Experimental Philosophy of Connexivity

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Abstract While Classical Logic (CL) used to be the gold standard for evaluating the rationality of human reasoning, certain non-theorems of CL—like Aristotle's ($\sim(A \to \sim A)$) and Boethius' theses ($(A \to B) \to \sim(A \to \sim B)$)—appear intuitively rational and plausible. Connexive logics have been developed to capture the underlying intuition that conditionals whose antecedents contradict their consequents, should be false. We present results of two experiments (total $n = 72$), the first to investigate connexive principles and related formulae systematically. Our data suggest that connexive logics provide more plausible rationality frameworks for human reasoning compared to CL. Moreover, we experimentally investigate two approaches for validating connexive principles within the framework of coherence-based probability logic [29]. Overall, we observed good agreement between our predictions and the data, but especially for Approach 2.

1 Introduction

Connexive logics have been developed to capture the intuition that a conditional, whose antecedent contradicts its consequent, should be false. For example, *if it rains, then it does not rain* not only appears intuitively odd but also false. If, however, the conditional $R \to \sim R$ is interpreted as a material conditional (with classical negation) $R \supset \neg R$, then it is contingent, since it is logically equivalent to the disjunction *it does not rain or it does not rain*. Formally, $(R \supset \neg R) \equiv (\neg R \vee \neg R) \equiv \neg R$. Since Classical Logic (CL) is Post-complete, simply adding the negation of $A \supset \neg A$ as an axiom or a theorem to CL would lead to unwanted trivializations [33]. Connexive logics, however, typically evaluate the negated conditional $\sim(A \to \sim A)$ as logically true and

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are hence non-classical logics (for overviews, see, e.g., [20, 33]). Various principles based on this basic intuition, the most prominent of which are *Aristotle's* theses, *Boethius* theses, and *Abelard's First Principle*, were proposed in the connexive literature (see Table 1).

While connexive logicians often refer to the intuitive plausibility of these principles, few empirical studies investigated their actual psychological plausibility. We will review the results on previous experimental work in Section 2. Moreover, we present two experiments, which—for the first time—investigate key connexive principles in a systematic way (sections 3 and 4). Specifically, we empirically study 29 formulae, including connexive principles usually discussed in the connexive literature [33], and other formulae which serve to assess the quality of the experimental material like contingent conditionals ($A \to B$) or self-negated conditionals ($A \to \sim A$; see Table 1).

While many semantics have been proposed to study connexivity, we focus on two recent approaches developed within the framework of coherence-based probability logic [28, 29]. These two approaches are based on the idea that conditionals can be interpreted as conditional probability assertions and that zero is the only coherent assessment of the conditional probability $p(\neg A|A)$, which readily captures the basic intuition of the minimum believability of $A \to \sim A$. In a series of psychological experiments spanning the last two decades, the conditional probability hypothesis (i.e., that people interpret conditionals by conditional probability assertions) has been corroborated (see, e.g., [1, 5, 7, 15, 18, 19, 21, 22, 23, 24, 25, 26, 31]).

In Approach 1, connexive principles are interpreted in terms of probabilistic constraints on conditional events. In a nutshell, the conditional $A \to B$ is interpreted by the constraint $p(B|A) = 1$, while the negated conditional $\sim(A \to B)$ is interpreted by the constraint $p(B|A) \neq 1$. Approach 1 differentiates iterated and non-iterated connexive principles, where *Iterated connexive principles* (e.g., *Boethius* theses) are those connexive principles characterized by conditionals connected via another conditional. *Non-iterated connexive principles* are, for example, *Aristotle's* theses (which are just negated (atomic) conditionals) or *Abelard's First Principle* (where conditionals are connected by a conjunction).

According to Approach 1, non-iterated connexive principles are validated iff "the probabilistic constraint associated with the connexive principle is satisfied by every coherent assessment on the involved conditional events" [29, Def. 4, p. 678]. Aristotle's Thesis, for example, is validated since $p(A|\neg A) \neq 1$ is the associated probabilistic constraint of $\sim(\sim A \to A)$ and every coherent assessment $p(A|\neg A)$ is such that $p(A|\neg A) \neq 1$.

Iterated connexive principles are validated iff the probabilistic constraint of the conclusion is satisfied by every coherent extension to the conclusion from any coherent probability assessment satisfying the constraint of the premise [29, Def. 5, p. 679]. *Boethius' Thesis* ($(A \to B) \to \sim(A \to \sim B)$), for example, is interpreted s.t. the premise constraint is $p(B|A) = 1$, which implies the conclusion constraint $p(\neg B|A) \neq 1$. Since $p(B|A) = 0$ is the unique coherent extension from $p(\neg B|A)$,

Table 1 Formulas and connexive principles investigated in Experiment 1 ($n_1 = 26$) and in Experiment 2 (first group: $n_2 = 21$, second group: $n_3 = 25$). Within each block, they are listed in the order of presentation to the participants, which was randomized using *random.org*. Response-predictions according to Classical Logic (CL), Approach 1, and Approach 2 as to whether a sentence "holds" *h*, "does not hold" *dnh* or one "cannot tell" *ct*. Principles often discussed in the connexive literature ([33]) are marked by ⋆.

Name	Formula	CL	Approach 1	Approach 2
Introductory examples (Experiments 1 and 2)				
Excluded Middle	$A \vee {\sim}A$	h	h	h
Contradiction	$A \wedge {\sim}A$	dnh	dnh	dnh
Contingent Conjunction	$A \wedge B$	ct	ct	ct
Block 1: *Basic principles* (Experiments 1 and 2)				
Negated Identity	${\sim}(A \to A)$	dnh	ct	dnh
Conjunction Elimination	$(A \wedge B) \to A$	h	h	h
Contingent Conditional	$A \to B$	ct	ct	ct
Self-negated Conditional	$A \to {\sim}A$	ct	dnh	dnh
Identity	$A \to A$	h	h	h
Arbitrary Fallacy	$A \to (A \wedge B)$	ct	ct	ct
⋆ *Aristotle's Thesis′*	${\sim}(A \to {\sim}A)$	ct	h	h
⋆ *Aristotle's Thesis*	${\sim}({\sim}A \to A)$	ct	h	h
Block 2: *Conjunctive principles* (Experiment 1)				
Negated Abelard's First Principle	$(A \to B) \wedge (A \to {\sim}B)$	ct	ct	dnh
Contingent Conditionals	$(A \to B) \wedge (A \to B)$	ct	ct	ct
⋆ *Abelard's First Principle*	${\sim}((A \to B) \wedge (A \to {\sim}B))$	ct	h	h
⋆ *Aristotle's Second Thesis*	${\sim}((A \to B) \wedge ({\sim}A \to B))$	ct	ct	ct
Contradicting Conditionals	$(A \to B) \wedge {\sim}(A \to B)$	dnh	dnh	dnh
Block 3: *Iterated principles I* (Experiment 2, first group)				
Iterated Self-negated Conditional	$(A \to B) \to {\sim}(A \to B)$	ct	dnh	dnh
⋆ *Boethius' Thesis*	$(A \to B) \to {\sim}(A \to {\sim}B)$	ct	h	h
Iterated Aristotle's Thesis	${\sim}({\sim}(A \to B) \to (A \to B))$	ct	h	h
Iterated Identity	$(A \to B) \to (A \to B)$	h	h	h
⋆ *Reversed Boethius' Thesis*	${\sim}(A \to {\sim}B) \to (A \to B)$	h	ct	h
⋆ *Boethius Variation 3*	$(A \to B) \to {\sim}({\sim}A \to B)$	ct	ct	ct
Improper Transposition (1/2)	$(A \to B) \to ({\sim}A \to {\sim}B)$	ct	ct	ct
Block 4: *Iterated principles II* (Experiment 2, second group)				
Iterated Aristotle's Thesis′	${\sim}((A \to B) \to {\sim}(A \to B))$	ct	h	h
Improper Transposition (2/2)	$(A \to B) \to ({\sim}A \to {\sim}B)$	ct	ct	ct
Denying a Conjunct	${\sim}(A \wedge B) \to ({\sim}A \to B)$	ct	ct	ct
⋆ *Boethius' Thesis′*	$(A \to {\sim}B) \to {\sim}(A \to B)$	ct	h	h
⋆ *Reversed Boethius' Thesis′*	${\sim}(A \to B) \to (A \to {\sim}B)$	h	ct	h
Symmetry	$(A \to B) \to (B \to A)$	ct	ct	ct
⋆ *Boethius Variation 4*	$({\sim}A \to B) \to {\sim}(A \to B)$	ct	ct	ct

Boethius' Thesis is validated in Approach 1.

Approach 2 is based on the theory of logical operations on conditional events (see [8, 9, 10, 11, 12]). Here, connexive principles are evaluated in terms of suitable conditional random quantities: a connexive principle is validated iff "the associated conditional random quantity is constant and equal to 1" [29, Def. 6, p. 681]. This definition captures both iterated and non-iterated connexive principles.

For an intuitive understanding of conditional random quantities, recall that the conditional event $C|A$ is trivalent (*true*, if $A \wedge C$ is true; *false*, if $A \wedge \neg C$ is true; and *void*, if $\neg A$ is true) and the corresponding conditional probability cannot be iterated (i.e., formulas like $p((D|B)|(C|A))$ would lead to Lewis' well-known triviality results [16]). In the theory of logical operations among conditional events, however, the corresponding operations yield conditional random quantities which are characterized by more than three possible values and they do not trivialize (see, e.g., [8]). Moreover, they preserve the usual probabilistic properties. As an example, consider conjunction. The Fréchet-Hoeffding bounds are the best possible coherent bounds for conjunctions of conditionals and for conjunctions of unconditional events: i.e., when $p(D|B) = x$ and $p(C|A) = y$, then the coherent prevision of $(D|B) \wedge (C|A)$ is at least $\max\{x + y - 1, 0\}$ and at most $\min\{x, y\}$, which are just the same bounds as for the conjunction of two unconditional events, i.e., $p(A \wedge B)$ when $p(A) = x$ and $p(B) = y$.

In Approach 2, *Abelard's First Principle*, $\sim((A \rightarrow B) \wedge (A \rightarrow \sim B))$, for example, is formalized by the conditional random quantity $\neg((B|A) \wedge (\neg B|A))$, i.e., a negated conjunction, where $A \neq \bot$ [29]. In the theory of logical operations among conditional events, it holds that $(B|A) \wedge (\neg B|A) = (B \wedge \neg B)|A = \bot|A$ and $\bot|A$ is constant and equal to 0. Moreover, the negation of $\bot|A$ is constant and equal to 1. Therefore, *Abelard's First Principle* is validated in Approach 2. For more technical details and validation proofs of the formulae in Table 1 within both approaches, see [29].

Our psychological predictions in Table 1 are derived from the three interpretations (Classical logic, Approach 1, and Approach 2) presented in this paper: under the CL interpretation, participants respond with "holds", "does not hold", or "cannot tell" if the formula/principle is logically true, logically false, or contingent, respectively. Under the interpretation of Approach 1 and Approach 2, participants respond with "holds", "does not hold", or "cannot tell" if the value associated with the formula/principle is constant and equal to 1, 0, or not constant, respectively. The experimental material presented in sections 3.2 and 4.2 however, is formulated in a neutral manner with respect to possible semantic interpretations. Therefore, the results also allow for empirical evaluation of other interpretations and the reader is invited to apply their own predictions to the data.

For example, while all three of our interpretations predict "cannot tell" for *Aristotle's Second Thesis*, this formula might hold in some connexive logics. Moreover, by using a different notion of conjunction of conditional events, *Aristotle's Second Thesis* can be validated: specifically, when the conjunction $\wedge$ of conditional events is replaced by the Kleene-Lukasiewicz-Heyting conjunction $\wedge_K$ (see, e.g. [2]), then *Aristotle's Second Thesis* is validated in Approach 2 [28]. However, as pointed out in

[28], this comes with the cost of losing the usual probabilistic properties (i.e., under the conjunction $\wedge_K$, the Fréchet-Hoeffding bounds are violated). There is also a proof that *Aristotle's Second Thesis* can be validated in Approach 2 (under the usual definition of conjunction of conditional events $\wedge$), when the additional constraint $p(B) \neq 1$ is assumed [28].

Before we introduce our experiments, we present a brief overview of selected previous empirical work.

2 Previous Experiments

Systematic quantitative research on the judgement of connexive principles is scarce, as even the few studies investigating formulae pertaining to connexivity only featured a handful of relevant formulae each. In this section, we give a brief overview of five such studies, with Table 2 providing a summary of their results at one glance. We also sketch the differences in the methodology of each study, which should be kept in mind when reading the table.

As mentioned in Section 1, we did not include data (see [13], parts of [17], [31] and [32]) which pertain to the acceptance of inference patterns rather than of the related propositional formulae under consideration here.[1] Additionally, two further experiments ([3] and [14]), concerning intuitions about the negation of propositional formulae, may be of interest to the reader, but relate only indirectly to our aims in this paper.

In Pfeifer and Yama 2017 [32], 63 participants—students of Osaka City University, where the experiment was held in Japanese—were presented with a vignette story similar to our own (see Section 3.2), revolving around someone named Hanako working in a playing blocks factory. In a two-step process (for each task), participants were first asked to gauge whether the truth-status of the formula in question could be inferred at all (*Can Hanako infer at all, how sure she can be that the [sentence formulation of the formula] holds?*), and if they answered affirmatively, whether *Hanako can be sure that the sentence in the box holds* or *. . . does not hold*. In Table 2, we interpreted *sure that holds* as +, *sure that does not hold* as − and *cannot infer at all how sure* as ?.

In Pfeifer and Stöckle-Schobel 2015 [30], 40 participants—students at Augustina-Hochschule Neuendettelsau, where the experiment was held in German—were presented with task material similar to our own, but with counterfactual formulations of the conditionals: E.g., *If the playing block were red, then it would be made from rubber* (*Contingent Conditional*). In the familiar two-step fashion, then, participants were first asked whether the formula under consideration is contingent (*Can you conclude whether [the sentence formulation of the formula] holds?* and, if answering

[1] I.e., we focus on formulae like $(A \to B) \to (B \to A)$ instead of corresponding inference patterns *from* $A \to B$, *infer* $B \to A$.

affirmatively, whether it *holds* or whether it *does not hold*. In Table 2 we interpreted *cannot conclude* as ?, *holds* as + and *does not hold* as −.[2]

In Pfeifer 2012 [22], 40 participants—students of the University of Salzburg, where the experiment was held in German—were presented with a vignette story about someone named Hans hearing a knock on the door, which he knew to be coming either from Thea or Ida. Then, the participants were asked to consider the validity of sentences like *It is not the case, that: If Ida knocks, then Ida does not knock.* (*Aristotle's Thesis ′*). They were simultaneously presented three answer-options: (1) *The [sentence] is guaranteed to be false*, (2) [. . .] *to be true.* and (3) *One cannot infer whether the sentence is true or false*, which we classified as −, +, and ?, respectively, in Table 2 . This paper also contains another sample from the University of Salzburg ($n = 141$) in which the results of two formulations of each *Aristotle's thesis* and *Aristotle's thesis ′* were pooled: 67% of participants responded with +, 17% with − and 16% with ?.

In Pfeifer and Tulkki 2017 [31], 60 participants—students of the University of Helsinki, where the experiment was held in Finnish—were presented with a vignette story similar to our own (featuring a protagonist named Paula), but fine-tuned to investigate inference rules. By recasting them as inferences from an empty premise set, the study nevertheless served to investigate three formulae in the way that is of interest here. Participants were asked first whether the task was informative at all (*Based on* [the premises]*, can Paula infer at all how sure she can be that the* [conclusion] *holds?*), and if they answered affirmatively, whether one could be sure that the conclusion holds or does not hold (*Paula is very sure that the* [conclusion] *does (not) hold.*). In Table 2, we marked *sure that holds* with +, *sure that does not hold* with − and *cannot infer at all how sure* with ?.

In McCall 2012 [17], 89 participants—students at McGill University, where the experiment was held in English—were asked: *Whether Hitler is dead or not, are the following statements true?*[3] Then, the participants were presented with a list of sentences, e.g., *If Hitler is dead, then Hitler is dead* (Identity). For each they had the options of choosing *YES*, *NO*, *DON'T KNOW*, or not to answer entirely. In Table 2 we matched *YES* to +, *NO* to − and classified *DON'T KNOW* and absent answers as ?.

In sum, the data of previous experiments support selected connexive principles and violate predictions of CL. However, a systematic study comparing all connexive principles and related formulae is missing. The next two sections present such data using uniform task material for comparability.

[2] In [30] the formulae for *Aristotle's thesis* and *Aristotle's thesis ′* are swapped. As we follow [33], we adjusted the names—also in the data taken from other studies—in Table 2 accordingly.

[3] Given the task formulations making reference to the proposition *Hitler is dead.*, we have some worries about world-knowledge infecting McCall's results. Specifically, with such task materials we see the danger that participants ignore the logical form and evaluate the sentences only on the basis of their world knowledge, which is a kind of a belief bias [4]. Or, even worse, if the task material appears to be too silly, participants may even opt out of the task.

Table 2 Collection of data from previously published experiments in %, for formulae also investigated in our experiments. We omitted data pertaining to the *Introductory principles*, as they are both uncontroversial and widely available elsewhere. As these studies differ slightly in their exact phrasing and presentation of the material, we opted to use pre-theoretic symbols +, −, ? to categorize their results, the meanings of which are detailed for each source in Section 2. Formulae and predictions by Classical Logic and coherence-based probability logic can be found in Table 1.

Formula	**Study**														
	[31], n = 60			[22], n = 40			[17], n = 89			[30], n = 40			[32], n = 63		
	+	−	?	+	−	?	+	−	?	+	−	?	+	−	?
Negated Identity	12	75	13	10	88	2		NA		10	78	13	6	63	30
Conjunction Elimination		NA			NA		78	20	2		NA			NA	
Contingent Conditional		NA		0	13	88		NA			NA			NA	
Identity		NA		93	3	5	97	3	0		NA			NA	
Arbitrary Fallacy		NA			NA		6	88	7		NA			NA	
Aristotle's Thesis′	77	7	17	78	18	5	88	7	6	68	23	10	76	11	13
Aristotle's Thesis	72	12	17	80	13	8		NA		70	20	10	65	16	19
Boethius' Thesis		NA			NA		84	8	8		NA			NA	

3 Experiment 1

Our aim with this experiment was to start a systematic investigation of how participants reason about propositional formulae relevant to connexive logic. To avoid cognitive overloading, though, we chose to focus on non-iterated connexive principles and to test the task material with basic formulae. Specifically, after presenting participants with the *Introductory Examples*, we investigated the tasks related to Block 1 (*Basic principles*) and Block 2 *Conjunctive principles* (see Table 1). We used task materials which are as neutral as possible with respect to the semantics in order to secure the results as relevant to semantics beyond CL, Approach 1, and Approach 2.

3.1 Participants

A total of 26 participants partook in Experiment 1. They were sampled from students participating in philosophy courses taught by the first author at the University of Vienna (8) and the University of Regensburg (18).

Of the 26 participants, 14 answered their gender to be *female*, eight *male* and four *non-binary/genderqueer*. None of the participants chose *no gender*, *prefer not to answer*, or *not listed*, the last of which would have prompted a custom input field.

Eleven participants had previously taken logic classes[4]. Eighteen study philosophy in either their major or minor, while the remaining eight study a variety of

[4] We inquired this due to the prevalence of CL and its material-conditional interpretation in introductory logic classes.

subjects, namely physics, psychology, computer science, cultural studies, political science, and criminology. Participants who had previously partaken in similar studies were not included in the sample.

The participants' age ranged from 19 to 30 years old, with a mean of 23.35 years ($SD = 2.73$). They have been studying for 6.27 semesters on the average ($SD = 3.94$).

3.2 Material and Procedure

Each participant of Experiment 1 solved a total of 16 formulae, three of which are *Introductory examples*, and 13 of which are the tasks from blocks 2 (*Basic principles*), and 3 (*Conjunctive principles*), as listed in Table 1. The experiment was conducted in German, using online questionnaires hosted by `soscisurvey.de` and a layout designed for participation using mobile phones, which turned out to be well suited for laptop users as well. The participants used their private devices to to fill in the questionnaires.

Participants were informed that the purpose of this experiment was to study how humans understand the logical structure of (combined) sentences, and asked to imagine a scenario in which someone named Ida works at a machine which produces playing blocks. Each of these blocks was specified to have a shape (*cylinder, cube, ball*) and a size (*small, large*), and the machine to produce blocks in all combinations of these shapes and sizes. In terms of the semantics of CL, this specification was meant to communicate any semantic model of the task formula as equally plausible.

As described in Section 2, previous experiments with similar vignette stories have already been successfully used in experimental philosophy. We also chose this scenario to minimize the impact of potential prior beliefs held by the participants. Such background knowledge can potentially impact the responses. As we were interested in the participants' reasoning about logical form, we aimed at minimizing the impact of background knowledge in order to avoid *belief-biases* (see [6] p. 243f). For the same reason, we selected *size* and *shape* over *colour, material,* and *weight* as the properties corresponding to variables, lest the participants intuitively judge certain combinations of the latter to be more coherent or more probable than others (e.g., *metal* might be conceived as heavier than *wood*, and we aimed to avoid such common sense associations). For our purposes, ideally, the participants should use only the information explicitly conveyed in the instructions in their reasoning processes. To clearly differentiate between the two variables and for intelligibility, we also opted to formulate *size* in terms of adjectives and *shape* using nouns.

For each task, then, the instructions stated that Ida is waiting in front of the machine, deliberating whether a certain (combined) sentence holds. These sentences were presented in a structured manner, in order to help the participants grasping the composition of the target sentences: we presented parts of the combined sentence first, then we expressed the target sentence in a semi-formalised manner, and, finally, we fully spelled it out. The *Arbitrary Fallacy* task, for example, was thus phrased as follows:

> Ida is waiting in front of the machine and considers the following sentences:
>
> **(A)** The next playing block is *large*.
> **(B)** The next playing block is **both** *large*, **and** a *cube*.
>
> Now Ida considers the following, combined sentence:
>
> **(C) If (A), then (B).**
>
> Or spelled-out:
> **(C) If** the next playing block is *large*, **then** it is **both** *large*, **and** a *cube*.

As an additional, visual aid, we colour-coded the sentence parts—reddish purple for **(A)** and blue for **(B)**, and included a colour-mapping task at the end of the experiment to control for issues related to colour perception.[5] Colour turned out to be not an issue, as 25 out of the 26 participants matched the colours we used correctly.

Attempting to provide appropriate, natural language formulations of the investigated principles, we chose to write (German version in parentheses): $\wedge$ as *both . . . and* (*sowohl . . . als auch*), $\rightarrow$ as *if . . . then* (*falls . . . dann*), $\vee$ as *or* (*oder*). To make the formulations intelligible and have them adhere to natural language grammar, we sometimes wrote $\sim$ as *not* (*nicht*) and other times we used the phrasing *it is not the case, that* (*es ist nicht der Fall, dass*).

Following the introduction of the task sentence, the participants were first shown a question to distinguish between contingent and non-contingent sentences (Question 1), i.e.,

> Can Ida even know anything about whether the underlined sentence (C) holds?
> *Please pay attention solely to the structure of the sentence (C).*
>
> □ NO, as the underlined sentence (C) could hold or not hold.
> □ YES, Ida can know something about whether the underlined sentence (C) holds.

and if they answered it affirmatively, the follow-up Question 2 differentiating between tautological and contradictory sentences appeared as follows:

[5] We approximated the RGB colours 0, 114, 178 and 204, 121, 167, as suggested in [34], using `soscisurvey.de`'s proprietary tools.

What can Ida know about whether the underlined sentence (C) holds?
Please pay attention solely to the structure of the sentence (C).

□ The underlined sentence (C) does NOT hold.
□ The underlined sentence (C) holds.

We chose this two-step approach to clearly establish contingency to be a fully fledged option and to counteract a tendency—observed during the pretest—of participants to differentiate solely between tautological and non-tautological sentences, which could be a pragmatic effect that we aimed to avoid. The two-fold task descriptions were visible for the participants throughout answering either question type. When first presented with any task, a timer ensured that the participants took a minimum of 10 seconds to familiarize themselves with the new sentences. For follow-up questions, we reduced the timer duration to three seconds to avoid possible frustrations. Throughout the experiment, the online questionnaire allowed us to track how much time each participant spent answering each question.

The three simple and uncontroversial[6] formulae in the block *Introductory examples* were presented as part of a tutorial section prior to the rest of the experiment, to help familiarize the participants with the material. We recorded the first, untainted answers for each example, but once completed, the participants were informed of a brief explanation of why their original answer was (not) correct. For the first example formula, *Excluded Middle*, we offered the following explanation:

Your answer is correct.

In this case, Ida can know, **solely based on the structure** of the sentence, that it holds.

Because: No matter the composition of the next playing block, it is *large* **or not** *large* in every case.

In the case of incorrect answers, the participants were asked to attempt the individual task again:

Your answer is not correct.

[6] All three of the logics investigated in this experiment agree in their predictions of the appropriate answers for these formulae.

In this case, Ida can know, **solely based on the structure** of the sentence, that it holds.

Because: No matter the composition of the next playing block, it is *large* **or not** *large* in every case.

Please try again!

After the three introductory examples, participants were given the opportunity—which only one of the 26 participants decided to take—to re-read the introduction or start the experiment immediately.

After answering the eight *Basic principles* tasks and five *Conjunctive principles* tasks in the order listed in Table 1, participants were asked for demographic information[7], as well as to give some additional insights into their response behaviour. Using percentage sliders, they were also asked to rate how clear (from *very UNCLEAR* to *very CLEAR*) and difficult (from *very difficult* to *very easy*) they perceived the tasks to be, and to rate their own confidence in their answers (from *confident, that they are INCORRECT* to *confident, that they are CORRECT*).

3.3 Results and discussion

Using *Fisher-Freeman-Halton exact tests*[8], we determined that in all tasks but *Negated Abelard's First Principle*[9], the p-values were clearly above the significance level of 0.05. Therefore, we decided to pool the two samples consisting of students from the Universities of Vienna and Regensburg, despite minor adjustments we made in the questionnaire design between taking the two samples.

Table 3 shows the response frequencies for the formulae investigated in Experiment 1. We observed clear evidence in favour of connexive reasoning: all connexive principles (marked by $\star$), except for the (controversial) *Aristotle's Second Thesis*, were judged to hold by the majority of participants. Whenever the predictions by CL diverged from those of both connexive approaches, the response frequencies

[7] When inquiring the participants' gender, we followed the 2021 *Guideline for the sensitive collection of gender in questionnaires* by the Institute of Psychology of Humboldt-Universität zu Berlin, `https://www.psychologie.hu-berlin.de/de/institut/organisation/gleichstellung/leitfaden-sensible-erhebung-von-geschlecht-in.pdf`, last accessed on September 6, 2022.

[8] In order to avoid frustrations during the third introductory example (*contingent conjunction*) we opted to only ask participants the question pertaining to the contingency of the sentence, immediately presenting them with an explanation upon their answer. As a result, we did not collect data distinguishing between *does not hold* and *cannot say* and used the Fisher-exact test without the *Freeman-Halton extension* for this specific task.

[9] *Negated Abelard's First Principle* assumes double negation elimination in its name.

favoured the latter. These results corroborate the armchair intuitions from the Introduction, as well as the experimental data presented in Section 2. In fact, Approach 2 even managed to correctly predict the response frequencies for every single task.

We also analysed which approach best-predicted the within-participant response patterns over the 13 tasks, and the results are similarly clear:[10] 85% of participants best agreed with Approach 2, compared to 19% for Approach 1, and 15% for CL. That is, of the 26 participants in n_1, CL best predicted four participants, Approach 2 was the winning predictor for 17 participants, while the remaining five were a tie between approaches one and two. We omitted the individual percentage numbers of each participant-approach pair here, due to the high overlap between the predictions of the three approaches (see Table 1). We observed no significant correlation between participants' experience with logic classes and their agreement with CL.

We obtained this evidence despite the high complexity of the target sentences, that made us originally decide against testing all formulae from Table 1 using a single questionnaire. Because of this complexity, we expected the modal response of the participants' self-assessments to tend towards *very difficult* and towards the low clarity end of the scale. However, this did not occur, see Table 4 for the self-assessment data. We observed medium mean difficulty responses with high standard deviations: while some participants indeed found the task material *very difficult*, others found it rather easy (Table 4). Additionally, the mean clarity responses were much higher than expected. The participants' confidence in the correctness of their responses, as well as the mean time spent on the entire questionnaire, is presented in Table 4.[11]

Over the roughly 15 minutes mean that participants took to complete the experiment, we observed them to quickly *warm up* to the task material: While it took them a considerable amount of time to answer the simple *Introductory examples*, the more complex *Basic principles* and *Conjunctive principles* were answered comparatively quickly, reflecting a need to familiarize themselves with the task material. Even for the most complex task, *Contradicting Conditionals*, participants took less than 45 seconds on average. Table 5 shows how the mean response time per task scales non-linearly with task complexity, as measured by the word-count of the fully spelled-out versions of the combined sentences.[12] We take this as evidence that the participants had an easier time with the questionnaire than we originally anticipated.

Only three participants responded that they considered primarily the semi-formalised formulations of the target sentences, 16 preferred the fully spelled out

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[10] Thanks to Nicole Cruz for suggesting a within-participants analysis of our data.

[11] Dwell time per participant is corrected for breaks using the *TIME_SUM* algorithm documented at `https://www.soscisurvey.de/help/doku.php/en:results:variables`, last accessed on September, 10, 2022.

[12] For each task, participants were only presented with questions of type two if they found the formula not to be contingent. Hence, only the time spent on question type one could sensibly figure into this between-task comparison.

Table 3 Response frequencies (in %) in Experiment 1 with $n_1 = 26$. The formatting marks predictions by Classical Logic, *Approach 1* and **Approach 2**. Standard connexive principles according to [33] are marked by ⋆.

Name	Holds	Does not hold	Cannot tell
	Introductory examples		
Excluded middle	***69.23***	3.85	26.92
Contradiction	3.85	***61.54***	34.62
Contingent Conjunction	—	—	***53.85***
	Block 1: *Basic principles*		
Negated Identity	19.23	**65.38**	*15.38*
Conjunction elimination	***88.46***	11.54	0.00
Contingent conditional	11.54	26.92	***61.54***
Self-negated Conditional	0.00	***84.61***	15.38
Identity	***92.31***	0.00	7.69
Arbitrary Fallacy	7.69	15.38	***76.92***
⋆ *Aristotle's Thesis′*	***57.69***	23.08	19.23
⋆ *Aristotle's Thesis*	***53.85***	34.62	11.54
	Block 2: *Conjunctive principles*		
Negated Abelard's first principle	15.38	**65.38**	*19.23*
Contingent Conditionals	15.38	19.23	***65.38***
⋆ *Abelard's first principle*	***50.00***	26.92	23.08
⋆ *Aristotle's second Thesis*	30.77	11.54	***57.69***
Contradicting Conditionals	23.08	***46.15***	30.77

Table 4 Self-assessment (confidence, difficulty and clarity from 0 to 100) and total dwell times with $n = 26$. For the response times per task, see Table 5.

Value	Minimum	Mean	SD	Maximum
Confidence	11.00	52.46	23.76	88.00
Difficulty	9.00	42.58	20.26	91.00
Clarity	7.00	59.81	26.95	100.00
Total Time (in mm:ss)	08:55	14:52	02:54	20:14

formulation, and seven indicated that they considered both formulations equally. Contrary to our expectation, having taken logic classes did not significantly correlate with a preference for the semi-formalised phrasings of the target sentences.

Of the 18 participants in the survey at the University of Regensburg that were presented with these additional questions, 14 found Question 1 (concerning contingency) more difficult, one person found Question 2 (*holds/does not hold*) more difficult, and three found them equally difficult. This matches the mean response time differences between questions 1 and 2 for each task as seen in Table 5, and is further corroborated by 16 out of 18 participants replying that they already decided upon an answer to Question 2 while deliberating Question 1.

Table 5 Mean dwell time (in seconds) per task for question type 1 and 2 respectively; Ratio of the time spent on question type one and the complexity of each task as a measure of the word count in the lower, longer sentence formulation; $n_1 = 26$.

Principle	Question 1	Question 2	Number of words	Ratio
Introductory examples				
Excluded Middle	51.88	24.68	8	6.49
Contradiction	31.62	7.71	11	2.87
Contingent Conjunction	24.92	—	10	2.49
Block 1: *Basic principles*				
Negated Identity	43.77	18.09	18	2.43
Conjunction Elimination-law	27.19	9.81	16	1.70
Contingent Conditional	24.73	10.70	11	2.25
Self-negated Conditional	16.46	6.09	11	1.50
Identity	17.62	7.63	10	1.76
Arbitrary Fallacy	30.50	8.50	15	2.03
⋆ *Aristotle's Thesis′*	30.88	9.14	18	1.72
⋆ *Aristotle's Thesis*	21.88	9.22	17	1.29
Block 2: *Conjunctive principles*				
Negated Abelard's First Principle	31.35	7.38	32	0.98
Contingent Conditionals	36.65	8.33	31	1.18
⋆ *Abelard's First Principle*	27.46	9.25	33	0.83
⋆ *Aristotle's Second Thesis*	40.77	7.64	33	1.24
Contradicting Conditionals	44.54	15.44	34	1.31

4 Experiment 2

The results concerning perceived complexity and mean dwell times from Experiment 1 bolstered us to present the more complicated, iterated formulae to participants in this second experiment. To assure comparability of the data, however, the participants were presented with the same introduction, including the first block, as in Experiment 1.

4.1 Participants

A total of 46 participants partook in Experiment 2. They were sampled from philosophy courses at the universities of Regensburg and Münster. As solving every task corresponding to the iterated connexive principles and formulae (see Table 1) would have required too much time and effort from participants, we decided to split these tasks into Block 3 and Block 4. Hence, the participants were randomly assigned into two groups, using `socscisurvey.de`'s randomization urn. The participants of the first group ($n_2 = 21$) were presented with the tasks of blocks 1–3, while the participants of the second group (n_3 = 25) were presented with the tasks of blocks 1, 2, and 4.

Twelve answered their gender to be *female*, 29 *male*, one *non-binary/genderqueer*, and four *preferred not to answer*. None of the participants chose *no gender*, or *not listed*, the latter of which would have prompted a custom input field.

Thirty-three participants had previous experience with logic classes. Thirty study philosophy in either their major or minor, while the remaining 16 study a variety of subjects, namely physics, computer science, mathematics, chemistry, political science, psychology and nanoscience. Participants who had previously partaken in similar studies were not included in the sample.

The participants' age ranged from 19 to 30 years old, with a mean of 24.20 (SD = 5.11). They have been studying for a mean of 6.04 semesters (SD = 5.02).

4.2 Material and Procedure

We used the same procedure as in Experiment 1 and adapted the experimental material accordingly. Thus, Experiment 2 started out with the *Introductory examples* and the *Basic principles*. From there, Group 1 received the tasks of Block 3 (*Iterated principles I*) and Group 2 received the tasks of Block 4 (*Iterated principles II*; see Table 1). For each participant, we used `socscisurvey.de`'s randomization urn to assign to them either of the questionnaires. Each participant was presented with 18 tasks, including the introductory examples. We provided feedback to participants for their answers to the introductory examples.

We used the same strategy to formulate the task material as in Experiment 1. The iterated connexive principle *Boethius' Thesis*, for example, was formulated as follows:[13]

> Ida is waiting in front of the machine and considers the following sentences:
>
> **(A) If** the next playing block is a *ball*, **then** it is *large*.
> **(B) If** the next playing block is a *ball*, **then** it is **not** *large*.
>
> Now Ida considers the following, combined sentence:
>
> **(C) If (A), then not (B).**
>
> Or spelled-out:
> **(C) If**, **if** the next playing block is a *ball*, then it is *large*, **then** it is **not** the case, that **if** the next playing block is a *ball*, **then** it is **not** *large*.

[13] We again made use of colour codes for sentences (A) and (B) to increase clarity and intelligibility.

By testing one formula (*Improper Transposition*, $(A \to B) \to (\sim A \to \sim B)$) twice, we sought to investigate possible impacts of the noun/verb order: in Block 3, we formulated A in terms of shape (*ball*) and B in terms of size (*large*), and in Block 4 we inverted it to A as size (*large*) and B as shape (*cylinder*).

We also newly included a question about fluency in German, which prompted a percentage scale if the participants selected that they were *non-native speakers*. Otherwise, both material and conduction of the experiment followed the description in Section 3.2.

4.3 Results and discussion

Tables 6, 7, and 8 show the results from Experiment 2, akin to tables 4, 3, and 5 in Section 3.3 for Experiment 1.

They further strengthen the case of connexive logics: not just for the more well known principles like *Aristotle's* and *Boethius'* theses did the response frequencies favour approaches 1 and 2 over CL, but in every case where the former two mutually disagreed with the latter. Except for *Symmetry*, Approach 2 again perfectly predicted the modal responses to each task. *Symmetry* is indeed exceptional, in that all three approaches predicted it to be contingent, but only 40% of participants responded as predicted, while 44% judged that *Symmetry* holds. This is somewhat puzzling and calls for further research. For every task that we retested from Experiment 1, Experiment 2 reproduced the same modal responses.

For the within-participant response predictions among the 21 participants in n_2, CL won out with three, Approach 1 with one and Approach 2 with 12 of the 21 participants. One participant was tied between CL and Approach 2, two between approaches 1 and 2 and one participant between all three approaches. CL was the winning predictor for three of the 25 participants in n_3, Approach 1 for two and Approach 2 for 16. Additionally, three participants were tied between approaches 1 and 2, and one participant's responses were equally well predicted by all three approaches. Among this sample was also the only participant whose responses were perfectly predicted by any one approach, namely by Approach 2. There was no significant correlation between participants ($n_2 + n_3 = 46$) having taken logic classes and agreeing with the predictions by CL.

Fourteen participants considered primarily the semi-formalised phrasings, 21 the fully spelled out phrasings, and 11 considered both phrasings of the target sentences equally. Twenty participants found Question 1 (contingency) more difficult, seven found Question 2 (*holds/does not hold*) more difficult, and 19 found them equally difficult. This fits well with the mean response time differences between Question 1 and 2 for each task (see Table 8), and is further corroborated by 36 of 46 participants replying that they already decided upon an answer to Question 2 while deliberating Question 1.

Table 6 Self-assessment (confidence, difficulty and clarity from 0 to 100) and sum of dwell times with $n_2 + n_3 = 46$; See Table 8 for the response times per task.

Value	Minimum	Mean	SD	Maximum
Confidence	5.00	55.24	25.43	91.00
Difficulty	5.00	39.35	16.26	85.00
Clarity	0.00	66.91	26.44	100.00
Time (in mm:ss)	10:06	14:27	02:15	18:22

Compared to Experiment 1, a larger percentage of participants oriented themselves around the semi-formalised phrasings of the target sentences (increase from 12% to 30%), where they previously more strongly preferred the fully spelled-out formulations (decrease from 62% to 46%). A similar proportion in each experiment was ambivalent between the two phrasings (slight decrease from 27% to 24%). With this in mind, we conclude it is indeed helpful to offer both phrasings (semi-formalised and fully spelled-out) to facilitate the cognitive processing of complex formulae. Like in Experiment 1, we observed no significant correlation between participants' experiences with logic classes and them preferring the semi-formalised phrasings.

Testing the formula *Improper Transposition* twice with different formulations points to consistency across formulations, as the clear majority of responses put the formula in the category *cannot tell*. Thus, there was no impact of noun/verb orders. This hints a high reliability of the data but, of course, a proper replication study is needed, which would go beyond the scope of this paper.

Despite being presented with two more tasks compared with Experiment 1, the mean response time per participant of Experiment 2 slightly decreased from 14 minutes and 52 seconds to 14 minutes and 27 seconds. With *Iterated Aristotle's Thesis′* though, Experiment 2 contained a task with a mean dwell time of over one minute (62 seconds).

Additionally, all 46 participants correctly identified the colours used to increase visual clarity of the sentence structures. 45 identified themselves as native speakers, with one participant evaluating their competency level of German to be 90%.

5 Concluding remarks

We investigated a total of 29 formulae (of which we tested one twice), which we sorted into 5 blocks by complexity and structure. Aiming to strike a compromise between achieving a reasonable sample size and not imposing too much cognitive demand on our participants, we split these 30 tasks into three distinct questionnaires, each featuring the *Introductory examples* and *Basic principles*, but only one each of the *Conjunctive principles*, *Iterated principles I* and *Iterated principles II* (see

Table 7 Response frequencies (in %) in Experiment 2 ($n_2 + n_3 = 46$); The formatting marks predictions by Classical Logic, *Approach 1* and **Approach 2**.

Name	Holds	Does not hold	Cannot tell
	Introductory examples, $n_2 + n_3 = 46$		
Excluded middle	***73.91***	4.35	21.74
Contradiction	6.52	***73.91***	19.57
Contingent Conjunction	—	—	***56.52***
	Block 1: *Basic principles*, $n_2 + n_3 = 46$		
Negated Identity	31.74	**63.04**	*15.22*
Conjunction elimination	***86.96***	2.17	10.87
Contingent conditional	0.00	8.70	***91.30***
Self-negated Conditional	0.00	***80.43***	19.57
Identity	***86.96***	8.70	4.35
Arbitrary Fallacy	2.17	10.87	***86.96***
★ *Aristotle's Thesis′*	***56.52***	30.43	13.04
★ *Aristotle's Thesis*	***67.39***	26.09	6.52
	Block 3: *Iterated principles I*, $n_2 = 21$		
Iterated Self-negated Conditional	4.76	***66.67***	28.57
★ *Boethius' Thesis*	***57.14***	28.67	14.29
Iterated Aristotle's Thesis	***47.62***	23.81	28.57
Iterated Identity	***61.90***	4.76	33.33
★ *Reversed Boethius' Thesis*	**71.43**	9.52	*19.05*
★ *Boethius Variation 3*	28.57	14.29	***57.14***
Improper Transposition (1/2)	14.29	9.52	***76.19***
	Block 4: *Iterated principles 2*, $n_3 = 25$		
Iterated Aristotle's Thesis′	***52.00***	12.00	36.00
Improper Transposition (2/2)	8.00	24.00	***68.00***
Denying a Conjunct	0.00	16.00	***84.00***
★ Boethius' Thesis′	***48.00***	24.00	28.00
★ Reversed Boethius' Thesis′	**64.00**	16.00	*20.00*
Symmetry	44.00	16.00	***40.00***
★ *Boethius Variation 4*	32.00	16.00	***52.00***

Table 1). Despite the high complexity of the target sentences and the difficulty of processing conditionals, negations, and especially nested conditionals, we were positively surprised by the good agreement between the data and the predictions of connexive logic in general. In particular, we observed good agreement between the data and the predictions of coherence-based probability semantics for connexive principles, and specifically with those of Approach 2.

Experimental work allows for arbitration among different semantics: if two semantics share similar formal qualities, but differ with respect to their prediction of experimental data, the one that better predicts ought to be preferred. For instance,

Table 8 Mean dwell time (in seconds) per task for question type 1 and 2 respectively; Ratio of the time spent on question type one and the complexity of each task as a measure of the word count in the lower, longer sentence formulation; $n_2 + n_3 = 46$.

Principle	Question 1	Question 2	Number of words	Ratio
Introductory examples, $n_2 + n_3 = 46$				
Excluded middle	50.00	21.39	8	6.25
Contradiction	39.76	9.00	11	3.61
Contingent Conjunction	20.07	NA	10	2.01
Block 1: *Basic principles*, $n_2 + n_3 = 46$				
Negated Identity	33.80	12.74	18	1.88
Conjunction elimination-law	25.30	8.20	16	1.58
Contingent conditional	18.52	7.5	11	1.68
Self-negated Conditional	18.28	5.70	11	1.66
Identity	14.00	6.66	10	1.40
Arbitrary Fallacy	22.78	8.17	15	1.52
Aristotle's Thesis′	21.11	9.05	18	1.17
Aristotle's Thesis	20.87	7.74	17	1.23
Block 3: *Iterated principles I*, $n_2 = 21$				
Iterated Self-negated Conditional	40.81	7.33	30	1.36
Boethius' Thesis	35.57	8.06	31	1.15
Iterated Aristotle's Thesis	35.57	5.93	41	0.87
Iterated Identity	17.43	5.71	24	0.73
Reversed Boethius' Thesis	28.24	6.59	31	0.91
Boethius Variation 3	37.71	8.33	31	1.22
Improper Transposition (1/2)	28.67	6.40	25	1.15
Block 4: *Iterated principles 2*, $n_3 = 25$				
Iterated Aristotle's Thesis′	62.32	14.00	36	1.73
Improper Transposition (2/2)	36.64	10.50	25	1.47
Denying a Conjunct	33.12	29.50	34	0.97
Boethius' Thesis′	39.72	11.22	31	1.28
Reversed Boethius' Thesis′	41.44	6.10	36	1.15
Symmetry	29.12	11.00	24	1.21
Boethius Variation 4	44.36	7.58	30	1.48

while there exist semantics which validate *Aristotle's Second Thesis*, this principle was not empirically supported by our data, as approaches 1 and 2 predicted.

We paid special attention to avoid belief biases [4] by stressing that the participants should focus on the form of the sentences and by presenting vignette stories, in which world knowledge does not provide an immediate solution to the tasks. Moreover, to avoid potential unwanted pragmatic effects, we used a two-step response format which frames *cannot tell* as a viable response option. Thereby, we aimed to avoid the pragmatic oddness of giving a (alleged) non-informative response within an informative task setting. Finally, the cover stories are flexible enough to shed light on the acceptance of principles and formulae independently of specific semantics or logics, or even beyond studies of connexivity altogether.

We suggest that future work should deepen the understanding of *Symmetry*. Moreover, while our study focuses on formulae, it would be interesting to shift our focus next to inference schemes. While all formulae of Table 1 can be treated as conclusions derived from the empty premise set, iterated connexive principles can also be presented to participants as inferences from their antecedents (as premises) to their consequents (as conclusions). This yields a blueprint for further experiments on connexive reasoning. Future work is also required to deepen the understanding of related argument schemes like *Contraposition* (first experimental results indicate that most people reject contraposition [27]).

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