Abstract. Aristotle counts as the founder of formal logic. The logic he develops dominated until Frege and others introduced a new logic. This new logic is taken to be more powerful and better capable of capturing inference patterns. The new logic differs from Aristotelian logic in significant respects. It has been argued by Fred Sommers and Hanoch Ben-Yami that the new logic is not well equipped as a logic of natural language, and that a logic closer to Aristotle’s is better suited for this task. Each of them developed their own formalism—Sommers in form of term logic, Ben-Yami in form of his Quantified Argument Calculus (QUARC). I discuss Aristotle’s logic—a term logic—and attempt a comparison between Aristotelian logic and (i) the new logic, (ii) Sommers’ term logic, and (iii) Ben-Yami’s QUARC. I consider differences between the systems, and show how they are related to and diverge from the new logic.
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1 Introduction

It is widely agreed that Aristotle is the inventor of formal logic. The logic he develops remains the dominant one until Gottlob Frege introduces his logical language in form of the Begriffsschrift (1879). As one might expect, their logics and formal languages are strikingly different. Aristotle develops a term logic, i.e., a logic which concerns the relation between terms. Terms can be affirmed or denied of terms, and can be assigned different quantities.

Fregean languages, on the other hand, distinguish different elements, such as predicates-symbols, individual-constants, variables, and logical symbols. This goes beyond the language of term logic in several respects. In particular, terms most closely correspond to certain predicate-symbols, but not every predicate-symbol can easily be considered to be a term. Moreover, the Fregean language knows quantifiers which directly indicate something like the quantity in question, whereas Aristotle’s term logic does not include them.

Fregean languages are a success-story. Ever since their introduction, they almost completely superseded term languages. The power and flexibility of Fregean languages made the term approach pretty much obsolete—which is also one of the main reasons to prefer Fregean languages. This, however, does not mean that there is no competition; and it is the competition that we are interested in here.

Two of the competitors are Fred Sommers’s so-called Term Functor Logic (TFL) and Hanoch Ben-Yami’s so-called QUantified ARGument Calculus (QUARC). Both Sommers and Ben-Yami point towards Aristotelian logic as a potential ally, and as a reason to reject Fregean languages. This is why I consider Aristotle’s approach as a base for both TFL and QUARC. Moreover, as both systems attempt to replace the Fregean approach, it is necessary to compare them to it. Overall, we are interested in a somewhat four-fold comparison between Aristotle’s logic, the Fregean approach, TFL, and QUARC.

This paper is structured as follows. Section 2 discusses Aristotle’s logic. Section 3 provides a generic picture of the Fregean approach as currently understood, but in a form more suitable for our purposes. Section 4 introduces Ben-Yami’s QUARC and Section 5 Sommers’s TFL; Section 6 compares the systems, though I also compare the approaches within the previous sections. Section 7 concludes the paper.

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1 Frege (1879) introduces what we now consider a second-order language; however, only the first-order fragment is relevant here.

2 Note that this does not mean that Aristotle and Aristotle’s logic do not know quantification.
2 Aristotle’s Logic

In Raab [2018], I reconstruct Aristotle’s assertoric logic in a subsystem of QUARC, and show that the reconstruction is very close to the original text. The target of the reconstruction is only the first few chapters of Aristotle’s Prior Analytics (viz., AP A1–6), but I suggest how to introduce complex terms (2018: §3.5) which are not to be found in those chapters. This original extension is the relevant one for our purposes, and there is some textual evidence that that’s the version Aristotle had in mind (see Section 2.4). We encounter the formalism in Section 2.9.

To arrive there, I don’t just consider Aristotle’s Prior Analytics, but the whole so-called Organon. One question to be asked (but, unfortunately, not really answered) is why Aristotle developed a term logic. Another question is what counts as a term to begin with. In order to answer these questions, I reconstruct parts of the Organon, though I cannot discuss every aspect.

In the following, I put the quotations of cited passages—including the Greek text—into footnotes (and I’d suggest ignoring them for the most part).[3]

2.1 Ti Kata Tinos

The general picture is something like this. In On Interpretation, Aristotle distinguishes between words (ὀνόμα/onoma)4 and verbs (ῥῆμα/rhêma) (Int 1, 16a1)5 both of which can then be considered to be terms (ὅρος/horos) (Int 3, 16b1f.6, APr A1, 24b1f.7). Terms on their own do not constitute a sentence and are neither true nor false, yet they are mean-

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4The literal translation is ‘name’, but what’s meant is something like ‘word’.

5First we must settle what a word is and what a verb is [Πρῶτον δεῖ θέσθαι τί ὄνομα καὶ τί ῥῆμα].

6When uttered just by itself a verb is a word and signifies something [αὐτὰ μὲν οὖν καθ’ αὑτὰ λεγόμενα τὰ ῥήματα ἀνάμικτα ὀνόμα τις καὶ σημαίνει τι’].

7I call a term that into which a premiss is resolved [Ὅρον δὲ καλῶ εἰς ὅν διαλύεται ἢ πρότασις]".
A sentence (λόγος/logos) is constituted by the combination of a word and a verb, i.e., by combining appropriate terms. However, not every sentence is significant, i.e., true or false (Int 4, 17a31f.), but every significant sentence must include a verb (Int 5, 17a9f., Int 10, 19b12f.).

More importantly, a simple sentence affirms something of something (τί κατὰ τινός/ti kata tinos) or denies something of something (τί ἀπὸ τινός/ti apo tinos) (Int 5, 17a20f.; see also, e.g., APo Α2, 72a13f.)—a structure also appearing in Aristotle’s Metaphysics (e.g., Met Z17, 1041a20–22). Aristotle also speaks of ‘compounded’ sentences (Int 5, 17a21f.), though it does not appear that he is concerned with them again throughout the Organon (with, maybe, a few exceptions; see below).

A sentence is made up of terms which signify something (Int 4, 16b26f., Int 6, 17a25f.), but, as the ‘ti kata/apo tinos’ suggests,

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8. Thus words and verbs by themselves—for instance ‘man’ or ‘white’ when nothing further is added—are like the thoughts that are without combination and separation; for so far they are neither true nor false [τὰ μὲν οὖν ὀνόματα αὐτὰ καὶ τὰ ῥήματα ἔοικε τῷ ἄνευ συνθέσεως καὶ διαιρέσεως νοήματι, οἷον τὸ ἄνθρωπος ή λευκόν, ὅταν μὴ προστεθῇ τι ὑπὸ γὰρ φεύγων οὔτε ἀληθῆς ποιῇ].

9. In APr, Aristotle uses a different word; see below and cf. Kneale and Kneale (1962: 34f.).

10. There is not truth or falsity in all sentences: a prayer is a sentence but is neither true nor false [οὐκ ἐν ἅπασι δὲ ὑπάρχει οἷον ἡ εὐχὴ λόγος μέν, ἀλλ᾿ οὔτ᾿ ἀληθῆς οὔτε ψευδῆς].

11. Every statement-making sentence must contain a verb or an inflexion of a verb [ἀνάγκη δὲ πάντα λόγον ἀποφαντικὸν ἐκ ῥήματος εἶναι ἢ πτώσεως].

12. Without a verb there will be no affirmation or negation [ἄνευ δὲ ῥήματος οὐδεμία κατάφασις οὐδ᾿ ἀπόφασις].

13. As has often been noted, Aristotle’s writings are ambiguous as to whether claims are about linguistic expressions or about things expressed by those expressions (see, e.g., Kneale and Kneale 1962: §II.2). That’s less of a problem in De Interpretatione, but certainly so in the Categories; I generally take Aristotle to be interested in things, not linguistic items.

14. Of these the one is a simple statement, affirming or denying something of something [τούτων δ᾿ ή μὲν ἀπλὴ ἐστὶν ἀπόφασις, οἷον τι κατὰ τινὸς ή τι ἀπὸ τινός].

15. The part of a contradictory pair which says something of something is affirmation; the part which takes something from something is a denial [μόριον δ᾿ ἀντιφάσεως τὸ μὲν τί κατὰ τινὸς κατάφασις, τὸ δὲ τί ἀπὸ τινὸς ἀπόφασις].

16. However, one could ask why a man is such a kind of animal. It is clear that this is not to ask why one who is a man is a man. So what one asks is why it is that one thing is affirmed of another [ζητήσει δ᾿ ὅτι ὁ άνθρωπος ἂν ζῶν τοιοῦτον, τούτῳ μὲν τοῖς δήν ἂν, διτί οὐ ζητεῖ διὰ τί ὃς ἂν ζήσω γὰρ άνθρωπος άνθρωπος ἃντιν].

17. The ‘ti kata tinos’ is important enough to become the title of Tugendhat (1958, 2003).

18. The other is compounded of simple statements and is a kind of composite sentence [η δ᾿ ἐκ τούτων συγχειμένη, οἷον λόγος τῆς ἡδῆ σύνθεσιος].

19. A sentence is a significant spoken sound some part of which is significant in separation [Λόγος δὲ ἂν ἐτί σφεν σημαντική, ἢς τῶν μερῶν τί σημαντικὸν ἂντι κε-χωρισμένον].

20. An affirmation is statement affirming something of something, a denial is a state-
terms need to be combined (via copula) in order to affirm or deny (Int 4, 16b28ff). Given this basic structure, we can distinguish between simple sentences which affirm or deny something of a subject, and complex sentences which are compounds of simple sentences (Int 5, 17a20ff). However, as far as I can tell, Aristotle does not mention compounded sentences again, and he does not specify modes of composition (though see Section 2.4).

The basic unit is a sentence which contains two terms, viz., a subject and a verb, where the verb is said/predicated of the subject. Given this basic unit, a few more distinctions are possible. Aristotle distinguishes between things (πράγματα/pragmata) which are universal (καθόλου/katholou) and those which are particular (καθ᾿ ἕκαστον/kath hekaston).

The distinction that Aristotle draws is between universal and particular things. He calls ‘things’ like human being ‘universal’, and ‘things’ like Callias or Socrates ‘particular’. The distinction is drawn by considering what something can be said of: universal things can be said of several things, particulars cannot (Int 7, 17a38–b1)—more on that in Section 2.5.

### 2.2 Universals and Universally

Both universal and particular things can be the subject of sentences so that things can be said of them (Int 7, 17b1ff)—and, in the case of universal things, that in either of two ways, viz., universally (καθόλου

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21: I mean that ‘animal’, for instance, signifies something, but not that it is or is not (though it will be an affirmation or denial if something is added) [λέγω δέ, οἷον ἄνθρωπος σημαίνει τι, ἀλλ᾿ οὐχ ὅτι ἔστιν ἢ οὐκ ἔστιν (ἀλλ᾿ ἔσται κατάφασις ἢ ἀπόφασις ἐάν τι προστεθῇ)].

22: Of these the one is a simple statement, affirming or denying something of something [κατάφασις δε ἐστιν ἀπόφασις τινὸς κατὰ τινός, ἀπόφασις δε ἐστιν ἀπόφασις τινὸς ἀπὸ τινός].

23: ... and this is either universal or particular or indeterminate [οὗτος δὲ ἢ καθόλου ἢ ἐν μέρει ἢ ἀδιόριστος].

24: Now of actual things some are universal, others particular (I call universal that which is by its nature predicated of a number of things, and particular that which is not; man, for instance, is a universal, Callias an particular [Ἐπεὶ δέ ἐστι τά μὲν καθόλου τῶν πραγμάτων τὰ δὲ καθ᾿ ἐκαστὸν, – λέγω δὲ καθόλου μὲν ὃ ἐπὶ πλειόνων τέρμων κατηγορεῖται, καθ᾿ ἐκαστὸν δὲ δ ἡ, οἷον ἁνθρωπος μὲν τῶν καθόλου Καλλίας δὲ τῶν καθʻ ἐκαστὸν].

25: So it must sometimes be of a universal that one states that something holds or does not, sometimes of an particular [ἀνάγκη δο ἀποφαίνεσθαι ός ὑπέρχει τι ἢ μὴ, ὅτε μὲν τῶν καθόλου τοι, ὅτε δὲ τῶν καθʻ ἐκαστὸν].
Aristotle, Term Logic, & QUARC August 7, 2024 §2.2 Universals and Universally

ἀποφαίνηται (katholou apophainéta) or not (Int 7, 17b3ff).

Examples of something being said universally of a universal are ‘every human being is white’ and ‘no human being is white’ (Int 7, 17b5ff). It is of a universal thing, because ‘human being’ signifies one; and it is said universally, because it is said of every/none of those things. The first of these two sentences counts as affirming something of something (ti kata tinos), whereas the latter as denying something of something (ti apo tinos) as the mode of predication changes, though the latter is not the negation of the former (see Section [2.3]).

Something is said of a universal not universally when the subject is a universal thing, but the predication is not universally. The examples Aristotle provides are ‘human being is white’ and ‘human being is not white’ (Int 7, 17b8ff). The examples are of universals as ‘human being’ signifies a universal thing. However, the predications are not universal, because of the quantity of the subject. Regarding this, Aristotle also insists: “every” does not signify the universal but that it is taken universally’ (Int 7, 17b11ff cf. also Int 10, 20a9ff). This indicates where the quantity is meant to be applied to. Aristotle rejects that sentences such as ‘every human being is every animal’ (Int 7, 17b15ff) can ever be true (Int 7, 17b12–13). The quantity is meant to indicate of ‘how much’ of the subject-term the predicate-term is said.

It is less clear how Aristotle thinks about subjects which are particulars. He affirms that the sentences ‘Socrates is white’ and ‘Socrates is not white’ are contradictions (Int 7, 17b26–29), but he does not mention

26“Now if one states universally of a universal that something holds or does not [ἐὰν μὲν οὖν καθόλου ἀποφαίνηται ἐπὶ τοῦ καθόλου ὅτι ὑπάρχει ή μή]”.
27“Examples of what I mean by ‘stating universally of a universal’ are ‘every man is white’ and ‘no man is white’ [λέγω δὲ ἐπὶ τοῦ καθόλου ἀποφαίνεσθαι καθόλου, οἷον πᾶς ἄνθρωπος λευκός, οὐδεὶς ἄνθρωπος λευκός]”.
28Note that the ‘no human being is white’ can actually be rendered differently, making the universal character explicit: ‘every human being is not white’. In this formulation, it is clear that something is predicated universally—and that’s the more appropriate way to understand it in the general subject/predicate structure together with the quantity and positive/negative copula involved; in the example sentence, it is every human being of whom white is not said, combining universal quantity with ‘negative’ predication, i.e., denial (ti apo tinos).
29“Examples of what I mean by ‘stating of a universal not universally’ are ‘a human being is white’ and ‘a human being is not white’ [λέγω δὲ τὸ μὴ καθόλου ἀποφαίνεσθαι ἐπὶ τῶν καθόλου, οἷον ἔστι λευκός άνθρωπος, οὐκ ἔστι λευκός άνθρωπος]”.
30“τὸ γὰρ πᾶς οὐ τὸ καθόλου σημαίνει ἀλλ’ ὅτι καθόλου”.
31“Of contradictory statements about a universal taken universally’ it is necessary for one or the other to be true or false; similarly if they are about particulars,
anything like a quantity in such cases. Indeed, such sentences only occur very sparingly and do not get a proper discussion (see also Section 2.6).

### 2.3 Affirmation, Denial, and Truth

What Aristotle tells us, though, is how affirmation and denial are related:

> the denial must deny the same things as the affirmation affirmed, and of the same thing, whether an individual or a universal (taken either universally or not universally). (Int 7, 17b38–18a1)

A sentence is only then a denial of another sentence if the terms are the same; the denial of ‘every human being is white’ is ‘not every human being is white’ i.e., we keep the terms as they are, and, in a sense, we also keep the quantity, though the negation acts on it. Aristotle does not discuss the complex case, but only suggests the following sentences as examples: ‘Socrates is white’ has as denial ‘Socrates is not white’ (Int 7, 18a2f). The correct denial of the more complex sentences is arrived at after further discussion (see, e.g., Int 10, 19b14–18).

The underlying idea is still that of 

\[ ti\; kata\; tinos \]

saying something of something. ‘A kata B’ has as its denial ‘A apo B’; the terms remain the same. Aristotle does not specify the denial of ‘A apo B’, though we can take the ‘A kata B’ as its denial, assuming the only options to be 

\[ ti\; kata\; tinos \quad \text{and} \quad ti\; apo\; tinos. \]

Given this picture, Aristotle suggests when sentences are true and false:

> For it is true to say that it is white or is not white, it is necessary for it to be white or not white; and if it is white or is not white, then it was true to say or deny this. If it is not the case it is false, if it is false it is not the case. (Int 9, 18a39–b)

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35-36. \( \text{τὸ γὰρ αὐτὸ δεῖ ἀποφῆσαι τὴν ἀπόφασιν ὅπερ κατέφησεν ἡ κατάφασις, καὶ ἀπὸ τοῦ αὐτοῦ, ἢ τῶν καθ’ ἐκκατά τινος ἢ ἀπὸ τῶν καθ’ ἐκκατά τινος, ἢ ὡς καθόλου ἢ ὡς μὴ καθόλου}. \]

37-38. \( \text{λέγω δὲ οἷον ἔστι Σωκράτης λευκός – οὐκ ἔστι Σωκράτης λευκός}. \]

39. \( \text{εἰ γὰρ ἀληθὲς εἰπεῖν ὅτι λευκὸν ἢ οὐ λευκόν ἐστιν, ἀνάγκη εἶναι λευκὸν ἢ οὐ λευκόν, καὶ εἰ ἔστι λευκὸν ἢ οὐ λευκόν, ἀληθὲς ἢ φάναι ἢ ἀποφάναι: καὶ εἰ μὴ ὑπάρχει, ἕσεσθαι, καὶ εἰ ἕσεσθαι, οὐχ ὑπάρχει}. \)
This understanding of truth is pretty much the same as that in his *Metaphysics* (Met Γ7, 1011b25ff). The general idea is that if we have a sentence, there are two terms involved, and one term is affirmed/denied of the other. Now, a sentence is true, if what is said actually obtains, and it is false if not. Moreover, under certain conditions, if a sentence is false, its denial is true—since the denial keeps the terms etc. intact, and similarly the other way around. Also, if $B$ is $A$, it is true to make a corresponding claim (‘$A$ kata $B$’), and false to assert the corresponding denial (‘$A$ apo $B$’); and if $B$ is not $A$, it is true to deny that $B$ is $A$ (‘$A$ apo $B$’), and false to affirm it (‘$A$ kata $B$’).

### 2.4 Complex Terms

We can also note that, in *On Interpretation*, Aristotle allows *negated terms*, i.e., it is not only sentences which are denials, but we can have affirmations involving negated terms. One of the examples is ‘not-human being’ (e.g., Int 10, 19b37ff); another is a negated verb: ‘not-just’ (Int 10, 19b28ff). Thus, we can form affirmations out of negated terms: every non-human being is not-just. (Cf. also, e.g., Top E6, 136a33ff.)

Furthermore, Aristotle also does not exclude the possibility of further complex terms. His standard example is ‘cloak’ (ἵματιον/himation) as word for something more complex (an example also occurring at Met Z4, 1029b25–28). For example, Aristotle suggests to introduce the term ‘cloak’ for the complex ‘horse and man’, though he denies a certain unity to sentences containing such terms; he rather thinks they are equivalent to compounded sentences (Int 8, 18a19–23).

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40. This will be plain if we first define what truth and falsehood are: for to say that that which is is not or that which is not is, is a falsehood; and to say that that which is and that which is not is not, is true; so that, also, he who says that a thing is or not will have the truth or be in error [δῆλον δὲ πρῶτον μὲν ὁρισαμένοι τί τὸ ἀληθὲς καὶ ψεῦδος. τὸ μὲν γὰρ λέγειν τὸ ὂν μὴ εἶναι ἢ τὸ μὴ ὂν εἶναι ψεῦδος, τὸ δὲ τὸ ὄν εἶναι καὶ τὸ μὴ ὄν μὴ εἶναι ἄληθὲς, ὥστε καὶ ὁ λέγων εἶναι ἢ μὴ ἄληθεύσει ἢ ψεύσεται].

41. “τὸ οὐκ ἄνθρωπος”.

42. “οὐ δίκαιος”.

43. “Thus (e.g.) inasmuch as animate is a property of living creature, animate will not be a property of not-living creature [οἷον ἐπεῖ τοῦ ζῶου ἰδιόν τὸ ἐμψυχον, οὐκ ἐν εἰς τοῦ μὴ ἦν ἰδιόν τὸ ἐμψυχον].”

44. “We must see, therefore, whether there is a formula of what being is for each of these compounds, and whether these too have a what-being-is, e.g. a white man. Suppose ‘cloak’ to be a word for this [σκεπτέον ἀρ’ εἶστι λόγος τοῦ τί ἦν εἶναι ἐκάστῳ αὐτῶν, καὶ υπάρχει καὶ τοῦτος τοῦ τί ἦν εἶναι, οἷον λευκῷ ἀνθρώπῳ [τί ἦν λευκῷ ἀνθρώπῳ]. ἔστω δὴ ἄρα ὄνομα αὐτῷ ἱμάτιον].”

45. “Suppose, for example, that one gave the word ‘cloak’ to horse and man; ‘a cloak is white’ would not be a single affirmation. For to say this is no different from saying ‘a horse and a man is white’, and this is no different from saying ‘a horse is white and a man is white’ [οἷον εἰ τις θείοι ὄνομα ἱμάτον ἵππῳ καὶ ἀνθρώπῳ, τοῦ ἦν ἱμάτον λευκόν, αὕτη οὖ κατάφασις οὐδὲ ἀπόφασις μία: οὐδὲν γὰρ διαφέρει τοῦτο εἴπειν ἢ ἔστιν ἴππος καὶ ἄνθρωπος λευκός, τοῦτο δ’ οὐδὲν διαφέρει τοῦ εἴπειν ἔστιν ἴππος λευκός καὶ ἔστιν ἀνθρώπος λευκός].”
Aristotle does not say much more about these complexes, though he does say more about the relationship of sentences involving negated terms and denials:

‘No human being is just’ follows from ‘every human being is not-just’, while the contradictory of this, ‘not every human being is not-just’, follows from ‘some human being is just’.

Thus, if the predicate-term is negated, the former sentence implies a denial with unnegated predicate-term; and the positive sentence, likewise, implies a denial with negated predicate-term.

Aristotle also suggests the following:

‘every not-man is not-just’ signifies the same as ‘no not-man is just’.

This suggests the equivalence of denial and affirmation with negated predicate-terms, though one direction is problematic (see n. 49).

2.5 Categories of Terms

Potentially moving away from Aristotle’s formal logic, let us consider his Categories which categorizes the terms. Aristotle notes that terms can be said in or without combination (Cat 2, 1a16f.), and it is the classification of terms without combination—that is: the terms, not sentences resulting from their combination—that he is interested in.

The categorization is based on two concepts:

(i) being said of a subject, and (ii) being in a subject.

Applying these concepts gives rise to a four-fold categorization:

(1) being said of a subject and being in a subject ((i) and (ii))

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46-ἀκολουθοῦσι δ᾿ αὕτη, τῇ μὲν πᾶς ἐστὶν ἄνθρωπος οὐ δίκαιος ἢ οὐδεὶς ἐστιν ἄνθρωπος δίκαιος, τῇ δὲ ἐστὶ τις δίκαιος ἄνθρωπος ἢ ἀντικειμένη ὃτι οὐ πᾶς ἐστὶν ἄνθρωπος οὐ δίκαιος.

47-These are captured by one of the semantics in Section 2.9; see Theorem 16 (p. 28).

The former claim is an instance of (1) (where $\|B\|_{\text{MN}} = \|B\|_{\text{MN}}$), the latter of (3).

48-τὸ δὲ πᾶς οὐ δίκαιος οὐκ ἄνθρωπος τῷ οὐδεὶς δίκαιος οὐκ ἄνθρωπος ταὐτὸ σημαίνει.

49-Only one direction holds in one of the semantics of Section 2.9; the other not; see Theorem 16 (2). The other semantics validates both directions, but clashes with different claims of Aristotle; see n. 47.

50-Of things that are said, some involve combination while others are said without combination [Τῶν λεγομένων τὰ μὲν κατὰ συμπλοκὴν λέγεται, τὰ δὲ ἄνευ συμπλοκῆς].

51-For example: “knowledge is in a subject, the soul, and is also said of a subject, knowledge-of-grammar [ἡ ἐπιστήμη ἐν ψυχῇ καθ’ ψυχῇ ὑποκειμένου δὲ λέγεται τῆς γραμματικής]” (Cat 2, 1b1ff.).
2.6 Particulars and Syllogistic

The immediate relevance for us is that *particulars do not occur as terms in Aristotle’s syllogistic*—and particulars are not the only examples of such terms. There is a certain symmetry. In his *Prior Analytics*, Aristotle suggests a term hierarchy. At the bottom of the hierarchy, there are terms—particulars (*καθ᾿ ἕκαστα/kath hekasta*)—which cannot be predicated:

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52 For example: “human being is said of a subject, the particular human being, but is not in any subject [οἷον ἄνθρωπος καθ᾿ ὑποκειμένου μὲν λέγεται τοῦ τινὸς ἀνθρώπου, ἐν ὑποκειμένῳ δὲ οὐδενί ἐστιν]” (Cat 2, 1a21f.).

53 For example: “the particular knowledge-of-grammar is in a subject, the soul, but is not said of any subject [ἡ τὶς γραμματικὴ ἐν ὑποκειμένῳ μὲν ἐστι τῇ ψυχῇ, καθ᾿ ὑποκειμένου δὲ οὐδενὸς λέγεται]” (Cat 2, 1a25ff.).

54 For example: “the particular human being or particular horse [ὁ τὶς ἄνθρωπος ἢ ὁ τὶς ἵππος]” (Cat 2, 1b4f.).

55 Of the several ways in which substance is spoken of, there are at any rate four which are the most important: the substance of a thing seems to be what being is for that thing, and its universal and its genus, and fourthly the subject. The subject is that of which other things are predicated while it itself is predicated of nothing further [Λέγεται δ᾿ ἡ οὐσία, εἰ μὴ πλεοναχῶς, ἀλλ᾿ ἐν τέτταρι ζητοῖται καὶ γὰρ τὸ τί ἦν εἶναι καὶ τὸ καθόλου καὶ τὸ γένος οὐσία δοκεῖ εἶναι ἑκάστου, καὶ τέταρτον τούτον τὸ ὑποκειμένον. τὸ δ᾿ ὑποκειμένον ἐστι καθ᾿ ὑποκειμένον ὑπάρχουσιν, ἐκεῖνο δὲ αὐτὸ μηρέται κατ᾿ ἄλλῳ].

56 A substance—that which is called a substance most strictly, primarily, and most of all—is that which is neither said of a subject nor in a subject, e.g. the particular man or the particular horse [Οὐσία δὲ ἐστιν η̱ κυριώτατα τε καὶ πρώτως καὶ μᾶλλον λεγομένη, ἢ μὴ τε καὶ τὴν ὑποκειμένον τοῦς λέγεται μὴ τε ὑποκειμένον τούτων ἐστιν, οὖν ὁ τὶς ἄνθρωπος ἢ ὁ τὶς ἵππος].

57 The species in which the things primarily called substances are, are called secondary substances, as also are the genera of these species [δεύτεραι δὲ οὐσίαι λέγονται, ἐν οἷς εἴδεσθαι αἱ πρῶταις οὐσίαι λεγόμεναι ὑπάρχουσιν, ταύτα τε καὶ τὰ τῶν εἰδῶν τούτων γένη].
That some things are by nature such as to be said of nothing else is clear, for more or less every perceptible thing is such as not to be predicated of anything except accidentally—for we do sometimes say that the white thing there is Socrates, or that what is approaching is Callias. (APr A27, 43a32–36⁵⁸)

Aristotle even insists that

of all the things there are, some are such that they cannot be predicated truly and universally of anything else (for instance, Cleon or Callias, that is, what is particular and perceptible)]. (APr A27, 43a25ff.⁵⁹)

Taken together, it seems as if Aristotle is saying that particulars—and pretty much all perceptible things—cannot be predicated. The latter passage just suggests that they cannot be predicated ‘truly and universally’, but the former suggests something stronger.

This also suggests that Aristotle does not seem to consider identity statements such as ‘Socrates is Callias’ or even ‘Socrates is Socrates’. Whatever the reason, Aristotle does not consider something like ‘is Socrates’ or just ‘Socrates’ as a predicate-term.

This situation is mirrored at the top of the hierarchy. Starting from the particular, we reach another limit:

But that one also comes to a halt if one goes upwards, we will explain later [at APo A22, 83b24–31⁶⁰; for the moment let this be assumed. (APr A27, 43a36f.⁶¹)]

Both ends of the hierarchy consist of terms which are not the target of Aristotle’s syllogistic. Aristotle is explicit (my translation):

⁵⁸ ὅτι μὲν οὖν ἔνια τῶν ὄντων κατ᾿ οὐδενὸς πέφυκε λέγεσθαι, δῆλον· τῶν γὰρ αἰσθητῶν σχεδὸν ἔκαστόν ἐστι τοιοῦτον ὡστε μὴ κατηγορεῖσθαι κατὰ μηδενός, πλὴν ὡς κατὰ συμβεβηκός· φαμέν γάρ ποτε τὸ λευκὸν ἐκεῖνο Σωκράτην εἶναι καὶ τὸ προσιὸν Καλλίαν".

⁵⁹ Ἀπάντων δὴ τῶν ὄντων τὰ μέν ἐστι τοιαῦτα ὡστε κατὰ μηδενὸς ἄλλου κατηγορεῖσθαι ἄλληθος καθόλου (οἶον Κλέων καὶ Καλλίας καὶ τὸ καθ᾿ ἑκάστον καὶ αἰσθητόν)".

⁶⁰ Τὸις μὲν δὲ καθ᾿ ἑνὸς οὐτ᾿ εἰς τὸ ἄνω ἤν καὶ ἐν καθ᾿ ἑνὸς οὐτ᾿ εἰς τὸ κάτω ὑπάρχειν λεγέσθαι. καὶ οὐκ ὃν γὰρ λέγεται τὰ συμβεβηκότα, ὅσα ἐν τῆς ὁμοίας ἑκάστου, ταῦτα δὲ οὐκ ἔπειρα· ἄνω δὲ ταῦτα ταῦτα καὶ τὰ συμβεβηκότα, ἀμφότερα οὐκ ἔπειρα. ἀνάγκη ἄρα εἰς τὸ ὃν προῆτον τι κατηγορεῖται καὶ τοῦτον ἄλλο, καὶ τοῦτο ἱστασθαι καὶ εἶναι τί ὃ οὐκέτι οὔτε κατ᾿ ἄλλου προτέρου οὔτε κατ᾿ ἑκάστου ἄλλο πρότερον κατηγορεῖται"

⁶¹ ὅτι δὲ καὶ ἐπὶ τὸ ἄνω πορευομένοις ἴσταται ποτὲ, πάλιν ἐροῦμεν· νῦν δ᾿ ἐστι τούτω κείμενον".
Clearly, the things inbetween admit of both (for they can be predicated of others and others of them). And more or less the arguments and investigations are especially about them. (APr A27, 43a40–43)

Note that there are two occurrences of ‘σχεδόν’ (‘schedon’), viz., at APr A27, 43a33 and at APr A27, 43a42, which have been translated as ‘more or less’; they suggest the possibility of exceptions. For the former occurrence, the exception is already made explicit. Regarding the latter, Aristotle does not indicate what the exception is meant to be.

Without the exceptions, the admissible terms for Aristotle’s syllogistic are those that (i) can be predicated of other terms and (ii) have other terms predicated of them (APr A27, 43a41f.). This also makes sense once we consider the conversion rules (Sections 2.7–2.9). But Aristotle sometimes seemingly uses individuals in syllogisms. As far as I can tell, there are only three passages of this sort (in the Organon); let me quote the first in full (the second and third in footnotes 65 and 66 respectively):

For example, if A is said of B and B of C—one might think that when the terms are so related, there is a syllogism, but in fact nothing necessary comes about, nor a syllogism. For let A designate always being, B, thinkable Aristomenes, and C, Aristomenes. Clearly it is true that A belongs to B, for Aristomenes is always thinkable. And it is also true that B belongs to C, for Aristomenes is a thinkable Aristomenes. But A does not belong to C, since Aristomenes is perishable. For no syllogism resulted from terms related in this way; rather, the premiss AB should have been taken as universal. But this is false—to claim that every thinkable Aristomenes always is, given that Aristomenes is perishable. (APr A33, 47b18–29; see also APr A33, 47b29–37 and APr B27, 62a)
The general point Aristotle makes in the first two passages is that there is a certain danger when it comes to modal syllogisms involving necessity (APr A33, 47b15–18)—hence the modal flavour of these passages. They also involve syllogisms from the first figure, so no conversion occurs. The particular only occurs in subject-term position. It is not clear which mood of the first figure is concerned, and Aristotle’s remark that ‘the premiss AB should have been taken as universal’ does not concern the particular Aristomenes who seems to be chosen just to illustrate the modal point.

The third passage involves a third-figure syllogism whose proofs all rely on conversion. However, the discussion is about enthymemes (ἐνθύμημα/enthymêma) which stem from the probable (εἰκός/eikos)—where “the probable is a reputable statement” (APr B27, 70a3f). Enthymemes are syllogisms involving the probable (APr B27, 70a1), and so might be considered to not entirely fit the discussion of the syllogisms as developed in the first chapters of the Prior Analytics.

In his Posterior Analytics, Aristotle seems to confirm the point that particulars are not said of anything (APo A1, 71a23f). Moreover, when he explains what ‘in itself’ (καθ᾿ αὑτά/kath hauta) means, Aristotle reiterates that there are things which are not said of anything else (APo A4, 73b5–10), and he insists that “every term is always universal” (APo B13, assumed, there was no syllogism.  

It often happens that we are deceived about syllogisms because of the necessity, as we said before. But sometimes it is due to the similarity in the position of terms. This must not escape our notice.  

An enthymeme is a syllogism starting from probabilities or signs ['Ενθύμημα δὲ ἐστὶ συλλογισμός ἐξ εἰκότων ἢ σημείων']. I’m leaving out the ‘sign’ in the discussion.  

In particular, the inference seems rather to be an induction than a deduction, inferring from a particular case to a general one.

This occurs when the items are in fact particulars and are not said of any underlying subject [ὅσα ἤδη τῶν καθ᾿ ἕκαστα τυγχάνει ὄντα καὶ μὴ καθ᾿ ὑποκειμένου τινός].”  

Again, certain items are not said of some other underlying subject: e.g. whereas what is walking is something different walking (and similarly for what is white), substances, i.e. whatever means this so-and-so, are not just what they are in virtue
Since terms corresponding to particulars are not universal—as particulars are exactly those things which aren’t universal (Int 7, 17a38–b1)—relevant terms are not those of particulars.

### 2.7 The Syllogistic

With these preliminaries out of the way, let’s consider the syllogistic. Aristotle develops it in his *Prior Analytics*; our focus is the assertoric part. Aristotle starts by suggesting which notions need to be introduced: *sentence/premiss* (πρότασις/protasis), *term* (ὅρος/horos), *syllogism* (συλλογισμός/syllogismos), *this (not) being in that as in a whole* (τὸ ἐν ὅλῳ (μὴ) εἶναι τόδε τῶδε/to en holô (mê) einai tode tôde), *predicate of all* (κατὰ παντὸς κατηγορεῖσθαι/kata pantos katêgoreisthai), and *predicate of none* (κατὰ μηδενὸς κατηγορεῖσθαι/kata mèdenos katêgoreisthai) (APr A1, 24a11–15).

Syllogisms consist of sentences/premisses, and Aristotle defines sentences/premisses as *affirming or denying something of something* (APr A1, 24a16ff.)—bringing back the *ti kata/apo tinos* structure. Both the ‘*ti*’ and the ‘*tinos*’, i.e., the predicate and the subject, respectively, are terms, as sentences/premisses are resolved in terms which are combined by a positive or negative copula (APr A1, 24b16ff.).

There are different ways of affirming/denying something of something, viz., *universally* (καθόλου/katholou), *particularly* (ἐν μέρει/en merei), and *indeterminately* (ἀδιόριστος/adioristos) (APr A1, 24a17). As noted in Section 2.1, the quantity is put as ‘ἐν μέρει’ (en merei), which can be translated as ‘in part’. This contrasts with the universal predication which does not just predicate ‘in part’, but universally. Aristotle characterizes these as follows:
By ‘universal’ I mean belonging to all or to none of something; by ‘particular’, belonging to some or not to some, or not to all; by ‘indeterminate’, belonging without universality or particularity, as in ‘of contraries there is a single science’ or ‘pleasure is not a good’.

(APr A1, 24a18–22)

The universal affirmation and denial say something of all of the subject; the universal affirmation/denial says of all of the subject that a term applies/does not apply to it. The particular affirmation/denial says only of part of the subject (hence, the ἐν μέρει/en merel-phrasing) that a term does/does not apply to it. The ‘indeterminate’ case just does not indicate whether all or only part of the subject is meant; it doesn’t play much of a role for us.

Given that terms built up sentences/premisses which say something of something, a syllogism is

an argument in which, certain things being posited, something other than what was laid down results by necessity because these things are so. (APr A1, 24b18ff).

Put differently, a syllogism is a valid argument (Read ms §1), which is not trivial, i.e., something new has to be concluded (cf. SE 1, 164b27–165a2).

Aristotle makes it clear that there must be a logical relationship between the sentences/premisses in order for a syllogism to obtain, a relationship that concerns the terms constituting the premisses (APr A1, 24b20ff).

The relationship Aristotle singles out is being in another as in a whole which he explains as follows: A is in B as in a whole iff B is predicated of all of A. Moreover, he explains: B is predicated of all of A iff there is no A that is not B (i.e., all A are B). Similarly, B is predicated of none of A iff there is no A that is B (i.e., no A are B) (APr A1, 24b26–31).

78λέγω δὲ καθόλου μὲν τὸ παντὶ ἢ μηδενὶ ὑπάρχειν, ἐν μέρει δὲ τὸ τινὶ ἢ μὴ τινὶ ἢ μὴ παντὶ ὑπάρχειν, ἀδιόριστον δὲ τὸ ὑπάρχειν ἢ μὴ ὑπάρχειν ἀνευ τοῦ καθόλου ἢ κατὰ μέρος, οἷον τὸ τῶν ἐναντίων εἶναι τὴν αὐτὴν ἐπιστήμην ἢ τὸ τὴν ἡδονὴν μὴ εἶναι ἀγαθόν;

79συλλογισμὸς δὲ ἐστι λόγος ἐν ᾧ τεθέντων τινῶν ἕτερόν τι τῶν κειμένων ἐξ ἀνάγκης συμβαίνει τῷ ταῦτα εἶναι.

80For a syllogism rests on certain statements such that they involve necessarily the assertion of something other than what has been stated, through what has been stated [ὁ μὲν γὰρ συλλογισμὸς ἐκ τινῶν ἐστι τεθέντων ὡστε λέγειν ἕτερον ἐξ ἀνάγκης τοῖς τῶν κειμένων διὰ τῶν κειμένων].

81By ‘because these things are so’ I mean that it results through these, and by ‘resulting through these’ I mean that no term is required from outside for the necessity to come about [λέγω δὲ τῷ ταῦτα εἶναι τὸ διὰ ταῦτα συμβαίνειν, τὸ δὲ διὰ ταῦτα συμβαίνειν τὸ μηδενὸς ἓξωθεν ὄρον προσδεῖν πρὸς τὸ γενέσθαι τὸ ἀναγκαῖον].

82For one thing to be in another as in a whole is the same as for the other to be predicated of all of the first. We speak of ‘being predicated of all’ when nothing can be found of the subject of which the other will not be said and the same account holds for ‘of none’ [τὸ δὲ ἐν ὅλω εἶναι ἐτερον ἐτέρῳ καὶ τὸ κατὰ παντὸς κατηγορεῖσθαι
Overall there are four sentence-types, depending on quantity and mode. The quantity can either be universal or particular (‘in part’), and the mode can be positive (‘kata’, affirming) or negative (‘apo’, denying). These account for the relation of the two terms involved in sentences. Given two terms $A$ and $B$, we get a sentence $AB$ whose predicate-term is $A$ and whose subject-term is $B$. The sentence $AB$ can be either

(a) universal-affirmative (“all $B$ are $A$”; $AaB$), or

(i) particular-affirmative (“some $B$ are $A$”; $AiB$), or

(e) universal-negative (“all $B$ are not $A$”; $AeB$), or

(o) particular-negative (“some $B$ are not $A$; $AoB$).

Aristotle calls sentence-types $e$ and $o$ privative ($στερητικός$/sterêtikos); he does not think of them as involving what we would understand as a negation. Indeed, he has different ways of referring to the same type. On the one hand, a sentence can be an affirmation ($κατάφασις$/kataphasis) and a denial ($ἀπόφασις$/apophasis) (and, derivatively, sentences can be affirmative ($καταφατικός$/kataphatikos) and negative ($ἀποφατικός$/apophatikos)), and he refers to the sub-types as universal and particular. On the other hand, he refers to the denials as privative; e.g., he speaks of the “universal privative premiss” (APr A2, 25a5f.83). The ‘privative’ applies to the copula—it suggests a negative copula—the ‘universal’ to the subject—indicating the quantity of the subject.

This second way singles out the subject (universally or particularly) and notes the privation, i.e., that a term does not apply to it. For example, some human beings are not healthy, i.e., lack health, and so health is privative to those human beings. The whole sentence is a denial (ti apo tinos), and the predicate-term is privative (apo), i.e., the subject lacks the corresponding property.

Since both constituents of a sentence are terms, there is a natural question as to their relationship. Aristotle notes that three of the sentence-types convert ($ἀντιστρέφειν$/antistrephein), viz., $a$, $i$, and $e$. Sentence-type $o$, however, does not.

Converting a sentence means interchanging the predicate-term and subject-term; the sentence $AB$ converts to $BA$. Sentence-types $i$ and $e$ convert to the same sentence-type; sentence-type $a$ on the other hand, converts to an $i$-type sentence (APr A2, 25a5–13). The conversions can be summarized as follows (symbolizing ‘converts to’ as ‘⇝’):

\[
\text{θατέρου θάτερον ταὐτόν ἐστιν. λέγομεν δὲ τὸ κατὰ παντὸς κατηγορεῖσθαι ὅταν μηδὲν ἤ λαβεῖν [τοῦ ὑποκειμένου] καθ’ οὗ θάτερον οὐ λεχθήσεται· καὶ τὸ κατὰ μηδενὸς ὡσαύτως δὲ.}
\]
83 “... it is necessary for the universal privative premiss of belonging to convert with respect to its terms. So, for instance, if no pleasure is a good, then neither will any good be a pleasure. And the positive premiss converts necessarily, though not universally, but to the particular; for instance, if every pleasure is a good, it is necessary that some good be also a pleasure. Of the particular premisses
Aristotle claims that these sentence-types convert—and that without any suggestion of a restriction in place—suggests that predicate- and subject-terms are on a par as worked out in Section 2.6. Suppose Aristotle allowed particulars into his syllogistic. Then sentences with such particulars cannot convert, since particulars cannot play the role of predicates; the validity of the conversions rules out terms denoting particulars.

With all these preliminaries out of the way, Aristotle goes on to introduce three figures and to establish their syllogisms. The figures come about by considering the different roles three terms, \(A, B, C\), can play. Sentences of the form \(AB\) have \(B\) as their subject-term and \(A\) as their predicate-term. Since the syllogisms come about via the relation of the terms, one term has to occur in two premisses as to establish a relation between the other terms. The three figures encode exactly that.

The first figure has one term—the so-called middle term (\(μέσον/\)meson)—occurring as predicate-term in one premise and subject-term in the other, i.e., the premises are \(AB\) and \(BC\) (APr A4, 25b35f\(^{86}\)). The conclusion concerns the other terms \(A\) and \(C\)—the so-called extremes (\(ἀκρα/akra\)) (APr A4, 25b36f\(^{87}\)).

The first-figure syllogisms are the following:

\[
\begin{align*}
\text{(Barbara)} & \quad AaB \quad BaC \\
\text{(Darii)} & \quad AaB \quad BiC \\
\text{(Celarent)} & \quad AeB \quad BaC \\
\text{(Ferio)} & \quad AeB \quad BiC \\
\end{align*}
\]

The second figure has the middle term only as predicate-term (APr A5, 26b34–37\(^{88}\)), and comprises the following syllogisms:

the affirmative necessarily converts to the particular, for if some pleasure is a good, then some good will also be a pleasure; but for the privative premiss this is not necessary. For it is not the case that, if man does not belong to some animal, then animal also does not belong to some man [\(τὴν \text{ μὲν} \text{ ἐν} \text{ ήττ} \text{ ὑπάρχειν καθόλου στερητικὴν ἀνάγκη τοῖς ὅροις ἀντιστρέφειν, οἷον εἰ μηδεμία ἡδονὴ ἀγαθόν, οὐδ᾿ ἡδονὴ οὐδὲν ἔσται ἡδονή· τὴν δὲ κατηγορικὴν ἀντιστρέφειν μὲν ἀναγκαῖον, οὐ χη ἡδονὴ ἀγαθόν ἔσται ἡδονή· τῶν δὲ ἐν μέρει τὴν μὲν καταφατικὴν ἀντιστρέφειν ἀνάγκη κατὰ μέρος (εἰ γὰρ ἡδονὴ τις ἀγαθόν, καὶ ἀγαθόν τι ἔσται ἡδονή), τὴν δὲ στερητικὴν ὦκχ ἀναγκαίαν. (οὐ γὰρ εἰ ἄνθρωπος μὴ ὑπάρχει τοι τῷ ὤνι, καὶ ὤνι μὴ ὑπάρχει τοι ἀνθρώπων).\]

\(^{85}\)I’m ignoring the fourth figure that Aristotle does not mention and which is not necessary to establish all the syllogisms.

\(^{86}\)I call ‘middle’ the term that is itself in another and in which there is also another—one that also has the middle position [\(καθόλου μὲν \text{ ἐν} \text{ καθόλου καὶ \underline{ἀλλο} ἐν \overline{αὐτῷ} \text{ ἐστὶν, ὅ καὶ τῇ \text{ λέξει} \text{ γίνεται μέσου}\)].

\(^{87}\)Extremes are what is in another and that in which there is another [\(ἀυτῆς \text{ ἐν} \text{ ἄλλῳ} \text{ καὶ \underline{ἐν} \overline{αὐτῷ} \text{ ἐστὶν}\)].

\(^{88}\)When the same thing belongs to all of one and none of the other, or to all or none
Aristotle is taken to endorse the square of opposition, though he does not state it explicitly (Kneale and Kneale 1962: 56). The four vertexes of the square are labelled by the four sentence-types, and the relations between these types are captured by edges.

The possible relations are contradictories, contraries, subcontraries, and subalternation. Consider two sentences $\varphi$ and $\psi$. They are contradictories iff exactly one of them is true; they are contraries iff they cannot both be true, but can both be false; they are subcontraries iff they cannot both be false, but can both be true; and $\varphi$ is a subaltern of $\psi$ iff $\varphi$ implies $\psi$. For example, an $\text{a}$-type sentence has the corresponding $\text{o}$-type sentence as its contradictory, the corresponding $\text{e}$-type sentence as its contrary, and the corresponding $\text{i}$-type sentence as its subaltern.

Figure 1 pictures the square. Except for subalternation, the relations are symmetrical; only subalternation is directed. Moreover, given (a-i\_conv) and (i-i\_conv), we can account for the sentence-type $\text{i}$ being the subaltern of sentence-type $\text{a}$ by (a-i\_conv). $\text{AaB}$ implies (converts to) $\text{BiA}$ which, by (i-i\_conv), implies (converts to) $\text{AiB}$.
Moreover, given the other relations, we can see that the \( AeB \) type sentence has the \( AaB \) type sentence as its subaltern. If \( AeB \) holds, then \( AaB \) cannot hold as its contrary. Thus, as exactly one of \( AaB \) and \( AoB \) has to be true, it follows that \( AoB \) must be true.

Aristotle provides the definitions for contradictories and contraries in *On Interpretation* (Int 7, 17b16–20\(^{90}\) Int 7, 17b20–23\(^{91}\) respectively), and he notes that contradictories cannot, but contraries can be \( \text{true false} \) together (Int 7, 17b23–29\(^{92}\)). Moreover, in his *Topics*, Aristotle suggests the subalternations (Top B1, 109a3–4\(^{93}\)).

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\(^{90}\)I call an affirmation and a negation contradictory opposites when what one signifies universally the other signifies not universally, e.g. ‘every man is white’ and ‘not every man is white’, ‘no man is white’ and ‘some man is white’ [\( ἀντικεῖσθαι μὲν κατάφασιν ἀποφάσει λέγω ἀντιφατικῶς τὴν τὸ καθόλου σημαίνουσαν τῷ αὐτῷ ὅτι οὐ καθόλου, οἷον πᾶς ἄνθρωπος λευκός – οὐ πᾶς ἄνθρωπος λευκός, οὐδεὶς ἄνθρωπος λευκός – ἕστι τις ἄνθρωπος λευκός\)].

\(^{91}\)But I call the universal affirmation and the universal denial contrary opposites, e.g. ‘every man is just’ and ‘no man is just’ [\( ἐναντίως δὲ τὴν τοῦ καθόλου κατάφασιν καὶ τὴν τοῦ καθόλου ἀπόφασιν, οἷον πᾶς ἀνθρώπου δίκαιος – οὐδεὶς ἀνθρώπου δίκαιος\)].

\(^{92}\)So these cannot be true together, but their opposites may be both true with respect to the same thing, e.g. ‘not every man is white’ and ‘some man is white’. Of contradictory statements about a universal taken universally it is necessary for one or the other to be true or false; similarly if they are about particulars, e.g. ‘Socrates is white’ and ‘Socrates is not white’ [\( διὸ ταύτας μὲν οὐκ οἷον τὰ ἄλλα ἱλήθη πολλάκις, τὰς δὲ ἀντικείμενα σύμμετρα ἐνδέχεται ἐπὶ τοῦ καθολικοῦ, οἷον οὐκ ἄνθρωπος λευκός, καὶ ἓστι τις άνθρωπος λευκός. ὅσοι μὲν οὐκ άνθρωπος τῶν καθολικῶν εἰσὶ καθόλου, ἀνάγκη τὴν ἐπέφερεν ἀλήθεια ἡ ζευγδή, καὶ ἓστι εἰπὼν τῶν καθολικῶν ἕκαστα, οἷον οὐκ ἦστε Σωκράτης λευκός – οὐκ ἦστε Σωκράτης λευκός\)].

\(^{93}\)for when we have proved that a predicate belongs in every case, we shall also have proved that it belongs in some cases. Likewise, also, if we prove that it does not
Lastly, the *subcontraries* result from the established relations as well. For, in cases where both the \( \text{\textit{a}} \) and \( \text{\textit{e}} \)-type sentence are false—as they can be as contraries—their contradictories must be true, i.e., the \( \text{\textit{o}} \) \( \text{\textit{i}} \)-type sentence is true as the contradictory of the \( \text{\textit{a}} \) \( \text{\textit{e}} \)-type sentence. Moreover, one of the \( \text{\textit{i}} \) and \( \text{\textit{o}} \)-type sentence has to be true. Suppose that the \( \text{\textit{i}} \)-type sentence is false. Then its contradictory \( \text{\textit{a}} \)-type sentence is true, which implies the corresponding \( \text{\textit{o}} \)-type sentence. For the same reason, one of the \( \text{\textit{a}} \) and \( \text{\textit{e}} \)-type sentence has to be false.

Aristotle is also aware that you can use the square to refute sentences. For, if you want to refute an \( \text{\textit{a}} \)-type sentence, it suffices to establish the corresponding \( \text{\textit{o}} \)-type sentence; and similarly with \( \text{\textit{e}} \) and \( \text{\textit{i}} \)-type sentences (Top B3, 110a32–37).

### 2.9 Formal Syllogistic

In order to have a comparison base, let me briefly introduce some formalism capturing Aristotle’s syllogistic. The presentation is based on, but also differs from, [Raab (2018)](Raab2018) where more discussion and details can be found.

**Definition 1 (The Language \( \mathcal{L}_A \))**

The language of Aristotelian Syllogistic (\( \mathcal{L}_A \)) consists of the following:

- a countable set \( \text{\textit{STerm}}_{\mathcal{L}_A} \) of (simple) terms,
- the set of logical symbols including ‘\( \neg \)’, ‘\( \bot \)’, ‘\( \land \)’, ‘\( \lor \)’, ‘\( \rightarrow \)’, ‘\( \leftrightarrow \)’, ‘\( \forall \)’, and ‘\( \exists \)’, and
- the set of auxiliary symbols including ‘(‘ and ‘)’.

The ‘\( \bot \)’-symbol is used to distinguish term-negation from a negative copula.\(^{95} \) The remaining symbols are to be understood as indicated below.

Since we allow complex terms, let us introduce them:

**Definition 2 (Complex \( \mathcal{L}_A \)-Terms)**

The (full) set of terms (\( \text{\textit{Term}}_{\mathcal{L}_A} \)) is recursively defined as follows:

\(^{94}\) Of course, in refuting a statement there is no need to start the discussion by securing any admission, whether the attribute is said to belong to all or to none of something; for if we prove that in any case whatever the attribute does not belong, we shall have refuted the universal assertion of it, and likewise if we prove that it belongs even in a single case, we shall refute the universal denial of it [πλὴν ἀνασευκάζοντι μὲν οὐδὲν δεῖ ἐξ ὁμολογίας διαλέγεσθαι, οὔτ᾿ εἰ παντὶ οὔτ᾿ εἰ μηδενὶ ὑπάρχειν εἴρηται· ἐὰν γὰρ δείξωμεν ὅτι οὐχ ὑπάρχειν ὁμοίως δὲ κἂν ἑνὶ δείξωμεν ὕπαρχεν ὁμοίως ἐπικάλεσθαι τὸ παντὶ ὑπάρχειν· ὁμοίως δὲ κἂν ἑνὶ δείξωμεν ὅτι οὐχ ὑπάρχειν ὁμοίως δὲ κἂν ἑνὶ δείξωμεν ὕπαρχεν ὁμοίως ἐπικάλεσθαι τὸ μηδενὶ ὑπάρχειν].

\(^{95}\) Including ‘\( \bot \)’ differs from, but is equivalent to, the set-up in Raab (2018 §3.5).
Given the language and the terms, we can define the formulas:

**Definition 3 (\(L_A\)-Formulas)**
The set of \(L_A\)-formulas (\(\text{Form}_{L_A}\)) is defined as follows:

- If \(A, B \in \text{Term}_{L_A}\), then
  - \((\forall A)B \in \text{Form}_{L_A}\)
  - \((\exists A)B \in \text{Form}_{L_A}\)
  - \((\forall A)\neg B \in \text{Form}_{L_A}\)
  - \((\exists A)\neg B \in \text{Form}_{L_A}\).

The formulas are to be read as follows: ‘\((\forall A)B\)’ as “all \(A\) are \(B\)”, ‘\((\exists A)B\)’ as “some \(A\) are \(B\)”, ‘\((\forall A)\neg B\)’ as “all \(A\) are not \(B\)” or “no \(A\) is \(B\)”, and ‘\((\exists A)\neg B\)’ as ‘some \(A\) are not \(B\)”. Note that, according to Definition 2, complex terms are covered by Definition 3. For example, formulas of the form ‘\((\forall (A \land B))\neg C\)’ are allowed, and should correspondingly be read as “all not-(\(A\)-and-\(B\)) are not-\(C\)” or “no not-(\(A\)-and-\(B\)) is not-\(C\)”.

The absence/occurrence of a negation-symbol ‘\(\neg\)’ indicates whether the formula is affirming or denying, respectively; it represents the copula. According to Definition 3, at most one negation-symbol occurs in a formula. Negation does not act on sentences, and we need to ensure in a different way how the sentences are related with respect to affirmation and denial; as in [Raab (2018)], this achieved via positive ('\(+\)') and negative ('\(\neg\)') semantic clauses (Definitions 6–7).

The quantifier-symbols indicate the quantity, i.e., whether a sentence is universal ('\(\forall\)') or particular ('\(\exists\)') (or, whether the predication is universally or particularly)—and that’s all they are doing: They are just a means to make explicit what kind of sentence is represented; instead of Aristotle’s way of suggesting that, e.g., ‘\(BA\)’ is universal affirming/denying sentence, we directly depict it as ‘\((\forall A)B\)’/‘\((\forall A)\neg B\)’.

Given this understanding, let me introduce a model-theoretic semantics. I provide two ways of doing so. One interpretation allows empty terms, i.e., the structure can assign the empty extension as interpretation of terms (I refer to it as ‘the empty semantics’); the other interpretation forces the simple terms to be non-empty (shown to be inadequate below).

**Definition 4 (\(L_A\)-Model)**
Let \(L_A\) be a language of Aristotelian Syllogistic. An \(L_A\)-model is a tuple \(M_A = (D, \| \cdot \|_{M_A})\) such that

1. \(D\) is a non-empty set (the universe)
(2) $\| \cdot \|_{M_A}$ is an interpretation-function of $M_A$ such that
\begin{enumerate}[(a)]
  \item if $A \in STerm_{\mathcal{L}_A}$, $\| A \|_{M_A} \subseteq D$;
  \item if $A \in Term_{\mathcal{L}_A}$ is a complex term of the form $'B'$ for some $B \in\ Term_{\mathcal{L}_A}$, then $\| A \|_{M_A} = D \setminus \| B \|_{M_A}$;
  \item if $A \in Term_{\mathcal{L}_A}$ is a complex term of the form $'(B \land C)'$ for some $B, C \in Term_{\mathcal{L}_A}$, then $\| A \|_{M_A} = \| B \|_{M_A} \cap \| C \|_{M_A}$.
\end{enumerate}

Since Definition 4 allows for empty terms, we have to specify that the domain $D$ is non-empty. Negated terms are interpreted as the set-theoretic difference between the extension of a term and the domain. Complex terms are treated as expected; Definition 4 only specifies the clause for conjunctive terms (‘$\land$’); the others are definable given clauses (2b)–(2c).

**Definition 5 (NE-$\mathcal{L}_A$-Model)**

Let $\mathcal{L}_A$ be a language of Aristotelian Syllogistic. A non-empty $\mathcal{L}_A$-model is a tuple $M_{ne} = \langle D, \| \cdot \|_{M_{ne}} \rangle$ such that
\begin{enumerate}[(1)]
  \item $D$ is a set (the universe);
  \item $\| \cdot \|_{M_{ne}}$ is an interpretation-function of $M_{ne}$ such that
    \begin{enumerate}[(a)]
      \item if $A \in STerm_{\mathcal{L}_A}$, $\emptyset \neq \| A \|_{M_{ne}} \subseteq D$;
      \item if $A \in Term_{\mathcal{L}_A}$ is a complex term of the form $'B'$ for some $B \in\ Term_{\mathcal{L}_A}$, then $\| A \|_{M_{ne}} = D \setminus \| B \|_{M_{ne}}$;
      \item if $A \in Term_{\mathcal{L}_A}$ is a complex term of the form $'(B \land C)'$ for some $B, C \in Term_{\mathcal{L}_A}$, then $\| A \|_{M_{ne}} = \| B \|_{M_{ne}} \cap \| C \|_{M_{ne}}$.
    \end{enumerate}
\end{enumerate}

In contrast to Definition 4, Definition 5 does not need to enforce the domain to be non-empty, as clause (2a) effectively takes care of it. The remaining clauses are the same as in Definition 4.

Given the different models, we can introduce corresponding satisfaction relations. Since $\mathcal{L}_A$ does not contain sentence-negation, we need to ensure that, for example, an $a$-type sentence has an $o$-type sentence as its contradictory by introducing positive (‘+’) and negative (‘−’) clauses. Moreover, since we take the validity of the square of opposition as a condition for any adequate satisfaction relation, we need to interpret the sentences accordingly. This results in different clauses for the $a$- and $o$-type sentences. (Note that, as Lemma 3.6 of Raab 2018 shows, the number of clauses is reducible to four.)

**Definition 6 (Satisfaction $\models_{\mathcal{L}_A}$)**

Let the satisfaction-relation $M_A \models_{\mathcal{L}_A} \varphi$ for $\mathcal{L}_A$-formulas $\varphi$ and $\mathcal{L}_A$-model $M_A$ be defined as follows: Let $A, B \in Term_{\mathcal{L}_A}$, then:

\footnote{Note that Definition 4 does not enforce $STerm_{\mathcal{L}_A}$ to be non-empty. If $STerm_{\mathcal{L}_A} = \emptyset$, clause (2a) does still not produce a problem as the clause is then vacuous.}
(a+) \( M_A \models_A (\forall A)B \) iff \( \| A \|_{\exists A} \cap \| B \|_{\exists A} = \| A \|_{\exists A} \) and \( \| A \|_{\exists A} \neq \emptyset \);

(a-) \( M_A \not\models_A (\forall A)B \) iff \( M_A \models_A (\exists A)\neg B \).

(i+) \( M_A \models_A (\exists A)B \) iff \( \| A \|_{\exists A} \cap \| B \|_{\exists A} \neq \emptyset \).

(i-) \( M_A \not\models_A (\exists A)B \) iff \( M_A \models_A (\forall A)\neg B \).

(e+) \( M_A \models_A (\forall A)\neg B \) iff \( \| A \|_{\exists A} \cap \| B \|_{\exists A} = \emptyset \).

(e-) \( M_A \not\models_A (\forall A)\neg B \) iff \( M_A \models_A (\exists A)B \).

(o+) \( M_A \models_A (\exists A)\neg B \) iff \( \| A \|_{\exists A} \cap \| B \|_{\exists A} \neq \| A \|_{\exists A} \) or \( \| A \|_{\exists A} = \emptyset \).

(o-) \( M_A \not\models_A (\exists A)\neg B \) iff \( M_A \models_A (\forall A)B \).

In order for the square of opposition to hold, we must ensure that an \( o \)-type sentences imply \( i \)-type sentences. The usual way to do so is by only allowing non-empty terms as in Definition 5, but Definition 4 allows for empty terms. Thus, a model \( M_A \) can only satisfy an \( a \)-type sentence if the term happens to be non-empty, i.e., if \( \| A \|_{\exists A} = \emptyset \), no sentence of the form ‘(\( \forall A \))B’ can be satisfied. Since \( a \)-type sentences have \( o \)-type sentences as their contradictories, the satisfaction-clause \( o+ \) needs to include the cases in which the subject-term is empty.

As the non-empty \( L_A \)-models \( M_{ne} \) don’t allow non-empty terms, the clauses are simpler than those of Definition 6. However, as shown in Theorem 14, there is no fully general formal analogue of \( a-i-conv \) and so the square of opposition does not follow.

**Definition 7 (NE-Satisfaction \( \equiv_{ne} \))**

Let the non-empty satisfaction-relation \( M_{ne} \models_{ne} \varphi \) for \( L_A \)-formulas \( \varphi \) and non-empty \( L_A \)-model \( M_{ne} \) be defined as follows: Let \( A, B \in \text{Term}_{\exists A} \), then:

\( a_{ne} \) \( M_{ne} \models_{ne} (\forall A)B \) iff \( \| A \|_{\exists A} \cap \| B \|_{\exists A} = \| A \|_{\exists A} \).

\( a_{ne} \) \( M_{ne} \not\models_{ne} (\forall A)B \) iff \( M_{ne} \models_{ne} (\exists A)\neg B \).

\( i_{ne} \) \( M_{ne} \models_{ne} (\exists A)B \) iff \( \| A \|_{\exists A} \cap \| B \|_{\exists A} \neq \emptyset \).

\( i_{ne} \) \( M_{ne} \not\models_{ne} (\exists A)B \) iff \( M_{ne} \models_{ne} (\forall A)\neg B \).

\( e_{ne} \) \( M_{ne} \models_{ne} (\forall A)\neg B \) iff \( \| A \|_{\exists A} \cap \| B \|_{\exists A} = \emptyset \).

\( e_{ne} \) \( M_{ne} \not\models_{ne} (\forall A)\neg B \) iff \( M_{ne} \models_{ne} (\exists A)B \).

\( o_{ne} \) \( M_{ne} \models_{ne} (\exists A)\neg B \) iff \( \| A \|_{\exists A} \cap \| B \|_{\exists A} \neq \| A \|_{\exists A} \).

\( o_{ne} \) \( M_{ne} \not\models_{ne} (\exists A)\neg B \) iff \( M_{ne} \models_{ne} (\forall A)B \).

Given a notion of satisfaction, we can introduce the usual notions:
**Definition 8**

Let $T \subseteq \text{Form}_{L_A}$, $\models \{ \models_A, \models_{ne} \}$, and $M_{ne} = \{ \begin{array}{ll} M_A & \text{if } \models \text{ is } \models_A \\ M_{ne} & \text{if } \models \text{ is } \models_{ne} \end{array} \}$. 

1. $\varphi$ is a logical consequence of $T$ ($T \models \varphi$) iff for all $L_A$-models $M_{ne}$, if $M_{ne} \models \psi$ for all $\psi \in T$, then $M_{ne} \models \varphi$.

2. If $T = \{ \varphi_1, \ldots, \varphi_n \}$, we write ‘$\varphi_1, \ldots, \varphi_n \models \varphi$’ for ‘$\{ \varphi_1, \ldots, \varphi_n \} \models \varphi$’.

3. $T$ is satisfiable iff there is an $L_A$-model $M_{ne}$ such that $M_{ne} \models \varphi$ for all $\varphi \in T$.

Given these definitions, we can formulate some results. First, we can note that the empty $L_A$-models see sentence-types $\box{a}$ and $\box{e}$ as contraries:

**Lemma 9 (Contraries)**

$(\forall A)B$ and $(\forall A)\neg B$ are contraries in $L_A$-models $M_A$:

1. $\{ (\forall A)B, (\forall A)\neg B \}$ is not satisfiable;

2. there are $L_A$-models $M_A$ such that $M_A \models \neg_A (\forall A)B$ and $M_A \models \neg_A (\forall A)\neg B$.

**Proof.** Let $M_A$ be an $L_A$-model.

1: Suppose that $M_A \models (\forall A)B$ and $M_A \models (\forall A)\neg B$. Then, by $\box{a}$. 

$\|A\|_{\text{ne}} \cap \|B\|_{\text{ne}} = \|A\|_{\text{ne}} \neq \emptyset$, and, by $\box{e}$, $\|A\|_{\text{ne}} \cap \|B\|_{\text{ne}} = \emptyset$, a contradiction.

2: Let $\|A\|_{\text{ne}} = \{a, b\}$ and $\|B\|_{\text{ne}} = \{a\}$. Then, $\|A\|_{\text{ne}} \cap \|B\|_{\text{ne}} \neq \emptyset$, i.e., by $\box{a}$, $M_A \models (\exists A)B$. And, since $\|A\|_{\text{ne}} \cap \|B\|_{\text{ne}} \neq \|A\|_{\text{ne}}$, by $\box{e}$, $M_A \models (\exists A)\neg B$. By $\box{a}$ and $\box{e}$, respectively, the result follows.

Two characteristics of the semantics are the following:

**Theorem 10**

The following hold.

1. $\not\models_A (\exists A)A$ 
2. $(\forall A)B \models_A (\exists A)B$
3. $\not\models_{ne} (\exists A)A$ 
4. $(\forall A)B \not\models_{ne} (\exists A)B$

**Proof.** Let $M_A$ be an $L_A$-model such that $\|A\|_{\text{ne}} = \emptyset$. Then, $\|A\|_{\text{ne}} \cap \|A\|_{\text{ne}} = \emptyset$, i.e., by $\box{e}$, $M_A \models (\forall A)\neg A$. Thus, by $\box{a}$, $M_A \not\models_A (\exists A)A$. 

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Let $\mathfrak{M}_A \models_A (\forall A)B$. Then, by (a-an), $\|A\|_{\mathfrak{M}_A} \cap \|B\|_{\mathfrak{M}_A} = \|A\|_{\mathfrak{M}_A} \neq \emptyset$. Therefore, by i-hi, $\mathfrak{M}_A \models_A (\exists A)B$.

Let $\mathfrak{M}_{ne}$ be a non-empty $\mathcal{L}_A$-model such that $\|A\|_{\mathfrak{M}_{ne}} = D$. Then, $\|A\|_{\mathfrak{M}_{ne}} = \emptyset$. Thus, $\|\overline{A}\|_{\mathfrak{M}_{ne}} = \emptyset$, so, by (e-an) and i-e, $\mathfrak{M}_{ne} \models_{ne} (\forall A)\overline{A}$. By i-ne, $\mathfrak{M}_{ne} \not\models_{ne} (\exists A)A$.

Consider the model in (3). Since $\|\overline{A}\|_{\mathfrak{M}_{ne}} \cap \|B\|_{\mathfrak{M}_{ne}} = \|\overline{A}\|_{\mathfrak{M}_{ne}}$, by (a-an) and i-e, $\mathfrak{M}_{ne} \models_{ne} (\forall A)\overline{A}B$. However, since $\|\overline{A}\|_{\mathfrak{M}_{ne}} \cap \|B\|_{\mathfrak{M}_{ne}} = \emptyset$, by (e-an), $\mathfrak{M}_{ne} \models_{ne} (\forall A)\overline{A}$, and so, by (i-ne), $\mathfrak{M}_{ne} \not\models_{ne} (\exists A)B$.

Theorem 10 (3–4) imply that the non-empty semantics does not validate the square of opposition; for example, sentence-types $\Box$ and $\Diamond$ fail to be contraries.

**Corollary 11**

In the non-empty semantics, $(\forall A)B$ and $(\forall A)\overline{A}$ are not contraries. In general, given a non-empty $\mathcal{L}_A$-model $\mathfrak{M}_{ne}$, if $\|A\|_{\mathfrak{M}_{ne}} = \emptyset$, then $\{(\forall A)B, (\forall A)\overline{A}\}$ is satisfiable in $\mathfrak{M}_{ne}$.

**Proof.** Consider the proof of Theorem 10 (4). The model $\mathfrak{M}_{ne}$ is such that both $\mathfrak{M}_{ne} \models_{ne} (\forall A)B$ and $\mathfrak{M}_{ne} \models_{ne} (\forall A)\overline{A}$. $lacksquare$

The empty semantics has formal analogues of the conversions:

**Theorem 12 (Conversion)**

The following conversions hold:

(a-i-conv$\models ne$) $(\forall A)B \models_{ne} (\exists B)A$

(i-i-conv$\models ne$) $(\exists A)B \models_{ne} (\exists B)A$

(e-e-conv$\models ne$) $(\forall A)\overline{B} \models_{ne} (\forall B)\overline{A}$

The non-empty semantics only validates two such conversions:

**Theorem 13 (NE-Conversion)**

The following conversions hold:

(i-i-conv$\models ne$) $(\exists A)B \models_{ne} (\exists B)A$

(e-e-conv$\models ne$) $(\forall A)\overline{B} \models_{ne} (\forall B)\overline{A}$

**Proof.** Let $\mathfrak{M}_A$ be an $\mathcal{L}_A$-model.

(a-i-conv$\models ne$) Suppose that $\mathfrak{M}_A \models_A (\forall A)B$. Then, by (a-an) and i-e, $\|A\|_{\mathfrak{M}_A} \cap \|B\|_{\mathfrak{M}_A} = \|A\|_{\mathfrak{M}_A} \neq \emptyset$. Thus, by (i-ne), $\mathfrak{M}_A \models_{ne} (\exists B)A$. 

(1.9) Formal Syllogistic
(i-i-conv) Suppose that $\mathcal{M}_A \models A (\exists A) B$. Then, by (i-i), $\|A\|_{2\mathcal{R}_A} \cap \|B\|_{2\mathcal{R}_A} \neq \emptyset$, i.e., $\|B\|_{2\mathcal{R}_A} \cap \|A\|_{2\mathcal{R}_A} \neq \emptyset$ and so, by (i-i), $\mathcal{M}_A \models A (\exists A) B$.

(e-e-conv) Suppose that $\mathcal{M}_A \models A (\forall B) \neg B$. Then, by (e-e), $\|A\|_{2\mathcal{R}_A} \cap \|B\|_{2\mathcal{R}_A} = \emptyset$, so also $\|B\|_{2\mathcal{R}_A} \cap \|A\|_{2\mathcal{R}_A} = \emptyset$, i.e., by (e-e), $\mathcal{M}_A \models A (\forall B) \neg B$.

(i-i-conv) and (e-e-conv) are shown in the same way. \(\square\)

The non-empty semantics does not validate the third conversion.

**Theorem 14 (NE-Conversion-Failure)**

The formal analogue of (a-i-conv) fails for the non-empty semantics:

(\(\forall A) B \not\models_{ne} (\exists B) A\)

**Proof.** Let $\mathcal{M}_{ne}$ be a non-empty $\mathcal{L}_A$-model such that $\|C\|_{2\mathcal{R}_{ne}} = D$. Then, by Definition 5 and 6, $\|C\|_{2\mathcal{R}_{ne}} = D \setminus \|C\|_{2\mathcal{R}_{ne}} = D \setminus D = \emptyset$.

Now, let $\|A\|_{2\mathcal{R}_{ne}} = \|C\|_{2\mathcal{R}_{ne}}$, and suppose that $\mathcal{M}_{ne} \models_{ne} (\forall A) B$. Then, by (a-i-ne), $\|A\|_{2\mathcal{R}_{ne}} \cap \|B\|_{2\mathcal{R}_{ne}} = \|A\|_{2\mathcal{R}_{ne}}$, i.e., $\|A\|_{2\mathcal{R}_{ne}} \cap \|B\|_{2\mathcal{R}_{ne}} = \emptyset$, so also $\|B\|_{2\mathcal{R}_{ne}} \cap \|A\|_{2\mathcal{R}_{ne}} = \emptyset$. By (e-e-ne), $\mathcal{M}_{ne} \not\models_{ne} (\forall B) \neg A$. Thus, by (i-i-ne), $\mathcal{M}_{ne} \not\models_{ne} (\exists B) A$.

Therefore, $\not\models_{ne} (\forall A) B \not\models_{ne} (\exists B) A$. \(\square\)

All we get is a restricted version:

**Theorem 15 (Restricted NE-a-i-Conversion)**

Let $\mathcal{M}_{ne}$ be a non-empty $\mathcal{L}_A$-model. Then:

(\(\exists A) (\forall A) B \models_{ne} (\exists B) A\)

I take this as evidence that Definitions 3 and 6 are the correct ones since the problem arises already with negative terms—which Aristotle explicitly discusses in his *Organon*, even though not in his *Prior Analytics*. Other complex terms might end up empty, too, even though the simpler terms are not. Let $\mathcal{M}_{ne}$ be a non-empty $\mathcal{L}_A$-structure. Suppose that $\emptyset \subseteq \|A\|_{2\mathcal{R}_{ne}} \subseteq \|D\|_{\overline{X}}$. Then, $\emptyset \subseteq \|\overline{A}\|_{2\mathcal{R}_{ne}} \subseteq \|D\|_{\overline{X}}$. However, $\|\overline{A} \land \overline{\overline{\overline{A}}}\|_{2\mathcal{R}_{ne}} = \emptyset$. Therefore, $\|\overline{A} \land \overline{\overline{\overline{A}}}\|_{2\mathcal{R}_{ne}} \cap \|B\|_{2\mathcal{R}_{ne}} = \emptyset$. Thus, by (a-i-ne), $\mathcal{M}_{ne} \models_{ne} \overline{\overline{\overline{A}}}$, but $\mathcal{M}_{ne} \not\models_{ne} \overline{\overline{\overline{A}}}$. This also means again that $\not\models_{ne} (\forall C) D \not\models_{ne} (\exists C) D$.

Of course, one could still insist on the non-emptiness of terms. One option, though I don’t take it to be particularly plausible, is to only allow terms which don’t lead to empty ones. That, of course, rules out simultaneously having negated and conjunctive terms. Another option is to do the same as in Definition 3, though then there is no reason to

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97 If we only consider negated terms, the option has some plausibility. Given the discussion of Section 2.6 assigning the whole domain as interpretation of a term might push us to the top of the term-hierarchy and, thus, to terms that Aristotle
assume that terms are non-empty to begin with. There might be different options available, but I take Definitions 4 and 6 to be the correct ones. Nevertheless, the semantics to be developed in the following sections are more like the non-empty one from Definitions 5 and 7.

Regarding the empty semantics, there is a difference between denials and affirmations modulo negated terms:

**Theorem 16 (Negation)**

The following hold:

1. $(\forall A)B \models_A (\forall A)\neg \overline{B}$;
2. $(\forall A)\neg \overline{B} \not\models_A (\forall A)B$;
3. $(\exists A)B \models_A (\exists A)\neg \overline{B}$;
4. $(\exists A)\neg \overline{B} \not\models_A (\exists A)B$.

Proof. Let $\mathcal{M}_A$ be an $\mathcal{L}_A$-model.

1. Let $\mathcal{M}_A \models_A (\forall A)B$. By $(A_+)\parallel B\parallel_{2\mathcal{L}_A} \cap \parallel B\parallel_{\mathcal{L}_A} = \parallel A\parallel_{\mathcal{L}_A} \neq \emptyset$. By Definition 4 (2b), $\parallel B\parallel_{\mathcal{L}_A} = D \setminus \parallel B\parallel_{2\mathcal{L}_A}$, i.e., $\emptyset = \parallel A\parallel_{\mathcal{L}_A} \cap \parallel B\parallel_{2\mathcal{L}_A}$. Therefore, $\emptyset = \parallel A\parallel_{\mathcal{L}_A} \cap \parallel B\parallel_{\mathcal{L}_A} = \parallel A\parallel_{\mathcal{L}_A} \cap \parallel B\parallel_{\mathcal{L}_A}$. Thus, by $(e_+)$, $\mathcal{M}_A \models_A (\forall A)\neg \overline{B}$.

2. Let $\parallel A\parallel_{\mathcal{L}_A} = \emptyset$. Then, $\parallel A\parallel_{\mathcal{L}_A} \cap \parallel B\parallel_{\mathcal{L}_A} = \emptyset$, i.e., by $(e_+)$, $\mathcal{M}_A \not\models_A (\forall A)\neg \overline{B}$. Also, as $\parallel A\parallel_{\mathcal{L}_A} = \emptyset$, by $(o_+)$, $\mathcal{M}_A \models_A (\exists A)\neg \overline{B}$. Therefore, by $(a_-)$, $\mathcal{M}_A \not\models_A (\forall A)B$.

3. Let $\mathcal{M}_A \models_A (\exists A)B$. By $(o_+)$, $\parallel A\parallel_{\mathcal{L}_A} \cap \parallel B\parallel_{\mathcal{L}_A} \neq \emptyset$. By Definition 4 (2b), $\parallel B\parallel_{\mathcal{L}_A} = D \setminus \parallel B\parallel_{2\mathcal{L}_A}$, so $\parallel A\parallel_{\mathcal{L}_A} \cap \parallel B\parallel_{2\mathcal{L}_A} \neq \parallel A\parallel_{\mathcal{L}_A}$. Therefore, by $(o_+)$, $\mathcal{M}_A \models_A (\exists A)\neg \overline{B}$.

4. Let $\parallel A\parallel_{\mathcal{L}_A} = \emptyset$. Then, by $(o_+)$, $\mathcal{M}_A \not\models_A (\exists A)\neg \overline{B}$. Also, $\parallel A\parallel_{\mathcal{L}_A} \cap \parallel B\parallel_{\mathcal{L}_A} = \emptyset$, so, by $(e_+)$, $\mathcal{M}_A \not\models_A (\forall A)\neg \overline{B}$. Thus, by $(i)$, $\mathcal{M}_A \not\models_A (\exists A)B$.

As shown in Lemma 3.29 of Raab (2018), the non-empty-semantics validates that sentence-types (i.e., $\neg (qA) \models (qA)\neg B$ iff $\models \neg (qA)\overline{B}$ (or $\not\in \{\forall, \exists\}$)) whereas the empty semantics only validates the direction from sentence-type (ii) to sentence-type (i), i.e., $(qA)\overline{B} \models (qA)\neg B$, but $(qA)\neg B \not\models (qA)\overline{B}$.

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dismisses as relevant for his syllogistic. Thus, if the only complex terms are negated terms, we might change Definition 5 (2b) to

(2) $(b^*)$ if $A \in STerm_{\mathcal{L}_A}$, $\emptyset \subsetneq \parallel A\parallel_{\mathcal{L}_A} \subseteq D$

which resolves the problem as for any $A \in Term_{\mathcal{L}_A}$, $\overline{A} \parallel_{\mathcal{L}_A} \not\models \emptyset$.  

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2.10 Identity

The syllogistic is lacking any treatment of identity. As suggested in Section 2.6, Aristotle does not consider something like ‘is Socrates’ to be a term; the Organon does not seem to include any identity claims.

Yet, Aristotle formulates some principles to test for the (non-)identity of terms. Given two terms $A$ and $B$, we can compare them with respect to other terms $C$. One principle Aristotle suggests is that if $A$ and $B$ are identical, then if $A$ is identical to $C$, $B$ must also be identical to $C$ (Top H1, 152a31f.99). Whereas finding differences breaks identities (cf. Top A18, 108b2ff.100), being identical to something else suffices for identity, i.e., if $A$ is identical to $C$, and $B$ is identical to $C$, then $A$ and $B$ are identical too (SE 6, 168b31f.101).

Aristotle does not say much more about this, and its not entirely clear of what sort of things he claims identity, though he seems to formulate a (more general) version of Leibniz’s law:

Speaking generally, one ought to be on the look-out for any discrepancy anywhere in any sort of predicate of each term, and in the things of which they are predicated. For all that is predicated of the one should be predicated also of the other, and of whatever the one is a predicate, the other should be a predicate as well.
(Top H1, 152b25–29102)

I put it in terms of terms above. It should be clear that there are no (explicit) principles to establish identities between terms, but the principles allow us to break some (see also Top H1, 152b34f.103). Suppose that we establish that $(\forall A)C$ and $(\exists B)\neg C$. Then $A$ and $B$ cannot be identical. Also, if we establish that $(qC)A$ and $(qC)\neg B$ $(q \in \{\forall, \exists\})$, then $A$ and $B$ cannot be identical. However, if $A$ and $B$ are identical, we can conclude from $(qC)A$ that $(qC)B$, as well as $(qC)B$ from $(qA)C$. As $A$ and $B$ are terms, they can occur both in subject- and predicate-position, and the principle Aristotle suggests is meant to check both options.

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98 Note that Aristotle does not speak about terms, but I take it to apply to them.
99- Again, look and see if, supposing the one to be the same as something, the other also is the same as it; for if they are not both the same as the same thing, clearly neither are they the same as one another [Πάλιν σκοπεῖν εἰ ᾧ θάτερον ταὐτόν, καί θάτερον εἰ γάρ μη ἁμισότερα τῷ αὐτῷ ταὐτά, δῆλον ὅτι οὐδ᾿ ἄλληλοιοι].”
100- for when we have found any difference whatever between the things proposed, we shall have shown that they are not the same thing [εὑρόντες γὰρ διαφορὰν τῶν προκειμένων ὁποιανοῦν δεδειχότες ἐσόμεθα ὅτι οὐ ταὐτόν].”
101- for we claim that things that are the same as one and the same thing are also the same as one another [τὰ γὰρ ἐνὶ καὶ ταὐτῷ ταὐτά καὶ ἄλληλοις ἄξιομεν εἶναι ταὐτά].’”
102- Καθόλου δ᾿ εἰπεῖν ἐκ τῶν ὁπωσοῦν ἐκατέρων κατηγορομένων καὶ ἣν ταῦτα κατηγορεῖται σκοπεῖν εἰ που διαφωνεῖ ὅσα γὰρ θάτερον κατηγορεῖται, καὶ θάτερον κατηγορεῖσθαι δεῖ, καὶ ὧν θάτερον κατηγορεῖται, καὶ θάτερον κατηγορεύεσθαι δεῖ.”
103- Moreover, see whether the one can exist without the other; for, if so, they will not be the same [Ἔτι εἰ δυνατὸν θάτερον ἀνευ θάτερον εἶναι· οὐ γὰρ ἂν εἶ ταὐτόν].”
only then identical if the same terms are predicated of them \((qA)C\) and \((qB)C\) and they are predicated of the same terms \((qC)A\) and \((qC)B\).

3 A Fregean Approach

3.1 Background

Aristotle’s logical system remained the dominant system until Gottlob Frege developed his *Begriffsschrift* ([1879](#footnote1)). That does not mean, though, that the syllogistic did not undergo any changes at all. One notable change is the treatment of particulars as terms in a way analogous to other terms (see Parkinson’s introduction in Leibniz [1966](#footnote1)). Given the formalism from Section 2.9, a model \(M_A\) interprets such terms \(A\) as \(\|A\|_{M_A} = \{a\}\) for an \(a \in D\). Thus, for any term \(B\), \((\exists A) B \models_A (\forall A) B\), i.e., the \(i\)-type sentence implies the \(a\)-type sentence. And, as already encoded in the square of opposition (Section 2.8), the latter also implies the former. For example, if \(a \in D\) is Socrates and we blur the line between predicates and individuals, then ‘some Socrates is human’ implies ‘every Socrates is human’, and vice versa.

However, Aristotelian syllogistic is limited in its expressive power. In particular, two limitations are generally pointed out, viz., Aristotelian syllogistic does not know relational terms, and, based on this, cannot deal with several quantificational phrases (see, e.g., Frege [1879](#footnote1), Carnap [1930/31/50](#footnote1), Russell [1946/2004](#footnote1): ch. 22, Kneale and Kneale [1962](#footnote1) 31, 487, and Link [2009](#footnote1) 10).

The main limitation is the syllogistic’s restriction to terms which we can take to correspond to unary predicates so that it cannot account for relations. According to the *ti kata tinos*, the basic structure of sentences is subject-predicate. This means that the syllogistic cannot—in its current form—account for relational statements such as ‘point \(a\) lies between point \(b\) and point \(c\)’. Frege overcomes this limitation by replacing the “concepts subject and predicate by argument and function” ([1879](#footnote1) 7, his emphases). Of course, the most basic structure is still that of subject-predicate and is captured by a function applying to an argument—something that presupposes individual-constants that are not included in the syllogistic as presented above—but that immediately generalizes once the function is allowed to take more than one argument. Moreover, the subject-predicate structure is broken up once we consider sentences within the range of application of the syllogistic; for example, a sentence like ‘all human beings are mortal’ is not taken to have ‘all human beings’ or ‘human beings’ as its subject (depending on how one understands the quantity indicated by ‘all’), but is analysed in terms for quantifiers, variables, connectives, and functions applying to arguments.

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104 As I’ve mostly treated terms as plural, it would be better to say ‘some/every Socrates are human’.
The other limitation is what might be called *nested quantification* (aka *multiply general propositions*). The subject-predicate structure does not rely on quantifiers, but the quantity of its subject is somehow indicated; Aristotle specifies it explicitly by saying, e.g., ‘let $AB$ be a universal affirmative sentence’, and the proposed formalism from Section 2.9 captures it by including a quantifier-symbol in front of the subject-term. Thus, the syllogistic can capture sentences like ‘all human beings are mortal’, but it lacks the means to express ‘all human beings have someone they like’ or ‘some human beings like all human beings’. What’s lacking is another way to even attach a quantity, and, as we have seen in Section 2.2, Aristotle does not think that sentences can be true if a quantity is assigned to more than the subject-term.

The Fregean approach with its function-argument analysis, on the other hand, has the means to assign quantities to several parts of sentences. Indeed, the *quantifiers* are treated as proper constituents of sentences. Only considering the first-order fragment, we can see that given arguments $a_1, \ldots, a_n$ and an $n$-ary function-symbol $f$, we can form a sentence ‘$f(a_1, \ldots, a_n)$’ in which every argument-place allows to be quantified in. For example, ‘$\forall x_1 \exists x_2 f(x_1, x_2)$’ is a sentence with nested quantifiers which can capture a sentence properly outside of Aristotelian syllogistic.

Overall, Frege captures a sentence like ‘all human beings are mortal’ as consisting of a quantifier (‘$\forall$’) binding a variable (‘$x$’) and acting on a *complex formula* with a conditional (‘$\rightarrow$’) as its main connective whose antecedent and consequent are *functions* applied to an argument (‘$H(x)$’, ‘$M(x)$’). None of these explicitly appears in the original sentence, and Frege is well aware that his formal language departs from ordinary language (1879: 6); he thinks he introduces a tool for “certain scientific purposes” (1879: 6), comparing it to the introduction of a microscope to better the human eye. Similarly, Carnap compares natural language to a “crude, primitive pocketknife” which is “useful for a hundred different purposes” (1963: 938), but not so much for specific purposes requiring greater precision. In this sense, we can—and will—understand the formal languages and their formalisms as *explications*.

One strength of the Fregean explication is that it allows for fairly simple solutions to the aforementioned limitations of Aristotelian syllogistic. For, as sentences are not forced to have subject-predicate structure, it is possible to allow relational predicates like ‘$x$ lies between $y$ and $z$’ (‘$B(x, y, z)$’), and nest quantifiers. For example, we can render a sentence like ‘every point $a$ lies between some points $b$ and $c$’ as ‘$\forall x \exists y \exists z (B(x, y, z))$’.

### 3.2 A Formalism

To make the approach formally precise and to have a basis for comparison, let me introduce a convenient formalism (which more or less follows
It should be clear that the following exposition is not following Frege in any detail, and is geared toward better comparison between the different formalisms to be introduced in the following and the one introduced in Section 2.9. Nevertheless, the following can rightly be claimed to expose, or explicate, a Fregean formalism.

The following exposition is not entirely standard, though it does not deviate much from a standard exposition. Insofar as it deviates, it is geared towards running in parallel with the exposition of the QUARC (Section 4.2). As I explain much of what’s going on here already, I can present the QUARC-formalism succinctly while just pointing out the QUARC-specific features.

We start by specifying the vocabulary of a Fregean language.

**Definition 17 (Fregean Language)**

A Fregean language ($L_F$) consists of the following:

- a countably infinite set $\text{Var}_{L_F} = \{v_0, v_1, v_2, \ldots \}$ of (individual-) variables,
- a countable set $\text{Const}_{L_F} = \{c_0, c_1, c_2, \ldots \}$ of (individual-) constants,
- for every $n > 0$, a countable set $\text{Pred}^n_{L_F} = \{P^n_0, P^n_1, P^n_2, \ldots \}$ of $n$-ary predicate-symbols,
- the set of auxiliary symbols including ‘(’, ‘)’, and ‘,’.

The sets are assumed to be disjoint. Let $\text{Pred}_{L_F} = \bigcup_{n>0} \text{Pred}^n_{L_F}$.

The basic vocabulary of a Fregean language $L_F$ is fairly standard and extends the language of Aristotelian Syllogistic $L_A$ in several ways. Firstly, $L_F$ contains what can be taken to correspond to $L_A$-terms, but also predicate-symbols of any arity. It also contains individual-constants and individual-variables, making it a first-order language. Lastly, $L_F$ has an additional logical symbol, viz., ‘=’, and it lacks the term-negation ‘¬’.

Even though the languages overlap significantly (as we can consider $L_A$ to be a sublanguage of $L_F$), the formation rules for $L_F$ are significantly different from those of $L_A$.

**Definition 18 ($L_F$-Formula)**

Let $L_F$ be a Fregean language. The set of $L_F$-formulas ($\text{Form}_{L_F}$) is recursively defined by:

1. given $n > 0$ $L_F$-constants $c_1, \ldots, c_n$ and $P \in \text{Pred}^n_{L_F}$, $P(c_1, \ldots, c_n) \in \text{Form}_{L_F}$;
2. given $L_F$-constants $c_1$ and $c_2$, $(c_1 = c_2) \in \text{Form}_{L_F}$;

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(3) if \( \phi \in \text{Form}_{L_F} \), then \( \neg \phi \in \text{Form}_{L_F} \);
(4) if \( \phi, \psi \in \text{Form}_{L_F} \) and \( \circ \in \{\wedge, \vee, \to, \leftrightarrow\} \), then \( (\phi \circ \psi) \in \text{Form}_{L_F} \);
(5) if \( \phi(c) \in \text{Form}_{L_F} \), \( x \in \text{Var}_{L_F} \), and \( q \in \{\forall, \exists\} \), then \( qx\phi[x/c] \in \text{Form}_{L_F} \).

Definition 18, in contrast to Definition 3, introduces a recursion to generate all the formulas. Clause (1) captures the function-argument structure, viz., the elements of \( \text{Const}_{L_F} \) are the arguments to the functions contained in \( \text{Pred}_{L_F} \). Additionally, the symbol ‘=’ figures as binary function/predicate. As usual, we can consider the formulas obtained by clauses (1)–(2) to be atomic and the constituents of complex formulas arrived by the remaining clauses.

Clause (5) allows for nested quantification for which clauses (1)–(2) provide the places. For example, for \( P \in \text{Pred}^2_{L_F} \) and \( c_1, c_2 \in \text{Const}_{L_F} \), clause (1) guarantees that \( P(c_1, c_2) \in \text{Form}_{L_F} \). Applying clause (5) to it, \( x \in \text{Var}_{L_F} \) and \( \forall \), leads to \( \forall x \phi(x, c_2) \in \text{Form}_{L_F} \). Applying it again to this, \( y \in \text{Var}_{L_F} \) and \( \exists \), we get \( \exists y \forall x P(x, y) \in \text{Form}_{L_F} \).

Definition 18 is non-standard insofar as it does not allow for open formulas. Clause (5) is the only clause introducing variables, and those variables are bound. Because of this and in order to have a better comparable formalism, we understand quantification as substitutional and treat it accordingly below.

Given this increase in complexity, the interpretations of such Fregean languages have to be more complex, too, though the underlying model remains the same; we just make more use of it.

**Definition 19 (\( L_F \)-Model)**

Let \( L_F \) be a Fregean language. A model for \( L_F \) (\( L_F \)-model) is an ordered pair \( M_F = \langle D, \| \cdot \|_{M_F} \rangle \) such that

1. \( D \) is a set (the domain of \( M_F \));
2. \( \| \cdot \|_{M_F} \) is an interpretation-function of \( M_F \) such that
   a. if \( c \in \text{Const}_{L_F} \), then \( \|c\|_{M_F} \subseteq D \);
   b. if \( P \in \text{Pred}^1_{L_F} \), then \( \emptyset \neq \|P\|_{M_F} \subseteq D \);
   c. if \( n > 1 \) and \( P \in \text{Pred}^n_{L_F} \), then \( \|P\|_{M_F} \subseteq D^n \).

Clause (2b) forces unary predicates to be assigned a non-empty extension. This has been done in order for a smoother comparison with QUARC. Moreover, in this way we also generate better comparability to (non-empty) \( L_A \)-models.

Moreover, as QUARC relies on substitutional quantification, we understand it similarly here. Hence, in order to correctly interpret the formulas involving quantification, we need to make sure that the interpretation does not rely on the specific choice of \( \text{Const}_{L_F} \). In order to do so, we first expand the underlying language (Definition 20), enriching it with further
individual-constants, and then making sure that the interpretation keeps up (Definition 21). With these in place, we can specify when a model satisfies a formula (Definition 22).

**Definition 20** (*L*$_F$-A-Expansion)

Let *L*$_F$ be a Fregean language. Let $\mathcal{M}_F = \langle D, \parallel \cdot \parallel_{\mathcal{M}_F} \rangle$ be an $\mathcal{L}_F$-model. Let $A \subseteq D$. The $\mathcal{L}_F$-A-expansion of $\mathcal{L}_F$ is the language $\mathcal{L}'_F := \mathcal{L}_F \cup \{c_a | a \in A\}$ where the $c_a$ are new (individual-)constants not contained in $\mathcal{L}_F$.

If $A = \{a\}$, we call $\mathcal{L}'_F$ an $\mathcal{L}_F$-a-expansion.

The idea is that we consider part of the domain of a model $\mathcal{M}_F$ and introduce new names for the elements of the chosen part. The new symbols need to be interpreted in the correct way too, which cannot be done in the original model $\mathcal{M}_F$ so that we have to expand it to $\mathcal{M}'_F$ in the following way.

**Definition 21** (*L*$_F$-Model Expansion)

Let $\mathcal{L}_F$ be a Fregean language and $\mathcal{M}_F = \langle D, \parallel \cdot \parallel_{\mathcal{M}_F} \rangle$ an $\mathcal{L}_F$-model. Let $A \subseteq D$, and $\mathcal{L}'_F$ be an $\mathcal{L}_F$-A-expansion. The A-expansion of $\mathcal{M}_F$ to $\mathcal{L}'_F$ is the model $\mathcal{M}'_F = \langle D', \parallel \cdot \parallel_{\mathcal{M}'_F} \rangle$ such that

1. $D' = D$;
2. $\parallel \cdot \parallel_{\mathcal{M}'_F} \subseteq \parallel \cdot \parallel_{\mathcal{M}_F}$;
3. $\parallel c_a \parallel_{\mathcal{M}'_F} = a \in A$ for every new symbol $c_a$.

The domains of the model $\mathcal{M}_F$ and its expansion $\mathcal{M}'_F$ are the same. The new constants are interpreted according to how they have been introduced. Since $A \subseteq D$ and $D' = D$, $A \subseteq D'$, and the new symbols $c_a$ for $a \in A$ just provide names for the elements $a \in D$.

Lastly, the expanded interpretation-function $\parallel \cdot \parallel_{\mathcal{M}'_F}$ extends the interpretation-function $\parallel \cdot \parallel_{\mathcal{M}_F}$, i.e., it leaves unaltered the interpretations of the original model $\mathcal{M}_F$. In particular, suppose that $\parallel P \parallel_{\mathcal{M}_F} = \{a\}$ for $a \in D$, but there is no $c \in \text{Const}_{\mathcal{L}_F}$ such that $\parallel c \parallel_{\mathcal{M}_F} = a$. We can then expand the language to $\mathcal{L}'_F$ to include $c_a \in \text{Const}_{\mathcal{L}'_F}$ without altering $\parallel P \parallel_{\mathcal{M}_F}$; all that the expansion does is give a name to a (potentially) unnamed object without altering the interpretation of the $P \in \text{Pred}_{\mathcal{L}_F}$.

With this machinery, we can define the corresponding satisfaction-relation. It also suffices to expand the language by one individual-constant at a time as we quantify over *all* such expansions so that no element of $D$ gets missed.

**Definition 22** (Satisfaction $\models_\mathcal{F}$)

Let the Fregean satisfaction-relation $\mathcal{M}_F \models_\mathcal{F} \varphi$ for $\varphi \in \text{Form}_{\mathcal{L}_F}$ and $\mathcal{L}_F$-model $\mathcal{M}_F = \langle D, \parallel \cdot \parallel_{\mathcal{M}_F} \rangle$ be recursively defined as follows:

1. $\mathcal{M}_F \models_\mathcal{F} P(c_1, \ldots, c_n)$ iff $\langle \parallel c_1 \parallel_{\mathcal{M}_F}, \ldots, \parallel c_n \parallel_{\mathcal{M}_F} \rangle \in \parallel P \parallel_{\mathcal{M}_F}$;
2. $\mathcal{M}_F \models_\mathcal{F} c_1 = c_2$ iff $\parallel c_1 \parallel_{\mathcal{M}_F} = \parallel c_2 \parallel_{\mathcal{M}_F}$.
The definition is mostly standard. Given clauses (3)–(4), we can define the clauses for the remaining connectives in the usual way. In contrast to objectual quantification which interprets the quantifiers via variable assignments, here the quantifiers are interpreted substitutionally; instead of considering all the possible values for the variables, the base model $M_F$ satisfies a formula of the form $'\forall x \varphi' \text{ if all expansions } M_F'$ satisfy $'\varphi[c_a]'$ where the new constants $'c_a'$ are substituted for the variable $'x'$. By Definitions [20] [21] every element of the domain $D$ is considered so that the truth of $'\forall x \varphi'$ does not depend on the particular choice of $\text{Const}_{\mathcal{L}_F}$.

We can define a corresponding notion of logical consequence analogous to Definition [8], just substitute $'\mathcal{L}_F'$ for $'\mathcal{L}_A'$, $'M_F'$ for $'M_m'$, and $'|=F'$ for $'|=L$'. With that at hand, one peculiarity of the above is the following.

**Theorem 23**
Let $P \in \text{Pred}_{\mathcal{L}_F}$. Then: $|=F \exists x P(x)$.

*Proof.* Let $P \in \text{Pred}_{\mathcal{L}_F}$. Let $M_F = \langle D, \parallel \parallel \parallel \parallel \rangle$ be an $\mathcal{L}_F$-model. By Definition [19] (2b), $0 \neq \parallel P \parallel_{\mathcal{M}_F} \subseteq D$. Let $a \in \parallel P \parallel_{\mathcal{M}_F}$. Let $L_F'$ be an $\mathcal{L}_F$-$a$-expansion of $\mathcal{L}_F$, and $M_F'$ be an $a$-expansion of $M_F$ to $L_F'$. By Definition [21] (2), $\parallel P \parallel_{\mathcal{M}_F} \subseteq \parallel P \parallel_{\mathcal{M}'_F}$ so that $a \in \parallel P \parallel_{\mathcal{M}'_F}$. By Definition [21] (3), $\parallel c_a \parallel_{\mathcal{M}'_F} = a \in \parallel P \parallel_{\mathcal{M}'_F}$. Thus, by Definition [22] (1), $M_F' |=F P(c_a)$. Therefore, by Definition [22] (3), $M_F |=F \exists x P(x)$.

This also means that universal quantification implies the existential one.

**Corollary 24**
$\forall x P(x) |=F \exists x P(x)$.

We can also note that the quantifiers behave as expected.

**Theorem 25**
The following equivalences hold:

(1) $|=F \forall x \varphi \leftrightarrow \neg \exists x \neg \varphi$;  
(2) $|=F \exists x \varphi \leftrightarrow \forall x \neg \varphi$;
(3) $|=F \neg \exists x \varphi \leftrightarrow \forall x \neg \varphi$;  
(4) $|=F \neg \forall x \varphi \leftrightarrow \exists x \neg \varphi$.

Furthermore, because of the non-emptiness requirement in Definition [19] (2b), analogues of conversion hold.
**Theorem 26 (Conversion)**

The following conversions hold:

\[ (a\text{-conv}) \forall x(A(x) \rightarrow B(x)) \models F \exists x(B(x) \land A(x)) \]

\[ (i\text{-conv}) \exists x(A(x) \land B(x)) \models F \exists x(B(x) \land A(x)) \]

\[ (e\text{-conv}) \forall x(A(x) \rightarrow \neg B(x)) \models F \forall x(B(x) \rightarrow \neg A(x)) \]

**Proof.** I only show the interesting case.

**\(a\text{-conv} \)**: Let \(M_F\) be an \(L_F\)-model such that \(M_F \models F \forall x(A(x) \rightarrow B(x))\). Then, by Definition 22 (5), for all \(a\)-expansions \(M'_F\) of \(M_F\), \(M'_F \models F A(c_a) \rightarrow B(c_a)\). By Definition 19 (2b), \(\emptyset \neq \|A\|_{M'_F}\). Let \(a \in \|A\|_{M'_F}\). Then, for the \(a\)-expansion \(M'_F\) of \(M_F\), \(M'_F \models F A(c_a) \rightarrow B(c_a)\). By Definition 21 (2), \(a \in \|A\|_{M'_F} \subseteq \|A\|_{M_F}\), i.e., \(a \in \|A\|_{M_F}\). Thus, for the \(a\)-expansion \(M'_F\) of \(M_F\), \(M'_F \models F A(c_a)\). Also, for the \(a\)-expansion \(M'_F\) of \(M_F\), \(M'_F \models F B(c_a)\). Therefore, for the \(a\)-expansion \(M'_F\) of \(M_F\), \(M'_F \models F B(c_a) \land A(c_a)\). By Definition 22 (5), \(M_F \models F \exists x(B(x) \land A(x))\).

This much suffices in terms of exposition of a Fregean language and its semantics. As we are only interested in semantics, there is no need to introduce a proof system.

## 4 Ben-Yami’s QUARC

### 4.1 Background

In recent years, Hanoch Ben-Yami has introduced a novel logical system called the *Quantified Argument Calculus* (QUARC). The underlying motivation is to find a formal system that captures more adequately the semantics of natural language. Section 3.1 already suggests that Frege’s main motivation is not to come up with a formal language to capture the semantics of natural language; however, the elegance and strength of his formal language surpassed anything else known and so was a natural candidate to be used outside its original intended range of application.

Ben-Yami introduces an early version of QUARC in his book *Logic & Natural Language* (2004)—which is the main focus of this brief exposition. Note, though, that certain of Ben-Yami’s views have developed and changed since the book was published in 2004; my concern here is not to

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105Hanoch prefers ‘logic of natural language’ (personal communication). I stick to ‘semantics’: Where it is clear to me that natural language has a semantics (and, potentially, several), it is less clear to me that it has a logic. My views are not settled, but am inclined to deny that there is the logic of natural language.
paint an accurate picture of his current views (for some of those see Yin and Ben-Yami 2023), though I mention some in footnotes.

Since then, he published an article exposing QUARC (2014), and considered how it treats the Barcan formulas and necessary existence (2020a) as well as how QUARC compares to natural logic (2020b). There have also been discussions with respect to generalized quantifiers (Ben-Yami 2009, 2012, and Westerståhl 2012).

Moreover, QUARC’s logical properties have been investigated. Lanzet and Ben-Yami (2004) provide an early assessment in model-theoretic terms, Raab (2016) consider QUARC’s relationship to classical logic and so does Lanzet (2017) in a three-valued setting. There are completeness results for the QUARC in different settings (e.g., Lanzet and Ben-Yami 2004, Raab 2016, Ben-Yami and Pavlović 2022), and treatments based on many-valued truth-valuational semantics (Yin and Ben-Yami 2023).

QUARC has also been investigated proof-theoretically (Pavlović 2017, Pavlović and Gratzl 2019a, 2019b, 2021a) as well as axiomatically (Pasquucci 2023). Moreover, Pavlović and Gratzl also consider abstract forms of quantification within QUARC (2023a) and investigate into decidable fragments (2023b). Several further aspects of QUARC are currently investigated.

Ben-Yami (2004) rejects Fregean languages when investigating the semantics of natural language. He suggests two main reasons, viz., the treatment of reference and quantification. I do not go into detail with all the subtleties, but focus on some general points.

Regarding reference, Ben-Yami notes that natural language contains plural referring expressions. Fregean languages, on the other hand, only allow singular reference. In the Fregean languages, this is achieved solely via the variables and individual-constants. Thus, as detailed in Section 3.1, a sentence like ‘All human beings are mortal’ is captured as ‘∀x(H(x) → M(x))’, quantifying singularly over everything. However, the surface structure of the sentence sees ‘all human beings’ as the subject of the sentence and ‘human beings’ refers plurally to human beings while ‘all’ specifies the relevant quantity of what’s being referred to.

Based on the treatment of reference as singular, Ben-Yami (2004, 2) also argues that Fregean languages misconstrue predication and quantification in natural language. Ben-Yami (2004, 8) suggests that Fregean languages understand singular terms to be the sole source of reference, and common nouns as logical predicates. Ben-Yami (2004, 8), on the other hand, argues that common nouns are used to refer to (pluralities of) particulars too. Given this understanding, he claims to arrive, among others, at a “radically different analysis of quantification” (2004, 12).

Ben-Yami’s main point is that “quantification involves reference to a

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106 Hanoch’s current views changed regarding reference which dropped out of the picture; indeed, he insists that the notion of reference is irrelevant for QUARC (personal communication).
 plurality” (2004: 59). In the example sentence above, ‘human beings’ refers to a plurality of human beings, and the quantifier ‘all’ specifies how much of that plurality is relevant, i.e., a “quantifier is attached to a noun that is used to refer to a plurality” (2004: 59f.) and these elements together “form a noun phrase” (2004: 60). Such noun phrases—called quantified arguments—can function as subjects of sentences; they can be put in the argument places of predicates. In the example sentence, ‘mortal’ is predicated of ‘all human beings’, i.e., the quantified argument ‘all human beings’ is put into the argument-slot of the predicate ‘mortal’. Ben-Yami (2004: 62) claims that this is in agreement with Aristotle’s understanding of predication.

One topic of concern for Ben-Yami is that of the expressive power of systems. Ben-Yami (2004: 78) notes that Aristotelian logic is not expressive enough to handle relations and nested quantification. His goal is to develop a system that is able to handle these, and suggests that “[a]ny alternative logic should have comparable power” (2004: 78) to Fregean logic with its predicate calculus. The QUARC is meant to have that.

In order to achieve that, QUARC needs a device to establish cross-reference; it captures it by the incorporation of anaphora. Moreover, natural language contains active and passive constructions, and different ways of negating, viz., negating a whole sentence (‘it is not the case that Socrates is mortal’) and negative predication (‘Socrates is not mortal’); all this is incorporated in the QUARC too. The QUARC also includes identity, though it treats it slightly different from the way it is in Fregean languages, as predication is understood differently (Ben-Yami 2004: 142). All these elements are incorporated in the formalism below.

4.2 A Formalism

Let me make the QUARC formalism precise. I generally follow the exposition of Section 3.2 and comment only on the QUARC-specific details of the formalism.

First, again, let’s specify the underlying vocabulary.

**Definition 27 (QUARC-Language)**

A QUARC-language ($\mathcal{L}_Q$) consists of the following:

- a countably infinite set $\text{Ana}_{\mathcal{L}_Q} = \{\alpha_0, \alpha_1, \alpha_2, \ldots\}$ of anaphors,
- a countable set $\text{SA}_{\mathcal{L}_Q} = \{s_0, s_1, s_2, \ldots\}$ of singular arguments,
- for every $n > 0$, a countable set $\text{Pred}_n^\mathcal{L}_Q = \{P_0^{1\ldots n}, P_1^{1\ldots n}, P_2^{1\ldots n}, \ldots\}$ of $n$-ary predicate-symbols,
- for every $n > 0$, for every $i \geq 0$, for every $P_i^{1\ldots n} \in \text{Pred}_n^\mathcal{L}_Q$, a set $\text{Reord}_n^\mathcal{L}_Q = \{P_i^{\pi(1)\ldots\pi(n)} | \pi: \{1, \ldots, n\} \to \{1, \ldots, n\} \text{ a permutation}\}$ of $n$-ary reorders,
• the set of **logical symbols** including ‘¬’, ‘∧’, ‘∨’, ‘→’, ‘↔’, ‘=’, ‘∀’, and ‘∃’, and

• the set of **auxiliary symbols** including ‘(’, ‘)’, and ‘,’.

For every \( n \geq 1 \), \( \text{Pred}^n_Q \subseteq \text{Reord}^n_Q \); all other sets are assumed to be disjoint. Let \( \text{Pred}_Q := \bigcup_{n>0} \text{Pred}^n_Q \) and \( \text{Reord}_Q := \bigcup_{n>0} \text{Reord}^n_Q \).

Compared to Definition 17, Definition 27 is more complex. Firstly, what’s analogous to the Fregean language, a QUARC-language contains **anaphors** which play a similar role to the **variables** of Fregean languages. However, Fregean languages need variables to achieve quantification, QUARC does not as witnessed by formulas of the form ‘(∀P)Q’ for \( P, Q \in \text{Pred}^n_Q \).

Moreover, the QUARC-language contains **singular arguments** which correspond to Fregean (individual-)constants. The logical and auxiliary symbols of the languages are the same. However, QUARC specifies its predicates differently. In particular, I put the members of \( \text{Pred}^n_Q \) as ‘\( P_1^{1 \ldots n} \)’, not just indicating the arity \( n \), but also the order of the slots. The reason for this is that this guarantees that they are identical to reorderings. For any \( P \in \text{Pred}^n_Q \), there are \( n! \)-many reorders, generated by permutations on the predicate’s argument-places. However, I just write ‘\( P^n \)’ instead of ‘\( P^{\pi(1) \ldots \pi(n)} \)’ (\( P \in \text{Pred}^n_Q \)) to indicate the reorder if it is relevant.

Since \( \text{Pred}^n_Q \subseteq \text{Reord}^n_Q \), we can often work with the latter in setting up the formalism; this helps reducing some complexity in specifying the QUARC-formulas.

**Definition 28 (\( \mathcal{L}_Q \)-Formula)**

Let \( \mathcal{L}_Q \) be a QUARC-language. The set of \( \mathcal{L}_Q \)-formulas (Form\( _{\mathcal{L}_Q} \)) is recursively defined by:

1. given \( n > 0 \) \( s_1, \ldots, s_n \in \text{SA}^n_Q \) and \( P \in \text{Reord}^n_Q \), then \( (s_1, \ldots, s_n)P \in \text{Form}^n_Q \);  
2. given \( s_1, s_2 \in \text{SA}^n_Q \), \( (s_1, s_2) = \in \text{Form}^n_Q \) (usually written as ‘\( (s_1 = s_2) \)’);  
3. given \( n > 0 \), \( s_1, \ldots, s_n \in \text{SA}^n_Q \), \( P \in \text{Reord}^n_Q \), and \( * \) a string of negation-symbols ‘¬’, \( (s_1, \ldots, s_n)P \in \text{Form}^n_Q \);  
4. if \( \varphi \in \text{Form}^n_Q \), then ‘¬\( \varphi \)’ \( \in \text{Form}^n_Q \);  
5. if \( \varphi, \psi \in \text{Form}^n_Q \) and \( \circ \in \{ \land, \lor, \rightarrow, \leftrightarrow \} \), then ‘\( \varphi \circ \psi \)’ \( \in \text{Form}^n_Q \);  
6. if \( \varphi \in \text{Form}^n_Q \) contains, from left to right, \( s_1, \ldots, s_n \) (\( n \geq 2 \)) occurrences of \( s \in \text{SA}^n_Q \), none of which is the source of \( \beta \in \text{Ana}^n_Q \) that occurs in \( \varphi \), and \( \varphi \) does not contain \( \alpha \in \text{Ana}^n_Q \), then

\(^{107}\) Hanoch (personal communication) prefers to think of \( \pi \) as an **operator** acting on predicates \( P \in \text{Pred}^n_Q \) so that the predicate stays the same, but gets reordered.
φ[s_α/s_1, α/s_2, . . . , α/s_n] ∈ Form_{LQ} where φ[s_α/s_1, α/s_2, . . . , α/s_n] is the result of substituting α for the occurrences s_2, . . . , s_n of s;

(7) if φ[s] ∈ Form_{LQ}, q ∈ {∀, ∃}, P ∈ Pred^1_{LQ}, then φ[qP/s] ∈ Form_{LQ} if qP governs φ (see Definition 30).

Let QA_{LQ} be the set of quantified arguments, i.e., expressions of the form qP for q ∈ {∀, ∃} and P ∈ Pred^1_{LQ}.

Clauses (1)–(2) correspond to Definition 18’s (1)–(2); the only difference is that QUARC takes predicates from Reord_{LQ} and that we write the arguments in front of the predicate-symbol. Moreover, clauses (4)–(5) are standard too. Let me comment on the remaining clauses.

Clause (3) is QUARC-specific. It allows arbitrarily many negation-symbols inbetween a predicate-symbol’s argument-slots and predicate-sign. Thus, we allow, e.g., ‘(s)¬¬¬P’ as an LQ-formula.

Clause (6) allows for the introduction of anaphors. If a formula contains several occurrences of a singular argument s, we can replace all but the first by new anaphors. For example, we can move from ‘(s, s)P’ to ‘(s_α, α)P’. As long as no quantified arguments are involved, these anaphors are not necessary, but they are once cross-reference is needed.

Clause (7), finally, allows the introduction of quantified arguments, i.e., expressions combining quantifiers with unary predicates so that quantification is understood as plural. These expressions can replace singular arguments given that they satisfy a certain condition, viz., that the quantified argument governs the formula—which we define below. As the quantified arguments can take the place of a singular argument that has anaphors referring to it, we also define the notion source of anaphora.

**Definition 29 (Source of Anaphora)**
If an anaphor is introduced according to clause (6), then the term s is the source of α (indicated as ‘s_α’) if it is the rightmost occurrence of s that is to the left of the anaphor α; if such a term is replaced by a t ∈ QA_{LQ} due to an application of clause (7), then t is the source of α (indicated as ‘qP_α’ if t = qP).

**Definition 30 (Governance)**
Let φ be a string of symbols and t ∈ QA_{LQ}. Then, t governs φ if it is the leftmost quantified argument and φ does not contain any other string of symbols ψ such that ψ ∈ Form_{LQ} contains t and all the anaphors of all arguments in ψ.

Given Definition 30 Definition 28 (7) is well-defined now. Roughly, the idea is that we can introduce quantified arguments if they are the main symbol, i.e., when breaking up the formula, one has to start with it.

As in Section 3.2 all formulas are closed. The mechanism to introduce anaphors and quantified expressions is via substitution and so we
treat quantification substitutionally. This follows the treatment from Section 3.2. In particular, we use models to interpret QUARC-languages.

**Definition 31 (\(L_Q\)-Model)**

Let \(L_Q\) be a QUARC-language. A model for \(L_Q\) (\(L_Q\)-model) is an ordered pair \(M_Q = \langle D, \| \cdot \|_{M_Q} \rangle\) such that

1. \(D\) is a set (the domain of \(M_Q\));
2. \(\| \cdot \|_{M_Q}\) is an interpretation-function of \(M_Q\) such that
   a. if \(s \in SA_{L_Q}\), then \(\|s\|_{M_Q} \in D\);
   b. if \(P \in \text{Pred}_{L_Q}^1\), then \(\emptyset \neq \|P\|_{M_Q} \subseteq D\);
   c. if \(n > 1\) and \(P \in \text{Pred}_{L_Q}^n\), then \(\|P\|_{M_Q} \subseteq D^n\);
   d. if \(n \geq 1\) and \(P^{\pi} \in \text{Reord}_{L_Q}^n\) for permutation \(\pi: \{1, \ldots, n\} \rightarrow \{1, \ldots, n\}\), then \(\|P^{\pi}\|_{M_Q} = \{\langle \|s_1\|_{M_Q}, \ldots, \|s_n\|_{M_Q}\rangle \mid \|s_1\|_{M_Q}, \ldots, \|s_n\|_{M_Q} \in \|P\|_{M_Q}\}\).

This definition corresponds to Definition 19. The only QUARC-specific part is clause (2d) which interprets reorders in the obvious way. As a reorder \(P^{\pi} \in \text{Reord}_{L_Q}^n\) comes from reordering the argument-places of a predicate \(P \in \text{Pred}_{L_Q}^n\), the interpretation does the same.

As before, we do not want to be held hostage to the particular choice of what individuals the language can name, i.e., to the specific \(SA_{L_Q}\), so we expand the language (Definition 32), and specify the corresponding model expansions (Definition 33). With that, we can define the satisfaction-relation (Definition 34).

**Definition 32 (\(L_Q\)-A-Expansion)**

Let \(L_Q\) be a QUARC-language and \(M_Q = \langle D, \| \cdot \|_{M_Q} \rangle\) be an \(L_Q\)-model. Let \(A \subseteq D\). The \(L_Q\)-A-expansion of \(L_Q\) is the language \(L'_Q := L_Q \cup \{s_a \mid a \in A\}\) where the \(s_a\) are new singular arguments not contained in \(L_Q\).

If \(A = \{a\}\), we call \(L'_Q\) an \(L_Q\)-a-expansion.

**Definition 33 (\(L_Q\)-Model Expansion)**

Let \(L_Q\) be a QUARC-language and \(M_Q = \langle D, \| \cdot \|_{M_Q} \rangle\) an \(L_Q\)-model. Let \(A \subseteq D\), and \(L'_Q\) be an \(L_Q\)-A-expansion. The \(A\)-expansion of \(M_Q\) to \(L'_Q\) is the model \(M'_Q = \langle D', \| \cdot \|_{M'_Q} \rangle\) such that

1. \(D' = D\);
2. \(\| \cdot \|_{M'_Q} \subseteq \| \cdot \|_{M_Q}\);
3. \(\|s_a\|_{M'_Q} = a \in A\) for every new singular argument \(s_a\).

**Definition 34 (Satisfaction \(\models_{L_Q}\))**

Let the QUARC satisfaction-relation \(M_Q \models_{L_Q} \varphi\) for \(\varphi \in \text{Form}_{L_Q}\) and \(L_Q\)-model \(M_Q = \langle D, \| \cdot \|_{M_Q} \rangle\) be recursively defined as follows:
(1) $\mathcal{M}_Q \models \varphi \iff (s_1, \ldots, s_n)P \iff \langle \langle s_1\|\alpha_1, \ldots, s_n\|\alpha_n \rangle \in \|P\|_{\mathcal{M}_Q} \rangle 
\quad (P \in \text{Reord}^{\mathcal{L}_Q})$

(2) $\mathcal{M}_Q \models s_1 = s_2 \iff \|s_1\|_{\mathcal{M}_Q} = \|s_2\|_{\mathcal{M}_Q}$

(3) $\mathcal{M}_Q \models \neg \varphi \iff \text{it is not the case that }\mathcal{M}_Q \vdash \varphi$ \quad (where $\mathcal{M}_Q \not\vdash \varphi$);

(4) $\mathcal{M}_Q \models \varphi \wedge \psi \iff \mathcal{M}_Q \models \varphi$ \text{ and } $\mathcal{M}_Q \models \psi$;

(5) $\mathcal{M}_Q \models ((s_1, \ldots, s_n)\neg P) \iff (s_1, \ldots, s_n)'P')$ \text{ (where }$\neg$, $'$ \text{ are possibly empty strings of negation-symbols})

(6) $\mathcal{M}_Q \models \varphi[\alpha_1/s_1, \alpha_2/s_2, \ldots, \alpha_n/s_n] \iff \mathcal{M}_Q \models \varphi$

(7) $\mathcal{M}_Q \models \varphi[\exists P\alpha] \iff \text{for some } a\text{-expansion }\mathcal{M}'_Q \text{ of }\mathcal{M}_Q \text{ such that }a \in \|P\|_{\mathcal{M}_Q}, \mathcal{M}'_Q \models \varphi[(s_1)\alpha/\exists P\alpha]$ \quad (where $\exists P$ governs $\varphi$ and is the source of $\alpha \in \text{An}a_{\mathcal{L}_Q}$ if there is one);

(8) $\mathcal{M}_Q \models \varphi[\forall P\alpha] \iff \text{for all } a\text{-expansions }\mathcal{M}'_Q \text{ of }\mathcal{M}_Q \text{ such that }a \in \|P\|_{\mathcal{M}_Q}, \mathcal{M}'_Q \models \varphi[(s_1)\alpha/\forall P\alpha]$ \quad (where $\forall P$ governs $\varphi$ and is the source of $\alpha \in \text{An}a_{\mathcal{L}_Q}$ if there is one).

Since the QUARC-models are pretty much the same as the Fregean-models, the satisfaction-relation is quite similar too. Clauses (1)–(4) correspond to Definition 22’s (1)–(4). The remaining clauses are QUARC-specific.

Clause (5) concerns predicate-negation. As long as no quantified arguments occur in a formula, we just move the negation-symbols from the predicate-negation into sentence-negation; the resulting formulas are in the range of clause (3).

Clause (6) concerns anaphora. As the anaphors are just referring to whatever their source refers to, we interpret them accordingly. That is, as long as no quantified arguments occur, they refer to what their source singular argument refers. That is, a model satisfies it in exactly the same circumstances as when they are replaced by their source.

Clauses (7)–(8) concern the quantified arguments. The general idea is the same as it was in the case of Fregean languages, i.e., as specified in Definition 22’s (1)–(6). However, as QUARC does not allow unrestricted quantification, we have to restrict the expansions in consonance with the quantified argument, consisting of a quantifier and unary predicate. Thus, instead of considering some or all $a$-expansions, we only consider those such that $a$ is an element of the interpretation of the restricting unary predicate. If $qP \in QA_{\mathcal{L}_Q}$, we only consider those $a \in D$ such that $a \in \|P\|_{\mathcal{M}_Q}$, i.e., if for some (all) of these the expanded model $\mathcal{M}'_Q$ satisfies a formula $\varphi$, then the base model $\mathcal{M}_Q$ satisfies the formula involving the quantified argument $\exists P$ ($\forall P$), i.e., it satisfies that some $P$ (all $P$) satisfy the formula.
Given the QUARC-language, its models, and the satisfaction-relation, we can define logical consequence etc. as in Definition 8 and obtain the QUARC-specific results below.

**Theorem 35**
\[ \models_Q (\exists P)P. \]

*Proof.* Let \( \mathcal{M}_Q \) be an \( \mathcal{L}_Q \)-model. By Definition 31 (2b), \( \emptyset \neq \| P \|_{\mathcal{M}_Q} \subseteq D \) for \( P \in \text{Pred}_{\mathcal{L}_Q} \). Let \( a \in \| P \|_{\mathcal{M}_Q} \), and let \( \mathcal{L}'_Q \) be the \( \mathcal{L}_Q \)-a-expansion of \( \mathcal{L}_Q \). Let \( \mathcal{M}'_Q \) be the \( a \)-expansion of \( \mathcal{M}_Q \) to \( \mathcal{L}'_Q \). Then, \( \mathcal{M}'_Q \models_Q (s_a)P \) since \( \| s_a \|_{\mathcal{M}'_Q} = a \in \| P \|_{\mathcal{M}_Q} \subseteq \| P \|_{\mathcal{M}_Q} \) by Definition 33 (2)–(3). Therefore, by Definition 34 (7), \( \mathcal{M}_Q \models_Q (\exists P)P. \)

**Theorem 36**
\[ \( \forall P)Q \models_Q (\exists P)Q. \]

*Proof.* Let \( \mathcal{M}_Q \) be an \( \mathcal{L}_Q \)-model such that \( \mathcal{M}_Q \models_Q (\forall P)Q \). By Definition 34 (8), for all \( a \)-expansions \( \mathcal{M}'_Q \) of \( \mathcal{M}_Q \) such that \( a \in \| P \|_{\mathcal{M}_Q} \), \( \mathcal{M}'_Q \models_Q (s_a)Q \). Moreover, by Definition 31 (2b), \( \| P \|_{\mathcal{M}_Q} \neq \emptyset \). Thus, there is an \( a \)-expansion \( \mathcal{M}'_Q \) of \( \mathcal{M}_Q \) such that \( a \in \| P \|_{\mathcal{M}_Q} \), \( \mathcal{M}'_Q \models_Q (s_a)Q \). Then, by Definition 34 (7), \( \mathcal{M}_Q \models_Q (\exists P)Q. \)

The quantifiers still behave as one would expect them to:

**Theorem 37**
The following equivalences hold:

1. \[ \models_Q (\forall P)S \iff \neg ((\exists P)\neg S); \]
2. \[ \models_Q (\exists P)S \iff (\forall P)\neg S; \]
3. \[ \models_Q (\exists P)S \iff (\forall P)\neg S; \]
4. \[ \models_Q (\forall P)S \iff (\exists P)\neg S. \]

*Proof.* I only illustrate part of one case:

3: Let \( \mathcal{M}_Q \models_Q \neg (\exists P)S \). Then, by Definition 34 (3), \( \mathcal{M}_Q \not\models_Q (\exists P)S \), i.e., by (7), it is not the case that for some \( a \)-expansion \( \mathcal{M}'_Q \) of \( \mathcal{M}_Q \) such that \( a \in \| P \|_{\mathcal{M}_Q} \), \( \mathcal{M}'_Q \models_Q (s_a)S \) iff for all \( a \)-expansions \( \mathcal{M}'_Q \) of \( \mathcal{M}_Q \) such that \( a \in \| P \|_{\mathcal{M}_Q} \), \( \mathcal{M}'_Q \not\models_Q (s_a)S \), i.e., by [3], for all \( a \)-expansions \( \mathcal{M}'_Q \) of \( \mathcal{M}_Q \) such that \( a \in \| P \|_{\mathcal{M}_Q} \), \( \mathcal{M}'_Q \models_Q \neg (s_a)S \), and so, by (8), for all \( a \)-expansions \( \mathcal{M}'_Q \) of \( \mathcal{M}_Q \) such that \( a \in \| P \|_{\mathcal{M}_Q} \), \( \mathcal{M}'_Q \models_Q (s_a)\neg S \). Thus, by (8), \( \mathcal{M}_Q \models_Q (\forall P)\neg S. \)

QUARC also validates the conversions.

**Theorem 38 (Conversion)**
The following conversions hold:

\[ (a \text{-i-conv}_{\text{\models}_Q}) \quad (\forall A)B \models_Q (\exists B)A \]
Theorem 39 generalizes to cases including quantified arguments. Applied repeatedly, we get that if ‘*’ contains an even number of negation-symbols, then ‘((s₁, ..., sₙ) * P)’ is equivalent to ‘((s₁, ..., sₙ) P)’ and if it contains an odd number, it is equivalent to ‘((s₁, ..., sₙ)¬P)’, and so, by Definition 34 (5), to ‘¬(s₁, ..., sₙ)P’.

This finishes the exposition of QUARC.

5 Sommers’s Term Logic

5.1 Background

Fred Sommers is also not satisfied with the common approach to the semantics of natural language. He develops his Term Functor Logic (TFL)
as an alternative approach. In this brief exposition, I focus on his book *The Logic of Natural Language* [1982], and only consider a few points that suggest themselves for comparison here (for a nice exposition, see Englebretsen [2016]).

Sommers’s conviction is that traditional formal logic is especially suited to the task of making perspicuous the logical form of sentences in the natural languages that are actually used in deductive reasoning and that, in virtue of this, traditional logic provides models for the study of what actually happens when we reckon the premisses and arrive at conclusion. (1982: 4)

Given that we generally reason in natural language, traditional formal logic is in a better position to make explicit how we do so; Fregean languages, with their machinery, rather distort this. In this context, Sommers emphasizes that the

traditional logician emphasized syntactic simplicity, requiring of a canonical sentence that it have a straightforward noun-phrase verb-phrase structure (or be a compound of such ‘categorical’ sentences). (1982: 9)

The simple noun-phrase verb-phrase structure can be found in Aristotle’s logic, though needs to be extended to overcome the syllogistic’s shortcomings. Indeed, Sommers is concerned in constructing a language that is similarly powerful as Fregean languages while maintaining the basic analysis of sentences.

The basic analysis is into noun-phrase and verb-phrase; both are considered to be terms. Additionally, the noun-phrase as well as all other subject expressions are assigned a quantity (1982: 67). The general form of a sentence is then ‘every/some S is (are)/is (are) not P’ where ‘every/some’ is the quantity of the subject S (e.g., 1982: 95).

In order to increase the expressive power, Sommers introduces proterms and allows complex terms. As in Aristotle’s logic, terms can play the role of both subject and predicate in sentences (1982: 116). Moreover, Sommers also allows n-ary terms (1982: 139), construed in a way so that the subject-predicate structure remains via nesting them (1982: 148). As terms can play several roles, Sommers (1982: 116f.) argues that there is no need to include identity in the way Fregean languages do. This also means that TFL is more parsimonious than Fregean languages are with respect to their primitive symbols.

Overall, Sommers claims that his TFL, already in a more basic form which he calls ‘Primitive Term Logic’ (PTL), is

roughly equivalent to that of a standard first-order logic whose logical particles consist of the existential quantifier and the signs for conjunction, negation and identity. (1982: 174)
He goes on to amplify PTL to full TFL. However, for the purposes of comparing the systems, I stick to the more basic system, though even depart from Sommers’s presentation and particular claims regarding it. Moreover, I continue the model-theoretic approach which is significantly different from Sommers’s algebraic treatment of term logic.

5.2 A Formalism

**Definition 40 (TFL-Language)**

A TFL-language \( \mathcal{L}_T \) consists of the following:

- a countably infinite set \( \text{PTerm}_{\mathcal{L}_T} = \{ \alpha_0, \alpha_1, \alpha_2, \ldots \} \) of proterms,
- a countable set \( \text{ITerm}_{\mathcal{L}_T} = \{ t_0, t_1, t_2, \ldots \} \) of individual-terms,
- for every \( n > 0 \), a countable set \( \text{STerm}^n_{\mathcal{L}_T} = \{ T^n_0, T^n_1, T^n_2, \ldots \} \) of (simple) \( n \)-ary term-symbols,
- the set of logical symbols including ‘\( \neg \)’, ‘\( \rightarrow \)’, ‘\( \forall \)’, and ‘\( \exists \)’, and
- the set of auxiliary symbols including ‘(’, ‘)’, and ‘, ’.

All the sets are assumed to be disjoint. Let \( \text{STerm}_{\mathcal{L}_T} := \bigcup_{n>0} \text{STerm}^n_{\mathcal{L}_T} \cup \text{ITerm}_{\mathcal{L}_T} \).

The TFL-language \( \mathcal{L}_T \) is different from the one Sommers actually uses, and changes certain aspects. What’s left are proterms \( \text{PTerm}_{\mathcal{L}_T} \) which play a similar role to Fregean variables and QUARC-anaphora. The language does not contain anything like individual-constants or singular arguments, but only terms. One kind of term are the individual-terms \( \text{ITerm}_{\mathcal{L}_T} \)—playing a similar role as individual-constants—another \( n \)-ary terms \( \text{STerm}^n_{\mathcal{L}_T} \). Similar to the language \( \mathcal{L}_A \) and in contrast to \( \mathcal{L}_F \) and \( \mathcal{L}_Q \), \( \mathcal{L}_T \) does not include an identity-symbol ‘\( = \)’ among its logical symbols, but includes a second negation-symbol ‘\( \neg \)’ which figures in the introduction of complex terms.

One important difference to Definition 1 of \( \mathcal{L}_A \) is that \( \mathcal{L}_T \) includes \( n \)-ary term. These are necessary to capture relational predications that Aristotle’s syllogistic misses.

Given this basic vocabulary, we can introduce the complex terms.

**Definition 41 (Complex \( \mathcal{L}_T \)-Terms)**

For each \( n > 0 \), the set of complex \( n \)-ary \( \mathcal{L}_T \)-terms \( \text{CTerm}^n_{\mathcal{L}_T} \) is recursively defined as follows:

1. if \( t \in \text{ITerm}_{\mathcal{L}_T} \), then \( t \in \text{CTerm}^1_{\mathcal{L}_T} \);
2. if \( A \in \text{STerm}^n_{\mathcal{L}_T} \), then \( A \in \text{CTerm}^n_{\mathcal{L}_T} \);
3. if \( A \in \text{CTerm}^n_{\mathcal{L}_T} \), then \( \overline{A} \in \text{CTerm}^n_{\mathcal{L}_T} \);
(4) if \( A, B \in \text{CTerm}_{\mathcal{T}}^n \), then \((A \circ B) \in \text{CTerm}_{\mathcal{T}}^n\) (\(\circ \in \{\land, \lor, \to, \leftrightarrow\}\));

(5) if \(1 \leq i \leq n - 1\), \(t_1, \ldots, t_i \in \text{ITerm}_{\mathcal{T}}^n\), \(q_1, \ldots, q_i \in \{\forall, \exists\}\), and \(A \in \text{CTerm}_{\mathcal{T}}^n\), then \((q_1t_1, \ldots, q_it_i, \ldots, -i+1, \ldots, -n)A \in \text{CTerm}_{\mathcal{T}}^{n-i}\) and so are all ways of putting the \(i\) terms into the \(n\) slots of \(A\) (where \(\_k\) indicates the \(k\)th argument-slot of \(A\), \(1 \leq k \leq n\));

(6) if \(A \in \text{CTerm}_{\mathcal{T}}^n\) and \(\pi: \{1, \ldots, n\} \to \{1, \ldots, n\}\) is a permutation, then \(A^{\pi} \in \text{CTerm}_{\mathcal{T}}^n\) where \(\_A^{\pi}\) is the result of permuting \(A\)'s slots according to \(\pi\).

Terms generated by clause (5) are called \textit{n-ary reduced terms} (\(\text{RTerm}_{\mathcal{T}}^n\)). Let \(\text{RTerm}_{\mathcal{T}} := \bigcup_{n \geq 1} \text{RTerm}_{\mathcal{T}}^n\).

Clauses (2)-(4) are analogous to the clauses (1)-(3) of Definition 2 of \(\text{Term}_{\mathcal{L}}\), just generalized from only \textit{unary} terms to \textit{n-ary} terms. These allow to capture relational predications in more complex settings. Moreover, clause (1) includes the individual terms among the \textit{unary} complex terms.

Clause (5) additionally allows to form further terms, reducing an \textit{n-ary} term \(A\) to an \textit{m-ary} term \(B\) by filling up slots with elements from \(\text{ITerm}_{\mathcal{T}}^n\).

In the spirit of TFIL, each term is assigned a \textit{quantity}. However, as the particular quantity does not make a difference for the individual-terms, both \(\forall\) and \(\exists\) are allowed as quantities.

Note, too, that clause (5) also sticks to the QUARC convention to place the argument-places to the left of the term symbol.

Clause (6), finally, allows for \textit{reordered} terms analogous to QUARC’s reorderings in Definition 27. The clause allows to reorder reorderings, but it is clear that there are only \(n!\)-many different ones. For example, \('(-1, -2)A'\) only leads to \('(-2, -1)A^{\pi}\)' as \(A^\pi = A\).

Given the vocabulary and the set of terms, we can define what counts as formula in a way mirroring Definition 28 of \(\text{Form}_{\mathcal{L}}\).

**Definition 42 (\(\mathcal{L}_T\)-Formula)**

Let \(\mathcal{L}_T\) be a TFIL-language. The \textit{set of \(\mathcal{L}_T\)-formulas} (\(\text{Form}_{\mathcal{T}}\)) is recursively defined by:

1. if \(n \geq 1\), \(A \in \text{CTerm}_{\mathcal{T}}^n\), \(t_1, \ldots, t_n \in \text{ITerm}_{\mathcal{T}}^n\), \(q_1, \ldots, q_n \in \{\forall, \exists\}\), and * a possibly empty string of negation-symbols \(\neg\), then 
   \(((q_1t_1, \ldots, q_nt_n) \ast A) \in \text{Form}_{\mathcal{T}}\);
2. if \(\varphi \in \text{Form}_{\mathcal{T}}\), then \(\neg \varphi \in \text{Form}_{\mathcal{T}}\);
3. if \(\varphi, \psi \in \text{Form}_{\mathcal{T}}\), then \((\varphi \circ \psi) \in \text{Form}_{\mathcal{T}}\) (\(\circ \in \{\land, \lor, \to, \leftrightarrow\}\));
4. if \(\varphi \in \text{Form}_{\mathcal{T}}\) contains, from left to right, \(t_1, \ldots, t_m\) (\(m \geq 2\)) occurrences of \(\ell \in \text{ITerm}_{\mathcal{T}}\), none of which is the source of \(\beta \in \text{PTerm}_{\mathcal{T}}\), that occurs in \(\varphi\), and \(\varphi\) does not contain \(\alpha \in \text{PTerm}_{\mathcal{T}}\), then \(\varphi[t_{m}/t_1, \alpha/t_2, \ldots, \alpha/t_m] \in \text{Form}_{\mathcal{T}}\) (which is the result of substituting \(\alpha\) for the occurrences \(t_2, \ldots, t_m\) of \(\ell\));
Definition 43

The interpretation of terms is mirrored by Definition 41. Indeed, government in clause (5) is to be understood analogous to Definition 42.

Moreover, an analogue of Definition 29 applies to the proterms in clause (6) and once the $t \in \mathcal{L}_\mathcal{T}$ gets substituted by $A \in \text{CTerm}^1_{\mathcal{T}}$. Also, we collapsed Definition 28 (1) and (3) into one clause (1).

What’s TFL-specific in Definition 42 is the assignment of quantities to all terms. Thus, the basic formulas are $n$-ary terms applying to $n$ individual-terms $t_i \in \mathcal{I}$, while assigning them a quantity, i.e., one of the quantifiers. As these terms are such that the particular quantifier does not make a difference, both are allowed. Definition 42 does not introduce wild quantities, but just assigns both quantities and the rest will be taken care by the interpretation.

Moreover, we allow for the usual combination of sentences via clauses (2)–(3). Terms for which the quantity makes a difference are only introduced in the last clause (5), and they always replace individual terms for which they are substituted—and this includes individual terms used in the reduced terms $R \in \mathcal{R}$.

As before, all formulas are closed, i.e., sentences. The complexity introduced by Definition 41 is mirrored in the interpretation of terms.

**Definition 43 (L_T-Model)**

Let $\mathcal{L}_\mathcal{T}$ be a TFL-language. An $\mathcal{L}_\mathcal{T}$-model is a tuple $\mathcal{M}_\mathcal{T} = \langle D, || \cdot ||_{\mathcal{M}_\mathcal{T}} \rangle$ such that

1. $D$ is a set (the universe)
2. $|| \cdot ||_{\mathcal{M}_\mathcal{T}}$ is an interpretation-function of $\mathcal{M}_\mathcal{T}$ such that
   a. if $t \in \mathcal{I}$, then $|| t ||_{\mathcal{M}_\mathcal{T}} = \{a\}$ for an $a \in D$;
   b. if $n = 1$ and $A \in \text{STerm}^n_{\mathcal{T}}$, then $\emptyset \neq || A ||_{\mathcal{M}_\mathcal{T}} \subseteq D$;
   c. if $n > 1$ and $A \in \text{STerm}^n_{\mathcal{T}}$, then $|| A ||_{\mathcal{M}_\mathcal{T}} \subseteq D^n$;
   d. if $n > 0$ and $A \in \text{CTerm}^n_{\mathcal{T}}$ is of the form $\neg B$ for a $B \in \text{CTerm}^n_{\mathcal{T}}$, then $|| A ||_{\mathcal{M}_\mathcal{T}} = \{ \langle a_1, \ldots, a_n \rangle \in D^n | \langle a_1, \ldots, a_n \rangle \notin || B ||_{\mathcal{M}_\mathcal{T}} \}$;
   e. if $n > 0$ and $A \in \text{CTerm}^n_{\mathcal{T}}$ is of the form $(B \circ C)$ for $B, C \in \text{CTerm}^n_{\mathcal{T}}$, then $|| A ||_{\mathcal{M}_\mathcal{T}} = \{ \langle a_1, \ldots, a_n \rangle \in D^n | \langle a_1, \ldots, a_n \rangle \in || B ||_{\mathcal{M}_\mathcal{T}} \circ \langle a_1, \ldots, a_n \rangle \in || C ||_{\mathcal{M}_\mathcal{T}} \} (\circ \in \{ \land, \lor, \rightarrow, \iff \})$;
   f. if $n > 0$ and $A \in \mathcal{R}$ stemming from $B \in \text{CTerm}^m_{\mathcal{T}}$ ($m > n$), $i = m - n$ individual terms $t_1, \ldots, t_i \in \mathcal{I}$ and $q_1, \ldots, q_i \in \{ \forall, \exists \}$ such that $A$ is of the form $(q_1 t_1, \ldots, q_i t_i, \ldots)_B$, then $|| A ||_{\mathcal{M}_\mathcal{T}} = \{ \langle a_1, \ldots, a_n \rangle \in D^n | \bigcup || t_i ||_{\mathcal{M}_\mathcal{T}}, a_1, \ldots, a_n \rangle \in || B ||_{\mathcal{M}_\mathcal{T}} \} $; similarly for all other ways of generating an $A \in \mathcal{R}$.
(g) if \( n > 0 \) and \( A^\pi \in \text{CTerm}_{\mathcal{L}_T}^n \) for permutation \( \pi \), then \( \| A^\pi \|_{\mathcal{M}_T} = \{ \langle \| t_{\pi(1)} \|_{\mathcal{M}_T}, \ldots, \| t_{\pi(n)} \|_{\mathcal{M}_T} \rangle \in D^n \| \| t_1 \|_{\mathcal{M}_T}, \ldots, \| t_n \|_{\mathcal{M}_T} \}) \in \| A \|_{\mathcal{M}_T} \} \).

The \( \mathcal{L}_T \)-models \( \mathcal{M}_T \) are similar to the models seen so far. However, similar to the \( \mathcal{L}_A \)-models \( \mathcal{M}_A \), they have to take care of the interpretation of the complex terms.

Clause (2a) interprets individual terms as terms, i.e., as a set; they are individual as the sets are singletons.

In line with how I introduced it before, unary terms are interpreted by non-empty sets. The reason is again to facilitate comparison with QUARC.

Complex \( n \)-ary terms are interpreted analogous to how \( \mathcal{L}_A \)-models \( \mathcal{M}_A \) interpreted complex unary terms; clause (2c) just generalizes from unary to \( n \)-ary terms, i.e., from subsets of the domain to \( n \)-ary relations on the domain.

Clause (2f) interprets the reduced terms. These are \( n \)-ary terms generated out of \( m \)-ary terms \( (m > n) \) by filling up slots with individual terms. These individual terms have quantities assigned, though as they are individual, the quantity does not make a difference. Thus, they are simply interpreted as \( \bigcup \| t \|_{\mathcal{M}_T} \), \( t \in \text{ITerm}_{\mathcal{L}_T} \). If \( \| t \|_{\mathcal{M}_T} = \{ a \} \), \( \| t \|_{\mathcal{M}_T} = a \).

The last clause (2g) is analogous to Definition 31 (2d), i.e., it interprets reorders by considering what they reorder; simply apply the permutation \( \pi \) to the \( n \)-tuples in the interpretation of term \( A \) in order to get the interpretation of \( A^\pi \).

Since we treat quantification substitutionally and \( \text{ITerm}_{\mathcal{L}_T} \) plays the role of individual-constants, we need to make sure that the particular choice of \( \text{ITerm}_{\mathcal{L}_T} \) does not lead to problematic results; we do that as before by expanding the language.

**Definition 44 (\( \mathcal{L}_T \)-A-Expansion)**

Let \( \mathcal{L}_T \) be a TFL-language and \( \mathcal{M}_T = \langle D, \| \cdot \|_{\mathcal{M}_T} \rangle \) be an \( \mathcal{L}_T \)-model. Let \( A \subseteq D \). The **\( \mathcal{L}_T \)-A-expansion of \( \mathcal{L}_T \)** is the language \( \mathcal{L}'_T := \mathcal{L}_T \cup \{ t_a | a \in A \} \) where the \( t_a \) are new individual-terms not contained in \( \mathcal{L}_T \).

If \( A = \{ a \} \), we call \( \mathcal{L}'_T \) an **\( \mathcal{L}_T \)-a-expansion**.

Once the language is expanded, we need to make sure that the interpretation keeps up.

**Definition 45 (\( \mathcal{L}_T \)-Model Expansion)**

Let \( \mathcal{L}_T \) be a TFL-language and \( \mathcal{M}_T = \langle D, \| \cdot \|_{\mathcal{M}_T} \rangle \) be an \( \mathcal{L}_T \)-model. Let \( A \subseteq D \) and \( \mathcal{L}'_T \) be an \( \mathcal{L}_T \)-A-expansion. The **A-expansion of \( \mathcal{M}_T \) to \( \mathcal{L}'_T \)** is the model \( \mathcal{M}'_T = \langle D', \| \cdot \|_{\mathcal{M}'_T} \rangle \) such that

1. \( D' = D \);
2. \( \| \cdot \|_{\mathcal{M}_T} \subseteq \| \cdot \|_{\mathcal{M}'_T} \);
3. \( \| t_a \|_{\mathcal{M}_T} = \{ a \} \subseteq A \) for every new individual term \( t_a \).
As in the cases before, Definition 45 keeps the domain the same, and extends the interpretation-function to $\parallel \cdot \parallel_{M^T}$ so that the new individual-terms $t_a$ are interpreted in alignment as they have been introduced. In accordance with Definition 43 (2a), these are not elements of the domain, but singleton-subsets.

**Definition 46 (Satisfaction $\models_T$)**

Let the TFL satisfaction-relation $M_T \models_T \varphi$ for $\varphi \in \text{Form}_{L_T}$ and $L_T$-model $M_T = \langle D, \parallel \cdot \parallel_{M_T} \rangle$ be recursively defined as follows:

1. $M_T \models_T (q_1t_1, \ldots, q_nt_n)A$ iff $\bigcup \parallel t_1 \parallel_{M_T}, \ldots, \bigcup \parallel t_n \parallel_{M_T} \in \parallel A \parallel_{M_T}$;  
2. $M_T \models_T (q_1t_1, \ldots, q_nt_n)\neg A$ iff $M_T \models_T \neg(q_1t_1, \ldots, q_nt_n)A$;  
3. $M_T \models_T \neg \varphi$ iff it is not the case that $M_T \models_T \varphi$ $(M_T \not\models_T \varphi)$;  
4. $M_T \models_T \varphi \land \psi$ iff $M_T \models_T \varphi$ and $M_T \models_T \psi$;  
5. $M_T \models_T \varphi[t_1/\alpha, \ldots, t_n/\alpha]$ iff $M_T \models_T \varphi$;  
6. $M_T \models_T \varphi[\exists A]$ iff for some $a$-expansions $M_T'$ of $M_T$ such that $a \in \parallel A \parallel_{M_T}$, $M_T' \models_T \varphi[\exists t_a]$;  
7. $M_T \models_T \varphi[\forall A]$ iff for all $a$-expansion $M_T'$ of $M_T$ such that $a \in \parallel A \parallel_{M_T}$, $M_T' \models_T \varphi[\forall t_a]$.

As already done in Definition 43 (2), individual-terms are interpreted regardless of their specific quantity as done in clause (1); individual-terms are pretty much treated as individual-constants in Definition 22 (1), as is predication.

As QUARC, TFL allows for negative predication; clause (2) is analogous to clause (5) of Definition 34. The negation-symbols $\neg$ are moved in front of formulas and then interpreted via clause (2) as long as only individual-terms are involved.

The remaining clauses are analogous to those of QUARC in Definition 34. In particular, we interpret quantifiers via the expansions, where, as in the QUARC-case given in Definition 34 (7–8), we consider appropriate expansions, i.e., expansions which expand with elements in the interpretation of the subject-term $A$ and consider as many as the quantity $q$ of $A$ specifies.

As before, we can define logical consequence as done in Definition 8. Given these notions, we can formulate the TFL-specific treatment of individual-terms.

**Theorem 47**

For $t \in \text{ITerm}_{L_T}$, $\models_T (\exists t)A \leftrightarrow (\forall t)A$.

*Proof.* Let $M_T$ be an $L_T$-model and $t \in \text{ITerm}_{L_T}$. Let $M_T \models_T (\exists t)A$. By Definition 46 (1), $\bigcup \parallel t \parallel_{M_T} \in \parallel A \parallel_{M_T}$ and so $M_T \models_T (\forall t)A$. \(\square\)
Moreover, we get a similar result regarding non-emptiness as Theorem 35, though extended to include individual-terms.

**Theorem 48**

For $A \in \text{ITerm}_{L_T} \cup \text{STerm}_{L_T}^1$, $\models_T (\exists A)A$.

**Proof.** Let $\mathcal{M}_T$ be an $L_T$-model.

- If $A \in \text{ITerm}_{L_T}$, by Definition 43 (2a) $\|A\|_{3r_T} = \{a\}$ for an $a \in D$. Thus, $\bigcup\|A\|_{3r_T} = a \in \|A\|_{3r_T}$. Therefore, by Definition 46 (1), $\mathcal{M}_T \models_T (\exists A)A$.

- If $A \in \text{STerm}_{L_T}^1$, by Definition 43 (2b) $\|A\|_{3r_T} \neq \emptyset$. Let $a \in \|A\|_{3r_T}$. Then, for some $a$-expansion $\mathcal{M}'_T$ of $\mathcal{M}_T$ such that $a \in \|A\|_{3r_T}$, $\mathcal{M}'_T \models_T (\exists t_a)A$. By Definition 46 (6), $\mathcal{M}_T \models_T (\exists A)A$.

However, as we allow for complex terms, this does not hold in general.

**Theorem 49**

$\not\models_T (\exists A)A$.

**Proof.** Let $\mathcal{M}_T$ be an $L_T$-model. Consider $A \in \text{STerm}_{L_T}^1$ such that $\|A\|_{3r_T} = D$. Then, by Definition 43 (2d), $\| \overline{A} \|_{3r_T} = \emptyset$. Thus, there is no $a$-expansion $\mathcal{M}'_T$ of $\mathcal{M}_T$ such that $a \in \|A\|_{3r_T}$, so $\mathcal{M}_T \not\models_T (\exists A)A$. □

For similar reasons, we get that the universal doesn’t imply the particular.

**Corollary 50**

$(\forall A)B \not\models_T (\exists A)B$.

**Proof.** Consider the model in the proof of Theorem 49. Since there is no $a$-expansion $\mathcal{M}'_T$ of $\mathcal{M}_T$ such that $a \in \|A\|_{3r_T}$, it follows that for all $a$-expansions $\mathcal{M}'_T$ of $\mathcal{M}_T$ such that $a \in \|A\|_{3r_T}$, $\mathcal{M}'_T \models_T (\forall t_a)B$, i.e., by Definition 46 (7), $\mathcal{M}_T \models_T (\forall A)B$. However, as there are no $a$-expansions $\mathcal{M}'_T$ of $\mathcal{M}_T$ such that $a \in \|A\|_{3r_T}$, $\mathcal{M}_T \not\models_T (\exists A)B$. □

Of course, as in the case of Theorem 15, we obtain a restricted version.

**Theorem 51**

$(\exists A)A, (\forall A)B \models_T (\exists A)B$.

Overall, as was to be expected, the $L_T$-models $\mathcal{M}_T$ behave similar to the rejected non-empty $L_A$-models $\mathcal{M}_{ne}$.

Moreover, the quantifiers still behave as expected.
The following equivalences hold:

\[(1) \models T (\forall A)B \leftrightarrow \neg((\exists A)\neg B); \quad (2) \models T (\exists A)B \leftrightarrow \neg((\forall A)\neg B);\]

\[(3) \models T \neg(\exists A)B \leftrightarrow (\forall A)\neg B; \quad (4) \models T \neg(\forall A)B \leftrightarrow (\exists A)\neg B.\]

Given the way term-negation ‘\(^\neg\)’ is interpreted, it is equivalent to a negative predication.

**Theorem 53**
\[\models T (qA)\neg B \leftrightarrow (qA)\overline{B} \ (q \in \{\forall, \exists\}).\]

*Proof.* Let \(M_T\) be an \(L_T\)-model such that \(M_T \models T (qA)\neg B\). Then, by Definition 46 (6)/(7), for some/all \(a\)-expansions \(M'\) of \(M_T\) such that \(a \in \parallel A\|_{M_T}\), \(M_T' \models T (q't_a)\neg B\). Thus, by Definition 46 (2), for some/all \(a\)-expansions \(M'\) of \(M_T\) such that \(a \in \parallel A\|_{M_T}\), \(M_T' \models T (\neg(q't_a)B)\), i.e., by Definition 43 (2d), \(\parallel ta\|_{M_T'} \notin \parallel B\|_{M_T'}\), i.e., for some/all \(a\)-expansions \(M'\) of \(M_T\) such that \(a \in \parallel A\|_{M_T}\), \(M_T' \models T (q't_a)\overline{B}\). Thus, by Definition 46 (6)/(7), \(M_T \models T (qA)B\). \(\square\)

Similar again to the non-empty models of Aristotelian syllogistic, only two conversions hold generally, and the third one with a restriction in place.

**Theorem 54 (Conversion)**

The following conversions hold:

\[(a-i\text{-conv}^{\models T} \ | \ \exists A) \ (\exists A)A, (\forall A)B \models T (\exists B)A\]

\[(i-i\text{-conv}^{\models T}) \quad (\exists A)B \models T (\exists B)A\]

\[(e-e\text{-conv}^{\models T}) \quad (\forall A)\neg B \models T (\forall B)\neg A\]

Lastly, negation works as expected as well.

**Theorem 55**

The following hold (‘\(*\)’ being a possibly empty string of negation-symbols ‘\(^\neg\)’):

\[(1) \models T (q_1t_1, \ldots , q_nt_n)^{\neg \cdots \neg} A \leftrightarrow (q_1t_1, \ldots , q_nt_n)^{\neg \cdots \neg} A;\]

\[(2) \models T (q_1t_1, \ldots , q_nt_n)^{\neg \cdots \neg} \overline{A} \leftrightarrow (q_1t_1, \ldots , q_nt_n)^{\neg \cdots \neg} A;\]

\[(3) \models T (q_1t_1, \ldots , q_nt_n)^{\neg \cdots \neg} \overline{A} \leftrightarrow (q_1t_1, \ldots , q_nt_n)^{\neg \cdots \neg} A.\]
6 Comparison

Having sketched the different systems, let’s compare them. Aristotle’s syllogistic and Fregean logic function as base; we consider how QUARC and TFL compare to them and differ from each other. The comparison, however, does not account for all the subtleties and differences between QUARC and TFL, but is restricted to more general points. It also remains open to see whether QUARC can be developed along TFL-lines and vice versa. For this reason, among others, I do not argue for the superiority of either of these systems when it comes to the question of which one better captures the semantics of natural language—the underlying motivation of both QUARC and TFL. The comparison is rather meant to consider potential differences which might lead to further development of either of these approaches along the lines of the other.

6.1 Aristotelian Roots

As we have seen in Sections 4.1 and 5.1, both Sommers and Ben-Yami claim a strong connection to Aristotelian logic. Ben-Yami (2004: 62) sees his understanding of predication as fundamentally in agreement with that of Aristotle, and Sommers considers several of Aristotle’s points throughout the development of TFL.

In the version of TFL developed in Section 5.2, I excluded many of Sommers’s more specific points that show a strong similarity to Aristotle’s logical discussions. For example, I did not include categories and, as a consequence, excluded Sommers’s discussion of contrariety (see, e.g., Sommers 1982: 80).

TFL, in contrast to QUARC, takes the subject-predicate structure of (basic) sentences to be fundamental. The formalism from Section 5.2 does not fully reflect that, though takes some steps towards it with the introduction of reduced terms collected in \( \text{RTerm}_L \) in Definition 41 (5). This allows to reduce \( n \)-ary terms to unary terms which can be the predicate in the subject-predicate structure. For example, a binary predicate like ‘loves’ can be reduced to a unary predicate ‘loves \( t \)’ (\( t \) a term) serving as predicate to a subject. Similarly, we can iterate this and use reduced terms to reduce further terms. This can account for the intended nesting of terms to keep the subject-predicate structure intact (cf., e.g., Sommers 1982: 113ff.).

Moreover, TFL does not include individual-constants, but does include individual-terms in form of \( \text{ITerm}_L \). As all the descriptive signs are terms, each term can play the role of subject and predicate. This is reflected in the conversions (Theorem 54) which only hold in QUARC for the unary predicates.

Each term in subject position is assigned a quantity—indicated by a quantifier. In the case of individual terms, the quantity does not make a difference (Theorem 47). In the formalism of Section 5.2 a quantity is
assigned, but not in the form of a wild quantity (as in Sommers 1982: 18). Given \( t \in \text{ITerm}_{\mathcal{L}_T} \), \( \mathcal{L}_T \)-models interpret them accordingly as singletons which puts them on a par with other unary terms. Indeed, Definition 43 allows for unary terms to be interpreted as singletons, too. The difference between an \( A \in \text{STerm}_{\mathcal{L}_T}^1 \) and a \( t \in \text{ITerm}_{\mathcal{L}_T} \) would only show up once the system is modalized; \( t \) would still be interpreted as singleton, \( A \) might not.

QUARC follows the Fregean line of dividing the language into individual-constants (\( \text{Const}_{\mathcal{L}_F} \))/singular arguments (\( \text{SA}_{\mathcal{L}_Q} \)) and \( n \)-ary predicates (\( \text{Pred}^n_{\mathcal{L}_F}/\text{Pred}^n_{\mathcal{L}_Q} \)). The Aristotelian root that Ben-Yami sees for QUARC is when it comes to predication. The sentences of the syllogistic (\( \text{Form}_{\mathcal{L}_A} \)) follow the subject-predicate pattern, where the predication can be universally or particularly and so assign the subject a quantity. This general structure is not kept for all the QUARC-sentences though, but only for those with unary predicates. In particular, only quantified sentences come with the assignment of quantities, not all sentences. TFL, in contrast, takes every sentence to come with a quantity.

Relational predcations, on the other hand, are treated by QUARC as they are in Fregean languages. This contrasts with TFL-sentences which keep the subject-predicate structure also for those. However, \( \text{Form}_{\mathcal{L}_T} \) also allows for sentences involving connectives so that complex sentences without this subject-predicate structure are included, too, but such complex sentences bottom out in sentences with subject-predicate structure in TFL; in QUARC, they do not.

### 6.2 Identity

Sommers (1982: ch. 6) argues that there is no need to include an identity-symbol ‘\( = \)’ into TFL. Rather, we can understand Aristotle’s basic notion of predicated of all/none (Section 2.7) as providing us with a substitution principle so that identity becomes superfluous. This substitution principle can be taken to be a formal rendering of (Barbara) which allows to conclude \( AaC \) from \( AaB \) and \( BaC \). In the languages of TFL and QUARC, this can be captured as (where ‘\( \models \)’ is either ‘\( \models_{\mathcal{T}} \)’ or ‘\( \models_{\mathcal{Q}} \)’)

\[
(\forall C)B, (\forall B)A \models (\forall C)A.
\]

However, TFL comprises more notions here as we are allowed to use individual-terms. QUARC, on the other hand, only allows unary predicates, and so the formal rending of (Barbara) does not apply to individuals as such. Nevertheless, as there is nothing ruling out unary predicates which are interpreted as singletons—i.e., as the \( t \in \text{ITerm}_{\mathcal{L}_T} \)—it can be taken to apply indirectly, via establishing a connection between the singular arguments and specific predicates.

Moreover, TFL allows this substitution also in cases where the predi-
cate is \(n\)-ary. For example, if \(B \in \text{CTerm}_{\mathcal{L}_T}^n\), we get from
\[
(q_1A_1, \ldots, q_{i-1}A_{i-1}, \forall A_i, q_{i+1}A_{i+1}, \ldots, q_nA_n)B
\]
and
\[
(\forall C)A_i
\]
that
\[
(q_1A_1, \ldots, q_{i-1}A_{i-1}, \forall C, q_{i+1}A_{i+1}, \ldots, q_nA_n)B.
\]

Even though QUARC can validate such consequences too, \(\mathcal{L}_Q\) contains an identity symbol ‘\(\equiv\)’ among its logical constants. Given the different understanding of predication, though, it behaves slightly different compared to the Fregean case. As Fregean languages quantify unrestrictedly over individuals, it can capture that everything is self-identical (‘\(\forall x(x = x)\)’). QUARC, on the other hand, cannot (see Section 6.4), though identity works similar. For example, given two singular arguments \(s_1, s_2 \in \text{SA}_{\mathcal{L}_Q}\), ‘\(s_1 = s_2\)’ is a QUARC-sentence. However, for \(\alpha, \beta \in \text{Ana}_{\mathcal{L}_Q}\), ‘\(\alpha = \beta\)’ would not be well-formed (and neither would be ‘\(\forall \alpha \alpha = \beta\)’ or something similar). Anaphora can only be introduced by replacing singular arguments; see Definition 28 (6). Thus, ‘\(s = s\)’ can lead to ‘\(s_\alpha = \alpha\)’ which, in turn, can lead to ‘\(\forall \alpha \alpha = \alpha\)’.

As \(\mathcal{L}_T\) does not contain any individual-constants or variables, identity cannot be introduced as in \(\mathcal{L}_F\) or \(\mathcal{L}_Q\). Nevertheless, in principle, it could be introduced as restricted to individual-terms. For example, if \(t_1, t_2 \in \text{ITerm}_{\mathcal{L}_T}\), ‘\(t_1 = t_2\)’ could be interpreted via Definition 46 (1), i.e., an \(\mathcal{L}_T\)-model \(\mathfrak{M}_T\) satisfies it iff \(\bigcup \|t_1\|_{\mathfrak{M}_T} \cup \|t_2\|_{\mathfrak{M}_T} \bigcup \|x\|_{\mathfrak{M}_T}\) (or, equivalently, \(\bigcup \|t_1\|_{\mathfrak{M}_T} = \bigcup \|t_2\|_{\mathfrak{M}_T}\). One could then also show that \((\forall t_1)t_2 \models t_1 = t_2\) (and so use ‘\((\forall t_1)t_2\)’ as definition of ‘\(t_1 = t_2\)’). In principle, this could also be achieved in QUARC.

### 6.3 Negation

As in the case of \(\mathcal{L}_A\), several ways to negate have been introduced into the systems. In the syllogistic, terms can be negated (‘\(\neg A\)’) and sentences can be negative (‘\((qA)\neg B\)’). Fregean languages, on the other hand, only contain sentence-negations (‘\(\neg \varphi\)’).\(^{108}\)

The version of QUARC presented in Section 4.2 incorporates sentence- and predication-negation. The former works as it does in Fregean languages, the latter negates predication and so compares to the negative sentences of the syllogistic (‘\(\tau i apotinos\)’). What is captured by predicate-negation is that a predicate such as ‘friendly’ can be affirmed or denied. However, as long as there is no quantification involved, these are treated as equivalent to sentence-negations as specified in Definition 34 (5).

Similarly, TFL, as presented in Section 5.2, contains both sentence- and predicate-negation. Additionally, it contains negated terms as the syllo-
gistic does. As I did not incorporate categories and contrariety into the
formalism, these are also treated as equivalent as shown in Theorems 53
and 55.

There is no reason to treat predicate-negation as equivalent to sen-
tence-negation in quantifier-free cases. Following Sommers’s discussion,
we can understand predicate-negation as connected to categories and
category mistakes. For example, the number 2 is neither friendly nor not
friendly; the sentences ‘it is not the case that the number 2 is friendly’
(which is true) and ‘the number 2 is not friendly’ (which is false) come
apart. This, too, could be incorporated into QUARC.

Given TFL’s additional negative terms, TFL can also treat predicate-
negation as introduced, and construe the negated terms as connected to
categories directly. It could also understand the predicate-negation so
and the term-negation as introduced. The different ways to negate open
different possibilities to introduce where negation can “go wrong”.

In the empty semantics for the syllogistic, on the other hand, negative
predication and negated terms are not equivalent; this is shown in
Theorem 16. The reason is that the L_A-models M_A allow terms with
empty extensions which rule out the validation of a-type sentences by
[a_a]. In the alternative semantics M_ne, simple terms are taken to be
non-empty—as are the simple terms in TFL according to Definition 43
(2b) as well as the (Fregean/QUARC) unary predicates according to Def-
nition 19 (2b)/Definition 31 (2b). If incorporated into TFL or QUARC,
this opens different ways of interpreting the different ways to negate.

## 6.4 Quantification

As the ‘QUAR’ in ‘QUARC’ suggests, Ben-Yami considers QUARC’s
treatment of quantification as one of its major divergences from Fregean
languages. Firstly, Ben-Yami (2004 §9.8) argues that quantification
comes with what he calls ‘referential import’ in his book (‘instantiation’
in his 2014). However, in my presentation of the Fregean language,
I incorporated this already; see Definition 19 (2b), Theorem 23 and
Corollary 24.

Secondly, Ben-Yami (2004 §6.1) argues that Fregean languages pre-
suppose a domain of quantification whereas QUARC does not. Rather,
quantification in natural language is always combined with a specifica-
tion as to what is quantified over, i.e., a plurality is identified and the
quantifier specifies how much of that plurality is relevant. For example,
in ‘all human beings are mortal’, ‘human beings’ refers to a plurality of

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109By now he prefers ‘instantial import’. He also insists (personal communication)
that there are two issues that are mixed together, viz., unary predicates are not
empty as to keep QUARC bivalent, and instantial import is about quantification,
viz., a sentence of the form ‘φ [VF]’ can only be true or false if there are Fs. Hanoch
points out that he has been clear about the distinction since after the publication
of his (2014).
human beings (i.e., reference is to be construed \textit{plurally}) and ‘all’ suggests how much of that plurality is relevant. The Fregean analysis, on the other hand, quantifies over the whole domain which, therefore, has to be presupposed.

Since TFL considers sentences to have subject-predicate structure where the subject is assigned a quantity, the treatment aligns with that of QUARC, viz., quantification is always \textit{restricted} by a term. Thus, insofar as QUARC’s treatment differs from that of Fregean languages, TFL’s does too.

However, Sommers treats ‘human beings’ as the \textit{subject} of the sentence, whereas Ben-Yami takes it to be ‘all human beings’. The ‘all’ only indicates the quantity of the subject, but does not figure as part of it in TFL. Formally, this does not make a difference, though, as can be seen by Definitions 34 (7)–(8) and 16 (6)–(7). Nevertheless, the underlying understanding of \textit{reference} is a different one, though one that I do not discuss here.

Another difference is that every sentence comes with quantities according to TFL but not to QUARC. The reason is that TFL takes all the descriptive signs to be \textit{terms} for which one can specify quantities. QUARC, on the other hand, follows the Fregean approach. However, as treated in Definition 16 (1), the quantity does not make a difference for individual terms; we might as well reformulate Definition 12 so as to allow \( t \in \text{ITerm}_L \) to occur \textit{without} quantifier in \( L_T \)-formulas. Similarly, we could reformulate Definition 28 (1) to \textit{include} quantifiers which don’t affect the interpretation.

\section*{6.5 Expressive Power}

Both Sommers and Ben-Yami are concerned with the \textit{expressive power} of their systems. Indeed, both consider expressive power as an adequacy criterion when it comes to alternatives to the Fregean approach. As Aristotelian syllogistic is clearly inferior in this respect, it fails to meet

\footnote{I have to admit that I—still; see Raab (2018, n. 29, p. 315)—don’t fully grasp Ben-Yami’s claim that domains are not needed. As interpreted here via Definitions 31 and 34 (7)–(8), it is true that all quantification is \textit{restricted} by the interpretation of the quantified argument: \( M_Q \models \varphi [\forall / \exists P] \) iff for all/some \( a \)-expansions \( M'_Q \) of \( M_Q \) such that \( a \in ||P||_{M_Q} \), \( M'_Q \models \varphi [s_a] \). However, that still \textit{presupposes} a domain in which \(||P||_{M_Q} \) lives.

Lanzet likewise claims to develop a “domain-free semantics” (2017: 550) and goes on to suggest that when “reference is made to the domain of an interpretation \( \mathcal{M} \), what will be meant is the domain of \( \mathcal{M} \) as a \textit{function}” (2017: 565, his emphasis). However, unless the \textit{function} maps \textit{into} somewhere—its range or our domain—it is not a function, and so the model would not be well-defined.

One might suggest that the problem is the \textit{model-theoretic} approach, but I don’t see how the problem disappears by going for a \textit{valuational semantics} (seemingly, Ben-Yami’s preferred approach). Whether I presuppose for each predicate \( P \) what exactly is referred to or whether I presuppose a domain and then restrict it to predicates seems to me to amount to the same (with the latter option to be in many cases more convenient and expressively richer; see Section 6.5).}
Both QUARC and TFL have a legitimate claim as to satisfy the criterion. QUARC achieves the expressive power by including anaphora and reorders of any arity; TFL by including proterms and complex terms of any arity. Both systems also have formal results to show their expressive power in comparison to Fregean languages. Sommers claims that “the expressive power of [PTL] is that of a standard language of modern predicate logic” (1982: 176), i.e., of a Fregean language; see also (1982: Appendix A). Given that TFL extends PTL, it is clear that TFL does not fall behind with respect to its expressive power.

QUARC, too, has been investigated with respect to its expressive power compared to a Fregean language. Once we expand $L_Q$ by a unary predicate $T$ such that $\|T\|_{M_Q} = D$, all $\varphi \in \text{Form}_{L_F}$ can be translated into QUARC and vice versa. One way to introduce such a predicate is to allow complex predicates into QUARC; see my (2016: ch. 5) and, for a fuller treatment, my (ms). For, we can then define $(\cdot)T$ as $(\cdot)(P \lor \neg P)$. This, then, allows to capture quantified sentences which don’t have restricting predicates such as ‘$\forall x(x = x)$’; QUARC captures it as ‘$\forall T_\alpha = \alpha$’. A similar approach works when showing that TFL can capture all $\varphi \in L_F$.

What has not been investigated is how exactly TFL and QUARC compare. Once translations between the systems and a Fregean language have been introduced, they can be used to establish the relation between them. However, this has not been done yet. Nevertheless, if the formal systems that have been introduced here are adequate representations of the intended systems, translations between them suggest themselves. Since the presentation of TFL has been quite diminished compared to Sommers’s developments, I would think that TFL is the most expressive systems among those considered here. However, there does not seem to be a principled reason to suggest that QUARC couldn’t similarly developed further to match this expressive richness.

7 Conclusion

I have developed four formalisms here, one for each of Aristotelian syllogistic, Fregean languages, QUARC, and TFL. Both QUARC and TFL are meant to favourably compare to Aristotle’s logic. QUARC’s understanding of predication and quantification and TFL’s understanding of terms and the subject-predicate structure of basic sentences is claimed to be close to Aristotle’s understanding of these. Moreover, both systems have been developed as a better way to the semantics of natural language compared to what Fregean languages are capable. Again, it’s the Aristotelian root that does much of the heavy lifting.

The expressive power of Fregean languages remains one of the main arguments to adopt the Fregean approach. However, QUARC and TFL have a claim to match this power, and so undermine at least the argument from expressive power. On the other hand, the availability of
translations of both TFL and QUARC into Fregean languages also shows that the expressive power alone cannot decide here. One major way in which the case is made for either QUARC or TFL is by the syntactic similarity of their formal grammars compared to that of natural language. Given that those formal grammars differ from one another while claiming to fit that of natural language well, it needs to be seen in which ways these formalisms can be extended to capture more and more of natural language. But even once that is done, if we can establish the precise relationship between these systems, it might well be that both can be developed to incorporate parts of the other so that nothing might decide between the two. As it stands, it’s focus on the terms and the subject-predicate structure of basic sentences means that TFL is a more radical alternative to Fregean languages; whether it is a better one than QUARC, I leave the readers to decide for themselves.

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