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What is Frege’s ‘concept *horse* problem’?

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Elizabeth Anscombe once complained of the ‘wild irrelevance’ of almost all that had previously been published on the *Tractatus*. This unfortunate state of affairs, she supposed, arose from a neglect of Frege (1959: 12). Sensitive to these concerns, Peter Geach went on to urge Frege’s importance as the source of much of what the *Tractatus* has to say about the distinction between what can be said in language and what can only be shown, and about the inexpressibility of logical category distinctions in particular. The *Tractatus*, he says, simply ‘takes over’ certain Fregean doctrines in this area, adding its own modifications and extensions where appropriate (1976: 55–6).

These claims seem worth evaluating. To do so it will be necessary to look carefully at Frege’s thought to see whether we can find there the seeds of the morals Wittgenstein drew, and, if so, to gauge how far Frege himself developed them. The ultimate purpose of such a detailed investigation is to distinguish the possible claim that Wittgenstein *derived* his views on the inexpressibility of logical category distinctions and the say/show distinction from Frege from the view that he merely *arrived at* these views by reflecting on issues to which Frege’s work gives rise. I will argue that while the second claim is plausibly correct, the first is not supported by the texts.

Although this is our ultimate goal, our immediate purpose is to get clearer about the nature of the concept *horse* problem, and to gauge how far Frege thought it could be solved. The concept *horse* problem is a natural starting point for any discussion of Frege’s views on logical category distinctions, for here our classificatory practices seem to run into trouble. Something we should like to categorize as a concept turns out—in Frege’s view—not to be so categorizable. The puzzle, however, has numerous aspects or sub-problems that must be carefully distinguished if its philosophical and historical significance is to be properly gauged. In what follows I shall attempt to distinguish these sub-problems more sharply than has been the custom in the scholarly literature. By so doing I hope to work towards a better understanding of which sub-problems Frege took seriously and which he regarded as superficial—or even just ignored. With those matters clarified, it will be easier to judge how far these problems can be addressed,

and—most importantly for the purposes of this volume—how far, and in what ways, Frege's reflections in this area are likely to have influenced Wittgenstein.

1 First problem: a breach of custom

Frege introduces the concept *horse puzzle* in his article 'On Concept and Object' (1892b). This article was written as a response to certain criticisms that had been leveled against Frege's position in *The Foundations of Arithmetic* by a prominent student of Brentano's, the Strasburg philosopher, Benno Kerry. Kerry wrote a series of eight articles appearing during the years 1885 to 1891 in the journal *Vierteljahrsschrift für Wissenschaftliche Philosophie* under the title 'Über Anschauung und ihre psychische Verarbeitung' ('On intuition and its psychical elaboration'). The concept *horse problem* is posed in the fourth of these articles (Kerry 1887) in the context of a discussion of Frege's fundamental tenet that the distinction between concepts and objects is exclusive. The article 'On Concept and Object', which contains Frege's defence of that thesis, then appeared in the same journal.

Kerry maintains against Frege's exclusivity thesis that the properties of being an object and of being a concept are relative rather than absolute: just as someone can be a father relative to one person and a son relative to another, something can be a concept relative to one way of regarding it and an object relative to another (1887: 272). He offers two arguments to back up this contention. The first seeks to demonstrate the incompatibility of the exclusivity thesis with Frege's criterion for being an object. That criterion takes the form of a sufficient condition: if an expression results from pre-fixing the definite article or a demonstrative pronoun to a singular occurrence of a concept word, it is a proper name (1884: §§51, 66), hence the name of an 'object' in Frege's sense of that term (1884: §68, n. 1). To pose his problem Kerry draws attention to the judgment, 'The concept I am now talking about is an individual concept.' He regards it as obvious that in this judgment, which he treats as true, the subject—i.e. what we would call the referent of the subject term—is a concept, hence, by the exclusiveness of the distinction, no object. But the subject term is of the form 'the *F*', and so by Frege's criterion its referent must be an object. We thus arrive at a contradiction, and so one of Frege's two assumptions must be given up.

By Kerry's own lights this first argument is not decisive against the exclusivity thesis, since he in any case rejects Frege's criterion on the ground that logical distinctions cannot be founded upon linguistic distinctions (1887: 273). Accordingly, he offers a second direct or 'positive' argument for the thesis that some concepts are objects. Kerry adduces the judgment, 'The concept "horse" is a concept easily attained,'¹ and assumes (without argument) that whatever falls under the concept expressed by the words 'is a

¹ Kerry uses quotation marks where Frege uses italics.

concept easily attained' must be an object. He concludes that the concept 'horse' must be both a concept and an object (1887: 274).

In response Frege allows that the subject term in each of Kerry's two examples may indeed refer to a 'concept', but not in the logical sense with which he is concerned (1892b: 182, 185). Kerry's argument, he thus suggests, falls at the first fence.

As directed against the two examples just discussed, Frege's response is convincing. The sort of concept that is 'attainable' is (something like) a psychological capacity, and the 'individual concepts' of which Kerry speaks would seem to be a species of psychological representation. Both of Kerry's examples therefore involve 'concepts' only in a psychological sense, and these would clearly count as 'objects' in the logical sense Frege is concerned with. Nonetheless, this response may not be altogether fair. For Frege neglects to mention that in his second argument Kerry offers a better example for his critical purposes. This involves one of Frege's own examples from the *Foundations*, viz., the second-level concept '[is] equinumerous with the concept *F*' (Kerry 1887: 274). Obviously, if Frege were to accept Kerry's assertion that the entities that fall under this concept are concepts, he would have to accept that they are concepts in the logical sense.

Although Frege fails to credit Kerry with this more promising criticism, he clearly recognizes its force. For he proceeds to acknowledge as a problem for his view an example that is naturally taken to involve concepts in the logical sense, viz., 'The concept *horse* is a concept'. His response is to deny the truth of this sentence, and beyond that, to assert its negation. The cost of this response, as he acknowledges, is an unavoidable 'awkwardness of language' manifested in a 'breach of custom'. We customarily say, 'The city Berlin is a city' and 'The volcano Vesuvius is a volcano', but if Frege is right, we must also say, 'The concept *horse* is not a concept'. More fully, the difficulty we face, if we follow Frege, is that, as a rule, we take instances of the following schema to be true: 'The *K T* is a *K*', where '*K*' is replaceable by a kind term and '*T*' by a term for something belonging to the kind *K*. But when '*K*' is replaced by the word 'concept' and '*T*' by the word '*horse*' the result is a falsehood. Let us call this the 'the breach of custom problem'.

Perhaps surprisingly, Frege does not try to solve this problem by arguing that the breach of custom is *only* apparent. Instead, he accepts it at face value, commenting, somewhat meekly, that 'Language is here in a predicament that justifies the departure from custom' (1892b: 186). The nature of the 'predicament' is clear enough: when I try to make a concept the subject of discourse the sentence I end up with turns out, against my intentions, to have an object as its subject. But while it might be reasonable to think that the predicament *necessitates* the departure from custom, it is hard to see how it could be supposed to justify it. After all, as Frege would later himself observe (1979: 177), the expression 'the concept *prime*' is (plausibly) 'constructed in a way which precisely parallels [the expression] "the poet Schiller"'. If that is so, then, since the sentence 'the poet Schiller is a poet' seems to be true in virtue of its form, the same

would have to apply to the sentence 'the concept *prime* is a concept', in which case the departure from custom would be wholly unjustified.

The unfortunate fact is that in the essay in which he introduces the breach of custom problem Frege fails to engage with it in any sustained or convincing way.² Nonetheless, we may ask on Frege's behalf how the problem might be addressed within a Fregean setting. To this end it will be useful to begin by considering an interesting proposal once made by Terence Parsons.

1.1 Parsons on the proxy object solution

According to Parsons (1986), a consistent application of Frege's ideas actually requires him to have said that the concept horse *is* a concept. Parsons is prompted to say this by Frege's remark in 'On Concept and Object' that in order to say something about a concept 'an object must go proxy [*vertreten*] for it' (1892b: 186). What is meant to justify the idea that the object referred to by 'the concept *horse*' goes proxy for the concept is Frege's view that in asserting something of this object one thereby expresses the very same thought that would be expressed by asserting something else of the corresponding concept. So, to use Frege's own example (1892b: 188), in asserting something of a certain *object* in:

[1] The concept *square root of 4* is realized,

one thereby expresses the very same thought that one would express by asserting something else of a *concept* in:

[2] There is at least one square root of four.

And for precisely the same reasons, according to Frege's way of thinking, the first-level predicate expressed by the last two words of [1] can be thought to represent or go proxy for the second-level predicate expressed by the first five words of [2]. That being so, the question naturally arises: Why not treat the sentence 'the concept *horse* is a concept' as making an assertion about a proxy object in order to express a truth about a concept? That is to say, why not treat the first-level predicate expressed by 'is a concept' as being used to make an assertion about an object, which assertion expresses precisely the same thought that would be expressed by asserting the corresponding

² Joan Weiner disagrees. She judges Frege's reply an 'appropriate and adequate' response to Kerry (1990: 246). I am not entirely sure what she takes that reply to be, but her main idea would seem to be that Frege is entitled to be untroubled by the paradoxical nature of sentences such as 'the concept *horse* is not a concept', since this would only be a problem, from Frege's point of view, if it prevented the sentence from playing what she takes to be its elucidatory role of '[getting] his readers to understand [Begriffsschrift] regimentation' (1990: 256). And Weiner seems to suppose that this sentence is *not* hampered in its elucidatory role. One has to wonder, however, how this sentence could help one to appreciate (for example) that the Begriffsschrift's second-order variables range over the referents of such expressions as 'is a horse'. It bears mentioning, moreover, that when Frege claims that 'the concept *horse* is not a concept' his immediate aim is not to get his readers to understand his Begriffsschrift regimentation, but rather to rebut Kerry's charge that the distinction between concept and object is not exclusive.

second-level predicate directly of a concept. Let's call this proposal 'the proxy object solution to the custom breach problem'.

In order to evaluate it³ we will need to consider, first, whether we can really make sense of this second way of expressing the thought in question, and, second, whether such a solution, if strictly thought through, would be consistent with Frege's other commitments. There is reason for pessimism on both scores, but these misgivings will take some time to develop.

As a preliminary, let's return to the simple illustrative case of claim [1]. If [1] is genuinely to express a true or false thought, it must, according to Frege, be expressible in his formalized language or *Begriffsschrift*. For the *Begriffsschrift* is supposed to be a tool for the expression of scientific thought: it is supposed to be capable of expressing any thought that has a claim to truth or falsehood. But how would [1] be expressed in a *Begriffsschrift*? It has seemed to some commentators—notably among them Jean van Heijenoort (1977: 104)⁴—that it would go over into a *Begriffsschrift* as: '∃*x* (*x* is a square root of 4)'. Such a formulation, after all, would seem to bring out perspicuously Frege's view that 'existence is a property of concepts' (1884: §53). But this is not, in fact, Frege's view. As he sees it, [1] is a significant sentence containing a proper name in the argument place of a first-level concept word. Accordingly, since Frege holds that every genuine concept must be true or false of any object whatsoever,⁵ [1]'s significance implies that the result of replacing its component phrase 'the concept *square root of 4*' with any other Fregean proper name must also make sense. And, indeed, consistently with this demand, Frege treats the sentence

[3] Julius Caesar is realized

as *false* rather than nonsensical (1892b: 189). If van Heijenoort were right, this sentence could not be rendered in *Begriffsschrift*, for in that case a predication of non-emptiness would be paraphrasable by a *Begriffsschrift* sentence only when the proper name of which the predication was being made was of the form 'the concept *F*'. And so, on the plausible assumption that all significant, truth-valued sentences should be capable of being rendered in a *Begriffsschrift*, we would have to deem [3] nonsensical. The fact that Frege treats it as making sense therefore suggests that he would have taken the first-level predicate 'ξ is realized' to be significantly predicable even of objects that (for all we know) do not proxy any concept. Frege would have seen this predicate as holding

³ That is to say, in order to evaluate it *as a historical proposal*. On its own terms it runs very quickly into the paradoxes. Or rather it does so if it is taken to involve the claim that *every* concept has a proxy object associated with it. For Russell's antinomy, in its Fregean setting, shows that some concepts do not have objects associated with them. Consider, for example, the concept of being the proxy object of a concept that fails to hold of its own proxy object. That concept cannot have a proxy object associated with it, on pain of contradiction.

⁴ Less notably, before thinking hard about 'On Concept and Object' I held this view myself.

⁵ This is just the requirement that concepts should be sharply defined. In the review of Schoenflies Frege glosses the requirement as saying: 'for every object it holds that it either falls under the concept or does not do so' (1979: 179).

true of just those objects that are the proxy objects associated with non-empty concepts. A natural suggestion for a Begriffsschrift expression that would express its sense perspicuously is therefore:

$$[4] \exists f [\xi = \overset{\cdot}{\epsilon} f(\epsilon) \ \& \ \exists x fx].$$

Here ‘ $\overset{\cdot}{\epsilon} f(\epsilon)$ ’ is the name of the extension of the concept f , and the predicate as a whole, informally speaking, expresses the property of being the extension of some realized concept.

It must be conceded that Frege nowhere suggests that the proxy object associated with a concept is its extension, and indeed he seems to have remained deliberately agnostic on that point.⁶ All the same, there can be no doubt that in a Fregean setting it is at least extremely natural to equate the object proxying a concept with its extension. Such a decision seems to do no obvious violence to Frege’s views. At any rate, in order to develop the proxy object solution, I shall suppose that the expression ‘the concept *horse*’ may be regarded as adequately expressed in a Begriffsschrift by the singular term: ‘ $\overset{\cdot}{\epsilon}$ (*horse* ϵ).’ The success of the proxy object solution will depend on whether we can suppose that the first-level predicate ‘ ξ is a concept’ is itself expressible in a Begriffsschrift. If the answer should turn out to be ‘no’, it will be unclear with what right we take ourselves to have expressed a genuine truth in uttering the words ‘the concept *horse* is a concept’.

Using as our model the Begriffsschrift rendering of the first-level predicate that goes proxy for the second-level predicate of ‘being realized’, we arrive at the following idea. Why not take the second-level predicate that we unsuccessfully⁷ try to express in English with the words ‘ Φ is a first-level concept of one argument’ to be proxied by the first-level predicate expressing the concept of being the value range of a first-level concept of one argument. In symbols:

$$[5] \exists f [\xi = \overset{\cdot}{\epsilon} f(\epsilon) \ \& \ C_x(f_x)]$$

Here ‘ $C_x(\varphi_x)$ ’ is some Begriffsschrift expression for the proxied second-level concept, and contains an argument place open to first-level functional expressions of one argument. It does not matter that the concept to be proxied forms a part of the complex concept that proxies it. Our goal is not to eliminate the predicate that expresses the proxied concept, but only to earn the right to suppose that in natural language an assertion about an object could serve to effect talk about the proxied concept. The idea would be that the sentence ‘ $\exists f [\overset{\cdot}{\epsilon}$ (*horse* (ϵ)) = $\overset{\cdot}{\epsilon} f(\epsilon) \ \& \ C_x(f_x)$ ’ expresses the very same thought as ‘ $C_x(\text{horse}_x)$ ’ despite its radically different logical articulation.

This solution would be acceptable only if ‘ $C_x(\varphi_x)$ ’ could itself be framed in the vocabulary of a Begriffsschrift. But *can* such a predicate be framed? It is often said that

⁶ A detailed discussion of this point is contained in chapter 3 of Sullivan (1989).

⁷ The reasons why this predicate is unsuccessful are discussed below.

Frege’s Begriffsschrift contains no symbols corresponding to the English idioms ‘is a function’ or ‘is a concept’.⁸ That observation, for what it’s worth, is correct, since the Begriffsschrift obviously does not provide for the expression of anything corresponding to such would-be *type-neutral* expressions. But the more interesting question is whether Frege would have taken the type-specific concept of ‘being a first-level concept of one argument’ to be expressible in a Begriffsschrift. And the answer to this question is that, at one stage, he would.

In his letter to Russell of the 29 June 1902 Frege introduces a Begriffsschrift symbol for a primitive second-level function that holds only of first-level functions of one argument, namely, the so-called rough-breathing function ‘ $\acute{\epsilon}\varphi(\epsilon)$ ’.⁹ After acknowledging the unavoidable inadequacy—or as he says ‘imprecision’—of the *natural-language* expression ‘... is a function’,¹⁰ Frege continues:

In a conceptual notation we can introduce a precise expression for what we mean when we call something a function (of the first level with one argument), e.g.:

‘ $\acute{\epsilon}\varphi(\epsilon)$ ’. Accordingly, ‘ $\acute{\epsilon}(\epsilon . 3 + 4)$ ’ would express precisely what is expressed imprecisely in the proposition ‘ $\zeta . 3 + 4$ is a function’. (1980: 136)

It is straightforward along these lines to frame a Begriffsschrift symbol for the concept of being a first-level concept of one argument. One simply frames a symbol for being a first-level *function* of one argument whose value for any object as argument is a truth-value, thus:

$$[6] \quad \acute{\epsilon}\varphi(\epsilon) \ \& \ \forall x \ (\varphi x = \text{---} \varphi x)$$

The predicate [6] succeeds in expressing the concept we unsuccessfully try to express in natural language with the words ‘ φ is a first-level function of one argument whose value for any argument is a truth-value’. The latter is a failed formulation of a second-level predicate because there is no satisfactory way of generating an instance. Suppose, for example, that we tried to use such a predicate to say that ‘plus two’ is a first-level function of one argument. We will find that replacing ‘ φ ’ with either of the obvious

⁸ See Weiner 1990: 259 and Conant 2000: 181.

⁹ Notice that the diacritic here is a Greek rough breathing, which faces in the opposite direction to the smooth breathing familiar from its occurrence in Frege’s terms for value ranges. This use of the rough breathing sign is not to be confused with another that occurs in Frege’s correspondence with Russell. In his letter to Russell of 13 November 1904, for the purposes of discussing Russell’s proposal that names of functions should be regarded as interchangeable with names of objects, Frege uses ‘ $\acute{\epsilon}\varphi(\epsilon)$ ’ for a definite description purporting to refer to a function. Here the rough-breathing sign is treated as combining with a predicate to form a definite description. Thus, for example, ‘ $\acute{\epsilon}(\epsilon + 1)1$ ’ would be read as ‘the result of applying to the number one that function whose value for any argument ϵ is $\epsilon + 1$ ’ (see Frege 1980: 160–2).

¹⁰ The context makes clear that when Frege says an imprecise expression is ‘unavoidable’ he just means ‘unavoidable in language’, not ‘unavoidable in a Begriffsschrift’, and it is equally clear that by ‘language’ here he means ‘natural language’. A typical passage in which Frege plainly uses ‘language’ to mean ‘natural language’ occurs in his article ‘Negation’: ‘The criterion for distinguishing between a negative and an affirmative judgment... cannot be derived from language; *for languages are unreliable on logical questions*. It is indeed not the least of the logician’s tasks to indicate the pitfalls laid by language in the way of the thinker’ (1918: 381, emphasis added).

candidate substituends—viz., ‘+ 2’ and ‘ζ + 2’—will yield only nonsense.¹¹ The Begriffsschrift predicate [6], on the other hand, does not face this problem because it carries with it variables to fill the argument place of any first-level functional expression that occupies its argument place. It turns out, then, that by Frege’s lights, at least, the predicate [5] *can* be expressed in Begriffsschrift and Parsons’s proposal must therefore be taken seriously. Nonetheless, it faces several difficulties, presented here in order of increasing acuteness.

First, although the exercise we have just gone through shows that Begriffsschrift is not a filter that will consign the natural-language sentence ‘the concept *F* is a concept’ to the bin of nonsense, nonetheless what we have come up with as its rendering in Begriffsschrift involves a wholly redundant functional expression. Even to grasp the predicate ‘ $\forall x (\varphi x = \neg\varphi x)$ ’ we must already understand that ‘ φ ’ has the feature we express using the predicate ‘ $\epsilon\varphi(\epsilon)$ ’; so [6] could be replaced simply by ‘ $\forall x (\varphi x = \neg\varphi x)$ ’ on its own. The expression ‘ $\epsilon\varphi(\epsilon)$ ’, in short, is a mere redundancy: it could never do anything for you in a proof.

The second difficulty is that the proxy predicate, once rendered in Begriffsschrift, turns out to be true of a lot of things we didn’t want it to be true of. The expression ‘... is a concept’ is treated as meaning ‘... is the value range of a first-level concept of one argument’, and—given Frege’s definition of number in *The Basic Laws of Arithmetic*—this holds, among other things, of the number 1. So numbers turn out to be concepts (or, if we refuse to acquiesce in the home language, we might say that the proxy predicate turns out to hold of numbers), which is precisely not a result Frege desires.

Third, and finally, if the proxy solution solves the narrow custom-breach problem, it does so only at the cost of sharpening the general threat to the exclusivity thesis. For now it comes out true both that the concept horse is an object and that it is a concept.

In light of these problems it seems worth trying another tack.

1.2 A second attempt at a solution

A natural line of response to the breach of custom problem would involve pressing the question whether ‘the concept *horse* is a concept’ is really an instance of the schema ‘The *K T* is a *K*’. If not, then in affirming that the concept *horse* is not a concept Frege will not after all be committing a breach of custom. What, then, is Frege’s view of the structure of this sentence? Since he takes the parallel with ‘The volcano Vesuvius is a volcano’ seriously, one must suppose that he sees the phrase ‘The concept *horse*’ as involving two noun phrases in apposition, namely: ‘The concept’ and ‘*horse*’. And one must suppose that Frege takes the italicization of ‘*horse*’ to signal that it functions as a proper name, presumably a name of the same object that is named by the expression ‘The concept *horse*’. But this reflection is already enough to show that we do not really

¹¹ Cf. ‘[A] function-name can never occupy the place of a proper name, because it carries with it empty places that answer to the unsaturatedness of the function’ (1893: §21).

have something of the form: ‘The K T is a K , where T is a member of the kind K . For objects do not belong to the kind ‘concepts’. This seems like a promising thing to say about the breach of custom problem. But, even if it should prove successful, matters cannot be left there, for Frege’s criterion creates deeper problems of which the one currently under consideration is just a symptom.

2 Second problem: self-stultification

Consider the claim: ‘The concept *horse* is unsaturated’. This is just the sort of utterance to which Frege might resort in attempting to convey his idea that concepts differ fundamentally from objects in having a kind of gappiness. But, by his own lights, it must be viewed as false, since the entity referred to by ‘The concept *horse*’ is an object and so *not* unsaturated. So Frege is not out of the woods yet. Although he doesn’t raise it explicitly himself, he faces a wider problem, illustrated by this claim, of which the custom-breach problem is just an instance: the form in which we attempt to refer to concepts sometimes makes what we want to say about them false. Let’s call this more general difficulty ‘the problem of self-stultification’.

2.1 The ‘exception’ solution

The next solution to be considered attempts to address this broader problem. It begins with the observation that when, in ‘On Concept and Object’, Frege discusses his criterion for being a proper name he acknowledges at least one class of exceptions to it. He recognizes that when they occur in generic constructions definite descriptions cannot be taken to refer to objects. So, for example, in the sentence ‘The horse is a four-legged animal’—used with its ordinary sense—the phrase ‘the horse’ cannot with any plausibility be regarded as a singular term referring to an individual horse. Recognizing this problem, Frege in effect proposes that generic occurrences of definite descriptions be treated as what Russell would later call ‘incomplete symbols’: that is to say, expressions that have no meaning in isolation but which are paraphrastically eliminable from every sentential context in which they occur. In this spirit ‘The horse is a four-legged animal’ is to be taken to mean: ‘All properly constituted horses are four-legged animals’ (1892b: 185).


Frege claims that counter-examples to his criterion for proper name-hood are so easily recognized as special cases that the value of that criterion is not impaired. He is right, I think, to suppose that we are good at recognizing generic uses of definite descriptions, even if, at the same time, he is wrong in thinking that his analysis of generics will work across the board. (His treatment will not work, for example, in familiar problem cases such as ‘The duck-billed platypus lays eggs’.) But because Frege fails to consider any possible exceptions beyond generics,¹² we cannot suppose him to

¹² I read Frege’s example of ‘The Turk besieged Vienna’ as one to which he *does* take the criterion to apply, since ‘The Turk’ in Frege’s view is the proper name of a people (1892b: 185).

have successfully stated even a sufficient condition for being a proper name. And of course, there are several other apparent exceptions that do need to be considered, for example, cases in which a definite description gets its meaning from an adverb of quantification, e.g. 'The owner of a Porsche is often smug'.¹³

That said, it may be that the best way to view Frege's criterion is as making a point about the onus of proof: we might take him to be saying that, in the absence of powerful reasons for thinking that a phrase of the form 'the *F*' is not functioning as a singular term, it ought to be treated as one.

If that is how Frege is thinking, then the question immediately arises why he should not have sought to address the breach of custom problem by treating the appearance of a breach as itself evidence that 'the concept *horse*' is functioning in the relevant context not as a proper name but rather as an incomplete symbol. The answer, I take it, is that Frege must have realized that we cannot in fact treat expressions of the form 'the concept *F*' as everywhere eliminable by paraphrase. For while there are many contexts from which this idiom can be eliminated, the sentences 'The concept *horse* is unsaturated' and 'The concept *horse* is not an object' are not among them.

Geach once suggested that we ought to write off such recalcitrant contexts as so much 'philosopher's nonsense,' on a par with 'The concept *man* is timeless' (1981: 221, 229–30). The suggestion is appealing, if rather draconian: it would counsel throwing overboard, along with the obvious junk, certain sentences that seem to have a serious use in semantics, among them: 'What "is a horse" stands for is the *concept horse*'. Against this, Geach might reply that what is threatened by his proposal is not so much semantics per se, as one way of doing it. For one might reject these formulations as nonsensical while claiming that the semantic truths they are the misfired attempts to convey are properly expressed, in Davidsonian fashion, by such claims as: 'For all *x*,  satisfies the predicate "γ is a horse" iff *x* is a horse.' The solution, however, comes at a cost. At the end of the day, it seems just too high-handed to consign whole realms of discourse to the litter-bin of nonsense just to avoid the concept *horse* problem. So we must conclude that a wholly satisfying, broadly Fregean solution to the self-stultification problem remains elusive.

3 Third problem: the frustration of referential intentions

The third member of the family of difficulties associated with the concept *horse* problem is seldom properly distinguished from the problem of self-stultification. This problem, although it is illustrated by statements about whose truth-value we may be unsure, such as 'The concept *F* is not a concept,' and 'The concept *F* is unsaturated', is not confined to them. It is, instead, a general problem for any statement containing an

¹³ For an interesting discussion of this case, see Fara 2001.

expression of the form ‘the concept F ’. Frege presents this more general problem in a celebrated passage occurring toward the close of ‘On Concept and Object’. He says:

... with the use of my terms [‘concept’] and [‘object’] I have got hold of a distinction of the highest importance. I admit that there is a quite peculiar obstacle in the way of an understanding with my reader. By a kind of necessity of language, my expressions, taken literally, sometimes miss my thought, in that [*indem*] I mention an object, when what I intend is a concept. I fully realize that in such cases I was relying upon a reader who would be ready to meet me half-way—who does not begrudge a pinch of salt. (1892b: 193)

There is, of course, room for doubt about whether Frege is really entitled to claim that he has successfully latched on to the distinction between concepts and objects. But there can be no doubt that he *believes* he has done so. The difficulty he describes here does not involve grasping that distinction so much as communicating an understanding of certain things—which things being not immediately clear. That point is plain enough, but it is a rather more subtle matter to identify the kind of obstacle to understanding that Frege has in mind. It has seemed to some commentators that he may be suggesting that there are inexpressible thoughts which would give expression to the distinction between concept and object if, *per impossibile*, they could be expressed.¹⁴ But such an interpretation is undermined by a careful reading of the passage—a fact that becomes apparent when one attends to Frege’s account of what precisely the phenomenon of his words’ ‘missing’ his thought consists in. My words, he says, miss my thought *in that* I mention an object when what I intend (to mention) is a concept. Frege’s all-important explanation is obscured by the standard translation of this passage, which suppresses the German word *indem*. But when that nuance is restored it becomes clear that the obstacle Frege sees as standing in the way of an understanding with his reader *consists in* the fact that he fails to refer to what he intends to refer to.

Two points about this ‘obstacle to understanding’ require emphasis. First, and most obviously, the obstacle Frege describes is specific to sentences containing singular terms. So Frege is *not* discussing difficulties that attend general sentences, not even a sentence that tends nonetheless to be treated by commentators as a possible focus of this passage, viz: ‘no concept is an object’. Second, the obstacle is present even in those cases in which a thought *is* expressed. It is present, for example, in the sentence ‘The concept *horse* is realized’ even though this sentence, expressing as it does the same thought as ‘There are horses’, is, by Frege’s lights, uncontroversially significant. Oddly, then, my words can ‘miss my thought’ in the precise sense explained in the pinch of salt

¹⁴ One commentator who has flirted with such a reading of the ‘pinch of salt’ remark—insofar as he describes it as a ‘possible reading’—is James Conant. After quoting that remark, Conant continues: ‘[Frege’s] words miss his thought (and end up being nonsense); so there is a thought they are aiming at: an understanding of what his words intend to say depends upon his reader latching onto the thought his words fail properly to express. This failure is due, according to Frege, to “a kind of necessity of language”. If he is to convey the thought he here seeks to convey he has no alternative but to have recourse to (elucidatory) nonsense’ (2000: 188). I shall argue, to the contrary, first, that for Frege elucidations are *not* nonsense, and, second, that the expressions he is referring to here are not, by his lights, nonsense.

passage in spite of the fact that in the ordinary sense they *succeed* in expressing the thought intended. It appears that Frege must be using the phrase 'miss my thought' in a rather idiosyncratic way. He cannot mean *in general* that when the obstacle he mentions is present his words fail to express his intended thought. Still less can he mean that they are nonsense. What he does mean, surely, is just that the thought expressed by a sentence containing the phrase 'the concept *F*' is expressed misleadingly because the sentence expressing it creates the misleading impression that the subject of discourse is a concept when it is really an object. In requesting his pinch of salt in the above passage Frege is just asking leave to say things about concepts by mentioning objects. He is not asking us somehow to latch on to absolutely inexpressible thoughts.

In 'On Concept and Object' this problem, which, in order to do justice to its generality, I shall refer to as 'the frustration of referential intentions problem', is presented as inevitable. But in his unpublished paper 'Comments on Sense and Meaning', which is either contemporary with or—more likely—post-dates, 'On Concept and Object',¹⁵ Frege no longer presents it this way. He writes:

[A concept being essentially unsaturated] there is now a great obstacle in the way of expressing ourselves correctly and making ourselves understood. If we want to speak of a concept, language, with an *almost* irresistible force, compels me to use an inappropriate expression which obscures—I might *almost* say falsifies—the thought. (1979: 119, emphases added)

The obscuring tendency of language (i.e. natural language)¹⁶ is here presented as powerful but not *wholly* irresistible. Frege neither acknowledges nor explains this softening in his stance, but it may well be connected with his willingness in these comments to take seriously the possibility that natural language might after all contain expressions that are capable of referring to a concept while occupying subject position. For having remarked that we ought to outlaw the expression 'the meaning of the concept-word *A*,' he continues:

It would be better to confine ourselves to saying 'what the concept word *A* means,' for this at any rate is to be used predicatively: [as in] 'Jesus is what the concept word "man" means', [which has] the sense of 'Jesus is a man'. (1979: 122)

Geach once proposed (1973: 156) that this suggestion—which constitutes a candidate solution to the failure of referential intentions problem—should be used to provide a solution to the self-stultification problem (though he didn't call it that). The idea

¹⁵ The exact date of the comments is unknown, but we know they must have been written after 1892, the date of publication of 'On Sense and Meaning'. On 'Concept and Object' was also published in 1892.

¹⁶ Notice that here, in speaking of an obstacle in the way of making oneself understood, Frege's considered view is that language is *obscuring* rather than *falsifying* (if one might *almost* say that *p*, then one can't quite say that *p*). This suggests that those cases in which the obstacle arises are principally those in which a true thought is expressed, albeit in a potentially misleading way. And these, I take it are such cases as 'The concept *horse* is realized' and 'The concept *mammal* is wider than the concept *whale*'. I take this as some evidence supporting a similar reading of the obstacle mentioned in 'On Concept and Object'.

would be to treat the thought we attempt to express when we utter the sentence ‘The concept *horse* is a concept’ as properly expressed by the sentence:

[7] What the concept word ‘horse’ means is a concept,

where here the predicate ‘... is a concept’ is understood to refer to a concept of *second* level.

Were this proposal to prove successful, it could double as a solution to the custom-breach problem, for then the sentence ‘The concept *horse* is a concept’ could be regarded merely an abbreviation of its form-revealing paraphrase [7], and so could be treated as straightforwardly true.

But the proposal, unfortunately, is not persuasive. It relies on our being able to hear the phrase ‘what *X* means’ as meaning something other than ‘the meaning of *X*’. And that is not an easy thing to do. Frege, by alleging the meaningfulness of

[8] Jesus is what the concept word ‘man’ means,¹⁷

tries to make it seem as if the phrase following the word ‘is’ in [8] occurs both in the role of a predicate and, at the same time, as the subject of an assertion. For in [8] the phrase ‘Jesus is’ might be taken to express a second-level predicate true of just those concepts that hold of Jesus, which is here predicated of the first-level concept expressed by ‘what the concept word “man” means’. But in practice it is hard to hear [8] as making sense, unless it is construed—against Frege’s intentions—as meaning ‘Jesus falls under the concept *man*.’

It is unclear whether Geach’s idea ever occurred to Frege, but even if it did, it cannot have been his considered view. For, when discussing the inappropriateness of the expressions ‘the concept’ and ‘the function’ in an unpublished article of 1914, Frege reverts to the unqualified claim that ‘Language forces an inappropriate expression on us’ (1979: 239). And before that, in 1906, he had said that the *only* way language can fit the designation for a concept into its grammatical structure is as a proper name (1979: 177). But although, on reflection, the problem came to seem inevitable to Frege, it is equally clearly something he thought we could learn to live with. The passage from 1914 continues: ‘This is a situation which, unfortunately, can hardly be avoided, *but we can render it harmless* by always bearing the inappropriateness of language in mind’ (1979: 239, emphasis added). The inappropriateness of language, then, poses no more of a threat to the philosophy of mathematics than does the moon illusion to astronomy. So long as we bear the predicament of language in mind, we are no more likely to be misled in our *judgments* in the former case than we are in the latter—or so Frege seems to suppose.

¹⁷ I have removed the comma the translators leave in place, since it is required in German but not in English. Cf. ‘Mann ist, was er isst.’

4 Fourth problem: the inexpressibility of logical category distinctions

Frege's difficulties over the concept *horse* are often associated with a worry about our inability to put logical category distinctions into words. The connection is usually established as follows: One observes that if the expression 'the concept *horse*' is a proper name, then the predicate 'is a concept' must be a first-level predicate true or false of objects. But—if one rejects the proxy object solution—it follows that this predicate must be false of anything of which it can be significantly predicated. And then one realizes that there are claims one might wish to make using this predicate, for example, that no first-level concept is an object, which cannot be made. Or rather, one realizes that, although such claims can be made, they fail to do the kind of work we wanted them to do. The claim that no first-level concept is an object for example, comes out as *vacuously* true if analyzed as meaning, 'for any object, if it is a first-level concept then it is not an object'. But someone who utters the sentence 'no first-level concept is an object' typically intends to express some non-vacuous truth; so this proposition cannot be what is intended; and we know this even though, obviously, we are unable to cite any particular proposition as the one that is intended.¹⁸

Frege does not himself explicitly discuss the claim, 'no first-level concept is an object', but his writings abound with remarks that face a similar predicament. In 'On Concept and Object', for example, he says:

[9] Second-level concepts, which concepts fall under, are essentially different from first-level concepts, which objects fall under. (1892b: 190)

The intended contrast can be spelled out more precisely in the claim:

[10] Any first-level concept, but no second-level concept, is such that any object either falls under that concept or under its negation.

Frege's remarks in §21 of the *Basic Laws of Arithmetic* suggest another example of this broad kind, one which attempts to distinguish a different pair of logical categories:

[11] Objects are fundamentally different from first-level functions because any object is saturated but no [first-level] function is.¹⁹

Each of these claims aims to ground a distinction between logical categories in a difference between their properties. The canonical criticism of such an ambition is found in the *Tractatus*:

¹⁸ The steps just rehearsed are essentially those followed by Weiner (1990: 254–5).

¹⁹ Frege actually says: 'Functions of two arguments are just as fundamentally different from functions of one argument as the latter are from objects. For whereas objects are wholly *saturated*, functions of two arguments are saturated to a lesser degree than functions of one argument, which too are already *unsaturated*' (1893: §21). Cf. '[functions] may be called "unsaturated", and in this way we mark them out as fundamentally different from numbers' (1904: 292).

One cannot distinguish forms from one another by saying that one has this property, the other that: for this assumes that there is sense in asserting either property of either form. (4.1241)

Here the word ‘form’ operates as a synonym for ‘type’. Wittgenstein is drawing attention to a consequence of the thesis he advances in the *Notes Dictated to Moore* when he says that ‘a THEORY of types is impossible’ (NM 109). The consequence is that one cannot theorize about types by using (the contrapositive of) the indiscernibility of identicals.

The argument turns on the idea that any predicative symbol—as the *Tractatus* puts it—‘contains the prototype of its argument’ (3.333). In other words, predicative symbols cannot be identified and individuated without paying attention to the logical type of their argument places. Taking that idea seriously, we find that each of the sentences [9] to [11] involves an equivocation. In [11], for example, what is denied of first-level functions is not the same as what is asserted of objects. It therefore *fails* to say that objects have a property that first-level functions lack—or rather, strictly speaking, although it does perhaps say *something*, there is nothing it tries but fails to say. The words in [11] do not miss a thought as an arrow might miss its target. Instead, there is only the illusion of a thought at which one who utters [11] imagines himself to aim.

Properly speaking, of course, *Tractatus* 4.1241 is only the misfired attempt to amalgamate all such particular criticisms into one general argument.²⁰ It attempts to say²¹ that there is no establishing the distinctness of any two entities at different levels of Frege’s hierarchy by citing a property that one of them has but that the other lacks. But by the lights of the *Tractatus* this very claim is itself nonsensical, since the phrase beginning ‘any two entities’ attempts to quantify across types. So here is one place at which the reader of the *Tractatus* finds himself standing on a ladder that must, sooner or later, be kicked away.

The reasoning of *Tractatus* 4.1241—which for convenience we shall call ‘the forms argument’—grants for the sake of argument the meaningfulness of such phrases as ‘is saturated’, ‘is a first-level function of one argument’, and ‘is an object’. It does not show—and does not purport to show—that such claims as [9]–[11] are nonsense or ill-formed. The argument’s conclusion is rather just that these sentences fail to draw the contrasts their authors intend—or rather, that they *imagine* they intend. One who utters them, if he is reflective, will not recognize them as expressions of any thought he intended, though he may come to regard them as expressing some other thought.

Although I take this (i.e. the inexpressibility of logical category distinctions) to be a deep problem, it can seem otherwise. It can appear to arise merely from an ill-judged adherence to Frege’s criterion for proper names. If that criterion is abandoned, the

²⁰ The alert reader will have noticed that even the description just offered of the ambitions of *Tractatus* 4.1241 is itself nonsensical, because there is no sense to be made of the generality involved in the phrase ‘all such particular criticisms’.

²¹ Or, because the ‘attempt’ is made in the clear knowledge that it must fail, we might say that it ‘makes as if to say’ this.

problem—or one manifestation of it—can seem to be trivially resolvable. For if we allow that definite descriptions may, after all, refer to functions, then the contrastive claims we seek to express can seem straightforwardly expressible. Suppose we were to say, for example, that what we loosely refer to as 'the function $x + 2$ ' can be more perspicuously referred to by a definite description ' $\lambda x.x + 2$ ' (read as 'that function whose value for any argument x is $x + 2$ '). This singular term would then be able to fit in the argument place of the predicates ' ξ is an object' and ' ξ is a function', so that the claim 'Socrates is an object but $\lambda x.x + 2$ is not', for example, would make full sense. And there would be no difficulty in expressing this logical category distinction in words.

One limitation of this proposal is that it is unclear how it could be extended to deal with those functions that are concepts. The phrase 'the function whose value for any argument x is x is a horse' may make sense in Fregean syntax (as, indeed, ' $2 + 2 = 4$ is a horse' does), but it makes no sense in English. A second, deeper, problem with the proposal is that it involves abandoning the Fregean hierarchy. For, if we no longer have syntactic criteria for recognizing an expression as a name of an object, it will be unclear how the types of expressions (and so of their referents) are to be fixed. Certainly, it will be of no use to say that types are fixed by the nature of the referent of an expression, for then the statement 'The concept *horse* is a function but Caesar is not a function', since its two occurrences of the predicate 'is a function' would have different types, would still fail to express the intended contrast. But if the hierarchy is dismantled, the paradox whose resolution it provides for—namely, the 'propositional functions' version of Russell's paradox—will be reinstated.²²

Frege, as we have seen, makes remarks that are vulnerable to the criticism presented in *Tractatus* 4.1241. This may not amount to a criticism of *him*, for it may be that his remarks are made with a certain degree of self-consciousness. But just how self-conscious are they? If we go on the evidence of the texts, rather than assumptions about what Frege *must* have thought, the answer is: 'not very'. He does note that what is said of a concept can never be said of an object, and he says that the attempt to make such predications is 'impossible' and 'without sense' [*sinnlos*] (1892b: 189). But, surprisingly, he fails to infer from this that statements such as [9] and [11], to the extent that they make sense, fail to draw the contrasts they seem intended to draw. Nor does he explicitly register any difficulty with so starkly problematic a claim as:

Objects stand opposed to functions. Accordingly I count as *objects* everything that is not a function. (1893: §2)

The trouble with this claim is that it aspires to a kind of generality that, according to Frege, does not exist. For, as we have already noted, Frege has no general conception

²² Wittgenstein's resolution of that paradox, as commentators have observed, makes use of nothing beyond the simple Fregean hierarchy; see Sullivan 2000.

of ‘being a function’ but only one of ‘being an n -level function of m arguments,’ for particular n and m .

Might Frege’s admission that at times he has to rely on ‘hints’ and ‘elucidations’ be evidence that he does register the problem of the inexpressibility of type distinctions? Again, a careful examination of the relevant passages suggests not. In ‘What is a Function?’ Frege says:

The peculiarity of functional signs which we here called ‘unsaturatedness’, naturally has something answering to it in the functions themselves. They too may be called ‘unsaturated’, and in this way we mark them out as fundamentally different from numbers. Of course this is no definition; but likewise none is here possible. I must confine myself to hinting at what I have in mind by means of a metaphorical expression, and here I rely on my reader’s agreeing to meet me half-way. (1904: 292)

There is no suggestion here that Frege’s appeal to his reader to ‘meet [him] half-way’ is prompted by the belief that he has failed to draw the intended contrast. It is prompted, rather, by his acknowledgment that the characterization he employs to establish the contrast—a contrast he supposes he has *succeeded* in drawing—fails to amount to a *definition*. This characterization cannot qualify as a definition, Frege makes clear, on account of the *metaphorical* character of the ‘hints’ deployed. This is exactly what Frege *should* say, since metaphors according to him are expressions that lack sharp boundaries. (That is so, presumably because they have the sense of their corresponding similes, and one thing may be like another without there being any definiteness about *how much* like the other it is.²³) Frege’s frequent recourse to hints, no less than his requests for cooperative understanding, is prompted by the need to convey the meaning of a word for something simple (or primitive) without the use of a definition. Thus toward the beginning of ‘On Concept and Object’ he remarks: ‘On the introduction of a name for something logically simple, a definition is not possible; there is nothing for it but to lead the reader or hearer, by means of hints, to understand the word as is intended’ (1892b: 183). And in this essay too, the impossibility of framing a definition is attributed to the need to resort to *metaphorical* hints: “‘Complete’ and “‘unsaturated’” are of course only figures of speech; but all that I wish or am able to do here is to give hints’ (1892b: 194).

There is no evidence here, then, that Frege regards hints as what we come up with when we attempt get across *absolutely* inexpressible logical category distinctions. They are rather presented as something we must resort to when attempting to convey the meaning of primitives, and Frege takes it that these meanings *can* be conveyed—so long as the reader is sufficiently cooperative.

Precisely parallel observations may be made in connection with Frege’s discussion of the role played by ‘elucidations’ in his article ‘The Foundations of Geometry II’. There elucidation is recommended as an appropriate way of bringing it about that

²³ I am grateful to Tom Ricketts for this point.

investigators attach a shared meaning to a term which, being logically primitive, is incapable of being explained through a definition:

Once the investigators have come to an understanding about the primitive elements and their designations, agreement about what is logically composite is easily reached by means of definition. Since definitions are not possible for primitive elements, something else must enter in. I call it elucidation. It is this, therefore, that serves the purpose of mutual understanding among investigators, as well as the communication of the science to others. (1906: 300)

He goes on to say we may need to rely on a little goodwill and cooperative understanding because 'frequently we cannot do without a figurative mode of expression' (1906: 301).

Elucidations are, in general, supposed to contain the primitive terms to be explained. So if Frege is not to be committed—in violation of the context principle—to nonsensical sentences that contain meaningful terms, he cannot have regarded elucidations as nonsensical. He must have regarded them, instead, as significant explanations that lack *scientific* standing because, being metaphorical, they lack the precision of definitions. This conclusion receives some confirmation both from Frege's remark in the 'Foundations of Geometry II', that 'we can demand from the originator of an elucidation that he himself know for certain what he means' (1906: 301), and from his remark in a letter to Hilbert that, 'Unlike definitions, . . . elucidatory propositions cannot be used in proofs *because they lack the necessary precision*' (1980: 37, emphasis added). Although elucidations are non-scientific, there is no suggestion in Frege that they are the misfired attempts to put into words distinctions that cannot be drawn.

All in all, then, it seems that neither Frege's remarks about hints nor what he has to say about elucidations provides any evidence of self-consciousness on his part about the failure of claims such as [9] and [11] to draw the contrasts at which they aim.

I conclude that Geach overstates his case when he claims that Wittgenstein 'accept [ed] from Frege' the view that 'there are logical category-distinctions which will clearly show themselves in a well-constructed formalized language, but which cannot properly be asserted in language' (1976: 55–56). In his published works, Frege had not gone so far as even to raise the problem, let alone provide for its solution by appealing to the say–show distinction. Indeed, a more likely source of the showing idea—or one aspect of it—is Russell's remark in the *Principles of Mathematics* that 'to mention anything is to show that it is' (1903: §427). That idea is, after all, closely related to the idea of *Tractatus* 4.1211, that a proposition '*f**a*' shows that the object *a* occurs in its sense.

To be sure, Frege did eventually come to have qualms about the natural language words 'function' and 'concept'. In his letter to Russell of 28 July 1902 he recommends their rejection on the grounds that 'they present themselves linguistically as names of first-level functions' whereas 'logically they should be names of second-level functions' (1980: 141). And in his draft review of Schoenflies, from 1906, he rejects the word 'concept' as 'defective' on the ground that 'the phrase "is a concept" requires a proper name as a grammatical subject; and so, strictly speaking, it requires something contradictory, since no proper name can designate a concept; or perhaps better still,

something nonsensical' (1979: 177–8).²⁴ But it is important to keep in mind—and especially so when considering Frege's possible influence on Wittgenstein—that remarks of this kind occur only in Frege's unpublished writings—and so in writings to which Wittgenstein is unlikely to have had access.²⁵

There is, then, scant reason to suppose that Wittgenstein would have *derived* his views on the inexpressible directly from Frege. That being so, it is nonetheless perfectly possible that *reflection* on certain remarks in Frege's writings—statements [9] and [11], among them—would have led him to these ideas. Indeed, in the light of Wittgenstein's comments in the Preface to the *Tractatus*, acknowledging his indebtedness to the 'great works' of Frege for the stimulation of his thoughts, such an hypothesis is plausible. At any rate, in the course of our discussion we have seen one route by which reflecting more thoroughly on the more general problems that receive only narrow solutions in Frege can lead in the direction of Wittgenstein's thought.

5 Conclusion

We have distinguished four sub-problems that might reasonably be taken to fall under the general rubric 'Frege's concept *horse* problem'. It has been argued that the first—'the custom-breach problem'—is plausibly soluble, but that this is a local and shallow solution, since a second, deeper and more general problem—the problem of 'self-stultification'—remains unsolved. The third problem, concerning 'the frustration of referential intentions', is one about which Frege's most considered position is just that it is an inevitable, yet harmless, awkwardness of natural language. The last problem—that of the inexpressibility of logical category distinctions—is a deep and deeply intractable problem. It is not one, however, to which Frege paid much, if any, attention. Although intractable, the related problem of how strictly inexpressible logical category distinctions may nonetheless somehow be indirectly communicated or got across—and of what that achievement consists in—is one on which some progress has been made in recent work.²⁶ A discussion of that progress will, however, have to wait for a future occasion.²⁷

²⁴ Frege's meaning is clear enough: in order for the sentence 'the concept *F* is a concept' to be true what is required is that the expression 'the concept *F*' should be the proper name of a concept, but a concept is just the kind of thing that cannot be given a proper name, so the condition the predicate attempts to impose is contradictory. Frege's afterthought is that it is perhaps better to say that what is required is something nonsensical because we can't really make sense of the idea of a symbol designating a concept in the manner of a proper name: after all, a proper name just is something that, if it designates at all, designates an object.

²⁵ It is possible, of course, that Russell might have mentioned these views to him, or that Frege might have communicated them to him directly in conversation, but conjectures of this kind would be a slim reed on which to hang Geach's interpretation.

²⁶ Notably by Adrian Moore (2003).

²⁷ Thanks to Imogen Dickie, Warren Goldfarb, Bernard Katz, Boris Kment, Martin Lin, Ray Monk, Michael Potter, Thomas Ricketts, Göran Sundholm, and Jessica Wilson. I owe a special debt of gratitude to Peter Sullivan for his advice and encouragement. Versions of this chapter were presented at the University of

References

- Anscombe, G. E. M. (1959) *An Introduction to Wittgenstein's Tractatus*. London: Hutchinson University Library.
- Conant, James (2000) 'Elucidation and Nonsense in Frege and the Early Wittgenstein', in A. Crary and R. Read, eds., *The New Wittgenstein*. London and New York: Routledge, 174–217.
- Fara, Delia Graff (2001) 'Descriptions as Predicates', *Philosophical Studies* 102, 1–42. (Originally published under the name 'Delia Graff'.)
- Frege, Gottlob (1884) *Die Grundlagen der Arithmetik*. Translated by J. L. Austin as *The Foundations of Arithmetic*. Evanston, Ill.: Northwestern University Press, 2nd revised edition 1978.
- (1892a) 'On Sense and Meaning'. Reprinted in Frege (1984), 157–77.
- (1892b) 'On Concept and Object'. Reprinted in Frege (1984), 182–94.
- (1893) *Grundgesetze der Arithmetik*, vol. 1. Part translated by M. Furth as *The Basic Laws of Arithmetic*. Berkeley and Los Angeles: University of California Press, 1964.
- (1904) 'What is a Function?'. Reprinted in Frege (1984), 285–92.
- (1906) 'On the Foundations of Geometry: Second Series'. Reprinted in Frege (1984), 293–340.
- (1918) 'Negation'. Reprinted in Frege (1984), 373–89.
- (1979) *Posthumous Writings*, ed. Hans Hermes *et al.*, trans. P. Long and R. White. Oxford: Basil Blackwell.
- (1980) *Philosophical and Mathematical Correspondence*, ed. G. Gabriel *et al.*, trans. H. Kaal. Chicago: The University of Chicago Press.
- (1984) *Collected Papers on Mathematics, Logic, and Philosophy*, ed. B. F. McGuinness. Oxford: Basil Blackwell.
- Geach, Peter (1973) 'Frege', in G. E. M. Anscombe and P. T. Geach, *Three Philosophers*. Oxford: Basil Blackwell.
- (1976) 'Saying and Showing in Frege and Wittgenstein', in J. Hintikka *et al.*, eds., *Essays in Honour of G. H. von Wright*, *Acta Philosophica Fennica* 28, 54–70.
- (1981) *Logic Matters*. Blackwell: Oxford.
- Kerry, Benno (1887) 'Über Anschauung und ihre psychische Verarbeitung', Part 4, *Vierteljahrsschrift für Wissenschaftliche Philosophie* 11, 249–307.
- Moore, A. W. (2003) 'Ineffability and Nonsense', *Proceedings of the Aristotelian Society*, suppl. vol. 77, 169–93.
- Parsons, Terence (1986) 'Why Frege should not have said "The concept horse is not a concept"', *History of Philosophy Quarterly* 2, 449–65.
- Russell, Bertrand (1903) *The Principles of Mathematics*. Cambridge: Cambridge University Press.
- Sullivan, Peter (1989) *Formal Concepts*, Oxford D.Phil. thesis.
- (2000) 'The Totality of Facts', *Proceedings of the Aristotelian Society* 100, 175–92.
- Van Heijenoort, Jean (1977) 'Frege on Sense Identity', *Journal of Philosophical Logic* 6, 103–8.
- Weiner, Joan (1990) *Frege in Perspective*. Ithaca, NY: Cornell University Press.

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Wittgenstein, Ludwig (1914) 'Notes Dictated to G. E. Moore in Norway' (NM), in *Notebooks 1914–1916*, 2nd edition, ed. G. H. von Wright and G. E. M. Anscombe, trans. G. E. M. Anscombe. Oxford: Basil Blackwell, 1979.

——(1922) *Logisch-Philosophische Abhandlung/Tractatus Logico-Philosophicus*, trans. C. K. Ogden, London: Kegan Paul Ltd, 2nd edition 1933; trans. D. F. Pears and B. F. McGuinness, London: Routledge & Kegan Paul Ltd, 1961, 2nd edition 1974.