A formal semantics for Wittgenstein's builder language

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> "The language is meant to serve for communication between a builder A and an assistant B. A is building with building-stones: there are blocks, pillars, slabs and beams. B has to pass the stones, and that in the order in which A needs them. For this purpose they use a language consisting of the words 'block', 'pillar', 'slab', 'beam'. A calls them out;—B brings the stone which he has learnt to bring at such-and-such a call.—Conceive this as a complete primitive language.. . .
>
> Let us look at an expansion of the language. Besides the four words 'block', 'pillar', etc., let it contain a series of words used [as numerals] (it can be the series of letters of the alphabet); further, let there be two words, which may as well be 'there' and 'this' (because this roughly indicates their purpose), that are use in connection with a pointing gesture; and finally a number of colour samples. A gives an order like: 'd–slab–there'. At the same time he shews the assistant a colour sample, and when he says 'there' he points to a place on the building site. From the stock of slabs B takes one for each letter of the alphabet up to 'd', of the same colour as the sample, and brings them to the place indicated by A.—On other occasions A gives the order 'this–there'. At 'this' he points to a building stone." (Ludwig Wittgenstein, *Philosophical Investigations*, §2 and §8)

Wittgenstein then asks: "Now what do the words of this language *signify*?—What is supposed to shew what they signify, if not the kind of use they have?"

Might one answer that rhetorical question by giving a compositional semantics for Wittgenstein's builder language? It wouldn't necessarily challenge any observations of Wittgenstein.[1] But in so far as Wittgenstein's example was supposed to cast doubt on the "ideal language" approaches of Frege and Russell (and early Wittgenstein), I think it'd be interesting to show how the builder language is perfectly amenable to contemporary formal analysis. It should be a fairly trivial exercise. The language does, however, harbour some complexities—it, at least, involves imperatives, demonstratives, and some form iconicity, it arguably makes use of generalised quantifiers and predicate modification, and arguably involves covert structure. The formal methods developed by logicians and analytic philosophers were originally developed for certain specific projects (e.g. the reduction of mathematics to logic), and given these aims it would indeed be the context-insensitive declarative fragment of natural language text that would be most yielding to treatments by such methods. Yet, natural language semantics has developed considerably over the last one-hundred years, and now covers a wide range of cases that go beyond the austere language of arithmetic. My main purpose here is just to sketch one natural way carry out the task—I will not draw any sweeping metatheoretical conclusions from this exercise. (And I won't in fact fully complete the exercise.)

[1] "But assimilating the descriptions of the uses of words in this way cannot make the uses themselves any more like one another. For, as we have seen, they are absolutely unlike." §10

Wittgenstein doesn't give us all the details about the structure of the language, but there is a clear description of the sorts of utterances the builders make and a clear description of what the builders take them to "mean". I will specify a grammar that generates the sorts of sentences Wittgenstein seem to have in mind, and then sketch a compositional semantics for the language. I don't of course claim that this is the only way to do this.

Let's call the builder language $\mathcal{B}$. What are the basic symbols of the language? First there are the four nouns: 'block', 'pillar', 'slab', 'beam'. The language also contains the "letters of the alphabet", which somehow indicate quantity. For ease let's just assume there are four of these: 'a','b','c','d'. I will call these the *quantifiers*. There are also the *colour samples*. I won't dwell here on semiotics, Peircian icons, or crossmodal coordination, or whatever. For now let's just assume there are two colour samples that serve to somehow modify the required colour of the stone: 🟩, 🟥. Finally there are the demonstratives 'there' and 'this' accompanied by pointing gestures. These elements are employed to make utterances where either (i) a builder produces the simple 'slab', (ii) a builder produces 'd slab there', while pointing to a place on the building site and at the same time holding up a colour sample, or (iii) a builder produces 'this there' while first pointing to a stone and then a place on the building site. Thus on the face of it there are three general sorts of utterances, which we might display as the following:

(1) slab

(2) 🟥 c slab there☞

(3) this☜ there☞

First question: How should we analyse the pointing gestures and colour samples? Should these be treated as part of the linguistic representation or should they be understood as part of the context of use. Both options are tenable, but I think it is most natural to assume that the gestures and samples are both aspects of the representation. That is, the pointing gestures and colour samples are construed as signs of the language and thus will be interpreted by the semantic theory. (Of course, Kaplan and the subsequent literature has a lot to say about the demonstratives and demonstrations.)

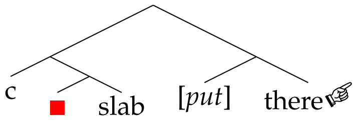
Second question: Where is the verb? In terms of the intended interpretations, these utterances would most naturally be understood as missing a verb (cf. Wittgenstein §19-20). That is, an utterance of the builder language such as (4) would seem to be adequately translated by the English imperative (5).

(4) 🟩 c slab there☞

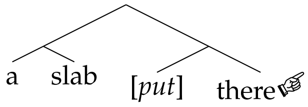
(5) Put exactly three green slabs there.

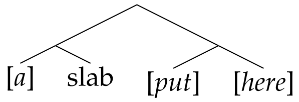
Given this I suggest that we posit some covert structure. We could pack the "missing" material into, say, the lexical entry for 'there' (so it roughly means $\lambda x.\lambda y.x$ put y there), or into 'c slab', or even into the composition rule that combines 'there' with 'c slab'. But I insist on keeping 'this' and 'there' as basic demonstratives, and don't want to posit multiple composition rules—in fact, I'd like to follow the standard assumption that all composition proceeds via functional application (i.e. Frege's conjecture) as far as possible.

I will assume that the basic structure of the sentences consist of (i) a verb phrase made up of a covert verb 'put' combined with a locational demonstrative— '[put] there'—and (ii) a determiner phrase. In the most complex cases the determiner phrase will be the result of, e.g., ‘c’ combined with ‘slab’ modified by a colour sample 🟥. Thus, the complex structure, which corresponds to (2), has this form:[2]

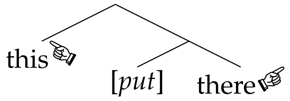


But what about the structure of (1), which is the simple utterance ‘slab’? I will assume that the lexical item ‘slab’ is the same as that which occurs in (2). Given this I think it is natural to assume that (1) and (2) have the same underlying logical form—its just that in utterance (1) the number is unpronounced and the location is identified via an implicit ‘here’ . So its like saying ‘a slab *there*’, but using ‘here’ instead.





Lastly, the structure corresponding to (3) will employ the simple demonstrative as its determiner phrase:



Those are the paradigm cases. To generate these structures let the well-formed sentences of $\mathcal{B}$ be provided by the following grammar:

⟨*sentence*⟩ ::= ⟨*determiner-phrase*⟩⟨*verb-phrase*⟩
⟨*determiner-phrase*⟩ ::= ⟨*quantifier*⟩⟨*noun-phrase*⟩ | this⟨*pointer*⟩
⟨*verb-phrase*⟩ ::= [*put*] there⟨*pointer*⟩ | [*put*] [*here*]
⟨*quantifier*⟩ ::= a | b | c | d
⟨*noun*⟩ ::= block | slab | pillar | beam
⟨*noun-phrase*⟩ ::= ⟨*noun*⟩ | ⟨*colour-sample*⟩⟨*noun*⟩
⟨*colour-sample*⟩ ::= 🟩 | 🟥
⟨*pointer*⟩ ::= ☜ | ☞

I think that provides a reasonable formalisation of the structure of the builder language. Finally, here is a sketch of the semantics for $\mathcal{B}$ under standard assumptions about models and types.[3]

[2] We could delay the modification of the colour sample so that it combines with ‘c slab there’, I just stick it in where it does its work so the types don’t have to carry the extra complication up the tree. The other way works too with some fiddling.

[3] Let c be a context, and let c_r be the addressee of the context, c_l the location, and c_a is the speaker.

Verb:

$[\![\mathit{put}]\!]^c = \lambda x_e.\lambda y_e.\ c_r$ puts y at x

Quantifiers:

$[\![\text{a}]\!]^c = \lambda P_{et}.\lambda Q_{et}.\ \#(P \cap Q) = 1$
$[\![\text{b}]\!]^c = \lambda P_{et}.\lambda Q_{et}.\ \#(P \cap Q) = 2$
$[\![\text{c}]\!]^c = \lambda P_{et}.\lambda Q_{et}.\ \#(P \cap Q) = 3$
$[\![\text{d}]\!]^c = \lambda P_{et}.\lambda Q_{et}.\ \#(P \cap Q) = 4$

Nouns:

$[\![\text{block}]\!]^c = \lambda x_e.\ x$ is a block
$[\![\text{slab}]\!]^c = \lambda x_e.\ x$ is a slab
$[\![\text{pillar}]\!]^c = \lambda x_e.\ x$ is a pillar
$[\![\text{beam}]\!]^c = \lambda x_e.\ x$ is a beam

Colour samples:

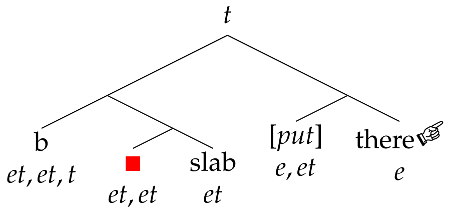
$[\![$🟩$]\!]^c = \lambda P_{et}.\lambda x_e.\ x$ is green and $x \in P$
$[\![$🟥$]\!]^c = \lambda P_{et}.\lambda x_e.\ x$ is red and $x \in P$

Demonstratives:

$[\![$this☞$]\!]^c$ = the object located at $[\![$☞$]\!]^c$
$[\![$there☜$]\!]^c$ = the location identical to $[\![$☜$]\!]^c$
$[\![$☜$]\!]^c$ = the location in c met by a straight line extending from the left index finger of c_a
$[\![$☞$]\!]^c$ = the location in c met by a straight line extending from the right index finger of c_a
$[\![\mathit{here}]\!]^c = c_l$

Given this one can look at some derivations:

(6)



$$[\![(6)]\!]^c = \#(\lambda x.\ x \text{ is red and } x \text{ is a slab} \cap \lambda y.\ c_r \text{ puts } y \text{ at ☞}) = 2$$

That completes my sketch of a compositional semantics for the builder language. We still need to say something about the fact that these are imperatives! We do get out a proposition (well once we add in the intensional types) describing the addressee's performance of an action. This is what will be appealed to in order to do the further semantic/pragmatic work that accounts for the imperatival aspect. For example, something like Portner's account where an imperative adds an action to the addressee's To-Do List would be a natural candidate. But I'll leave this task to others. . . "It is one of the chief skills of the philosopher not to occupy himself with questions which do not concern him." (Wittgenstein 1915)