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Theoretical Implications of the Study of Numbers and Numerals in Mundurucu

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Developing earlier studies of the system of numbers in Mundurucu, this paper argues that the Mundurucu numeral system is far more complex than usually assumed. The Mundurucu numeral system provides indirect but insightful arguments for a modular approach to numbers and numerals. It is argued that distinct components must be distinguished, such as a system of representation of numbers in the format of internal magnitudes, a system of representation for individuals and sets, and one-to-one correspondences between the numerosity expressed by the number and its metrics. It is shown that while many-number systems involve a compositionality of units, sets and sets composed of units, few-number languages, such as Mundurucu, do not have access to sets composed of units in the usual way. The nonconfigurational character of the Mundurucu language, which is related to a property for which we coin the term 'low compositionality power', accounts for this and explains the curious fact that Mundurucus make use of marked one-to-one correspondence strategies in order to overcome the limitations of the core system at the perceptual/motor interface of the language faculty. We develop an analysis of a particular construction, parallel numbers, which has not been studied before, elucidating the whole system. This analysis, we argue, sheds new light on classical philosophical, psychological and linguistic debates about numbers and numerals and their relation to language, and more particularly, sheds light on few-number languages.

Keywords: Approximation; Compositionality Power; Few-Number Language; Metrics; Mundurucu; Parallel Numbers; Symmetry; Weber law

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1. Introduction

The relationship between number and space has been an important topic in the history of mathematics, especially in ancient Greece, where a notion of RATIONAL NUMBER emerged from the necessity of counting, and the very same notion was used to measure space. The confrontation of numbers and measure is at the core of the crisis of the irrational numbers. Through the centuries, mathematics has divided into algebra and geometry. Descartes united both branches in his analytical geometry, but there nevertheless have always remained two flavors in mathematics: one for numbers and one for surfaces. Even so, numbers have space-like properties, something that earlier Pythagoreans already noticed and is evident in terms like ‘square’ for numbers N which are equal to n^2 for some n.

According to many authors (see, among others, Dehaene, 1997), a sense of (approximate) number exists before the language faculty develops, at least for small numbers upto three. The language faculty provides then an interface with various modules from which sets composed of units can arise (Spelke, 2003). On the other hand, Hauser, Chomsky and Fitch (2002) make the stronger claim that the language faculty gives numbers their full power, i.e., their recursive structure. In other words, the formation of exact numbers can only exist because of the recursive structure provided by language. Of course, they do not necessarily claim that this condition is sufficient, but the existence of a population that would possess a faculty of language and yet only few lexical numbers poses an apparent problem for this hypothesis, to the extent that these numbers only designate cardinalities approximately. This is the case for the Mundurucu, whose numerical cognition has been studied previously in at least two papers, Pica et al. (2004) (Pica, Lemer, Izard, & Dehaene, 2004, henceforth PLID) and Dehaene et al. (2007) (Dehaene, Izard, Lemer, & Pica, 2007, henceforth DILP). This paper addresses the challenge that this type of system poses.

It is claimed that the understanding of the complex properties of the Mundurucu number system can be related to several modules. The case of Mundurucu shows in particular that external trading (Murphy, 1960) or other external cultural factors do not suffice to trigger the emergence of exact numbers and, on the contrary, supports the hypothesis that the association of numbers with objects is related to structural properties of the human mind/brain, such as the configurational or nonconfigurational character and the compositional power of a given language.

This article not only tries to solve an obvious tension between PLID (2004) and DILP (2007) about the role of the competence/performance distinction and the role of language in the process of number crystallization (that is, the development of a rich numeral system using clearly distinct and autonomous nonapproximate numbers) but also paves the way to an understanding of how spatial/perceptual representations can be used to overcome some of the constraints inherent to the properties of the Mundurucu core system of numbers. For example, while Mundurucu core numbers can only be interpreted approximately, they allow certain basic calculations and show a clear one-to-one correspondence effect between the number of syllables and the cardinality of the number, suggesting a tension between exactitude and approximation.

The paper is organized as follows. In section 2, we recall the background of our observations. In section 3, we recall the main results of PLID and DILP and consider some questions that arise from these results that have not been addressed previously. We also provide an explanation for the fact that small numbers upto 'four' in Mundurucu are related to a *metric structure*, where the number of slots expresses the *exact cardinality* to which the numeral refers. This is the case up to 'four' (see PLID and DILP). This system stops with *pũg põgbi*, which has a vaguer meaning, as the vocalic alternation expresses (see Crofts, 1971),[1] closer to 'a handful' than to 'five'. In section 4, we address a way of using numbers which had never been observed before: we coin the term "parallel numbers" for the kind of numbers revealed in this usage, exhibiting a one-to-one correspondence of the same quantity over the two sides of a symmetrical axis, often corresponding to the sagital axis of the body. If simple reduplication was used to generate small numbers up to 'four', it is this time, a double reduplication which is used, suggesting as we shall see that *the metric system is linked to numerosity* in both types of reduplication (simple and double). This new type of numeral that involves both the syntactic and phonological components of the grammar is somewhat curiously only used by experts, in activities which can be assimilated to games. In section 5, going back to core numbers, that is, numbers up to 'four', we show that in fact symmetry properties are already (at least partly) present in the core system of numerals, which we claim involves the lexicon and morpho-phonology of Mundurucu. In this demonstration, the apparent anomaly of *pũg põgbi* is explained, and other properties of the Mundurucu language are explored. Finally, we come back to expressions which involve more than 'one hand', such as *pũg põgbi xepxep bodi* (literally, 'one handful and two on the (other) side') which were considered idiomatic or complex in both PILD and DILP. In the conclusion, we consider what these observations tell us about the nature of numbers and the pervasive role of symmetry, which tends not to be limited to some domain of core knowledge and appears to be transversal with regard to several cognitive modules involving both language and perception.

2. Background

As already mentioned, this article elaborates on PLID and DILP. Both papers showed that approximate number words exist, but mainly for very small numbers (± one, ± two, ± three, ± four); for other quantities, people use only approximate words of quantity, like 'a few', 'a handful' or 'many'. The Mundurucu system exhibits, in all aspects, the signature of approximation, as manifested by *Weber's law*, according to which the imprecision of judgment grows with the quantity expressed. The authors also showed that these facts did not prevent the Mundurucus from estimating quantities and performing approximate calculations. While the lack of many number words was related to a mere lexical gap in PILD, it was rather related to a performance phenomenon in DILP.[2]

3. Mundurucu Numerals: A First Approximation

In PLID and DILP, the absence of exact numbers in Mundurucu was left unexplained, and some have even denied the cognitive interest of this fact without grounds (Gomes, 2006) so that the status of this phenomenon and its alleged relation to the Mundurucu language remained mysterious. In fact, Mundurucu numerals are indeed often very long, having as many syllables as the corresponding quantity, with the exception of *pũg põgbi*, whose meaning varies considerably from 'five' to 'a handful'. Moreover, words for 'three' and 'four' are polymorphic: *e-ba-pũg* $= 2 + 1$, *e-ba-dip-dip* $= 2 + 1 + 1$, where *e-ba* means 'your (two) arms'[3]. While it was suggested that this could be related to a system of base 2 common in Tupi languages, we suggest an alternative hypothesis in terms of perceptual sets related to symmetry, in section 5 below. Earlier, we also noted, without providing an explanation, that this system was not productive, for example, expressions such as **e-ba e-ba dip* or **e-ba e-ba pũg* do not exist.

The hypothesis of a base 2 left unexplained such expressions as *xepxep põgbi* (10) (literally, 'two handfuls') or *ebadipdip põgbi* (15) (literally, 'four handfuls') limited to two hands and feet. These expressions could suggest a base 5, but we will see in section 5 below that such a hypothesis is doubtful and that both base-2 and -5 effects derive from general constraints of symmetry.

Such a lack of productivity within a base-2 system would indeed be surprising. The absence of *number crystallization* was related either to a lexical gap, due to an as yet unexplained property of the Mundurucu lexicon (PILD), or to a *performance phenomenon* (DILP), which was curious in view of the omnipresence of a one-to-one correspondence of the metric structure with the cardinality of the number (see Table 1).

We agree on the basic hypothesis of PLID and DILP, according to which Mundurucus only possess *approximate* numbers, as illustrated by the fact that these numbers can combine with the collective suffix *ayũ* without much change of meaning, as in *xepxepayũ*, meaning roughly 'a group of approximately two' (where *ayũ* means 'a group of'), as compared with *xepxep*, which means 'approximately two'. In this paper we suggest that the complexity of the Mundurucu number system is far more interesting and deep than PLID or DILP suggests. We even propose that

Table 1. The core Mundurucu number system

Pũg	± One	One syllable, with vocalic nasalization
xep-xep	± Two	Two syllables, without vocalic alternation
E-ba-pũg	± Three	Three syllables, with vocalic nasalization
E-ba-dip-dip	± Four	Four syllables, without vocalic alternation
Pũg-põg-bi	a handful, ± Five	Three syllables, with vocalic alternation on the second syllable

"collective approximation" might not always be a necessary ingredient of the Mundurucu numeral system and that various types of approximation could be at stake, although the topics deserve further investigation.

We would like to suggest that the absence of exact numbers in Mundurucu is closely related to the fact that, although Mundurucus recognize sets and individuals, they do not make sets *out of* individuals through distributive quantification, much in the sense of Spelke's (2003) interesting study of infants before the emergence of the language. According to Spelke (2003), the formation of the concept of SET composed of units depends on the use of compositionality.[4] We may therefore conjecture that a language with a limited compositionality power (as seems to be the case in polysynthetic languages like Mundurucu) is a handicap for forming such concepts. If this is so, it would also explain the lack of a notion of a VARIABLE which could range over a set, and would open the possibility of getting a set by simply binding it, as indicated in the usual notation for a set of individuals satisfying a property P: $\{x \mid P(x)\}$. The poor *compositionality power* of polysynthetic languages is particularly evident when we observe that even a numeral and the name it modifies do not form a constituent. This is illustrated in Mundurucu by a sentence such as *xep xep pa osodop akoba* ('two remained bananas' (two bananas remained (there))), as the non-contiguity between the numeral and the noun it modifies illustrates. It is this restricted compositionality, which in our terms poses serious problems for a developed representation of number and limits the Mundurucu system to few numbers that are part of core systems of numbers (small approximate numbers).

This in turn raises intriguing questions about important features of the Mundurucu grammar, such as the absence of a singular/plural distinction, the nature of reduplication, or the absence of *distributive quantification*.[5] More generally, it raises questions about the nature of the faculty of language and its relation to other modules of the mind/brain in nonanalytic languages.

The fact that the number of syllables corresponds to the cardinality of the number nevertheless strongly suggests that Mundurucu numerals have a system according to which one accent is associated with each syllable. In other words, the metric grid associated with each word is a so-called *counting meter*, as illustrated in Figure 1 below. The bracket indicates that the operation of reduplication takes place at the linguistic level at which the syllable is represented and expresses the fact that the last syllable is an affix copy of the preceding syllable. This analysis strongly suggests that (in the case of single reduplication) the role of reduplication is to provide the appropriate number of metrics slots, to express in an exact way the cardinality of the numeral.

x	x	x	x
e	ba	[dip	dip

Figure 1. Exact correspondence between the number of metric slots and the cardinality the number expresses in Mundurucu, here for the expression 'ebadipdip' meaning 'approximately four'.

This exact correspondence that we can now express between the number of metric slots and the cardinality of the number is curious, in that it indicates (if we try to reconciliate this observation with the results of both DILP and PILD, according to which the core system of Munducucu numerals have an approximate meaning) that such a correspondence is not enough to give rise to exact numbers. Yet our analysis of Mundurucu numerals is supported by the fact that, in *pũg-põg-bi*, which escapes from the one-to-one correspondence system, the approximate character of the number is marked by the alternation of the vowel {ũ ≥ õ}. This is a phonological process which has the meaning of vagueness in Mundurucu, as we already stressed in note 1 above. This analysis raises the question of why all numerals in Mundurucu are not marked by such a vocalic alternation. We shall deal with this issue by further examining in the next section the role of the morpho-phonological properties of numerals in Mundurucu, and in particular the role of reduplication, where a special usage of number is addressed.

4. Parallel Numbers and Parallel Constructions

We believe that an explanation of the tension between exactness (of the correspondence) vagueness and approximation (of the evaluation of quantities) can be given in the light of two phenomena which enable us to understand better the role of reduplication.

1. The role of metrics, in particular of double reduplication and its interpretation at the interface level.
2. The analysis of a specific use of numerals which we shall call "*parallel numbers*," numbers that are used to express quantities on both sides of a virtual symmetrical axis.

4.1. On Metrics and (Double) Reduplication

We have seen in the previous section that some expressions of Mundurucu are obtained by simple reduplication, like

xep-xep (simple reduplication of the syllable *xep*)
e-ba-dip-dip (simple reduplication of the syllable *dip*).

But there are also numerical expressions obtained by double reduplication:

xep-xep-xep (double reduplication of the syllable *xep*)
eba-pũg-pũg-pũg (double reduplication of the syllable *pũg*)
e-ba-dip-dip-dip (double reduplication of the syllable *dip*).

The literal meanings of these expressions are given in Table 2, where we can see that the double reduplication means 'on both sides'. This is why we term these marked numbers "parallel numbers": they are numerals which express numerosity as conceived on the opposite vertical sides of an axis. The specific use of parallel numbers has not to our knowledge been recognized before or, as has been noted

Table 2. Mundurucu Parallel numbers

Pũg-pũg-pũg	One on both sides
Xep-xep-xep	Two on both sides
E-ba-pũg-pũg-pũg	Three on both sides
E-ba-dip-dip-dip	Four on both sides
Pũg-põg-bi-bi-bi	Five on both sides
Xep-xep-põg-bi-bi-bi	Ten on both sides

in passing, as an argument against our view, as a special case of multiplication (Gomes, 2006). But why should *double reduplication* exist at all? Let us first note that each reduplication of this peculiar type projects one affix that is a total of two affixes which are not interpreted as being parts of the base, but rather as two locations on opposite sides of a virtual axis. The metric structure associated with the core number (that is the number expressed by the base and simple reduplication) is then associated with each side/slot of the axis creating a new one-to-one correspondence, as illustrated in Figure 2 below.[6]

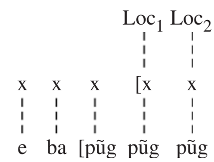

Figure 2. Paralell number expressed through double reduplication: the representation of eba-pũg-pũg-pũg 'three on both sides'.

If simple *syllable reduplication* is defined as an operation which applies at the phonological level, where syllable is expressed, by adding to the word a copy of its last syllable (whether or not the syllable has any sense by itself) as the first reduplication expresses, double reduplication poses a challenge to any phonological theory we aware of. Double reduplication, a phenomenon strongly marked and rarely observed, apparently performs this operation twice, in such a way that the first reduplication is, in some sense, legitimated parasitically by the second one. This analysis is confirmed by the fact that while *peburũrũrũ* (for 'how much on both sides?') is accepted, with a double reduplication of the interrogative adverbial *pebũrũ* (whose meaning is 'how much') the simple reduplication of *pebũrũ* in **pebũrũrũ* is agrammatical.

How does the meaning 'on both sides' arise? To understand this, let us look further at the way double reduplication is achieved at all. While it is possible to express the first reduplication at the syllable level, as illustrated in Figure 1 above, we know a single bracketing is not sufficient to legitimate *two* affixes.

This suggests, much as in the case of visual poetry (see Halle (1987) that the second affix is legitimated within the metrical pattern itself according to principles that are not linguistic proper, but rather involve the perceptual/motor interface. The second bracketing and the first affix it is related to are interpreted as a symmetric pattern involving two symmetrical locations (as the two arrows indicate) associated with the two sides of the body ('soatpu' (['person,' 'body'] which means literally 'all hands and feet, fingers and toes) main axis. As we stressed earlier, the phonological affixes are not entirely part of the base *ebapũg* (three), to which the initial quantity is associated.

The speaker does not express 'six' (as could be expected), but the idea of putting a set of three objects in one-to-one correspondence with a similar set, thus ensuring that the numerosity is really 'three' (on each side). The meaning is expressed by the interpretation of the metrical grid itself and the relation of the mental/perceptual image obtained with the two sides of the human body.

We can therefore interpret parallel numbers as involving two one-to-one correspondence procedures: (1) one related to the number of syllables of the base and the cardinality of the number; and (2) another as involving the quantity expressed in the two sides (two locations on the side of a virtual axis). Remember that this analysis, amounts to say that the last phonological affix is legitimated at the perceptual level (and is not really part of the base). Rather, it expresses a *symmetrical* pattern. This analysis is supported by the fact that the same effect can be achieved by using a double reduplicated noun associated with a nonreduplicated numeral, as illustrated in phrase 1:

e-ba-dip-dip ip-ip-ip ('four poles on both sides') (1)

where it is the noun *ip* ('pole') which is double reduplicated, as illustrated in Figure 3. It is interesting to note that if our analysis is correct, parallel numerals and parallel constructions such as phrase 1 involve two one-to-one correspondences (namely the one-to-one correspondence of the number with its cardinality and the correspondence expressed by both sides of the metric image) from one side to the other side and are, namely, limited to two sides (!), as illustrated by the ungrammaticality of phrase 2:

**e-ba-dip-dip ip-ip-ip*, *warat kadi e-ba-pũg*
('four poles on both sides, four on the other side') (2)

4.2. Parallel Numbers and Symmetry

As expected by our interpretation of double reduplication, parallel numerals are seriously constrained by mental-symmetry constraints—which we believe are supported by the body. What the numeral word expresses is then really one set of the two sets to which an exact cardinality is associated: in fact, through a new symmetrical one-to-one correspondence and potentially the gesture associated with it.

While it is true that this kind of system, which involves both morphological and phonological components, and the syntactic component of the grammar, overcomes the limits of core number representations in the absence of a counting system, as pointed out by Spelke (personal communication), it is somewhat puzzling that the

Loc$_1$ Loc$_2$

x x x x x [x x

e ba dip dip [ip ip ip

Figure 3. Representation of a double-reduplicated noun associated with a nonreduplicated numeral, *e ba dip dip ip ip ip*, 'four poles on both sides'.

construction is limited to very few nouns or objects, such as *ip* ('pole'), *bu* ('finger') or *pu* ('line') and, more generally, nouns referring to body parts. While the topic deserves further study, we tentatively suggest that this is due to a perceptual constraint again: if reduplication is altogether considered a symmetrical repetition (or copy) of something, then the objects which are reduplicated must be able to provide a natural and fixed symmetric axis. While this is natural for poles, (fishing) lines, and body parts, which can be easily placed in a symmetric way, it is most natural with an exact quantity of fingers displayed on both sides of the body—as illustrated in Figure 3 above and Figure 4.

5. Mundurucu Numerals: A Closer Approximation

5.1. Counting with Small Numbers and the Low Power of Compositionality

We think that our analysis of parallel constructions shows that a single one-to-one correspondence of the number of metric slots with the cardinality is not sufficient to ensure the exact denotation of the number. Parallel constructions do partly overcome this difficulty by adding a new one-to-one correspondence between the two sides of a virtual vertical axis. The question of how similar parallel numbers are to simple Mundurucu numbers now arises. Remember that Mundurucu is a nonconfigurational language and that it has low compositionality power, as illustrated by the lack of a noncollective plural, the lack of intersective adjectives or of nonadjoined relative clauses (see note 7 and Hale [1976] for relevant observations about Warlpiri).

We would like to suggest that the fact that Mundurucu lacks the power of developing a rich system of numbers is neither (entirely) due to a performance factor nor due to accidental lexical/cultural gaps (as opposed to the analysis proposed for Warlpiri by Hale, 1975), but to the fact that Mundurucus do not have access to individuals through sets without appealing to the notion of LOCATION.[8] These sets must in fact be seen as kinds of "topological sets," without any access to a fine-grained granularity.[9] Ingredients of these topological sets are simple neighborhoods in such a way that it is difficult to *separate* points. It seems that Mundurucus do not identify objects by means of a membership relation but by means of a part-whole relation (in the sense of Baker, 1999), or as invariants in topological transformations where wholes are configurations of locations giving symmetrical sets.

Figure 4. Mundurucu woman showing the pattern associated with *ebapũg pũg pũg* ('three on both sides') with her fingers and hands, Iaperep (January 2007) (copyright Pierre Pica/CNRS).

Although the topic goes far beyond the scope of this article, we would like to suggest that numbers involving simple reduplication can be analyzed as a special and idiomatic case of double reduplication. This is supported by the fact that, when the number refers to a natural collection/set (that is, two hands, two eyes, etc.), the last phonological affix can be dropped. This is illustrated by a sentence like *ag̃okatkat xep xep pa pa* (*pa*) ('every man possesses two arms'), where the last phonological affix can be dropped. This suggests that simple reduplication might arise, at least in some cases, from a process of grammaticalization (from double reduplication). Such an analysis explains both the collective and approximate reading and the idiomatic-like character of core numbers in Mundurucu.

Going more deeply to the question of simple reduplication, it is interesting to note that small numbers in Mundurucu involve, in the general case, only one hand and that symmetry is also subjacent to numbers like *e-ba-pũg* and *e-ba-dip-dip*. This is illustrated by *e-ba*, which means (perhaps metaphorically) 'your two arms'. The fact that *eba* refers to a natural group, the collection of (two) arms, already implies a one-to-one correspondence between the two metric slots associated with the reduplicated syllables and the cardinality expressed by the number, as in the case of *e-ba-dip-dip* ('four').

This alleged relationship between the body and the reduplication system suggests that the basis of the metric single reduplication itself is again related to the *symmetry* of the body (the representation of the two arms) although it involves this time another support: the hand. The hand, we propose, is in the case of Mundurucu core numbers chunked into a coordinated natural collection/set. But this raises the question of how Mundurucu has odd numbers at all, such as 'three'. The gesture associated with 'three' made by joining the thumb and the little finger shows that the middle metric slot associated with it represents (at least metaphorically) the main body with the two arms symmetric to the body axis.

This analysis is confirmed by the fact that the word for 'four', *e-ba-dip-dip* (literally formed by single reduplication of the last syllable of *ebadip* [with the meaning 'companion']) can be seen as a set of two collections articulated around the axis associated with the open bracket, which now licenses single reduplication where the base and the phonological reduplication affix both refer to two natural collections of two symmetrical elements, ultimately interpreted at the perceptual/sensory interface see Figure 5 below.

Loc_1	Loc_2	$Loc_{1'}$	$Loc_{2'}$
x	x	x	x
(e	ba)	[dip	dip

Figure 5. Representation of *eba-dip-dip*, 'four', referring to four locations through an operation of chunking.

Our analysis might explain why the Mundurucus do not have a *counting routine*, since there is no one finger that can really serve to express the successor function, +1. It furthermore explains the approximate character of even the core numbers (the numbers from ± one to ± four), sealed by the linguistic system which does not allow sets made out of individuals, as expressed by the absence of distributive quantifiers such as 'each' in Mundurucu, which lacks the combinatory power of analytic languages. This again indicates that the exactness of the *metric system* with the cardinality expressed by the number is not sufficient to ensure the exact character of the number, suggesting it might be only visible at the perceptual/sensory interface, where the metric grid is interpreted as a set of natural collections.

Such a conspiracy between phonological, nonlinguistic, and linguistic factors and factors linked to the representation of numerosity in Mundurucu (such as the Weber law) explains the robustness of the Mundurucu system. If our article is on the right track, such systems involve deep and interesting properties of various submodules. Mundurucu involves a core approximate system which stops at around 'three' and 'four'. The nature of *pũg põgbi*, which involves vocalic alternation, clearly shows the limit of this system based on chunking of symmetric (natural collection) and part-whole relationship, which forces long idiomatic words upto three/four.

5.2. Extended Symmetry

The simple observation, according to which symmetry is operating even in the field of 'one hand', allows us to understand why it is difficult for Mundurucus to use expressions that only involve symmetry through complex chunking or involve more than two approximate sides. Numbers like *pũg põgbi bodi* (literally, 'a handful and a side', i.e., 'six' or 'seven') are accepted by all. In contrast, *pũg põgbi xepxep bodi* (literally, 'a handful and two on the other side') (that is 'seven') are more marked since they might involve complex chunking into two (or more) coordinated constituents related to two natural collections, as in (*pũg põgbi*) (*xepxep*), hence respecting symmetry in a complex way (in this case expressed by the two respective sides related by the two hands). Expressions respecting symmetry in a more simple way but expressing larger quantities such as *xepxep põgbi* (literally, 'two handfuls'—i.e., 10) are employed, to the puzzlement of PILD and DILP, which had no explanation for the marked character of expressions like *pũg põgbi xepxep bodi* and considered them to be idiomatic. In the double-hands register, we must notice that some expressions are impossible simply because they cannot be associated with a natural collection or any one-to-one correspondence, such as **pũg põgbi xepxep bodidi* ('a handful and two on both sides', in other words, 'seven on two sides'), which are indeed not accepted, as predicted by our analysis according to which the Mundurucus use several parts of their body as a natural support for representing numerosity.

6. Conclusion

In conclusion, the Mundurucu numeral system involves various subsystems and various types of numbers, each of them being related to a specific organization involving various representations related to various modules of the mind/brain. While we have stressed that the absence of a system with many numbers is related to the weak compositionality power of the language and to the absence of sets composed of units, we have shown that Mundurucus apply various strategies making use of a special type of chunking and various one-to-one correspondence systems ultimately related to a perceptual interpretation.

It is worth considering to which extent the Mundurucu system could shed light on the nature of the Weber law and on the development of number concept in children as discussed by Leslie and Chen (2007). We need to keep in mind that in the case of Mundurucu, symmetry, which seems to play a role in all number representations,[10,11,12] might be an essential element for their survival, as suggested in Khalabatari and Pica (2007, June 16), who also pointed out that many purported cultural phenomena can be reconsidered in cognitive terms. Moreover, the nature of symmetry constraints, which underlines the role of perceptual representations of sets in all constructions (such as parallel numbers), might shed light on those constructions which can be seen as *mathematically elaborate structures used by experts* that go beyond functional utility (see Chomsky, 2003). We do not find this type of phenomenon in societies that possess an advanced technology since indigenous societies might involve a kind of external variation that is poorly understood. If it is right that these practices involve all sorts of games with language or even some modification of the actual language, we understand better their marked character and variation across cultures, as suggested by ongoing work on more elaborated numerical patterns (see Butterworth & Reeve, this issue).

It is our belief that these constructions, which involve many components of the mind/brain, are good candidates to provide new insights into core systems preserved in small-scale societies with limited technology. Nonconfigurational languages, which have only a weak power of compositionality (as our interpretations of Hale's, 1983, work suggests), are often spoken in this type of society. Such culture-specific activities as "parallel numbers" have not been studied intensively enough as of today and will certainly deserve further and extensive studies in the future. Our analysis nevertheless suggests that their nature is relevant for diverse constructions involving various supports (one hand, two hands or the whole body), indicating that similar mechanisms might be at work in few-number languages in a more general way. The analysis developed above tentatively suggests that few-number languages might be an impressive tool for the study of the notion of number and its relation to the faculty of language. Our study suggests furthermore that few-number languages might elucidate the still badly understood relationship of the notion of NUMBER and its relation to perception and space and the nature of the general constraints that apply therein. It might also shed light on the role of perceptual constraints in nonpolysynthetic languages, where the approximate system seems to be dormant, as suggested by Hurford's (2001) observations according to which all languages treat number from 1 to 4 differently.

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A Note on Materials and Methods

Psychological experiments dealt mainly with approximation and involved numerosity-naming tests, based on stimuli involving sets of dots that participants were asked to evaluate or to compare, displayed on a solar-powered portable computer.

While several Mundurucus speak Portuguese, we concentrated this study on a part of the population that is not bilingual and had very restricted access to education. Most linguistics tests were devised in accordance with Hale's (1983) seminal work on Aborigine languages of Australia. These tests led us to discover some syntactic constructions, which were further studied by constructing new examples and asking Mundurucu speakers to give grammaticality judgments. It might be interesting to note that these judgments never contradicted but enriched the results based on the psychological tests, hereby suggesting that a fruitful collaboration is indeed possible between linguistics and a psychological entreprise making use of psycho-physiological experiments.

Notes

[1] This phenomenon is also illustrated by a minimal pair such as *i-rem rem* ('blue') vs. *i-rem rũm* ('blueish').

[2] See Hale (1975) for the idea that a cultural gap should indeed be distinguished from performance phenomena.

[3] We use in an informal way the character '–' to any relevant syllable syllable of a numeral or a reduplicated noun.

[4] Spelke (2003) argues that compositionality provided by the language faculty is the basic key to the interaction between different modules of the mind/brain, providing the basis of sets composed of units. We restrict our attention here to compositionality within nonconfigurational languages, arguing that not all languages possess the same degree of compositionality power. Let us remember that constituency is a device particularly well suited for expressing the composition of several predicates applied to the same variable, such as 'a beautiful dancer'. This translates not only into 'an x such that x is a dancer and x is beautiful' but also into 'an x such that x, being a dancer, dances beautifully'. Interestingly this last reading is impossible in Mundurucu, something which might be due to the absence of phi features, such as number; see also Bouchard (2002), and Baker (1993).

[5] This is demonstrated by the absence of any distributive quantifier in Mundurucu syntax (as illustrated by the fact that 'soat ayacat' 'all women' cannot be interpreted as 'each woman'. See also Baker (1993) for relevant observations about Mohawk.

[6] We leave aside, for obvious reason of space, the relation of parallel numbers with a marked interpretation according to which parallel numbers numbers and the focus marker *ma*, mark a kind of reiteration as in 'pũg põg bi bi bi ma iumap cebe ip (literally, 'five to give to all of them'). These constructions do not involve distributive quantification, but rather a one-to-one correspondence procedure, much as in ('five on both sides'). This is a topic to which we shall return elsewhere.

[7] As the following sentence illustrates, relative clauses in mundurucu are adjoined, in the sense of Hale (1976) (as a representation in terms of constituants already illustrates): *osubadobuxik [ajoba ba iat] pa* (lit. he cl-found [[something (which is)] [in form of a arm] arm] litteraly he found something in form of an arm, an arm-like object.

[8] The fact that Mundurucu does not possess sets made up of individuals but can refer to locations might be illustrated by such sentences as 'puybit oğum pe pe pe ma' (food, I gave to all three of them) where the preposition is repeated three times to express the fact that each individual is intended individually. It is tempting to related this property, and the fact that numbers cannot express exact quantities (even though the metric slots express the exact cardinalities to which the number refer) to the nature of the Mundurucu logarithmic mental line, as developed in some details in Dehaene et al. (2008).

[9] Notice that Mundurucu have a mass/count distinction, which is illustrated by the fact that *xepxep e it* (literally, 'two honeys') means 'two kinds of honey'.

[10] This is already suggested by the syntax of words for quantity, such as ade (many), in which simple reduplication, 'adede' (literally, 'many on both sides'), means really 'a lot'. See also pũg (one) vs. pũg pũg (literally 'one on both sides', that is, 'some'). This intriguing and

refined strategy is reminiscent of the notion of CHUNKING developed in Feigenson & Halberda (2004), according to which infants overcome their limit on tracking multiple objects by binding individuals into sets, an analysis which goes back to Miller's seminal article (Miller, 1956) where this type of phenomenon was treated in terms of performance (limitation of memory).

[11] Note that, as predicted by our analysis 'core numbers' which involves grammaticalized (dormant) symmetry can be combined with any nouns irrespective of the fact that their property provide or not a symmetrical axis.

[12] It is worth noting that expressions referring to 'two hands register' are mainly used for counting, and are rarely associated with any nouns, as the deviance of 'pũg põg bi xep xep bodi ag̃okatkatayũ) ('five + two men') attests.

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