# An extended TOPSIS for multi-attribute decision making problems with neutrosophic cubic information

**Surapati Pramanik[1], Partha Pratim Dey[2], Bibhas C. Giri[3], and Florentin Smarandache[4]**

[1] Department of Mathematics, Nandalal Ghosh B.T. College, Panpur, P.O.-Narayanpur, District - North 24 Parganas, Pin Code-743126, West Bengal, India. E-mail: sura_pati@yahoo.co.in

[2] Department of Mathematics, Jadavpur University, Kolkata-700032, West Bengal, India. E-mail: parsur.fuzz@gmail.com

[3] Department of Mathematics, Jadavpur University, Kolkata-700032, West Bengal, India. E-mail: bcgiri.ju.math@gmail.com

[4] University of New Mexico. Mathematics & Science Department, 705 Gurley Ave., Gallup, NM 87301, USA. Email: fsmarandache@gmail.com

**Abstract**. The paper proposes a new technique for dealing with multi-attribute decision making problems through an extended TOPSIS method under neutrosophic cubic environment. Neutrosophic cubic set is the generalized form of cubic set and is the hybridization of a neutrosophic set with an interval neutrosophic set. In this study, we have defined some operation rules for neutrosophic cubic sets and proposed the Euclidean distance between neutrosophic cubic sets. In the decision making situation, the rating of alternatives with respect to some predefined attributes are presented in terms of neutrosophic cubic information where weights of the attributes are completely unknown. In the selection process, neutrosophic cubic positive and negative ideal solutions have been defined. An extended TOPSIS method is then proposed for ranking the alternatives and finally choosing the best one. Lastly, an illustrative example is solved to demonstrate the decision making procedure and effectiveness of the developed approach.

**Keywords:** TOPSIS; neutrosophic sets; interval neutrosophic set; neutrosophic cubic sets; multi-attribute decision making.

## 1 Introduction

Smarandache [1] proposed neutrosophic set (NS) that assumes values from real standard or non-standard subsets of] $^{-}0$, $1^{+}$[. Wang et al. [2] defined single valued neutrosophic set (SVNS) that assumes values from the unit interval [0, 1]. Wang et al. [3] also extended the concept of NS to interval neutrosophic set (INS) and presented the set-theoretic operators and different properties of INSs. Multi-attribute decision making (MADM) problems with neutrosophic information caught much attention in recent years due to the fact that the incomplete, indeterminate and inconsistent information about alternatives with regard to predefined attributes are easily described under neutrosophic setting. In interval neutrosophic environment, Chi and Liu [4] at first established an extended technique for order preference by similarity to ideal solution (TOPSIS) method [5] for solving MADM problems with interval neutrosophic information to get the most preferable alternative. Şahin, and Yiğider [6] discussed TOPSIS method for multi-criteria decision making (MCDM) problems with single neutrosophic values for supplier selection problem. Zhang and Wu [7] developed an extended TOPSIS for single valued neutrosophic MCDM problems where the incomplete weights are obtained by maximizing deviation method. Ye [8] proposed an extended TOPSIS method for solving MADM problems under interval neutrosophic uncertain linguistic variables. Biswas et al. [9] studied TOPSIS method for solving multi-attribute group decision making problems with single-valued neutrosophic information where weighted averaging operator is employed to aggregate the individual decision maker's opinion into group opinion.

In 2016, Ali et al. [10] proposed the notion of neutrosophic cubic set (NCS) by extending the concept of cubic set to neutrosophic cubic set. Ali et al. [10] also defined internal neutrosophic cubic set (INCS) and external neutrosophic cubic set (ENCS), $\frac{1}{3}$ -INCS ( $\frac{2}{3}$ -ENCS), $\frac{2}{3}$ -INCS ( $\frac{1}{3}$ -ENCS) and also proposed some relevant properties.

In the same study, Ali et al. [10] proposed Hamming distance between two NCSs and developed a decision making technique via similarity measures of two NCSs in pattern recognition problems. Jun et al. [11] studied the notions of truth-internal (indeterminacy-internal, falsity-internal) neutrosophic cubic sets and truth-external (indeterminacy-external, falsity-external) neutrosophic cubic and investigated related properties. Pramanik et al. [12] defined similarity measure for neutrosophic cubic sets

and proved its basic properties. In the same study, Pramanik et al. [12] developed multi criteria group decision making method with linguistic variables in neutrosophic cubic set environment.

In this paper, we develop a new approach for MADM problems with neutrosophic cubic assessments by using TOPSIS method where weights of the attributes are unknown to the decision maker (DM). We define a few operations on NCSs and propose the Euclidean distance between two NCSs. We define accumulated arithmetic operator (AAO) to convert neutrosophic cubic values (NCVs) to single neutrosophic values (SNVs). We also define neutrosophic cubic positive ideal solution (NCPIS) and neutrosophic cubic negative ideal solution (NCNIS) in this study. The rest of the paper is organized in the following way. Section 2 recalls some basic definitions which are useful for the construction of the paper. Subsection 2.1 provides several operational rules of NCSs. Section 3 is devoted to present an extended TOPSIS method for MADM problems in neutrosophic cubic set environment. In Section 4, we solve an illustrative example to demonstrate the applicability and effectiveness of the proposed approach. Finally, the last Section presents concluding remarks and future scope research.

## 2 The basic definitions

**Definition: 1**
**Fuzzy sets [13]:** Consider $U$ be a universe. A fuzzy set $\Phi$ over $U$ is defined as follows:
$$\Phi = \{ \langle x,\ \mu_\Phi(x) \rangle \mid x \in U \}$$
where $\mu_\Phi(x) : U \to [0,\ 1]$ is termed as the membership function of $\Phi$ and $\mu_\Phi(x)$ represents the degree of membership to which $x \in \Phi$.

**Definition: 2**
**Interval valued fuzzy sets [14]:** An interval-valued fuzzy set (IVFS) $\Theta$ over $U$ is represented as follows:
$$\Theta = \{ \langle x,\ \Theta^-(x), \Theta^+(x) \rangle \mid x \in U \}$$
where $\Theta^-(x), \Theta^+(x)$ denote the lower and upper degrees of membership of the element $x \in U$ to the set $\Theta$ with $0 \le \Theta^-(x) + \Theta^+(x) \le 1$.

**Definition: 3**
**Cubic sets [15]:** A cubic set $\Psi$ in a non-empty set $U$ is a structure defined as follows:
$$\Psi = \{ \langle x, \Theta(x), \Phi(x) \rangle \mid x \in U \}$$

where $\Theta$ and $\Phi$ respectively represent an interval valued fuzzy set and a fuzzy set. A cubic set $\Psi$ is denoted by $\Psi = \langle \Theta, \Phi \rangle$.

**Definition: 4**
**Internal cubic sets [15]:** A cubic set $\Psi = \langle \Theta, \Phi \rangle$ in $U$ is said to be internal cubic set (ICS) if $\Theta^-(x) \le \mu(x) \le \Theta^+(x)$ for all $x \in U$.

**Definition: 5**
**External cubic sets [15]:** A cubic set $\Psi = \langle \Theta, \Phi \rangle$ in $U$ is called external cubic set (ECS) if $\mu(x) \notin (\Theta^-(x), \Theta^+(x))$ for all $x \in U$.

**Definition: 6**
Consider $\Psi_1 = \langle \Theta_1, \Phi_1 \rangle$ and $\Psi_2 = \langle \Theta_2, \Phi_2 \rangle$ be two cubic sets in $U$, then we have the following relations [15].

1. (Equality) $\Psi_1 = \Psi_2$ if and only if $\Theta_1 = \Theta_2$ and $\mu_1 = \mu_2$.

2. (P-order) $\Psi_1 \subseteq_P \Psi_2$ if and only if $\Theta_1 \subseteq \Theta_2$ and $\mu_1 \le \mu_2$.

3. (R-order) $\Psi_1 \subseteq_R \Psi_2$ if and only if $\Theta_1 \subseteq \Theta_2$ and $\mu_1 \ge \mu_2$.

**Definition: 7**
**Neutrosophic set [1]:** Consider $U$ be a space of objects, then a neutrosophic set (NS) $\chi$ on $U$ is defined as follows:
$$\chi = \{ x, \langle \alpha(x), \beta(x), \gamma(x) \rangle \mid x \in U \}$$
where $\alpha(x), \beta(x), \gamma(x) : U \to \, ]0,\ 1^+[$ define respectively the degrees of truth-membership, indeterminacy-membership, and falsity-membership of an element $x \in U$ to the set $\chi$ with $^-0 \le \sup \alpha(x) + \sup \beta(x) + \sup \gamma(x) \le 3^+$.

**Definition: 8**
**Interval neutrosophic sets [9]:** An INS $\Gamma$ in the universal space $U$ is defined as follows:
$$\Gamma = \{ x, \langle [\Gamma_\alpha^-(x), \Gamma_\alpha^+(x)], [\Gamma_\beta^-(x), \Gamma_\beta^+(x)], [\Gamma_\gamma^-(x), \Gamma_\gamma^+(x)] \rangle \mid x \in U \}$$
where, $\Gamma_\alpha(x)$, $\Gamma_\beta(x)$, $\Gamma_\gamma(x)$ are the truth-membership function, indeterminacy-membership function, and falsity-membership function, respectively with $\Gamma_\alpha(x)$, $\Gamma_\beta(x)$, $\Gamma_\gamma(x) \subseteq [0,\ 1]$ for each point $x \in U$ and $0 \le \sup \Gamma_\alpha(x) + \sup \Gamma_\beta(x) + \sup \Gamma_\gamma(x) \le 3$.

**Definition: 9**
**Neutrosophic cubic sets [15]**
A neutrosophic cubic set (NCS) $\Xi$ in a universe $U$ is presented in the following form:

$$\Xi = \{ \langle x, \Gamma(x), \chi(x) \rangle \mid x \in U \}$$

where $\Gamma$ and $\chi$ are respectively an interval neutrosophic set and a neutrosophic set in $U$.
However, NSs take the values from] ‾0, 1⁺[ and single-valued neutrosophic set defined by Wang *et al.* [2] is appropriate for dealing with real world problems. Therefore, the definition of NCS should be modified in order to solve practical decision making purposes. Hence, a neutrosophic cubic set (NCS) $\Xi$ in $U$ is defined as follows:

$$\Xi = \{ \langle x, \Gamma(x), \chi(x) \rangle \mid x \in U \}$$

Here, $\Gamma$ and $\chi$ are respectively an INS and a SVNS in $U$ where $0 \le \alpha(x) + \beta(x) + \gamma(x) \le 3$ for each point $x \in U$. Generally, a NCS is denoted by $\Xi = <\Gamma, \chi>$ and sets of all NCS over $U$ will be represented by NCS$^U$.

**Example 1.** Assume that $U = \{u_1, u_2, u_3, u_4\}$ be a universal set. An INS $A$ in $U$ is defined as
$\Gamma = \{< [0.15, 0.3], [0.25, 0.4], [0.6, 0.75] >/ u_1 + < [0.25, 0.35], [0.35, 0.45], [0.4, 0.65] >/ u_2 + < [0.35, 0.5], [0.25, 0.35], [0.55, 0.85] >/ u_3 + < [0.7, 0.8], [0.15, 0.45], [0.2, 0.3] ]>/ u_4\}$
and a SVNS $\chi$ in $U$ defined by
$\chi = \{< 0.35, 0.3, 0.15 >/ u_1, < 0.5, 0.1, 0.4 >/ u_2, < 0.25, 0.03, 0.35 >/ u_3, < 0.85, 0.1, 0.15 >/ u_4\}$
Then $\Xi = <A, \chi>$ is represented as a NCS in $U$.

**Definition: 10**
**Internal neutrosophic cubic set** [10]: Consider $\Xi = <\Gamma, \chi> \in$ NCS$^U$, if $\Gamma_\alpha^-(x) \le \alpha(x) \le \Gamma_\alpha^+(x)$ ; $\Gamma_\beta^-(x) \le \beta(x) \le \Gamma_\beta^+(x)$ ; and $\Gamma_\gamma^-(x) \le \gamma(x) \le \Gamma_\gamma^+(x)$ for all $x \in U$, then $\Xi$ is said to be an internal neutrosophic cubic set (INCS).

**Example 2.** Consider $\Xi = <\Gamma, \chi> \in$ NCS$^U$, if $\Gamma(x) = <$ [0.65, 0.8], [0.1, 0.25], [0.2, 0.4] $>$ and $\chi(x) = < 0.7, 0.2, 0.3 >$ for all $x \in U$, then $\Xi = <\Gamma, \chi>$ is an INCS.

**Definition: 11**
**External neutrosophic cubic set** [10]: Consider $\Xi = <\Gamma, \chi> \in$ NCS$^U$, if $\alpha(x) \notin (\Gamma_\alpha^-(x), \Gamma_\alpha^+(x))$ ; $\beta(x) \notin (\Gamma_\beta^-(x), \Gamma_\beta^+(x))$ ; and $\gamma(x) \notin (\Gamma_\gamma^-(x), \Gamma_\gamma^+(x))$ for all $x \in U$, then $\Xi = <\Gamma, \chi>$ is said to be an external neutrosophic cubic set (ENCS).

**Example 3.** Consider $\Xi = <\Gamma, \chi> \in$ NCS$^U$, if $\Gamma(x) = <$ [0.65, 0.8], [0.1, 0.25], [0.2, 0.4] $>$ and $\chi(x) = < 0.85, 0.3, 0.1 >$ for all $x \in U$, then $\Xi = <\Gamma, \chi>$ is an ENCS.

**Theorem 1. [10]**
Consider $\Xi = <\Gamma, \chi> \in$ NCS$^U$, which is not an ENCS, then there exists $x \in U$ such that
$\Gamma_\alpha^-(x) \le \alpha(x) \le \Gamma_\alpha^+(x)$ ; $\Gamma_\beta^-(x) \le \beta(x) \le \Gamma_\beta^+(x)$ ; or $\Gamma_\gamma^-(x) \le \gamma(x) \le \Gamma_\gamma^+(x)$.

**Definition: 12**
$\frac{2}{3}$ **-INCS($\frac{1}{3}$ -ENCS) [10]:** Consider $\Xi = <\Gamma, \chi> \in$ NCS$^U$, if $\Gamma_\alpha^-(x) \le \alpha(x) \le \Gamma_\alpha^+(x)$ ; $\Gamma_\beta^-(x) \le \beta(x) \le \Gamma_\beta^+(x)$ ; and $\gamma(x) \notin (\Gamma_\gamma^-(x), \Gamma_\gamma^+(x))$ or $\Gamma_\alpha^-(x) \le \alpha(x) \le \Gamma_\alpha^+(x)$ ; $\Gamma_\gamma^-(x) \le \gamma(x) \le \Gamma_\gamma^+(x)$ and $\beta(x) \notin (\Gamma_\beta^-(x), \Gamma_\beta^+(x))$ or $\Gamma_\beta^-(x) \le \beta(x) \le \Gamma_\beta^+(x)$ ; and $\Gamma_\gamma^-(x) \le \gamma(x) \le \Gamma_\gamma^+(x)$ and $\alpha(x) \notin (\Gamma_\alpha^-(x), \Gamma_\alpha^+(x))$ for all $x \in U$, then $\Xi = <\Gamma, \chi>$ is said to be an $\frac{2}{3}$ -INCS or $\frac{1}{3}$ -ENCS.

**Example 4.** Consider $\Xi = <\Gamma, \chi> \in$ NCS$^U$, if $\Gamma(x) = <$ [0. 5, 0.7], [0.1, 0.2], [0.2, 0.45] $>$ and $\chi(x) = < 0.65, 0.3, 0.4 >$ for all $x \in U$, then $\Xi = <\Gamma, \chi>$ is an $\frac{2}{3}$ -INCS or $\frac{1}{3}$ -ENCS.

**Definition: 13**
$\frac{1}{3}$ **-INCS ($\frac{2}{3}$ -ENCS) [10]:** Consider $\Xi = <\Gamma, \chi> \in$ NCS$^U$, if $\Gamma_\alpha^-(x) \le \alpha(x) \le \Gamma_\alpha^+(x)$ ; $\beta(x) \notin (\Gamma_\beta^-(x), \Gamma_\beta^+(x))$ ; and $\gamma(x) \notin (\Gamma_\gamma^-(x), \Gamma_\gamma^+(x))$ or $\Gamma_\gamma^-(x) \le \gamma(x) \le \Gamma_\gamma^+(x)$ ; $\alpha(x) \notin (\Gamma_\alpha^-(x), \Gamma_\alpha^+(x))$ and $\beta(x) \notin (\Gamma_\beta^-(x), \Gamma_\beta^+(x))$ or $\Gamma_\beta^-(x) \le \beta(x) \le \Gamma_\beta^+(x)$ ; $\alpha(x) \notin (\Gamma_\alpha^-(x), \Gamma_\alpha^+(x))$ and $\gamma(x) \notin (\Gamma_\gamma^-(x), \Gamma_\gamma^+(x))$ for all $x \in U$, then $\Xi = <\Gamma, \chi>$ is said to be an $\frac{1}{3}$ -INCS or $\frac{2}{3}$ -ENCS.

**Example 5.** Consider $\Xi = <\Gamma, \chi> \in$ NCS$^U$, if $\Gamma(x) = <$ [0. 5, 0.8], [0.15, 0.25], [0.2, 0.35 ] $>$ and $\chi(x) = < 0.55, 0.4, 0.5 >$ for all $x \in U$, then $\Xi = <\Gamma, \chi>$ is an $\frac{1}{3}$ -INCS or $\frac{2}{3}$ -ENCS.

**Definition: 14 [10]**
Consider $\Xi_1 = <\Gamma_1, \chi_1>$ and $\Xi_2 = <\Gamma_2, \chi_2>$ be two NCSs in $U$, then

1. (Equality) $\Xi_1 = \Xi_2$ if and only if $\Gamma_1 = \Gamma_2$ and $\chi_1 = \chi_2$.

2. (P-order) $\Xi_1 \subseteq_P \Xi_2$ if and only if $\Gamma_1 \cong \Gamma_2$ and $\chi_1 \cong \chi_2$.

3. (R-order) $\Xi_1 \subseteq_R \Xi_2$ if and only if $\Gamma_1 \cong \Gamma_2$ and $\chi_1 \supseteq \chi_2$.

For convenience, $p = < ([\Gamma_\alpha^-, \Gamma_{\alpha_1}^+], [\Gamma_\beta^-, \Gamma_{\beta_1}^+], [\Gamma_\gamma^-, \Gamma_\gamma^+])$, $(\alpha, \beta, \gamma) >$ is said to represent neutrosophic cubic value (NCV)

**Definition: 15**

**Complement [10]:** Consider $\Xi = < \Gamma, \chi >$ be an NCS, then the complement of $\Xi = < \Gamma, \chi >$ is given by

$$\Xi^C = \{ \langle x, \Gamma^{\tilde{C}}(x), \chi^{\tilde{C}}(x) \rangle \mid x \in U \}.$$

## 2.1 Several operational rules of NCVs

Consider $p_1 = < ([\Gamma_{\alpha_1}^-, \Gamma_{\alpha_1}^+], [\Gamma_{\beta_1}^-, \Gamma_{\beta_1}^+], [\Gamma_{\gamma_1}^-, \Gamma_{\gamma_1}^+])$, $(\alpha_1, \beta_1, \gamma_1) >$ and $p_2 = < ([\Gamma_{\alpha_2}^-, \Gamma_{\alpha_2}^+], [\Gamma_{\beta_2}^-, \Gamma_{\beta_2}^+], [\Gamma_{\gamma_2}^-, \Gamma_{\gamma_2}^+])$, $(\alpha_2, \beta_2, \gamma_2) >$ be two NCVs in $U$, then the operational rules are presented as follows:

1. The complement [10] of $p_1$ is $p_1^C = < ([\Gamma_{\gamma_1}^-, \Gamma_{\gamma_1}^+], [1-\Gamma_{\beta_1}^+, 1-\Gamma_{\beta_1}^-], [\Gamma_{\alpha_1}^-, \Gamma_{\alpha_1}^+]), (\gamma_1, 1-\beta_1, \alpha_1) >$.

2. The summation between $p_1$ and $p_2$ is defined as follows:

$$p_1 \oplus p_2 = < ([\Gamma_{\alpha_1}^- + \Gamma_{\alpha_2}^- - \Gamma_{\alpha_1}^- \Gamma_{\alpha_2}^-, \Gamma_{\alpha_1}^+ + \Gamma_{\alpha_2}^+ - \Gamma_{\alpha_1}^+ \Gamma_{\alpha_2}^+], [\Gamma_{\beta_1}^- \Gamma_{\beta_2}^-, \Gamma_{\beta_1}^+ \Gamma_{\beta_2}^+],$$
$$[\Gamma_{\gamma_1}^- \Gamma_{\gamma_2}^-, \Gamma_{\gamma_1}^+ \Gamma_{\gamma_2}^+]), (\alpha_1 + \alpha_2 - \alpha_1 \alpha_2, \beta_1 \beta_2, \gamma_1 \gamma_2) >.$$

3. The multiplication between $p_1$ and $p_2$ is defined as follows:

$$p_1 \otimes p_2 = < ([\Gamma_{\alpha_1}^- \Gamma_{\alpha_2}^-, \Gamma_{\alpha_1}^+ \Gamma_{\alpha_2}^+], [\Gamma_{\beta_1}^- + \Gamma_{\beta_2}^- - \Gamma_{\beta_1}^- \Gamma_{\beta_2}^-, \Gamma_{\beta_1}^+ + \Gamma_{\beta_2}^+ - \Gamma_{\beta_1}^+ \Gamma_{\beta_2}^+], [\Gamma_{\gamma_1}^- + \Gamma_{\gamma_2}^- - \Gamma_{\gamma_1}^- \Gamma_{\gamma_2}^-, \Gamma_{\gamma_1}^+ + \Gamma_{\gamma_2}^+ - \Gamma_{\gamma_1}^+ \Gamma_{\gamma_2}^+]), (\alpha_1 \alpha_2, \beta_1 + \beta_2 - \beta_1 \beta_2, \gamma_1 + \gamma_2 - \gamma_1 \gamma_2) >.$$

4. Consider $p_1 = < ([\Gamma_{\alpha_1}^-, \Gamma_{\alpha_1}^+], [\Gamma_{\beta_1}^-, \Gamma_{\beta_1}^+], [\Gamma_{\gamma_1}^-, \Gamma_{\gamma_1}^+])$, $(\alpha_1, \beta_1, \gamma_1) >$ be a NCV and $\kappa$ be an arbitrary positive real number, then $\kappa p_1$ and $p_1^\kappa$ are defined as follows:

   (i) $\kappa p_1 = < ([1-(1-\Gamma_{\alpha_1}^-)^\kappa, 1-(1-\Gamma_{\alpha_1}^+)^\kappa], [(\Gamma_{\beta_1}^-)^\kappa, (\Gamma_{\beta_1}^+)^\kappa], [(\Gamma_{\gamma_1}^-)^\kappa, (\Gamma_{\gamma_1}^+)^\kappa]), (1-(1-\alpha_1)^\kappa, (\beta_1)^\kappa, (\gamma_1)^\kappa) >;$

   (ii) $p_1^\kappa = < ([(\Gamma_{\alpha_1}^-)^\kappa, (\Gamma_{\alpha_1}^+)^\kappa], [1-(1-\Gamma_{\beta_1}^-)^\kappa, 1-(1-\Gamma_{\beta_1}^+)^\kappa], [1-(1-\Gamma_{\gamma_1}^-)^\kappa, 1-(1-\Gamma_{\gamma_1}^+)^\kappa]), ((\alpha_1)^\kappa, 1-(1-\beta_1)^\kappa, 1-(1-\gamma_1)^\kappa) >.$

**Definition: 16 [10]**

Consider $\Xi_1 = < \Gamma_1, \chi_1 >$ and $\Xi_2 = < \Gamma_2, \chi_2 >$ be two NCSs in $U$, then the Hamming distance between $\Xi_1$ and $\Xi_2$ is defined as follows:

$$D_H(\Xi_1, \Xi_2) = \frac{1}{9n} \sum_{i=1}^{n} (|\Gamma_{1\alpha}^-(x_i) - \Gamma_{2\alpha}^-(x_i)| + |\Gamma_{1\beta}^-(x_i) - \Gamma_{2\beta}^-(x_i)| + |\Gamma_{1\gamma}^-(x_i) - \Gamma_{2\gamma}^-(x_i)| + |\Gamma_{1\alpha}^+(x_i) - \Gamma_{2\alpha}^+(x_i)| + |\Gamma_{1\beta}^+(x_i) - \Gamma_{2\beta}^+(x_i)| + |\Gamma_{1\gamma}^+(x_i) - \Gamma_{2\gamma}^+(x_i)| + |\alpha_1(x_i) - \alpha_2(x_i)| + |\beta_1(x_i) - \beta_2(x_i)| + |\gamma_1(x_i) - \gamma_2(x_i)|).$$

**Example 7:** Suppose that $\Xi_1 = < \Gamma_1, \chi_1 > = < ([0.6, 0.75], [0.15, 0.25], [0.25, 0.45]), (0.8, 0.35, 0.15) >$ and $\Xi_2 = < \Gamma_2, \chi_2 > = < ([0.45, 0.7], [0.1, 0.2], [0.05, 0.2]), (0.3, 0.15, 0.45) >$ be two NCSs in $U$, then $D_H(\Xi_1, \Xi_2) = 0.1944$.

**Definition: 17**

Consider $\Xi_1 = < \Gamma_1, \chi_1 >$ and $\Xi_2 = < \Gamma_2, \chi_2 >$ be two NCSs in $U$, then the Euclidean distance between $\Xi_1$ and $\Xi_2$ is defined as given below.

$$D_E(\Xi_1, \Xi_2) = \sqrt{\frac{1}{9n} \sum_{i=1}^{n} \left( \begin{array}{l} (\Gamma_{1\alpha}^-(x_i) - \Gamma_{2\alpha}^-(x_i))^2 + (\Gamma_{1\alpha}^+(x_i) - \Gamma_{2\alpha}^+(x_i))^2 + (\Gamma_{1\beta}^-(x_i) - \Gamma_{2\beta}^-(x_i))^2 + \\ (\Gamma_{1\beta}^+(x_i) - \Gamma_{2\beta}^+(x_i))^2 + (\Gamma_{1\gamma}^-(x_i) - \Gamma_{2\gamma}^-(x_i))^2 + (\Gamma_{1\gamma}^+(x_i) - \Gamma_{2\gamma}^+(x_i))^2 + \\ (\alpha_1(x_i) - \alpha_2(x_i))^2 + (\beta_1(x_i) - \beta_2(x_i))^2 + (\gamma_1(x_i) - \gamma_2(x_i))^2 \end{array} \right)}$$

with the condition $0 \leq D_E(\Xi_1, \Xi_2) \leq 1$.

**Example 8:** Suppose that $\Xi_1 = < \Gamma_1, \chi_1 > = < ([0.4, 0.5], [0.1, 0.2], [0.25, 0.5]), (0.4, 0.3, 0.25) >$ and $\Xi_2 = < \Gamma_2, \chi_2 > = < ([0.5, 0.9], [0.15, 0.3], [0.05, 0.1]), (0.7, 0.1,$

0.15) > be two NCSs in $U$, then $D_E(\Xi_1, \Xi_2) = 0.2409$.

## 3 An extended TOPSIS method for MADM problems under neutrosophic cubic set environment

In this Section, we introduce a new extended TOPSIS method to handle MADM problems involving neutrosophic cubic information. Consider $B = \{B_1, B_2, \ldots, B_m\}$, $(m \geq 2)$ be a discrete set of $m$ feasible alternatives and $C = \{C_1, C_2, \ldots, C_n\}$, $(n \geq 2)$ be a set of attributes. Also, let $w = (w_1, w_2, \ldots, w_n)^T$ be the unknown weight vector of the attributes with $0 \leq w_j \leq 1$ such that $\sum_{j=1}^{n} w_j = 1$. Suppose that the rating of alternative $B_i$ $(i = 1, 2, \ldots, m)$ with respect to the attribute $C_j$ $(j = 1, 2, \ldots, n)$ is described by $a_{ij}$ where $a_{ij} = < ([\Gamma_{\alpha_{ij}}^-, \Gamma_{\alpha_{ij}}^+], [\Gamma_{\beta_{ij}}^-, \Gamma_{\beta_{ij}}^+], [\Gamma_{\gamma_{ij}}^-, \Gamma_{\gamma_{ij}}^+]), (\alpha_{ij}, \beta_{ij}, \gamma_{ij}) >$. The proposed approach for ranking the alternatives under neutrosophic cubic environment is shown using the following steps:

### Step 1. Construction and standardization of decision matrix with neutrosophic cubic information

Consider the rating of alternative $B_i$ $(i = 1, 2, \ldots, m)$ with respect to the predefined attribute $C_j$, $(j = 1, 2, \ldots, n)$ be presented by the decision maker in the neutrosophic cubic decision matrix ( See eqn. 1).

$$\langle a_{ij} \rangle_{m \times n} = \begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1n} \\ a_{21} & a_{22} & \ldots & a_{2n} \\ . & . & \ldots & . \\ . & . & \ldots & . \\ a_{m1} & a_{m2} & \ldots & a_{mn} \end{bmatrix} \quad (1)$$

In general, there are two types of attributes appear in the decision making circumstances namely (i) benefit type attributes $\in J_1$, where the more attribute value denotes better alternative (ii) cost type attributes $\in J_2$, where the less attribute value denotes better alternative. We standardize the above decision matrix $\langle a_{ij} \rangle_{m \times n}$ in order to remove the influence of diverse physical dimensions to decision results.

Consider $\langle s_{ij} \rangle_{m \times n}$ be the standardize decision matrix, where

$$s_{ij} = < ([\dot{\Gamma}_{\alpha_{ij}}^-, \dot{\Gamma}_{\alpha_{ij}}^+], [\dot{\Gamma}_{\beta_{ij}}^-, \dot{\Gamma}_{\beta_{ij}}^+], [\dot{\Gamma}_{\gamma_{ij}}^-, \dot{\Gamma}_{\gamma_{ij}}^+]),$$
$$(\dot{\alpha}_{ij}, \dot{\beta}_{ij}, \dot{\gamma}_{ij}) >,$$

where

$s_{ij} = a_{ij}$, if the attribute j is benefit type;

$s_{ij} = a_{ij}^c$, if the attribute j is cost type.

Here $a_{ij}^c$ denotes the complement of $a_{ij}$.

### Step 2. Identify the weights of the attributes

To determine the unknown weight of attribute in the decision making situation is a difficult task for DM. Generally, weights of the attributes are dissimilar and they play a decisive role in finding the ranking order of the alternatives. Pramanik and Mondal [16] defined arithmetic averaging operator (AAO) in order to transform interval neutrosophic numbers to SVNNs. Based on the concept of Pramanik and Mondal [16], we define AAO to transform NCVs to SNVs as follows:

$$NC_{ij} < \dot{\Gamma}_{\alpha_{ij}}, \dot{\Gamma}_{\beta_{ij}}, \dot{\Gamma}_{\gamma_{ij}} > =$$

$$NC_{ij} \left\langle \frac{\dot{\Gamma}_{\alpha_{ij}}^- + \dot{\Gamma}_{\alpha_{ij}}^+ + \dot{\alpha}_{ij}}{3}, \frac{\dot{\Gamma}_{\beta_{ij}}^- + \dot{\Gamma}_{\beta_{ij}}^+ + \dot{\beta}_{ij}}{3}, \frac{\dot{\Gamma}_{\gamma_{ij}}^- + \dot{\Gamma}_{\gamma_{ij}}^+ + \dot{\gamma}_{ij}}{3} \right\rangle$$

In this paper, we utilize information entropy method to find the weights of the attributes $w_j$ where weihgts of the attributes are unequal and fully unknown to the DM. Majumdar and Samanta [17] investigated some similarity measures and entropy measures for SVNSs where entropy is used to measure uncertain information. Here, we take the following notations:

$$T_{\Omega_P}(x_i) = \left[ \frac{\dot{\Gamma}_{\alpha_{ij}}^- + \dot{\Gamma}_{\alpha_{ij}}^+ + \dot{\alpha}_{ij}}{3} \right], \quad I_{\Omega_P}(x_i) = \left[ \frac{\dot{\Gamma}_{\beta_{ij}}^- + \dot{\Gamma}_{\beta_{ij}}^+ + \dot{\beta}_{ij}}{3} \right],$$

$$F_{\Omega_P}(x_i) = \left[ \frac{\dot{\Gamma}_{\gamma_{ij}}^- + \dot{\Gamma}_{\gamma_{ij}}^+ + \dot{\gamma}_{ij}}{3} \right]$$

Then we can write $\Omega_P = \left\langle T_{\Omega_P}(x_i), I_{\Omega_P}(x_i), F_{\Omega_P}(x_i) \right\rangle$.

The entropy value is given as follows:

$$E_i(\Omega_P) = 1 - \frac{1}{n} \sum_{i=1}^{m} (T_{\Omega_P}(x_i) + F_{\Omega_P}(x_i)) \left| I_{\Omega_P}(x_i) - I_{\Omega_P}^c(x_i) \right|$$

which has the following properties:

(i). $E_i(\Omega_P) = 0$ if $\Omega_P$ is a crisp set and $I_{\Omega_P}(x_i) = 0$, $F_{\Omega_P}(x_i) = 0 \; \forall \; x \in E$.

(ii). $E_i(\Omega_P) = 0$ if $\left\langle T_{\Omega_P}(x_i), I_{\Omega_P}(x_i), F_{\Omega_P}(x_i) \right\rangle = < T_{\Omega_P}(x_i),$ 0.5, $F_{\Omega_P}(x_i) >, \; \forall \; x \in E$.

(iii). $E_i(\Omega_P) \geq E_i(\Omega_Q)$ if $\Omega_P$ is more uncertain than $\Omega_Q$ i.e.

$$T_{\Omega_P}(x_i) + F_{\Omega_P}(x_i) \leq T_{\Omega_Q}(x_i) + F_{\Omega_Q}(x_i)$$

and $\left| I_{\Omega_P}(x_i) - I_{\Omega_P}^c(x_i) \right| \leq \left| I_{\Omega_Q}(x_i) - I_{\Omega_Q}^c(x_i) \right|$.

(iv). $E_i(\Omega_P) = E_i(\Omega_P^c), \; \forall \; x \in E$.

Consequently, the entropy value $Ev_j$ of the j-th attribute can be calculated as as follows:.

$Ev_j = 1 - \frac{1}{n}\sum_{i=1}^{m}(T_{ij}(x_i) + F_{ij}(x_i))\left|I_{ij}(x_i) - I_{ij}^{C}(x_i)\right|$, i = 1, 2, …, m; j = 1, 2,…, n.

We observe that $0 \leq Ev_j \leq 1$. Based on Hwang and Yoon [18] and Wang and Zhang [19] the entropy weight of the j-th attribute is defined as follows:

$w_j = \dfrac{1 - Ev_j}{\sum_{j=1}^{n}(1 - Ev_j)}$ with $0 \leq w_j \leq 1$ and $\sum_{j-1}^{n}w_j = 1$.

## Step 3. Formulation of weighted decision matrix

The weighted decision matrix is obtained by multiplying weights of the attributes ($w_j$) and the standardized decision matrix $\langle s_{ij}\rangle_{m\times n}$. Therefore, the weighted neutrosophic cubic decision matrix $\langle z_{ij}\rangle_{m\times n}$ is obtained as:

$$\langle z_{ij}\rangle_{m\times n} = w_j \otimes \langle a_{ij}\rangle_{m\times n} = \begin{bmatrix} w_1 s_{11} & w_2 s_{12} & \cdots & w_n s_{1n} \\ w_1 s_{21} & w_2 s_{22} & \cdots & w_n s_{2n} \\ . & . & \cdots & . \\ w_1 s_{m1} & w_2 s_{m2} & \cdots & w_n s_{mn} \end{bmatrix}$$

$$= \begin{bmatrix} z_{11} & z_{12} & \cdots & z_{1n} \\ z_{21} & z_{22} & \cdots & z_{2n} \\ . & . & \cdots & . \\ . & . & \cdots & . \\ z_{m1} & z_{m2} & \cdots & z_{mn} \end{bmatrix}$$

where

$z_{ij} = < ([\ddot{\Gamma}_{\alpha_{ij}}^{-}, \ddot{\Gamma}_{\alpha_{ij}}^{+}], [\ddot{\Gamma}_{\beta_{ij}}^{-}, \ddot{\Gamma}_{\beta_{ij}}^{+}], [\ddot{\Gamma}_{\gamma_{ij}}^{-}, \ddot{\Gamma}_{\gamma_{ij}}^{+}]), (\ddot{\alpha}_{ij}, \ddot{\beta}_{ij}, \ddot{\gamma}_{ij}) >$

$= < ([1 - (1 - \dot{\Gamma}_{\alpha_{ij}}^{-})^{w_j}, 1 - (1 - \dot{\Gamma}_{\alpha_{ij}}^{+})^{w_j}],$

$[(\dot{\Gamma}_{\beta_{ij}}^{-})^{w_j}, (\dot{\Gamma}_{\beta_{ij}}^{+})^{w_j}], [(\dot{\Gamma}_{\gamma_{ij}}^{-})^{w_j}, (\dot{\Gamma}_{\gamma_{ij}}^{+})^{w_j}]), (1 - (1 - \dot{\alpha}_{ij})^{w_j},$

$(\dot{\beta}_{ij})^{w_j}, (\dot{\gamma}_{ij})^{w_j}) >$

## Step 4. Selection of neutrosophic cubic positive ideal solution (NCPIS) and neutrosophic cubic negative ideal solution (NCNIS)

Consider $z^U = (z_1^U, z_2^U, \dots, z_n^U)$ and $z^L = (z_1^L, z_2^L, \dots, z_n^L)$ be the NCPIS and NCNIS respectively, then $z_j^U$ is defined as follows:

$z_j^U = < ([(\ddot{\Gamma}_{\alpha_j}^{-})^U, (\ddot{\Gamma}_{\alpha_j}^{+})^U], [(\ddot{\Gamma}_{\beta_j}^{-})^U, (\ddot{\Gamma}_{\beta_j}^{+})^U], [(\ddot{\Gamma}_{\gamma_j}^{-})^U,$

$(\ddot{\Gamma}_{\gamma_j}^{+})^U]), ((\ddot{\alpha}_j)^U, (\ddot{\beta}_j)^U, (\ddot{\gamma}_j)^U) >$

where

$(\ddot{\Gamma}_{\alpha_j}^{-})^U = \{(\max_i \{\ddot{\Gamma}_{\alpha_{ij}}^{-}\}| j \in J_1), (\min_i \{\ddot{\Gamma}_{\alpha_{ij}}^{-}\}| j \in J_2)\}$,

$(\ddot{\Gamma}_{\alpha_j}^{+})^U = \{(\max_i \{\ddot{\Gamma}_{\alpha_{ij}}^{+}\}| j \in J_1), (\min_i \{\ddot{\Gamma}_{\alpha_{ij}}^{+}\}| j \in J_2)\}$,

$(\ddot{\Gamma}_{\beta_j}^{-})^U = \{(\min_i \{\ddot{\Gamma}_{\beta_{ij}}^{-}\}| j \in J_1), (\max_i \{\ddot{\Gamma}_{\beta_{ij}}^{-}\}| j \in J_2)\}$,

$(\ddot{\Gamma}_{\beta_j}^{+})^U = \{(\min_i \{\ddot{\Gamma}_{\beta_{ij}}^{+}\}| j \in J_1), (\max_i \{\ddot{\Gamma}_{\beta_{ij}}^{+}\}| j \in J_2)\}$,

$(\ddot{\Gamma}_{\gamma_j}^{-})^U = \{(\min_i \{\ddot{\Gamma}_{\gamma_{ij}}^{-}\}| j \in J_1), (\max_i \{\ddot{\Gamma}_{\gamma_{ij}}^{-}\}| j \in J_2)\}$,

$(\ddot{\Gamma}_{\gamma_j}^{+})^U = \{(\min_i \{\ddot{\Gamma}_{\gamma_{ij}}^{+}\}| j \in J_1), (\max_i \{\ddot{\Gamma}_{\gamma_{ij}}^{+}\}| j \in J_2)\}$,

$(\ddot{\alpha}_j)^U = \{(\max_i \{\ddot{\alpha}_{ij}\}| j \in J_1), (\min_i \{\ddot{\alpha}_{ij}\}| j \in J_2)\}$,

$(\ddot{\beta}_j)^U = \{(\min_i \{\ddot{\beta}_{ij}\}| j \in J_1), (\max_i \{\ddot{\beta}_{ij}\}| j \in J_2)\}$,

$(\ddot{\gamma}_j)^U = \{(\min_i \{\ddot{\gamma}_{ij}\}| j \in J_1), (\max_i \{\ddot{\gamma}_{ij}\}| j \in J_2)\}$;

and $z_j^L$ is defined as given below

$z_j^L = < [(\ddot{\Gamma}_{\alpha_{ij}}^{-})^L, (\ddot{\Gamma}_{\alpha_{ij}}^{+})^L], [(\ddot{\Gamma}_{\beta_{ij}}^{-})^L, (\ddot{\Gamma}_{\beta_{ij}}^{+})^L], [(\ddot{\Gamma}_{\gamma_{ij}}^{-})^L,$

$(\ddot{\Gamma}_{\gamma_{ij}}^{+})^L], ((\ddot{\alpha}_{ij})^L, (\ddot{\beta}_{ij})^L, (\ddot{\gamma}_{ij})^L) >$

where $(\ddot{\Gamma}_{\alpha_j}^{-})^L = \{(\min_i \{\ddot{\Gamma}_{\alpha_{ij}}^{-}\}| j \in J_1), (\max_i \{\ddot{\Gamma}_{\alpha_{ij}}^{-}\}|$ $j \in J_2)\}$, $(\ddot{\Gamma}_{\alpha_j}^{+})^L = \{(\min_i \{\ddot{\Gamma}_{\alpha_{ij}}^{+}\}| j \in J_1), (\max_i \{\ddot{\Gamma}_{\alpha_{ij}}^{+}\}|$ $j \in J_2)\}$, $(\ddot{\Gamma}_{\beta_j}^{-})^L = \{(\max_i \{\ddot{\Gamma}_{\beta_{ij}}^{-}\}| j \in J_1), (\min_i \{\ddot{\Gamma}_{\beta_{ij}}^{-}\}|$ $j \in J_2)\}$, $(\ddot{\Gamma}_{\beta_j}^{+})^L = \{(\max_i \{\ddot{\Gamma}_{\beta_{ij}}^{+}\}| j \in J_1), (\min_i \{\ddot{\Gamma}_{\beta_{ij}}^{+}\}|$ $j \in J_2)\}$, $(\ddot{\Gamma}_{\gamma_j}^{-})^L = \{(\max_i \{\ddot{\Gamma}_{\gamma_{ij}}^{-}\}| j \in J_1), (\min_i \{\ddot{\Gamma}_{\gamma_{ij}}^{-}\}|$ $j \in J_2)\}$, $(\ddot{\Gamma}_{\gamma_j}^{+})^L = \{(\max_i \{\ddot{\Gamma}_{\gamma_{ij}}^{+}\}| j \in J_1), (\min_i \{\ddot{\Gamma}_{\gamma_{ij}}^{+}\}|$ $j \in J_2)\}$, $(\ddot{\alpha}_j)^L = \{(\min_i \{\ddot{\alpha}_{ij}\}| j \in J_1), (\max_i \{\ddot{\alpha}_{ij}\}| j \in J_2)\}$, $(\ddot{\beta}_j)^L = \{(\max_i \{\ddot{\beta}_{ij}\}| j \in J_1), (\min_i \{\ddot{\beta}_{ij}\}| j \in J_2)\}$, $(\ddot{\gamma}_j)^L = \{(\max_i \{\ddot{\gamma}_{ij}\}| j \in J_1), (\min_i \{\ddot{\gamma}_{ij}\}| j \in J_2)\}$.

## Step 5. Calculate the distance measure of alternatives from NCPIS and NCNIS

The Euclidean distance measure of each alternative $B_i$, i = 1, 2, …, m from NCPIS can be defined as follows:

$D_{E_i}^{+} =$

$$\sqrt{\frac{1}{9n}\sum_{j=1}^{n}\begin{pmatrix} (\ddot{\Gamma}_{\alpha_{ij}}^{-} - (\ddot{\Gamma}_{\alpha_j}^{-})^U)^2 + (\ddot{\Gamma}_{\alpha_{ij}}^{+} - (\ddot{\Gamma}_{\alpha_j}^{+})^U)^2 + (\ddot{\Gamma}_{\beta_{ij}}^{-} - (\ddot{\Gamma}_{\beta_j}^{-})^U)^2 + \\ (\ddot{\Gamma}_{\beta_{ij}}^{+} - (\ddot{\Gamma}_{\beta_j}^{+})^U)^2 + (\ddot{\Gamma}_{\gamma_{ij}}^{-} - (\ddot{\Gamma}_{\gamma_j}^{-})^U)^2 + (\ddot{\Gamma}_{\gamma_{ij}}^{+} - (\ddot{\Gamma}_{\gamma_j}^{+})^U)^2 + \\ (\ddot{\alpha}_{ij} - (\ddot{\alpha}_j)^U)^2 + (\ddot{\beta}_{ij} - (\ddot{\beta}_j)^U)^2 + (\ddot{\gamma}_{ij} - (\ddot{\gamma}_j)^U)^2 \end{pmatrix}}$$

Similarly, the Euclidean distance measure of each alternative $B_i$, i = 1, 2, …, $m$ from NCNIS can be written as follows:

$D_{E_i}^- =$

$$\sqrt{\frac{1}{9n}\sum_{j=1}^{n}\begin{pmatrix}(\ddot{\Gamma}_{\alpha_{ij}}^- - (\ddot{\Gamma}_{\alpha_j}^-)^L)^2 + (\ddot{\Gamma}_{\alpha_{ij}}^+ - (\ddot{\Gamma}_{\alpha_j}^+)^L)^2 + (\ddot{\Gamma}_{\beta_{ij}}^- - (\ddot{\Gamma}_{\beta_j}^-)^L)^2 + \\ (\ddot{\Gamma}_{\beta_{ij}}^+ - (\ddot{\Gamma}_{\beta_j}^+)^L)^2 + (\ddot{\Gamma}_{\gamma_{ij}}^- - (\ddot{\Gamma}_{\gamma_j}^-)^L)^2 + (\ddot{\Gamma}_{\gamma_{ij}}^+ - (\ddot{\Gamma}_{\gamma_j}^+)^L)^2 + \\ (\ddot{\alpha}_{ij} - (\ddot{\alpha}_j)^L)^2 + (\ddot{\beta}_{ij} - (\ddot{\beta}_j)^L)^2 + (\ddot{\gamma}_{ij} - (\ddot{\gamma}_j)^L)^2 \end{pmatrix}}.$$

**Step 6. Evaluate the relative closeness co-efficient to the neutrosophic cubic ideal solution**

The relative closeness co-efficient $RCC_i^*$ of each $B_i$, i = 1, 2, …, $m$ with respect to NCPIS $z_j^U$, j = 1, 2, …, $n$ is defined as follows:

$$RCC_i^* = \frac{D_{E_i}^-}{D_{E_i}^+ + D_{E_i}^-}, \text{ i = 1, 2, …, } m.$$

**Step 7. Rank the alternatives**
We obtain the ranking order of the alternatives based on the $RCC_i^*$. The bigger value of $RCC_i^*$ reflects the better alternative.

## 4.   Numerical example

In this section, we consider an example of neutrosophic cubic MADM, adapted from Mondal and Pramanik [20] to demonstrate the applicability and the effectiveness of the proposed extended TOPSIS method.
Consider a legal guardian desires to select an appropriate school for his/ her child for basic education [20]. Suppose there are three possible alternatives for his/ her child:
(1) $B_1$, a Christian missionary school
(2) $B_2$, a Basic English medium school
(3) $B_3$, a Bengali medium kindergarten.
He/ She must take a decision based on the following four attributes:
(1) $C_1$ is the distance and transport,
(2) $C_2$ is the cost,
(3) $C_3$ is the staff and curriculum, and
(4) $C_4$ is the administrative and other facilities
Here $C_1$ and $C_2$ are cost type attributes; while $C_3$ and $C_4$ are benefit type attributes. Suppose the weights of the four attributes are unknown. Using the the following steps, we solve the problem.

**Step 1.** The rating of the alternative $B_i$, i = 1, 2, 3 with respect to the alternative $C_j$, j = 1, 2, 3, 4 is represented by neutrosophic cubic assessments. The decision matrix $\langle a_{ij}\rangle_{3\times4}$ is shown in Table 1.

Table 1. Neutrosophic cubic decision matrix

|       | $C_1$ | $C_2$ |
|-------|-------|-------|
| $B_1$ | < ([0.3, 0.4],[0.1, 0.2],[0.2, 0.35]), (0.3, 0.4, 0.25) > | < ([0.6, 0.7],[0.05, 0.1],[0.2, 0.3]), (0.5, 0.1, 0.25) > |
| $B_2$ | < ([0.8, 0.9],[0.1, 0.2],[0.15, 0.3]), (0.7, 0.15, 0.3) > | < ([0.3, 0.5],[0.1, 0.4],[0.3, 0.5]), (0.4, 0.3, 0.2) > |
| $B_3$ | < ([0.6, 0.7],[0.2, 0.4],[0.25, 0.4]), (0.5, 0.3, 0.3) > | < ([0.2, 0.35],[0.1, 0.25],[0.2, 0.3]), (0.3, 0.3, 0.4) > |

|       | $C_3$ | $C_4$ |
|-------|-------|-------|
| $B_1$ | < ([0.5, 0.6],[0.2, 0.4],[0.1, 0.3]), (0.5, 0.3, 0.4) > | < ([0.4, 0.6],[0.1, 0.25],[0.1, 0.3]), (0.5, 0.2, 0.4) > |
| $B_2$ | < ([0.4, 0.5],[0.2, 0.35],[0.05, 0.2]), (0.35, 0.1, 0.1) > | < ([0.2, 0.3],[0.2, 0.35],[0.1, 0.25]), (0.4, 0.1, 0.1) > |
| $B_3$ | < ([0.4, 0.7],[0.1, 0.3],[0.15, 0.25]), (0.5, 0.2, 0.2) > | < ([0.5, 0.7],[0.1, 0.2],[0.2, 0.25]), (0.3, 0.1, 0.2) > |

**Step 2.** Standardize the decision matrix.
The standardized decision matrix $\langle s_{ij}\rangle_{3\times4}$ is constructed as follows (see Table 2):

Table 2. The standardized neutrosophic cubic decision matrix

|       | $C_1$ | $C_2$ |
|-------|-------|-------|
| $B_1$ | < ([0.2, 0.35],[0.8, 0.9],[0.3, 0.4]), (0.25, 0.6, 0.3) > | < ([0.2, 0.3],[0.9, 0.95],[0.6, 0.7]), (0.25, 0.9, 0.5) > |
| $B_2$ | < ([0.15, 0.3],[0.8, 0.9],[0.8, 0.9]), (0.3, 0.85, 0.7) > | < ([0.3, 0.5],[0.6, 0.9],[0.3, 0.5]), (0.2, 0.7, 0.4) > |
| $B_3$ | < ([0.25, 0.4],[0.6, 0.8],[0.6, 0.7]), (0.3, 0.7, 0.5) > | < ([0.2, 0.3],[0.75, 0.9],[0.2, 0.35]), (0.4, 0.7, 0.3) > |

|       | $C_3$ | $C_4$ |
|-------|-------|-------|
| $B_1$ | < ([0.5, 0.6],[0.2, 0.4],[0.1, 0.3]), (0.5, 0.3, 0.4) > | < ([0.4, 0.6],[0.1, 0.25],[0.1, 0.3]), (0.5, 0.2, 0.4) > |
| $B_2$ | < ([0.4, 0.5],[0.2, 0.35],[0.05, 0.2]), (0.35, 0.1, 0.1) > | < ([0.2, 0.3],[0.2, 0.35],[0.1, 0.25]), (0.4, 0.1, 0.1) > |
| $B_3$ | < ([0.4, 0.7],[0.1, 0.3],[0.15, 0.25]), (0.5, 0.2, 0.2) > | < ([0.5, 0.7],[0.1, 0.2],[0.2, 0.25]), (0.3, 0.1, 0.2) > |

**Step 3.** Using AAO, we transform NCVs into SNVs. We calculate entropy value $E_j$ of the j-th attribute as follows:
    $Ev_1 = 0.644, Ev_2 = 0.655, Ev_3 = 0.734, Ev_4 = 0.663.$
The weight vector of the four attributes are obtained as:
    $w_1 = 0.2730, w_2 = 0.2646, w_3 = 0.2040, w_4 = 0.2584.$

**Step 4.** After identifying the weight of the attribute ($w_j$), we multiply the standardized decision matrix with $w_j$, j = 1, 2, …, $n$ to obtain the weighted decision matrix $\langle z_{ij}\rangle_{3\times4}$ (see Table 3).

Table 3. The weighted neutrosophic cubic decision matrix

|  | C₁ | C₂ |
|---|---|---|
| B₁ | < ([0.059, 0.110], [0.941, 0.972], [0.720, 0.779]) (0.075, 0.87, 0.72) > | < ([0.057, 0.090], [0.972, 0.986], [0.874, 0.91]), (0.073, 0.972, 0.832) > |
| B₂ | < ([0.043, 0.093], [0.941, 0.972], [0.941, 0.972]), (0.093, 0.957, 0.907) > | < ([0.09, 0.168], [0.874, 0.972], [0.727, 0.832]), (0.057, 0.910, 0.785) > |
| B₃ | < ([0.076, 0.13], [0.87, 0.94 I], [0.87, 0.907]), ( 0.093, 0.907, 0.828) > | < ([0.057, 0.090], [0.928, 0.972], [0.653, 0.757]), (0.126, 0.910, 0.727) > |

|  | C₃ | C₄ |
|---|---|---|
| B₁ | < ([0.132, 0.17], [0.720, 0.830], [0.625, 0.782]), (0.084, 0.625, 0.625) > | < ([0.124, 0.21 I], [0.552, 0.699], [0.552, 0.733]), (0.164, 0.660, 0.789) > |
| B₂ | < ([0.100, 0.132], [0.720, 0.807], [0.543, 0.720]), (0.084, 0.625, 0.625) > | < ([0.056, 0.088], [0.66, 0.762], [0.552, 0.699]), (0.124, 0.552, 0.552) > |
| B₃ | < ([0.100, 0.218], [0.625, 0.782], [0.679, 0.754]), (0.132, 0.720, 0.720) > | < ([0.164, 0.267], [0.552, 0.660], [0.660, 0.699]), (0.088, 0.522, 0.660) > |

**Step 5.** From Table 3, the NCPIS $z_j^U$, j = 1, 2, 3, 4 is obtained as follows:

$z_1^U$ = < ([0.043, 0.093], [0.941, 0.972], [0.941, 0.972]), (0.075, 0.957, 0.907) >,

$z_2^U$ = < ([0.057, 0.09], [0.972, 0.986], [0.874, 0.91]), (0.057, 0.972, 0.832) >,

$z_3^U$ = < ([0.132, 0.218], [0.625, 0.782], [0.543, 0.72], (0.132, 0.625, 0.625)>,

$z_4^U$ = < [0.164, 0.267], [0.552, 0.66], [0.552, 0.699], (0.66, 0.552, 0.552)>;

The NCNIS $z_j^L$, j = 1, 2, 3, 4 is determined from the weighted decision matrix (see Table 3) as follows:

$z_1^L$ = < [0.076, 0.13], [0.87, 0.941], [0.72, 0.779], (0.093, 0.87, 0.72)>,

$z_2^L$ = < [0.09, 0.168], [0.874, 0.972], [0.653, 0.757], (0.126, 0.91, 0.727)>,

$z_3^L$ = < [0.1, 0.132], [0.72, 0.83], [0.679, 0.782], (0.084, 0.782, 0.83)>,

$z_4^L$ = < [0.056, 0.088], [0.66, 0.762], [0.66, 0.733], (0.088, 0.66, 0.789)>.

**Step 6.** The Euclidean distance measure of each alternative from NCPIS is obtained as follows:

$D_{E_1}^+$ = 0.1232, $D_{E_2}^+$ = 0.1110, $D_{E_3}^+$ = 0.1200.

Similarly, the Euclidean distance measure of each alternative from NCNIS is computed as follows:

$D_{E_1}^-$ = 0.0705, $D_{E_2}^-$ = 0.0954, $D_{E_3}^-$ = 0.0736.

**Step 7.** The relative closeness co-efficient $RCC_i^*$, i = 1, 2, 3 is obtained as follows:

$RCC_1^*$ = 0.3640, $RCC_2^*$ = 0.4622, $RCC_3^*$ = 0.3802.

**Step 8.** The ranking order of the feasible alternative according to the relative closeness co-efficient of the neutrosophic cubic ideal solution is presented as follows:

$$B_2 > B_3 > B_1$$

Therefore, $B_2$ i.e. a Basic English medium school is the best option for the legal guardian.

## 5 Conclusions

In the paper, we have presented a new extended TOPSIS method for solving MADM problems with neutrosophic cubic information. We have proposed several operational rules on neutrosophic cubic sets. We have defined Euclidean distance between two neutrosophic cubic sets. We have defined arithmetic average operator for neutrosophic cubic numbers. We have employed information entropy scheme to calculate unknown weights of the attributes. We have also defined neutrosophic cubic positive ideal solution and neutrosophic cubic negative ideal solution in the decision making process. Then, the most desirable alternative is selected based on the proposed extended TOPSIS method under neutrosophic cubic environment. Finally, we have solved a numerical example of MADM problem regarding school selection for a legal guardian to illustarte the proposed TOPSIS method. We hope that the proposed TOPSIS method will be effective in dealing with different MADM problems such as medical diagnosis, pattern recognition, weaver selection, supplier selection, etc in neutrosophic cubic set environment.

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