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MAPPING THE UNDERSTANDING COMPLEX IN RUSSELL'S THEORY OF KNOWLEDGE

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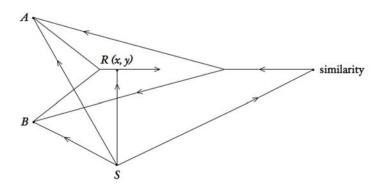
Anyone familiar with Russell's work on the multiple-relation theory of judgment will at some point have puzzled over the map of the five-term understanding complex at the end of Chapter I, Part II of his *Theory of Knowledge* (1913). Russell presents the map with the intention of clarifying what goes on when a subject *S* understands the "proposition" that *A and B are similar*. But the map raises more questions than it answers. In this paper I present and develop some of the central issues that arise from Russell's map, and I offer an interpretation of it that reflects his evolving views in the manuscript. I argue that multiple lines in the map are not meant to represent many relations, but rather one comprehensive multiple relation of understanding. And I argue that such a relation relates in a complex way due to the distinctive nature of its relata.

I. INTRODUCTION

t the end of Chapter I, Part II of his *Theory of Knowledge* manuscript, Bertrand Russell presents us with a map of the five-term understanding complex. The map plays a very important role for him. He says that it will help make clearer what goes on when, as he puts it, a subject understands a proposition, specifically, when S understands that A and B are similar. Although Russell hoped the map would illuminate the results of his previous investigations, it is quite puzzling. Many scholars remain unsure about how

russell: the Journal of Bertrand Russell Studies The Bertrand Russell Research Centre, McMaster U. n.s. 36 (winter 2016–17): 101–27 ISSN 0036–01631; online 1913–8032

it is meant to be interpreted. Here is Russell's map on page 118 of *Theory of Knowledge*:



Map A

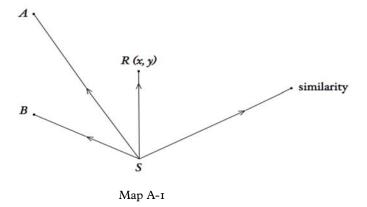
In this paper, I will present and develop some of the central conceptual questions that the map raises, and suggest answers to those questions that are most in keeping with Russell's views in Theory of Knowledge. I begin with a brief sketch of his shifting views on propositions from 1903 to 1913. I then examine two central questions about the sorts of relations which Russell wanted to depict with his map, and I argue in favour of the interpretation that takes all the lines in the map to stand for *one* multiple comprehensive relation of understanding. Next, I address questions concerning the way the lines (or arrows) of the map depict the nature and exact role of the relation of understanding itself—how it is meant to be different from the relation of acquaintance; how it is connected to the logical form; and whether it is meant to incorporate in some way the so-called "position relations" Russell introduced. Finally, I briefly consider the question of whether the map was in fact meant to represent the specific five-term understanding complex, or the *form* of such a complex.

I must warn readers that my concern in this paper has *not* been to advance large historical and interpretative theses about Russell's multiple-relation theory of judgment, its development, and the reasons why he abandoned the theory. Much valuable work has been and continues to be done in this area.¹ My aim has also *not* been to trace the

See, to name just a few: NICHOLAS GRIFFIN, "Russell's Multiple Relation Theory

historical development of Russell's maps of judgment and how such development reflects his changing views. Important and illuminating work has been done in this area too.² My aim in this paper has been modest, but with potentially very fruitful effects for the abovementioned avenues of research. Through close reading and analysis of the relevant passages of Theory of Knowledge, I have tried to highlight the most pressing questions about Russell's map of the understanding complex as well as the tensions in the text which make answering those questions challenging. Thus, even if the reader does not end up agreeing with my interpretation of his map, I hope that it will be clear that a proper understanding of the map is vital to any interpretation of Theory of Knowledge which seeks to explain his most mature version of the multiple-relation theory of judgment.

In Map A, we see four straight lines linking the subject S to terms A, B, similarity, and the general form of dual complexes R(x, y). But this is not all that Russell chooses to represent with Map A, for if this were the case his map would have looked much simpler. It could have looked like this:



of Judgment" (1985); Gregory Landini, "A New Interpretation of Russell's Multiple Relation Theory of Judgment" (1991); PETER HANKS, "How Wittgenstein Defeated Russell's Multiple Relation Theory of Judgment" (2007); ROSALIND CAREY, Russell and Wittgenstein on the Nature of Judgment (2007); CHRISTOPHER PINCOCK, "Russell's Last (and Best) Multiple-Relation Theory of Judgment" (2008); JAMES CONNELLY, "On 'Props', Wittgenstein's June 1913 Letter, and Russell's 'Paralysis'" (2011).

See in particular Carey, "The Development of Russell's Diagrams for Judgment" (2003).

Map A-I represents the multiple relation of understanding relating the subject S to the terms A, B, similarity, and the form R(x, y), but this is all it does. Russell's Map A, on the other hand, has further lines which connect the terms of the multiple relation of understanding between themselves. It is on the status of these further lines that we will focus our attention in the next section. For now, I want to note that both diagrams above—Map A and Map A-I—are, on the face of it, in keeping with the view that Russell is expounding in *Theory of Knowledge*: namely, that when we understand the "proposition" we do not stand in a single dual relation to one entity, but rather in a multiple relation to a number of different entities, some of which may, but need not, constitute an existing complex.

This view departs significantly from Russell's views in the ten years prior. In *The Principles of Mathematics* (1903), he held that understanding is a dyadic relation between a subject and a single entity he called a "proposition". He thought of propositions as non-linguistic complexes composed of distinct sorts of entities ("terms")—concrete particulars such as Socrates ("things"), universals ("concepts"), and denoting concepts (also "denoting complexes"). Thus, for example, the proposition The most famous student of Plato is wise consists for Russell of the denoting concept the most famous student of Plato and the universal wisdom. The proposition Alice is wise contains the particular girl Alice and the universal wisdom. But what does the truth or falsehood of such a proposition consist in? The proposition Alice is wise exists whether or not it is true and there is no other complex Alice being wise that is the truth-maker. There is, therefore, really nothing informative that can be said about the truth of the proposition. In Russell's view, "truth" and "falsehood" are unanalyzable. Even more troublingly, false propositions exist. Let's say that I falsely judge that Alice is not wise. The false proposition Alice is not wise is in the world along with the true proposition Alice is wise. According to the 1903 view, the world is a very strange place indeed.

Circa 1904, during the time that Russell published the three instalments of "Meinong's Theory of Complexes and Assumptions", he seemed to be sympathetic to the Meinongian view that to judge a false proposition is simply to judge something that has being but does not

³ I use quotation marks here because, as we will see shortly, Russell of 1913 did not believe in the existence of propositions.

exist.⁴ After "On Denoting" (1905), however, his views with regard to propositions begin to shift. Definite descriptions are no longer regarded as indicating denoting concepts occurring in propositions. Russell eliminates ordinary proper names such as "Apollo", and both ordinary proper names and definite descriptions are given contextual definitions in the contexts of the statements in which they occur. The existence of such names no longer compels him to admit the Homeric gods into ontology.

Though Russell retained his ontology of propositions (with "truth" and "falsehood" as primitive), by 1906 he began to wonder whether contextual definitions of propositions might avoid having to accept them as *sui generis* entities, thus avoiding the problems of the 1903 view. Propositions would thus be treated within the context of statements of judgment—and thus when a subject S judges that Alice is wise, S is no longer to be regarded as standing in a judging relation to a proposition. Instead, S stands in a multiple relation to the entities which, if S judges truly, would compose a fact (complex) of *Alice being wise*.

By 1910, in "On the Nature of Truth and Falsehood", we find that Russell fully abandons his former propositions and endorses a multiple-relation theory of judgment. He states:

Every judgment is a relation of a mind to several objects, one of which is a relation; the judgment is *true* when the relation which is one of the objects relates the other objects, otherwise it is false.

(Philosophical Essays, p. 181/156; Papers 6: 122)

At this point, as well as in his subsequent work *The Problems of Philosophy* (1912), Russell held that judgment is not a dyadic relation

- ⁴ See "Meinong's Theory of Complexes and Assumptions I" (1904), p. 219, where Russell suggests that false propositions must have some kind of "extra-mental subsistence"; and "Meinong's Theory of Complexes and Assumptions III" (1904), pp. 519–24, for a more detailed discussion of this view by Russell (*Papers* 4: 445 and 469–74, respectively).
- ⁵ Russell's paper "On The Nature of Truth" (1906) is a valuable source for those who wish to trace the emergence of Russell's multiple-relation theory of judgment. Therein, in sec. III on pp. 44–9 (*Papers* 5: 450–4), he outlines two theories—one that admits of "objective falsehoods" and one that does not; it is the latter that will become his multiple-relation theory. He excluded the section in reprinting the paper as "The Monistic Theory of Truth" in *Philosophical Essays* (1910, 1966).

 $\mathcal{J}(S, p)$ but a multiple relation $\mathcal{J}(S, a, R, b)$. That is, when S judges that a stands in a relation R to b, the subject S stands in a multiple relation of judging to a and R and b. Accordingly, S judges truly if there is a corresponding complex aRb, and S judges falsely if there is no such corresponding complex.

Finally, in his 1913 manuscript *Theory of Knowledge*, Russell added the logical form R(x, y) of the would-be dual complex as one of the terms of the judging or understanding relation. Thus, instead of $\mathcal{J}(S, a, R, b)$, he has $\mathcal{J}(S, a, R, b, R(x, y))$. The same holds of the understanding relation: he has $U(S, a, R, b, \gamma)$, where γ is the form of a dual complex. It is this version of the view that finds its representation in Russell's Map A, on page 118 in *Theory of Knowledge*, to which we now turn.

2. TWO CENTRAL QUESTIONS ABOUT MAP A

The two main questions that will be examined here are: (1) in his representation of the understanding complex, why did Russell choose to draw Map A, rather than Map A-1? And more specifically, (2) in Map A, what is supposed to be the role of the additional lines that connect *similarity* to A and B, and that connect the form R(x, y) to A, B, and *similarity*? The hope is that through a close examination of these two questions, we will gain insight into how Russell is thinking about the multiple-relation theory of judgment in *Theory of Knowledge*.

(1) Why did Russell choose to draw Map A rather than Map A-1? The following passage can give us a clue as to his motivation:

In order to understand "A and B are similar", we must be acquainted with A and B and similarity, and with the general form of symmetrical dual complexes. [...] But these separate acquaintances, even if they all coexist in one momentary experience, do not constitute understanding of the one proposition "A and B are similar", which obviously brings the three constituents and the form into relation with each other, so that all become parts of one complex. It is this comprehensive relation which is the essential thing about the understanding of a proposition. Our problem is, therefore, to discover the nature of this comprehensive relation. (TK, p. 112; italics mine)

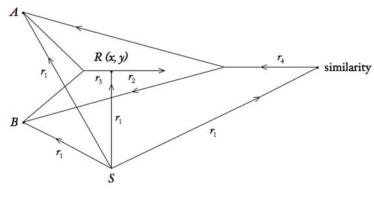
Thus, what seems to motivate Russell's choice of Map A is his desire to represent the relation of understanding as a "comprehensive relation", a relation very different from a number of separate but simultaneous acquaintance relations. Map A-I could be seen as representing either four separate acquaintance relations going from S to the terms A, B, similarity, and R(x, y), or as one multiple acquaintancetype relation going from S to the four terms. In the above passage, Russell explicitly rules out the identification of the understanding relation with four separate acquaintance relations, but he also rules out the identification of the understanding relation with one multiple acquaintance-type relation. When he says that the understanding relation "brings the three constituents and the form into relation with each other", it is clear that the understanding relation is more than an acquaintance relation between a subject and objects of acquaintance, and that it is the nature of this "more" that Russell tries to represent with his Map A. We will be looking more closely at the difference between acquaintance and understanding in Russell in section 3, but for our purposes here, it is sufficient to point out that Russell did not think of the understanding relation as a special type of acquaintance relation. Let's now consider the second question.

(2) In Russell's Map A, what is the role of additional lines that connect similarity to A and B, and that connect the form R(x, y) to A, B, and similarity? Answering this question is made difficult by his own remarks. Right after the drawing of Map A on page 118, Russell proceeds to explain it as follows:

In this figure, one relation goes from S to the four objects; one relation goes from R(x, y) to similarity, and another to A and B, while one relation goes from similarity to A and B. (TK, p. 118; italics mine)

Let's refer to each of the so-called "relations" that Russell lists as follows: r_1 for the relation that goes from S to A, B, similarity, and R(x, y); r_2 for the relation from R(x, y) to similarity; r_3 for the relation from R(x, y) to A and B; and A for the relation from similarity to A and A. With these drawn in, his Map A looks like this:

Map A-2 represents r₄ (and r₃) as one line that then branches out to A and B. In so doing, I tried to stay faithful to Russell's Map A, on the one hand, and to his description of it on p. 118, on the other. For a discussion of the possibility that these branching lines are intended to represent position relation(s), see §3.3 below.



Map A-2

Now, if we are to take Russell's words in the passage on page 118 at face value, there are *four* relations in an understanding complex. But how can this be when Russell seems clearly committed, in the passage on page 112 quoted above, to there being just *one* multiple comprehensive relation of understanding which acts as the relating relation in the understanding complex? Let us consider the following five ways of resolving the tension between these two passages.

- (i) One might wish to claim that r_2 , r_3 , and r_4 are sui generis relating relations, independent of the relation of understanding. However, if this were the case, then the map would be representing S's understanding that A and B are similar (presumably with r_1) as well as the actual complex A being similar to B (with r_4). But this cannot be the case, since with the multiple-relation theory, Russell is supposed to allow us to judge a "proposition" without committing ourselves to its truth, that is, without committing ourselves to the existence of the
- An anonymous referee has suggested that, despite what Russell says on p. 118, r_2 and r_3 are better understood as one relation rather than two. The reason provided is that in Map A there is no arrow going from R(x, y) to A and B, whereas there is indeed an arrow pointing towards *similarity*. Thus, the suggestion is that it is more natural to view r_2 and r_3 as one relation that brings together the form R(x, y) with A, B, and *similarity*. I take this to be an interesting suggestion worth exploring. However, I must point out that such a reading would clash not just with what Russell writes on p. 118, but also with the diagrams found on the verso of the manuscript of Appendix A.5, in Appendix B.2, *Papers* 7: 200. Both diagrams in Appendix B.2 are taken to be precursors of Map A in TK, and they both contain arrows going from R(x, y) to A and B.

corresponding complex. Russell is very careful throughout Chapter I of Part II of Theory of Knowledge,⁸ as well as in other parts of Theory of Knowledge,⁹ to distinguish the would-be complex A being similar to B from the understanding complex S understands A and B are similar. Since understanding that A is similar to B can occur without A being similar to B, it seems safe to say that r_4 is not meant to represent a relating relation within an existing complex A being similar to B.

Similarly, r_2 and r_3 cannot be genuine relating relations, independent of the relation of understanding, for they would be relating the form R(x, y) to *similarity* and the form R(x, y) to A and B. It is far from clear what sorts of complexes this would generate and to what end. But, crucially, Russell states unambiguously that the logical form is only a constituent of an understanding complex (and not of a similarity complex, were it to exist), ¹⁰ thus ruling out the possibility that r_2 and r_3 are relating relations in non-cognitive complexes.

(ii) Could r_2 , r_3 , and r_4 stand for non-relating relations, independent of the relation of understanding? This too is extremely unlikely, for what would be the purpose of such relations? And how should they be thought of? If they were conceived to be non-relating relations such as similarity within the understanding complex, then they are clearly being misrepresented in the map; that is, they should be represented as terms, with dots and names for relations, rather than lines with arrows. But more to the point, Russell says nothing to suggest that subject's understanding of the "proposition" A and B are similar, generates any terms in addition to A, B, similarity, and the form R(x, y); that is, there is nothing in the manuscript to suggest that further non-relating relations need to be added as terms when S judges that A and B are similar.

Another possibility is that r_2 , r_3 , and r_4 were envisioned as non-relating relations of a different sort than *similarity*, hence the difference in representation. However, this too seems rather unlikely, since

⁸ See in particular pp. 116–17 of *TK*.

⁹ See, for instance, pp. 99 and 149 of TK.

Take, for example, the following passage: "In an actual complex, the general form is not presupposed; but when we are concerned with a proposition which may be false, and where, therefore, the actual complex is not given, we have only, as it were, the 'idea' or 'suggestion' of the terms being united in such a complex; and this, evidently, requires that the general form of the merely supposed complex should be given" (*TK*, p. 116).

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Russell says nothing about r_2 , r_3 , and r_4 's possible special role. Indeed, the fact that Russell does not take the time to separate out and explain the role of what I have here referred to as "relations r_2 , r_3 , and r_4 " indicates that he did not think that he was introducing any new sort of entity with a special new role at this point in the manuscript. It is thus much more likely that the map is merely illustrating the results of his previous discussion, rather than introducing new concepts.

(iii) If indeed r_2 , r_3 and r_4 are neither *sui generis* relating relations independent of the relation of understanding, nor non-relating relations independent of the relation of understanding, then what else can they be? One possibility is that r_2 , r_3 and r_4 are not relations at all and that Russell simply misspoke when he referred to them as such. According to this interpretation, of the four "relations" r_1 , r_2 , r_3 , and r_4 the only genuine relation is r_1 , and it is by virtue of this multiple relation r_1 holding between S and the four terms that S is able to understand the different roles that the other four terms play in the complex and their relationships to one another.

In fact, immediately preceding the map, Russell elaborates on the different ways in which the terms enter the understanding complex. He writes:

It is obvious, in the first place, that S is related to the four other terms in a way different from that in which any of the four other terms are related to each other. (It is to be observed that we can derive from our five-term complex a complex having any smaller number of terms by replacing any one or more of the terms by "something". If S is replaced by "something", the resulting complex is of a different form from that which results from replacing any other term by "something". This explains what is meant by saying that S enters in a different way from the other constituents.) It is obvious, in the second place, that R(x, y) enters in a different way from the other three objects, and that "similarity" has a different relation to R(x, y) from that which A and B have, while A and B have the same relation to R(x, y). Also, because we are dealing with a proposition asserting a symmetrical relation between A and B, A and B have each the same relation to "similarity", whereas, if we had been dealing with an asymmetrical relation, they would have had different relations to it. (*TK*, p. 117)

Thus one could make the case that Russell, in trying to come up with a visual representation of the way in which different terms occur in the understanding complex, helped himself to further lines and arrows and corresponding talk of "relations" (r_2 , r_3 , and r_4) in an attempt to capture those different roles. But all he really means here when he mentions "relations" is simply different relationships and ways of occurring. Thus with r_2 and r_3 Russell wishes to show that the form R(x, y) has a peculiar role: through S's understanding of the form, S understands that A, B, and similarity would constitute a dual complex of the type "something stands in some relation to something"; similarity enters in such a way that when S is related to it through understanding, S understands that it is a dyadic symmetrical relation, and so on.

The benefit of this interpretation of Map A is that it does not strongly conflict with Russell's main commitments in Chapter 1 of Part II of *Theory of Knowledge* or with that manuscript more broadly. However, it certainly leaves unanswered the following questions: why did he choose to represent r_2 , r_3 , and r_4 in the same way as he did r_1 , instead of, say, representing them with dotted lines (thus indicating their different ontological status)? And more to the point, why did Russell refer to them repeatedly as "relations" if he did not think of them as such? The next two interpretations weigh in on these questions.

(iv) In (iii) above, it was assumed that Russell misspoke when on page 118 of the manuscript, after presenting Map A, he referred to r_2 , r_3 , and r_4 as separate relations. But what if we interpret Russell as deliberately choosing to refer to a relation in each case, but as speaking somewhat loosely in referring to these as separate relations? On the latter interpretation, r_1 , r_2 , r_3 , and r_4 are all "parts" of one comprehensive cognitive relation of understanding—call it r, and by singling them out Russell was attempting to draw attention to different relationships that are established between the terms when the relation of understanding is exemplified in the complex.

Of course, the talk of "parts" of a relation should not be taken literally here: firstly, it is not clear what (if any) metaphysical sense can be made of a "part of a relation"; and secondly, Russell does not say anything that can be construed as a commitment to relations having parts. But there is certainly nothing incoherent about him drawing attention to parts of an understanding *complex* by appealing to different ways in which terms are related by the understanding relation.

To say that r_1 , r_2 , r_3 , and r_4 are all actually just *one* relation, is in keeping with his talk of a comprehensive relation of understanding in

the passage on page II2 of *Theory of Knowledge* quoted above. This reading has further textual support on page II4 where, in the context of a discussion of a definition of "proposition", he notes that when two different people judge that A and B are similar, the two judgment complexes do not just share the same objects (terms), but they also both "bring the objects into the same relation to each other". This talk of a cognitive relation "bringing together" the terms and putting them in relation "to each other" also appears in *The Problems of Philosophy* (1912). There Russell talks of the relation of *believing* being the sort of relation that "knits together" its terms, rather than standing between the subject and each of the terms individually $(PP_2, pp. 126-7)$.

One might worry that taking all the lines in Map A to represent just one relation of understanding, while in keeping with the passages on pages 112 and 114 in *Theory of Knowledge* and pages 126–7 in the *Prob*lems, leaves unresolved the tension with the passages on pages 117 and 118 where Russell refers to r_1 , r_2 , r_3 , and r_4 as distinct relations. But I wish to suggest that the tension only appears to be significant if we take Russell's remarks on pages 117–18 literally. If, however, we take him to be speaking loosely of different relations, the tension dissipates. And there is a lot to be said in favour of this "loose speak" interpretation. Firstly, as the discussion in (i) and (ii) above has shown, interpreting r_2 , r_3 , and r_4 as sui generis relations distinct from the relation of understanding plainly conflicts with much of what he argues for in Theory of Knowledge. Secondly, in the context of discussing a diagram, it would have been easy for him to slip from talking about different segments of a line to talking about different relations, especially when the line is in fact meant to represent a single relation and when Russell is trying to have the reader focus on different parts of an understanding complex brought about by the relation. And thirdly, as we will see in (v) below, there is hardly any support for an interpretation according to which r_1 , r_2 , r_3 , and r_4 are all distinct cognitive relations of understanding.

(v) Finally, we come to consider the possibility that r_1 , r_2 , r_3 , and r_4 are all distinct relations of understanding. According to this interpretation, we should take Russell's reference to distinct relations on pages 117–18 of Theory of Knowledge literally; but, contra (i) and (ii), r_2 , r_3 , and r_4 do not have an importantly different status from r_1 —they are all cognitive relations, i.e., they are all relations of understanding.

This reading does not sit well with the passages referred to above

that mention the understanding relation as *one* multiple comprehensive relation rather than many. But perhaps this could be gotten around by saying that r_1 is indeed the one multiple relation of understanding that relates S to the four subjects; whereas r_2 , r_3 , and r_4 are relations that come into being once r_1 is exemplified. That is, once S stands in an understanding relation to A, B, *similarity*, and the logical form R(x, y), S understands how *similarity* ought to relate A and B and what form the would-be complex takes.

As tempting as this interpretation might be, it is very difficult to make it work. Firstly, Russell did not want to allow for instantiation-in-stages of the understanding relation. For him, there seems to be no doubt that when subject S understands a proposition, the instantiation of the relation happens all at once. Secondly, for r_2 , r_3 , and r_4 to be cognitive, they cannot exist independently of the subject S; that is, it is not the form R(x, y) that understands how A, B, and similarity are to be put together, and it is not similarity that understands A and B. Thus if relations r_2 , r_3 , and r_4 are to qualify at all as cognitive relations of understanding, they must have a subject as one of their relata. But then r_2 , r_3 , and r_4 would have to be construed to somehow include r_1 , and that is not what passage on page 118 suggests.

It would also not help matters to start reinterpreting Russell's definition of a multiple relation as having to do with *multiple relations*— r_1 , r_2 , r_3 , and r_4 —rather than one relation relating multiple relata. Russell was quite clear on the fact that the "multiplicity" of a relation is tied to its adicity. In Chapter XII of the *Problems*, Russell characterizes multiple relations as relations that hold between more than two terms (i.e. their adicity is greater than two). His examples of such relations are: between ("A is between B and C"), jealousy, and—curiously—wishing to promote marriage ("A wishes B to promote C's marriage with D") (PP_2 , pp. 124–5). Thus, when Russell takes cognitive relations such as judging, believing, and understanding to be multiple relations, all he ever seems to want to say is that they relate more than two terms.

In light of all of the above, if Map A is to stand any chance at illuminating Russell's discussion of the multiple-relation theory of judgment, it would be best interpreted along the lines suggested in (iv) above. This is not to say that such a reading is perfectly clear, but merely that it is the most charitable to Russell and clashes the least with his other philosophical commitments in *Theory of Knowledge*.

3. ON THE NATURE OF THE COMPREHENSIVE RELATION OF UNDERSTANDING

Although the interpretation (iv) above has helped us to reconcile some tensions in Russell's *Theory of Knowledge*, the exact role and nature of the *one* multiple comprehensive relation of understanding in Map A remains far from clear. In this section, I will try to shed some light on this relation by considering the following questions: How is the relation of understanding different from the relation of acquaintance, according to Russell? Does logical form impact the understanding relation, and if so, in what way? And finally, what might be the relationship between the understanding relation and the position relations in Map A?

3.1. The understanding relation and acquaintance

According to Russell, in order to understand the "proposition" A and B are similar, we must be acquainted with A, B, similarity, and the form R(x, y). The requirement that we be acquainted with A, B, and similarity was something that he had already discussed at length in his 1911 "Knowledge by Acquaintance and Knowledge by Description". For instance, in this paper Russell states his principle as follows: "Whenever a relation of supposing or judging occurs, the terms to which the supposing or judging mind is related by the relation of supposing or judging must be terms with which the mind in question is acquainted." He thought the truth of this principle to be evident, for it was obvious to him that "we cannot make a judgment or a supposition without knowing what it is that we are making our judgment or supposition about" (ibid.).

By the time that Russell was engaged in writing the *Theory of Knowledge* manuscript, the main change from "Knowledge by Acquaintance and Knowledge by Description" and the *Problems* had to do with the entities to which the principle was supposed to apply. That is, the form of the would-be complex was added as one of the constituents with which we must be acquainted if we are to understand a

¹¹ Russell states this frequently. On p. 112 of *TK*, he writes: "In order to understand 'A and B are similar', we must be acquainted with A and B and similarity, and with the general form of symmetrical dual complexes." The same thought is repeated on p. 114 of *TK*.

Proceedings of the Aristotelian Society 11 (1910–11): 118; Papers 6: 155.

proposition. I will discuss in more detail below what the addition of the form brings to understanding, but here I'd like to briefly discuss a different issue. When Russell says that "when we understand, those objects with which we must be acquainted when we understand, and those only, are object-constituents [...] of the understanding-complex" (*TK*, p. 117), he does not clearly state whether acquaintance with the constituents is supposed to happen prior to understanding the proposition, at the same time as understanding the proposition, or indeed if the acquaintance relation is in some sense a constituent of the understanding relation. How should we understand him on this issue?

Initially, Russell is very careful to distinguish between acquaintance and understanding. On page 108 of Theory of Knowledge he stresses that the two relations are "very widely different in logical form, and give rise to quite different logical problems". Acquaintance is a dual relation that holds between a subject and an object; understanding, on the other hand, is a multiple relation that holds between a subject and multiple objects. Another crucial difference has to do with truth and falsehood. Acquaintance involves objects that are not true or false, but simply "are" (p. 108), whereas understanding is a propositional cognitive relation concerned with truth and falsehood. From all this, it looks as if Russell is thinking of the understanding relation as importantly distinct from the acquaintance relation. It would appear that regardless of whether or not acquaintance with the constituents of the "proposition" in fact happens prior to or at the same time as understanding, it is the understanding relation and not the acquaintance relation(s) that feature in the understanding complex.

However, what starts to muddle the distinction between the relation of understanding and the relation of acquaintance is Russell's interchangeable use of the two terms in his discussion of logical form. For instance, on page 113 he writes that he wishes to consider the question how we can be sure that *acquaintance* with the "form" is involved in *understanding* a proposition. But then, when on page 116 he comes to consider this question, he talks of *understanding* the form being required for understanding the proposition; in fact, at this point, he even puts the initial question as: "What is the proof that we must *understand* the 'form' before we can *understand* the proposition?" (italics mine).

I suggest, however, not taking these examples as evidence of Russell's general lack of clarity about the distinction between *acquaintance*

and *understanding* relations in the context of understanding a proposition, but rather as his lack of clarity on the exact type of cognitive relation to be invoked when it comes to logical forms. In fact, he even confesses to as much in the chapter on logical data in *Theory of Knowledge*, when he says: "It should be said, to begin with, that 'acquaintance' has, perhaps, a somewhat different meaning, where logical objects are concerned, from that which it has when particulars are concerned" (p. 97). Later, when he returns to this issue within the context of discussion of various examples of understanding, Russell concludes that he does not think that there is in fact any difference between understanding and acquaintance when it comes to the logical form "something has some relation to something" (p. 130).

3.2. The understanding relation and logical form

It is now time to consider the special role that the logical form is supposed to play in our understanding of a "proposition" and how the form might affect the relation of understanding itself. Let's begin by looking at the way that Russell motivates the introduction of the form in *Theory of Knowledge*:

I held formerly that the objects alone sufficed, and that the "sense" of the relation of understanding would put them in the right order; this, however, no longer seems to me to be the case. Suppose we wish to understand "A and B are similar". It is essential that our thought should, as is said, "unite" or "synthesize" the two terms and the relation; but we cannot actually "unite" them, since either A and B are similar, in which case they are already united, or they are dissimilar, in which case no amount of thinking can force them to be united. The process of "uniting" which we can effect in thought is the process of bringing them into relation with the general form of dual complexes. The form being "something and something have a certain relation", our understanding of the proposition might be expressed in the words "something, namely A, and something, namely B, have a certain relation, namely similarity". [...] In an actual complex, the general form is not presupposed; but when we are concerned with a proposition which may be false, and where, therefore, the actual complex is not given, we have only, as it were, the "idea" or "suggestion" of the terms being united in such a complex; and this, evidently, requires that the general form of the merely supposed complex should be given. More simply, in order to understand "A and B are similar", we must know what is supposed to be done with A and B and similarity, i.e. what it is for two terms to have a relation; that is, we must understand the form of the complex which must exist if the proposition is true.

(*TK*, p. 116; italics mine)

As Russell points out in this passage, he no longer holds that the relation of understanding has "direction" or "sense". While in the Problems he thought of a relation of understanding "arranging" the subject and objects "in a certain order" (PP2, p. 127), the relation of understanding does no such thing in the Theory of Knowledge. In Chapter VII, Part I Russell spends time trying to convince the reader that the "from-and-to" character that we tend to attribute to relations such as "before" and "after" is merely apparent. Relations are not like goods trucks, with a hook in front and an eye behind. Such metaphors, Russell warns, are "positively misleading" since "before" and "after" only differ linguistically; thus, he concludes, "whatever a relation is, it must be symmetrical with respect to its two ends" (TK, p. 86). Applying this to the relation of understanding, we see how Russell thought that it too can have no essential "sense" or "from and to" character as part of its nature, nothing that would ontologically discriminate between what sorts of entities can take up the "hook" slot and what sorts of entities can take up the "eye" slot.

What, then, is meant to replace the "sense" of the relation of understanding? In the above passage, Russell suggests that in the case of symmetrical dual complexes such as "A and B are similar", it is the form "something and something have a certain relation" that is crucial to our understanding of "A and B are similar". By understanding or being acquainted with the general form of dual complexes, we understand "what it is for two terms to have a relation" (TK, p. 116).

But how exactly is the form meant to perform this role and what is the nature of such an entity? With respect to the nature of logical form, Russell is certain that it must be "something exceedingly simple" (*TK*, p. 114), i.e., it must not contain any constituents (*ibid*.). Forms for him are what different groups of complexes have in common—the form of all subject-predicate complexes is "something has some predicate", the form of all dual complexes is "something has some relation to something", etc. But despite these linguistic descriptions of different forms, we shouldn't think of them as linguistic entities or indeed as entities that *have* structure. The form, for Russell, "*is* a structure" (*TK*, p. 114), and if we are struggling to grasp what exactly he might

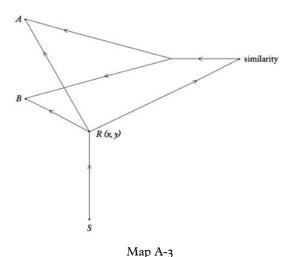
mean by this, it is probably because, as he puts it, "the language is not well adapted for speaking of such objects" (*ibid*).

In what way is logical form supposed to aid our understanding of propositions and how does it replace the "sense" previously attributed to the relation of understanding? Russell does not give us much guidance on this, and where one would expect an explanation of some kind, we are merely presented with the map. Let's take another look at Map A, on page 102 above. I have already argued in favour of the interpretation of the map according to which all of the lines represented in it stand for only one relating relation—the multiple comprehensive relation of understanding. However, the multiplicity of lines and arrows in Map A suggests that the relation of understanding relates its constituents in a complex way. This way of relating does not appear to be for Russell a product of the nature of the relation of understanding itself; rather, the relation of understanding seems to be affected (or *informed*, so to speak), by the nature of the terms being related, and in particular by the logical form. Thus we can interpret Map A as attempting to represent that: (1) standing in an understanding relation to the form R(x, y) helps S understand that the would-be complex is dual; (2) standing in an understanding relation to A and B and the form helps S understand that A and B are two terms that in a would-be complex are related by a dual relation; and (3) standing in an understanding relation to similarity and the form helps S understand that similarity is the dual symmetrical relating relation in a would-be complex.

This interpretation makes repeated reference to the form and its centrality in the understanding of the "proposition". With this, I do not wish to indicate, however, that the logical form R(x, y) is supposed by itself to structure A, B, and *similarity* in the understanding complex. I find little evidence for such an interpretation of the role of the form in Russell. Although the logical form is a structure for him and is found to be common to many complexes, there is no reference to the form as somehow "structuring" the other constituents of the cognitive complex.

It also seems incorrect to interpret the role of the logical form R(x, y) as a template through which the understanding relation "reaches out" and unifies the constituents of the would-be complex. First, this interpretation does not fare well with the map itself, which, although it *does* present the form R(x, y) as central, it certainly does *not* present

it in the forefront of the complex with radiating further lines. This is what I have in mind:



Secondly, this sort of interpretation would imply that understanding the form would need to happen first, i.e. before the understanding relation reached the other constituents, and there is no indication of such sequencing in Russell. And thirdly, he is clear on the fact that each of the constituents occurs differently in the complex, thus indicating that it cannot be the logical form all by itself that somehow determines the structure of the complex.

Therefore, I suggest, each of the constituents, together with the form and the understanding relation that relates them all, contributes to the particular structure of the understanding complex. This reading aligns nicely with the passage on page 117 of Theory of Knowledge, quoted in section 2 above, where Russell, immediately before presenting us with Map A, points out different ways in which each of the five constituents of the complex—S, A, B, similarity, and the form R(x,y)—occur in it. With these different "ways of occurring" he wants to show how each of the constituents contributes in its own unique way to the resulting structure of the understanding complex.

Additional support for this reading can be found in the following passage from Chapter III, Part II of Theory of Knowledge where Russell is reflecting on what matters the most in how we characterize and classify mental facts:

In the classification of mental facts, various different considerations may be brought to bear. There is, first, the logical form of the mental fact concerned—whether it is a dual or treble or quadruple ... relation. Then there is the logical character of the objects concerned. Then, when both the form of the mental fact and the logical character of the objects are given, there is the actual relating relation which is a constituent of the fact. In considering these grounds of classification, one would naturally have expected that the form of the mental fact would be the source of the most fundamental divisions. In obedience to this supposition, we began with dual relations, acquaintance, attention, etc.; and when we came to propositional thought, it seemed at first as if the change was due to the fact that here the cognitive relations concerned were multiple. This whole point of view, however, is erroneous. The classification of mental facts by the logical character of the objects involved turns out to be far more important than their classification by their own logical form.

(TK, p. 131; italics mine)

In this passage, Russell admits that initially (perhaps in the early stages of writing *Theory of Knowledge*), he thought that the distinguishing feature of mental facts was conferred by the adicity of the cognitive relation concerned. Acquaintance was supposed to be a dual relation, while understanding was supposed to be a multiple relation; hence, he thought, it is the *adicity* of the relation that determines the nature of the cognitive fact. However, perhaps during the very writing of Chapter I of Part II (i.e., the chapter which concludes with Map A), there was a shift on this issue in Russell; he begins to realize that the real impact on the nature of the cognitive fact is conferred by the "logical character" of *all* of the objects involved. I have argued here that it is this interplay of the understanding relation with the entities—each of which has a very different logical character—that gives rise to the understanding complex, and that it is the complex logical structure of such a complex that Russell is at pains to represent with his Map A.

3.3. The understanding relation and position relations

For the sake of simplicity, for most of Chapter I, Part II of *Theory of Knowledge*, Russell chooses to discuss the understanding of a "proposition" which involves a symmetrical relation of *similarity*. What makes this sort of case easier to discuss is the fact that no different complexes result from interchanging A and B in the complex A being *similar to B*; in Russell's words, such a complex is "non-permutative".

This, however, is not the case where asymmetrical relations such as precedes are involved: A precedes B is indeed quite a different complex from B precedes A. Complexes of this sort are called "permutative".

In Chapter v, Part II of Theory of Knowledge Russell attempts to solve the problem of permutation, i.e., the problem of describing unambiguously the complex corresponding to the belief or judgment which contains asymmetrical relations like precedes. He tackles this problem within the framework of the correspondence theory of truth and argues that a belief or judgment of the form $\mathcal{J}(S, F, R, x_1, x_2)$ is true when there is a corresponding complex consisting of the objects of that belief or judgment, and otherwise false. It is clear how this becomes problematic when the relation R in our example is asymmetrical—in such a case, two different complexes result from the interchange of terms. As Russell sees it, the problem then becomes: "when several complexes can be formed of the same constituents, to find associated complexes unambiguously determined by their constituents" (TK, p. 145).

His solution to this problem relies on: (1) the introduction of position relations that hold between the terms and the complex; and (2) providing the description of the so-called "associated complexes" which are non-permutative and hence, according to Russell, unambiguously determined by their constituents.

Position relations, for Russell, hold between a constituent of the complex x_1 and the complex y, where y refers to a complex which has as constituents x_1 , x_2 , and R. Thus, the first associated complex with γ is x_1 standing in the position relation C_1 to γ (i.e., $x_1C_1\gamma$) while the second associated complex is x_2 standing in a different position relation C_2 to γ (i.e., $x_2C_2\gamma$). Russell takes it that the associated complexes are free from the problem of permutation, since the logical character of the constituents prevents the possibility that, say, γ could stand in a position relation $C_{\rm I}$ to $x_{\rm I}$. He can now provide an unambiguous description of a complex that involves an asymmetrical relation by providing descriptions of the associated complexes. In our example, the judgment $\mathcal{J}(S, F, R, x_1, x_2)$ comes out true if there is a complex γ such that $x_1C_1\gamma$ and $x_2C_2\gamma$.

It is beyond the scope of this paper to engage with the problem of permutation and Russell's solution to it in any more detail. I mention it here only because Russell seems committed to having position relations in both permutative and non-permutative complexes, and Map

A was meant to represent a dual non-permutative complex. Thus, it is reasonable to ask: were position relations meant to be represented by Russell in Map A? And if so, in what way?

Take again the example of the judgment $\mathcal{J}(S, F, R, x_1, x_2)$. If R is asymmetrical, there are two distinct position relations— C_1 and C_2 —holding between terms x_1 and x_2 and the corresponding complex; but in the case of R being symmetrical, there is only one such relation C that holds between each of the terms x_1 and x_2 and the corresponding complex. Russell is explicit about this when he writes: "If R is symmetrical with respect to two constituents, two of the C's will be identical. If R is heterogeneous with respect to two constituents, two of the C's will be incompatible" (TK, p. 146).

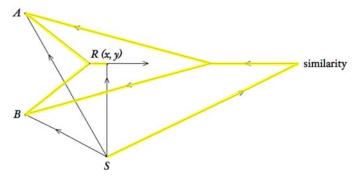
Moreover, it is important to note that there seems to be nothing more to Russell's notion of "position" than "standing in a position relation to a complex", i.e. there is no indication of positions being thought of by him as analogous to empty slots or blank spaces in a complex (or on sides of relations). Thus, the image of something like "__ is similar to __" is far from what he seems to have in mind when it comes to the notion of "position". With this in mind, consider the following passage:

A complex may be called "symmetrical" with respect to two of its constituents if they occupy the *same position* in the complex. Thus in "A and B are similar", A and B occupy the *same position*. A complex is "unsymmetrical" with respect to two of its constituents if the two occupy different positions in the complex.

(TK, pp. 122–3; italics mine)

If we thought of "positions" in analogy with "slots" in a complex, then this passage would commit Russell to holding that there is only *one slot* in the complex of the form *A being similar to B* and that there are two entities somehow sharing (or crowding) that "slot". But if we dispense with such metaphors and take him at his word when he defines positions in terms of position *relations*, then what we have is simply two entities standing in the same relation to the complex they help constitute.

From these considerations about position relations, it follows that it is possible to read into Russell's Map A, two identical position relations in which A and B stand to the complex. Consider it again, but now with the following highlighted segments:



Map A-4

It is possible that the branching line connecting *similarity* to A and B is meant to represent S's understanding that A and B are related by an identical position relation to the would-be complex and that such a relation is determined in the would-be complex by the symmetrical relating relation of *similarity*. The very same thing is represented also by the branching line connecting A and B to the form R(x, y).

If this is correct, then Russell might have been trying to represent that understanding of *similarity* gives the subject two things rather than one: an understanding that the relation of similarity is a dual relation as well as the understanding that it is symmetrical. For it is due to the fact that the relating relation in the would-be complex is symmetrical that the position relations involved in such a complex are identical. Russell even says as much when immediately preceding the presentation of Map A, he writes: "because we are dealing with a proposition asserting a symmetrical relation between A and B, A and B have each the same relation to "similarity", whereas, if we had been dealing with an asymmetrical relation, they would have had different relations to it" (TK, p. 117; italics mine).

This is all, of course, rather speculative; instead of endorsing the interpretation that sees Russell's Map A as representing, amongst other things, position relations, I merely wanted to draw attention to the textual support in favour of such an interpretation. Though this reading draws nicely on some parts of Russell's discussion of position relations, it also raises a number of questions. For example: if position relations are indeed to play a part in Map A, why didn't Russell argue

that acquaintance with position relations is necessary for understanding the "proposition" "A and B are similar"? And why didn't a position relation make it into Map A as a separate term, rather than as an unmarked segment of a line? Furthermore, if position relations are meant to hold between terms and the would-be complex, why would their representation in Map A (as well as the quotation above that precedes it) imply that they relate the terms to the relation of similarity?

It is not clear to me what the right answer to these questions might be, or even that there is one. As we know, there are many unfinished interesting threads in Russell's manuscript, and this might be one of them.

4. WHAT IS THE MAP A MAP "OF"?

This paper has thus far assumed that with Map A Russell was trying to represent a particular complex—the five-term understanding complex "S understands that A and B are similar". But could it be that Map A was actually meant to represent the *form* of the five-term understanding complex rather than the complex itself? There is certainly some textual support for this reading.

First, Russell presents the map at the end of the section in which he is addressing the question: "What is the *logical structure* of the fact which consists in a given subject understanding a given proposition?" (*TK*, p. 113; italics mine). As we have seen in §3.2, Russell referred to the logical form of the dual complex as *being a structure*; thus, it would not be a jump to read the question about the *logical structure* of the understanding complex as a question about its *logical form*.

Secondly, Russell explicitly mentions his concern with the logical form of the understanding complex on the page that leads up to Map A. He writes:

Thus a first symbol for the complex will be $U\{S, A, B, \text{ similarity}, R(x, y)\}$.

This symbol, however, by no means exhausts *the analysis of the form* of the understanding complex. There are many kinds of five-term complexes, and we have to decide what the kind is.

(TK, p. 117; italics mine)

This would suggest that what Russell takes himself to be engaging in is the analysis of the form of the understanding complex. Why is it that the symbol U(S, A, B, similarity, R(x, y)) is inadequate? What does it leave out? Clearly, it does give us the adicity of the understanding relation—i.e., it shows us that the complex in question is a five-term one, the same way that R(x, y) shows us that the would-be complex is dual. What more does Russell want the form to do?

At this point of the text, Russell starts to focus on the constituents of the complex and the peculiar way in which each of them enters into the complex. To highlight different ways in which the subject S enters into the complex, Russell suggests replacing it with "something". He writes: "If S is replaced by 'something' the resulting complex is of a different form from that which results from replacing any other term by "something". This explains what is meant by saying that S enters in a different way from the other constituents" (TK, p. 117). Similarly, Russell notes, R(x, y) enters in a different way from the other constituents, and the same is the case with A, B, and *similarity*.

These comments can be seen as revealing the tension in Russell's views on the form of understanding complex. On the one hand, thinking of logical form as an entity that solely gives us the adicity of the complex seems insufficient to him. But on the other hand, building into the logical form of the complex the particular way in which each of the constituents contributes to the understanding complex, seems difficult (or even impossible) to do. Could it be that Russell's Map A was his experimental attempt at capturing this more encompassing notion of the form? Was he in the process of realizing that merely giving the adicity of the understanding complex, by, say, replacing the terms at the nodes of the map with "something" would not capture the true logical structure of the complex? And if so, should his holding onto the specific terms of the understanding relation be seen as his realization of this fact and as an illustration of the particular contribution that each of the terms brings to the form of the complex?

Whatever the answer to this question may be, it is clear that a certain shift in his thinking does occur in these pages of Theory of Knowledge. For by Chapter III of Part II Russell has decided that the classification of cognitive facts "by the logical character of the objects involved" is "far more important than their classification by their own logical form" (TK, p. 131). Thus, even if he had contemplated the idea

of somehow building more than adicity into the logical form of the complex, by this point he seems to have abandoned it.

5. CONCLUSION

The purpose of this paper has been to draw the reader's attention once more to the map that plays a prominent role in Russell's presentation of the multiple-relation theory of judgment in Theory of Knowledge. I have tried to bring out different tensions in the interpretation of the map, and I have advanced an interpretation that sees Map A as representing a comprehensive multiple understanding relation relating the subject S to A, B, similarity, and the form R(x, y). I have argued that such a multiple comprehensive relation relates its terms in a complex way, and that this cannot be due to the nature of the relation of understanding itself. Rather, the relation of understanding's complex way of relating is affected by the nature of the terms being related. Thus, Map A represents that: (1) standing in an understanding relation to the form R(x, y) helps S understand that the would-be complex is dual; (2) standing in an understanding relation to A and B and the form helps S understand that A and B are two terms that in a wouldbe complex are related by a dual relation; and (3) standing in an understanding relation to similarity and the form helps S understand that similarity is the dual symmetrical relating relation in a would-be complex. I have further pointed out that there is textual evidence to support the thesis that Map A also represents the identical position relations that relate terms A and B to similarity, though such a reading would raise other questions for Russell. And finally, I have wondered whether the map of the specific five-term understanding complex is supposed to be a representation of the *form* of such a complex for him. With the above discussion, I hope to have shown that Russell's map deserves at least another close look and that it might hold the key to a better understanding of his multiple-relation theory of judgment. ¹³

¹³ I am most grateful to my colleague Gregory Landini for our lively discussions of the map and many passages of *TK*. Without his support and enthusiasm, I might have never thought that this topic was worth exploring in such detail.

WORKS CITED

- Carey, Rosalind. "The Development of Russell's Diagrams for Judgment". *Russell* 23 (2003): 27–41.
- Russell and Wittgenstein on the Nature of Judgment. New York: Continuum, 2007.
- CONNELLY, JAMES. "On 'Props', Wittgenstein's June 1913 Letter, and Russell's 'Paralysis'". Russell 31 (2011): 141-66.
- GRIFFIN, NICHOLAS. "Russell's Multiple Relation Theory of Judgment". *Philo*sophical Studies 47 (1985): 213–47.
- Hanks, Peter. "How Wittgenstein Defeated Russell's Multiple Relation Theory of Judgment". *Synthese* 154 (2007): 121–46.
- LANDINI, GREGORY. "A New Interpretation of Russell's Multiple Relation Theory of Judgment". *History and Philosophy of Logic* 12 (1991): 37–69.
- PINCOCK, CHRISTOPHER. "Russell's Last (and Best) Multiple-Relation Theory of Judgment". *Mind* 117 (2008): 107–39. RUSSELL, BERTRAND. *PoM*.

- —. "Meinong's Theory of Complexes and Assumptions 1". Mind 13 (1904): 204– 19. Papers 4: 17.
- —. "Meinong's Theory of Complexes and Assumptions III". *Mind* 13 (1904): 509–24. *Papers* 4: 17.
- —. "On the Nature of Truth". *Proceedings* of the Aristotelian Society 7 (1906–07): 28–49. Secs. I–II as "The Monistic Theory of Truth" in *Philosophical Essays* (London: Longmans, Green, 1910); rev. edn., (London: Allen & Unwin, 1966). Secs. I–III in *Papers* 5: **14**.
- —. "On the Nature of Truth and False-hood". In *Philosophical Essays. Papers* 6: 12.
- —. "Knowledge by Acquaintance and Knowledge by Description". Proceedings of the Aristotelian Society 11 (1910–11): 108–28. Papers 6: 15.
- —. *PP*₂.
- —. Theory of Knowledge: the 1913 Manuscript. Papers 7.