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References and Notes

9. Materials and methods are available as supporting material on Science Online.

Exact and Approximate Arithmetic in an Amazonian Indigene Group

Pierre Pica,1 Cathy Lemer,2 Véronique Izard,2 Stanislas Dehaene2*

Is calculation possible without language? Or is the human ability for arithmetical dependent on the language faculty? To clarify the relation between language and arithmetic, we studied numerical cognition in speakers of Munduruku, an Amazonian language with a very small lexicon of number words. Although the Munduruku lack words for numbers beyond 5, they are able to compare and add large approximate numbers that are far beyond their naming range. However, they fail in exact arithmetic with numbers larger than 4 or 5. Our results imply a distinction between a nonverbal system of number approximation and a language-based counting system for exact number and arithmetic.

All science requires mathematics. The knowledge of mathematical things is almost innate in us…. This is the easiest of sciences, a fact which is obvious in that no one’s brain rejects it; for laymen and people who are utterly illiterate know how to count and reckon.

Roger Bacon (1214–1294), English philosopher and scientist

Where does arithmetic come from? For some theorists, the origins of human competence in arithmetic lie in the recursive character of the language faculty (1). Chomsky, for instance, stated that “we might think of the human number faculty as essentially an ‘abstraction’ from human language, preserving the mechanisms of discrete infinity and eliminating the other special features of language” (2). Other theorists believe that language is not essential—that humans, like many animals, have a nonverbal “number sense” (3), an evolutionarily ancient capacity to process approximate numbers without symbols or language (4–6) that provides the conceptual foundation of arithmetic. A third class of theories, while acknowledging the existence of nonverbal representations of numbers, postulates that arithmetical competence is deeply transformed once children acquire a system of number symbols (7–9). Language would play an essential role in linking up the various nonverbal representations to create a concept of large exact number (10–12).

To elucidate the relations between language and arithmetic, it is necessary to study numerical competence in situations in which the language of numbers is either absent or reduced. In many animal species, as well as in young infants before they acquire number words, behavioral and neurophysiological experiments have revealed the rudiments of arithmetic (6, 13–16). Infants and animals appear to represent only the first three numbers exactly. Beyond this range, they can approximate “ numerosity,” with a fuzziness that increases linearly with the size of the numbers involved (Weber’s law). This finding and the results of other neuroimaging and neuropsychological experiments have yielded a tentative reconciliation of the above theories: Exact arithmetic would require language, whereas approximation would not (12, 17–21).

This conclusion, however, has been challenged by a few case studies of adult brain-lesioned or autistic patients in whom language dysfunction did not abolish exact arithmetic; such a finding suggests that in some rare cases, even complex calculation may be performed without words (22).

In the final analysis, the debate cannot be settled by studying people who are raised in a culture teeming with spoken and written symbols for numbers. What is needed is a language deprivation experiment, in which neurologically normal adults would be raised
reports

without number words or symbols. Although such an experiment is ethically impossible in our Western culture, some languages are intrinsically limited in their ability to express number, sometimes using a very narrow set of number words ("one, two, many") (23). These often endangered languages present a rare opportunity to establish the extent and limits of nonverbal arithmetic abilities.

Here, we studied numerical cognition in native speakers of Mundurukú, a language that has number words only for the numbers 1 through 5 (24, 25). Mundurukú is a language of the Tupi family, spoken by about 7000 people living in an autonomous territory in the Pará state of Brazil (Fig. 1). Following regular research stays since 1998, and two pilot

Fig. 1. Location of indigene territories of Brazil (top) and of the main Mundurukú territory where our research was conducted (bottom). Colored dots indicate the villages where participants were tested. The legend at bottom gives the sizes of the six groups of participants and their average age. [Maps adapted with permission from R. Beto, Ed., Povos indígenas no Brasil (Instituto Socioambiental, São Paulo, Brazil, 2000). pp. 161, 461].

studies in 2001 and 2002, one of us (P.P.) traveled through several villages during 2003 and was able to collect data from 55 speakers of Mundurukú in a computerized battery of numerical tests. Ten native speakers of French (mean age 50) served as controls.

The Mundurukú have some contact with nonindigenous culture and individuals, mainly through government institutions and missionaries. Thus, several of them speak some Portuguese, and a few, especially the children, receive some instruction in basic school topics (26). To evaluate the potential impact of these variables, we formed two groups of strictly monolingual adults and children without instruction, and we compared their performance with that of more bilingual and educated participants (Fig. 1). Using a solar-powered laptop computer, we collected a large amount of trials in classical arithmetical tasks, including a chronometric comparison test. This allowed us to test whether competence for numbers is present in the absence of a well-developed language for number.

A first task explored the verbal expressions for numbers in Mundurukú (26). Participants were presented with displays of 1 to 15 dots in randomized order, and were asked in their native language to say how many dots were present. This task permitted an objective analysis of the conditions of use of number words. No systematic variation across groups was identified, except for lack of use of the word for “5” in the younger children, and the results were therefore pooled across all groups (Fig. 2). The results confirm that Mundurukú has frozen expressions only for numbers 1 to 5. These expressions are long, often having as many syllables as the corresponding quantity. The words for 3 and 4 are polymorphemic: ebap = 2 + 1, ebadip = 2 + 1 + 1, where “eba” means “your (two) arms.” This possibly reflects an earlier base-2 system common in Tupi languages, but the system is not productive in Mundurukú (expressions such as “eba eba dip” or “eba eba ebapū” are not used and are judged meaningless).

Above 5, there was little consistency in language use, with no word or expression representing more than 30% of productions to a given target number. Participants relied on approximate quantifiers such as “some” (adesū), “many” (ade), or “a small quantity” (būrūmaku). They also used a broad variety of expressions varying in attempted precision, such as “more than one hand,” “two hands,” “some toes,” all the way up to long phrases such as “all the fingers of the hands and then some more” (in response to 13 dots).

The Mundurukú did not use their numerals in a counting sequence, nor to refer to precise quantities. They usually uttered a numeral without counting, although (if asked to do so) some of them could count very slowly and nonverbally by matching their fingers and toes to the set of dots. Our measures confirm that they selected their verbal response on the basis of an apprehension of approximate number rather than on an exact count. With the exception of the words for 1 and 2, all numerals were used in relation to a range of approximate quantities rather than to a precise number (Fig. 2). For instance, the word for 5, which can be translated as “one hand” or “a handful,” was used for 5 but also 6, 7, 8, or 9 dots. Conversely, when five dots were presented, the word for 5 was uttered on only 28% of trials, whereas the words for 4 and “few” were each used on about 15% of trials. This response pattern is comparable to the use of
round numbers in Western languages, for instance when we say “10 people” when there are actually 8 or 12. We also noted the occasional use of two-word constructions (e.g., “two-three seeds”), analogous to references to approximate quantities in Western languages (27). Thus, the Munduruku are different from us only in failing to count and in allowing approximate use of number words in the range 3 to 5, where Western numerals usually refer to precise quantities.

If the Munduruku have a sense of approximate number, they should succeed in approximation tasks with quantities beyond the range for which they have number words. If, however, concepts of numbers emerge only when number words are available, then the Munduruku would be expected to experience severe difficulties with large numbers. We tested this alternative with the use of two estimation tasks. First, we probed number comparison. Participants were presented with two sets of 20 to 80 dots, controlled for various severe difficulties with large numbers. We used a nonsymbolic version of the approximate addition task, which is thought to be independent of language in Western participants (12, 17, 18). Participants were presented with simple animations illustrating a physical addition of two large sets of dots into a can (Fig. 3B). They had to approximate the result and compare it to a third set. All groups of participants, including monolingual adults and children, performed considerably above chance (minimum 80.7% correct, P < 0.0001). Performance was again solely affected by distance (F_{3,152} = 78.2, P < 0.0001); there was no difference between groups, nor a group × distance interaction (31). If anything, performance was higher in this addition + comparison task than in the previous comparison task, perhaps because the operation was represented more concretely by object movement and occlusion. In brief, Munduruku participants had no difficulty in adding and comparing approximate numbers, with a precision identical to that of the French controls.

Finally, we investigated whether the Munduruku can manipulate exact numbers. The number sense view predicts that in the absence of spoken or written symbols, number can only be represented approximately, with an internal uncertainty that increases with number (Weber’s law). Beyond the range of 3 or 4, this system cannot reliably distinguish an exact number n from its successor n + 1. Thus, the Munduruku should fail with tasks that require manipulation of exact numbers such as “exactly six.” To assess this predicted limitation of Munduruku arithmetic, we used an exact subtraction task. Participants were asked to predict the outcome of a subtraction of a set of dots from an initial set comprising one to eight items (Fig. 3, C and D). The result was always small enough to be named, but the operands could be larger (e.g., 6–4). In the main experiment, for which we report statistics below, participants responded by pointing to the correct result among three alternatives (0, 1, or 2 objects left). The results were also replicated in a second version in which participants named the subtraction result aloud (Fig. 3D).

In both tasks, we observed a fast decrease of performance with the size of the initial number (F_{3,336} = 44.9, P < 0.0001). This decrease was significant in all Munduruku groups, although a significant group effect (F_{5,48} = 3.81, P = 0.005) and a marginal group × size interaction (F_{35,336} = 1.40, P = 0.07) indicated that performance was slightly better in the more bilingual and educated group, especially when fewer than five dots were present (see Fig. 3D). However, all Munduruku groups performed much worse

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**Fig. 2.** Number naming in Munduruku. Participants were shown sets of 1 to 15 dots in random order and were asked to name the quantity. For each quantity on the x axis, the graph shows the fraction of times that it was named with a given word or locution. We only present the data for words or locutions produced on more than 2.5% of all trials. For numbers above 5, frequencies do not add up to 100%, because many participants produced rare or idiosyncratic locutions or phrases such as “all of my toes” (a complete list is available from the authors).

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www.sciencemag.org | SCIENCE | VOL 306 | 15 OCTOBER 2004 | 501
A. Comparison
Indicate the larger set

B. Approximate addition and comparison
Indicate which is larger: n1+n2 or n3

C. Exact subtraction
Point to the result of n1- n2

D. Exact subtraction
Name the result of n1- n2

Fig. 3. Performance in four tasks of elementary arithmetic. In each case, the left column illustrates a sample trial (see movie S1). The graphs at right show the fraction of correct trials, in each group separately (M, monolinguals; B, bilinguals; NI, no instruction; I, instruction) as well as averaged across all the Munduruk and French participants (right graphs). The lowest level on the scale always corresponds to chance performance. For number comparison (A and B), the relevant variable that determines performance is the distance between the numbers, as measured by the ratio of the larger to the smaller number (e.g., n1/n2 if n1 > n2, n2/n1 otherwise). For exact subtraction (C and D), the relevant variable is the size of the initial number n1. The fits are based on mathematical equations described in (26).

than the French controls, in whom performance was only slightly affected by number size (F, 7,63 = 2.36, P < 0.033). Thus, we observed a highly significant effect of language group (French versus Munduruk, F,1,62 = 25.7, P < 0.0001) and a language × size interaction (F,7,434 = 6.80, P < 0.0001).

The Munduruk’s failure in exact subtraction was not due to misunderstanding of the instructions, because they performed better than chance (indeed, close to 100% correct) when the initial number was below 4. Success within this range might reflect exact verbal coding, or it might reflect a nonverbal parallel individuation of small sets, as also found in preverbal infants (13) and nonhuman primates (14). Performance also remained above chance for higher values of the initial number (e.g., 49.6% correct for $8 - n$ problems, chance = 33.3%, P < 0.0001). The entire performance curve over the range 1 to 8 could be fitted by a simple psychophysical equation that supposes an approximate Gaussian encoding of the initial and subtracted quantities, followed by subtraction of those internal magnitudes and classification of the fuzzy outcome into the required response categories (0, 1, or 2). Thus, the Munduruk still deployed approximate representations, subject to Weber’s law, in a task that the French controls easily resolved by exact calculation.

Together, our results shed some light on the issue of the relation between language and arithmetic. They suggest that a basic distinction must be introduced between approximate and exact mental representations of number, as also suggested by earlier behavioral and brain-imaging evidence (12, 18) and by recent research in another Amazon group, the Pirahë (23). With approximate quantities, the Munduruk do not behave qualitatively differently from the French controls. They can mentally represent very large numbers of up to 80 dots, far beyond their naming range, and do not confuse number with other variables such as size and density. They also spontaneously apply concepts of addition, subtraction,
and comparison to these approximate representations. This is true even for monolingual adults and young children who never learned any formal arithmetic. These data add to previous evidence that numerical approximation is a basic competence, independent of language, and available even to preverbal infants and many animal species (6, 13–16). We conclude that sophisticated numerical competence can be present in the absence of a well-developed lexicon of number words. This provides an important qualification of Gordon’s (23) version of Whorf’s hypothesis according to which the lexicon of number words drastically limits the ability to entertain abstract number concepts.

What the Mundurukú appear to lack, however, is a procedure for fast apprehension of exact numbers beyond 3 or 4. Our results thus support the hypothesis that language plays a special role in the emergence of exact arithmetic during child development (9–11). What is the mechanism for this developmental change? It is noteworthy that the Mundurukú have number names up to 5, and yet use them approximately in naming. Thus, the availability of number names, in itself, may not suffice to promote a mental representation of exact number. More crucial, perhaps, is that the Mundurukú do not have a counting routine. Although some have a rudimentary ability to count on their fingers, it is rarely used. By requiring an exact one-to-one pairing of objects with the sequence of numerals, counting may promote a conceptual integration of approximate number representations, discrete object representations, and the verbal code (10, 11). Around the age of 3, Western children exhibit an abrupt change in number processing as they suddenly realize that each count word refers to a precise quantity (10, 11).

Separate Neural Systems

Value Immediate and Delayed Monetary Rewards

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When humans are offered the choice between rewards available at different points in time, the relative values of the options are discounted according to their expected delays until delivery. Using functional magnetic resonance imaging, we examined the neural correlates of time discounting while subjects made a series of choices between monetary reward options that varied by delay to delivery. We demonstrate that two separate systems are involved in such decisions. Parts of the limbic system associated with the midbrain dopamine system, including paralimbic cortex, are preferentially activated by decisions involving immediately available rewards. In contrast, regions of the lateral prefrontal cortex and posterior parietal cortex are engaged uniformly by intertemporal choices irrespective of delay. Furthermore, the relative engagement of the two systems is directly associated with subjects’ choices, with greater relative fronto-parietal activity when subjects choose longer term options.

In Aesop’s classic fable, the ant and the grasshopper are used to illustrate two familiar, but disparate, approaches to human inter-temporal decision making. The grasshopper luxuriates during a warm summer day, indifferent to the future. The ant, in contrast,