A veritist's reply to the Swamping Problem

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Abstract

Veritism says that the fundamental source of epistemic value for a doxastic state is the extent to which it represents the world correctly—that is, its fundamental epistemic value is determined entirely by its truth or falsity. The Swamping Problem says that Veritism is incompatible with two pre-theoretic beliefs about epistemic value (Zagzebski, 2003; Kvanvig, 2003):

(I) a true justified belief is more (epistemically) valuable than a true unjustified belief;

(II) a false justified belief is more (epistemically) valuable than a false unjustified belief.

In this paper, I consider the Swamping Problem from the vantage point of decision theory. I note that the central premise in the argument is what Stefánsson & Bradley (2015) call Chance Neutrality in Richard Jeffrey's decision-theoretic framework. And I describe their argument that it should be rejected. Using this insight, I respond to the Swamping Problem on behalf of the veritist.

Veritism, as I will use the term, is a thesis about the purely epistemic value of doxastic states, such as beliefs and credences. In this paper, I will focus only on beliefs, but much of what I say will apply also to credences. Veritism says that the fundamental source of epistemic value for a doxastic state is the extent to which it represents the world correctly—that is, its fundamental epistemic value is determined entirely by its truth or falsity. According to Veritism, any further source of value for a belief is derivative, not fundamental—that is, it is valuable because of the extent to which it promotes or otherwise serves the fundamental source. The Swamping Problem is an objection to Veritism. It says that Veritism is incompatible with two pre-theoretic beliefs about epistemic value (Zagzebski, 2003; Kvanvig, 2003):

(I) a true justified belief is more (epistemically) valuable than a true unjustified belief;

(II) a false justified belief is more (epistemically) valuable than a false unjustified belief.

Now, Veritism is compatible with the claim that a *true* belief is more (epistemically) valuable than a *false* belief. Indeed, it entails that claim.

And it is also compatible with the claim that a *justified* belief is more valuable than a *unjustified* belief. After all, if we give an account of justification on which a justified belief is more likely to be true than an unjustified belief, as we will below, it follows that, for a veritist, a justified belief has greater expected epistemic value than an unjustified belief; and something with more expected value is standardly taken to be more valuable than something with less. For instance, if I find a lottery ticket on the street that I know comes either from a 10-ticket lottery or from a 100-ticket lottery, both of which pay out the same amount to the holder of the winning ticket, then the situation in which the ticket is from the smaller lottery—and thus boasts a 10% chance of winning—is more valuable than the situation in which it is from the 100-ticket lottery—and thus boast only a 1% chance of winning—since the former has higher expected monetary value than the latter, and we might suppose that monetary value is the sole fundamental source of value for a lottery ticket.

However, according to the Swamping Problem, Veritism is not compatible with the view that a *true justified* belief is strictly more valuable than a *true unjustified* belief, and it is not compatible with the view that a *false justified* belief is strictly more valuable than a *false unjustified* belief. The argument runs as follows: Suppose I possess a particular item—it might be a lottery ticket; it might be a belief. I take myself to have identified the fundamental source of value for this sort of item; that is, I have identified the variable whose value, once fixed, fixes the fundamental value of the item—if it's a lottery ticket, the fundamental source of its value might be its monetary value; if it's a belief, it might be its truth value, as the veritist contests. Now suppose we are comparing such an item in two different hypothetical situations. In both situations, let's say, the fundamental value is the same—that is, the variable that fixes it takes the same value in both. But the item differs in some other features between these two situations. Then, according to the Swamping Problem, the item cannot be more valuable in one situation than in the other—they must be equally valuable in both. If not, then there must be some other fundamental source of value that accounts for the difference in value—one of the features in which they differ. Their similarity in the feature that determines fundamental value must 'swamp' their difference in any other feature. For instance, suppose again that I find a lottery ticket on the street that I know comes from a 10-ticket lottery or from a 100-ticket lottery, both of which pay out the same amount to the holder of the winning ticket. Then the situation in which the ticket is *the winning ticket* from the smaller lottery is surely exactly as valuable as the situation in which it is *the winning ticket* from the 100-ticket lottery. Although the former had higher expected monetary value than the latter, the fact that it is the winning ticket in both situations, and thus has the same fundamental value in both situations, must 'swamp' any facts about the chance that it would be the winning ticket.

To respond to the Swamping Problem, I want to introduce considerations from decision theory. Although the Swamping Problem is based on considerations of value, it is rarely treated in the decision theory framework. As we will see, considerations from that framework will provide a solution.

In what follows, we will work with a particular sort of account of justification, which I will call a *truth-promoting account*. Examples include William Alston's indicator reliabilism (Alston, 1988, 2005) and Alvin Goldman's process reliabilism (Goldman, 1979, 2008). On such accounts, a belief in a proposition is justified iff, conditional on some feature of the belief, the proposition believed is likely to be true—that is, its objective probability conditional on that feature is above some threshold $0 < t < 1$. On Alston's account, the feature is the ground on which the belief is based. So a belief in a proposition is justified iff, conditional on the agent having the ground on which she in fact bases that belief, the proposition believed is likely to be true. On Goldman's account, the feature is the process by which the belief is formed. So a belief in a proposition is justified iff, conditional on the agent forming the belief using the process she does, the proposition believed is likely to be true.

The objective probability to which such accounts allude is a measure of some sort of chance. I won't say more about what sort of chance it is—actual or hypothetical frequentist, perhaps, or something else—since my discussion will not depend on the answer.

Of course, truth-promoting accounts of justification are controversial. So why focus on them? Because, by doing so, we can illustrate one way in which Veritism might co-exist with (I) and (II). Now, for veritists who reject such accounts of justification it might bring little solace to know that what they take to be a false account of justification witnesses the compatibility of Veritism with (I) and (II). But the reasoning we use may well apply also to a number of different accounts. And if it does not apply to your favoured account? Well, in that case, the Swamping Problem may be a mark against your account.

Let's combine truth-promoting accounts of justification with Veritism. Given Veritism, a true belief is more valuable than a false belief because it has greater fundamental epistemic value. And given the conjunction of Veritism with a truth-promoting account of justification, a justified belief is more valuable than an unjustified belief because it has greater objective expected fundamental epistemic value—that is, greater expected fundamental epistemic value when that is calculated relative to the objective probabilities. But surely a true justified belief is not more valuable than a true unjustified belief, and this for the same reason that a winning ticket from the smaller lottery above is not more valuable than a winning ticket from the larger lottery above—after all, although it has greater objective expected value, that fact is surely 'swamped' by the fact that it is the winning ticket in both cases and so its monetary value is fixed.

In the presence of a truth-promoting account of justification, the central assumption of the Swamping Problem is this: once the actual fundamental value of an item is fixed, this swamps any facts about the chance that it would have that value. In the decision-theoretic context, H. Orri Stefánsson and Richard Bradley call this assumption Chance Neutrality (Stefánsson & Bradley, 2015). They state it precisely within the framework of Richard Jeffrey's decision theory (Jeffrey, 1983). In that framework, the relevant features of your attitudes are represented by a desirability function V and a credence function c, both of which are defined on an algebra of propositions $\mathcal{F}$. For a proposition A in $\mathcal{F}$, $V(A)$ measures how strongly you desire A, or how greatly you value it, while $c(A)$ measures how strongly you believe A, or your credence in A. The central principle of the decision theory is this:

Desirability Suppose the propositions $X_1, \ldots, X_n$ form a partition. Then

$$V(X) = \sum_{i=1}^{n} c(X_i|X)V(X \,\&\, X_i)$$

That is, roughly, your value for X is your expectation of its value over some partition of X.

Now, suppose the algebra on which V and c are defined includes some propositions that concern the objective probabilities of other propositions in the algebra. Then we suppose throughout that your credence function c obeys David Lewis' Principal Principle (Lewis, 1980):

Principal Principle Suppose the propositions $X_1, \ldots, X_n$ form a partition. And suppose $0 \leq \alpha_1, \ldots, \alpha_n \leq 1$ and $\sum_{i=1}^{n} \alpha_i = 1$. Then

$$c(X_j | \bigwedge_{i=1}^{n} \text{Objective probability of } X_i \text{ is } \alpha_i) = \alpha_j$$

That is, your credence in X_j, conditional on information that gives the objective probability of X_j and other members of a partition to which X_j belongs, should be equal to the objective probability of X_j.

In this framework, Chance Neutrality can be stated as follows:

Chance Neutrality Suppose $X_1, \ldots, X_n$ form a partition. And suppose $0 \leq \alpha_1, \ldots, \alpha_n \leq 1$ and $\sum_{i=1}^{n} \alpha_i = 1$. Then

$$V(X_j \,\&\, \bigwedge_{i=1}^{n} \text{Objective probability of } X_i \text{ is } \alpha_i) = V(X_j)$$

That is, the actual outcome of the chance process that picks between $X_1, \ldots, X_n$ 'swamps' information about the chance process itself in your evaluation, which is recorded in your value or desirability function V. A simple consequence of this: if $0 \leq \alpha_1, \alpha'_1 \ldots, \alpha_n, \alpha'_n \leq 1$ and $\sum_{i=1}^{n} \alpha_i = 1$ and $\sum_{i=1}^{n} \alpha'_i = 1$, then

$$V(X_j \,\&\, \textstyle\bigwedge_{i=1}^{n} \text{Objective probability of } X_i \text{ is } \alpha_i) =$$

$$V(X_j \text{ \& } \bigwedge_{i=1}^{n} \text{Objective probability of } X_i \text{ is } \alpha_i')$$

That is, X_j coming about as the result of one chance process is exactly as valuable as X_j coming about as the result of some different chance process.

Now consider the particular instance of Chance Neutrality that is cited in the Swamping Problem. Suppose I believe X. I assign greater value to this belief being true and justified than I do to it being true and unjustified; and I assign greater value to it being false and justified than I do to it being false and unjustified. Thus, we have the following restatements of (I) and (II) from above, where we assume that it's given that you believe X.

(I*) $V(X \text{ \& Objective probability of } X \text{ is at most } t) <$

$$V(X \text{ \& Objective probability of } X \text{ is greater than } t)$$

(II*) $V(\neg X \text{ \& Objective probability of } X \text{ is at most } t) <$

$$V(\neg X \text{ \& Objective probability of } X \text{ is greater than } t)$$

In both cases, t is the threshold for justification—if the objective probability of X, conditional on whatever feature of the belief figures in the truth-promoting account of justification, then it is justified; if it does not, it is unjustified. Both (I*) and (II*) violate Chance Neutrality.

Thus, in this framework, the argument of the Swamping Problem is just that the following three claims are inconsistent: (I*), a truth-promoting account of justification, and Chance Neutrality. And similarly with (II*) in place of (I*). Thus, if we are to solve the Swamping Problem and retain a truth-promoting account of justified belief, we must reject Chance Neutrality. To do this, we turn to an argument due to Stefánsson and Bradley (Stefánsson & Bradley, 2015, Section 3). They show that, in the presence of Desirability and the Principal Principle, Chance Neutrality entails a principle called Linearity.[1] Then they claim that Linearity is not a requirement of rationality.

Linearity is the following principle:

[1]We make the following abbreviation: P_X^α is the proposition *The objective probability of X is α*. By Desirability,

$$V(\bigwedge_{i=1}^{n} P_{X_i}^{\alpha_i}) = \sum_{j=1}^{n} c(X_j | \bigwedge_{i=1}^{n} P_{X_i}^{\alpha_i}) V(X_j \text{ \& } \bigwedge_{i=1}^{n} P_{X_i}^{\alpha_i})$$

Linearity

$$V(\bigwedge_{i=1}^{n} \text{Objective probability of } X_i \text{ is } \alpha_i) = \sum_{i=1}^{n} \alpha_i V(X_i)$$

That is, the value of a lottery is the objective expected value of its outcome. Now, as is well known, real agents often violate Linearity (Buchak, 2013). The most obvious cases are ones like these: let K_1 be a situation in which you receive £20 for sure; and let K_2 be a situation in which a fair coin is tossed and you receive £100 if the coin lands heads and nothing if it lands tails. Thus:

	Heads	Tails
K_1	£20	£20
K_2	£100	£0

Many people value K_1 more than K_2. That is, $V(K_1) > V(K_2)$. But K_1 has lower expected monetary value than K_2. In such cases, defenders of Linearity often note that your utility need not be linear in money. That is, the difference between the utility you assign to £100 and the utility you assign to nothing need not be five times the difference between the utility you assign to £20 and the utility you assign to nothing. If, instead, the former is less than twice the latter, Linearity would then lead you to assign higher expected utility to K_1 than to K_2. But such a defence does not seem to cover every case in which we prefer a sure thing to an uncertain gamble. Intuitively, it seems rational even for someone whose utility is linear in money to prefer the sure thing to the gamble on the grounds that she is risk-averse and the gamble risks leaving you empty-handed while the sure thing does not.

What's more, there are cases in which we cannot ensure conformity with Linearity by making the utility function concave in money. The most famous is given by the *Allais pref-*

By the Principal Principle,

$$c(X_j | \bigwedge_{i=1}^{n} P_{X_i}^{\alpha_i}) = \alpha_j$$

By Chance Neutrality,

$$V(X_j \ \& \ \bigwedge_{i=1}^{n} P_{X_i}^{\alpha_i}) = V(X_j)$$

Therefore,

$$V(\bigwedge_{i=1}^{n} P_{X_i}^{\alpha_i}) = \sum_{i=1}^{n} \alpha_i V(X_i)$$

which is just Linearity, as required.

erences (Allais, 1953). Suppose there are 100 tickets numbered 1 to 100. One ticket will be drawn and you will be given a prize depending on which option you have chosen from L_1, $\ldots$, L_4. The table below shows how the four options pay out.

	Tickets 1-89	Tickets 90-99	Ticket 100
L_1	£1,000,000	£1,000,000	£1,000,000
L_2	£1,000,000	£5,000,000	£0
L_3	£0	£1,000,000	£1,000,000
L_4	£0	£5,000,000	£0

Each ticket has an equal chance of winning. Now, it turns out that many people have preferences recorded in the following desirability function V:

$$V(L_1) > V(L_2) \quad \text{and} \quad V(L_3) < V(L_4)$$

That is, they strictly prefer L_1 to L_2 and L_4 to L_3. When there is an option that guarantees them a high payout (£1m), they prefer that over something with 1% chance of nothing (£0) even if it also provides 10% chance of much greater payout (£5m). On the other hand, when there is no guarantee of a high payout, they prefer the chance of the much greater payout (£5m), even if there is also a slightly greater chance of nothing (£0). The problem is that there is no way to assign values to V(£0), V(£1m), and V(£5m) so that V satisfies Linearity and also these inequalities.[2]

[2]Suppose, for a reductio, that there is. By Linearity,

$$\begin{aligned} V(L_1) &= 0.89V(\text{£1m}) + 0.1V(\text{£1m}) + 0.01V(\text{£1m}) \\ V(L_2) &= 0.89V(\text{£1m}) + 0.1V(\text{£5m}) + 0.01V(\text{£0m}) \end{aligned}$$

Then, since $V(L_1) > V(L_2)$, we have:

$$0.1V(\text{£1m}) + 0.01V(\text{£1m}) > 0.1V(\text{£5m}) + 0.01V(\text{£0m})$$

But also by Linearity,

$$\begin{aligned} V(L_3) &= 0.89V(\text{£0m}) + 0.1V(\text{£1m}) + 0.01V(\text{£1m}) \\ V(L_4) &= 0.89V(\text{£0m}) + 0.1V(\text{£5m}) + 0.01V(\text{£0m}) \end{aligned}$$

Then, since $V(L_3) < V(L_4)$, we have:

$$0.1V(\text{£1m}) + 0.01V(\text{£1m}) < 0.1V(\text{£5m}) + 0.01V(\text{£0m})$$

And this gives a contradiction.

Stefánsson and Bradley show that, in the presence of Desirability and the Principal Principle, Chance Neutrality entails Linearity; and they argue that there are rational violations of Linearity (such as the Allais preferences); so they conclude that there are rational violations of Chance Neutrality. That is, it is sometimes rational to value a winning ticket from a smaller lottery differently from a winning ticket from a larger lottery that has the same prize money. And it is sometimes rational to value a true belief that was very likely to be true differently from a true belief that was very unlikely to be true. In sum: the central assumption of the Swamping Problem, at least as an objection to a truth-promoting account of justification, is false.

Now, we might be tempted to leave our discussion there. After all, the Swamping Problem says that Veritism is incompatible with (I*) and (II*). Chance Neutrality is the key premise in the argument for that conclusion. But by showing that this premise is false, we don't thereby show that the conclusion is also false. To do that, we must say how to set the epistemic values

- $V(X \text{ \& the objective probability of } X \text{ is } \alpha)$
- $V(\neg X \text{ \& the objective probability of } X \text{ is } \alpha)$

when I have a belief in proposition X, and check that it has the consequences we would like it to have.

To do this, let's consider how we set the values of $V(£20 \text{ \& } K_1)$, $V(£100 \text{ \& } K_2)$, and $V(£0 \text{ \& } K_2)$ in the example from above—recall, in K_1, you receive £20 with probability 100%; in K_2, you receive £100 with probability 50% and you receive nothing with probability 50%. If we wish to preserve the preference $V(K_1) > V(K_2)$, without appealing to a non-linear utility function, we must look to a risk-sensitive decision theory, such as Lara Buchak's risk-weighted expected utility theory (Quiggin, 1982, 1993; Buchak, 2013).

Buchak states her theory by retaining Chance Neutrality and dropping Desirability instead. But Pettigrew (2015) shows how to state it while retaining Desirability and dropping Chance Neutrality instead. The full details of Buchak's theory need not detain us here. I will describe how it works only for cases in which there are just two outcomes: the toss of a coin, for instance, as in the case of K_1 and K_2; or the truth or falsity of a belief, as in our central case. According to Buchak, you have not only a credence function c and a desirability function V, but also a risk function r that encodes your attitudes to risk. r takes certain of the probabilities assigned by your credence function and skews them. If r is a risk-averse risk function, it skews the probability by reducing it; if it is a risk-seeking risk function, it increases it; if it is risk-neutral, it keeps it fixed, and standard expected utility is the special case of Buchak's theory in which r is the risk-neutral risk function. Buchak assumes that r is a strictly increasing and continuous function with $r(0) = 0$ and $r(1) = 1$. To see it in action, consider K_2. According to Pettigrew's reformulation of Buchak's theory, we have:

- $V(£0 \,\&\, K_2) = \frac{r(1)-r(\frac{1}{2})}{1-\frac{1}{2}} V(£0 \,\&\, \overline{£0})$
- $V(£100 \,\&\, K_2) = \frac{r(\frac{1}{2})}{\frac{1}{2}} V(£100 \,\&\, \overline{£100})$

where $\overline{£k}$ is the situation in which you receive $£k$ for certain—that is, with probability 100%. So, as you can see, the more risk-averse you are, the lower $r(\frac{1}{2})$ is, and so the greater $V(£0 \,\&\, K_2)$ is and the less $V(£100 \,\&\, K_2)$ is. Also, note that, if you are risk-averse, it is better to receive £0 as a result of a process—such as K_2—that gave you a chance to have something more than it is to receive £0 for sure, while it is worse to receive £100 as a result of a process—such as K_2—that gave you a chance to have something less than it is to receive £100 for sure. Clearly, in this notation, $V(K_1) = V(£20 \,\&\, \overline{£20})$.

Now, according to Desirability, we have:

$$V(K_2) = c(£100|K_2)V(£100 \,\&\, K_2) + c(£0|K_2)V(£0 \,\&\, K_2)$$

What's more, by the Principal Principle, we can then derive:

$$\begin{aligned}
& V(K_2) \\
= \; & \frac{1}{2}V(£100 \,\&\, K_2) + \frac{1}{2}V(£0 \,\&\, K_2) \\
= \; & \frac{1}{2}\frac{r(1)-r(\frac{1}{2})}{1-\frac{1}{2}}V(£0 \,\&\, \overline{£0}) + \frac{1}{2}\frac{r(\frac{1}{2})}{\frac{1}{2}}V(£100 \,\&\, \overline{£100}) \\
= \; & (1-r(\frac{1}{2}))V(£0 \,\&\, \overline{£0}) + r(\frac{1}{2})V(£100 \,\&\, \overline{£100})
\end{aligned}$$

which is the value assigned by Buchak's theory. Thus, for instance, if the value you assign to receiving a sum of money for sure is linear in the amount of money, so that $V(\pounds k \ \& \ \overline{\pounds k}) = k$, then $V(K_2) = 100r(\frac{1}{2})$ and $V(K_1) = 20$. So, if $r(\frac{1}{2}) < \frac{1}{5}$, then $V(K_2) > V(K_1)$. This will hold, for instance, if $r(x) = x^3$, which is a risk-averse risk function. If you are risk-neutral, on the other hand, $r(\frac{1}{2}) = \frac{1}{2}$, and thus, $V(K_2) < V(K_1)$.

Let's now apply this to the case of beliefs. Let's suppose that you have a belief in proposition X. Then, according to Buchak's theory, we have:

- $V(\neg X \ \& \ P_X^\alpha) = \frac{r(1)-r(\alpha)}{1-\alpha} V(\neg X \ \& \ P_X^0)$
- $V(X \ \& \ P_X^\alpha) = \frac{r(\alpha)}{\alpha} V(X \ \& \ P_X^1)$

where P_X^α means *the objective probability of X is* α. So $V(X \ \& \ P_X^1)$ is the utility of being right for certain and $V(\neg X \ \& \ P_X^0)$ is the utility of being wrong for certain. Let $R = V(X \ \& \ P_X^1)$—'R' is for 'right'. And let $W = V(\neg X \ \& \ P_X^0)$—'$W$' is for 'wrong'. We need a risk function r, together with R and W, for which the following hold:

(I*) $V(X \ \& \ P_X^{\alpha'}) = \frac{r(\alpha')}{\alpha'} R < \frac{r(\alpha)}{\alpha} R = V(X \ \& \ P_X^\alpha)$, for all $0 < \alpha' < \alpha < 1$.

That is, a true belief with higher probability α of being true is more (epistemically) valuable than a true belief with lower probability α' of being true.

(II*) $V(\neg X \ \& \ P_X^{\alpha'}) = \frac{1-r(\alpha')}{1-\alpha'} W < \frac{1-r(\alpha)}{1-\alpha} W = V(\neg X \ \& \ P_X^\alpha)$, for all $0 < \alpha' < \alpha < 1$.

That is, a false belief with higher probability α of being true is more (epistemically) valuable than a false belief with lower probability α' of being true.

(III*) $V(X \ \& \ P_X^\alpha) = \frac{r(\alpha)}{\alpha} R > \frac{1-r(\alpha)}{\alpha} W = V(\neg X \ \& \ P_X^\alpha)$, for all $0 < \alpha < 1$.

That is, a true belief with probability α of being true is more (epistemically) valuable than a false belief with probability α of being true.

Suppose you set $R > W > 0$. Then we can cook up a risk function r for which (I*), (II*), (III*) hold as follows: $r(x) = (ax + (1-a))x$, where $0 < a < \frac{R-W}{R}$.[3] Note that, together, (I*),

[3] *Proof.*

- $r(0) = (a \cdot 0 + (1-a)) \cdot 0 = 0$ and $r(1) = (a \cdot 1 + (1-a)) \cdot 1 = 1$;

(II*), (III*) entail the following: a true belief is more valuable than a false belief; a justified belief is more valuable than an unjustified belief; a true justified belief is more valuable than a true unjustified belief; and a false justified belief is more valuable than a false unjustified belief.

This, I contend, answers the Swamping Problem. Given a truth-promoting account of justification of the sort that will be particularly appealing to the veritist, we can see why a true justified belief is more valuable than a true unjustified belief. Monetary value is the fundamental source of value for a lottery ticket, but we nonetheless value a winning ticket with lower probability of winning less than a winning ticket with higher probability of winning, since we are risk-averse, and valuing them in these ways is the only way to secure our risk-averse preferences in the presence of Desirability and the Principal Principle; similarly, while truth is the fundamental source of value for beliefs, as the veritist contends, we nonetheless value a true belief with lower probability of being true less than a true belief with higher probability of being true.

One interesting upshot of this treatment of the Swamping Problem is that it does not make it *compulsory* to prefer a justified true belief to an unjustified true belief; it only makes it *permissible*. After all, it is not compulsory to be risk-averse, and certainly not compulsory to be risk-averse in one of the ways that ensure (I*), (II*), and (III*) from above. If you are risk-seeking (so that $r(x) > x$, for all x) or even risk neutral (so that $r(x) = x$, for all x), for instance, you will not have that preference.

Thus, I conclude: the Swamping Problem can be answered. It relies on Chance Neutrality, but that is not a requirement of rationality. And once we see why it fails, we can give an account of a desirability function in Richard Jeffrey's decision-theoretic framework that measures epistemic value and that captures (I) and (II), as required.

- r is continuous and strictly increasing on $[0, 1]$;
- $\frac{r(x)}{x}R = (ax + (1-a))R$. This is increasing, which gives (I*). And it lies in $[R(1-a), R]$, which we'll use below in the proof of (III*).
- $\frac{1-r(x)}{1-x}W = \frac{1-(ax+(1-a))x}{1-x}W = (ax+1)W$. This is increasing, which gives (II*). And it lies in $[W, W(1+a)]$, which we'll use below in the proof of (III*).
- If $a < \frac{R-W}{R}$, then $a < \frac{R-W}{W}$, and thus $R(1-a) > W$ and $R > W(1+a)$. Since both functions are linear, this gives (III*).

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