A Scheme Foiled: A Critique of Baron's Account of Extra-mathematical Explanation

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Abstract. Extra-mathematical explanations explain natural phenomena primarily by appeal to mathematical facts. Philosophers disagree about whether there are extra-mathematical explanations, the correct account of them if they exist, and their implications (e.g., for the philosophy of scientific explanation and for the metaphysics of mathematics) (Baker 2005, 2009; Bangu 2008; Colyvan 1998; Craver and Povich 2017; Lange 2013, 2016, 2018; Mancosu 2008; Povich 2019, 2020; Steiner 1978). In this discussion note, I present three desiderata for any account of extra-mathematical explanation and argue that Baron's (2020) U-Counterfactual Theory fails to meet each of them. I conclude with some reasons for pessimism that a successful account will be forthcoming.

1. Introduction

Extra-mathematical explanations[1] explain natural phenomena primarily by appeal to mathematical facts. Philosophers disagree about whether there are extra-mathematical explanations, the correct account of them if they exist, and their implications (e.g., for the philosophy of scientific explanation and for the metaphysics of mathematics (Baker 2005, 2009; Bangu 2008; Colyvan 1998; Craver and Povich 2017; Lange 2013, 2016, 2018; Mancosu 2008; Povich 2019, 2020; Steiner 1978). In this discussion note, I present three desiderata for any account of extra-mathematical explanation and argue that Baron's (2020) U-Counterfactual Theory fails to meet each of them.

In section 2, I briefly elaborate on extra-mathematical explanation and present the three desiderata: the modal, distinctiveness, and directionality desiderata. In section 3.1, I explain Baron's (2020) recent U-Counterfactual Theory, and in sections 3.2-3.4, I argue that it fails to meet each of the desiderata. In section 4, I conclude with some reasons for pessimism that a successful account will be forthcoming.

2. Extra-mathematical Explanations

Extra-mathematical explanations work primarily by showing a natural explanandum to follow in part from a mathematical fact. Many[2] extra-mathematical explanations thus show that the explanandum had to happen, in a sense stronger than any ordinary causal law can supply. As a paradigmatic example, consider Terry's trefoil knot (Lange 2013). The explanandum is the fact that Terry failed to untie his knot. The explanantia are the empirical fact that the knot is a trefoil

[1] Also called distinctively mathematical explanations (Lange 2013, 2016, 2018).

[2] I will be noncommittal here about whether they all work this way.

knot and the mathematical (knot theoretic) fact that the trefoil knot is distinct from the unknot (i.e., mathematically cannot be untied). The unknot is a single closed loop (think torus or donut), while the trefoil knot has three crossing loops. That the trefoil knot is distinct from the unknot, and so, mathematically, cannot be untied, means that there are no 'admissible' moves of twisting, lifting, or crossing strands without cutting them (the so-called Reidemeister moves) that can transform the trefoil knot into the unknot. Thus, the explanantia ensure mathematically that Terry will fail to untie his knot; his success is mathematically impossible.

This example illustrates three desiderata for an account of extra-mathematical explanation: modality, distinctness, and directionality.

> *The Modal Desideratum*: an account of extra-mathematical explanation should accommodate and explicate the modal import of some extra-mathematical explanations. (Baron 2016)

Terry's failure is modally robust— he could not succeed. An account of extra-mathematical explanation should capture and explicate this modal robustness. (Note that this desideratum allows that some extra-mathematical explanations are not modally robust; see fn. 2).

> *The Distinctiveness Desideratum*: an account of extra-mathematical explanation should distinguish uses of mathematics in explanation that are extra-mathematical from those that are not. (Baron 2016)

Bromberger's (1966) flagpole[3] is an example of an explanation that uses mathematics but is not an extra-mathematical explanation. The explanandum is the fact that the length of a flagpole's shadow is L. The explanantia are the empirical facts that the angle of elevation of the sun is θ

[3] The example actually comes from Salmon (1989), who gives it the name "Bromberger's flagpole". Bromberger (1966) himself uses slightly different examples to make the same point.

and that the height of the flagpole is H and the mathematical fact that tan $\theta = H/L$. Thus, there are two ways an account of extra-mathematical explanation might fail to meet the distinctiveness desideratum: it might count as extra-mathematical an explanation that is not, and it might count as not extra-mathematical an explanation that is.

> *The Directionality Desideratum*: an account of extra-mathematical explanation should accommodate the directionality of extra-mathematical explanation. (Craver and Povich 2017; Povich and Craver 2018)

Craver and Povich argue that, analogously to Bromberger's flagpole explanation, the explanation of Terry's trefoil knot can be 'reversed'[4] to form an argument that fits Lange's (2013) account of extra-mathematical explanation but is not explanatory. In fact, there's an algorithm for such a reversal: Simply take the explanandum and the empirical premise, swap them, and negate them, akin to turning a modus ponens into a modus tollens. Thus, change the explanandum to "Terry's knot is not trefoil." Change the empirical premise to "Terry untied his knot." The mathematical premise is the same: the trefoil knot is distinct from the unknot. This reversal should not count as an explanation; Terry's untying his shoelace doesn't explain why his knot is non-trefoil.

These desiderata should not be controversial: the first two were proposed by Baron himself, and the third has been widely accepted in philosophical discussions of explanation since Bromberger (1966). They also help to show why extra-mathematical explanations are distinctive and explanatory. They are arguably constitutive of extra-mathematical explanation. An account

[4] Craver-Povich reversals in this sense are not strict reversals – simple swaps of explanandum and explanans – like the well-known reversal of Bromberger's flagpole. Henceforth, I will drop the scare quotes.

of extra-mathematical explanation that does not meet *further* desiderata – such as, e.g., that the account should comport well with intra-mathematical explanation – would not be ideal, but an account that violates the modality, distinctiveness, or directionality desiderata is arguably not an account of extra-mathematical explanation at all.[5]

3.1 Baron's U-Counterfactual Theory

Baron (2020) has recently presented what he calls the U-Counterfactual Theory of extra-mathematical explanation ('U' for unifying or unification). The U-Counterfactual Theory makes use of countermathematicals – counterfactuals with mathematically impossible antecedents, which I assume for the sake of argument are not trivially or vacuously true (Baron, Colyvan, and Ripley 2017). Baron's central explanatory concept, which demarcates explanatory from non-explanatory countermathematicals, is the 'generalized counterfactual scheme'. According to the U-Counterfactual Account, roughly, a countermathematical is explanatory just when it is an instance of a generalized counterfactual scheme.

A generalized counterfactual scheme (similar to Kitcher's [1989] argument schemes) consists of 1) a counterfactual in which some or all of the non-logical expressions have been replaced with variables, 2) a set of filling instructions specifying the values the variables can take, and 3) a classification, which explains how an instance of the scheme is to be evaluated (Baron 2020).

On Baron's full account, a counterfactual CF, featuring a mathematically impossible antecedent, is explanatory just when:

(i) CF is an instance of a counterfactual scheme CS such that:

[5] Thanks to an anonymous reviewer for pressing me here.

> (1) All of the instances of CS are true.
>
> (2) For at least two instances of CS, CF1 and CF2, CF1 and CF2 are nomically distinct.
>
> (ii) There is no other counterfactual scheme CS* such that:
>
> (1) All of the instances of CS* are true.
>
> (2) For each instance of CS with consequents $c_1 \ldots, c_n$, there is a true instance of CS* with exactly that consequent.
>
> (3) For each instance of CS*, none of the antecedents of those instances involve a mathematical impossibility.
>
> (4) Each instance of CS is true, because the mathematical twiddles that realize each counterfactual's antecedent change the physical features in CS* that are responsible for unification in that scheme. (Baron 2020, p. 556)

CF1 and CF2 are nomically distinct when the physical laws relevant to the evaluation of those counterfactuals are different. The degree to which a counterfactual is explanatory is proportional to the number of nomically distinct instances of its associated generalized counterfactual scheme (Baron 2020, p. 549).

Baron (2020) uses the well-known cicada case (Baker 2005) to show how the U-Counterfactual Theory works. As Baron presents the case[6], the explanandum is the fact that two subspecies of cicada possess life cycles of 13 and 17 years, respectively. The explanation relies crucially on the number-theoretic fact that 13 and 17 are both co-prime with each of 2, 3, 4, 6, 7,

[6] In Baker (2005, p.230), the explanandum is slightly different: the fact that cicada life cycle periods are prime.

8, and 9. To fit this example to the U-Counterfactual Theory, we need a generalized counterfactual scheme, such as:

(CS1) If $x_1 \ldots x_n$ had not been co-prime with y_1, y_2, …, or y_m, the p_1, p_n would not have had x_n U Cs.

The filling instructions are:

(1) The p_n are periodical phenomena within any actual or physically possible system S that is under pressure to optimize some feature and where that feature is optimized just when for periodical phenomena $p^*_1 \ldots p^*_m$ that are in S and that are distinct from the p_n, the frequency of intersection between the p_n and the p^*_m is minimized.

(2) The x_i are numbers that are bijectively mapped to the p_n.

(3) The y_i are numbers that are bijectively mapped to the p^*_m.

(4) U is the unit of the p_n (e.g. years).

(5) The Cs are the type of period that characterizes the p_n (e.g. life cycles). (Baron 2020, p. 550)

Now consider this countermathematical:

(CF1) If 13 and 17 had not been co-prime with 2, 3, 4, 5, 6, 7, 8 or 9, then North American cicadas would not have had 13- or 17-year life cycles. (Baron 2020, p. 542)

This countermathematical is an instance of the abovementioned generalized counterfactual scheme CS, reached by the abovementioned filling instructions. Furthermore, all of the instances of CS are true, and there is, according to Baron, plausibly no other counterfactual scheme that meets the criteria in (ii) above. I am skeptical of this last claim and will return to it in section 3.3.

Furthermore, the U-Counterfactual Theory requires that there be at least two instances of CS that are nomically distinct. Baron's second instance uses an example of rotating gears. In this case, the explanandum is the fact a hypothetical company that aims to manufacture the longest lasting engine they can, manufactures an engine with large gears with either 13 or 17 teeth. The explanation relies on the number theoretic fact that 13 and 17 are both co-prime with each of 2, 3, 4, 6, 7, 8, and 9. Supposing the company is constrained to manufacture small gears with between 2 and 9 teeth per gear and large gears with between 12 and 18 teeth per gear, large gears with either 13 or 17 teeth minimize wear on the small gears, maximizing the engine's longevity (Baron 2020, p. 546). This leads to the second instance of CS:

> (CF2) If 13 and 17 had not been co-prime with 2, 3, 5, 6, 7, 8 or 9, the large gears in the company's engine would not have had 13 or 17 period rotations. (Baron 2020, p. 550)

CF1 and CF2 are nomically distinct, according to Baron (p. 551), because the evaluation of CF1 involves the laws of evolution and natural selection, while the evaluation of CF2 involves the laws of mechanics. (One might deny that there are laws of evolution and natural selection and that these cases are nomically distinct. Here I assert only the conditional: if these two cases are nomically distinct, then so too are the two problem cases in section 3.3 below.)

Since there are at least two nomically distinct instances of CS and all other conditions of the U-Counterfactual Theory are satisfied, CF1 and CF2 count as explanatory countermathematicals, and the cicada and gear cases count as extra-mathematical explanations.

3.2 The Modal Desideratum

Though Baron (2020) does not consider whether the U-Counterfactual Theory meets the modal desideratum he presented in earlier work (Baron 2016), it seems to me that it does not. There is nothing necessary about Baron's explananda, the instances of 'the p_1, p_n have x_n U Cs' or 'the length of P is A U'. Recall that perhaps not all explananda of extra-mathematical explanations are necessary. Perhaps these explananda – these *sets* of explananda, since these descriptions contain variables that can be filled in specific cases – are contingent. But even if the explanandum were necessary – and Baron thinks *some* explananda are – there is nothing in the U-Counterfactual Theory that explicates its necessity. Thus, even if Baron's account adequately handles extra-mathematical explanations with contingent explananda, it cannot handle those with necessary explananda, and thus is incomplete as an account of extra-mathematical explanation.[7]

3.3 The Distinctiveness Desideratum

Baron's theory also fails to meet the distinctiveness desideratum, for two reasons: 1) it incorrectly counts his own paradigm example of extra-mathematical explanation as not extra-mathematical, and 2) it incorrectly counts Bromberger's flagpole example as an extra-mathematical explanation.

Recall that Baron asserts that there is no other counterfactual scheme that meets the criteria in (ii) above. I can now explain why I am skeptical of this. Consider a scheme that Baron says is not explanatory because its unifying power traces to the existence of an underlying physical twiddle: If x/y had not equalled z, then c would not have ended at B*. Baron says this scheme has these two nomically distinct instances: 1) if 10/10 had not equalled 1, then train T's journey would not have ended at 3 p.m., and 2) if 50/1 had not equalled 50, then Suzy's refuelling of her car would not have ended at 70 litres (p. 555). The scheme is not explanatory because its unifying power is 'due to an underlying physical correlate – an exchange rate [i.e., a rate of change; in the train case it is kilometers per hour and in the fuel case it is dollars per liter]

[7] It will not do to say that being the consequent of a true countermathematical explicates the requisite necessity (when the explanandum is in fact necessary), since that would falsely imply that every true countermathematical has a necessary consequent. The countermathematicals throughout this paper are plausibly true and have contingent consequents. Here is an unrelated, uncontroversial example: If Hobbes had squared the circle, sick children in the mountains of South America at the time would not have cared (Nolan 1997).

– that we can get at by twiddling the mathematics' (p. 558). Baron then claims that 'There is no general physical twiddle that we can make to both the cicada system and the L-Engine system that would have the same upshot for both cases as the one produced by altering the co-primeness of 13 and 17' (2020, pp. 558-9). But it strikes me that if rate of change can count as an underlying physical correlate we can get at by twiddling the mathematics in the train and fuel instances, then so can frequency of intersection in the cicada and gear instances. The relevant counterfactual scheme would be something like[8]:

> (CS1*) If the minimum frequency of intersection between the p_n and the p^*_m had been different, the p_1, p_n would not have had x_n U Cs.

[8] I think the following also works, but might be a bit more controversial:

> (CS1**) If $x_1 \ldots x_n$ U Cs had not minimized the frequency of intersection with y_1, y_2, …, or y_m U Cs, the p_1, p_n would not have had x_n U Cs.

with instances

> (CF1**) If 13- and 17-year life cycles had not minimized the frequency of intersection with 2-, 3-, 5-, 6-, 7-, 8- or 9-year life cycles, then North American cicadas would not have had 13- or 17-year life cycles.

> (CF2**) If 13 and 17 period rotations had not minimized the frequency of intersection with 2, 3, 5, 6, 7, 8 or 9 period rotations, the large gears in the company's engine would not have had 13 or 17 period rotations.

I say these might be more controversial because one might think that the antecedents are mathematically impossible, but I do not think they are. They look superficially like mathematical impossibilities, but they are statements of *physical* impossibility that contain numerals. Compare: 'If 2 sets of 2 o had not resulted in 4 o, then…', where 'o' is an object variable. This antecedent is also a statement of physical impossibility that contain numerals and looks superficially like a mathematical impossibility. Perhaps in such a world a new object appears or disappears whenever 2 sets of 2 objects are gathered. In CF1** and CF2**, perhaps at certain times cicadas/gears appear or disappear or entire years/rotations appear or disappear. Such a world would be a strange world indeed, a physically impossible world certainly, but not mathematically impossible.

where the filling instructions for the relevant variables are the same, yielding the following instances:

(CF1*) If the minimum frequency of intersection between North American cicadas and their predators had been different, then North American cicadas would not have had 13- or 17-year life cycles.

(CF2*) If the minimum frequency of intersection between large and small gears had been different, the large gears in the company's engine would not have had 13 or 17 period rotations.

Note that the minimum frequency of intersection must change if the mathematical twiddling in CS1 is to do its work. Baron makes much of this point for the train and fuel cases. If the minimum frequency of intersection between the p_n and the p^*_m does not change when the mathematical twiddling occurs, then the gear and cicada explananda remain the same, making the relevant instances of CS1 false. Changes in co-primeness have – and can only have – their intended effects on the explananda *because* these changes alter the minimum frequency of intersection. Thus, CF1* and CF2* are true, and CF1 and CF2 are true *because* CF1* and CF2* are true, as required by condition ii.4[9]. Thus, Baron's theory fails to meet the distinctiveness desideratum because it incorrectly counts his own paradigm example of an extra-mathematical explanation as not extra-mathematical.

Now I argue that Baron's theory incorrectly counts the case of Bromberger's flagpole as an extra-mathematical explanation. I present below a generalized counterfactual scheme and

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[9] In fact, I am suspicious of condition ii.4 in general, because I think *any* mathematical twiddling must have *some* physical correlate, not just for each instance of a scheme, but even a very general, 'scheme-level' correlate, if the twiddling is not to be explanatorily idle. But I neither argue for nor rely on this thesis here. See footnote 8 for further discussion.

filling instructions by which a countermathematical can be deduced that, were it explanatory, would make Bromberger's flagpole an extra-mathematical explanation. Since it is agreed by all parties to the debate on extra-mathematical explanation that Bromberger's flagpole is not one, the countermathematical I will present is not explanatory, and the U-Counterfactual Theory fails to meet the distinctiveness desideratum.

Suppose that a flagpole casts a 15 foot shadow, that the angle of the sun's elevation is 40 degrees, and that the flagpole is 12.59 feet tall (approximately). Now consider this counterfactual scheme:

(CS2) If tan z had not equaled x/y, then the length of P would not have been A U.

And these filling instructions:

(1) θ is an acute angle in a Euclidean right triangular system S, O is the length of the side opposite θ in S, and A is the length of the side adjacent to θ in S.

(2) x is a non-negative real number mapped to O.

(3) y is a positive real number mapped to A.

(4) z is a non-negative real number mapped to θ.

(5) P is the adjacent side of a S.

(6) U is a unit of length (e.g., feet).[10]

The following countermathematical is an instance of the generalized counterfactual scheme, reached by following the filling instructions:

(CF3) If tan 40 had not equaled 12.59/15, then the length of the flagpole's shadow would not have been 15 feet.

[10] For simplicity, I am going to ignore the angular units for θ.

Furthermore, the generalized counterfactual scheme CS2 is applicable across nomically distinct systems, since it applies to all right triangular systems, regardless of the physical laws governing those systems, and thus regardless of the physical laws relevant to the evaluation of CS2's instances. Here is another such instance. Suppose a painter is commissioned to paint the spandrel on the right side of a large archway at her local cathedral. She practices on a right triangular canvas which is 15 feet long, 12.59 feet tall, and has an internal angle of 40 degrees. The following countermathematical is an instance, using this example, of the same generalized counterfactual scheme CS2, reached by following the same filling instructions:

> (CF4) If tan 40 had not equaled 12.59/15, then the length of the canvas would not have been 15 feet.

The evaluation of CF3 involves the laws of optics governing the rectilinear motion of light, while the evaluation of CF4 involves the laws of mechanics. Furthermore, all of the instances of CS2 will be true, given that the filling instructions specify that only information pertaining to right triangles can be entered, and there is plausibly no other counterfactual scheme, CS2*, that meets the criteria in (ii) above. Thus, the U-Counterfactual Theory incorrectly counts CF3 and CF4 as explanatory and so counts Bromberger's flagpole and the canvas case as extra-mathematical explanations.

I just stated that there is plausibly no other counterfactual scheme, CS2*, that meets the criteria in (ii) above. However, consider the following[11]:

> (CS3) If the space S occupies had not been locally Euclidean, then the length of P would not have been A U.

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[11] I thank an anonymous reviewer for this suggestion.

with the same filling instructions for the relevant variables. I do not think this will work. It does not seem to be the case that every instance of CS3 is true. It may be true in the standard flagpole case where the length of the shadow is the explanandum, if we imagine keeping the position of the sun and height of the flagpole fixed and curving the space where the shadow is cast, much like curving the ground; then the length of the shadow will change. However, it does not generally seem to be the case that changing the curvature of space results in a change in the lengths of objects occupying it. A meter long rod is still a meter long when slightly curved.

I do not think my response is conclusive, because there are some difficult conceptual issues surrounding the evaluation of this counterfactual. For example, I claimed that a meter long rod is still a meter long when slightly curved. But this depends on what we mean by 'length'. I am relying on a non-Euclidean notion of length that, so to speak, 'follows the curve' of the rod. But if by 'length of the rod' we mean the distance of the *Euclidean* straight line connecting two ends of the rod, then a slightly curved rod is slightly shorter. When imagining the truth of the antecedent, what notion of length should we employ when evaluating the consequent: Euclidean or non-Euclidean? If Kripke (1980, p. 77) is right that in counterfactual reasoning we continue to use our actual conceptual conventions, it seems as though we should employ a Euclidean notion of length rather than a non-Euclidean one. On the other hand, Kocurek, Jerzak, and Rudolph (2020) have provided convincing counterexamples to Kripke's rule. Instead of trying to resolve these conceptual issues here, though, it is enough for me simply to say this. 1) If Baron keeps criterion ii.4, then the cicada case is not an extra-mathematical explanation, since the frequency of intersection is an underlying physical correlate of both the cicada and gear cases that we can get at by twiddling the mathematics. The cicada case is his *paradigm* extra-mathematical explanation, so this constitutes a failure to meet the distinctiveness desideratum. Furthermore, depending on the conceptual issues surrounding the evaluation of CS3 just mentioned, the flagpole case may count as an extra-mathematical explanation, which also constitutes a failure to meet the distinctiveness desideratum. 2) If Baron drops criterion ii.4, then the cicada case remains an extra-mathematical explanation, but the flagpole case now certainly counts as an extra-mathematical explanation, which constitutes a failure to meet the distinctiveness desideratum. Either way, Baron's theory fails to meet the distinctiveness desideratum.

3.4 The Directionality Desideratum

With trivial changes to the flagpole countermathematical CF3, we can show that the U-Counterfactual Theory also incorrectly counts the *reversal* of Bromberger's flagpole as an extra-mathematical explanation. Take the height of the flagpole as the explanandum and simply change CF3 to:

> (CF5) If tan 40 had not equaled 12.59/15, then the height of the flagpole would not have been 12.59 feet.

Could Baron adopt Lange's (2018) proposed solution to Craver-Povich reversals here? According to Lange, the fact described in the empirical premise in Craver-Povich reversals is not understood to be 'constitutive of the physical task or arrangement at issue'. In the 'forward' case, it is understood to be constitutive of Terry's knot that it is trefoil. In contrast, in the reversal, it is not understood to be constitutive of Terry's knot that he untied it.

This response will not work for Baron. First, there is nothing in Baron's account remotely like this – there are no empirical premises/explanantia that could be understood as constitutive of the physical task or arrangement at issue. Second, even if Lange's proposal could somehow be grafted ad hoc onto Baron's account, it is unclear whether it would succeed (see Povich's 2020 response to Lange 2018). Third, this reversal is not of the Craver-Povich type, which is designed to target Lange's account and that Lange's response is supposed to avoid. This is a version of the standard flagpole reversal (see footnote 4 above and the paragraph in which the footnote occurs). Thus, Baron's U-Counterfactual Theory cannot satisfy the directionality desideratum.

4. Conclusion

Baron's (2020) U-Counterfactual Theory cannot satisfy the desiderata on an account of extra-mathematical explanation. What goes wrong with the account? What does it get right? And is there a general ground for pessimism that any account can satisfy these desiderata? First, what does it get right? If there are extra-mathematical explanations, a counterfactual theory is a promising place to look (e.g., Pincock 2015; Povich 2019; Reutlinger 2016), and Baron's previous work with Colyvan and Ripley (Baron, Colyvan, and Ripley 2017) on the evaluation of countermathematicals provides a key step in the development of any viable counterfactual account of extra-mathematical explanation. However, what goes wrong, in my opinion, is the emphasis on unification and lack of emphasis on anything ontic. Recall that appeal to something ontic, namely causation, secures the directionality or asymmetry of explanation in the flagpole case (Craver 2014, Salmon 1989). Ontic accounts like Pincock's (2015) and Povich's (2019) are well-suited to meet the directionality and modal desiderata by tying extra-mathematical explananda to necessary mathematical facts. On the other hand, the Platonism of extant ontic accounts saddles them with well-known metaphysical and epistemological problems, and more. Kuorikoski (2021) has recently argued that ontic accounts require a 'same-object' condition to ensure that the countermathematicals are really describing explanatory (rather than merely epistemic) dependence relations. However, Kuorikoski argues, this requirement cannot be met because, when evaluating countermathematicals, we cannot distinguish whether we are conceiving of a change in a given mathematical structure or conceiving of a *different* mathematical structure. Much work remains to find a successful account of extra-mathematical explanation.

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