Universal yearning for understanding

Individuals do not set the course of events; it is the social force.

F. William Lawvere (1998)

The singular purpose of education is to nurture the universal yearning for understanding. Here we show how a nurturing pedagogy naturally resolves the "learning crisis", with the solution manifesting as human development (alluded to in Muralidharan and Singh, 2021). In doing so, we spell out--for further discussion--how the historic Indian education reforms embodied in the National Education Policy 2020 (NEP; https://www.education.gov.in/sites/upload_files/mhrd/files/NEP_Final_English_0.pdf) can be implemented. Furthermore, the nurturing pedagogy that we advocate invariably results in the culture of research excellence that the national education policy calls for (NEP, pp. 45-46).

First, what is understanding? Understanding is organized knowledge, wherein bits and pieces of knowledge are organized into a cohesive body of understanding "so that the new ideas and methods collected and developed as one goes through life can find their appropriate places" (Lawvere and Schanuel, 1997, p. xiii). As such, our education reforms calls for placing: "emphasis on conceptual understanding rather than rote learning" (NEP, p. 5).

Aren't concepts abstract and therefore difficult to understand? Yes, concepts and ideas are abstract. But, "we all begin gathering mathematical ideas in early childhood, when we discover that our two hands match, and later when we learn that other children also have grandmothers, so that this is an abstract relationship that a child might bear to an older person, and then that 'uncle' and 'cousin' are of this type also" (Lawvere and Schanuel, 1997, p. xiii). This natural fluency in abstract thinking turns into fear of mathematics as a result of the teaching-to-test education.

We (> 50 years old) all remember memorizing multiplication tables all the way up to 16 x 16, not to mention being academically rewarded for the speed with which we recited--without pausing--the multiplication tables correctly in the class. With technological advances (e.g. calculators), this knowing by heart is of little value. A more fruitful exercise--with lasting value--would have been asking: Why 2 x 3 = 6? Yes, it is a sensible question (ibid, p. 9). A first step in answering this question is to state explicitly what we did in answering 2 x 3 = ? We picked a number (6) from all the numbers that there are. In order to pick one out of many, the picked one must be distinct from the rest, i.e. it must be unique. Further reflection reveals that that uniqueness is with respect to the given operation (multiplication) and the given factors (2 and 3), which simply means that if we change the operation to addition, i.e. 2 + 3 = , then the number on the right-hand side would be different (5); so is the case with changing the factors, i.e. 3 x 4 is no longer 6. The key point to note here is the uniqueness of sums and products. Working slowly along these lines leads to the definitions of PRODUCT and SUM, which capture the mathematical content of everyday concepts 'AND' and 'OR', respectively (ibid, pp. 216-222).

In searching for basic notions and in constructing fundamental concepts, mathematicians think slowly. In contrast, pre-university college students are encouraged to calculate, for example, the number of subsets (or parts) of a set X--at lightning speed--using the formula 2^|X|, but rarely nudged to pause and ponder: Where did the 2 in the formula 2^|X| come from? 2 is the two-element set {false, true}. This relationship between sets and logic is not visible in the current mathematics curriculum, where set theory and logic are taught as separate subjects. Conceptual Mathematics transcends these artificial boundaries separating algebra, arithmetic, calculus, geometry, and logic, thereby bringing the unity of mathematics into clear focus for all to see and use (ibid, p. xiii). In fact, "explicit use of the unity and cohesiveness of mathematics sparks the many particular processes whereby ignorance becomes knowledge" (Lawvere, 1991, p. 2; see also Ehresmann, 1966).

What exactly is the unity of mathematics? For example, logic is the algebra of parts (Lawvere and Rosebrugh, 2003, pp. 193-212; Lawvere and Schanuel, 1997, pp. 335-352; see also https://philpapers.org/rec/POSSAL). Algebra is the opposite of geometry (Lawvere and Schanuel, 2009, pp. 81-85, 369-371; Lawvere, 2016). In everyday terminology, if we think of geometry as looking at [the back of] a car in front of us, algebra is like looking into the rear-view mirror to see [the front of] a car behind us. Notice that we can overtake the car in front of us to see its front in the rear-view mirror just as we can let the car behind us overtake us to see its back, which is an informal description of how we solve difficult geometric problems by translating them into algebra (and vice versa). Next, how does the unity--transcending artificial disciplinary boundaries--of mathematics contribute to the advancement of mathematics? Upon recognizing the unity of mathematics, Professor F. William Lawvere (https://www.acsu.buffalo.edu/~wlawvere/) showed that the foundation of mathematics is within mathematics, i.e. mathematical foundation does not need a language distinct from that of mathematics (Lawvere and Rosebrugh, 2003, pp. ix-x, 235-236; Lawvere, 2003). In this spirit, our new education policy emphasizes multidisciplinarity and unity of knowledge (NEP, p. 5).

Taking cognizance of the fact that "young children learn and grasp nontrivial concepts more quickly in their mother tongue" (NEP, p. 13), our national education policy calls for teaching mathematics in students' mother tongue. The medium of instruction in government high schools is, in fact, students' mother tongue, but the medium of instruction of all higher education (beginning with undergraduate engineering) is English. As a result of which, even bright students, who graduate from government high schools with excellent grades, all of whom are from low-income families, find it difficult to excel in higher education. In an effort to remove this additional burden placed on the already [financially] disadvantaged student population, our national education policy calls for setting up of Indian Institute of Translation and Interpretation (IITI) dedicated to translating advanced scientific texts into students' mother tongues, so that all students, irrespective of their family financial status, can excel in higher education (NEP, pp. 53-55).

In implementing any policy, especially one of vast reach and immense impact such as the Indian national education policy (NEP), the first step is more than a step: it is the foundation on which the future of India rests; it is the rocket that launches the national education policy satellite into an orbit of research excellence! In light of the gravity of the first step in implementing our national education policy, we find that Bengaluru is well placed to house IITI. Why Bengaluru? Bengaluru is reflective of the diversity of India. Kannadigaru, with their gracious hospitality, transformed Bengaluru into a home of all Indian languages. With people speaking every Indian language living and making a living in Bengaluru, Bengaluru has all the resources needed for fulfilling the mandate of IITI.

Another equally immediate question that the IITI needs to address is: Which advanced scientific text to translate? We suggest translating Conceptual Mathematics (Lawvere and Schanuel, 2009) into Kannada as ಪರಿಕಲ್ಪನಾ ಅಧ್ಯಯನ (Parikalpanā Adhyayana, based on the meaning of the etymological roots of the word: mathematics), and making the Kannada textbook ಪರಿಕಲ್ಪನಾ ಅಧ್ಯಯನ (Parikalpanā Adhyayana) freely available online and in print to students attending government schools. Why Conceptual Mathematics or ಪರಿಕಲ್ಪನಾ ಅಧ್ಯಯನ (Parikalpanā Adhyayana)? Before we answer this question, we must note that Lawvere and Schanuel (2009) Conceptual Mathematics is a first introduction to categories (or universes of discourse). A universe of discourse, say, a category of cats is one in which every cat in the category has the essence (in the sense of the way parts of a whole stick together; see Lawvere and Schanuel, 2009, p. 146; Lawvere, 2015) 'catness', and every transformation of any cat is a natural transformation (e.g. transformation of a small cat into a big cat, preserves the catness that characterizes the category of cats). Therefore, what we refer to as Conceptual Mathematics is also known as Category Theory or Theory of Naturality (Eilenberg and MacLane, 1945; see also Lawvere, 2006, p. 2).

Now we address--from a broad perspective--the question:

Why Conceptual Mathematics or ಪರಿಕಲ್ಪನಾ ಅಧ್ಯಯನ (Parikalpanā Adhyayana)?

There is one thing that we have been doing since the dawn of free will, which is socializing. We have been telling stories and we have been listening to songs. In participating in the conscious practice of communicating with one another, we abstracted the essence--grammar--of language, which guides the practice of expressing our thoughts in words and sentences, such that they are understood in the intended sense. Recognizing the importance of understanding one another in sustaining social unity, we teach grammar early in education.

In interacting with our fellow human beings, we are also in constant interaction with reality. This interaction, which includes telling and listening, takes the form of thinking about things and making things we think of (for example, pots, music, and friendships). Participation in the conscious practice of thinking-and-making led to the abstraction of its essence--science in general and mathematics in particular--that we can use to live in harmony with nature. Conceptual Mathematics or ಪರಿಕಲ್ಪನಾ ಅಧ್ಯಯನ (Parikalpanā Adhyayana) is all about this grammar of life: the essence of thinking about things and making things we think of (see Lawvere, 1994a, 2003; http://www.mat.uc.pt/~picado/lawvere/interview.pdf, p. 6). Just as we teach grammar--the essence of telling and listening--so that we can communicate and collaborate to sustain our society, we also need to teach Conceptual Mathematics or ಪರಿಕಲ್ಪನಾ ಅಧ್ಯಯನ (Parikalpanā Adhyayana)--the essence of thinking and making--so that we can continue to search for evermore refined answers to the most basic question: How should we live our lives in harmony with reality?

Now, we present a few examples--all too familiar in our everyday experience--of what we learn in Conceptual Mathematics or ಪರಿಕಲ್ಪನಾ ಅಧ್ಯಯನ (Parikalpanā Adhyayana; see also https://philpapers.org/rec/POSSOK).

The way we think about things depends on the nature of the things. For example, I cannot be both dead and alive, but India's boundary is both India and not India (Lawvere, 1994a, p. 48; Lawvere, 2003, pp. 214-215; Lawvere and Rosebrugh, 2003, p. 201).

All that there is to know about any object is in its relationships (Lawvere and Schanuel, 2009, pp. 369-371). For example, you can find all about me by looking at my contacts.

A part of a whole is both itself and its relationship to the whole (Lawvere, 1994a, p. 53). For example, my head is not only my head but also includes its connections to my body.

Being determines Knowing (Lawvere, 2003, p. 217). For example, I cannot find the name of a child, who does not understand English, by asking: What is your name?

What is it good for? Imagine a child pointing to a pen and asking: what is it? We describe PEN in terms of 'what it is good for', which is WRITING. This is also how we define mathematical objects (Lawvere and Rosebrugh, 2003, pp. 26-29; Lawvere and Schanuel, 2009, p. 334).

Equipped with a conceptual understanding, which develops slowly, of these examples and their vast scientific reach (Lawvere, 1992), students of Conceptual Mathematics or ಪರಿಕಲ್ಪನಾ ಅಧ್ಯಯನ (Parikalpanā Adhyayana) can extract the mathematical (meaning: teachable and learnable; Lawvere, 1999) content of any subject matter. (This is not unlike the scenario, wherein learning grammar is useful in writing epics, executive summaries, etc.) As such, teaching Conceptual Mathematics or ಪರಿಕಲ್ಪನಾ ಅಧ್ಯಯನ (Parikalpanā Adhyayana) to pre-university college, undergraduate, post-graduate, and doctoral students will endow them with the intellectual freedom needed to formulate and answer original scientific questions.

How does this nurturing pedagogy translate into research excellence? It is in teaching--teaching in an understandable manner--that we see the world anew: "the need to explain daily for students is often the source of new mathematical discoveries" (Lawvere in http://www.mat.uc.pt/~picado/lawvere/interview.pdf, p. 13; see also Lawvere, 2003, 2005a). One such discovery is the category of sets (Lawvere, 1994b, 2005b). Along these lines, in explaining that "the motion of a rock means more than its track" (Lawvere and Schanuel, 2009, p. 3), we find that that "more" is the space containing the track (ignoring this obvious "more" delayed the discovery of mathematical categories and, in turn, Conceptual Mathematics). Additional explanation of motion involves stating the even more obvious: rock remains rock during its motion. More broadly, every change of any object [of a category] preserves its essence (as alluded to earlier). For example, the transformation of young Posina into old Posina preserved Posina, and hence is a natural transformation (or Becoming consistent with Being; see Lawvere and Rosebrugh, 2003, pp. 135, 173, 235-236, 241; Lawvere and Schanuel, 2009, pp. 152, 369-370, 378). Simply put, aging didn't tear me apart (ibid, p. 146, 210).

Is there a major outstanding scientific problem that can be addressed using Conceptual Mathematics/Category Theory/Theory of Naturality or ಪರಿಕಲ್ಪನಾ ಅಧ್ಯಯನ (Parikalpanā Adhyayana)? Yes, there is (see Fodor, 1998a; Núñez et al., 2019). In fact, category theory has already been applied to model various scientifically intractable notions such as the self that are encountered in consciousness studies (Ehresmann and Vanbremeersch, 2007; see also https://link.springer.com/journal/10516/volumes-and-issues/19-3). Professor Andrée Ehresmann (https://ehres.pagesperso-orange.fr/) has been developing category theoretic constructs specifically designed to meet the demands of consciousness studies (Ehresmann, 1997, 2002).

Category theory has also been of immense value in the foundational studies of automata/machines (Arbib and Manes, 1975, 1980), which can guide the development of a theoretical basis for the contemporary artificial/machine intelligence. In a true multidisciplinary spirit that our NEP aspires, Professor Michael Arbib made foundational contributions to many disciplines (https://scholar.google.com/citations?user=it1vhYAAAAAJ&hl=en).

Reality consists of not only things and their motions and transformations, but also their reflections in our minds (as ideas or concepts) and consciousness (as percepts; Posina, 2019, 2020a; see also Lawvere, 1994a, 2004; Lawvere and Schanuel, 2009, pp. 84-85). Thus, we need a Category of Reflecting (with adjoint functors as objects; see ibid, pp. 369-377; Lawvere, 2016; http://www.math.union.edu/~niefiels/13conference/Web/; Posina, 2020b), in addition to the more familiar Categories of Being (or unity/cohesion objectified as reflexive graphs; Lawvere, 2007; see also https://zenodo.org/record/3924474#.YP9yCr0zbZ4) and Becoming (or change/variation objectified as dynamical systems; see Lawvere and Schanuel, 2009, pp. 135-141), to scientifically reconstruct reality by synthesizing ontology and epistemology into which reality is analyzed (cf. Posina, 2020c).

The kinship between science and cognition has been recognized time and again by scientists working in diverse disciplines such as mathematics, philosophy, and physics. For example, "science is nothing more than a refinement of everyday thinking" (Einstein, 1936/2003, p. 23; see also Fodor, 2006, p. 93; Lawvere, 2013a; Posina, 2020b; Posina, Ghista, and Roy, 2017; Schapira, 2016). When viewed from the perspective of this propinquity between scientific knowing and ordinary cognition, it is clear that we now have a good opportunity to build cognitive science (Fodor, 1998b) based on the "precise mathematical model for a very general scientific process of concept formation" (Lawvere, 2013b, p. 7; see also Lawvere, 2002, pp. 267-268; Lawvere and Rosebrugh, 2003, pp. 235-236; Lawvere and Schanuel, 2009, pp. 127-129, 380).

More than anything else, we need intellectual investment! We need to invest in learning, teaching, and translating Conceptual Mathematics/Category Theory/Theory of Naturality into ಪರಿಕಲ್ಪನಾ ಅಧ್ಯಯನ (Parikalpanā Adhyayana) so that we too can contribute our share to the world of science as envisioned in our education reforms (NEP, pp. 45-46). Translating Conceptual Mathematics (Lawvere and Schanuel, 2009) into ಪರಿಕಲ್ಪನಾ ಅಧ್ಯಯನ (Parikalpanā Adhyayana) appears all the more significant in light of the ideal assessment of one's understanding:

Write in your own words!

Most important of all, we wholeheartedly thank Professor Manjul Bhargava--on behalf of all Indian students, which include us--for taking his valuable time to draft our National Education Policy (https://www.ias.edu/news/2020/indian-nep).

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