Against Second-Order Primitivism

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> The quantifiers ‘$(\exists x)$’ and ‘(x)’ mean ‘there is some entity x such that’ and ‘each entity x is such that’ (Quine 1953/1999b, 102)

The language of first-order logic has served analytic philosophy well. The language has a clear recursive syntax and semantics. It is powerful enough to formulate a wide variety of novel theories. The consequences of these theories are determined by a complete proof theory. Paraphrase of other natural language sentences into the language of first-order logic has aided in resolving ambiguity and clarifying entailments. The language can also be used to expose the fact that distinct syntactic categories of natural language have a similar semantic function. One source of the appeal of first-order logic is that it can be regimented with a single style of explicit variable and fully general quantifiers. This enables the first-order logician to capture the truth conditions of the various quantificational claims of natural language. If the first-order variables are fully general—if they are absolutely unrestricted, then one can frame statements and questions about things without knowing very much about them. For instance, one may ask whether *something* causes the expansion of the universe without making presuppositions about what *kind* of thing this would be.

Rather than being a mere device of mechanical calculation, the sentences of first-order logic are comprehensible. They constitute a well-understood, unambiguous fragment of natural language (with a few “opportunistic departures” such as variables and parentheses for ease of identifying cross-reference and structure). A formula of first-order logic such as $\exists x(Person(x) \wedge Swims(x))$ is readily pronounceable by a clunky, but comprehensible, sentence of (nearly) natural language such as ‘there is something$_x$ such that it_x is a person and it_x swims’.

Yet, a recent trend—prominently advocated by Timothy Williamson among others—pulls away from first-order logic and towards higher-order logics. Williamson argues for the necessity of theorizing in a higher-order language from two premises. One premise is that some first-order quantifiers are unrestricted. Very roughly: for anything there is, it is something. The other premise is that it is possible to generalize over the interpretation of every possible predicate of the language of first-order logic. The truth of a sentence such as 'Annabel swims' requires (roughly) that the interpretation of 'swims' applies to referent of 'Annabel'. This interpretation applies to an object if and only if the object swims. Williamson shows that many first-order formulations of these two assumptions are inconsistent. They lead to a version of Russell's paradox.

Williamson avoids the paradox by denying that predicate interpretations are values of first-order variables. The predicate interpretations are to be specified in the language of second-order logic. This language includes a collection of second-order variables syntactically marked with an adicity. Second-order variables specify the interpretations of first-order predicates. The syntactic positions these second-order variables can occupy are restricted so that their values cannot be said to be identical to the values of first-order variables. *Prima facie*, this move violates the idea that some first-order quantifiers are absolutely general. There are things—the predicate interpretations or the values of the second-order variables—that are not identical to anything.

This paper will examine strategies to make this position coherent. Williamson adopts a strategy that follows Frege (1951) in distinguishing first- and second-order quantification syntactically. Claims that would violate the unrestricted generality of first-order quantification are not syntactically well-formed. The grammar of the language prevents one from asking whether a given predicate interpretation is identical to a value of a first-order variable.

> The distinction [between first- and second-order quantification] must remain one of grammar and not of ontology, because one cannot use first-level and second-level expressions in the same grammatical context to articulate an ontological distinction without violating constraints of well-formedness. (Williamson 2003, 458)

Thus, in the language of second-order logic, one cannot state that the values of the second-order variables are things. The letter of unrestricted generality has been preserved.

I argue that this strategy is insufficient. Williamson himself concedes that whether the second-order quantifiers are restricted first-order quantifiers turns on how the former are *understood*. If they are understood as making claims about sets or about properties and relations, then they are actually restricted first-order quantifiers. So, I will argue that the reconciliation of the principle that first-order quantifiers are absolutely general and the claim that predicates have interpretations cannot rely on purely syntactic features of the language of second-order logic.

The other strategy is broadly semantic or ontological. It follows an historically influential interpretation of Russell's type theory and has been recently defended by Jones and Florio (2021).[1] On this view, there is no purely grammatical obstacle to substituting first- and second-order variables. The obstacle is semantic. Some predicates apply only to the values of first-order variables and others only to the values of second-order variables. Substituting a second-order variable for a first-order variable in a sentence results in semantic anomaly. The anomalous sentence either lacks a truth-value or fails to express a proposition. I will argue that this account too turns second-order quantifiers into restricted first-order quantifiers.

If the first-order quantifier is absolutely unrestricted, it would appear that second-order variables are restricted first-order variables. A remaining question is: how should the first-order logicians reply to the charge that they cannot frame the wanted generalizations over all interpretations of all possible predicates? I argue that these generalizations are not required to frame the notions of logical truth and consequence for first-order languages.

1 Unrestricted Generality

The thesis that some first-order quantifiers are unrestricted arises out of the development of first-order logic. The language of first-order logic is a lightly regimented fragment of natural language. Its vocabulary includes: n-ary predicates, terms (constants and variables), connectives ($\wedge$ and $\neg$), and quantifiers ($\forall$ and $\exists$). An n-ary predicate π preceding a sequence of terms $t_1,\ldots,t_n$ forms an atomic sentence $\pi t_1 \ldots t_n$. Complex sentences include the conjunction $\phi \wedge \psi$ and negation $\neg\phi$ of any sentences ϕ and ψ.

Semantically, a term refers to an object (relative to an assignment). An n-ary predicate applies to the sequences of n objects that satisfy some condition. An atomic sentence is true (relative to an assignment) if the predicate applies to the sequence consisting of the referents of the terms (under the assignment). A sentence Sa consisting of a monadic predicate S and a term a is true if and only if the predicate S applies to the referent of a. Suppose that the sentence Sa regiments the English sentence 'Annabel swims'. The term a refers to Annabel, and the predicate S applies to an object if and only if the object swims. The sentence Sa is true if and only if Annabel swims. A conjunction $\phi \wedge \psi$ is true if its conjuncts ϕ and ψ are true. A negation $\neg\phi$ is true if the sentence it negates ϕ is not true.

The apparatus of generalization in first-order logic deploys quantifiers ($\forall$ and $\exists$) and variables ($x_1,x_2,\ldots$).

[1] There is considerable disagreement about the conception of type theory in (Whitehead and Russell 1910/1957), with interpreters such as Sainsbury (1979), Landini (1998), and Klement (2010) offering broadly substitutional interpretations. However, I take it to be relatively uncontroversial that Russell's earliest type theory in (Russell 1903/1996, Appendix B) is broadly realist.

An existential generalization $\exists v\phi$ is true relative to an assignment σ, if ϕ is true relative to an assignment differing from σ at most in its interpretation of v. A universal generalization $\forall v\phi$ is true relative to an assignment σ, if ϕ is true relative to every assignment differing from σ at most in its interpretation of v.

1.1 Natural Language Quantification

Unlike the language of first-order logic, quantifiers in English are restricted both overtly and covertly. In the sentence 'every cat escaped', the quantifier 'every cat' is overtly restricted by the noun 'cat'. The sentence is true if and only if the predicate 'escaped' applies to every cat rather than to everything in the universe. Most utterances of this sentence are also covertly restricted. A speaker might utter this sentence to say that every cat they own escaped rather than that every cat in the universe escaped.

English quantifiers are formed by combining a determiner with a noun phrase that expresses an overt restriction. These determiner phrases then combine with verbs to yield sentences. Thus (1a-d) all contain a quantifier phrase formed by a determiner and a noun phrase.

(1)
  a. Every person dies.
  b. Every person who owns a cat dies.
  c. Every natural number is divisible by 1.
  d. The square of some real number is 2.

A naive approach might formalize these claims so that the noun phrase restrictor in each sentence introduces its own distinctive sort of variable, as in (1′).

(1′)
  a. $\forall p\ Dies(p)$
  b. $\forall c\ Dies(c)$
  c. $\forall n\ Divides(1, n)$
  d. $\exists r\ r^2 = 2$

The variable types p, c, n, and r are restricted to persons, cat owners, natural numbers, and real numbers, respectively. In these sentences, I more lightly regiment the predicates as words of English (and infixed '=') rather than as letters and use parentheses to aid interpretation.

Williamson (2003, 451) offers an argument against having every noun phrase restrictor introduce its own variable type. Expressions of the form $\ulcorner$Every $\phi\urcorner$ are mastered by finite agents. One does not individually learn the quantifier expressions of natural language ‘every person’, ‘every person who owns a cat’, ‘every natural number’, ‘every real number’ and so on. Rather, if one can understand the expressions ‘every’ and ϕ, then one can understand ⌜Every ϕ⌝.

Carnap (1950/1988) sorted variables in a less naive way. Entities divide into broad categories (person, space-time point, natural number, real number, and so on). These broad categories each introduce their own variable type. Thus, ‘person’ may introduce its own distinct type of variable, but ‘person who owns a cat’ need not. The class of people who own cats is a mere *subclass* of the category of people. The cat owners are the people who own cats. The claim that every person dies will still be regimented as (1′a), but the claim that every person who owns a cat dies will be regimented as $\forall p(CatOwner(p) \rightarrow Dies(p))$. Thus, not every natural language restrictor will introduce its own variable type. Rather, variable types will correspond only with the most general categories.

Quine (1960, §32; 1976a, 208) defended the simplicity and “striking economy” of regimenting these various quantifiers with fully general quantifiers ($\forall x$ and $\exists x$) that bind a single style of variable. Quine thought that some noun phrases are not associated with any particular category. As a result, Quine would deny that there is a substantive difference between a category such as *person* and a subclass such as *person who owns a cat*. These are all subclasses of the more general category of *thing*. On this approach, (1a-d) would have their more familiar regimentations (1″a-d).

(1″) a. $\forall x\ (Person(x) \rightarrow Dies(x))$

b. $\forall x\ ((Person(x) \wedge CatOwner(x)) \rightarrow Dies(x))$

c. $\forall x\ (Natural(x) \rightarrow Divides(1, x))$

d. $\exists x\ (Real(x) \wedge x^2 = 2)$

At very least, the truth conditions of (1a-d) could be stated as (1″a-d), respectively.[2]

Williamson offers a similar objection to a particular version of the Carnapian strategy which associates categories with principles of individuation. Just as Quine would deny that every expression is non-trivially associated with a category, Williamson (2003, 451) argues that “natural languages contain many nouns that are not associated with any non-trivial principle of individuation, even as they are coherently used in a particular context (‘thing’, ‘object’, ‘item’, ‘entity’, ‘member’, ‘element’, ‘instance’, ‘example’, ‘topic’, ‘compliant’,...).” Because Williamson is writing in a context where categories are associated with principles

[2] Compare (Russell 1903/1996, §87).

of individuation, this suggests that he agrees that certain quantificational expressions of natural language are not associated with distinctive categories.

Theorizing with an unrestricted variable allows one to postulate the existence of entities without knowing the categories to which they belong. That is, one can say that there is something or other in the next room without knowing whether it is a person, inanimate object, or whatever. Similarly, one can postulate laws—such as the law that everything is self-identical—that govern all entities, whether people or numbers. In order to formulate a law that says nothing moves faster than the speed of light, one need not consider in advance the range of *categories* of entities that may be relevant to the generalization (cf. Williamson 2003, 236-9).

The benefits of the first-order regimentation with fully general quantifier and variables extend to other quantificational constructions in natural language. The adverb 'somewhere' in (2) quantifies over locations.

(2) Annabel swam somewhere.

Approaches broadly following Davidson (1967/2001) derive the truth conditions for this sentence using straightforward first-order quantification over locations, specified by (3a,b).

(3) a. $\exists l \exists e$ (Swimming(e) $\wedge$ Agent (e,a) $\wedge$ Destination(e,l))

b. There is a location and an event such that the event is a swimming, Annabel is the agent of the event, and the location is the destination of the event.

Thus, just as the reduction to the single style of variable of the first-order logic can explain how speakers learn the variety of determiner phrase quantifiers in natural language, it can also explain how speakers are able to understand adverbs of quantification. The motivations for an unrestricted first-order variable do not stop at the analysis of any particular syntactic category such as determiner phrases of natural language.

1.2 Formulating Unrestricted Generality

The truth conditions of quantified formulae have been gradually explained in terms of unrestricted quantificational claims such as ($1''$a-d) and (2). This suggests the thesis of unrestricted generality: everything is a value of the unrestricted, first-order variable. Williamson expresses this view by requisitioning the word "thing" which is not associated with any category or nontrivial principle of individuation.

> Whatever is is a thing. If there were any non-things, they too would be things: so there are no non-things. In any sense of 'exist' in which there are non-existents, they are things just as much

as existents are. ... Any value of a variable is a thing, and everything is the value of a variable under at least one assignment[.] (2003, 420)

The most natural way to interpret Williamson's claim, I will call Unrestricted Generality.

Unrestricted Generality: $\forall \alpha\, Thing(\alpha)$ (where α is any style of variable).

Unrestricted Generality says that the predicate 'thing' is universal.

The English determiner phrases 'everything' (or 'every thing') and 'something' (or 'some thing') are unrestricted because the noun 'thing' is unrestricted in its application. These quantifiers are regimented by the first-order quantifiers $\forall$ and $\exists$, respectively, preceding an unrestricted first-order variable (x_1,x_2,...). The first-order variables (x_1,x_2,...) range over all *things*. Thus, whatever is a *thing* is a value of a first-order variable. If α is a thing, then α is the value of a first-order variable. So whatever is true of α will be true of *something*.

If $\exists \alpha (Thing(\alpha) \wedge \phi_\alpha)$, then $\exists x \phi_x$.

Similarly, if a claim is true of everything, then it will be true of all *things*. That is:

If $\forall x \phi_x$ and $\forall \alpha Thing(\alpha)$, then $\forall \alpha \phi_\alpha$.

Using these principles, one can use unrestricted first-order variables to universally instantiate to or existentially generalize on *things*, no matter what style of variable is used to designate them.

Krämer (2017) characterizes unrestricted generality in terms of identity. Everything that exists is identical to something. Or alternatively, there is no entity which is distinct from all the things.[3]

Unrestricted Generality$_1$: $\neg \exists \alpha \forall x (\alpha \neq x)$

Unrestricted Generality$_2$: $\neg \exists \alpha \forall x (\neg \phi \wedge \phi[x/\alpha])$

I will focus on the thesis Unrestricted Generality, although what I will say will extend to the other theses.

[3]Krämer adds subscripts to the quantifier itself rather than deploying distinct styles of variables. But Williamson's (2003, 454; 2013, 237-8) own semantics distinguishes first- and second-order quantification not in terms of the quantifiers but in terms of the variables. Presumably, the reason for this is similar to the argument given above: finite speakers need not to learn a new quantifier for each style of variable. It suffices that they understand the variables.

1.3 The Interpretations of Predicates

Williamson combines Unrestricted Generality with the thesis that it is possible to generalize over predicate interpretations. For motivation, Williamson (2003, 426-7) appeals to Tarski's (1936/1956) definition of logical consequence as truth preservation under any interpretation of the predicates. For simplification, I switch to the notion of logical truth. A logical truth is a sentence that is true on any interpretation of its predicates. Modern accounts extend this definition to require truth on every interpretation of non-logical vocabulary, although I focus on predicates for present purposes. Thus, sentence (4) is a logical truth, because it is true no matter how F is interpreted.

(4) $\forall x(Fx \vee \neg Fx)$

Williamson argues that there must be an interpretation for each "contentful" predicate.

> [W]hen we apply the definition of logical consequence [or logical truth], it must be possible to interpret a predicate letter according to any contentful predicate, since otherwise we are not generalizing over all the contentful arguments of the right form. (Williamson 2003, 426)

If a sentence is a logical truth, then any proper, uniform substitution of its predicates is true. Thus, if (4) is a logical truth, then (5) is true.

(5) $\forall x(Swims(x) \vee \neg Swims(x))$

The truth of (5) would follow from the fact that (4) is true on every interpretation of F provided that there is an interpretation of F whose extension exactly coincided with the extension of $Swims$.

Williamson infers that each predicate P has an interpretation that coincides in extension with any "contentful" predicate F. That is, the following is true: $\ulcorner P \urcorner$ applies to x on interpretation i if and only if Fx. More formally:

Interpretation: $\exists i\ \forall x\ (Applies(\ulcorner P \urcorner, x, i) \leftrightarrow Fx)$.[4]

From Interpretation and Unrestricted Generality, Williamson is able to derive a contradiction analogous to Russell's paradox.

Williamson defines a predicate, which I will call G. G applies to x if and only if $\ulcorner P \urcorner$ fails to apply to x on interpretation x.

[4] Williamson makes use of a function $I(.)$ which takes an expression to its unique interpretation.

Def: $\forall x(Gx \leftrightarrow \neg Applies(\ulcorner P \urcorner, x, x))$

Williamson next plugs G into Interpretation.

$$\exists i \forall x(Applies(\ulcorner P \urcorner, x, i) \leftrightarrow Gx)$$

Replacing G by its definition and instantiating for the interpretation i yields:

$$\forall x(Applies(\ulcorner P \urcorner, x, i) \leftrightarrow \neg Applies(\ulcorner P \urcorner, x, x)).$$

Unrestricted Generality guarantees that x can be instantiated to i, delivering:

$$Applies(\ulcorner P \urcorner, i, i) \leftrightarrow \neg Applies(\ulcorner P \urcorner, i, i).$$

This is a contradiction. $\ulcorner P \urcorner$ applies to i on interpretation i if and only if $\ulcorner P \urcorner$ does not apply to i on interpretation i. It has been generated by the assumption Unrestricted Generality that first-order variables are absolutely unrestricted and Interpretation that every predicate has an interpretation on which it "applies" to everything in the extension of the predicate.

Crucially, I assumed that interpretations are things. That is, I assume $\forall i\ Thing(i)$. Unrestricted Generality allows one to instantiate the variable x to the interpretation i. Even if interpretations are specified using a distinctive style of variable (as I have done by using i for interpretations and x for an unrestricted first-order variable), Unrestricted Generality would still entail that interpretations are values of first-order variables.

The paradox bears an obvious resemblance to Russell's paradox. Russell's paradox derives from a naive principle of Comprehension, that for any open sentence ϕ with free variables $y_1, \ldots, y_n$, there is a property or relation (or related entity) that is instantiated by $y_1, \ldots, y_n$.

Comprehension: $\exists x \forall y_1 \ldots y_n(Instantiate^{n+1}(x, y_1 \ldots y_n) \leftrightarrow \phi)$.

Russell considered the open sentence $\neg Instantiate(y, y)$, saying that y is not instantiated by y. Substituting this open sentence for ϕ in Comprehension yields the claim that there is something that is instantiated by all and only the things that don't instantiate themselves.

$$\exists x \forall y(Instantiate(x, y) \leftrightarrow \neg Instantiate(y, y))$$

Instantiating the existential and then universal quantifier yields a contradiction:

$$Instantiate(x, x) \leftrightarrow \neg Instantiate(x, x).$$

As before, this derivation could be blocked if the specification of properties and relations demanded special variables whose values were outside of the range of first-order variables. That is, the derivation of the paradox could be blocked by denying UNRESTRICTED GENERALITY.

2 The Language of Second-Order Logic

To resolve the paradox, Williamson turns to the language of second-order logic and uses it to reject the formulation of INTERPRETATION. Every predicate has an interpretation, but the interpretation i is a relation between a predicate and its range of application.[5] The language of second-order logic extends the language of first-order logic by containing:

Variables superscripted by numbers: for each n: $x_1^n, x_2^n, \ldots$,

Atomic formulae of the form $v^n t_1 \ldots t_n$, where v^n is variable with superscript n and $t_1, \ldots, t_n$ are n terms, and

Quantified formulae of the forms $\exists v^n \phi$ and $\forall v^n \phi$, where v^n is a variable superscripted by n and ϕ is a formula.

This language displays two important features. First, the variables of second-order logic are sorted.

SORTED VARIABLES: There are distinct styles of variable in the language of second-order logic.

First-order variables lack a numerical superscript: x_1,x_2,...

For each natural number n, there is a type of second-order variable indicated by a superscript n: x_1^n,x_2^n,...

Thus, in addition to first-order variables, we have distinguished second-order variables.

The second feature is that first- and second-order variables cannot occur in the same positions in atomic sentences. I will call this feature STRATIFICATION, following Quine (1953/1999c). Quine applies his notion of stratification to individual formulae. Adjusted for the language of second-order logic, we can say that a formula of is stratified under the following condition.[6]

[5]This proposal developed an idea in (Boolos 1998b, 80). Williamson (2013, 237-8) offers a semantics with a direct interpretation of higher-order variables, developing ideas in (Linnebo and Rayo 2012).

[6](Quine 1953/1999c) offers a definition of stratification for set theory which I have adapted to monadic type theory. With some adjustments to the characterization, one could characterize standard polyadic type theory by positing a mapping to more complex types.

Stratification: A formula ϕ is stratified if it is possible to assign every variable in ϕ to a number $0, 1, 2, \ldots$ satisfying the following constraints.

> If a variable v is assigned 0, then ϕ has no atomic subformula of the form $vt_1, \ldots, t_n$ for any $n > 0$.
>
> If a variable v is assigned number $n > 0$ and occurs in atomic subformula ψ of ϕ, then ψ is of the form $vt_1 \ldots t_n$ and no t_i is identical to v.

In the language of second-order logic, the formation rules for atomic sentences are specified in terms of the different categories of variable. That is, the mapping is given by the index on the variables. This means that the language of second-order logic is stratified in a particularly strong way. The variables are not only stratified within a sentence, but also across sentences. If a variable v occurs in the initial position of any atomic formula $vt_1 \ldots t_n$ of the language, then it will only occur in atomic formulae of the form $vc_1 \ldots c_n$, where the c_i are terms and no c_i is identical to v. The thesis that a language has sorted variables and the thesis that the language is stratified are easy to run together. But, the stratification of variables in atomic sentences does not follow from the fact that variables are divided into different sorts (cf. Allaire 1960, 14; Pickel 2017, §4.3).[7]

Importantly, Williamson's attempt to reconcile unrestricted generality with the view that the values of the second-order variables are not among the values of the first-order variables appeals to the fact that variables are stratified and not merely that they are sorted. As we have seen, if the second-order variables were merely sorted but could occupy the same syntactic positions as first-order variables, then Unrestricted Generality entails that their values are things just like the values of first-order variables.

2.1 Interpretation and Comprehension

Williamson specifies interpretations using second-order variables. Interpretation* replaces Interpretation.

> Interpretation: $\exists i\ \forall x\ Applies(\ulcorner P \urcorner, x, i) \leftrightarrow Fx$.
>
> Interpretation*: $\exists i^2\ \forall x(i^2(\ulcorner P \urcorner, x) \leftrightarrow Fx)$

Roughly speaking, interpretations are relations between predicates and what they apply to.

[7] Nor for that matter, does the thesis that the variables are sorted in a formula or language follow from the fact that they are stratified.

Adopting Interpretation* in place of Interpretation blocks the derivation of the paradox above. Williamson (2003, 454) argues that a predicate analogous to G above can no longer be constructed. Recall that G applies to x if and only if $\ulcorner P \urcorner$ does not apply to x on interpretation x. That is, $\forall x(Gx \leftrightarrow \neg Applies(\ulcorner P \urcorner, x, x))$. In this definition, the variable x occupies the place of both an interpretation and an object the interpretation applies to. In the formulation of Interpretation*, a variable for an interpretation i^2 occurs only in the initial position of an atomic formula and followed by exactly two terms, as in $i^2(t_1, t_2)$. The objects to which an interpretation applies are specified by variables that occur only in non-initial positions of atomic formulae. Thus $i^2(\ulcorner P \urcorner, x)$ says roughly that i^2 relates predicate $\ulcorner P \urcorner$ to x.

To construct a predicate analogous to G, a single variable would have to occupy both the initial position in an atomic sub-formula and a non-initial position. For example, one might attempt to construct a corresponding predicate G^*.

(6) $\quad \forall x(G^*x \leftrightarrow \neg x(\ulcorner P \urcorner, x))$

But (6) is not a sentence of the language of second-order logic. The variable x occurs in the initial position and non-initial position of the atomic subformula $\neg x(\ulcorner P \urcorner, x)$. Thus, the predicate G^* cannot be defined and substituted into Interpretation*.

The principle Interpretation* follows from a more general comprehension principle for second-order logic. Unlike Comprehension which led to Russell's paradox, Second-Order Comprehension specifies "properties" or "relations" using second-order variables.[8]

Second-Order Comprehension: $\exists x^n \forall y_1 \dots y_n(x^n y_1 \dots y_n \leftrightarrow \phi)$.

Comprehension leads to a paradox when the problematic open sentence $\neg Instantiate(x, x)$ is substituted for ϕ. In the language of second-order logic, the analogous open sentence $\neg xx$ is not well-formed because the same variable occupies both initial and subsequent position of a subformula. The open formula violates Stratification. Thus, the syntactic features that allow Williamson to block the derivation of a paradox from Interpreation* also enable him to block the derivation of a paradox from Second-Order Comprehension.

2.2 Second-Order Logic and Unrestricted Generality

Using the language of second-order logic, one can affirm the existence of an interpretation for each predicate without allowing the derivation of the contradiction above. Does this approach deliver unrestricted

[8]See (Shapiro 1991, 66-7).

GENERALITY? Recall that unrestricted generality was formulated as the thesis that everything is a thing. That is, for a variable α of any type, the following holds: $\forall\alpha\, Thing(\alpha)$.

One might worry at this point that UNRESTRICTED GENERALITY is in fact violated. First of all, we simply do not have (7).

(7) $\forall x^1\, Thing(x^1)$.

Second, suppose that Annabel swims is regimented as Sa. Consider the fact that (8) follows trivially from SECOND-ORDER COMPREHENSION.

(8) $\exists x^1 x^1 a$

This sentence might be glossed as saying that Annabel has some property. If everything—no matter what sort of variable use to specify it—is a thing, then one might be tempted to infer that this property, the thing that Annabel is, is also a thing. That is, one might be tempted to infer (9).

(9) $\exists x^1(x^1 a \wedge Thing(x^1))$

If (9) is true, then the values of the second-order variables are among the things ranged over by the first-order variables. But this result threatens to reintroduce the paradox.

Once again, the language of second-order logic does not contain (7) or (9). They are not well-formed because the variables of the language of second-order logic are *stratified*. If a variable occurs preceding n-terms in an atomic formula, then it can never occur in non-initial position in an atomic formula. Since (7) and (9) are not sentences, they cannot be affirmed or denied. Thus, the untruth of (7) and (9) does not give rise to a counterexample to UNRESTRICTED GENERALITY.

Williamson (2003, 458) formulates this as the thesis that first- and second-order quantification are *incommensurable* because they bind into different syntactic positions.

> ...quantification into predicate position is simply incommensurable with quantification into name position; the former presents no coherent threat to the absolute generality of the latter

So first- and second-order quantification are incommensurable because a single variable cannot occupy the initial position in one atomic sentence ("predicate position") and a non-initial position in another atomic sentence ("name position"). Thus, Williamson is appealing to the fact that the formulae of second-order logic are stratified to argue that the language is compatible with UNRESTRICTED GENERALITY. By this, I mean that Williamson's reconciliation relies on the fact that first- and second-order variables have different and incompatible distributions in atomic sentences. STRATIFICATION is a characterization of this distribution.

3 Syntactic Distribution

Williamson is clear that the distinction between "name position" and "predicate position" is a matter of syntax. As we saw, he says that the distinction between first- and second-order quantification "must remain one of grammar" (Williamson 2003, 458). So Williamson appeals to the different syntactic distributions of first- and second-order variables in atomic sentences in order both to block the derivation of the paradoxes and to maintain Unrestricted Generality.

Yet, Williamson himself concedes that this syntactic difference is not sufficient. In particular, standard proposals for pronouncing second-order quantifiers—as 'some set', 'some way', and so on—make them seem like restricted first-order quantifiers. Thus, the formula (8) might be pronounced using the English sentences (10a-c):

(8) $\exists x^1\ x^1 a$

(10) a. There is a property such that Annabel has it.

b. There is a set such that Annabel belongs to it.

c. There is a way Annabel is.

These English sentences correspond to first-order formulae asserting the existence of sets, properties, or ways such as $\exists x(Property(x) \wedge Has(a, x))$. Thus it would appear that the variables of second-order logic on this understanding are mere restrictions on the first-order variables, as Williamson agrees.

> the philosophical discussion of higher-order quantifiers . . . treated them as ranging over properties and relations. But 'property' and 'relation' are ordinary nouns. Quantifiers such as 'some property' and 'every relation' are simply restricted first-order quantifiers over a restricted domain. (Williamson 2013, 239)[9]

Williamson accepts that if second-order formulae regiment talk of properties, sets, or ways, then second-order variables are merely first-order variables ranging over a restricted domain.[10]

This fact raises two problems for Williamson.

[9]cf. (Burgess 2005, 212) and (Boolos 1998c).

[10]Prior (1971, 37) and more recently Rayo and Yablo (2001) appeal to adverbs of quantification such as 'somehow' to interpret higher-order languages. However, I agree with Williamson (2013, 258, footnote 98) that a sentence such as (11) containing an adverb of quantification is "uncomfortably close" to a sentence explicitly quantifying over ways such as (12).

(11) Annabel hurt him somehow.

(12) There is some way in which Annabel hurt him.

More generally, the broadly Davidsonian account of (11) in terms of (12) allows theoretical unification.

PROBLEM 1: It is natural to interpret the sentences of second-order logic along the lines of (10a-c). From this interpretation, it follows that second-order variables are merely first-order variables over a restricted domain. The problem is to explain how to interpret the formulae of second-order logic without lapsing into one of these natural interpretations.

PROBLEM 2: Whether or not the language of second-order logic is read as regimenting natural language constructions such as (10a-c) is not a syntactic matter. Therefore, the syntactic features of this language alone fail to reconcile UNRESTRICTED GENERALITY with the thesis that every predicate has an interpretation.

I will first explore Williamson's response to PROBLEM 1 with an eye towards a solution to PROBLEM 2. In particular, Williamson considers alternative English pronunciations of the language of second-order logic. I will focus on two of these interpretations: second-order quantification as plural and as post-copular quantification. Williamson (2003, 457-8; 2013, 242-3) sometimes points to these interpretations as proof that PROBLEM 1 can be solved even if he does not believe the interpretations are ultimately satisfactory in the details. I will argue that these approaches offer no model for solving PROBLEM 2.

Williamson ultimately prefers a *primitivist* interpretation of the language of second-order logic. According to this interpretation, the sentences of this language have no natural language pronunciation. They are understood in use. Again, this interpretation solves PROBLEM 1 but not PROBLEM 2. We are still left in the dark about what difference in syntactic features renders first- and second-order quantification incommensurable. In later sections, I will examine further syntactic and semantic features that might be proposed to render first- and second-order quantification incommensurable.

3.1 Plural quantification

Boolos (1998d) suggested an interpretation of (monadic) second-order logic in terms of plural constructions. The (monadic) variables of the language of second-order logic ($x^1_1, x^1_2, \ldots$) would be pronounced as English plural pronouns 'they', 'them', and 'their' depending on context. An existential quantifier followed by a monadic second-order variable $\exists x^1$ would be pronounced 'there are some things such that...'. The result of concatenating a (monadic) second-level variable x^1 and a term t would be pronounced $\ulcorner t$ is one of them$\urcorner$. Thus, a sentence such as (8) $\exists x^1\ x^1 a$ would be pronounced as 'There are some things such that Annabel is one of them'. This proposal solves PROBLEM 1 because second-order quantified claims are pronounced in natural language differently from first-order quantified claims.

But plural approaches fail to render first- and second-order quantifiers incommensurable. In particular, singular and plural expressions may occupy the same syntactic positions. Compare, for example, 'a wall surrounded the city' and 'some walls surrounded the city'.[11] Similarly, the pronunciation scheme would seem to offer a natural pronunciation for the sentence (9) $\exists x^1(x^1 a \wedge Thing(x^1))$.

(13) There are some things such that Annabel is among them and they are a thing.

The subsentence 'they are/are not a thing' is grammatical in English just as the sentence 'they are/are not a family' is grammatical.

If the advocates of plural interpretations of the language of second-order logic wish to deny that (9) or (13) is true, they should hold that some things are not a thing, $\exists x^1 \neg Thing(x^1)$. On this interpretation, plural interpretations of second-order logic deny Unrestricted Generality: $\forall\alpha\ Thing(\alpha)$. But this interpretation renders first- and plural second-order quantification commensurable. First- and second-order variables have different values just as the variables n and r might have different values because n is reserved for natural numbers and r for real numbers.

Williamson (2003, 456; cf. Williamson 2013, 243) himself criticizes the plural interpretation of the language of second-order logic on the grounds that "plurals are not predicative". This suggests that his worry is that they can occupy the positions associated with singular determiner phrases. Essentially, this means that they do not lend support to Stratification.[12]

3.2 Post-Copular Quantification

The other strategy for interpreting the language of second-order logic—associated with Prior (1971)—appeals to post-copular quantification. Post-copular quantification occurs when quantifier phrases such as 'something', 'everything', or 'many things' follow a copula, as in (14a-b).

(14) a. Mary is something that Susie is not.

b. Mary is everything that Susie is not.

In these sentences the quantifier expressions 'something', 'everything', or 'many things' do genuinely seem to occupy a syntactic position inaccessible to many other determiner phrases. This is perhaps why Williamson approvingly remarks, "the degree of nominalization involved seems slight" (2013, 255).

[11] Similar issues issues are discussed at length in (Oliver and Smiley 2016, chapter 4) and (Florio and Linnebo 2021, chapter 6). The limited point that I am making here is that the interpretation of the second-order variables as plurals does not in itself support stratification.

[12] The alternative is to say that plural quantifiers are unrestricted and singular first-order quantifiers are merely a restriction of the plural variables. See (Rayo 2002, §11) and (McKay 2006, pp. 13-20).

But, as Moltmann (2013, 101-2) argues, the fact that the quantifiers can bind into positions that are unusual for determiner phrases does not mean that these same expressions cannot also bind into more familiar positions.[13] Consider the following variants on Moltmann's examples.

(15) (a) Mary is something that is admirable.

(b) [Something$_t$ [t is admirable]] [Mary is t]

(16) (a) Mary is something that distresses John.

(b) [Something$_t$ [t distresses John]] [Mary is t]

Sentence (15) requires that 'something' binds the trace in 't is admirable'. Similarly, sentence (16) requires that 'something' binds the trace in 't distresses John'. These are both familiar positions for determiner phrases. So these constructions, again, do not support STRATIFICATION and, therefore, do not provide a solution to PROBLEM 2.

Lastly, it is worth pointing out that slight variants of (14a-b) suggest that the post-copular quantifiers decompose into a quantifier and the predicate 'thing'.

(17) Mary is many admirable things.

(18) Mary is many things that Susie is not.

This suggests that post-copular quantifiers range over *things* just as do first-order quantifiers. Generally, there cannot be many admirable things that Mary is, unless they are *things*.

3.3 Primitivism

Rather than endorsing alternative natural language pronunciations of the language of second-order logic, Williamson (2003; 2013) proposes that the language of second-order logic may still be comprehensible even if sentences of the language of second-order logic have no natural language rendering.

> Perhaps no reading in a natural language of quantification into predicate position is wholly satisfactory. If so, that does not show that something is wrong with quantification into predicate position, for it may reflect an expressive inadequacy in natural languages. (Williamson 2003, 459)

[13]Compare (Moltmann 2003) which develops a different semantics of nominalization.

According to this *primitivism*, the language of second-order logic may be understood through immersion. This process is justified by the need to reconcile the tension between the UNRESTRICTED GENERALITY and the existence of an interpretation for every predicate.

4 Extensions of the Language

While primitivism may respond to PROBLEM 1, it does nothing to address PROBLEM 2. What syntactic feature renders first- and second-order quantifiers incommensurable? To answer this question, we must further investigate why first- and second-order quantifiers would be commensurable on the assumption that second-order quantifiers regiment English claims about properties, sets, or ways.

If second-order quantifiers are read as 'for all/some properties/sets/ways', then the sub-language of English consisting of the fragment regimented by the language of second-order logic can easily be extended into a language that allows sentences that can be regimented as (9) $\exists x^1(x^1 a \wedge Thing(x^1))$. Even if (9) happens to be ungrammatical in the language of second-order logic (or the fragment of English that expresses second-order formulae), the language can easily be expanded without change in meaning to allow this sentence.[14] Even though the sentence 'there is a property that Annabel has and it is a thing' is not included in the fragment (sub-language) of English that is regimented by the language of second-order logic, the sentence is clearly still grammatical in the full language of English.[15]

This suggests one strategy that Williamson could offer to defend the incommensurability of first- and second-order quantification. Williamson could claim that the language of second-order logic could not be *easily* or *naturally* expanded to include sentences such as (9) $\exists x^1(x^1 a \wedge Thing(x^1))$. I will argue that there is no principled syntactic reason that the language could not be easily or naturally expanded to include sentences such as (9). This is analogous to Magidor's (2009b) claim that English could be naturally expanded to allow for so-called "type confusions" such as 'runs eats'. Magidor (2009b, 2) had argued that English could be *naturally* expanded to allow strings such as 'runs eats' to be grammatical and even truth-evaluable.

I will not be arguing for anything so strong as Magidor. In particular, I will concede that one could not *naturally* expand ordinary language to allow strings of predicates such as 'runs eats' to form grammatical or truth-evaluable sentences. Since the predicates of first-order logic regiment natural language predicates,

[14] Compare (Jones 2016, 136, footnote 11) and (Trueman 2015, 1891) who argue that purely syntactic restrictions on first-order predicates (and presumably second-order variables) would be arbitrary.

[15] This interpretation makes sense of one of Quine's (1960, 268, footnote 2) argument against type theory. Quine complains that ruling unstratified formulae as ungrammatical preserves COMPREHENSION (and thus the principle that every predicate has an interpretation) in letter, but not in spirit. The approach merely "excises" certain open sentences that should be allowed. If the sentences of the language of second-order logic are understood as expressing sentences from an arbitrarily restricted fragment of English, then this charge seems appropriate.

I will also concede that one could also not naturally expand the language of first-order logic to allow for a formula such as $Thing(S)$ where S is a predicate. Either the predicate S or the predicate $Thing$ would have a different grammatical profile or meaning as it occurs in this new sentence.

Instead, I will argue for the weaker thesis that these restrictions do not extend to the variables of second-order logic. That is, I will argue that for the artificial language of second-order logic, there is no natural reason that the language could not be expanded to include an unstratified formula such as (9) $\exists x^1(x^1a \wedge Thing(x^1))$. At any rate, the characterization of the language of second-order logic offers no principle to limit such an expansion.

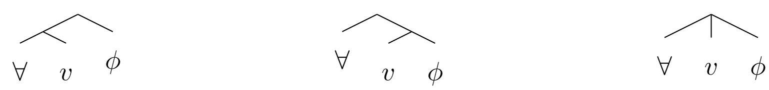
4.1 Variables as Predicates

Given that I concede that the language of first-order logic could not naturally be expanded to allow predicates into the position of terms, it would be natural for Williamson to argue that the variables of second-order logic are relevantly like predicates of first-order logic. As a result, the second-order variables also cannot be placed following a first-order predicate such as '$Thing$'. The proposal is that second-order variables are relevantly like the predicates of the language of first-order logic. Indeed, Williamson repeatedly emphasizes the claim that the variables of second-order logic are *predicative*. They are relevantly like the predicates of the language of first-order logic and unlike the terms: constants and variables.

But in what respect are first-order predicates and second-order variables relevantly alike? One respect in which they are alike is that both predicates and second-order variables occur in the initial positions of atomic sentences. It might be tempting to conclude from this that predicates of the language of first-order logic and second-order variables are playing the same syntactic function. But this does not follow without auxiliary assumptions. The fact that two expressions occur in the same ordinal position in the surface structure of corresponding sentences does not, in itself, show that the sentences were generated by a single syntactic construction rule. Thus the fact that the predicate S (regimenting the verb 'swims') is followed by the term a (regimenting 'Annabel') in Sa and the fact that the second-order variable x^1 is also followed by the term a in x^1a do not entail that the predicate and the second-order variable are of the same syntactic category or that they perform the same syntactic function in these two sentences.

With the help of certain auxiliary premises, one could argue that the expressions must be of the same syntactic category. In a very strict categorial grammar such as (Adjukiewicz 1967), the syntactic type of an expression is constrained by the syntactic types of the expressions surrounding it to form a sentence. On this view, since both Sa and x^1a are sentences (of syntactic type S) that combine an expression S or x^1 with a term (of type N) it follows that S and x^1 are of the same syntactic type. Each is typed to take a term and yield a sentence, or (S/N). Strict categorial grammars of this kind have fallen out of favor in the analysis of the syntax of natural language, but Williamson might argue that they are appropriate for a logical language.

Once one looks beyond atomic sentences, one sees that first-order predicates and second-order variables are unalike in an important respect. In particular, a quantified formula is formed by combining a sentence ϕ, a quantifier $\forall$, and a variable v of any sort. I suppose that quantified formulae have one of the following structures.



Regardless of which structure is chosen, we have a difference between predicates and second-order variables. A variable can combine with quantifiers to form a quantified sentence, a predicate cannot. On the other hand, first- and second-order variables do occupy corresponding positions in quantified formulae. If an expression's relative position in a formula strictly dictates its syntactic type, then we should conclude that first- and second-order variables are syntactically alike. So, while second-order variables and predicates are alike in their ordinal position in atomic sentences, second-order variables and first-order variables are alike in their position in quantified formulae.

To be fully clear, I am suggesting that the syntactic construction and semantic evaluation rules governing constructions of the form $v^n t_1 \dots t_n$ may be different from the corresponding rules governing constructions of the form $\pi t_1 \dots t_n$, where v^n is a second-order variable, π is a predicate of the language of first-order logic, and t_i is a term for each i.[16] The syntactic construction rules are different because the categories of variable and predicate are different.[17] These latter categories are different because second-order variables, but not predicates, may combine with a quantifier and an open sentence to form a quantified sentence. Of course, once one has introduced separate syntactic construction rules for sentences of the form $\pi t_1 \dots t_n$ and $v^n t_1 \dots t_n$, one can specify these sentences disjunctively: $\gamma t_1 \dots t_n$ is a sentence if γ is *either* a predicate *or* n marked variable.[18]

The semantic evaluation rule for constructions of the form $v^n t_1 \dots t_n$ might then be specified as follows:

$$v^n t_1 \dots t_n \text{ is true on assignment } \sigma \text{ if and only if } Instantiate^{n+1}(\sigma(v^n), \sigma(t_1), \dots, \sigma(t_n)).$$

[16] Compare (Menzel This volume, §1.4).

[17] There is a parallel issue in Kaplan's (1986, 234-235) rejection of Quine's argument that "the failure of substitution implies the incoherence of quantification".

[18] This disjunctively specified construction rule would be analogous to a construction rule specified using a type ambiguous notation in the language of higher-order logic.

In the statement of this evaluation rule, the variable v^n occurs as a restricted first-order variable, ranging over only n-ary relations. Once constructions of the form $v^n t_1 \ldots t_n$ are added to the language, their semantic clauses can even be given homophonically:

> $v^n t_1 \ldots t_n$ is true relative to assignment σ if and only if for all x^n, $y_1, \ldots, y_n$, if $\sigma(v^n) = x^n$, $\sigma(t_1) = y_1$,...,$\sigma(t_n) = y_n$, then $x^n(y_1, \ldots, y_n)$. (cf. Williamson 2013, 237-8)

Again, in this statement of the semantic evaluation rule, the variable x^n is a restricted first-order variable. It does not occur in the position of a predicate, because predicates and variables belong to different syntactic categories.

I have argued for the coherence of distinguishing predicates and second-order variables from the fact that first-order variables and second-order variables can occur in the same syntactic positions in quantificational constructions. This argument could be resisted if one posited distinct first- and second-order quantifiers in addition to distinct first- and second-order variables. But this approach would require language users to learn a distinct quantifier for each syntactic type. We saw that Williamson himself raised learnability objections against views of this type, according to which there are many restricted first-order quantifiers and no overarching semantics. Williamson (2013; 236-8) wants to give a uniform semantics for the quantifier. For this reason, Williamson's semantics makes use of a single style of quantifier with distinct styles of variable. As a consequence, first- and second-order variables are put in the same syntactic positions following a quantifier. So, if expressions that can occupy the same syntactic position are of the same syntactic type, then first- and second-order variables are of the same syntactic type.

It might be tempting to draw a stronger conclusion from the fact that first- and second-order variables can occur in the same position. In particular, it might be tempting to argue polemically that first- and second-order variables must have the same grammatical function since they can occur in the same grammatical position. This argument might be offered if one accepts the principle that if two expressions may substitute grammatically in one context, then they are of the same grammatical category and there is no principled reason they cannot substitute everywhere.

The inference to this stronger claim might seem problematic because first-order predicates and names have different syntactic distributions in the language of first-order logic. In particular, in the language of first-order logic sentences of the form $\exists x \phi$ are well-formed, but sentences of the form $\exists n \phi$ are not well-formed, where n is a name. Thus, the argument might be taken to show (problematically) that names and first-order variables belong to different syntactic categories.[19] While I am not offering the stronger argument, I do not

[19] Thanks to Peter Fritz and Nicholas Jones for raising the possibility of this response.

believe that the problem is decisive against it. I see no principled reason that the language could not be expanded to allow for sentences such as $\exists n\phi$, as Carnap (1934/1959, §54) once considered. More recently, the idea that names of natural language function as variables has been defended in a number of places.[20] But for the purposes of this paper, it is sufficient to claim that the fact that first- and second-order variables do not in fact occupy the same syntactic positions in the atomic formulae of the language of second-order logic does not reveal some underlying impossibility.

4.2 Variables as Verbs

In claiming that second-order variables and predicates may be treated differently, I oppose Quine (1986, 66-7) who says that understanding second-order quantified claims as assertions about properties, sets, or ways would lead us to misunderstand the nature of predicates. Suppose that α is an individual constant, v^1 is a second-order variable, and π is a predicate. Quine's idea seems to be that if we read constructions of the form $v^1\alpha$ as $\ulcorner\alpha$ has $v^1\urcorner$, then we are are required to read constructions of the form $\pi\alpha$ as $\ulcorner\alpha$ has the property of π-ing$\urcorner$. Thus, if we read v^1a as saying $\ulcorner$Annabel has (property) $v^1\urcorner$, then we should read Sa as saying 'Annabel has the property of swimming' rather than 'Annabel swims'.

However, the fact that first-order predicates and second-order variables both occur in atomic sentences followed by n constants does not entail that they make the same syntactic and semantic contribution. There is nothing improper about reading constructions of the form $v^1\alpha$ as $\ulcorner\alpha$ has $v^1\urcorner$ and constructions of the form $\pi\alpha$ as $\ulcorner\alpha$ πs$\urcorner$. Thus, v^1a is read as $\ulcorner$Annabel has (property) $v^1\urcorner$ and Sa as 'Annabel swims'. In other words, to allow quantifications such as $\exists v^1\ v^1\alpha$ is not to treat predicates as names unless the second-order variables are already construed as having the same syntactic status as predicates.

Quine's point might be bolstered by an argument from Williamson, who had suggested that a plural reading of v^1a as as $\ulcorner$Annabel is one of the $v^1\urcorner$ would "impose more structure than appears to be present in the object-language" (Williamson 2003, 456). Williamson would likely suggest that reading v^1a as $\ulcorner$Annabel has (property) $v^1\urcorner$ also imposes more structure on this sentence than appears to be present. The formal sentence juxtaposes only a term and a variable. The English sentence also contains the verb 'has'. But, this argument would be too quick. The regimentation of an English sentence will often not contain the full syntactic structure of the original. Regimentation should expose only the logical structure necessary.[21] To regiment 'Benjamin gave it to Annabel' as the atomic sentence $Gbxa$ does not require treating $Gbxa$

[20] The general idea that names of natural language should be modeled as variables has been articulated or defended by Yagisawa (1984), Dever (1998), Cumming (2008), Pickel (2015), and Schoubye (2020), among others. Pickel and Rabern (forthcoming) argue that this position should be read into Frege.

[21] Compare Quine 1960, §33.

as implicitly containing the preposition 'to'. Analogously, pronouncing $v^1\alpha$ as $\ulcorner\alpha$ has $v^1\urcorner$ does not require supposing that the sentence $v^1\alpha$ has hidden structure corresponding to 'has'.

One might argue, however, that if v^1 is an individual variable restricted to properties, then the sequence v^1a really does need additional syntactic structure to form a sentence. Sentences in most natural languages require some combination of noun-like expressions and verb-like expressions. As Plato has the stranger say in *Sophist* (262c):

> ...the simplest and smallest kind of speech, I suppose—would arise from that first weaving of name and verb together. (Cooper and Hutchinson 1997, 286)

In English, the strings 'lion stag horse' (*Sophist*, 262c) and 'the number 2 the concept prime number' (Frege 1951) are not sentences, in part, because they lack verbs. One might generalize this observation to suggest that verbs and nouns each play distinctive roles in the *syntactic* formation of a sentence and that the sentence must somehow designate syntactically which expressions play each of these roles. If v^1a is meant to be read aloud as an English sentence, then a string of first-order variables will lack a constituent that is pronounced as a verb. The verb is required to unite v^1 and a into a sentence.

But, even if it is agreed that there must be *something* to unite the expressions in a sentence, it does not follow that the unifier is itself an expression. The syntactic relation might itself do the work of uniting the expressions. This point has been made by (Anscombe 1971, 36-7; 101-2) and (Sellars 1962) in their respective interpretations of the *Tractatus*. Even if we assume that in natural language, the verb does the work of uniting other expressions into an atomic sentence, there is nothing that prevents the syntactic relation from uniting a variable v^1 of any type and a name a into the atomic formula v^1a in a formal language.

5 Semantic Anomaly

To review, Williamson agrees that if second-order quantified statements are read as making assertions about properties, relations, ways, or what have you, then second-order quantifiers are restricted first-order quantifiers. First- and second-order quantification would be commensurable in the sense that an unstratified sentence such as (9) $\exists x^1(x^1a \wedge Thing(x^1))$ could be added to the language of second-order logic without changing the meaning of any expressions. We would then face the difficult choice of either denying that the quantifier is unrestricted—by affirming $\exists x^1(x^1a \wedge \neg Thing(x^1))$—or restricting the principle that every predicate has an interpretation.

In §4, I rejected attempts to show that syntactic features of second-order variables prohibit this type of expansion of the language. In this section, I will examine the possibility that semantic considerations render first- and second-order quantification incommensurable. According to this view, there is no purely grammatical reason that the language lacks formulae violating type restrictions such as (9) and its constituent $Thing(x^1)$. However, there are semantic reasons that the language lacks sentences violating these type restrictions. In particular, Jones and Florio (2021) would argue that as the language of second-order logic is currently structured, this formula would not express a proposition or determine a truth-value.

Jones and Florio suggest that a predicate such as 'Swims' or 'Thing' has a range of significance, a notion they adopt from Russell (1908). A predicate determines a proposition (or truth-value) only when followed by a term (or terms) with the right kind of semantic value (at an assignment). The predicates 'Swims' and 'Thing' determine propositions when combined with the value of a first-order variable. Thus, the open sentences $Swims(x)$ and $Thing(x)$ express propositions relative to any assignment of values to the first-order variable x. However, the value of a second-order variable is outside of the range of significance for these predicates. Thus, $Swims(x^1)$ and $Thing(x^1)$ do not determine truth-values (or express propositions) relative to any assignment of values to x^1.

It will be useful to talk about the range of significance, not merely of a predicate, but of a *sentential context*. A sentential context is a pair of a sentence ϕ and a variable v that occurs in ϕ. Suppose a sentence ϕ contains a free variable v of any type. Then, an item d is in the range of significance of the sentential context defined by ϕ and v (relative to assignment σ) if there is some variable v' and assignment τ differing from σ at most in assigning v' to d such that the result of substituting v' for v in ϕ (or $\phi[v'/v]$) is true or is false (relative to τ).

The sentential context that the first-order variable x occupies in the open formulae x^1x does not include the values of second-order variables in its range of significance. Thus, x^1x^1 does not determine a proposition relative to any assignment of values to variables. Analogously, Jones and Florio will say that the initial position in an atomic sentence does not determine a proposition for the value of a first-order variable. Thus, xx will be anomalous for any value of x. Jones and Florio would argue that even if formulae violating type restrictions could be grammatically introduced into the language, they would not express propositions unless there was some change either in the meaning of the constituent expressions or in the semantic composition rules.

To preserve the Unrestricted Generality of first-order quantification, Jones and Florio argue that quantification is always restricted to the range of significance of the sentential context it binds into.

> [I]t is impossible for a predicate F to be true or false of things that cannot be meaningfully said to be F. So the domain of quantification of $\forall vF(v)$ must be included in the range of significance of F. (Jones and Florio 2021, 51)

Jones and Florio would therefore say that the quantified formula (9) $\exists x^1(x^1 a \wedge Thing(x^1))$ fails to determine a proposition for any (nonempty) domain of quantification because the embedded open sentence $x^1 a \wedge Thing(x^1)$ does not determine a proposition for any value of x^1.

The automatic restriction of quantified claims to the range of significance of a sentential context also gives Jones and Florio an account of UNRESTRICTED GENERALITY. If the domain of a quantifier $\forall x$ extends beyond the range of significance defined by formula ϕ and the variable x, then $\forall x\phi$ will not determine a proposition (or truth-value).

> When [domain] d extends beyond the range of significance of F, it contains something for which there is no such singular proposition. So $\forall vF(v)$ does not express a proposition when "interpreted" over d: meaningful quantification never goes beyond range of significance. (Jones and Florio 2021, 51, footnote 9)

The first-order quantifiers are absolutely general because they range over *everything in the range of significance* of the positions they bind into.

5.1 Semantic Anomaly and Restriction

Limitations on ranges of significance are difficult to constrain once introduced. That is, a range of significance is limited whenever a sentence containing a variable fails to express a proposition relative to an assignment. Entity d is outside of the range of the sentential context defined by $\phi(x)$ and x just in case for no variable v assigned to d does $\phi(v)$ express a proposition. The problem is that many semantic phenomena have been explained by appeal to semantic anomaly. Consider, for instance, the predicate 'uproot'. It is tempting to hold that this predicate can be meaningfully applied only to an object that has roots. Sentence (19a) determines a proposition, but sentence (19b) does not, because dogs lack roots.

(19) a. I uprooted the tree.

b. I uprooted my dog.

c. I uprooted everything in the garden. (Adapted from Shaw 2015.)

Thus, dogs are not in the range of significance of the sentential context defined by the open sentence 'I uprooted x' and the variable x. As a result, a quantifier binding into subject position, as in (19c), will be unrestricted even though it does not range over whatever dogs may be in the garden. This is the view of Shaw (2015), from whom I take example (19c). Shaw agrees with Jones and Florio that a quantifier is automatically restricted to the range of significance of the sentential context into which it binds.

Similarly, standard treatments of presuppositions make use of partial functions (cf. Heim and Kratzer 1998, §4.4). A construction of the form 'regrets that ϕ' does not yield a truth-value (or express a proposition) unless it is applied to a subject for which ϕ is true. Thus, 'regrets sleeping through the meeting' will not yield a proposition unless it is combined with a subject who did sleep through the meeting. If John did not sleep through the meeting, then (20a) is truth-valueless.

(20) a. John regrets sleeping through the meeting.

b. Everybody regrets sleeping through the meeting.

Applying the strategy of Shaw (2015) and Jones and Florio (2021), the domain of the quantifier 'everybody' in (20b) would include only people who in fact slept through the meeting. Thus, if John did not sleep through the meeting, he is not relevant to the truth-value of (20b).[22]

If the range of significance of a predicate is restricted to the entities that satisfy its presuppositions, then this undermines the claim that an occurrence of a quantifier is unrestricted provided that its domain includes all of the entities in the range of significance of the sentential context that it binds into. It is strange to say that the quantifiers in (19c) or (20b) are unrestricted. Indeed, Shaw (2015) himself describes his proposal as introducing a new kind of quantifier domain *restriction*. Shaw's position is challenged by Mankowitz (2019) who argues that the quantifier in (19b) need not be restricted to the objects that have roots and that the restriction—when in force—can be generated by standard mechanisms of covert quantifier domain restriction. Regardless of mechanism, it seems appropriate to describe the quantifiers in (19c) or (20b) on the intended interpretation as restricted. A quantifier whose domain excludes John on account of his not sleeping through the meeting or a dog for lacking roots is restricted.

5.2 Semantic Anomaly and Type Theory

One might respond by appealing to Magidor's (2009a; 2013) rejection of any semantic anomalies, if construed as grammatically well-formed declarative sentences that nevertheless fail to determine a truth-value or express

[22] The point here isn't that there is a unified notion of presupposition failure that should be explained by truth-value gap but that truth-value gaps (or failure to express a proposition) have proved useful for analyzing some forms of presupposition failure.

a proposition. Magidor suggests that the unacceptability of sentences such as (19b) and (20a) is due to a kind of presupposition failure, where having a false presupposition is compatible with having a truth-value. However, this strategy is unavailable to Jones and Florio. Crucially, their proposal allows that the relevant unstratified sentences may be syntactically well-formed. According to Jones and Florio, if the sentences were well-formed, they would lack truth-values or fail to express propositions. I can understand and sympathize with Magidor's position that there are no grammatical semantic anomalies at all. But, as Magidor argues, this should put pressure on type theory as well because type theory is usually introduced into compositional semantics precisely to account for the possibility of syntactically well-formed but semantically anomalous sentences (cf. Heim and Kratzer 1998, especially §3.3, but also §§3.4-3.5 and 4.4.4). More generally, I do not understand a position that permits well-formed sentences that fail to determine a truth-value or express a proposition but restricts these semantic anomalies to the result of a narrow range of type clashes (cf. Magidor 2009b).[23]

6 Interpretations and Completeness

Williamson theorizes in the language of second-order logic in order to reconcile the UNRESTRICTED GENERALITY of first-order variables variables and the principle that every predicate has an interpretation. The second-order approach introduces sorted variables and makes a claim about the distribution of these variables. Williamson argued that the syntactic features of the language of second-order logic rendered first- and second-order quantification incommensurable. I have argued that a language with these same syntactic features may be interpreted so that the second-order quantifiers are restricted first-order quantifiers. Thus, the syntactic features of the language of second-order logic are insufficient to render first- and second-order quantification incommensurable. I then rejected an alternative proposal that appeals to semantic features of the language.

A better response, I suggest, would be to restrict the comprehension schema and thereby deny that every predicate has an interpretation (cf. Bennet and Karlsson 2008). For instance, we could follow Quine in restricting ourselves to stratified instances of the comprehension schema. Or, we could appeal to more familiar approaches such as ZFC.[24]

[23]It is noteworthy that Russell's early invocation of the notion of a range of significance was expanded in precisely this way. Russell held that the range of significance of certain propositional functions was restricted to classes or pluralities of individuals. He also allowed that two propositional functions could have different ranges of significance but the same "range of truth" (Russell 1903/1996, §497). Thus, even though one propositional function implicitly restricts the range of any quantifier that binds it, the other does not. So the restriction can be stated explicitly.

[24]Menzel This volume also suggests that comprehension should be restricted in response to the paradoxes.

But this leaves a substantive issue still unsettled. Williamson argued that Tarski's account of logical truth and logical consequence entails that every predicate has an interpretation. In particular, we saw that if a formula such as (4) is a logical truth, then any proper, uniform substitution of its predicates is a logical truth. Thus if (4) is a logical truth, then so is (5).

(4) $\forall x(Fx \vee \neg Fx)$

(5) $\forall x(Swim(x) \vee \neg Swim(x))$

If every predicate is coextensive with some interpretation, then this entailment is easy to explain. The fact that (4) is a logical truth entails that it is true however one interprets the predicate F. And, one way of interpreting this predicate is as applying to just those things that satisfy 'Swims'.

Yet, we do not need to derive the truth of (5) from the fact that (4) is a logical truth by this reasoning. In particular, part of the interest of Tarski's definition of logical truth (and consequence) is that it supports a completeness theorem for the language of first-order logic: a sentence can be "proved" if it is true on all interpretations. Here, 'proof' refers to using the familiar methods of first-order logic. This is essentially Quine's response.

> The word 'proved' here may be taken as alluding to some method of proof appearing in the logic textbooks; the completeness theorem can be established for each of various sorts of methods. Some of these methods, moreover, are visibly sound, visibly such as to generate only schemata that come out true under all substitutions. (Quine 1986, 54)

If sentence (4) is true on every interpretation, then it follows from visibly sound axioms. But these axioms equally apply to any instance of (5). Instances of (5) will follow from visibly sound axioms. As a result, we don't need to suppose that every possible predicate has an interpretation to deliver a satisfactory account of logical truth or consequence.

7 Conclusions

This paper has evaluated the argument that it is proper to theorize in the language of second-order logic because this language allows one to affirm the existence of an interpretation corresponding to each predicate without sacrificing the absolute generality of the variables of first-order logic. Yet, paradox is avoided only because the values of the second-order variables are not among the values of the first-order variables. This might seem to be a violation of the absolute generality of the first-order quantifiers since we cannot affirm that the values of the second-order variables are among the values of the first-order quantifiers.

Williamson and others respond that there is no counterexample here because it is nonsense to either affirm or deny that the values of second-order variables are among the values of the first-order quantifiers. According to Williamson, such a claim would be nonsense because it is not grammatical in the language of second-order logic. Yet—as Williamson concedes—the language of second-order logic can be interpreted as speaking about classes or properties and relations. So, the actual syntactic distribution of expressions in the language does not save absolute generality.

Jones and Florio offer a different reason for supposing that the claim is nonsense. They agree that there is no principled grammatical prohibition against first- and second-order variables occupying the same positions. But any such sentence would be a semantic anomaly, claim Jones and Florio. The sentence would not express a proposition or have a truth-value, because the values of the second-order variables are not in the range of significance of first-order predicates. However, this view suggests that the first-order quantified claims are restricted rather than unrestricted. It therefore does not preserve the absolute generality of first-order logic.

From the perspective of this paper, the second-order logician is responding to legitimate questions with silence. The silence is followed by the suggestion that there are syntactic or semantic prohibitions barring one from asking the questions. On examination, we have found the prohibitions unmotivated. Of course, one may choose to be silent about difficult matters. But such silence does not usually lend itself to productive theorizing.

References

Adjukiewicz, Kazimierz 1967. 'Syntactic Connection'. In McCall and Grushman (1967), pp. 207–231.

Allaire, Edwin B. 1960. 'Types and Formation Rules: A Note on Tractatus 3.334'. *Analysis*, 21, 1: 14–16.

Anscombe, G.E.M. 1971. *An Introduction to Wittgenstein's Tractatus*. Philadelphia: University of Pennsylvania Press.

Bennet, Christian and Karlsson, Martin Filin 2008. 'Williamson's Barber'. *Analysis*, 68, 4: 320–326.

Boolos, George 1998a. *Logic, Logic, and Logic*. Cambridge, MA: Harvard University Press.

———1998b. 'Nominalist Platonism'. In Boolos (1998a), pp. 73–87.

———1998c. ‘On Second-Order Logic’. In Boolos (1998a), pp. 37–53.

———1998d. ‘To Be is To Be the Value of a Variable (Or to Be Some Values of Some Variables)’. In Boolos (1998a), pp. 54–73.

Burgess, John P. 2005. *Fixing Frege*. Princeton, NJ: Princeton University Press.

Carnap, Rudolph 1934/1959. *The Logical Syntax of Language*. Chicago, IL: University of Chicago Press.

———1947/1988. *Meaning and Necessity: A Study in Semantics and Modal Logic*. Midway reprints, Chicago, IL: University of Chicago Press.

———1950/1988. ‘Empiricism, Semantics, and Ontology’. In Carnap (1947/1988), pp. 205–221.

Cooper, John M. and Hutchinson, D. S. 1997. *Plato: Complete Works*. Indianapolis, IN: Hackett Publishing Co.

Cumming, Samuel 2008. ‘Variabilism’. *Philosophical Review*, 117, 4: 525–554.

Davidson, Donald 1967/2001. ‘The Logical Form of Action Sentences’. In Davidson (2001), pp. 105–149.

———2001. *Essays on Actions and Events*. Oxford: Clarendon Press.

Dever, Josh 1998. *Variables*. Ph.D. thesis, University of California, Berkeley.

Florio, Salvatore and Linnebo, Øystein 2021. *The Many and the One: A Philosophical Study of Plural Logic*. Oxford University Press.

Frege, Gottlob 1951. ‘On Concept and Object’. *Mind*, 60, 238: 168–180.

Heim, I. and Kratzer, A. 1998. *Semantics in Generative Grammar*. Malden: MA: Blackwell Publishers.

Jones, Nicholas K. 2016. ‘A Higher-Order Solution to the Problem of the Concept Horse’. *Ergo: An Open Access Journal of Philosophy*, 3.

Jones, Nicholas K. and Florio, Salvatore 2021. ‘Unrestricted Quantification and the Structure of Type Theory’. *Philosophy and Phenomenological Research*, 102, 1: 44–64.

Kaplan, David 1986. ‘Opacity’. In Hahn, Lewis Edwin and Schilpp, Paul Arthur (eds.) *The Philosophy of W. V. Quine*. Open Court, pp. 229–289.

Klement, Kevin 2010. 'The functions of Russell's No-Class Theory'. *The Review of Symbolic Logic*, 3, 04: 633–664.

Krämer, Stephan 2017. 'Everything, and Then Some'. *Mind*, 126, 502: 499–528.

Landini, G. 1998. *Russell's Hidden Substitutional Theory.* Oxford: Oxford University Press.

Linnebo, Øystein and Rayo, Agustin 2012. 'Hierarchies Ontological and Ideological'. *Mind*, 121, 482: 269–308.

Magidor, Ofra 2009a. 'Category Mistakes Are Meaningful'. *Linguistics and Philosophy*, 32, 6: 553–581.

———2009b. 'The Last Dogma of Type Confusions'. *Proceedings of the Aristotelian Society*, 109: 1–29.

———2013. *Category Mistakes.* Oxford University Press.

Mankowitz, Poppy 2019. 'Triggering Domain Restriction'. *Mind and Language*, 34, 5: 563–584.

McCall, S. and Grushman, B. 1967. *Polish Logic, 1920-1939.* Oxford: Oxford University Press.

McKay, Thomas 2006. *Plural Predication.* Oxford University Press.

Menzel, Christopher This volume. 'Pure Logic: First-order, Type-free, Hyperintensional'.

Moltmann, Friederike 2003. 'Nominalizing Quantifiers'. *Journal of Philosophical Logic*, 32: 4445–481.

———2013. *Abstract Objects and the Semantics of Natural Language.* Oxford University Press.

Oliver, Alex and Smiley, Timothy 2016. *Plural Logic: Second Edition, Revised and Enlarged.* Oxford University Press.

Pickel, Bryan 2015. 'Variables and Attitudes'. *Noûs*, 49, 2: 333–356.

———2017. 'Naming, Saying, and Structure'. *Noûs*, 51, 3: 594–616.

Pickel, Bryan and Rabern, Brian forthcoming. 'Against Fregean Quantification'. *Ergo.*

Prior, A. N. 1971. *Objects of Thought.* Oxford: Clarendon Press.

Quine, W. V. O. 1953/1999a. *From a Logical Point of View.* Cambridge, MA: Harvard University Press.

———1953/1999b. 'Logic and the Reification of Universals'. In Quine (1953/1999a), pp. 102–129.

———1953/1999c. 'New Foundations for Mathematical Logic'. In Quine (1953/1999a), pp. 80–101.

———1960. *Word and Object*. Cambridge, MA: The MIT Press.

———1976a. ‘Carnap’s Views on Ontology’. In Quine (1976b), pp. 203–211.

———1976b. *The Ways of Paradox and Other Essays*. Cambridge, MA: Harvard University Press.

———1986. *Philosophy of Logic*. Cambridge, MA: Harvard University Press.

Rayo, Agustín 2002. ‘Word and Objects’. *Noûs*, 36, 3: 436–464.

Rayo, Agustin and Yablo, Stephen 2001. ‘Nominalism Through de-Nominalization’. *Noûs*, 35, 1: 74–92.

Russell, Bertrand 1903/1996. *The Principles of Mathematics*. New York, NY: W. W. Norton & Company.

———1908. ‘Mathematical Logic as Based on the Theory of Types’. In Russell (1992), pp. 57–102.

———1992. *Logic and Knowledge*. London: Routledge.

Sainsbury, R. M. 1979. *Russell*. Routledge.

Schoubye, Anders J. 2020. ‘Names Are Variables’. *Philosophical Review*, 129, 1: 53–94.

Sellars, Wilfrid 1962. ‘Naming and Saying’. *Philosophy of Science*, 29, 1: 7–26.

Shapiro, Stewart 1991. *Foundations Without Foundationalism: A Case for Second-Order Logic*. Oxford: Oxford University Press.

Shaw, James R. 2015. ‘Anomaly and Quantification’. *Noûs*, 49, 1: 147–176.

Tarski, Alfred 1936/1956. ‘On The Concept of Logical Consequence’. *Logic, semantics, metamathematics*: 409–420.

Trueman, Robert 2015. ‘The Concept Horse with No Name’. *Philosophical Studies*, 172, 7: 1889–1906.

Whitehead, Alfred North and Russell, Bertrand 1910/1957. *Principia Mathematica*. Cambridge: Cambridge University Press.

Williamson, Timothy 2003. ‘Everything’. *Philosophical Perspectives*, 17, 1: 415–465.

———2013. *Modal Logic as Metaphysics*. Oxford: Oxford University Press.

Yagisawa, Takashi 1984. ‘Proper Names as Variables’. *Erkenntnis*, 21, 2: 195–208.