of an identity, or otherwise paraphrased, mass and energy cannot be defined globally at all.

So, the flattening of pseudo-Riemannian space by quantum mechanics can be subdivided into two stages:

4.3.1. The “postulated flattening” due to the application of the axiom of choice after which the smooth continuum of pseudo-Riemannian space is “enumerated” into a countable sequence of space-time tensors for each point.

4.3.2. Each of those space-time tensors is flattened again in virtue of the axiom of choice into an infinite vector or to the unordered (“commutative”) set of its components in virtue of the unitarity of the separable complex (or qubit) Hilbert space.

As a result and interpreted physically, any entangled subsystem is projected as an equivalent, “flat” subsystem of the universe, for which the mass and energy can be defined uniformly to the Standard model, and within the framework of the classical quantum mechanics thoroughly. This finishes the transformation from special and general relativity to quantum mechanics information, after which Minkowski space is mapped into the one of the dual qubit Hilbert spaces.

However, it is not bijective, or in other words, it is bijective to the two certain bits (four “letters”) of classical information: they which are to be added necessarily to any qubit to be determined to which of the four subdomains of Minkowski space it belongs.

Thus, Einstein, Podolski, and Rosen (1935) would be correct in their statement, that the “description of quantum mechanics is incomplete” due to the lost of an infinite amount of classical information after “translating” all qubits (those four “letters” for each of them) of any wave function.

Nonetheless, the same information is not lost since it can be added as a still one qubit<sup>24</sup> to the infinite set of the rest qubits, and the description of quantum mechanics to be complete (as e.g. Niels Bohr stated, on the other hand).

So, the problem whether quantum mechanics is complete depends on additional assumptions missing in the initial formulation after which quantum mechanics is “both complete and incomplete”.

4.4. The “quantum-information ether” of quantum mechanics and information consists of qubits, a static, omnipresent and eternal medium, which is what generates space-time, matter, and energy therefore becoming visible at any point of the universe by means of the rest quantum information “visible only *invisibly*”, i.e. as *dark* matter and energy. This is all the amount of the “quantum-information calculations” necessary for the visible universe to appear on the “screen of space”: it represents physical quantities (or Hermitian operators in terms of quantum mechanics). The “dark universe”, the “quantum computer hidden behind the screen of space-time” possesses an energetic volume about twenty times greater than that of the “screen” *itself*, which one has identified as the universe *itself and even by itself* until now.

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<sup>24</sup> As well as: all qubits to be multiplied by 4 for each subdomain of Minkowski space or a quantum number of four values to be added in each qubit (or a certain “old” and known one to be recognized or interpreted as such). Whatever way has been chosen, the essence is that an infinite set can be bijectively mapped onto a true subset and unlike any finite set.

That new and ultimate (in a sense) quantum-information ether possesses still one symmetry (respectively its violation), or rather, makes it explicit being hidden or not-articulated in special and general relativity, on the one hand, and quantum mechanics and information, on the other hand, since it consists in the relation itself of those “two hands” of relativity and quantumness correspondingly, above:

This is the “still more general principle of relativity” after introducing the concept of “external (discrete or quantum) reference frame”, which postulates its indistinguishability from all the smooth manifold of all standard, arbitrarily accelerated reference frames investigated by general relativity and even all the volume of “reference frame” until now. That symmetry (respectively asymmetry) is embodied in any qubit representing its insufficiently emphasized essence: the symmetry (or asymmetry) of the chosen single point and all the qubit before the choice (or an empty qubit). It represents simultaneously two fundamental principles: the wave-particle duality and the local-global-space duality of the Standard model:

Indeed, the point chosen among all others can be interpreted (after postulating that symmetry or “duality” of quantumness and relativity) both as a particle and as containing a local space being the same as the global one correspondingly. Accordingly, its counterpart of the empty qubit itself stands both for the wave “peer” of the particle meant by the chosen point and for the global space consisting of an infinite set of copies of itself within each point of itself. For example, that symmetry (asymmetry) is able to explain extremely simply and concisely the flattening of pseudo-Riemannian space (being a curved 3D equivalent of a qubit in each moment of time or to an observer in any space-time point) into the qubit Hilbert space and thus, into the usual separable complex Hilbert space:

Initially, the point of an observer in a reference frame is represented as a choice of the point (where the observer is) among all points of the space-time arbitrarily deformed in virtue of a certain state of gravitational field. So two asymmetries are available at the same time due to (1) the choice of an observer (i.e. the point, in which it is situated); and (2), the nonhomogeneity of the gravitational field being always a deformation in definition (i.e. being really symmetric, it would be zero).

Well, the “flattening” consists in the summation of both symmetries into single one expressed as the asymmetry of the observer<sup>25</sup> in the “flat” Hilbert space of the classical mechanics, not less equivalent to the “flat” Minkowski space of special relativity, in which the observer is changed jump-likely and compensatorily.

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<sup>25</sup> The alternative summation of both symmetries as a single one, but expressed as a jump-likely and compensatorily changed gravitational field, in which is chosen a privileged reference frame, in which the observer is situated, as if in “the center of the universe” is not less relevant and correct in virtue of the “supersymmetry” (which is not the standard use of “supersymmetry”), which is considered. Thus, one can reverse the direction from the flattening the picture painted by general relativity: that is the description of quantum mechanics and information to be relevantly deformed according to the account of general relativity. In other words, one can acquire a gravitational redescription for any quantum phenomenon. The paper demonstrates the very important case to be reformulated the Standard model as a privileged reference frame.

Then, the quantum-relative symmetry (asymmetry) can be interpreted in one more way: any change of an observer to another can be represented as an equivalent change of what is observed as well as vice versa; not less, any superposition of the former, “subjective” and the “latter” objective “change” takes place under the condition of quantum conservation, to the continuum (interval) of which both, purely subjective and objective changes are the two extremes. Or in other words, a certain quantity of the subjective quantum information always corresponds to another certain quantity (or even: to the same under additional conditions) of the objective quantum information.

Speaking philosophically or more loosely, quantum information is able to unify not only the foundation(s) of physics and mathematics, but also: the two poles of Descartes’s “body - mind” dualism, respectively, the “subject - object” opposition of German classical philosophy, at that quantitatively (that is: in the framework of a single mathematical structure).

Thus, the quantum-information ether is able to unify or to restore the initial Newtonian unity of an immovable observer and an immovable medium, i.e. the ether (turning to be inconsistent to each other and so forcing either the former or the latter in the later theories, such as special and general relativity or quantum mechanics). Their unification is possible only in virtue of the conservation of quantum information. The relation of what the ether would be to be (i.e. whether an immovable observer or immovable medium) can be tracked in a few details in those theories:

The choice of special and general relativity is in favour of the light medium being the same postulatively to any possible observer who therefore is changeable or exchangeable. Relativity emancipates itself from Newtonian mechanics just opposing the absolute observer and reference frame.

On the contrary, quantum mechanics is forced to restore the absolute observer linked to the apparatus studying the system including both the apparatus and quantum entity. So, the ether of quantum mechanics can be identified with the apparatus unchangeable to changeable and measurable quantum entities.

The light ether of relativity and the apparatus ether of quantum mechanics are to be unified by the quantum-information ether and conservation therefore suggesting the option to be investigated quantitatively the natural ways for the “objective ether” of relativity to be transformed into the “subjective ether” of quantum mechanics or vice versa.

## VII THE RECONCILIATION OF THE STANDARD MODEL AND QUANTUM INFORMATION BY CONFINEMENT

One may return to the initial metaphor of many Plato’s “caves” situated anywhere in the universe. Their “walls” represented the temporal screens, on which the “chained people” observe different shadows. However, all “walls” as well as the depicted on them should share the same physical laws and the Einstein field equation and the Standard model in particular. However, if the Standard model is equivalent (as above) to a certain reference frame being unique for each cave according to its unique space-time location and motion in the universe, then the Standard model would be dependent on the corresponding reference frame it and thus different in general from a “cave” to another.

This is a particular reformulation of the trouble about the consistency or inconsistency of general relativity and quantum mechanics, where the former is postulated to be primary by the concept of the reference frame, and the latter is substituted by the Standard model closely linked to it: can both fundamental physical theories be universal and valid in any point of the universe simultaneously? A difference between the temporal screen including depicted on it and the nontemporal quantum information by itself seems to be unavoidable, and moreover, confirmed experimentally as dark matter and dark energy.

Fortunately, the Standard model contains the “backdoor” of confinement in strong interaction able to reconcile it with general relativity by the mediation of entanglement and (quantum) information in virtue of the symmetries of quantum information and the concept of external (discrete) reference frame, which allows for a single internal (continuous) reference frame to be attached to the Standard model unambiguously.

The status of confinement is “semi-legal” (as that of entanglement or gravitation): it is an additional interaction, for which nobody knows how to be reduced or expressed by the three “legal” ones: electromagnetic, weak and strong (just nobody knows the analogical way for entanglement and gravitation). It is postulated *ad hoc* only to explain why free quarks cannot be corroborated experimentally though they are almost the half (eight from seventeen) from the fundamental particles of the Standard model.

At that, an extremely extraordinary and counterintuitive property is granted distinguishing it radically from all three legal natural “forces”: it increases as the distance between the interacting entities, increases even exponentially-like so that tends to infinity at boundaries corresponding to the space-time magnitude e.g. of a nucleon therefore preventing quarks with whatever energy to leave it, to “emancipate as free”. The same extravagant relation to the environment does not allow for any experimental confirmations being situated definitively out of the size within which confinement acts.

Furthermore, the fundamental idea of local and global space (underlying the Standard model) is able to interpret confinement by its strange dependence on distance if the idempotent dual Hilbert spaces are identified as the pair of local and global ones therefore explaining the reciprocal relation to distance by the isometry of the dual *complex* Hilbert spaces. Thus, what happens at the infinitesimal boundary of confinement is the quantum leap between the two dual complex Hilbert spaces (in the “classical” quantum mechanics), but it is a certain state of entanglement in quantum information. Following the same conception suggested by the Standard model, one may develop a mathematical (consequently quantitative) formalism describing confinement in terms of entanglement, and adding the identification of entanglement and gravitation (as this paper advocates): that formalism would be to be interpreted as a gravitational “theory” (rather a hypothesis) of confinement.

Confinement can be related only to strong interaction or respectively to the symmetry  $SU(3)$  (and acceleration if the concept of external, quantum reference frame be established). Unlike position and velocity which can be considered as independent variables (as Hamiltonian mechanics does), acceleration (as “two velocities” both sharing the same property of being

*velocities*) does not allow for this<sup>26</sup> (in the framework of a single Hamiltonian mechanics in turn relevant to Lagrangian or Newtonian mechanics). Then, that “inseparability of acceleration” would imply for both velocities and positions or components of them not to be able to be observed free and independent of each other. Therefore, the local and global space of the Standard model would coincide as to electroweak interaction, but would be complementary to each other, as to strong interaction.

Returning to the metaphor by a “field of Plato’s caves” associated with each point of space-time unambiguously, the problem (of what is conserved and thus, is more fundamental as natural laws: general relativity or the Standard model) is resolved as follows. Each space-time “wall” obeys the Standard model independently of being associated with a different reference frame in general because they can differ from each other due to different unobservable states of confinement able to represent (or properly, to hide) any correspondent gravitational field on any “wall” including that caused by dark matter and dark energy, i.e. by entanglement. In other word or speaking loosely, the quantity of confinement can be defined as the difference between the actual reference frame of that cave (to a certain one) and that associable reference frame of the Standard model (to the same certain one). As a result, what would distinguish the physics in any “cave” (i.e. a cognitive niche associated with a space-time point) is only the “degree of impossibility” for free quarks to be observed directly: however, maybe only “differently infinite” and thus unobservable practically.

As a conclusion of this section and speaking rather figuratively, dark matter and dark energy are “necessary to equate” the space-time dependent general relativity to the space-time independent Standard model by means of entanglement equivalent to gravitation and its “projection” on the Standard model as confinement is interpreted.

## VIII CONCLUSIONS:

A preliminary note is to elucidate the “genre” of the paper, which is both philosophical and scientific, properly mathematical and physical. Anyway, it does not belong to philosophy of science as this area is implied at present: rather it can be related to the way as philosophy and science were closely linked to each other, in the dawn of modern science and philosophy, in the 17<sup>th</sup> century: e.g. after Galileo, Descartes, Newton, and Leibniz.

The main reason for that peculiarity of the present paper is due to postulating a fundamentally non-empirical entity (as what the totality is) to justify holism and its duality (or complementarity in terms of quantum mechanics, or the doubleness of local and global space as to the Standard model) to empirical and experimental experience or science. Properly, this is a philosophical and mathematical premise identifying any infinite hierarchy (e.g. that of a well-ordered set such as that of all natural numbers) considered as a complete infinite whole and

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<sup>26</sup> The hypothetical relevant mechanics in terms of Hamiltonian mechanics would need two twin copies of itself so that the one to be related to the pair of position and velocity, and the other, to acceleration properly. In fact the two dual spaces of the complex Hilbert space can be interpreted as related to those two copies of Hamiltonian mechanics therefore allowing for different ways the velocity of the former Hamiltonian mechanics to be linked to the first independent velocity of the latter.

any binary idempotent structure to which Boolean algebra is applicable (such as set theory or propositional logic in turn identifiable to each other in virtue of their isomorphism). Then, all basic properties implied by that postulate of the totality (enumerated in the present paragraph) can be found in the qubit Hilbert space relevant to quantum mechanics and its foundations as well as to Hilbert arithmetic relevant to the foundations of mathematics<sup>27</sup>.

The specific physical innovation of the present paper consists in: (1) the concept of information symmetry as the class of equivalence of all possible states of a bit or qubit identifying them with  $U(1) \otimes SU(2) \otimes SU(3)$  and the conservation of quantum information as well as implying the identification of gravitation and entanglement; (2) the introduction of the generalized, quantum-information “ether” as a relevant medium implying a generalized invariance to a wider class of reference frames including the newly-involved, external (or discrete) reference frames and corresponding to the conservation of quantum information; as well as in the tracking the concept of the ether in a few fundamental physical theories starting from Newtonian mechanics; (3) the concept of external or discrete reference frame allowing for the unity of general relativity and quantum mechanics to be considered in terms of the former as well as a relevant generalization of Einstein’s general principle of relativity in way to be valid to them as well; (4) the interpretation of the Standard model as a privileged discrete reference frame; (5) the interpretation of confinement as allowing for dark matter and dark energy being temporal representations of atemporal physical phenomena to be meant implicitly in the Standard model therefore “adjusting” general relativity and quantum mechanics to consistency by mediation of quantum information.

The specific philosophical innovation consists in the conjecture of nontemporal physical phenomena such as entanglement only mapped on the temporal screen originating from our experience, but being postulated as a necessary condition of physics and experimental science until now.

On the contrary, the very well confirmed existence of dark matter and energy demonstrates that the essentially prevailing physical phenomena do not occur in the course of time. In fact, they are indistinguishable from calculations of the universe interpreted as a quantum computer. Those calculations due to the unity of the foundation of physics and mathematics embodied in the dual qubit Hilbert space and “Hilbert arithmetic” (as the concept is defined and utilized in the paper) serve for the usual and well-known space-time physical phenomena to appear in the temporal screen of experience and experiments.

The bridge, connecting both aspects: scientific and philosophical, is the methodological or metascientific conception of “scientific transcendentalism” postulating the totality to be definable rigorously therefore admitting its refusal in virtue of the experimental rejection of corollaries following from it (e.g. in physics) or its inconsistency to other axioms (e.g. in mathematics).

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<sup>27</sup> The merging of those foundations, namely those of physics, mathematics, and philosophy is discussed in another paper (Penchev 2020 Oct 20) in detail, and furthermore as allowing for the deduction of dark matter and dark energy only from them.

Unlike the usual speculative transcendentalism originating from German classical philosophy, being unfalsifiable implicitly or explicitly and belonging to the treasure of fundamental ideas regardless of the scientific development in a certain epoch, scientific transcendentalism utilizes the same idea in a way to be applicable to cognition in a given historical moment, right ours.

Thus, the “totality” turns out to be linked to experience unambiguously though being by itself inaccessible empirically. Anyway, it can be tested or checked indirectly due to its conclusions. The only difference to many scientific notions also inaccessible directly consists in the suggestion that the immediate empirical or experimental inaccessibility of the totality is fundamental and definitive: in fact, a conjecture which can be rejected in the future.

Physical transcendentalism is used in the paper by the introduction of transcendental time in its framework. So, “transcendental time” is considered as a standard scientific idea able to be either confirmed or rejected, i.e. falsifiable, and generalizing the physical concept of time, first of all, its well-ordering in two ways: (1) as the class of well-orderings isometric (eventually, anti-isometric<sup>28</sup>) to each other rather than only sharing the same ordinal number; (2) as the unordered set of the same elements corresponding to the class meant in (1).

Thus, transcendental time can be adopted to be the group<sup>29</sup> counterpart of information symmetries granted to be joined to it in the title of the paper.

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<sup>28</sup> The reversible time of any quantum entity in coherent state as well as the space-like time utilized in special or general relativity can be considered as simultaneously confirming both isometry and anti-isometry of the relevant temporal sequences. The anti-isometry of time running “backwards” is embodied in quantum mechanics as that of the dual Hilbert space, and as the nonconvex pseudo-metric of time in relation to space coordinates in relativity. The simultaneously isometric and anti-isometric experimental considerations of any entity is forbidden postulatively: correspondingly, as the complementarity of experiment or by virtue of the non-exceedable velocity of light in vacuum in a gravitational field. Thus, the concept of transcendental time hints that both prohibitions are the same.

<sup>29</sup> Here, “group” admits it to be discrete not less than continuous or smooth (as a Lie group is).

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