Jan Pociej

SOLVING JÖRGENSEN'S DILEMMA, LIAR PARADOX, ROSS PARADOX AND PRIOR PARADOX

ABSTRACT: The dilemma and paradoxes mentioned in the title have long been a puzzle to logic. To solve them, it was necessary to make some philosophical decisions. The article provides these decisions and discusses ways to solve each of the problems.

KEY WORDS: solution, Jörgensen's dilemma, liar's paradox, Ross' paradox, Prior's paradox, logic, classical, imperative, deontic, values, logical, true, false, good, evil, philosophy, metaphysics, propositions, sentences, affirmative, evaluative, normative, imperative, evaluations, norms, precepts, prohibitions, declaratives, imperatives, interrogatives, state, affairs, reason, intellect, theoretical, practical, will, reasoning, willing, implication, constructive syllogism

1. A short history of research on the logic of non-declaratives

Traditional logic accepted only these as propositions in the logical sense which can be evaluated as true or false. “True” meant the agreement of the sentence with reality, and “false” – disagreement. As a consequence, the area of logic’s reasoning was limited to sentences stating the occurrence of states of affairs and to generally applicable principles and laws.

At the turn of the nineteenth and twentieth centuries, the situation changed thanks to the Polish logician Kazimierz Twardowski, who recognized that the category of propositions[1]

[1] „»Proposition« was replaced by »sentence« because it was a term too loaded with psychologism. Łukasiewicz still used this term in 1912. Only Kotarbiński explicitly abandoned it and replaced it with »sentence«. The issue may seem trivial, but propositions, as an expression of the mental activity of judging, could contain some subjective factors, which include values and norms. Sentences, understood in the spirit of neo-positivism, were

includes not only declarative expressions but also evaluative and normative ones. According to Twardowski, “These expressions are the subject of formal logic and are true and false in a logical sense. This value is not relative to the language or culture in which such expressions occur” (Pacewicz 2016). A similar position was taken by his students – Jan Łukasiewicz (1878-1956), Tadeusz Czeżowski (1889-1981) (Czeżowski 1949), Kazimierz Ajdukiewicz (1890-1963) (Ajdukiewicz 1975) and Tadeusz Marian Kotarbiński (1886-1981) (Kotarbiński 1986). They all considered propositions and norms to be propositions in the logical sense, as long as they could be reduced to a form that allowed them to be considered true or false. Their group was joined in the twentieth century by at least thirty other Polish thinkers who made a significant contribution to the theory of imperatives and norms (Jadacki 2012) initiated by the works of Marian Borowski (1879-1938). The issue of evaluative and normative statements gained the status of a separate section in the textbook *Logika praktyczna* by Zygmunt Ziembiński (1920-1996). Aleksander Peczenik (1937-2005) noted that “At the level of propositional calculus, there are no differences between the logic of norms and the logic of descriptive propositions. Boolean constants such as “or”, “and”, “if ... then” have the same meaning both when used as conjunctions that take propositions and when used as conjunctions that take norms” (Peczenik 1964).

Contemporary researchers in this field, Jacek Jadacki (1948-) and his collaborator Anna Brożek (1980-), postulate extending the concept of reasoning so that it includes not only operations on declarative sentences, i.e. declaratives, but also on imperative and interrogative sentences, i.e. imperatives and interrogatives (Jadacki, Brożek 2012). Their opinion on imperatives is shared by the Greek American logician Peter B.M. Vranas, defining inference as “a reasoning process that begins with the recognition of some declarative or imperative sentence (reasoning premises) and ends with the recognition of a declarative or imperative sentence (reasoning conclusion)”1.

Jadacki and Brożek are aware that their postulate will be met with disapproval concerning the validity of imperative and interrogative reasoning because commands and questions are neither true nor false in the logical sense[2]. Therefore, they propose a solution to the problem

supposed to describe only facts. This, in turn, could lead to the conclusion that evaluative and normative expressions are obviously not facts, and therefore have no logical value and belong to a completely different area than the object of interest of logic. Such thesis, however, was not put forward by the aforementioned authors, because it would significantly impede the possibility of practicing normative ethics as a science (Pacewicz 2016).

[2] The occurrence of interrogative and imperative reasoning “in practice” and the legitimacy of attempts to create a theory of them have been repeatedly questioned. In our opinion, this is related to the stereotypes about rational mental processes that have been established in the logical and philosophical tradition. The emotional

by identifying the analogons of truth and falsity that may be "inherited" by the arguments of the logical relation (imperative or interrogative) of consequence and then show that "The nature of the inferential relationships between interrogatives and imperatives is not ... fundamentally different from the relationships between declarations: in explicating these relationships we use the concept of logical truth" (Jadacki, Brożek 2012).

2. The need for philosophical decisions

Opponents of non-declarative logic cite in support of their position the Jörgensen dilemma, stated by the Danish logician Jörg Jörgensen (1899-1969), the Ross paradox, found by the Danish lawyer and legal philosopher Alf Niels Christopher Ross (1899-1979), and the Prior paradox, discovered by the New Zealand logician Arthur Norman Prior (1914-1969).

Jörgensen's dilemma is a simple statement of a strange state of affairs: here logically correct reasonings are carried out on norms (practical syllogisms), although according to our knowledge they cannot be carried out because norms are not sentences in the logical sense. Various attempts have been made to solve this dilemma. The Finnish philosopher and logician George Henry von Wright (1916-2003), the founder of deontic logic, recognized that propositions involve something beyond truth, but later concluded that all propositions can be reduced to true and false. Contemporary deontic logic tries two methods: (1) the first consists in making a distinction between a norm and a normative sentence; (2) the second one, which grew out of the investigations of conflict-tolerant deontic logics, uses input-output logic (I/O), developed to solve the problems related to the philosophy of standards (Makinson, van der Torre 2007). This logic brings interesting results (SEP 2023); however, Jörgensen's dilemma remains unresolved.

The strength of the Ross paradox comes from the implicative treatment of the domination rule for the disjunction, which in the Classical Propositional Calculus has the form $p \Rightarrow p \vee q$ (or $q \Rightarrow p \vee q$). In deontic logic it takes a similar form $OA \Rightarrow O(\vee B)$ which means that if a given action is binding, then a given action or some other is binding. For example, if the

---

and volitional spheres of the human mind used to be sharply separated from the rational sphere - as if they were governed by separate 'laws'. Although the differences between these spheres are unquestionable, we are convinced, firstly, that at least higher-order volitional processes (of wanting, of not wanting) are schematized, and secondly, that they are controlled by the rational sphere and therefore with this, it is possible to formulate their normative theory" (Jadacki, Brożek 2012).

variable “A” is defined as “Send a letter!”, the implication leads to the conclusion “Send a letter or burn it!”, which is unacceptable (Ross 1941). Some try to circumvent the problem with the notation $p \vee T \equiv T$, where “T” is supposed to mean “Truth” (tautology), but such a procedure cannot be considered legitimate.

Prior's paradox concerns the formula of deontic logic O~A=>O(A=>B). If we take it as an implication, we must accept the reasoning that committing a prohibited act obliges the perpetrator to do something more – committing theft, for example, obliges him to commit adultery. Von Wright recognized Prior's observation as “a real difficulty” and to solve the paradox he created the first system of dyadic deontic logic, which gave rise to an avalanche of further solutions, but without the expected result.

Describing the paradoxes of Ross and Prior, the Swedish logician Jörg Hansen states that “Since these paradoxes have troubled deontic logic for three generations, and called the whole enterprise of deontic logic into question, a solution would be extremely welcome” (Hansen 2006). The way to this solution is shown by the Polish school of logic. As Grzegorz Pacewicz says:

> ...formal logic only provides a formal system without limiting its applicability – after all, what is and what is not a proposition turns out to be a non-logical question in the sense that logical interpretation only assumes that the basic unit in formal logic takes two logical values. Whether evaluative sentences are encompassed by the values of truth and falsity is not decided by formal logic. The awareness of this state of affairs is one of the key achievements of the Polish school of logic, in which it was clearly stated that practicing logic is not possible without certain philosophical decisions (...).(Pacewicz 2016)

Let us, therefore, make the philosophical decisions postulated by the Polish school of logic. As the metaphysical basis of the normative theory of imperative reasoning, let us adopt the realistic theory of cognition.

3. The solution to Jörgensen's dilemma

Realistic metaphysics distinguishes two types of intellectual cognition – theoretical cognition (Greek θεωρείν - see, look, look at, review) and practical cognition (Greek πρακτικός - active). Depending on the type of cognition, reason[3] is called theoretical or practical. The purpose of cognitive acts of theoretical reason is only to consider the truth, and the purpose of cognitive acts of practical reason is action. As can be seen from this, the reason is one, and it is called theoretical or practical, depending on the purpose of the operations it performs[4].

Theoretical cognition comes from man's natural aspiration to cognition, which is mentioned by Aristotle in the first sentence of the introduction to his *Metaphysics*. Practical knowledge comes from the equally natural drive to act and is divided into two types, which we will call here (1) operational practical knowledge and (2) descriptive practical knowledge. Operational practical cognition concerns the actual implementation of the action, and descriptive practical cognition concerns the description of the action. Each of the listed types of practical cognition is closely related to the corresponding type of volition. Practical operational cognition runs in close connection with practical operational desire. Practical cognition and willing[5] form a dual unity of cooperation of reason and will in moral activity, which Thomas Aquinas described in detail, and the Polish philosopher and theologian, Jacek Woroniecki OP (1878-1949), summarized in the following scheme:

	REASON	WILL
Discernment	1. The idea (PL: pomysł) of an object as being good or bad (evaluation)	2. Liking or disliking (PL: upodobanie lub nieupodobanie) for the item
Intention	3. Plan (PL: zamysł) - thinking of the object as a goal	4. The intention (PL: zamiar) to achieve the object as a goal
	5. Deliberation (PL: namysł) - considering means to an end	6. Permission (PL: przyzwolenie) - allowing some means, rejecting others
	7. Intent (PL: rozmysł)	8. The choice (PL: wybór)

[3] I use the term “reason” here in the classical sense of the subject of the function of discursive (indirect) cognition. In the Thomistic tradition, it is sometimes used interchangeably with the term “intellect”. For a brief description of the relations between intellect, sense, wisdom, and reason see (Kalinowski 1973).

[4] Aquinas T., *Summa Theologiae*, I q. 79, a. 11, c.

[5] “There are certain mental processes in which at least some stages, connected by motivational relationships, consist in taking certain attitudes towards interrogatives or imperatives. These attitudes, however, are, in our opinion, not of a persuasive nature (recognition), but of a volitional nature. (…) When we utter an interrogative seriously, we reveal that we do not know something and at the same time we want to find out. When we express an imperative seriously, we betray that we want a certain state of affairs to occur” (Jadacki, Brożek 2012).

	- judging between means to an end	of one of the means
Execution	9. Order (PL: rozkaz) - decision or order of an act	10. Active execution (PL: wykonanie czynne)

The formulation of sentences belongs to the reason. Theoretical reason creates declaratives, practical reason – normatives and imperatives, and both create interrogatives, suppositives and evaluatives, with the difference that in the case of theoretical reason they are simple statements, and in the case of practical they move one to action. Therefore, if the same reason formulates all kinds of propositions, they should be subject to the same logic. As a consequence, if sentences formulated by theoretical reason assume the logical value of truth or falsity, then sentences formulated by practical reason should take analogous values, and since the equivalent of the "truth" cognited by the theoretical intellect is the "good" cognited by the practical intellect, practical sentences should take the values of "good" and "bad" in a logical sense. Logical "good" and "bad" should concern the realization of desirable or undesirable states of being. The sentence postulating the realization of the desired state of being should be called a logically good sentence, and the sentence postulating the realization of an undesirable state of being – a logically bad sentence, with the proviso that the logical values "refer not to the immutable nature of good or evil contained in things, but to how things relate to the purposes and aspirations of the man who makes knowledge" (Penczek 2012).

It is not difficult to see, however, that calling sentences "logically good" or "logically bad" would always imply a reference to their nature, and not to the postulating of a desirable or undesirable state of being contained in them. A similar situation also takes place concerning the logical values of truth and falsity, so the question should be asked where these names of values come from and what they mean. Their author is the pioneer of modern logic, Friedrich Ludwig Gottlob Frege (1848-1925). A closer look at the history of his thought reveals seven phases of searching for answers to our questions. In the first phase, the German logician introduced the categories of recognition (*bejahen*) and negation (*verneinen*) (Frege 1879), in the second – he replaced them with the categories of correctness and incorrectness (Besler 2010), in the third – he stated that there is an analogy between truth in logic and good in ethics (Frege 1884), and in the fourth he introduced truth and falsity as logical values that are "semantic correlates of a sentence" (Frege 1891). This step involved the necessity of rejecting the correspondence theory of truth (as the agreement of the mind with reality) and became the reason for numerous criticisms. The fifth phase brought the thesis on the indefinability of truth (Frege 1897), and the sixth phase – the thesis that one should speak of a significant relationship between logic and truth only for didactic reasons (Frege 1915). In the seventh and final phase, truth is for Frege a predicate predicated about thought (Frege 1918), but predicated based on of whether the names in the sentence (in which the thought is expressed) have their semantic correlates (Besler 2010).

As can be seen, the repeated changes in the formulations of what we call logical values testify to Frege's deep dilemma, which accompanied him throughout all the years of his scientific career. His lack of classical philosophical education was clearly to his detriment when, first, he could not distinguish truth as transcendental from truth as the agreement of mind with reality, declaring that truth is indefinable, and then stated that he spoke of truth in connection with logic only for reasons of didactics. So the question should be asked again: What do the values denoted by zeros and ones in the Classical Propositional Calculus mean? If they were to mean truth and falsehood following the correspondence concept of truth, how could we make such a sentence as “A child in the mother's womb is a human being” as universally true as the sentence “Warsaw is the capital of Poland”? Anyone who has come into contact with an abortionist knows that this is impossible. The same situation applies to the value of a logical function involving two sentences. For example, you can assign logical values to the following biconditional of two sentences:

p		q		p<=>q
If the child in the mother's womb is a human being,	1	then abortion is murder.	1	1
If the child in the mother's womb is a human being,	1	then abortion is not murder.	0	0
If the child in the mother's womb is not a human being,	0	then abortion is murder.	1	0
If the child in the mother's womb is not a human being,	0	then abortion is not murder.	0	1

The abortionist will answer that the child in the mother's womb is not human and will appeal to the same truth table. The conclusion is that the so-called truth tables can just as well be called falsity tables. So what we call the truth values of propositions and functions refers to something independent of true and false. What could it be? The continuation of the discussion with the abortionist will help us find the answer to this question. The logical response to his position is the proposition: “If you want an abortion, then abort yourself!”

p		q		p<=>q
If you want an abortion,	1	then abort yourself!	1	1
If you want an abortion,	1	then don't abort yourself!	0	0
If you don't want an abortion,	0	then abort yourself!	1	0
If you don't want an abortion,	0	then don't abort yourself!	0	1

As can be seen from the table, the logical values of two imperative sentences in the third column, expressing striving for two opposite states – desirable and undesirable, correspond to the logical values of two declarative sentences in the third column of the previous table, expressing the occurrence of two opposite states – existing and non-existent. This confirms Peczenik's thesis that logical constants have the same meaning, regardless of whether they appear as operators of declaratives or normatives. An affirmative sentence in the indicative mood says that there is a certain state of affairs, and in the imperative mood – that the existence of a certain state of affairs is desirable. Thus, both affirmative sentences have in common that they speak of the **existence** of states of affairs. On the other hand, a negative sentence in the indicative mood says that a certain state of affairs does not exist, and in the imperative mood – that it is desirable to a certain state of affairs that it does not exist. Thus, both negative sentences have in common that they speak of the **non-existence** of states of affairs. As a consequence, it can be concluded that the logical values common to the considered sentences are *isity* (Polish *istność,* marked with “I”) and *isnoty* (Polish *nieistność,* marked with “N”), and therefore the *isity tables* are common for declarative and imperative sentences. Moreover, if in the first column we put an interrogative instead of the declarative, the isity table will work as well as for the declarative, as can be seen from the well-known slogan “Have you been drinking? Do not drive!”

p		q		p<≠>q
Have you been drinking?	1	Drive!	1	0
Have you been drinking?	1	Don’t drive!	0	1
Haven't you been drinking?	0	Drive!	1	1

Haven't you been drinking?	0	Don’t drive!	0	0

There is, of course, a large group of people who adhere to the principle of “Obey your desires!” who do not consider driving under the influence of alcohol as something undesirable. Instead of the exclusive disjunction “Have you been drinking? Don’t drive!” these people will use the biconditional “Have you been drinking? Drive!”. Logic is powerless against such a choice because the choice of good or evil depends on the essential fitness of the mind, not on its logical fitness. For those who live responsibly, however, logic shows that it includes in its values not only declaratives, imperatives, and interrogatives, but also suppositives, i.e. sentences in the subjunctive mood, as shown in the table below:

p		q		p<=>q
If I had listened to good advice,	1	I would happily return.	1	1
If I had listened to good advice,	1	I wouldn’t happily return.	0	0
If I hadn’t listened to good advice,	0	I would happily return.	1	0
If I hadn’t listened to good advice,	0	I wouldn’t happily return.	0	1

Thus, the logical values of isity and isnoty can be stated for all kinds of propositions. Moreover, even evaluative exclamation sentences, such as “How beautiful it is here!” or “It's ugly in here!”, reveal such values. Therefore, it can be said that it is difficult to identify any sentences that would not have a logical value. Probably just sentence equivalents like “Guess what?” or “What?” seem devoid of them, but they too contain a connoting, accepting, doubting, questioning, or assuming relationship to logically valuable implicit propositions. So we can change the wording of the second axiom of logic from: “Every sentence in the logical sense is true or false – and only that” (Pociej 2021) to “Every sentence in the logical sense is isitive or isnotive – and only that.”

The values of isity and isnoty seem at first glance to encompass even statements that self-declare to be true or false, such as the famous utterance in the liar paradox: “The sentence I am saying is false”. Hearing someone utter such a sentence, one gets the impression that it is a real sentence. Just as true seems to be the opposite sentence: “The sentence I am saying is true.” We are inclined to assume the isity of both of these sentences based on the affirmative form of the copula “is” in them, and not based on the predicate. In the same way, we would probably be inclined to consider the statement “The sentence I am saying is isnotive” as a real proposition, but in this case, there is a visible contradiction between the affirmative copula "is" and the logical value "isnotive" stated in the predicate. This means that the sentence is self-contradictory. On the other hand, the statement "The sentence I am saying is isitive" seems to be consistent, as seems the statement "The sentence I am saying is true". The conclusion is that the real cause of the liar paradox is hidden below the verbal layer. Let us, therefore, reach for the metaphysical foundations of the sentences in question and consider the statement "The sentence I am uttering is isnotive" as a pure being. This being receives existence only thanks to the speaker, and already contains in its content a logical evaluation of itself. It has not yet come into being itself, and already it supposedly calls into existence something else. The real existence of such a being is not possible, because to be able to act, one must first exist. This proposition requires no proof, just as the proposition that a man cannot lift himself by his hair requires no proof. Therefore, meaningful self-asserting sentences (self-indicating, self-assuming) of their truth value are not possible. However, deceptive imitations of such sentences are possible, similar to imitations of names such as "square circle". It remains therefore to say that probably only *self-sentences* are not sentences in the logical sense.

Determining the truth values of propositional functions is as simple as determining the isity or isnoty of propositions. Systematic treatment of the logical values of all binary functions using the example of the imperatives "Sing!" and "Dance!" (Vranas 2010) are listed in the table below. "One" means logical isity, and "zero" – logical isnoty.

		State 1 singing p	State 1 singing p	Not state 1 not singing ~p	Not state 1 not singing ~p
		State 2 dancing q	Not state 2 not dancing ~q	State 2 dancing q	Not state 2 not dancing ~q
1	Both states coexist (conjunction) Sing-and-dance! p^q	1	0	0	0
2	At least one of the states is present (disjunction)	1	1	1	0

	Sing or dance! p∨q				
3	State 1 occurs but state 2 does not (strong and weak inhibition/non-implication) Sing but don't dance! p≠>q	0	1	0	0
4	State 2 occurs but state 1 does not (reverse, strong and weak inhibition/non-implication) Dance but don't sing! q≠>p	0	0	1	0
5	Both states coexist or co-nonexist (biconditional) If you dance, then sing! p<=>q	1	0	0	1
6	Both states do not coexist or do not co-nonexist (exclusive disjunction) If you dance, don’t sing! p<≠>q	0	1	1	0
7	Only state 1 does not occur, or both states coexist or do not occur (strong and weak, broad competition) Sing, nevertheless dance! p=>q	1	0	1	1
8	Only state 2 does not occur or both states coexist or do not occur (strong and weak, inverse broad competition) Dance, nonetheless sing! q=>p	1	1	0	1
9	Both states co-non-exist (binegation) Neither dance nor sing! p ↓ q	0	0	0	1
10	Both states do not coexist (disjunction) Either don't sing or don't dance! p ↑ q	0	1	1	1

As one can see, the arrangement of values in the isity table fully corresponds to the arrangement of values in the truth table. Thus, it can be said that thanks to the correct identification of the essence of logical values, Jörgensen's dilemma has been solved.

4. Relationship between the implication and the competition

The key to solving the paradoxes of Ross and Prior is also the correct identification of the essence, however, not the essence of logical values, but the essence of the logical function known since antiquity as implication. In the article *Resolving the Material Implication Paradox* (Pociej 2021), I showed that the supposed implication is actually a competition. The competition concerns the oppositional states of affairs, not consequential ones, and it uses oppositional connectives (e.g. “but”, “however”, “nevertheless”), not consequential ones (e.g. “if ... then”, “since”). Let's get to know how it works with two examples.

The first example concerns a legislator dealing with a provision in which an age range is to be defined allowing for a more lenient treatment of the offender. The legislator has two sentences at his disposal: (1) p = “The perpetrator is over the age of seventeen” and (2) q = “The offender is over the age of eighteen”. Applying the reasoning with the use of implication, he can conclude at most that (q=>p):

I.1/1: **If** the offender is over eighteen, **then** he is over seventeen.

By adding negation operators, he can say the same thing in negative terms (~p=>~q):

I.1/2: **If** the offender is under seventeen, **then** he is under eighteen.

It can be seen that the expression p=>q taken as an implication does not provide any information that would help the legislator to determine the age of the perpetrator. On the other hand, considering the same expression as a competition (in this case – a broad competition) with the addition of a negation operator, the legislator may state that (p=>~q):

I.1/3: The perpetrator is over seventeen, **but** not over eighteen.

Therefore, the broad competition allows for a good determination of the perpetrator's age range but requires the use of an additional operator. More economical than this is the inhibition (narrow competition, usually called non-implication), with the help of which the legislator can formulate the sentence (p≠>q), identical to I.1/3, but without the additional operator “~”.

I.1/4: The perpetrator is over seventeen **but not** over eighteen.

He could also state that:

I.1/5: The perpetrator is over seventeen **and not** over eighteen.

He would then use the biconditional $(p\neq>q)<=>(p\wedge\sim q)$, but in this case, he would again have to use an additional negation operator. Therefore, it can be assumed as a rule that for the logical description of a given situation, an appropriate logical function should be used with the smallest possible number of operators.

The second example concerns the construction syllogism (*modus ponendo ponens*). In the Classical Propositional Calculus, it has the form: $((p=>q)\wedge p)=>q$ and corresponds to the inference scheme given by Chrysippus of Soloia (279-208 BC): “If the first, then the second, and the first, so second.” In the implicational approach, it is illustrated with sentences describing two states of affairs connected by a cause-and-effect relationship, for example:

I.2/1: “If it snows, it is slippery; and the snow is falling – so it's slippery.”

The symbol “=> “ is treated as the operator of the alleged implication.

However, it is also possible to represent this syllogism in a form that includes the biconditional: $[(p<=>q)\wedge p]=>q$. This changed form can be illustrated by the same reasoning. Logic would therefore use two functions to describe the same situation. It was difficult to explain this duality, but it disappears without a trace when we treat the function using the operator “=>“ as the operator of the competition. Thus, one can illustrate the reasoning of version $((p=>q)\wedge p)=>q$ with the sentence:

I.2/2: “He likes to have fun, **but** he is moderate; and has fun – **but** remains modest.”

An illustration using the terms found in I.2/1 would lead to a paradox in this case because there is a cause-and-effect relationship between the state “it is snowing” and the state “it is slippery”, not an opposition. Thus, it can be seen that the choice of a logical function cannot be detached from the type of relationship between real states of affairs.

The replacement of the competition with the biconditional can be developed further. The reasoning in the version $((p<=>q)<=>p)=>q$ can be illustrated by the sentence:

I.2/3: “**If** you work hard, the reward is generous; **so** I work hard – **nevertheless**, the reward will be rich.”

Further, the reasoning in the version $((p<=>q)<=>p)<=>q$ can be illustrated with the sentence:

I.2/4: “**If** we keep calm, we will get chocolate for it, **so** we keep calm **because** we will get chocolate for it”, and in addition – if we change the arrangement of parentheses – the version $(p<=>q)<=>(p<=>q)$ will appear, which can be illustrated with the reasoning:

I.2/5: “**If** it snows, it is slippery, **so because** it snows, it is slippery”

All presented versions are tautologies and show an analogous structure of reasoning both with the use of biconditional and competition, therefore it seems that we can talk about four forms of construction syllogism: equivalent, competitive, and two mixed forms.

This close affinity between opposition and consequence, allowing the biconditional to be partly confused with the competition, makes it possible to understand why the competition was regarded for centuries as the one-sided biconditional and was termed implication. Competition is to biconditional as mathematical weak inequality ($\leq$) is to equality ($=$). Thus, the common element of both functions is “equality” – partial in the competition, full in the biconditional – and hence a partial similarity between the schemes of the oppositional and consequential reasoning.

5. Solving the Ross paradox

Starting to solve the Ross paradox, it should be noted that its paradoxicality is twofold:

- first, the sentence “Send the letter or burn it” describes two mutually exclusive states of affairs, so its formalization should take the form of an exclusive disjunction, not a simple disjunction. The disjunction should describe non-exclusive states of affairs, for example, “Send a letter or make a phone call!”

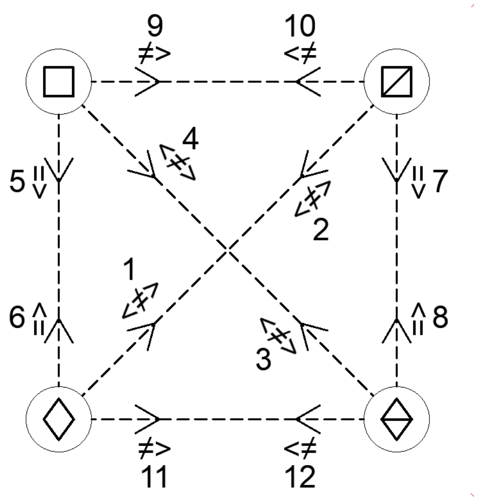
- secondly, every imperative is some command or prohibition, so it belongs to the sphere of deontic logic. The second of the axioms of this logic is $O(A=>B)=>(OA=>OB)$, which in the implicational version means: “If the implication from A to B is obligatory, then from the obligation of A follows the obligation of B”. According to this axiom, the formula $O(A=>(A\vee B))$ is transformed into the formula $OA=>O(A\vee B)$. As long as the transformation concerns the alleged implication operating with consequential sentences, the form of the transformed formula seems to be correct. However, if we consider the alleged implication as a competition operating with opposing sentences, then the opposition of sentences should be accompanied by the appropriate opposition of operators. So we need to change the formula $OA=>O(A\vee B)$ so that the operator O is opposed by one of the other deontic operators. Which one? Von Wright adopts three deontic operators – command (“O” for “obligatory”), prohibition (“F” for “forbidden”), and permission (“P” for “permitted”), noting that the operator P has two meanings. Thus, he follows the ancient tradition that identified the modalities of being – necessity, possibility, and impossibility – and the analogous modalities of action – “I must”, “I can” and “I must not” They all seem to originate in the four possible degrees of the desirability of states of affairs. Marking the desired state of affairs as “+” and the undesirable as “¯”, we get the following classification:

1	2	3	4	5
	States of affairs	Leibniz 1671	Von Wright and tradition	Proposed notation
+ +	Strongly desired	*debitum*	obligatory O or OB obligatory	obligation (PL: nakaz) □ should do
+ ¯	Weakly desired	*licitum*	permitted P or PE permissible	permission (PL: dopuszczenie) ◊ may do
¯ +	Weakly undesired	*indifferens*	non-obligatory N or OM omissible	omission (PL: odpuszczenie) ⟠ may not do
– –	Strongly undesired	*illicitum*	forbidden F or IM impermissible	prohibition (PL: zakaz) ⧄ should not do

Von Wright adopted the formula OA=>PA as the third axiom of deontic logic. He treated it as an implication, but inadvertently pointed out the way to the sought-for opposition, because, considering it as a competition, it can be treated as one of ten possible oppositions:

1	2	3	4
Propositional function	<≠>	=>	≠>
Contrast	contradiction	opposition	contrariety and subcontrariety
Formula	P1: ◊A<≠> ⧄A P2: ⧄A <≠> ◊A P3: ⟠ A<≠> □A P4: □A <≠> ⟠A	P5:□A => ◊A P6:◊A => □A P7:⧄A => ⟠A P8:⟠A => ⧄A	P9: □A ≠> ⧄A P10: ⧄A ≠> □A P11: ◊A≠> ⟠A P12: ⟠A ≠> ◊A

For easier memorization, they can be presented in the square of opposition:



In addition, due to the fact that the negation operator can stand on one or both sides of the deontic operator, there are many possibilities for switching operators. The twelve simplest are listed in the table below:

□	⧄	◊	⟠
N1:□~ <=> ⧄ N2: ~□<=> ⟠ N3: ~□~ <=> ◊	N4: ⧄~ <=> □ N5: ~⧄ <=> ◊ N6: ~⧄~ <=> ⟠	N7: ◊~ <=> ⟠ N8: ~◊ <=> ⧄ N9: ~◊~ <=> □	N10: ⟠~ <=> ◊ N11: ~⟠ <=> □ N12: ~⟠~ <=> ⧄

In the Ross paradox there is an opposition (P5). Therefore, we need to convert the formula O(A=>A∨B) into the formula OA=>P(A∨B), which can be read as follows:

PR.1: “You should send a letter, **however** you can send a letter **or** settle the matter by phone.”

So it is the same situation as in the case of compulsory military service:

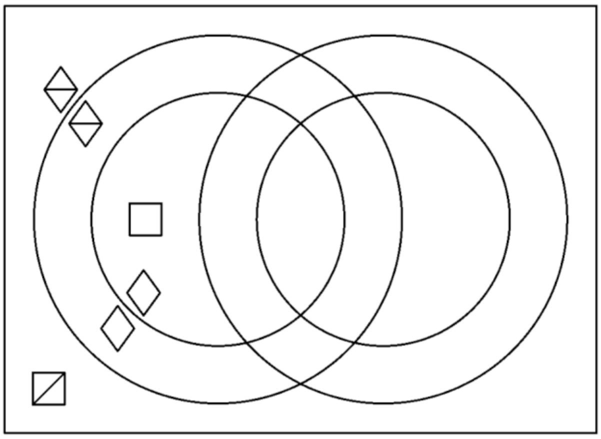
PR.2: “You should do required military service; **however**, you can do required military service **or** perform civil service instead.”

This is the solution to the Ross paradox. There is also the problem of expressing duty in the form of an imperative. The imperatives “Send a letter!” and “Go to the army!” seem to be sentences that do not allow any alternative, which agrees with Woroniecki's scheme, in which the imperative is an order at the end of the chain of activities of the practical intellect. An order prescribes the execution of an act, therefore it cannot be regarded as just one of the options at the same time, and it is precisely such a mixed juxtaposition of obligation and permission that we are dealing with in the present case. It must therefore be concluded that the imperative in the sentence “Send a letter, but you can nevertheless send a letter or deal with the matter by telephone” should be treated not as an order that must be strictly obeyed, but as an order that can be carried out in one way or another.

Out of pure cognitive curiosity, one can also ask a question: What would happen if instead of an obligation, a prohibition appeared in the formula: A(A=>A∨B)? According to the opposition (P6), the formula could take the form ⍁A=>⟠(A∨B). However, the sentence “You must not send a letter, but you may choose not to send a letter or notify by telephone” is confusing. The reason of the confusion is the lack of expansion of the consequent. After such an expansion, the formula ⍁A=>(⟠A∨⟠B) appears, which can be illustrated with the sentence: “You are not allowed to send a letter, but you may either not send a letter or not make a phone call.” The sentence already sounds good. Moreover, it can be seen that the negation contained in deontic operators affects the disjunction differently than the negation in the Classical Propositional Calculus. The negation of a disjunction in propositional calculus is equivalent to the conjunction of negations ~(p∨q)<=>(~p∧~q), while in deontic logic the omission of a disjunction takes the form of a disjunction of omissions: ⟠(A∨B)<=>(⟠A∨⟠B). The disjunction of negations in propositional calculus is equivalent to the alternative denial (~p∨~q)<=>(p ↾ q), so consequently the skeleton of the deontic formula appears in the form ~p=>(p ↾ q). This skeleton is a tautology, as is the starting formula. The deontic formula is built on it. Since the negation contained in the omission operators (“may **not**”, ⟠<=>◊~) was transferred to the alternative, the permission operator (“may”, ◊) appears in its place, and the whole deontic formula takes the form ⧄A=>◊(A ↑ B). Thus, it can be stated that the biconditional of ⟠(A∨B)<=>◊(A ↑ B) holds. Consequently, using an analogous method of inference, four laws of deontic logic can be formulated:

Rule	Formula	Meaning
PD1	⟠(A∨B)<=>◊(A↑B)	The omission of disjunction is the permission of alternative denial.
PD2	⧄(A∨B)<=>□(A↑B)	The prohibition of disjunction is the obligation of alternative denial
PD3	⟠(A∧B)<=>◊(A↓B)	The omission of conjunction is the permission of joint denial
PD3	⧄(A∧B)<=>□(A↓B)	The prohibition of conjunction is the obligation of joint denial

The relationships between deontic functions can be illustrated with Venn diagrams. These diagrams differ from the diagrams for the Classical Calculus by two additional circles and take the following form:



A list of Venn diagrams for basic deontic functions, containing two propositional arguments in the affirmative mood, is given in the appendix.

As can be seen, the introduction of opposition to deontic logic requires not only the customization of opposing operators, but also the introduction of adequate changes to logical functions.

6. Solving the Prior paradox

Let us begin the solution of Prior's paradox with the tautology on which Von Wright based his deontic formula. It has the form ~p=>(p=>q). The British logicians Bertrand Russell (1872-1970) and Alfred North Whiteahead (1861-1947) read it as the implication that "any sentence follows from a false proposition" (Curley 1975). This reading was probably intended to be a modern version of the *ex falso quodlibet* rule. Apparently, Von Wright did not take this seriously when he added obligation operators to the propositions on either side of the principal connective. Then, developing the formula O(A=>B), he applied the axiom O(A => B) =>(OA=>OB), which led to the formula O~A=>(OA=>OB), which concealed the Prior's aforementioned paradoxical conclusion.

Having explained the difficulties of the construction syllogism and the Ross paradox, the explanation of the Prior paradox is not too difficult and comes down to a few previously tested moves.

- first, the formula ~p=>(p=>q) should be read not as an implication, but as a competition. It can be illustrated with a travesty of a Polish funny saying: "What cannot be done, neverthless can be done; however – slowly and with caution" (Wszystko można, co nie można, byle z wolna i z ostrożna).

- secondly, the formula O~A => (OA => OB) should also be read as a competition,

- third, as in the case of the Ross paradox, the deontic operators must be adapted to the inverse of the function. Therefore, the formula O(~A=>(A=>B)) should be expanded to O~A=>P(A=>B). For the expansion of the expression P(A=>B) there is no specific rule for substituting deontic operators. The antecedent will take the form of PA, but the consequent may take the form of OB as well as PB, NB or FB, depending on the content of the sentences substituted for the variable B. The possible classification of these sentences and their relationships with the sentence in the antecedent goes beyond the scope of this article. As an example, let's choose the form O~A=>(PA=>OB). It can be illustrated with the sentence:

PP1: It is not allowed to enter the hospital in shoes; **however**, you can enter in shoes, **but** you must wear pads.

A more official example comes from Article 55 of the Constitution of the Republic of Poland (Constitution 2023):

PP2: The extradition of a Polish citizen is prohibited..., **however**, the extradition of a Polish citizen may be carried out..., **nevertheless**, the admissibility of extradition is decided by the court1.

This is the solution to Prior's paradox.

7. Summary

Summarizing the results of the conducted logical and metaphysical inquiries, it should be stated that:

Firstly, the solution to the problems of logic turned out to be possible based on realistic metaphysics. This confirms the thesis of the Polish school of logic that practicing logic is not possible without certain philosophical decisions.

Secondly, the identification of the true nature of truth values made it possible to conclude that all sentences except self-sentences are sentences in the logical sense. Thus, Jorgensen's dilemma and the liar paradox were resolved.

Third, an analysis of how the competition replaces the implication in a construction syllogism revealed the essence of the affinity between these functions.

Fourth, the replacement of implication with competition in deontic logic and the adjustment of deontic operators to the mutual relations of oppositional propositions made it possible to solve the Ross paradox and the Prior paradox.

Fifth, the inclusion of opposites in the scope of logical reasoning requires an adequate enlargement of the set of symbols and modification of the axioms.

Sixth, the inclusion of opposition in computer programming seems to enable the construction of artificial volition (AV), which together with artificial intelligence (AI) will allow the creation of robots with an artificial soul (AS).

Seventh, the described changes affect all branches of logic, so it can be said that we are dealing with a reform of logic.

Feci, quod potui, faciant meliora potentes.

BIBLIOGRAPHY

1. Ajdukiewicz K., 1975, *Logika pragmatyczna*, Warszawa 1975, p. 28.
2. Curley E.M., 1975, *The Development of Lewis' Theory of Strict Implication*, in: "Notre Dame Journal of Formal Logic" XVI, 4/1975, p. 517.
3. Czeżowski T., 1949, *Logika. Podręcznik dla studiujących nauki filozoficzne*, Warszawa 1949, p. 13.
4. Frege G., 2011, *Funktion, Begriff, Bedeutung. Fünf logische Studien*, Göttingen 2011.
5. Frege G., 1918, G. Frege: *Myśl — studium logiczne*, in: *Frege. Pisma semantyczne*. transl. B. Wolniewicz. Warszawa 1977, p. 103.
6. Frege G., 1884, *Die Grundlagen der Arithmetik. Eine logisch mathematische Untersuchung über den Begriff der Zahl,* Breslau 1884.
7. Frege G., 1897, *Logik*, p. 140. Unpublished text, transl. G. Besler.
8. Frege G., 1915, *Meine grundlegenden logischen Einsichten*, in: *Frege. Nachgelassene Schriften*, Hamburg 1983, s. 271—272.
9. Frege G., 1879, *Begriffsschrift, eine der arithmetischen nachgebildete Formalsprache des reinen Denkens*, Halle 1879).
10. G. Besler, 2010, *Gottloba Fregego koncepcja analizy filozoficznej,* Katowice 2010, p. 197.
11. Hansen J., 2006, *The Paradoxes of Deontic Logic: Alive and Kicking*, in: "Theoria" 72/3, Lund[Oxford] 2006, (pp. 221-232).
12. Hansen J, 2008, *Imperatives and Deontic Logic: On the Semantic Foundations of Deontic Logic*, PhD Dissertation, University of Leipzig.
13. Hansen J., 2013, *Imperatives Logic and Its Problems*, in: Gabbay et al. 2013, pp. 137–191.
14. Hedenius I., 1941, *Om Ratt och Moral* (*On Law and Morals*), Stockholm 1941.
15. Jadacki J., Brożek A., 2012, *Uogólnione pojęcie rozumowania*, in: "Studia Philosophica Wratislaviensia", VII, 1/2012, pp. 105-119.
16. Jadacki J., 2012, *Byt i powinność*, Warszawa 2012.
17. Kalinowski J., *Reason, sense and philosophy*, "Studia Philosophiae Christianae" 9/1 1973, pp. 29-54.

18. *Konstytucja Rzeczypospolitej Polskiej*, Legal status as of: 21.02.2023, Dz.U.1997.78.483 - *Konstytucja Rzeczypospolitej Polskiej z dnia 2 kwietnia 1997 r.* https://lexlege.pl/konstytucja-rzeczypospolitej-polskiej/art-55/
19. Kotarbiński T., 1986, *Elementy teorii poznania, logiki formalnej i metodologii nauk*, Warszawa 1986, p. 109.
20. Leibniz G. W., 1671, *Elementa Juris Naturalis*. Cf.: *Gottfried Wilhelm Leibniz: Sämtliche Schriften und Briefe*, t. 1. Reihe: Philosophische Schriften, Darmstadt 1930, pp. 465–485.
21. Makinson D., Leendert van der Torre, 2000, *Input/Output Logics*, in: "Journal of Philosophical Logic", 29(4)/2000, p. 383–408. doi:10.1023/A:1004748624537
22. Makinson D., Leendert van der Torre, 2001, *Constraints for Input/Output Logics*, "Journal of Philosophical Logic", 30(2)/ 2001, s. 155–185. doi:10.1023/A: 1017599526096
23. Makinson D., Leendert van der Torre, 2023, *What is Input/Output Logic? Input/Output Logic, Constraints, Permissions*, p. 1. Source: https://d-nb.info/ 991738675/ 34, access: 30.01.2023.
24. Murawski R., 2011, *Filozofia matematyki i logiki w Polsce międzywojennej*, Toruń 2011, pp. 240-253.
25. Pacewicz G., 2016, *Wartości i normy jako zdania w logice*, in: "Humanistyka i przyrodoznawstwo", 22, Olsztyn 2016, p. 294.
26. Parent X., Leendert van der Torre, 2013, *Input/output logic*, in: Gabbay et al. 2013, pp. 499–544.
27. Peczenik A., 1964, *Czy istnieje logika norm?*, in: "Ruch prawniczy, ekonomiczny i socjologiczny", 26/1964, Zeszyt 1, s. 132.
28. Penczek M., 2012, *Wola i intelekt w filozofii Tomasza z Akwinu*, Kraków 2012, p. 95.
29. Pociej J., *Rozwiązanie problemu implikacji materialnej*, 2021. Text published on "Academia", https://www.academia.edu/45653000/Rozwiazanie_problemu_ implikacji _ materialnej
30. Ross A., 1941, *Imperatives and Logic*, in: "Theoria " 7/1941, pp. 53–71. doi:10.1111/j. 1755-2567.1941.tb00034.x
31. SEP (Stanford Encyclopedia of Philosophy). Source: https://plato.stanford.edu/entries/ logic-deontic/index.html#ref-78%20Deontic%20 logic, access: 03.03.2023.
32. Van Fraassen, 1973, *Values and the Heart's Command*, "The Journal of Philosophy", 70(1)/ 1973, pp. 5–19. doi:10.2307/2024762

33. Von Wright, G.H., 1951, “Deontic Logic”, *Mind*, 60(237)/1951, pp. 1–15. doi:10.1093/mind/ LX.237.1
34. Von Wright, G. H., 1956, *A Note on Deontic Logic and Derived Obligation*, “Mind”, 65/1956, pp. 507–509.
35. Von Wright, G.H., 1963, *Norm and Action: A Logical Enquiry*, New York 1963.
36. Vranas P.B.M., *In Defence of Imperative Inference*, in: “Journal of Philosophical Logic”, 1 (39)/2010, p. 60. Transl. (Jadacki J., Brożek A. 2012)
37. Vranas P.B.M., *Imperatives. Logic of.* in: *The International Encyclopedia of Ethics*, 2010. Source: https://philarchive.org/archive/VRAILO, access: 06.02.2023.

APPENDIX

Table 1. Deontic diagrams for the obligation.

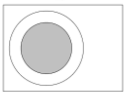
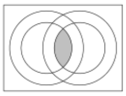
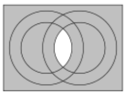
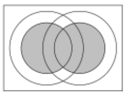
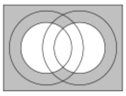
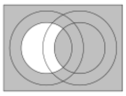
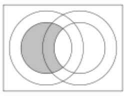
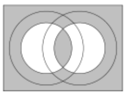
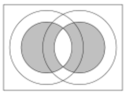
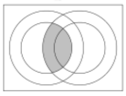
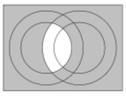
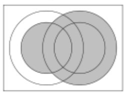
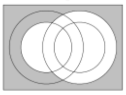
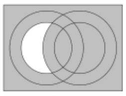
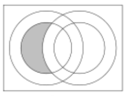
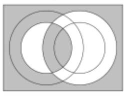
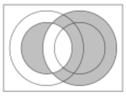
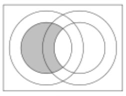
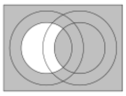
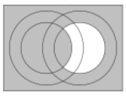
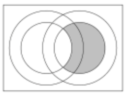
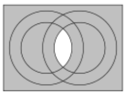
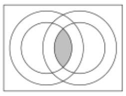
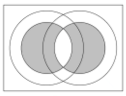
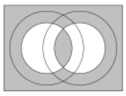
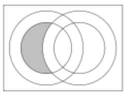
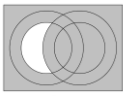
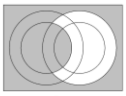
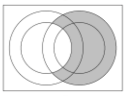
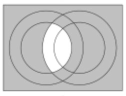
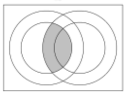
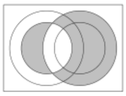
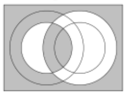
p	∧	↑	∨	↓	=>	≠>	<=>	<≠>
 □p	 □p∧□q <=> ⟠p↓⟠q □p≠>⟠q	 □p↑□q <=> ⟠p∨⟠q □p=>⟠q	 □p∨□q <=> ⟠p↑⟠q ⟠p=>□q	 □p↓□q <=> ⟠p∧⟠q ⟠p≠>□q	 □p=>□q <=> □p↑⟠q ⟠p∨□q	 □p≠>□q <=> □p∧⟠q ⟠p↓□q	 □p<=>□q <=> ⟠p<=>⟠q □p<≠>⟠q ⟠p<≠>□q	 □p<≠>□q <=> □p<=>⟠q ⟠p<=>□q ⟠p<≠>⟠q
	 □p∧◊q <=> ⟠p↓⍁q □p≠>⍁q	 □p↑◊q <=> □p=>⍁q ⟠p∨⍁q	 □p∨◊q <=> ⟠p↑⍁q ⟠p=>◊q	 □p↓◊q <=> ⟠p∧⍁q ⟠p≠>◊q	 □p=>◊q <=> □p↑⍁q ⟠p∨◊q	 □p≠>◊q <=> □p∧⍁q ⟠p↓◊q	 □p<=>◊q <=> ⟠p<=>⍁q □p<≠>⍁q ⟠p<≠>□q	 □p<≠>◊q <=> □p<=>⍁q ⟠p<=>◊q ⟠p<≠>⍁q
	 □p∧⟠q <=> ⟠p↓□q □p≠>□q	 □p↑⟠q <=> ⟠p∨□q □p=>□q	 □p∨⟠q <=> ⟠p↑□q ⟠p=>⟠q	 □p↓⟠q <=> ⟠p∧□q ⟠p≠>⟠q	 □p=>⟠q <=> □p↑□q ⟠p∨⟠q	 □p≠>⟠q <=> □p∧□q ⟠p↓⟠q	 □p<=>⟠q <=> ⟠p<=>□q □p<≠>□q ⟠p<≠>⟠q	 □p<≠>⟠q <=> □p<=>□q ⟠p<=>⟠q ⟠p<≠>□q
	 □p∧⍁q <=> ⟠p↓◊q □p≠>◊q	 □p↑⍁q <=> ⟠p∨◊q □p=>◊q	 □p∨⍁q <=> ⟠p↑◊q ⟠p=>⍁q	 □p↓⍁q <=> ⟠p∧◊q ⟠p≠>⍁q	 □p=>⍁q <=> □p↑◊q ⟠p∨⍁q	 □p≠>⍁q <=> □p∧◊q ⟠p↓⍁q	 □p<=>⍁q <=> ⟠p<=>◊q □p<≠>◊q ⟠p<≠>□q	 □p<≠>⍁q <=> □p<=>◊q ⟠p<=>⍁q ⟠p<≠>◊q

Table 2. Deontic diagrams for the omission.

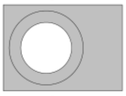
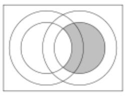
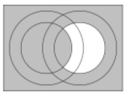
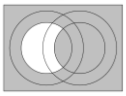
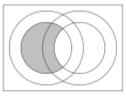
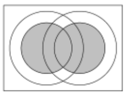
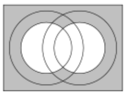
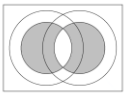
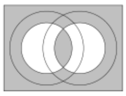
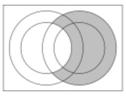
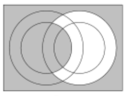
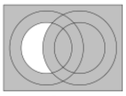
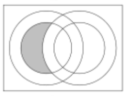
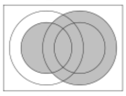
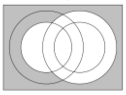
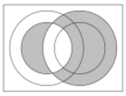
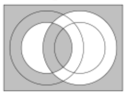
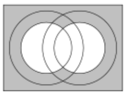
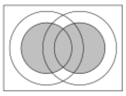
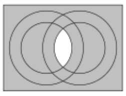
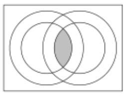
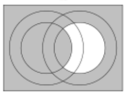
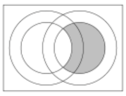
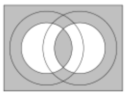
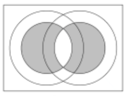
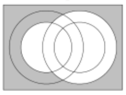
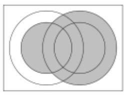
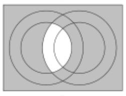
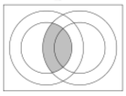
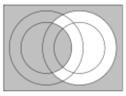
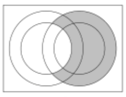
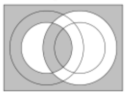
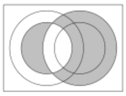
p	∧	↑	∨	↓	=>	≠>	<=>	<≠>
 ⟠p	 ⟠p∧☐q <=> ☐p↓⟠q ⟠p≠>⟠q	 ⟠p↑☐q <=> ☐p∨⟠q ⟠p=>⟠q	 ⟠p∨☐q <=> ☐p↑⟠q ☐p=>☐q	 ⟠p↓☐q <=> ☐p∧⟠q ☐p≠>☐q	 ⟠p=>☐q <=> ⟠p↑⟠q ☐p∨☐q	 ⟠p≠>☐q <=> ⟠p∧⟠q ☐p↓☐q	 ⟠p<=>☐q <=> ☐p<=>⟠q ☐p<≠>☐q ⟠p<≠>⟠q	 ⟠p<≠>☐q <=> ☐p<=>☐q ⟠p<=>⟠q ☐p<≠>⟠q
	 ⟠p∧◊q <=> ☐p↓⧄q ⟠p≠>⧄q	 ⟠p↑◊q <=> ☐p∨⧄q ⟠p=>⧄q	 ⟠p∨◊q <=> ☐p↑⧄q ☐p=>◊q	 ⟠p↓◊q <=> ☐p∧⧄q ☐p≠>◊q	 ⟠p=>◊q <=> ⟠p↑⧄q ☐p∨◊q	 ⟠p≠>◊q <=> ⟠p∧⧄q ☐p↓◊q	 ⟠p<=>◊q <=> ☐p<=>⧄q ☐p<≠>◊q ⟠p<≠>⧄q	 ⟠p<≠>◊q <=> ☐p<=>◊q ⟠p<=>⧄q ☐p<≠>⧄q
	 ⟠p∧⟠q <=> ☐p↓☐q ⟠p≠>☐q	 ⟠p↑⟠q <=> ☐p∨☐q ⟠p=>☐q	 ⟠p∨⟠q <=> ☐p↑☐q ☐p=>⟠q	 ⟠p↓ ⟠q <=> ☐p∧☐q ☐p≠>⟠q	 ⟠p=>⟠q <=> ⟠p↑☐q ☐p∨⟠q	 ⟠p≠>⟠q <=> ⟠p∧☐q ☐p↓⟠q	 ⟠p<=>⟠q <=> ☐p<=>☐q ☐p<≠>⟠q ⟠p<≠>☐q	 ⟠p<≠>⟠q <=> ☐p<=>⟠q ⟠p<=>☐q ☐p<≠>☐q
	 ⟠p∧⧄q <=> ☐p↓◊q ⟠p≠>◊q	 ⟠p↑⧄q <=> ☐p∨◊q ⟠p=>◊q	 ⟠p∨⧄q <=> ☐p↑◊q ☐p=>⧄q	 ⟠p↓⧄q <=> ☐p∧◊q ☐p≠>⧄q	 ⟠p=>⧄q <=> ⟠p↑◊q ☐p∨⧄q	 ⟠p≠>⧄q <=> ⟠p∧◊q ☐p↓⧄q	 ⟠p<=>⧄q <=> ☐p<=>◊q ☐p<≠>⧄q ⟠p<≠>◊q	 ⟠p<≠>⧄q <=> ☐p<=>⧄q ⟠p<=>◊q ☐p<≠>◊q

Table 3. Deontic diagrams for the permission.

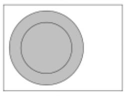
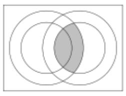
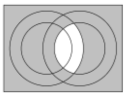
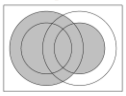
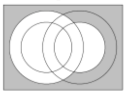
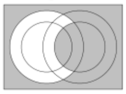
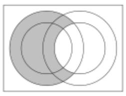
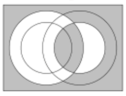
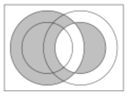
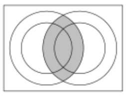
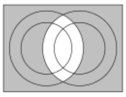
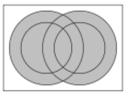
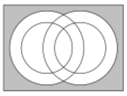
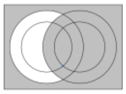
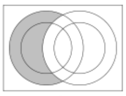
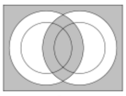
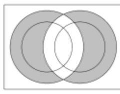
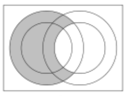
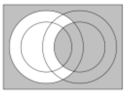
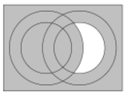
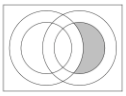
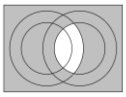
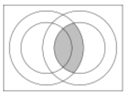
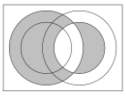
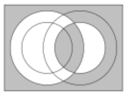
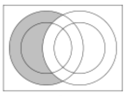
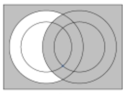
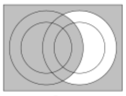
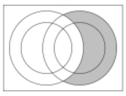
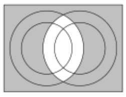
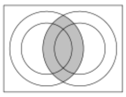
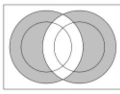
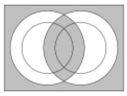
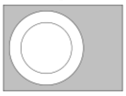
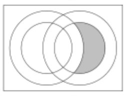
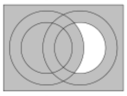
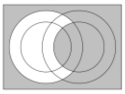
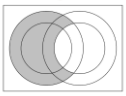
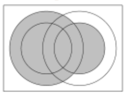
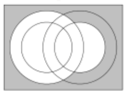
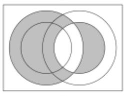
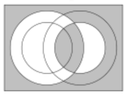
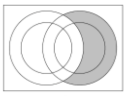
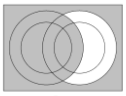
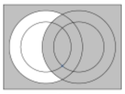
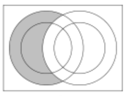
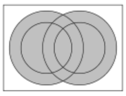
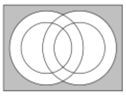
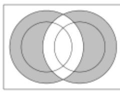
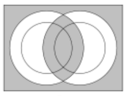
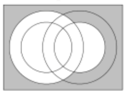
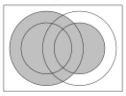
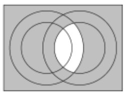
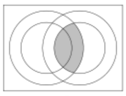
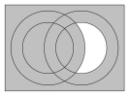
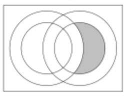
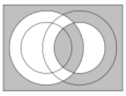
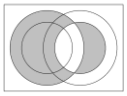
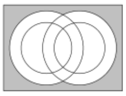
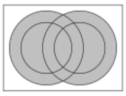
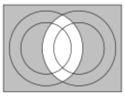
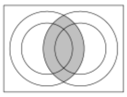
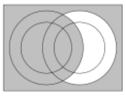
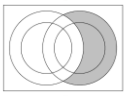
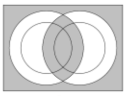
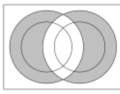
p	∧	↑	∨	↓	=>	≠>	<=>	<≠>
 ◊p	 ◊p∧□q <=> ⧄p↓⟠q ◊p≠>⟠q	 ◊p↑□q <=> ⧄p∨⟠q ◊p=>⟠q	 ◊p∨□q <=> ⧄p↑⟠q ⧄p=>□q	 ◊p↓□q <=> ⧄p↑⟠q ⧄p≠>□q	 ◊p=>□q <=> ◊p↑⟠q ⧄p∨□q	 ◊p≠>□q <=> ◊p∧⟠q ⧄p↓□q	 ◊p<=>□q <=> ⧄p<=>⟠q ◊p<≠>⟠q ⧄p<≠>□q	 ◊p<≠>□q <=> ◊p<=>⟠q ⧄p<=>□q ⧄p<≠>⟠q
	 ◊p∧◊q <=> ⧄p↓⧄q ◊p≠>⧄q	 ◊p↑◊q <=> ⧄p∨⧄q ◊p=>⧄q	 ◊p∨◊q <=> ⧄p↑⧄q ⧄p=>◊q	 ◊p↓◊q <=> ⧄p∧⧄q ⧄p≠>◊q	 ◊p=>◊q <=> ◊p↑⧄q ⧄p∨◊q	 ◊p≠>◊q <=> ◊p∧⧄q ⧄p↓◊q	 ◊p<=>◊q <=> ⧄p<=>⧄q ◊p<≠>⧄q ⧄p<≠>◊q	 ◊p<≠>◊q <=> ◊p<=>⧄q ⧄p<=>◊q ⧄p<≠>⧄q
	 ◊p∧⟠q <=> ⧄p↓□q ◊p≠>□q	 ◊p↑⟠q <=> ⧄p∨◊q ◊p=>□q	 ◊p∨⟠q <=> ⧄p↑□q ⧄p=>⟠q	 ◊p↓⟠q <=> ⧄p∧□q ⧄p≠>⟠q	 ◊p=>⟠q <=> ◊p↑□q ⧄p∨⟠q	 ◊p≠>⟠q <=> ◊p∧□q) ⧄p↓⟠q	 ◊p<=>⟠q <=> ⧄p<=>□q ◊p<≠>□q ⧄p<≠>⟠q	 ◊p<≠>⟠q <=> ◊p<=>□q ⧄p<=>⟠q ⧄p<≠>□q
	 ◊p∧⧄q <=> ⧄p↓◊q ◊p≠>□q	 ◊p↑⧄q <=> ⧄p∨□q ◊p=>◊q	 ◊p∨⧄q <=> ⧄p↑◊q ⧄p=>⧄q	 ◊p↓⧄q <=> ⧄p∧◊q ⧄p≠>⧄q	 ◊p=>⧄q <=> ◊p↑◊q ⧄p∨⧄q	 ◊p≠>⧄q <=> ◊p∧◊q ⧄p↓⧄q	 ◊p<=>⧄q <=> ⧄p<=>◊q ◊p<≠>◊q ⧄p<≠>⧄q	 ◊p<≠>⧄q <=> ◊p<=>◊q ⧄p<≠>◊q ⧄p<=>⧄q

Table 4. Deontic diagrams for the prohibition.

p	∧	↑	∨	↓	=>	≠>	<=>	<≠>
 ⧄p	 ⧄p∧□q <=> ◊p↓⟠q ⧄p≠>⟠q	 ⧄p↑□q <=> ◊p∨⟠q ⧄p=>⟠q	 ⧄p∨□q <=> ◊p↑⟠q ◊p=>□q	 ⧄p↓□q <=> ◊p∧⟠q ◊p≠>□q	 ⧄p=>□q <=> ⧄p↑⟠q ◊p∨□q	 ⧄p≠>□q <=> ⧄p∧⟠q ◊p↓⟠q	 ⧄p<=>□q <=> ◊p<=>⟠q ◊p<≠>□q ⧄p<≠>⟠q	 ⧄p<≠>□q <=> ◊p<=>□q ⧄p<=>⟠q ◊p<≠>⟠q
	 ⧄p∧◊q <=> ◊p↓⧄q ⧄p≠>⧄q	 ⧄p↑◊q <=> ◊p∨⧄q ⧄p=>⧄q	 ⧄p∨◊q <=> ◊p↑⧄q ◊p=>◊q	 ⧄p↓◊q <=> ◊p∧⧄q ◊p≠>◊q	 ⧄p=>◊q <=> ⧄p↑⧄q ◊p∨◊q	 ⧄p≠>◊q <=> ⧄p∧⧄q ◊p↓◊q	 ⧄p<=>◊q <=> ◊p<=>⧄q ◊p<≠>◊q ⧄p<≠>⧄q	 ⧄p<≠>◊q <=> ⧄p<=>⧄q ◊p<=>◊q ◊p<≠>⧄q
	 ⧄p∧⟠q <=> ◊p↓□q ⧄p≠>□q	 ⧄p↑⟠q <=> ◊p∨□q ⧄p=>□q	 ⧄p∨⟠q <=> ◊p↑□q ◊p=>⟠q	 ⧄p↓⟠q <=> ◊p∧□q ◊p≠>⟠q	 ⧄p=>⟠q <=> ⧄p↑□q ◊p∨⟠q	 ⧄p≠>⟠q <=> ⧄p∧□q ◊p↓⟠q	 ⧄p<=>⟠q <=> ◊p<=>□q ◊p<≠>⟠q ⧄p<≠>□q	 ⧄p<≠>⟠q <=> ◊p<=>⟠q ⧄p<=>□q ◊p<≠>□q
	 ⧄p∧⧄q <=> ◊p↓◊q ⧄p≠>◊q	 ⧄p↑⧄q <=> ◊p∨◊q ⧄p=>◊q	 ⧄p∨⧄q <=> ◊p↑◊q ◊p=>⧄q	 ⧄p↓⧄q <=> ◊p∧◊q ◊p≠>⧄q	 ⧄p=>⧄q <=> ⧄p↑◊q ◊p∨⧄q	 ⧄p≠>⧄q <=> ⧄p∧◊q ◊p↓⧄q	 ⧄p<=>⧄q <=> ◊p<=>◊q ◊p<≠>⧄q ⧄p<≠>◊q	 ⧄p<≠>⧄q <=> ◊p<=>⧄q ⧄p<=>◊q ◊p<≠>◊q