Secondary mathematics teachers' use of learners' responses to foster justification skills

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ABSTRACT

This study aimed to understand how secondary mathematics teachers engage with learners during the teaching and learning process. A sample of six participants was purposively selected from a population of ordinary level mathematics teachers in one urban setting in Zimbabwe. Field notes from lesson observations and audio-taped teachers' narrations from interviews constituted data for the study to which thematic analysis technique was then applied to determine levels of mathematical intimacy and integrity displayed by the teachers as they interacted with the students. The study revealed some inadequacies in the manner in which the teachers handled students' responses as they strive to promote justification skills during problem solving, in particular teachers did not ask students to explain wrong answers. The teachers indicated that they did not have sufficient time to engage learners in authentic problem-solving activities since they would be rushing to complete syllabus for examination purposes. On the basis of these findings, we suggest teachers to appreciate the need to pay special attention to the kinds of responses given by learners during problem-solving in order to promote justification skills among learners.

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1. INTRODUCTION

In spite of mathematics being regarded as a key to opening career opportunities for students and a bedrock of science and technology, we have observed that many students do shy away from mathematics. This can be attributed to some extent to learning problems such as reduced levels of concept mastery and lack of problem-solving abilities by students. This is supported by Webb [1] who suggests that the learning of mathematics has been characterised by some learning problems such as poor concept grasping due to reasons such as incompetence by mathematics teachers.

We have also observed that there are certain mathematical practices by teachers that impact on the learning of mathematics. Schoenfeld and Sloane [2] suggests that such mathematical practices include representing, justifying, and communicating mathematical ideas. These observations motivated us into raising questions about how mathematics teachers handle learners' responses during problem solving as well as the extent to which teachers engage with learners in an effort to promote justification skills. This is because teachers are key players to the enactment of effective mathematics learning and realisation of the mathematics curriculum objectives.

We hypothesize that the way mathematics teachers handle learners' responses during classroom interaction and problem solving has consequences on students' reasoning, exploring mathematical ideas, making conjectures and justifying solutions. In most instances teachers tend to give prominence to correct answers only. However, effective learning can spring from wrong answers as improvements can be suggested from wrong answers through directing students' thinking. Another observation we have made is that mathematics teachers rarely encourage justification of individual students' responses, however giving correct answers does not always imply students' understanding of concepts. Usually there is emphasis by teachers on reinforcement through positive comments and the common clapping of hands for a correct response which to some extent does not reflect or lead to effective and deep engagement with concepts during problem solving.

The manner in which a mathematics teacher treats learner's responses has important implications for subsequent learning. There is often lack of deep engagement with learners' responses with more emphasis on correct answers. However, constructive ideas can be generated from wrong answers. Further, pupils are rarely asked to explain and justify correct answers to sharpen their justification skills and promote reasoning which boosts direct benefit to individuals' cognitive abilities essential for mathematics learning. However, learning to justify solutions, whether wrong or correct will lead to deep learning according Manilla and Wallin [3]. Questions can thus be raised on the levels of intimacy and integrity exhibited by teachers as they engage with learners. That is, questions on how mathematics teachers handle learners' responses through probing or even directing student's thinking as well as providing insightful questions and encouraging comments that evidently promote an environment conducive for effective authentic problem solving. Effective learning can only take place when students are able to discover connections embedded in mathematical knowledge. Hence, this study seeks to assess the level of intimacy and integrity exhibited by secondary mathematics teachers when they handle leaners' responses during problem solving by addressing these research questions: i) How do mathematics teachers handle learners' responses during problem solving? ii) To what extent do mathematics teachers foster justification skills among learners?

We argue that a teacher's mathematical intimacy and level of mathematical integrity play a significant role in the promotion of students' procedural knowledge and conceptual understanding in problem solving. So, the focus of our discussion is on how these teacher qualities contribute to effective problem solving in the context of some of the existing problem-solving schemes. The most versatile and widely used schemes for problem solving are those formulated by Polya [4], Carlson and Bloom [5] and this study was influenced by ideas drawn from these problem solving schemes that we will now discuss.

Problem solving, as a domain of mathematical activity concerns the student's engagement on any mathematical task that is not judged procedural or the student does not have an initial overall idea on how to proceed in solving the task. Polya [4] enunciates four phases of problem solving; entry phase where the problem solver is called upon to understand the problem, designing a solution plan, implementing the plan and reflecting on one's solution attempt. Barb and Quinn [6] comment that Polya's problem solving model is a linear progression and emphasizes the importance of establishing how the various items are connected, that is, how the unknown is linked to the given information or data, in order to obtain the idea of the solution so as to make a plan. Kilpatrick [7] posits that the process of making mathematical connections allows students to discern learning strategies that can be used to evaluate how learners justify their answers in order to get a sense of their of line of reasoning.

Similar to Polya's problem solving scheme the multidimensional problem solving framework described by Carlson and Bloom [5] has four phases, each with the same four associated problem-solving attributes. The four phases are orienting, planning, executing, and checking whilst the four associated problem-solving attributes are resources, heuristics, affect and monitoring. Carlson and Bloom [5] state that mathematicians rarely solve a problem by working through it in linear fashion. Experienced problem solvers typically cycle through the plan-execute-check cycle several times when attempting one problem. Effective problem solvers express beliefs that doing mathematics requires persistent pursuit of a solution, as the solution process may require many attempts. Carlson and Bloom [5] elaborate that those problems that involve mathematical reasoning are enjoyable when mathematical ideas are understood instead of just memorised. Thus, learning mathematics requires sorting out information on one's own as a natural part of the problem-solving process. They also state that this cycle had an explicit execution and reflecting that are usually explicated in writing.

Debellis [8] describes mathematical intimacy as the deep, emotional engagement an individual may have with mathematics. Therefore, a teacher with mathematical intimacy displays a deeply-rooted emotional engagement for the purpose of developing mathematical meaning. Goldin [9] describes mathematical intimacy as consisting of a series of interactions and relationships. Intimacy will thus be revealed through the teachers' discussions of how they feel about mathematics. This can be described as the 'love' and 'excitement' and, on occasion, even 'pleasure', that arise when doing mathematics.

Mathematical integrity is defined by Seifert *et al.* [10] as an individual's fundamental commitment to mathematical truth, search for mathematical understanding, or moral character guiding mathematical study. According to Gomez-Chacon [11] mathematical integrity describes an individual's affective psychological posture in relation to when mathematics is 'right,' when a problem solution is satisfactory, when the learner's understanding suffices, or when mathematical achievement deserves respect or commendation. Thus, mathematical integrity involves a teacher's commitment to 'truth' and students' understanding. Mathematical integrity is therefore associated with a teacher's desire to foster justification skills among students. We can conjecture that, a teacher with a strong mathematical integrity has the potential to engage learners in deeper learning processes and problem-solving experiences.

2. RESEARCH METHOD

2.1. Research design

A qualitative design is suited for this study as it concerns meanings and in-depth descriptions of processes in explaining teacher practices during lesson delivery. Qualitative research is empathetic, striving to capture phenomena as experienced by the research participants themselves as suggested by Creswell [12]. Hillebrand and Berg [13] posit that qualitative research involves developing meanings, concepts, definitions, characteristics, from participants in a natural setting. Hence, according to Patton [14] a naturalistic approach that seeks to understand phenomena in context-specific settings, such as real-world settings was employed in this study. Furthermore, consistent with Patton's suggestion we took cognisance of the value-laden nature of our investigation that had a strong bearing on our relationship with the participants, as well as the situational constraints that shaped the inquiry.

Consequently, a case study method was used. According to Yin [15], a case study provides some of the most useful methods available in educational research due to its flexibility and adaptability to a range of contexts, processes, people, and foci. Case studies are differentiated from other types of qualitative research in that they are intensive descriptions and analyses of a single unit or bounded system, such as an individual, program, event, group, intervention or community. Additionally, Johnson and Stake [16] notes that, "It is the case we are trying to understand". With a case study, we were therefore able to explore and understand the underlying processes that characterise for instance, in individual teacher practices as they interact with learners. In other words, a case study enabled us to develop a deep understanding of the ways teachers handled students' responses as well as gaining insights into the manner in which teachers attempted to encourage students to justify decisions and actions taken during problem solving.

2.2. Population and sampling

From the population of 15 teachers available in the three schools a sample of six mathematics teachers, two from each school were purposively selected. Creswell [12] asserts that 'the logic and power of purposive sampling lies in selecting participants from whom in-depth information can be elicited". Hence, we believed that the six ordinary level mathematics teachers involved in the study had different mathematical backgrounds and experiences due to their varying professional qualifications and experiences. Hence, our rationale for selecting six teacher informants was that they had an average teaching experience at secondary level of more than five years. Besides relevant teaching experience, all participants were holders of a degree or diploma in education with a major in mathematics.

2.3. Data collection procedures

For each of the six ordinary level mathematics teachers, a lesson observation was conducted and audio taped. Video recordings were made of the teachers' dialogues with their students so as to get insights as to how the teacher handled learners' responses and promoted justification amongst learners. The first author of this article observed how the teachers probed, directed, and redirected students. The lesson observation guide also captured teachers' questioning and handling of learners correct or incorrect answers as learners explained their thinking. Video recordings allowed us to note non-verbal expressions and actions which can be directly observed such as nodding of the head and facial expressions. Further, field notes were written on: classroom activities, chalkboard presentations by learners and the teacher as indicators of mathematical intimacy and integrity.

After each class observation an interview with each of the six teachers was conducted to substantiate preliminary data analysis. Hence, the interview guide was informed by teachers' interactions with learners during instruction for the interview. During the interviews, the teachers were asked to explain their reasons for handling learners' responses in the way they did. The interviews were also audio-taped. For instance, a teacher could be asked to explain why he/she had a learner should discard the solution $x = -2$ when solving an equation involving $log\ x$.

Furthermore, informed by Molefe and Brodie [17] the interviews were semi-structured to allow us to understand what the participants' thoughts with respect to how they handled learners' responses, why they handled students' responses in the manner they did. On completion of data collection, we transcribed the data from observations and interviews. Video recordings enabled us to produce accurate transcriptions as we could play videos of lessons several times.

2.4. Ethical considerations

We employed the principle of informed consent. In this respect we assured the informants that participation in the study was voluntary and at that they could withdraw from the study at any time as suggested by Creswell [12]. An informed consent form was then signed by each teacher informant. Along with Berg [18] mathematics teachers involved in this study were assured of anonymity of their identities. In other words, participants were assured that lesson video recordings were solely for study purposes and that privacy and confidentiality of such videos would be strictly observed by the researchers. In this respect, we explained to the participants that we would use pseudonyms to report our research findings. The intent was to safeguard the professional and social standing teachers involved in the study.

2.5. Data analysis

Data analysis is a systematic search for meaning, through what Cohen *et al*. [19] describe as means of organising and interrogating data in ways that allow researchers to see patterns, identify themes, discover relationships, develop explanations, make interpretations, mount critiques, or generate theories. Along with Hatch [20] our first attempt was to develop a list of indicators of the two notions of integrity and intimacy as teachers handle learners' responses during the learning process. This systematic way of organising and gaining meaningful parts of data as it relates to the research question is called coding according to Creswell [12]. Our coding process evolved through an inductive analysis and was a cyclical process. We describe the coding process as being a cyclical process because it involved going back and forth between transcripts until we were satisfied with the final themes–the so called construct of theoretical saturation by Corbin [21]. We later on engaged in interpretation of these codes by comparing theme frequencies, identifying and displaying relationships between different themes. A coding framework for each of the participant' transcript was devised. Our aim was to attempt to go beyond surface meanings of the data to make sense of the data and tell an accurate story of what the data illuminated.

Along with Punch [22] we applied analytic induction to the textual data which involves scanning the data for categories of levels of mathematical intimacy and integrity among teacher informants. After scanning, we had to find relationships among the categories. We had to discern common practices across the lessons and interviews. Some categories were informed by the literature while others emerged from the data. We described and attempted to explain emerging themes from the teachers' lesson observation and interview data. Similarities as well as differences observed in the six teachers' practices were then noted and in-vivo codes were used to support our inferences from data as suggested by Braun and Clarke [23].

3. RESULTS AND DISCUSSION

In this section we present our research results per research question raised. In our presentation of results, we used pseudonyms to illustrate how participants involved in this study handled learners' responses in lessons observed and promote justification skills among learners. First, we present research findings pertaining to research question one: How do mathematics teachers handle learners' responses during problem solving?

The intent of raising such a question was to explore the levels of mathematics teachers' intimacy as they interacted with students during problem solving. Creswell [12] suggests the need to make sense of data and organizing it into categories or themes that cut across all sources building patterns, categories and themes from bottom up (inductive analysis). Along with Creswell [12], we developed themes from data from lesson observations and interviewing of the mathematics teachers. These themes served as indicators of the levels of mathematical intimacy displayed by teachers as they handled students' responses. Hence, the emerging categories as informed by existing literature and inductive categories that emerged from thematic analysis of qualitative data are now presented.

3.1. Emerging theme 1 on nature of engagement with pupils

All six teachers engaged pupils in class discussion through the development of procedural aspects of the solution. Lesson presentations generally comprised of a series of steps to get solutions, thus characterised by the absence of revealing picture of the embedded relationships within concepts. The level of engagement however differed according to the teacher and the respective students. Instruction was mostly characterised by teacher dominance of the development process and activity. It was only in one instance that Mr. Chimombe encouraged a student to present solution on the chalkboard. Another teacher Mrs. Bope, while she made an attempt to urge students to discuss problems assigned in pair, she did not move around to check, observe or listen to students' interactions. Similarly, Mrs. Chimuriwo moved around observing a few individuals' working without commenting and she seemed to rush through the lesson.

With regards to question distribution techniques, it was a common feature in lessons observed that it was mostly the same articulate students who raised up their hands first that were called upon to make a contribution. However, Pennant [24] suggests that when teaching using problem solving the teacher should call on students to think through, discuss and share and compare different ideas. The following were disturbing scenarios in lessons observed. Mrs. Mucheche and Mr. Chimombe allowed chorus answers and these teachers rarely gave attention to individual student's effort and progress. Furthermore, these teachers did not encourage students to talk about the problem and to restate it in their own words. In one incident involving Mr. Dzinouya, we observed that when a student had given a wrong response, he could shift his attention to another student without highlighting the inadequacy of the solution suggested by the student. On a positive note, Mr. Tasara encouraged class discussions which were coupled with positive comments to encourage students to participate and build on learners' confidence. Positive comments were also common in Mrs. Chimuriwo and Mrs. Bope's lessons.

Interview data revealed that Mrs. Bope just praised students who provide correct answers through use of words and phrases such as 'good', 'well-done', 'let's clap hands for him or her'. Similar comments apply to teacher, Mrs. Mucheche who pointed out that "if a response is correct, I praise,' and Mr. Dzinouya pointed the need to encourage with a 'yes', 'correct', 'good', or 'that's fine'. While such comments can help to motivate and instil confidence among students, they do not serve any significant purpose in explicating students' thoughts about mathematical ideas. An interview exchange with Mr. Tasara revealed the need to give real-life contexts to help the students to reduce the undesirable effects of procedural or conceptual flaws.

3.2. Emerging theme 2 on nature of probing by mathematics teachers

Mrs. Bope and Mrs. Mucheche did not ask students to clarify responses or explain incomplete responses and they did not pose follow-up questions. For instance, Mrs. Mucheche who was revising a test in which students had performed badly was supposed to use follow-up questions to clear students' misconceptions. Mr. Chimombe asked simple recall questions which did not require pupils to delve deeper in order to make sense of any conceptual relations. Mr. Tasara rephrased the questions and in some instances made some emphasis in an attempt to gain pupils' attention and encouraged them to participate. Mr. Dzinouya focused on a particular individual learner in an attempt to allow him/her to realize the wrong part of his/her answer before re-directing questions to the next pupil.

Probing of students during classroom activities demands good listening skills in order to direct the teacher on what to probe and how to probe. An interview exchange with Mr. Tasara revealed that listening to pupils is an essential aspect involved in handling students' responses because "in mathematics if a teacher fails to listen to students' feedback, he or she will not be able to deduce or determine whether they have mastered or internalised the communicated aspect". Hence, there is need for teachers to pay sufficient attention to feedback from students because it is from the responses that teacher can decide on what and how to probe to clarify missing links not captured. Just listening to pupils' feedback motivates them, because it will register in their minds that they are worthy contributors and their effort is being recognised as noted by Aarnes and Knudtzon [25]. Mr Dzinouya's interview data indicated that listening to learners provides an opportunity for students to be motivated as they become aware that the teacher pays attention to their progress, problems and needs.

3.3. Emerging theme 3 on asking insightful questions

Overall, teachers asked questions that were limited in terms of engaging students' low cognitive skills because the teachers asked simple, closed questions. For example, Mrs. Mucheche raised low order questions that required students to state the denominator and recall a mathematical procedure.

"What is the common denominator?"

Kazemi and Stipek [26] distinguish between high press and low press questions. High press questions encourage learners to include mathematical arguments in their explanations, while low press questions encourage procedural descriptions only. High press questions can create opportunities for learners to work cooperatively or in collaboration with each other, as they will be forced to share their thinking in preparation of convincing either the teacher or their fellow learners. Overall, low press questions such as the case of Mrs. Mucheche illustrated were dominant in lessons observed.

Mr. Tasara used strategies such as repeating key elements of the question as well as rephrasing questions in order to simplify and clarify question demands. Further, he also developed part questions from main questions so that learers could easily connect. Efforts to build mathematical connections were evident from lesson introductions. However, in their efforts teachers failed to give students opportunities to make sense of the concepts in question so as to make a plan and in turn test it. Teachers, however, considerably showed some significant intimacy.

3.4. Emerging theme 4 on encouraging learning from mistake

Only two out six made an attempt to encourage students to learn from their mistakes. For example, Mr. Dzinouya gave a student who had given a wrong response opportunity to correct own mistake before redirecting to the student. In Mr. Dzinouya and Mr. Tasara's lessons, students were urged to meditate their solution attempts and reflect also on a colleague's wrong answer. On the contrary, Mr. Chimombe disqualified wrong and incomplete responses without explanation and in extreme instances he would even dismiss wrong answers without comment. Similarly, Mrs. Mucheche displayed some impatience with learners as she took no time to understand why pupils had a common recurring misconception. Further, she could often interject before a student could finish presenting a response, which even implies she did not give much attention to students' feedback. Her focus was on a few outstanding individuals who repeatedly gave correct responses. The way a teacher handles learners' wrong responses can have adverse effects on individual's confidence and problem solving in mathematics. In particular wrong answers can be used as a springboard to new understanding. Therefore, low levels of mathematical intimacy were a key characteristic of lessons observed.

3.5. Emerging theme 5 on teacher focuses on learners' interactions

Mr. Chimombe did not at all move around to check on the students' progress as they were attempting the task assigned. In stark contrast, Mrs. Bope valued chalkboard presentations by students, sharing of ideas by the whole class and even in some instances signalled the whole class to pay attention and later on to comment on other students' solution attempts. However, Mrs. Bope imposed a time restriction of just two minutes on individual chalkboard demonstrations. Mrs Bope at some instance responded to a student's presentation by telling the whole class what mistake the particular student had made without first asking the students to identify the mistake. Mr Tasara listened to the students' answers and then would help the student to adjust and improve the response in order to make sense of mathematical ideas involved. In addition, Mr. Tasara encouraged students to raise questions. Mr. Dzinouya gave pupils opportunities to discuss and moved around checking on the students' efforts.

Overall, instruction generally was characterised by insignificant attempts to promote argumentation. Very few 'why', and 'how', questions were posed. Attempts to encourage students to justify their solutions in order to get the deeper meaning behind the response which in turn could reflect individual pupil understanding were rare. Students' behaviour revealed that the students were not used to being asked the 'why' questions as they failed to satisfactorily respond, some hesitation was noted. In these instances, teachers would intervene to explain upon releasing that the students got stuck.

The way teachers handle learners' responses through probing as means of directing student's thinking is evidence of teacher's level of mathematical intimacy during problem solving. A teacher concerned with students' mathematical reasoning and explanation skills should employ probing techniques. Further, consistent with Rubie-Davies [27] the teacher informants reported that during problem solving students could learn from mistakes. The concept of mathematical intimacy was used here to evaluate individual teachers' reasoning and actions as they engage students in problem solving. Most teachers displayed probing qualities, directed student's thinking and asked insightful questions, in some instances rephrasing them in order for allow tasks to be within the conceptual reach of students. Though interview data revealed that teachers were aware of the importance of using answers to shape students' understanding, lesson observed showed that teachers rarely, encouraged learning from mistakes, and few attempts were directed at promoting argumentation about mathematics ideas embedded in a given task.

Second, we present findings from research 2: To what extent do mathematics teachers foster justification skills among learners? The research goal was to determine aspects of mathematical integrity displayed by mathematics teachers during lesson delivery. Similarly, thematic analysis of field notes from: observations and interview transcriptions gave rise to categories now present:

3.5.1. Category 1: Mathematical integrity with respect to lesson planning and delivery

Mr. Chimuriwo had lesson notes that were written on the chalkboard and students were given time to note down for reference. The lesson began by a recap of content covered previously and lesson objectives were then highlighted. Timing of lesson activities by teacher informants was appropriate except for Mrs.

Mucheche, who failed to revise all test items as scheduled. With regards to lesson delivery, lesson observations revealed that there was limited individual teacher attempts to evaluate students' problem-solving abilities. Further, the teachers showed limitations in their capacity to clear some misconceptions. For example, Mr. Chimuriwo failed to guide students to explain concepts and definitions in their own words, but rather expected students to regurgitate definitions he had written on the chalkboard. Similar observations were made in Mrs. Bope and Mrs. Mucheche's lessons in which students were never given an opportunity to explain in own words the concepts involved in tasks assigned.

3.5.2. Category 2 on mathematical integrity: evidence of strong mastery of subject

All the teachers are qualified mathematics teachers with the least being a holder of Diploma in Mathematics Education with more than five years teaching experience. The teachers displayed strong mastery of subject as their explanations were rich indicating familiarisation with their mathematical knowledge. Quality of instruction however differed from class to class according to teacher. Mr. Chimuriwo indicated the need to intervene when a pupil gets stuck so that he or she can move along with others avoiding discouraging students. Such intervention requires sound mathematics grounding by the teacher—a trait Mr. Chmuriwo manifested.

3.5.3. Category 3 on guiding and fostering justification skills during problem solving

Following Polya's problem solving model it was anticipated that teachers could guide students devise a problem-solving plan. On the contrary with all the lessons observed the students were not guided into making solution plans. While lessons observed revealed that (four out of six) teachers did not make attempts to foster justification skills among the students, interview data revealed that they had a good understanding of the importance of promoting argumentation among the students. An excerpt by Mrs. Bope illuminated the importance of explaining one's solution attempt.

> *"Teachers just teach the front of it not teaching the back, so if a pupil is able to answer the 'how' and the 'why' he or she understands the back of knowledge which is very important"*

Glass and Maher [28] assert that justification and sense making is more than confirming whether a solution is correct or not, it allows students to gain insight into ways of thinking about problems and may then attain higher order thinking around mathematics concepts involved. This finding was similar to Stols *et al*.' [29] finding that enhancing students' justification skills promotes a deep understanding of concepts and relations involved. Hence, processes engaged in during problem solving should be justified so that those who might have doubts will have their doubts cleared. Swan [30] supports the idea of encouraging students to explain their reasoning. Swan [30] posits that getting students to explain themselves creates opportunities for discussions. Hence, according to Pentag *et al*. [31], students involved in discussions of the underlying concepts of the problem at hand can develop a deep understanding of subject matter.

Furthermore, questions intended to provoke higher order thinking among the students were rarely posed. Following Rasiman *et al*. [32], we noted that in this case that mathematics was not used as a vehicle for developing creativity among learners, creative reasoning n turn allows learners to build up arguments autonomously. We note that learning affordances that embrace the argumentation discourse can lead to improved mathematics learning.

4. CONCLUSION

With respect to research question 1, this study uncovered inconsistencies in levels of mathematical intimacy between results from lesson observations and those from interview data. For instance, while interview transcriptions revealed the need to allow students to learn from their mistakes, analysis from lesson observation showed that teachers rarely capitalized on wrong answers to promote learning of mathematics concepts but rather teachers were more disposed to offer comments about correct responses than they would for wrong responses. In addition, lesson delivery focused on procedural fluency rather than conceptual understanding emphasised by problem solving models. Furthermore, probing and directing was mainly done through low order questions, and in many instances, teachers repeated questions for emphasis. From the sentiments expressed by the participants we concluded that students were not given sufficient time to understand questions posed, make an alternative plan that promotes reflection on the problem-solving process as suggested by Polya's framework. With reference to research question 2 this study has also revealed mismatches between theory and practice. While interview narrations showed that teachers valued the need to promote justification in lesson delivery, analysis of lesson observations indicated that to a very limited extent the teachers managed to direct student's investigations as well as guiding students to explain some concepts and definitions in their own words. In particular students were not asked to explain correct answers in order to check their understanding. Rather emphasis was on "praising" the students with the goal of motivating them. This finding is in stark contrast with the current thinking in mathematics education that a justifying habit is a productive way of thinking that becomes common in the processes of mathematical inquiry and sense making.

Basing on the findings of this study, first we recommend that teachers and curriculum planners explore means to contain mismatches between theory and practice. In particular teachers need to foster argumentation among students by posing higher order (high press) questions to inculcate thinking as opposed to merely passing praising comments. In addition, the classroom socio-mathematical norms could include the enculturation of teachers into habits creating knowledge from wrong students' responses. Second, the current study has uncovered a propensity by teachers to promote procedural knowledge ahead of conceptual knowledge. We therefore recommend that instructors capitalise on key themes that emerged from this study to promote a balance between students' conceptual and instrumental knowledge. Further, inconsistencies between findings from interview data and those from lesson observations can be attributed to time for lesson delivery. Finally, we suggest that further research into factors that account for mismatches between lessons observed and interview narrations by teacher informants would be useful.

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