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Chapter 1. Introduction: Scientific Explanation, Mathematics, and Metaontology

Abstract: In this chapter I introduce themes that will recur throughout the book, including the ontic conception of scientific explanation, deflationary Carnapian metaontology, and the enhanced indispensability argument (EIA) for platonism.

Keywords: metaontology, mathematics, ontic conception, platonism, scientific explanation, indispensability

1.1. Introduction: Scientific Explanation

One central aim of science is to provide explanations of natural phenomena. Explanation is a scientific achievement distinct from description, prediction, and confirmation. What is the nature of the achievement of explanation? What role(s) does mathematics play in achieving this aim? How does mathematics contribute to the explanatory power of science? In this book, I defend answers to these questions. Specifically, I defend an ontic conception of scientific explanation, a normativist account of mathematics, and I combine these into a normativist-cum-ontic account of the distinctive role of mathematics in scientific explanation. In this section, I briefly expand on these views, which recur throughout the book.

The central idea of mathematical normativism, common though perhaps inchoate among many members of the Vienna Circle, is that mathematics contributes to the explanatory power of science by *expressing* conceptual or semantic[1] norms or rules, primarily rules for transforming

[1] Where irrelevant, I ignore the distinctions between terms and concepts, and between sentences and propositions. This shouldn't affect any of the points I make here.

empirical descriptions.[2] Mathematics should not be thought of as *describing*, in any substantive sense, anything, let alone an abstract realm of eternal mathematical objects, as traditional platonists have thought. A pure mathematical claim such as “3 is prime” should be thought of as expressing a semantic rule according to which it is *correct* to apply “is prime” when it is *correct* to apply “3”. We then use this rule to transform empirical descriptions such “there are three particles” into “there are a prime number of particles”.

In the following chapters, I update this normativist view of mathematics with contemporary philosophical tools like semantic deflationism, the idea, roughly, that the truth concept is governed solely by the ‘equivalence schema’: ‘*p*’ is true if and only if *p*.[3] Truth is thus not a substantive property. This is not to deny that there are substantive ways the world must be for some propositions to be true. But the substantiveness of the truth of “*p*” comes down to the substantiveness of the fact that *p* (and vice versa). For example, the truth of “snow is white” requires that the world be a certain way – such that snow is white; whereas the non-substantiveness of the truth of, say, “2 is prime” comes down to the non-substantiveness of the fact that 2 is prime. As I explain below, this truth and its associated fact are non-substantive because they are analytic. I argue that this combination of normativism and deflationism is compatible with the mainstream semantic theory, truth-conditional semantics. This allows the normativist to accept that there are mathematical truths and that they can play explanatory roles

[2] This is idea goes back at least to Wittgenstein (1956/1978, 2013).

[3] Or: the proposition that *p* is true if and only *p*. There are many ways of cashing out semantic deflationism (see, e.g., Horwich 1998a), and for my purposes I don’t need to commit to any of them.

in science, while resisting the platonistic idea that there exist abstract mathematical objects that explain such truths or explain the truth of such truths.

I combine this philosophy of mathematics with a particular account of the distinction between scientific explanations that are in some sense distinctively mathematical – that explain natural phenomena in some uniquely mathematical way – and those that are only standardly mathematical. In *standardly* mathematical explanations in science, the mathematics plays a merely representational role, i.e., it merely represents the explanatorily relevant features, such as the quantities and magnitudes associated with explanatorily relevant causes. In *distinctively* mathematical explanations (DMEs), the mathematics is supposed to do something more than this; it bestows upon the explanandum (i.e., the thing to be explained) a kind of necessity that mere effects of causes do not possess (Lange 2013a, 2016). In Chapter 2, I present desiderata for any account of DME and criticize competing accounts for their failure to meet them. In Chapter 8, I critique other prominent views in the philosophy of mathematics such as fictionalism, conventionalism, and neo-Fregeanism.

I call my account of DME – which will be presented in Chapter 4 – the Narrow Ontic Counterfactual Account (NOCA). NOCA is an ontic account of explanation, meaning that it takes the explanandum, explanans (i.e., the thing doing the explaining), and explanatory relations between them to be objective, "worldly" objects, properties, and relations. One might reasonably suspect that an ontic of DME would be committed to platonism. If mathematical facts are among the explanans in DMEs, and these facts are objective, (other-?)worldly facts, then platonism seems to follow inescapably. (Or perhaps there follows an empiricism according to which mathematical facts are concrete, empirical facts, but I won't be going that route either. More on empiricism in Chapter 6.) I will argue that platonism does not follow.

NOCA is an ontic conception of explanation. I hold an ontic conception of scientific explanation generally. That is, I think *all* scientific explanations appeal to objective relations between objective phenomena. In every case, the objective explanatory relation that holds between objective explanans and objective explanandum is a kind of counterfactual dependence relation. The explanandum counterfactually depends on the explanans – if the explanans hadn't occurred, then the explanandum wouldn't have occurred – and this counterfactual dependence holds in virtue of some objective relation between them, such as causation or constitution (or perhaps others). By "objective," I mean mind-independent, in the sense that whether the explanans, explanandum, and explanatory relation between them exist is not up to us explainers. This does not rule out the possibility of explanation in the cognitive sciences; brains, beliefs, and so on are perfectly objective in the sense that matters for explanation, and only on exceedingly controversial and exceedingly rare philosophical views can such things fail to enter into causal or other natural relations.

Thus, all scientific explanations follow a single pattern: they all exhibit (represent) relations of counterfactual dependence that hold in virtue of some ontic relation that holds between explanandum and explanans and in virtue of which they count as explanations.[4] I call

[4] Specifically, what matters is the representation of *patterns* of counterfactual dependence that hold in virtue of some ontic relation. It is not simply that x explains y if and only if y counterfactually depends on x in virtue of some ontic relation that holds between them. Preemption cases and others from the causation literature are relevant here. The breaking of the bottle does not counterfactually depend on Suzy's throwing her rock if Billy threw his right after (or if he threw his at the same time or if he would've thrown his if Suzy hadn't thrown hers). But

this the generalized ontic conception. NOCA thus portrays DME as a component in an intuitive typology of kinds of explanation that are individuated by the ontic relation in virtue of which the relation of counterfactual dependence holds between explanans and explanandum. When the relation of counterfactual dependence holds in virtue of a causal relation between explanans and explanandum, we have a causal explanation. When the relation of counterfactual dependence holds in virtue of a constitutive mechanistic relation between explanans and explanandum, we have a constitutive mechanistic explanation (see Craver 2007, Craver, Glennan, and Povich 2021). When the relation of counterfactual dependence holds between a mathematical explanans and a natural explanandum in virtue of some other relation – perhaps instantiation – we have a DME, but more on this in Chapter 4. NOCA thus unifies causal and non-causal, ontic and modal. We will also see in Chapter 5 that, for the normativist, DME is a very special kind of quasi-causal or quasi-mechanistic explanation.

So, that's the ontic conception, or my version of it. Why hold it? I believe it is the best account of what is distinctive about the scientific achievement of explanation. It satisfies two widely held desiderata for any adequate account of scientific explanation: 1) it demarcates

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this needn't undermine the claim that Suzy's throw explains the breaking of the bottle in these scenarios. For, as Woodward (2003, 86) points out, when it comes to causal explanation, "once we have been given information about the complete patterns of counterfactual dependence in [these kinds of] cases as well as a description of the actual course of events, it appears that nothing has been left out that is relevant to understanding why matters transpired as they did". The existence of preemption cases and others does not preclude a counterfactual account of causal (or other) explanation.

explanation from other scientific achievements, like description, prediction, and confirmation, and 2) it provides norms for evaluating explanations (Craver 2014, Craver and Kaplan 2020). Proponents of the ontic conception "believe one cannot satisfy these desiderata without taking a stance on the kinds of worldly (that is, ontic) relations that a putative explanation must reveal to count as explanatory" (Craver and Kaplan 2020, 294). (Note that it is no part of my understanding of the ontic conception that an explanation *is* something ontic, like a cause or a mechanism, and *not* a representation, text, model, etc.) There are questions surrounding what the norms of explanation are and whether an ontic conception of explanation supplies the right ones. For example, many have thought that an ontic conception implies that the more detailed an explanation, the better (e.g., Batterman and Rice 2014, Chirimuuta 2014). I think Craver and Kaplan (2020) have adequately dispelled this myth.

This is not to deny there is room for "pragmatic context" or "interests" in the evaluation of explanations. It is uncontroversial – and I think consistent with an ontic account – that whether and to what extent a putative explanation reveals relevant worldly facts depends on who is consuming it (Povich 2021). Some might argue that this gives up the game (e.g., Wright and van Eck 2018). If they want to call the generalized ontic conception "the generalized epistemic conception," that's fine. What matters is the view itself. Although many of the arguments in this book refer to "the generalized ontic conception," the substance of those arguments doesn't hinge at all on whether interests are included in the evaluation of explanations or call it "the generalized epistemic conception". For example, in Chapter 3, I argue against certain accounts of RG explanation and in favor of an account in line with the generalized ontic conception. My arguments concern how RG explanations work. Similarly, in Chapters 4, I argue against certain accounts of DME and in favor of an account in line with the generalized ontic conception (i.e., NOCA). In Chapter 5, I argue that NOCA remains consistent with the generalized ontic conception even after its platonistic language is normativistically deflated. All these arguments concern how DMEs work. Nothing at all of substance in this book changes if we include interests in the evaluation of explanations and call my view "the generalized epistemic conception". Wright and van Eck (2018), critics of the ontic conception, cite approvingly Bokulich's (2016) distinction between 'conceptions' of explanation, which concern what explanations *are*, and 'accounts' of explanation, which concern how they work. If you want to call the generalized ontic conception "the generalized ontic *account*," fine by me. I don't mind what you call it; I mind that it is a monistic, counterfactual view of scientific explanation that unifies causal explanations, RG explanations, and DMEs, and that it still covers DMEs once they are normativistically deflated. More on the generalized ontic conception in Chapter 3. For now, let us move on to metaontology.

1.2. Metaontology

I believe the package of views going by various names such as "pragmatism," "functional pluralism," and "deflationary (or minimalist) metaontology" (e.g., Brandom 1994, Price 2011, Thomasson 2014, 2020a) provides the most plausible, illuminating, and naturalistic picture of, well, everything. (Obviously these views are not the same and their proponents have disagreements.) In particular, of human beings and our practices. Functional pluralism is the thesis that not all declarative sentences have the function of describing or representing the world, in any substantive sense (Price 2011).[5] Usually, this functional pluralist thesis is combined with

[5] Sometimes one who believes that a class of terms doesn't describe is called an 'anti-representationalist' about that class.

the claim that thinking otherwise has led much philosophy astray and is the cause many philosophical problems and confusions. Deflationary or minimalist metaontology is basically the thesis that, at least for some things, existence is cheap (see, e.g., Linnebo 2018, Thomasson 2014, Warren 2020). Often, and historically, the cheapness of the existence of some class of entities (e.g., mathematical entities like numbers, sets, etc.) is cashed out in terms of some kind of dependence on language or conceptual scheme. In other words, the existence of such and such entities is cheap because they are a product of our language or conceptual scheme. Of course, we must be very careful with that kind of talk, because we usually *don't* want to say that such entities did not exist before human minds or language, or wouldn't have existed if human minds or language hadn't. I take "pragmatism" to be roughly the combination of functional pluralism and deflationary metaontology, and I take it to be roughly equivalent to what goes by the name "neo-Carnapianism" these days. Neo-Carnapianism is the metaontology with which I am most sympathetic,[6] so I will expand on it below.

Admission: unfortunately, I cannot give a thorough defense of the metaontological views I hold, and which will pop up frequently throughout the book. Certainly, I will respond to many objections along the way, but other controversial theses are simply assumed. The most significant, and central for my purposes, is the thesis that there are analytic or conceptual[7] truths. The analytic/synthetic (A/S) distinction has been seeing something of a comeback. This thesis

[6] I have also been influenced by Azzouni (2004), Balaguer (2021), Eklund (2013), Hirsch (2011), Putnam (1981, 1987), Sellars (see, e.g., his essays collected in Scharp and Brandom 2007), and Wittgenstein (1956/1978, 1976), among others.

[7] I use "analytic truth" and "conceptual truth" interchangeably.

plays a crucial role in my Carnapian style of pragmatism. (Some pragmatists such as Amie Thomasson embrace it; others such as Michael Williams eschew it.)[8] Analytic truths have traditionally been said to be those owing their truth to the meanings of their constituent terms alone. This is usually called the *metaphysical* sense of analyticity.[9] For example, the sentence "bachelors are unmarried" is true *because* the terms therein have certain meanings and *not* because of the way the (extra-linguistic) world is. This has intuitive appeal. After all, make any change in the extra-linguistic world you want and you will not change the truth-value of the sentence, but you can change its truth-value by changing the meanings of its constituent terms.[10] Synthetic truths have traditionally been said to be those owing their truth both to the meanings of their constituent terms *and* to the way the world is. The sentence "bachelors are unhappy" is true – if it is true – because the terms therein have certain meanings *and* because of the way the (extra-linguistic) world is. If that claim is false, it is synthetically, not analytically, false – false

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[8] Putnam (1981, 1987) and Hirsch (2011) are sometimes described as neo-Carnapians, although they don't rely on the A/S distinction. Perhaps a *neo*-Carnapian is one who says many similar things that Carnap says about (meta)ontology, but who rejects the A/S distinction, and one who says many similar things that Carnap says about ontology *and* accepts the A/S distinction is just a Carnapian.

[9] See Boghossian (1996) for the classic distinction between epistemic and metaphysical analyticity, a critique of the latter, and defense of the former.

[10] Obviously, this is also true of some non-analytic truths, such as a posteriori necessities like "water is H_2O," but normativists break such truths into an analytic and a synthetic component. See the refences in footnote 11. I discuss a posteriori necessities more in Chapter 9.

because the concepts therein have certain meanings and because of the way the (extra-linguistic) world is. The thesis that there are analytic truths in this traditional metaphysical sense has respectable defenses (e.g., Rabinowicz 2010, Russell 2008, Warren 2015b), and I will return to it below and in Chapter 8. According to the epistemic sense of analyticity, analytic truths are those knowable by grasp of their meanings alone. Thomasson has given the epistemic understanding of analyticity a normative twist, according to which, "mastery of the relevant linguistic/conceptual rules *entitles* one to accept the conceptual truth (without the need for any further investigation), and … rejecting it would be a *mistake*" (2014: 238–9, my emphasis; see also Thomasson 2007a for further defense of analytic truth). On this view, the claim that bachelors are unmarried is analytic because mere mastery of the terms involved entitles one to accept its truth.[11] Though I will not provide direct arguments for the thesis that there are analytic truths, you should see the book itself as an argument: look at all you can do if you accept analytic truth! Obviously, those unconvinced by the book will make the same exclamation, sarcastically.

I think both senses of analyticity are helpful for a proper defense of normativism. However, Boghossian (1996) famously argued against metaphysical analyticity, which he says is required to make sense of what he called the "linguistic" theory of necessity – the idea that necessity is explained by linguistic conventions. Normativism certainly says something like this, but we need to be careful. Normativism does not say that linguistic conventions are truthmakers

[11] As defenders of analyticity since Grice and Strawson (1956) have noted, acceptance of analyticity is compatible with Quine's claim that all statements are in principle revisable in the light of experience. A revision of an analytic statement results in a change of meaning, and we may alter the meanings of our terms because experience suggests that it would be useful to do so.

of necessities, and the dependence of necessity on convention that normativism does accept is not the usual counterfactual kind. Einheuser (2006, 2011), whose work features prominently in Chapter 5, has convincingly argued that conventionalists – among whom normativists would be included – need only the idea that adopting an alternative conceptual scheme would result in different necessities, as judged from *within* the alternative scheme. This notion of dependence is all that is required by the metaphysical sense of analyticity and by the "linguistic" theory of necessity. I return to this topic in Chapter 8.

Speaking of conventionalism – the thesis that mathematical and logical truths are in some sense conventional, based on convention, explained by convention, etc. – while writing this manuscript, I read Jared Warren's (2020) wonderful book, *Shadows of Syntax: Revitalizing Logical and Mathematical Conventionalism*. I think his book successfully rebuts most of the influential objections to analyticity and conventionalism, including Boghossian's. The philosophy of mathematics I present here is certainly of a piece with his. In fact, while I have disagreements with some of Warren's specific claims, which I will address in Chapter 8, I take Warren's conventionalism and mathematical normativism to be roughly equivalent, differing mainly in emphasis. Throughout this book, I will help myself to both normativistic and conventionalistic turns of phrase. Let me briefly explain both views, and why I will treat them as equivalent. A more elaborate presentation of normativism comes in Chapter 5.

Thomasson's (2019a, 2020a) modal normativism is somewhat similar to expressivism about metaphysical modality, the thesis that claims about what is metaphysically necessary or (im)possible do not *describe* anything, but *express* something. (We needn't worry here about what exactly *metaphysical* modality is.) Expressivists can disagree about *what* is expressed by terms of whatever class about which they are expressivists, though usually it is a mental state of some motivational, non-belief kind. In Thomasson's case, though, what is expressed is conceptual/semantic rules[12] or consequences thereof, i.e., conventional rules for how to use words and concepts. Note that a metaphysically modal claim is not *about* those rules. For example, according to Thomasson, a metaphysical necessity such as that a statue cannot survive being squashed is an expression of rules of use for our statue concept – the statue concept is not to be applied after squashing.[13] Although Thomasson's normativism concerns specifically *metaphysical* modality, it is easily generalizable to mathematics, which I will do in this book. According to mathematical normativism, mathematical claims do not describe, in any substantive sense, anything, but instead are *expressions* of conceptual rules or consequences thereof.

Thomasson's normativism is only "somewhat" similar to expressivism, because she accepts the existence of modal truths, facts, and descriptions as long as all of these things are understood in suitably deflationary senses (Thomasson 2020a; see also Baker and Hacker 2009). Since it's necessary that bachelors are unmarried, we can trivially derive that "it's necessary that

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[12] Rules which may include empirical variables to account for a posteriori necessities (Sidelle 1989; Thomasson 2020a; Warren 2022b), but these, as well as de re necessities, are irrelevant to the present work. However, see Chapters 8 and 9 for brief discussion of posteriori necessities.

[13] It may be more complicated than this. Perhaps there are circumstances where the statue concept might still apply after squashing, but normally we wouldn't say that the squashed clay is the *same* statue. But this – talk of persistence conditions, identity conditions, and so on – is all still expressing conceptual rules, rules about when concepts are to be applied and, in the case of persistence and identity conditions, reapplied.

bachelors are unmarried" is true, using the equivalence schema. Thus, there are modal truths. Deflationists often accept similar equivalence schemas, such as: it is a fact that p if and only if p. Since it's necessary that bachelors are unmarried, we can trivially derive that it is a fact that it's necessary that bachelors are unmarried. Thus, there are modal facts. The mathematical normativist is similarly capable of recognizing mathematical truths and mathematical facts. Thus, the problem of so-called "creeping minimalism" in metaethics (Dreier 2004) arises here as well. In metaethics, this is the problem of how to distinguish moral expressivism from moral realism once the expressivist adopts semantic minimalism or deflationism and is thereby able to say everything the realist says. There are several proposals for solving this problem in metaethics. Adjusting Simpson's (2020) solution in metaethics to the topic of mathematics, we could say that mathematical normativism differs from platonism in not having to appeal to mathematical facts to explain (the content of) mathematical language and thought (see also Brandom's 2008 explanation of modal language). For example, the normativist wouldn't (and can't) say that the mathematical facts make the mathematical truths true or explain why they are true. The mathematical truths and facts have been so deflated that no explanatory relation can hold between them. I think this is right, but to address the problem in modality and mathematics, I think the easiest solution is to appeal to analyticity, something usually not open to metaethicists, since most these days don't believe that moral truths are analytic.[14] In other words, both platonist and normativist say that numbers exist (for example), but the former takes this to be a synthetic claim and the latter takes it to be analytic. Avoiding the problem of creeping minimalism – i.e.,

[14] For a detailed exploration of the similarities and differences between philosophical problems of morality and mathematics, see Clarke-Doane (2020).

making the required distinctions between 'substantive' and 'non-substantive' reference, existence, etc. – will be harder, I think, for those deflationists (such as Michael Williams) who eschew analyticity.[15]

This is all quite similar to Warren's (2020) conventionalism, according to which all mathematical truths in a language are fully explained by (the validity of) the basic inference rules of that language. For Warren, for the basic inference rules to fully explain a mathematical truth is for the mathematical truth to be derivable solely from the basic inference rules. (I don't think Warren is clear enough about the sense of 'explanation' here, a point on which I expand in Chapter 8.)

Notably, according to Warren's (2020) conventionalism, it is not the case that mathematical truths *describe* conventions. You could say that arithmetical truths describe numbers because their terms refer to numbers, but such reference – and, therefore, existence – is a trivial byproduct of our arithmetical language. For example, let us assume our arithmetical language is formally modeled by first-order Peano arithmetic, one of whose basic inference rules allows the derivation of "N0" (i.e., "zero is a number") from no premises. From this, we can easily derive "there is a number" via the introduction rule for the existential quantifier. Thus, the existence of numbers is analytic, because it is a consequence of our basic inference rules; it is a trivial byproduct of our arithmetical language. Thomasson and Warren are both deflationary

[15] This 'substantive'/'non-substantive' distinction is the same one Linnebo (2018) is after with his distinction between thick and thin objects. He notes that the analytic/synthetic distinction, if workable, does the trick, but he prefers to make the distinction using abstractionism, which I discuss in Chapter 8.

“trivial realists” in mathematical ontology.

There are some obvious differences between Warren’s and Thomasson’s views, but I will treat them as equivalent when the differences are irrelevant. One difference is Warren’s emphasis on inference rules and Thomasson’s emphasis on application conditions. This is unimportant since Thomasson is aware that application conditions might not be the only kind of semantic rule that is expressed by necessary statements (e.g., other kinds of introduction rule for a term may be expressed), and Warren accepts that application conditions can be meaning-determining (Warren 2022a). They also both accept the legitimacy of implicit definition as a meaning-determining practice.

An apparently more significant difference is the fact that Thomasson is an expressivist and Warren is not – he’s an inferentialist, according to whom the meanings of mathematical terms are determined by their inference rules. (More on inferentialism in Chapter 6.) However, Thomasson is not an expressivist in the traditional sense, which is one reason she prefers the term ‘normativism’. Normativism is not a semantic or metasemantic thesis, like traditional expressivism, which has an ‘ideational’ (meta)semantics according to which the *meaning* of the relevant class of terms is determined by the mental states they express; normativism is a *functional* thesis, a thesis about the function of a class of terms. In fact, like Warren, Thomasson is also an inferentialist about meaning (2020, 79), and the normativist’s functional thesis is entirely open to Warren. This is why I take conventionalism and normativism to be roughly equivalent, differing mainly in emphasis. What for Warren is fully explained by (the validity of) basic inference rules, is for Thomasson an expression of conceptual rules or their consequences. This is not to say there are no important differences between Warren and Thomasson. For example, Thomasson's inferentialism is normative and Warren's isn't.[16] But this difference will not be relevant until Chapter 6, where I side with Thomasson (though my normative inferentialism is naturalistic, so maybe it isn't far from Warren's after all).

I said above I would come back to Carnap. Normativism and neo-Carnapianism are distinct theses. Normativism is a thesis about the function of a class of terms, and I think it is best to view neo-Carnapianism as a metaphilosophical thesis: it is (or entails) a view about the nature of philosophy, and metaphysics in particular. According to it, philosophical questions are resolvable via some combination of conceptual analysis, empirical investigation, and normative, pragmatic considerations. There is no *special* or distinctive metaphysical method. Philosophical questions that seem unresolvable are conceptually confused and require either conceptual repair or rejection.

Carnap's metaphilosophy is closely associated with the distinction between internal and external (I/E) questions. Carnap held that existence questions (e.g., "Are there numbers?"), conceived as internal to a framework, can be straightforwardly answered via conceptual analysis and empirical investigation. To answer the question, "Does Bigfoot exist?", conceived as an internal question, you need to determine what that means, i.e., what it would take for Bigfoot to exist, and then undertake empirical investigation to determine whether those conditions are actually met. Existence questions that are conceived as external to a framework are either pragmatic questions or senseless pseudo-questions. Note that the same question can be conceived internally or externally. What exactly it means to ask a question internal or external to a

[16] It also seems that they disagree about what to say about the contingent a priori (Thomasson 2020a, Warren 2022a) and quantifier variance (Thomasson 2014, Warren 2020).

framework depends on what frameworks are, and there is debate about what exactly a framework is (Eklund 2013, 2016). I won't discuss all the options, but I think everything I argue in this book is compatible with the idea that a framework is a fragment of language. On this understanding of a framework, to ask an internal question is simply to ask a question in a particular language (fragment), and to ask an external question is either to ask what language to use or to say something nonsensical like "Answer this question but ignore its actual meaning: are there numbers?"[17] (Eklund 2013, 232). As Eklund notes, so far this seems pretty trivial and doesn't seem immediately to have any deflationary metaontological implications. "There are numbers" may be true in some languages and false in others, but it doesn't mean the same thing in those languages. I think that's right – *so far* there are no deflationary metaontological implications.

This is why the A/S distinction is important. The way I see it, the entirety of Carnap's deflationary metaontology rests on two claims: the A/S distinction holds in natural languages like ours (Carnap 1955), and there is no such thing as the one true language. It is these two claims that allow Carnap to say that all questions are either answerable by conceptual analysis, empirical investigation, or normative pragmatic considerations, or they are senseless. Eklund (2013, 245) is understandably curious why, if this is right, Carnap (1950) focused on the I/E distinction, rather than the A/S distinction, in his anti-metaphysical arguments.[18] It's a great

[17] Compare explicit denials of analytic truths, like "Bachelors are married". These can only be interpreted as being about language (e.g., as being suggestions to change language) or as based on confusion (Burgess, Cappelen, and Plunkett 2020, Thomasson 2017). I return to this below.

[18] Basically the same thing happens in Section V of Carnap's (1937/2001) *Logical Syntax of Language*. He makes a distinction between what he calls "object-questions," which concern

question. I'm not sure; perhaps he assumed that readers knew that, according to him, frameworks come with an A/S distinction and that it determines how questions within a framework are answered. The fact that Carnap (1950) states that some questions can be answered by conceptual analysis alone shows that he was assuming the A/S distinction. Regardless of what Carnap's view was, I will present mine. I will make explicit the relation I see between ontological questions, external questions, and analyticity.

On the relation between the A/S and I/E distinctions, Eklund argues the following (2013, 237, original emphasis):

> Carnap is actually drawing a *tripartite* distinction: between questions internal to a framework, questions about which framework we should choose to employ, and the pseudo-questions—the supposed theoretical external questions. What Quinean criticism of the analytic/synthetic distinction threatens is the distinction between *the first two* categories: change in theory and change in language cannot be separated in the way Carnap assumes. But even if this distinction collapses, Carnap's critique of ontology still stands. For the *third* category, that of the supposed pseudo-questions, can remain untouched.

Of course, the very *idea* of pseudo-questions doesn't require the A/S distinction – one could make sense of the idea in other ways – but I think an intuitive understanding of what pseudo-

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extralinguistic objects, and "logical questions," which concern linguistic objects, that is similar to the I/E distinction; he argues that metaphysical questions are pseudo-object-questions that are actually logical questions; and the A/S distinction, although central throughout *Logical Syntax,* doesn't figure in his discussion.

questions are and why they can only sensibly be interpreted as questions about which language to use arises quite naturally from the A/S distinction. In other words, there is an important and motivated connection between the account of ontological questions, external questions, and analyticity. Eklund would argue otherwise – he would argue that a questioner asks a pseudo-question (i.e., a putatively *theoretical*, rather than practical, external question) when the questioner knows what's true in the language, and the questioner is not asking a practical question about which language to use. Here there is no mention of analyticity. True, but when the questioner knows what's true in the language and still asks an existence question, the question rests on the questioning of an analytic truth, i.e., they would consider their question answered if they accepted an analytic truth.[19] And it is the fact that their question rests on the questioning of an analytic truth that helps motivate the idea that they must really be asking a question about which language to use, since analytic truths express linguistic rules. If they aren't asking such a question, they are asking a pseudo-question. That is the connection between external questions and analyticity. Let me illustrate how when the questioner has been told what's true in the language and still asks an existence question, the question always rests on the questioning of an analytic truth. (To be clear, I mean here that an external question always rests on the questioning of something that is an analytic truth in the framework of the thing whose existence is being questioned.) A philosopher asks us "Are there numbers?" First, we see if they intend this as an internal question. Internal questions concern what's true in our language and are answerable via empirical and conceptual means. Considered as an internal question, it can be answered by

[19] Thus, I hold the position mentioned by Eklund (2013, 245) that "all properly ontological disputes turn on analytic claims".

purely conceptual means. We may explain to them why “there are numbers” is true in our language via the derivation of that claim from the Peano axioms. They persist in their questioning, which clearly rests on the questioning of an analytic truth, namely, the analytic truth that there are numbers. Obviously, if they accepted the analytic truth that there are numbers, they would consider their question answered. Since the question rests on the questioning of an analytic truth – an expression of a rule for the use of language – they can only sensibly be asking a pragmatic question about the use of language. The alternative is that they are asking a senseless pseudo-question.

You may think this can’t be an adequate account of everything Carnap regarded as a pseudo-question. For example, he regarded “Are there tables?”, construed as an external question not about language, as a pseudo-question, yet this question doesn’t rest on the questioning of an analytic truth. This is wrong. Construed as an external question, the question *does* rest on the questioning of an analytic truth, therefore, if it is not about language use, it is a pseudo-question. Let me illustrate. A philosopher asks us, “Are there tables?” First, we see if this is meant as an internal question. We explain to them why “there are tables” is true in our language. It doesn’t really matter whether they *agree* that “there are tables” is true in our language. If they continue to question whether there are tables, their question will rest on the questioning of an analytic truth. Suppose they agree that it is true in our language but continue to question. Then their question rests on the questioning of the analytic truth (A): if “there are tables” is true in our language, then there are tables. If they accepted (A), they would consider their question answered, since they accept the antecedent.

If they didn’t agree that “there are tables” is true in our language, they could accept (A). Their continued questioning would thus not rest on the questioning of *that* analytic truth. But it would still rest on the questioning of *some* analytic truth. Such a philosopher might say, "I accept (A). I just deny its antecedent – I don't think that "there are tables" *is* true in our language. I think that what it takes for it to be true in our language is for there to be a certain kind of macroscopic object, but I don't believe in macroscopic objects. I only believe in particles." They thus accept (A), but they still do not accept an analytic truth: that if there are particles arranged tablewise, then there are tables. If they accepted this analytic truth, their question would be answered, since they accept its antecedent. The same goes for other ontologists. The nihilist might say, "I accept (A). I just deny its antecedent – I don't think that it *is* true in our language. I think that what it takes for it to be true is for there to be a certain kind of object, but I don't believe in *anything*." But they still deny an analytic truth: that if it is tabling,[20] then there are tables. If they accepted this analytic truth, their question would be answered. Each of these antecedents is simply a *different way of describing* what it would take for "there are tables" to be true (cf. Heil 2003, 177, Rayo 2013, 31, and Thomasson 2014, 106-7). For a Carnapian, ontologies are languages. I think this goes for all ontologists who would deny that "there are tables" is true in our language. In fact, I think it's what distinguishes the skeptical ontologist from the delusional person. For the skeptical ontologist, as opposed to the delusional person, there is some p such that p analytically entails that tables exist and they believe that p.[21] (X analytically entails Y if and only if "if X, then Y" is an analytic truth.) Since they believe some such p and deny that tables exist, there is some analytic truth of the form "if p, then tables exist"

[20] This is the feature-placing language of ontological nihilists (Hawthorne and Cortens 1995).

[21] Thus, for the delusional person, for all p, if p analytically entails that tables exist, then they don't believe that p.

that they deny. Their questioning thus rests on the questioning of some analytic truth.[22]

Thus, all external questions rest on the questioning of an analytic truth. As I said, I don't think it's impossible to explicate the I/E distinction in a way that doesn't appeal to analyticity. However, I think appealing to analyticity can give a better account of *why* one who says, "I know 'there are tables' is true in our language, but are there tables?" (and similar things) can only sensibly be asking which language to use. It is because that question rests on the questioning of a truth that is analytic in the framework of the object whose existence is being questioned, a truth which is simply an expression of linguistic rules for the use of terms for that object. A truth, furthermore, that serves as an introduction rule for the relevant term – the analytic conditionals the questioner questions are precisely the kinds used to introduce new terms into a language. An external existence question thus questions the introduction of new terms. And the denial of such analytic conditionals needn't betray any conceptual incompetence; it can betray a refusal to adopt a linguistic framework (see Chapter 8 for elaboration of the points in this paragraph).

So, the A/S distinction supplies a direct connection between asking an external existence question and asking about a linguistic framework. Accounts of the I/E distinction that don't appeal to analyticity (e.g., Bird 2003, Eklund 2013) seem not to explain this. Or, if they do, they rely on something like inference to the best explanation (IBE): why can one who says, "I know

[22] The same line of reasoning in this paragraph also applies to the philosopher who questions the existence of numbers. They might disagree that "there are numbers" is true in our language. But there is certainly some *p* such that *p* analytically entails that numbers exist and they believe that *p*; for *all p*, *p* analytically entails that numbers exist, since "numbers exist" is analytic. Thus, there is certainly some analytic conditional of the form "if *p*, then numbers exist" that they deny.

‘there are tables’ is true in our language, but are there tables?” only sensibly be asking which language to use? Because there is no better explanation of what they mean. Now, I don’t think there’s anything wrong with IBE – in fact, I’m going to use it right now – but I think appealing to analyticity gives a better explanation of why someone who asks an external question is either asking a question about language use or asking a pseudo-question. I conclude that appealing to analyticity gives us the best account of external questions and why they are either questions about which language to use or pseudo-questions.

Now, you may be wondering how this all squares with an ontic account of DME. How can a Carnapian normativist hold an *ontic* account of DME? Regarding the compatibility of normativism and an ontic account of DME, I argue in Chapter 5 that NOCA does not cease to be an ontic account after being deflated by normativism. The short explanation is: normativism reconceives the metaphysical nature of the explanans and explananda of DMEs, and this allows the normativist to see ontic accounts of DME, including NOCA, as basically ontic accounts of what people think and say. After all, for the mathematical normativist, mathematical truths express conceptual/semantic rules – what people think and say is all there is to explain, and it can be explained ontically.

Regarding the compatibility of deflationary metaontology and the generalized ontic conception, I will say this. In this book, I am only concerned to deflate mathematics; I won’t be discussing deflationary metaontology in other areas. However, there is much debate over whether one can say some of the deflationary things I want to say in one area without it generalizing to a global deflationism and ultimately global anti-realism (see, e.g., Price, Blackburn, Brandom, Horwich, and Williams 2013). And the brand of deflationary metaontology with which I am most sympathetic is Carnap’s (1950), which is certainly global in character. One might therefore reasonably worry whether the generalized ontic conception requires a kind of metaphysical realism with which Carnapian metaontology is incompatible. I don't think they are incompatible. First, unlike the other metaontological deflationists just cited who worry about global anti-realism, I accept the analytic/synthetic distinction. For me, deflating mathematics means making it analytic. You cannot similarly deflate tables and chairs. The existence of tables and chairs is not analytic; and if you tried to make it so by stipulating the analyticity of "tables exist," you would simply change the meaning of the word (Warren 2020).[23] "But isn't the existence of tables relative to a linguistic framework for Carnap?" Not in any problematic sense. A framework in which "tables exist" is false is one in which "tables" (or "exists") means something different. For Carnap, there is a world out there, and we can talk about it in many different ways. Ontologies are languages, so while one philosopher may say that tables exist and another may say that only particles arranged tablewise exist, they are merely talking about the same thing in different languages (Dyke 2012, Heil 2003, Hirsch 2011, Putnam 1981, 1987, Rayo 2013, Thomasson 2014). The generalized ontic conception does not require ontological realism in the sense that there is one correct ontology, one correct language in which to describe the world. Take a straightforward causal explanation: the bottle broke because Suzy threw a rock at it. The ontology of rocks simply doesn't matter. It matters not a bit to the generalized ontic conception whether what Suzy threw was a substance, a bunch of particles arranged rock-wise, a

[23] As Warren (2020, 232-3) points out, this defuses a standard objection to us defenders of analytic existence claims: why can't we make the existence of God analytic? By all means, make "God exists" analytic. Unfortunately, you won't have established what you think you have.

part of the universe that was rock-ing, or whatever.[24] So, when the generalized ontic conception says that the explanans – here, the rock – must be objective in order to explain, it does not mean that rocks as such must figure in the one true ontology, or that the explanans must be described in a certain language. The generalized ontic conception requires what we might call "empirical objectivity" or "empirical realism" as opposed "ontological objectivity" or "ontological realism". There are many ontologically different but empirically equivalent ways of describing the real, mind-independent explanandum, explanans, and explanatory relation (Thomasson 2019b). In fact, ontologists often *insist* that different ontological theories are empirically indistinguishable (e.g., Merricks 2011, van Inwagen 1995). I take this to be obvious – e.g., no possible experience could distinguish between the truth of the claim that there are substantial rocks and the truth of the claim that there are only particles arranges rock-wise. If they were empirically distinguishable, ontologists would be doing empirical investigation.

Hofweber (2016) agrees that different ontological hypotheses are *phenomenologically* indistinguishable, but he thinks they are still empirically distinguishable. His argument is that our perceptual beliefs are about objects, not simples arranged object-wise (for example), and these beliefs are defeasibly justified. We may have justified beliefs about simples arranged object-wise, but these are not *perceptual* beliefs; these are beliefs downstream from our justified perceptual beliefs about objects. He writes that "The belief that there are simples arranged chair-wise is not

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[24] This idea does not imply that the bottle's breaking is wildly causally overdetermined, since these descriptions of the cause are just different ways of describing to the same thing. See also Thomasson (2007a) on the confusion of overdetermination and causal exclusion arguments against ordinary objects.

a perceptual belief at all, and it can't be in our perceptual system" (192). I'm not sure what he means by "it can't be in our perceptual system". It can't mean that we can't have perceptual beliefs with that content because we can't perceive individual simples, for perceiving simples arranged chair-wise needn't require that ability. Maybe he just means we can't have perceptual beliefs with that content because beliefs with that content are always downstream of perceptual beliefs about objects. If that's just a claim about us, as we actually are, then it's plausible.[25] However, I see nothing incoherent in the idea of a linguistic community that learns the language of simples arranged object-wise first, and only later comes to talk about objects. It seems plausible that the members of such a community would form perceptual beliefs about simples arranged object-wise, and their beliefs about objects would be downstream. That our own conceptual development didn't happen this way and that, perhaps for contingent social and neurological reasons, no community would conceptually develop this way, doesn't undercut the point. (See Thomasson 2019b on the development of language for ordinary objects.) I want to emphasize that I am arguing against the idea that ontological claims are empirically distinguishable; I am not arguing against the justification of our perceptual beliefs in ordinary objects.[26]

Now on to the book's central foil: the enhanced indispensability argument for platonism.

1.3. The Enhanced Indispensability Argument

[25] Though not unassailable. Brandom (2015, Chapter 2) discusses a Sellarsian account of perception that would allow one to perceive simples arranged object-wise.

[26] Hofweber (2016) has a second argument for empirical distinguishability, but it relies on the first for its plausibility.

Some of the most influential arguments for platonism have been and continue to be indispensability arguments. The thought is that we ought to be platonists because mathematics is indispensable to us, in some way that needs to be cashed out. According to the Quine-Putnam version of this argument, we ought to be platonists because our best scientific theories indispensably quantify over mathematical objects, where this means that every theory that doesn't quantify over mathematical objects is worse, by some standard (e.g., simplicity, fruitfulness, predictive power, etc.) (see, e.g., Quine 1976, Putnam 1979). Baker (2009, 613) christened the following version of this argument the "enhanced indispensability argument" (EIA), which focuses on *explanatory power*, i.e., DMEs:

(1) We ought rationally to believe in the existence of any entity that plays an indispensable explanatory role in our best scientific theories.

(2) Mathematical objects play an indispensable explanatory role in science [i.e., there are DMEs].

(3) Hence, we ought rationally to believe in the existence of mathematical objects.

Many critics of the EIA have denied the second premise, the existence of DMEs (e.g., Melia 2000, Daly and Langford 2009, Saatsi 2011). They insist that in all putative DMEs, the mathematics is playing a merely representational role. As far as I know, no one has denied the first premise. It expresses a widespread scientific realist attitude in contemporary philosophy of science. And no one, as far as I know, has argued that the EIA is invalid. But that's what I think, and that's what I will argue.

At least, I think it's invalid when properly formulated. For, the premises appear to be category mistakes. How could an *entity* play a role in a *theory*? Entities play roles – e.g., causal or functional roles – in the world, but not in theories. Instead, I think the argument is better rendered as something like (EIA'):

(1') We ought rationally to believe in the existence of any entity referred to by a concept that plays an indispensable explanatory role in our best scientific theories.

(2') Mathematical concepts play an indispensable explanatory role in science [i.e., there are DMEs].

(3') Hence, we ought rationally to believe in the existence of mathematical objects.

But this isn't valid. For, we need:

(2.5') Mathematical concepts refer to mathematical objects.

And here's why I think the EIA' is invalid: I argue that (2.5') is false, at least if reference is here understood as a substantial relation, as surely it must be for any proponent of the EIA'. If the reference of mathematical concepts can be got for cheap, then the existence of mathematical objects can be got for cheap, and there's no point in using the EIA' to secure their existence. The proponent of the EIA is after something more. After all, no proponent of the EIA' would be satisfied with the merely *analytic* truth of "mathematical concepts (successfully) refer to mathematical objects" or "there are mathematical objects," which normativists and conventionalists accept. For the EIA' proponent, reference is not cheap – it is by playing an indispensable explanatory role in our best scientific theories that we are entitled to believe that a concept succeeds in referring. Let us say that *X*s exist analytically (synthetically) when "*X*s exist" or "there are *X*s" is an analytic (synthetic) truth. With the analyticity of existence comes the analyticity of successful reference and vice versa: "*X*s exist" is an analytic (synthetic) truth if and only if "'*X*' refers successfully" is an analytic (synthetic) truth (holding fixed the actual meaning of '*X*' – it is given the actual meaning of '5' that "'5' refers successfully" is analytic). The proponent of the EIA is after the *synthetic* existence of mathematical objects and *synthetic* successful reference of mathematical concepts. A central aim of this book is to show how normativism can deflate even ontic accounts of DME, rendering the EIA' invalid when understood platonistically, i.e., when reference and existence are understood synthetically.

Note that (2.5') is not meant to imply *successful* reference. If it did, (2.5') alone would take us to platonism. The distinction between reference and successful reference is common. Obviously, we could do away with the distinction, treating reference as essentially successful, and restate the argument with (2.5') as "Mathematical concepts refer to mathematical objects, if they refer," or "Mathematical concepts purport to refer to mathematical objects," mutatis mutandis. I will stick with the distinction between reference and successful reference.

Given that the proponent of the EIA is after synthetic reference and synthetic existence, an even more explicit formulation is as follows (EIA*):

> (1*) We ought rationally to believe in the synthetic existence of any entity synthetically referred to by a concept that plays an indispensable explanatory role in our best scientific theories.
>
> (2*) Mathematical concepts play an indispensable explanatory role in science [i.e., there are DMEs].
>
> (3*) Mathematical concepts synthetically refer to mathematical objects.
>
> (4*) Hence, we ought rationally to believe in the synthetic existence of mathematical objects.

To believe in the synthetic existence of *X*s, I don't think it's necessary to believe that *X*s exist *and* to believe that "*X*s exist" is synthetic. You don't need the concept of the synthetic to believe in the synthetic existence of *X*s. To believe in the synthetic existence of *X*s is just to believe that *X*s exist, where the proposition that *X*s exist is a synthetic proposition. This doesn't require possession of the concept of the synthetic. The synthetic proposition that *X*s exist is different from the analytic proposition that *X*s exist because "*X*s exist" means different things depending on whether it is analytic or synthetic. In Chapter 6, I defend an inferentialist account of meaning according to which meaning is determined by inferential rules. Since "*X*s exist" has different inferential rules governing it depending on whether it is analytic or synthetic – e.g., if it's analytic, but not if it's synthetic, you are allowed to infer it anywhere in a proof – it means different things depending on whether it is analytic or synthetic.

Premise (3*) is simply the denial of normativism. To (purport to) refer synthetically is to refer successfully synthetically, if reference is successful at all. So, if mathematical concepts refer synthetically to mathematical objects, that implies that if they succeed, their successful reference is synthetic. In other words, according to (3*), if "'5' refers successfully to 5" is true, it is synthetically true. So, (3*) implies that sentences like "'5' refers successfully to 5" (and, so, "5 exists") are synthetic; but these are analytic according to the normativist. (3*) says that mathematics describes in the substantive sense denied by normativism. Thus, the EIA* is invalid without begging the question against the normativist. She can accept the existence of DMEs while denying platonism, because she denies (3*).

She could also deny (1*), of course, but she needn't. I am taking (1*) to be equivalent to "If a concept plays an indispensable explanatory role in our best scientific theories, we ought to believe in the synthetic existence of any entity it synthetically refers to".[27] Call this (1a*). The

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[27] And I take (1a*) to be equivalent to "If a concept plays an indispensable explanatory role in our best scientific theories, then, if it synthetically refers to an entity, we ought to believe in the synthetic existence of that entity (i.e., we ought to believe that the concept's synthetic reference

normativist can accept (1a*). Call a concept that plays an indispensable explanatory role in our best scientific theories an "i-concept". The normativist can agree with (1a*) that we ought to believe in the synthetic existence of any entity an i-concept synthetically refers to, because she thinks mathematical i-concepts don't synthetically refer to anything, so there's nothing to believe synthetically exists. Thus, the normativist can accept the scientific realist sentiment of (1) by accepting (1a*). One could instead take (1') to mean "If a concept plays an indispensable explanatory role in our best scientific theories, we ought to believe it synthetically refers successfully". Call this (1b*). The normativist would deny (1b*). She thinks mathematical concepts are i-concepts, but that they don't synthetically refer successfully. If (1b*) were used as the first premise, then the EIA* would seem to me valid without premise (3*), but still not valid without an anti-normativist premise, this time premise (1b*).

Let me stress that I think that many mathematical concepts are descriptive and in fact successfully describe, but only in applied contexts. I will give an account of their descriptive content in Chapter 6. But the applied uses of mathematical concepts are not the *distinctive* uses that figure in DMEs – the applied uses are merely representational uses. In DMEs, mathematical concepts appear in truths both of pure and applied mathematics, but it is the appearance of truths of pure mathematics that supposedly gives DMEs their ability to support platonism in the EIA.[28] In other words, the indispensable explanatory role appealed to in the EIA is not the representational role. Recall that those who deny the existence of DMEs do so by claiming that

is successful)". The normativist can accept this because she thinks mathematical concepts don't meet the second, embedded antecedent.

[28] I will leave the asterisk off when it doesn't matter which version of the EIA I'm referring to.

all uses of mathematics within them are representational. So, premise (3*) doesn't mean that mathematical concepts can be empirically applied, something no normativist need deny; it means that *pure* mathematics describes mathematical objects.

1.4. Conclusion

The idea that pure mathematics is not descriptive in any substantive sense is not new. As I mentioned, many of the positivists, especially Wittgenstein in different ways in different periods, held something like it. Their views have come under heavy fire over the decades though, and I believe that normativism provides the most plausible way of resurrecting their view from the ashes. I will discuss normativism in detail in Chapter 5. First, I must elaborate on our central topic: distinctively mathematical explanation (DME).