

Chapter 8

Ibn Ḥazm on Heteronomous Imperatives. A Landmark in the History of the Logical Analysis of Norms



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Abstract Ibn Ḥazm of Córdoba's (994–1064) defence of logic has lasting consequences for the logic of norms. His book *Facilitating the Understanding of the Rules of Logic and Introduction Thereto, with Common Expressions and Juristic Examples* is a demonstration of how Aristotelian logic may be applied in the religious sciences, especially law. Among other things, he thoroughly investigates deontic notions and their modal counterparts, assuring him a place among the fathers of the logic of norms. The basic units of Islamic deontic logic qualify the performance of actions as subject to either *reward*, or *sanction*, or *neither*; and they might therefore be called, indulging in terminological anachronism, *heteronomous imperatives*. With remarkable insight, Ibn Ḥazm pairs these with the natural modalities of necessity, possibility, and impossibility. Employing some features of Martin-Löf's Constructive Type Theory (CTT) to shape the logic of heteronomous imperatives thus emerging from Ibn Ḥazm's insights, the authors formulate a new approach to the logical analysis of deontic categories.

Keywords Arabic logic · Heteronomous imperative · Ibn Ḥazm · Deontic logic · Islamic jurisprudence

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8.1 Introduction¹

The passionate and staunch defence of logic of the controversial thinker Ibn Ḥazm, Abū Muḥammad Alī b. Aḥmad b. Sa īd of Córdoba (994–1064), had lasting consequences in the Islamic world.² Indeed his book *Facilitating the Understanding of the Rules of Logic and Introduction Thereto, with Common Expressions and Juristic Examples* (*Kitāb al-Taqrīb li-ḥadd al-manṭiq wa-l-mudkhal ilayhi bi-l-alfāz al-āmmiyya wa-l-amthila al-fiqhiyya*), composed in 1025–1029, was well known and discussed during and after his time; and it paved the way for the studies of his compatriots Ibn Bājja (d. 1138), Ibn Ṭufayl (d. 1185), and Ibn Rushd (d. 1198), who each gave demonstrative reasoning a privileged place within the methods of attaining knowledge.³

Unfortunately, as too often in the history of science, Ibn Ḥazm’s innovative perspectives and contributions in logic have been overlooked or considered with an attitude of contempt. On the one hand, his work has been seen, at best, as promulgating the benefits of studying Aristotle’s logic, so that his contribution is assessed as more didactical than conceptual. And on the other hand, those who do examine his innovations often consider them to be mistaken.⁴

However, a reassessment of his work on logic has since begun, by delving into the ways the thinker of Córdoba studied the links between deontic and modal qualifications of propositions.⁵ In this context an important contribution is Lameer’s (2013) paper on the logical sources of Ibn Ḥazm. He observes (p. 417, n. 1) that—although, strictly speaking, it was al-Fārābī who first drew the parallelism between deontic and modal concepts—it was Ibn Ḥazm who developed it and worked it out in a more precise manner.

In fact, as Lameer elsewhere points out (1994, p. 240), though Al-Fārābī, while paraphrasing the *de Interpretatione*, speaks of the possibility for rephrasing a command in terms of necessity, the convertibility strategy he follows does not achieve the reduction he is after. Indeed, his example: converting “Zayd come over here” into “Zayd must come over here,” does not seem to paraphrase away the deontic component of the command encoded by the expression “must.”

Independently of the success of these attempts, it seems to be the case that these passages by al-Fārābī ground Lameer’s remarks (1994, pp. 240–241, 2013, p. 417) that al-Fārābī’s and Ibn Ḥazm’s perspectives appear to be the earliest testimony on

¹ The present paper shares the formal analysis of deontic modalities with that of Rahman et al. (2019a), though it develops further the notion of necessity—both historically and formally.

² For a recent comprehensive volume on his work, see Adang et al. (2013).

³ For an overview of the reception and reshaping of the Aristotelian Peripatetic work on logic, see Hasnawi and Hodges (2016).

⁴ As indicated by Chejne (1984, p. 2), contempt towards the logical work of Ibn Ḥazm was also present in its reception by Eastern philosophers who accused him of deviating from Aristotelian logic and of dabbling in things beyond his capability.

⁵ See also Guerrero (1997, 2013), and Lameer (2013).

record of a transference from deontic to modal concepts.⁶ Lameer's observations can be seen as targeting Knuuttila's (1981) well-known overview of the history of deontic logic, situating its origins around the fourteenth century, and Von Wright's (1981, p. 3) assertion that deontic logic was born in Leibniz's *Elementa Juris Naturalis* of 1671. Von Wright's assertion is based on the fact that Leibniz explicitly states in that work that the *transference* between deontic and modal concepts can be carried out in the following way:

Modal	Deontic
Possible, it is intelligible.	(<i>licitum</i>) permissible
Necessary, its negation is not intelligible.	(<i>debitum</i>) obligatory
Possibly not, its negation is intelligible.	(<i>indebitum</i>) omissible
Impossible, it is not intelligible.	(<i>illicitum</i>) forbidden

It is known that Leibniz's work on legal reasoning was inspired by his studies in Roman Law and Stoic Logic.⁷ Indeed it was the Stoics who, from the background of a dynamic ontology constituted by events and actions, proposed to extend or perhaps even substitute Aristotle's relational approach to necessity with a propositional one,⁸ whereby connectives and inference rules played the role of the Aristotelian term-relation governed by the metaphysics of essences and the logic of the syllogism.⁹

⁶ Lameer (2014, p. 306) acknowledges Gutas (1988, p. 270) for the reference to Ibn Ḥazm.

⁷ Cf. Armgardt (2008, 2015), Magnier (2013), Rahman (2015).

⁸ For a thorough discussion on Aristotle's relational view on modalities see Malink (2013), who also proposes a formal reconstruction based on what he calls a *mereological pre-order semantics*.

⁹ Bénatouïl (2017) recalls that quite before Łukasiewicz's (1934) famous paper on the history of propositional logic, Brochard (1892, 1912) and Hamelin (1901) not only acknowledged the *propositional turn* promoted by the Stoics but they also discussed whether the Stoics' proposal amounted to a replacement—or rather an extension—of the Aristotelian metaphysical framework of essences with one rooted in physics and events. In fact, according to chapter 9 of the *Peri Hermeneias* the notion of modality as applied to events, individualized by some time structure, amounts to a predicative relation between cause and effect. Moreover, in such a context Aristotle seems to think the relation is from the event to the cause, rather than the other way round—e.g., *if there is rain, there is necessarily a cause (clouds)*—but rain is not necessary (for a lucid and thorough study on the subject, see Crubellier (2010)). The Stoics, who arguably preserved Aristotle's time dimension, undertook the task of constituting the Cause-Effect link by means of a propositional connective, rather than by developing some special predicative relation. The Leibnizian project (at least in the contemporary reconstruction promulgated by Kripke and Hintikka), substituted the time structure with the abstract and meta-logical structure of possible worlds. However, in this move causes are, in a manner of speaking, absorbed by the conditions defining a possible world. More precisely, in such an approach modality is not attached to causality *per se*. In fact, there is a growing consensus within the community of experts in ancient philosophy that traditional modalities cannot be understood as the operators studied in contemporary modal logic (see Bobzien 1993; Malink 2013).

The propositional perspective on causal necessity allowed the Roman Jurists, and Cicero in particular, to transfer different forms of natural causality into the realm of legal reasoning. This contributed to the inception of the notion of *ratio legis*, the cause grounding a juridical decision. Perhaps one could understand the Stoic theory of signs, not as some early kind of formalism, but as a way to gather a general notion of cause-effect applying to both norms and events.¹⁰ Now, however, two main problems arose.

1. While the predicative approach of Aristotle assured contentual relevance, the propositional construction made it difficult to tie cause and effect with bare truth-functional means. Recall the well-known disputes on how to define an implication that expresses causality.
2. The question was raised of the gap between norms as prescriptions (and their actualization), and norms as propositions understood as bearers of truth—particularly in the context of legal reasoning.

These gaps evoke the broader epistemological problem of how to link theory and experience, or theory and *praxis*: does it make sense to speak of a *practical syllogism*? The Arabic tradition, particularly sensitive to issues concerning *praxis*, developed the insight that the interface *theory-praxis* should be studied from the perspective of the dyad *prescription-actualization*, precisely in the contexts most cherished by the Stoics; namely, ethics and jurisprudence. This new insight of the Arabic tradition led to the following bold steps:

- Prescriptions are understood as prescriptions to **do** rather than prescriptions that take us from one state of affairs to another: *Tun Sollen* rather than *Sein Sollen*.
- Not only events but (performances of) actions are first-class denizens of the universe of discourse. Actions and prescriptions display a contentual link that yields a classification of types of actions. Deontic reasoning is reasoning with content.
- Prescriptions to do are embedded in a system of hypothetical judgements involving implications where actions—the actualizations of the prescriptions—are subjects of predication: actions are bearers of qualifications such as law-abiding or law-breaking.¹¹ Similarly, events are qualified as necessarily happening, or possibly happening, or not happening at all.
- Norms presuppose freedom of choice: a prescription to do presupposes the possibility of choosing between carrying out, or not carrying out, the action prescribed by the norm.

¹⁰ Bréhier (1997) proposes a *semiotic* reading of the logic of the Stoics that moves away from the naturalistic interpretations of Brochard (1892) and Hamelin (1901).

¹¹ The notion of *conditional assertions* provided the ground for further sophisticated developments within the Islamic tradition of implications (including bi-implications), or *sharṭiyya muttaṣila*, and disjunctives, or *sharṭiyya munfaṣila*. For a recent, thorough study of the notion of *sharṭiyya* see Hasnawi and Hodges (2016, section 2.4.3, pp. 63–65).

- A basic principle that *all actions are permissible unless proscribed by Law* negotiated with the development and application (among the majority of Sunni jurists) of correlational modes of argumentation, subsumed under the rubric of *qiyās*; these sought to regulate into gradually expanding corpora of substantive law the dynamic integration of explicit “updated” deontic qualifications for novel and unresolved problem-cases.¹²

To be certain, analogies between deontic, temporal, and modal concepts have a long and rich history before their resurgence in contemporary deontic logic.¹³ Important lacunae nevertheless exist in the literature on its historic development, even in the most recent overviews, and particularly so in relation to contributions developed within Islamic jurisprudence.¹⁴ This remains the case despite the fact that there has been work on the influence of Stoicism on Arabic thinkers in general, and on the moral classification of acts as *obligatory*, *forbidden*, *recommended*, *reprehended*, and *neutral*, including studies by van Ess (1966) and Jadaane (1968).¹⁵ In fact, Gutas (1994) shows that the conditions for a grounded assessment of the influence of Stoicism on Islamic thinkers are not yet available. Indeed, he makes it very clear that studies such as the ones just mentioned are not backed by evidence stemming from the sources.¹⁶

Be that as it may, it seems that it is precisely in the context of Islamic jurisprudence that the contribution of the Arabic tradition to modality and its logic should be studied and pondered. Avicenna, who was not particularly interested in the logic of jurisprudence, might have influenced contemporary studies which focused

¹² For an overview of the *qiyās*-theory of the Shāfiī legal theoretician Abū Ishāq al-Shīrāzī (d. 1083), see Young (2017). The archetypal form of *qiyās* is that which is based on a shared *illa*, meaning “cause” or “occasioning factor,” which is known or inferred to have triggered the juridical decision (such as *legally valid*) or deontic qualification (such as *forbidden* or *obligatory*) of a known, authoritative case, and thus allows it to be transferred to the new case which shares that same *illa*. The *illa* is in certain ways analogous to the *ratio legis* mentioned above.

¹³ In fact, Knuuttila (1993, p. 182) observes that Peter Abelard (1079–1144) and other early medieval philosophers often endorsed an inverted form of Leibniz’s reduction by defining modal concepts by means of deontic concepts. According to this characterization, necessity is taken to be what nature demands, possibility is identified with what nature allows, and impossibility with what nature forbids.

¹⁴ See, for example, Knuuttila (1981), and the otherwise excellent essay by Hilpinen and McNamara (2013, p. 14). Though these works discuss the occurrence of deontic concepts in classical Islamic jurisprudence, they do not mention early testimonies to the parallelism between deontic and modal concepts in that tradition.

¹⁵ Jadaane (1968, pp. 184–189) discusses and convincingly relativizes Van den Bergh’s strong assertion (1954, reprinted 1987, vol. II, p. 117 of the notes) that the *obligatory*, *recommended*, *reprehended*, and *forbidden* notions of Islamic jurisprudence correspond (respectively) to the Stoic notions of *recte factum*, *commodum*, *incommodum*, and *peccatum*. Van den Bergh also points out (1954, vol. II, p. 118 of the notes) that Islamic theologians coupled the deontic notion of *permissible* with the modality *not logically impossible*. He does not develop the issue any further, however. Notably, Gutas (1994) develops a thorough, critical analysis of the hasty assessments made by Van den Bergh and Jadaane.

¹⁶ This situation, so far as we know, has not changed substantially since Gutas’ remark.

on the developments of metaphysical rather than of deontic necessity. In fact, a primary aim of this paper is to stress the role of the work of Ibn Ḥazm in developing a notion of deontic necessity deeply rooted in legal normativity. According to our view, the basic units of Islamic deontic logic are what we might call, indulging in terminological anachronism, *heteronomous imperatives*.¹⁷ As it turns out, the heteronomy of imperatives within Islamic legal systems is closely related with those of the moral realm. In premodern Islam, as Hallaq (2009) has shown us, there was no real division between the moral and the legal.¹⁸ However, in the present paper we will focus on the heteronomous imperatives of legal systems rather than on the imperatives when applied to purely moral connections. In this context, the work of Ibn Ḥazm extends the parallelism, stressed by his predecessors, between the necessity of events and that of human actions. Roughly:

- The relation between *Cause* and *Effect* within natural events is paralleled with the relation between *Action* and *Legal Consequence* (e.g., *Reward* or *Sanction*) within the Law.¹⁹

According to our understanding, Ibn Ḥazm's parallelism can be rendered explicit formally by means of a *conditional* (or hypothetical) structure shared by both deontic and modal propositions. Thus,

while in the domain of alethic modalities, given some causal conditions, it makes sense to introduce the categories (*causally necessary*, *impossible* and *contingent events*) in order to distinguish an event that is more likely to happen from another; in the domain of deontic modalities, the categories *obligatory*, *forbidden* and *permissible (optional) actions*, can be deployed to signify how the performance of different kinds of actions leads to different ways of distributing *Reward* and *Sanction*.²⁰

While developing our point we will ourselves delve into the logical structure of the heteronomous imperatives. This distinguishes our contribution from the existing literature, such as the papers of Chejne (1984), Lameer (2013), and Guerrero (1997, 2013), which do not provide logical analyses of the deontic concepts put to work

¹⁷ We refer to Kant's distinction between an autonomous imperative, that prescribes to act virtuously for the sake of the virtue itself, and a heteronomous, or hypothetical, imperative, that prescribes to act virtuously in view of attaining some kind of benefit or reward. Von Wright (1981, p. 34) has already suggested that "traditional deontic logic is not a genuine 'logic of norms' but a logic of structures resembling what Kant called hypothetical imperatives." We agree with the last part of the quote and would add that this shows that the traditional logical analysis of norms is indeed quite different from current deontic logic.

¹⁸ This lack of division may even be observed in the work of the young Leibniz (1678), who was adamantly disposed to run together moral and legal concepts. Recall for example his proposal to define obligatory as "what is necessary for a good person to do."

¹⁹ As pointed out by Zysow (2013) and Young (2019), there was a historically much-debated parallelism between the natural, or "intellective" cause (*illa aqliyya*), and the legal cause (*illa shar'iyya*). Some argued that the latter could not be treated like the former, and others that it could.

²⁰ NB: throughout this study, "sanction" is to be understood in the sense of "penalty."

by Ibn Ḥazm.²¹ The true antecedent to the present paper is the work of Farid Zidani (2007, 2015), who, so far as we know, was the first to undertake such a task.

8.2 Ibn Ḥazm's Logic of Heteronomous Imperatives

8.2.1 The Main Definitions

Muslim jurists identified five deontic qualifications for an action. Ibn Ḥazm defines them as follows:²²

- 1 *wājib, farḍ, lāzim*. Obligatory action is the one which: If we do it we are rewarded. If we do not do it we are sanctioned.
- 2 *ḥarām, mahzūr*. Forbidden action is the one which: If we do it we are sanctioned. If we do not do it we are rewarded.
- 3 *mubāh mustahabb*. Recommended permissible action is the one which: If we do it we are rewarded. If we do not do it we are neither sanctioned nor rewarded.
- 4 *mubāh makrūh*. Reprehended permissible action is the one which: If we do not do it we are rewarded. If we do it we are neither sanctioned nor rewarded.
- 5 *mubāh mustawin*. Evenly permissible action is the one which: If we do it we are neither sanctioned nor rewarded. If we do not do it we are neither sanctioned nor rewarded.

Note that the classification assumes that *reward* and *sanction* are incompatible but not contradictory. Some actions can be neither rewarded nor sanctioned.

Interestingly, Ibn Ḥazm's classification of actions varies in relation to others, such as that of the prominent Mu tazilite Qāḍī Abd al-Jabbār (935–1025), in his *Mughnī* (vols. 11–14), and in his *al-Uṣūl al-Khamṣa* (pp. 79–96).²³ Abd al-Jabbār's classification distinguishes “evil” (*qabīh*) actions (the doer deserves blame)

²¹ When completing the final draft of the current article, the authors learned of Joep Lameer's then-forthcoming article “Deontic Modalities in Ibn Ḥazm” (2019). Though complementary to our study, and contributing important philosophical observations and historical remarks, we should note that Lameer's approach is quite different, employing modality as an operator that builds propositions from propositions in order to express deontic modality. Our approach, on the other hand, employs hypothetical judgements incorporating choice, reward, and sanction to performances of actions. This is, we believe, requisite for understanding Ibn Ḥazm's parallelism.

²² Ibn Ḥazm (1926–1930, vol. 3, p. 77); idem (1959, p. 86; 2003, pp. 83–4). Note that Ibn Ḥazm's extension of *mubāh*-permissibility into the categories of recommended and reprehended is atypical. To be certain, this innovative subdivision facilitates the comparison (see below) with nearly possible, distantly possible, and purely possible; but it also highlights the underlying values. All forms of “permissibility” have a value; that is, in terms of doing the recommended or not doing the reprehended, both surpass the neutral value of the “evenly permitted,” while not yet reaching the value of doing the obligatory and not doing the forbidden. At the same time, neither doing the reprehended nor neglecting the recommended descends below the neutral value of the “evenly permitted,” which latter, always above the status of doing the forbidden and neglecting the obligatory, remains steadfastly in the middle.

²³ We owe this citation to Hourani (1985, pp. 99–102), who extracted the following definitions from the cited texts.

from “good” (*ḥaṣan*) actions. Good actions are classified into (i) acts of grace (*tafaḍḍul*), or the recommended (*nadb*), if and only if the doer deserves praise, and the ommitter does not deserve blame; (ii) the merely permissible [or optional] (*mubāḥ*), if and only if neither the doer nor the ommitter deserves blame or praise; and (iii) the obligatory (*wājib*), if and only if the ommitter deserves blame. As pointed out by Hilipinen and McNamara (2013, p. 14), who briefly discuss this classification of Abd al-Jabbār, the set seems to be lacking the category of *reprehended* or *excusable* (*makrūh*) actions. However, Abd al-Jabbār (1962) and (1979), instead of distinguishing a special category for the *excusable*, introduces the category of *not obligatory* (*ghayr wājib*), characterizing all those actions for which the ommitter is not blamed—including the *evil*, the *permissible* and the *recommended*.²⁴

Be that as it may, Abd al-Jabbār’s formulation has, in relation to Ibn Ḥazm’s, certain desirable properties, but also one which is less felicitous.

- Abd al-Jabbār’s category of actions that can be omitted has the logical advantage that it allows logical inferences from categories of *the evil*, *the permissible*, and *the recommended* to the *not obligatory*. Ibn Ḥazm’s definitions, given above, are more static.
- The use of “deserve” in Abd al-Jabbār’s definitions is deontically charged. Already in his day, Abd al-Jabbār’s formulation was qualified as circular, so he attempted to respond to the objection by introducing the notion of *correspondence*. What we have, therefore, is that such actions as are obligatory are those that, if omitted, *correspond* to evil (cf. Hourani 1985, p. 102).

Obviously, Ibn Ḥazm’s definitions do not suffer from this form of circularity, mainly because *reward* and *sanction*—rather than *blame* and *praise*—are primitive incompatible notions. Notice too that one way to see the problem of Abd al-Jabbār’s definitions is that they are purported to define what “good” is, from both the moral and theological point of view.

8.2.2 Freedom and Heteronomy: Ought Presupposes Can

The following approach is based on the insight that the most salient characteristics of the deontic imperatives listed above are:

- **Assumption of freedom of choice, or *ikhtiyār*:** the fact that an action can be chosen to be performed or not.
- **The heteronomy of imperatives:** the fact that the way actions are qualified by reward or sanction depends upon the choices made.

²⁴ Hilipinen and McNamara (2013, p. 14) indicate the proximity of Abd al-Jabbār’s formulation with that of Alexius Meinong.

Both conditions are linked to the idea of responsibility that is at the core of Ibn Ḥazm's understanding of obligation. This point has been stressed by Hourani (1985, p. 175), as follows:

The fact that concerns us in a historical account is that in all ethical contexts [Ibn Ḥazm] regards man as responsible for his own actions and liable to Reward and Punishment accordingly.

In our understanding, responsibility manifests itself in the fact that a legally accountable individual can choose to do or not to do some kind of action. On the other hand, reward and sanction are both dependent on the choices made.

In fact, Islamic Jurisprudence makes explicit the presuppositions for the application of a deontic qualification. Indeed, classifications such as obligatory, forbidden, and permissible, grounding a juridical decision (*ḥukm*) for a particular action (e.g., *it is forbidden to eat pork*), presuppose that (a) the person who performs an action is legally accountable (*mukallaf*); (b) the action in question is one for which the liberty to choose between carrying it out or not has been given (the provision of this liberty of choice is called *takhyr*).

Notice that this approach is quite different from current studies in deontic logic that include, as axiom, the implication $OA \supset MA$, where “O” stands for “obligatory” and “M” for “possible,” known as the principle that *Ought Implies Can*, and also dubbed *Kant's Principle (Sollen-Können-Prinzip)*.²⁵ According to our analysis of the Islamic conception, however, we find that:

- Every deontic qualification, and not only the obligatory, presupposes ethical/legal liability.²⁶

So, in this sense, *ought presupposes can*, whereby *liability* is understood as the presupposition of being able to perform (or abort) the action prescribed by the norm.²⁷

The logical upshot of all this is that the underlying structure is that of a hypothetical, such that if we accept to make the choice between performing or not

²⁵ Cf. Prior (1958), von Wright (1963, pp. 108–116, 122–125), Hilpinen (1981a, pp. 14–15), Chellas (1974), al-Hibri (1978, pp. 18–21), Hilpinen and McNamara (2013, p. 38).

²⁶ Hintikka's (1981, p. 86) analysis of Kant's principle is quite close to our view of the role of *takhyr*—though he speaks of **non-logical consequence or deontic consequence** rather than presupposition.

²⁷ However, if *can* is understood as some general form of *permissibility*, then all actions qualified as *mandatory* are also permissible. In such contexts permissible is defined as including all those actions that qualify for reward (i.e., those that are rewarded when carried out and those that are rewarded when not carried out). In fact, in his *al-Iḥkām fī Uṣūl al-Aḥkām* (vol. 8, p. 101), Ibn Ḥazm seems to extend his deontic system with notions of *forbidden to do* and *obligatory not to do*, based only on what is *permissible to do* or *not to do*. Accordingly, *forbidden is all that is not permissible to do*, and *obligatory is all that is not permissible not to do*. Still, there is another sense of “*can*” involved in Kant's principle—namely, as *ability to fulfil the duty*—that triggers some known puzzles of current deontic logic, including Chisholm's *contrary to the duty obligations* (see Chisholm (1963b), Hilpinen and McNamara (2013, section 8)). In a recent paper, Rahman et al. (2019a) discuss Chisholm's puzzle in the context of Islamic deontic categories.

performing a certain action, we are rewarded or sanctioned (or neither) in relation to this choice.

This already suggests the main idea behind our analysis of judgements involving deontic and modal concepts. In a nutshell, our point is to analyse such judgements as a conjunction of two implications, such that the truth of the antecedent of each of these implications is dependent upon (i.e., presupposes) a disjunction.²⁸ Take the case of the *conditional* expressing an obligation. This conditional is constituted by the following implications:

- If an action x of type A is performed, then it will be rewarded; and if it is omitted, then it will be sanctioned (omitting to perform A has been established by the legal system as triggering a sanction, i.e. the contrary of reward), provided there exists the choice of performing or not performing an action of the type A —that is, provided the disjunction $A \vee \neg A$.

Similarly, for the case of necessity as applied to events, assuming that the cause is both necessary and sufficient (for the sake of simplicity we consider a conjunction of causes as a unity):

- If event E occurs, then it is the presence of C that causes E to happen. The absence of C precludes E from happening, provided C is contingent—that is, provided the disjunction $C \vee \neg C$.²⁹

Now, before we develop our logical analysis, we will have a closer look at the historic sources of this parallelism, including a translation of the original text from Ibn Ḥazm's *Taqrīb*.

8.3 On Natural and Deontic Necessity

There are two distinctive approaches to modality in the Arabic tradition; namely: (a) an approach closer to the relational view of Aristotle, whereby necessity designates some specific form of predication (including temporality),³⁰ and (b) one that is based on a propositional structure yielding some form of conditional necessity.³¹

Thinkers in the Arabic tradition proposed innovations to both approaches. Ibn Ḥazm's parallelism, on the other hand, seems to combine them. For while deontic necessity is characterized by the distribution of *Reward* and *Sanction*, both of them defined as predicates over performances of actions, natural necessity is

²⁸ Cf. Rescher (1963, pp. 76–78), and Jadaane (1968, pp. 117–21). For a recent, thorough study of the notion of *sharṭiyya*-conditionality, see Hasnawi and Hodges (2016, section 2.4.3, pp. 63–65).

²⁹ We will return to this point below.

³⁰ For an overview of work on the first approach, see the sections on modal logic in Hasnawi and Hodges (2016), Strobino and Thom (2016).

³¹ To this point our suggestion is only that: a suggestion. A thorough examination of the implications of the notion of conditional modality for Arabic studies on causality has yet to be undertaken.

characterized by means of the predicates *Causes* and *Precludes*, both of them defined over the presence or absence of some specific conditions. This provided roots for transference from the realm of actions to the realm of causally (determined) events.³² In other words, the mirroring of deontic and modal concepts is a result of:

1. understanding modalities as affecting both actions and events;
2. mirroring the freedom of choice assumed by the legal system of sanctions and rewards within the contingency of natural events (or actions), given some natural conditions;
3. the idea that both modal and deontic qualifications admit levels that can be put in correspondence with each other.

Witness to the first point is that when Ibn Ḥazm, like other Muslim thinkers before him, speaks about modalities, he refers to them as qualifications of all “things”, *ashyā’* (s. *shay’*), including, here, actions and events. All these “things” can be classified as necessary, possible, or impossible.³³ Let us have a look at the original text.

Ibn Ḥazm, *al-Taqrīb li-Ḥadd al-Manṭiq wa-l-Mudkhal ilayhi bi-l-Alfāz al-Āmmiyya wa-l-Amthila al-Fiḥhiyya*.³⁴

Extract Translated by Walter Edward Young.

Chapter on elements (<i>anāṣir</i>)	باب العناصر
Know that the elements (<i>anāṣir</i>) of all things (<i>ashyā’</i>)—that is, their classes with regard to making assertions (<i>ikhbār</i>) about them—are of three classes, there being no fourth.	اعلم أنّ عناصر الأشياء كلّها أيّ أقسامها في الإخبار عنها ثلاثة أقسام لا رابع لها
[They are] either necessary (<i>wājib</i>), being such as are necessary and manifest, or from among such as must be, like the rising of the sun each morning, and the like of that, this being called in God’s laws the ‘obligatory’ (<i>farḍ</i>) and the ‘binding’ (<i>lāzim</i>);	إمّا واجب وهو الذي قد وجب وظهر أو ما يكون مما لا بدّ من كونه كطلوع الشمس كلّ صباح وما أشبه ذلك وهذا يسمّى في الشرائع الفرض واللازم

(continued)

³² In fact, alethic modalities also affect actions. However, in this case it seems that those actions are considered as determined by natural causality.

³³ As we shall see, in general *causal possibility* amounts to *contingency*, while *permissibility* (deontic possibility) amounts in principle to *optional*. In other words, in principle, causal possibility excludes necessity, while permissibility excludes obligation.

³⁴ Ed. Aḥmad b. Farīd b. Aḥmad al-Mazīdī, (Beirut: Manshūrāt Muḥammad Alī Bayḍūn, Dār al-Kutub al- Ilmiyya, 2003), pp. 83–84.

<p>or possible (<i>mumkin</i>), being such as might be and might not be, like our anticipation that it will rain tomorrow, and the like of that, this being called in God's law the 'lawful' (<i>ḥalāl</i>) and the 'permitted' (<i>mubāḥ</i>);</p>	<p>وإما ممكن وهو الذي قد يكون وقد لا يكون وذلك مثل توقعنا أن تمطر غداً وما أشبه ذلك وهذا يسمى في الشرع الحلال والمباح</p>
<p>or impossible (<i>mumtani</i>) being such as to which there is no path, like a human's remaining under water for an entire day, or his living a month without food, or his walking in the air without some cunning artifice, and the like of that. And this is the type of thing that, if we saw it manifest in a human, we would know he is a prophet; and this class is called in God's laws the 'forbidden' (<i>ḥarām</i>) and the 'prohibited' (<i>maḥẓūr</i>).</p>	<p>وإما ممتنع وهو الذي لا سبيل إليه كبقاء الإنسان تحت الماء يوماً كاملاً أو عيشه شهراً بلا أكل أو مشيه في الهواء بلا حيلة وما أشبه ذلك وهذه التي إذا ظهرت من إنسان علمنا أنه نبي وهذا القسم يسمى في الشرائع الحرام والمحظور</p>
<p>Furthermore, the possible (<i>mumkin</i>) is divided into three classes, there being no fourth:</p>	<p>ثم الممكن ينقسم أقساماً ثلاثة لا رابع لها</p>
<p>the nearly possible (<i>mumkin qarīb</i>), like the possibility of occurrence of rain upon a condensing of clouds in the two months of <i>Kānūn</i>,³⁵ or the victory of a large number of the courageous over a small number of the cowardly;</p>	<p>ممكن قريب كما يمكن وقوع المطر عند تكاثف الغيم في شهري كانون وغلبة العدد الكبير من الشجعان العدد اليسير من الجبناء</p>
<p>and the distantly possible (<i>mumkin ba ṭd</i>), which is like the defeat of a large number of the courageous at the hands of a small number of the cowardly, and like a cupper (<i>ḥajjām</i>) [i.e., a practitioner of cupping] taking charge of the Caliphate, and the like of that;</p>	<p>وممكن بعيد وهو كانهزام العدد الكثير من الشجعان عند عدد يسير من جبناء وكحجّام يلي الخلافة وما أشبه ذلك</p>
<p>and the purely possible (<i>mumkin maḥḍ</i>), whose two extremes are equal, this being like one standing—either he will walk or he will sit—and the like of that.</p>	<p>وممكن محض وهو يستوي طرفاه وهو كالمرء الواقف إما يمشي وإما يقعد وما أشبه ذلك</p>

(continued)

³⁵I.e., December and January.

<p>And likewise we find that this middle class [i.e., the <i>mumkin</i>, corresponding to the <i>mubāḥ</i>] is, in God's laws, divided into three classes: recommended-permitted (<i>mubāḥ mustahabb</i>); reprehended-permitted (<i>mubāḥ makrūh</i>); and evenly permitted (<i>mubāḥ mustawin</i>) having no tendency towards one of the two sides.</p>	<p>وكذلك نجد هذا القسم المتوسط في الشرائع ينقسم أقساماً ثلاثة فمباح مستحب ومباح مكروه ومباح مستوي لا ميل له إلى أحد الجهتين</p>
<p>As for recommended-permitted (<i>mubāḥ mustahabb</i>), it is such that when you do it you are rewarded (<i>ujirta</i>), but if you neglect it you do not sin (<i>lam ta'tham</i>) and you are not rewarded; like praying two supererogatory prayer-cycles, voluntarily.</p>	<p>فأما المباح المستحب فهو الذي إذا فعلته أجرت وإذا تركته لم تأثم ولم تؤجر مثل صلاة ركعتين نافلة تطوعاً</p>
<p>And as for reprehended-permitted (<i>mubāḥ makrūh</i>), it is such that when you do it you do not sin and you are not rewarded, but if you neglect it you are rewarded; and that is like eating while reclining, and the like.</p>	<p>وأما المباح المكروه فهو الذي إذا فعلته لم تأثم ولم تؤجر وإذا تركته ولم تؤجر مثل صلاة ركعتين نافلة تطوعاً</p>
<p>And as for evenly permitted (<i>al-mubāḥ al-mustawī</i>), it is such that when you do it or you neglect it you do not sin and you are not rewarded; and that is like dyeing your garment whichever colour you please, and like your riding whichever beast of burden you wish, and the like.</p>	<p>وأما المباح المستوي فهو الذي [إذا] فعلته أو تركته لم تأثم ولم تؤجر وذلك مثل صبغك ثوبك أي لون شئت وركوبك أي حمولة شئت ونحوه</p>

As pointed out by such scholars as Chejne (1984), Guerrero (1997, 2013), Lameer (2013) and Puerta Vílchez (2013), Ibn Ḥazm's parallelism between deontic and modal notions emerged from the task of bridging the modal logic of Aristotelians and Post-Aristotelians with Islamic Jurisprudence.

The source of distinguishing between different levels of alethic modality within the natural realm seems to be Aristotle's distinction, in the *Peri Hermeneias* (chapter 9), between different cases of contingent events:³⁶

³⁶ We owe this reference to Carlo Natali (Università Ca' Foscari Venezia). The authors of the present paper enjoyed further assistance on this topic from Tony Street (University of Cambridge), and Robert Wisnovsky (McGill University).

*Thus it is clear that not everything is or takes place of necessity. Cases there are of contingency ; no truer is then the affirmative, no falser, than the negative statement. Some cases, moreover, we find that, at least, for the most part and commonly, tend in a certain direction, and yet they may issue at times in the other or rarer direction.*³⁷

Importantly, Ammonius, in treating this passage in his famous commentary on Aristotle, is strikingly close to Ibn Ḥazm's above-quoted passage on the possible:

The contingent is divided into three: one is called 'for the most part' (hōs epi to polu), for example that a man is born with five fingers or becomes gray with age (for things behaving otherwise are rare); another is 'for the lesser part' (hōs ep' elatton), for example that one digging comes upon a hoard; and the last is 'equally <often>' (ep' isēs), for example to bathe or not to bathe and to walk or not to walk.

[...]. *Concerning the contingent <which occurs> 'equally<often>' there is only choice, for example to go out or not to go out, to converse or not. Only this species of the contingent is called 'however it chances', because its existence is no more or less <frequent> than its non-existence, but whichever part of the contradiction it chances can equally occur.*³⁸

Now, despite the structural similarities, there is no evidence that Ibn Ḥazm's parallelism is rooted in Ammonius's commentary. Nor is there evidence as to what version of Aristotle's text was accessible to our author.³⁹

8.4 Deontic Imperatives and the CTT-Analysis of Hypotheticals

Per Martin-Löf's (1984) Constructive Type Theory (CTT) provides a thorough formal framework whereby categorical and hypothetical judgements can be explicitly distinguished at the object-language level without conflating judgements with the propositions that constitute them.⁴⁰

Since these distinctions are crucial for the formal reconstruction of traditional logic in general—and of the Arabic tradition in particular—we have chosen to employ the language of CTT for our logical study on the origins of deontic concepts. More precisely, the CTT-framework allows one to distinguish, at the language level, both the *taṣawwur* of a judgement, i.e., its *conceptualization* or (roughly)

³⁷ *Peri Hermeneias*, chapter 9, 19a18–22, in Aristotle (1962), p. 139.

³⁸ Ammonius (1998) 142.1, pp. 104–105.

³⁹ Another plausible source is Paul the Persian's summary of Ammonius's commentary (i.e., the translation from Persian into Syriac by Sebokht). Paul the Persian stresses the role of human choice in the context of moral issues in his "l'Élucidation du Peri Hermeneias," section 26. Translated from the Syriac by Henri Hugonnard-Roche (2013, p. 63). Robert Wisnovsky (McGill University) has pointed out to the authors of the present paper that Yahyā ibn Adī's work on the nature of the possible might also constitute a possible source (see Ehrig-Eggert 1990).

⁴⁰ See Martin-Löf (1984, pp. 9–10). For a short introductory survey, see Rahman et al. (2018, chapter II).

proposition, and its *taṣdīq*, or *assent* or *believing to be true*, i.e., the act of judgment itself, or, sometimes, the linguistic expression of that act.⁴¹

8.4.1 Quantifying Over Actions

The translation of the text suggests the following logical analysis of obligatory actions, with “*g*” for agent, “*O*” for obligation:

$O_g A$ iff

if *A* is carried out by *g*, *g* is rewarded & if *A* is not carried out by *g*, *g* is sanctioned & *g* is able to both carry out and not carry out *A*.

In principle, the overall logical structure of the analysis comes quite close to this conjunction. Nonetheless, we would like to

1. express at the object-language level that the primary subject of sanction or reward is the performance of *some specific type of action*. Sanction and reward are predicates (propositional functions) defined with regard to performance of actions, not just propositional variables. This requires performance of actions to be denizens of the domain of quantification. Thus, from this perspective, sanction and reward apply indirectly to the agent: the agent is sanctioned and rewarded for his actions. Perhaps it would be more accurate to display a double qualification: the performance of the action is law-breaking and the agent is therefore sanctioned. However, in order to keep our analysis as simple as possible, we will focus only on the agent’s performance, rather than on the agent.
2. render the ability to carry out *A* and not carry out *A* as a presupposition; namely, the legal presupposition of liability mentioned above. If the presupposition is not fulfilled the norm does not apply.

Let us now elaborate on these objectives.

As for the first: It has been well known since at least the time of Aristotle that examples such as *Some cobblers are good (as cobblers)*, cannot be analysed as a conjunction; in Aristotle’s framework, the distinction Subject and Predicate renders: *Good is said of some cobblers*. CTT, however, has a very distinctive approach to such kinds of “complex predicates;” namely, *dependent types*.

The idea is that the expression *Good(x)*, is a proposition (i.e., is of the type proposition), provided (under the hypothesis), that *x* is an element of the domain

⁴¹ Translating *taṣdīq* by *assent* follows the choice of Hasnawi and Hodges (2016, pp. 56–57), though the authors warn that such a choice might be controversial. In fact, Lameer (2014, p. 403, n. 16) strongly contests this translation and suggests *believing to be true*, such as *what we do when we believe in first principles* or *what we do not do when we accept (or adopt) a false proposition for the sake of an argument* (such as in a proof *per impossibile*). Perhaps *endorsement* would also be a good choice, meaning such acts by which we not only accept that a proposition is true but we employ it for our own inferences.

(set) of cobblers. In short, $Good(x)$ is a propositional function defined over the domain $Cobbler$. Thus, within CTT, the familiar notion of propositional function will be rendered as

$Good(x) : prop (x : Cobbler)$, given $Cobbler : set$

whereby “*prop*” stands for the dependent type proposition, “*Cobblers*” stands for the type *set*, and the colons admit the glosses “is an instance of”, or “is an element of.” Strictly speaking, because of the Curry-Howard isomorphism between types, sets, and propositions, $Good(x)$ and $Cobblers$ are both of the same type, *prop* or *set*.⁴² However, the former is a type dependent upon the latter: if there are no cobblers, there are no good cobblers. We use the denominations *set* and *prop* in order to stress the dependence of the latter upon the former.

Now, the existential *Some cobblers are good (as cobblers)*—called, in its general form, a *Sigma-type* (i.e. when conceived as the disjoint union of a family of sets)—receives the notation

$(\exists x : Cobbler) Good(x)$ **true**

which comes quite close to the subject-predicate structure of ancient logic.

What makes this *true* is a pair (its *proof-object*) of elements, such that the first element of the pair, a , is a cobbler (an element of the set of cobblers), and the second element, b , is a function that takes the first element (the chosen cobbler) and makes $Good$ cobbler true by rendering an instance of a good cobbler. Thus:

$(a,b) : (\exists x : Cobbler) Good(x)$

whereby

$Good(x) : prop (x : Cobbler)$, given $Cobbler : set$

$a : Cobbler$ (a is a cobbler/ a is an element of the set cobblers)

$b : Good(a)$ (b verifies that cobbler a is a good cobbler).

From the point of view of the elimination rules, suppose $(\exists x : Cobbler) Good(x)$. Then there is some (complex) proof-object making the existential true, say, c

$c : (\exists x : Cobbler) Good(x)$

given the same assumptions as before.

We know that “ c ” must be a pair. The functions, $p(c)$ and $q(c)$, called the left and right projections, render the first and the second element of the pair; namely,

⁴² More generally, within CTT a proposition is interpreted as a set the elements of which represent the proofs of the proposition, the solution to a problem, or the fulfilment of an expectation. It is also possible to view a set as a problem description in a way similar to Kolmogorov’s explanation of the intuitionistic propositional calculus. In particular, a set can be seen as a specification of a programming problem, the elements of the set are then the programs that satisfy the specification (see Martin-Löf 1984, p. 7). Furthermore, in CTT sets are also understood as types, so that propositions can be seen as data- (or proof-) types.

$p(c)$: *Cobbler* and $q(c)$: *Good(p(c))*

that evaluate as follows

$p(a,b) = a$: *Cobbler* and $q(a,b) = b$: *Good(p(c))*.

If we bind the free variable with a universal quantifier we obtain the following assertion, which, in its general form, is said to constitute a Π -type (the CTT-term for assertions where a universal quantifier occurs as the main logical operator—more generally, they stand for the cartesian product of a family of sets):

$(\forall x: \text{Cobbler})\text{Good}(x)$ *true*

What makes it *true* is a function expressing the fact that there is a method associating any chosen cobbler with the predicate *Good(x)*, given *Good(x): prop (x : Cobbler)*, given *Cobbler: set*.

Returning to our subject, the point is that the specificity of the sanction or law-breaking, as qualifying performances of, say, the type of action *A*, is to be understood as a dependent type. After all, sanctioning indiscriminate killing is quite different from sanctioning not paying the alms-tax (زكاة, *zakāt*) prescribed by the Law. In other words the formation rules for sanction are

Sanction(x): prop (x : A), given *A: set*

Thus, a first rendering of **OgA**, yields

$(\forall y : A) R(y) \wedge (\forall z : \neg A) S(z)$

Every performance of A is rewarded and every performance of $\neg A$ is sanctioned.

Notice that in our framework, omitting to perform an action that instantiates the action-proposition *A* (i.e., not doing it) is conceived of as frustrating the performance of an action of that type, e.g., *stopping (or inhibiting) eating or drinking when a day of fasting in Ramadan begins*. This interpretation is close to the notion of aborting a process that grounds the inferential notion of negation (see Martin-Löf 1984, p. 36).

So far, so good. However, we would like to express not only that being able to do *A* and $\neg A$ is a constitutive part of the distribution of reward and sanction prescribed by the norm, but also that performances of *A* are sanctioned (or rewarded) because $\neg A$ might have been chosen. This brings us to the second objective.

As for the second objective: Let us look once more at the notion of dependent proof-object and hypothetical judgements. The only means by which standard frameworks can deal with hypothetical judgments are implications. In CTT, the distinction between categorical judgments and hypothetical judgments is rendered in a quite straightforward manner:

A categorical judgment is true if there is an independent proof-object for the proposition involved. *C true* expresses a categorical if some proof-object *c* can be found that makes it true:

$c : C$.

A hypothetical judgment is true, if it is true under the proviso of some hypothesis (or hypotheses)

$B(x) \text{ true } (x : C)$

whereby

$C : \text{set}$ and $B(x) : \text{prop } (x : C)$

the bracketed expression to the right being the hypothesis conditioning the truth of the main proposition, the proof-object of which is a dependent object, that is a function

$b(x) : B(x) \text{ true } (x : C)$.

Clearly, an assertion involving a hypothetical judgement does not express that the condition C has been verified; but it asserts that if the condition is verified, then $B(x)$ is true—if a method can be found that transforms this verification into a verification of the main proposition.

Thus, if we have $b(x) : R(x) (x : A)$ as a premise, and we have as a second premise the fact that indeed there is a performance a of the action-proposition A (i.e., if we have as a premise $a : A$), then we can infer that performance a will be rewarded (i.e., $b(a) : R(a)$). In plain words, from the premises

- (i) Any performance x of an action will be rewarded, provided it is the performance of an action of the type A ,
- (ii) a is such a performance ($a : A$),

we can infer:

- (iii) Performance a is rewarded ($b(a) : R(a)$).

$$\frac{a : A \quad b(x) : R(x) (x : A)}{b(a) : R(a)}$$

The same holds for sanctions for performing forbidden actions, or for omitting to perform obligatory actions.

Relevant to our aims is the fact that the CTT analysis of hypothetical judgments also provides the means for analyzing presuppositions, such as are found in the classic example: *Sam stopped smoking* (which presupposes that Sam smoked):

$\text{Sam stopped smoking}(x) \text{ true } (x : \text{Sam smoked})$

$b(x) : \text{Sam stopped smoking}(x) (x : \text{Sam smoked})$

With regard to our subject, the presupposition of the predicates *Sanction* and *Reward* is that the actions to which they are predicated can be either carried out or not carried out. In other words, *Sanction* and *Reward* presuppose $A \vee \neg A$.

Now we need to express the dependence of the rewarding or sanctioning upon the presupposition that a choice has been made.⁴³ More precisely, if we are describing an *obligatory action*, we need to express the following:

Obligatory action:

- (i) *If the individual **g** made the choice to perform an action of type A (i.e., if there is a proof-object that makes the **left side** of the disjunction true) then this performance is rewarded.*
- (ii) *If the individual **g** made the choice of omitting an action of type A (i.e., if there is a proof-object that makes the **right side** of the disjunction true) then this omission is sanctioned.*

Finally, if we pull all this together, and employ the abbreviation $\{H\}$ for $x : A \vee \neg A$, we obtain:

$$b(x) : [(\forall y : A) \mathbf{left}^{\vee}(y) = \{H\}x \supset R(y)] \wedge [(\forall z : \neg A) \mathbf{right}^{\vee}(z) = \{H\}x \supset S(z)] (x : A \vee \neg A)$$

wherein the expressions “**left**[∨](y)” and “**right**[∨](z)” stand for the injections rendering the disjunction $A \vee \neg A$ *true*;⁴⁴ and “**left**[∨](y) = $\{H\}x$ ” stands for the choice of performing an action of the type (of the action-proposition) A ; and “**right**[∨](z) = $\{H\}x$ ” stands for the choice of not performing this type of action-proposition.⁴⁵ The expression “**right**[∨](z) = $\{H\}x$ ” where the identity relation “=” occurs can be glossed as follows:

Any performance of A (or act of omitting A) is identical to the proof-object that renders true the disjunction—by rendering true either the left or the right of $A \vee \neg A$.

Thus:

$$(\forall y : A) \mathbf{left}^{\vee}(y) = \{H\}x \supset R(y) \text{ true } (x : A \vee \neg A)$$

⁴³ **Caveat:** In the context of Islamic Law, omitting to perform an obligatory action, or performing a forbidden one, is sanctioned **if the omission z is not excused**; i.e., provided that $\neg E(z)$ applies (the prescribed fasting during Ramadan is not obligatory, for example, while travelling). Note that this kind of proviso is also very important in contemporary European Civil Law. It can be integrated into the hypothetical as follows:

- $(\forall y : A) [\mathbf{left}^{\vee}(y) = \{H\}x \supset R(y)] \wedge [(\forall z : \neg A) (\neg E(z) \wedge \mathbf{right}^{\vee}(z) = \{H\}x \supset S(z)) \text{ true } (x : A \vee \neg A)$.

In the present paper, however, we will leave out $E(x)$; it relates to defeasibility, an issue linked to the dynamics of argumentation which has been dealt with elsewhere (Rahman and Iqbal 2018), while Rahman et al. (2019b, chapter 2) discuss defeasibility in the context of legal argumentation in Islamic Law, though they do not combine it with an analysis of the deontic qualifications of actions.

⁴⁴ We have slightly changed the notation for injections, which when they occur as proof-objects of a disjunction usually take the notation $i(x)$ and $j(x)$ (see Ranta 1994, pp. 47). The injection **left**[∨](a), takes an element of A , namely a (in our case, the performance a of action A) and renders a as proof-object for the left side of the disjunction.

⁴⁵ Cf. Ranta (1994, pp. 52–53).

which reads

Assuming that, given the choice of performing or not performing an action of type A, performing it has been chosen (i.e., if the left side of the disjunction has been chosen to be performed), then, for any performance y within the set A that is identical to this choice (within the set $\{A \vee \neg A\}$), reward (for performing this action) follows.

And a similar reading admits

$$(\forall z : \neg A) \text{right}^\vee(z) = \{H\}.x \supset S(z) \text{ true } (x : A \vee \neg A).^{46}$$

Notice that the hypothesis or presupposition $x : A \vee \neg A$, carries a modal feature: sanction or reward apply only if the presupposition *can* be fulfilled. Indeed, within CTT, assertions under hypotheses are understood as modalities.⁴⁷

Moreover, if, as will be suggested below, the presupposition is bounded by a universal quantifier, we have all the universal scope that other frameworks capture by introducing an alethic necessity operator, such as Anderson's (1958) reduction OA iff $\Box[(A \supset R) \wedge (\neg A \supset S)]$.⁴⁸

The key intent of our approach is that we neither quantify over possible worlds nor cast action and reward (or sanction) as an implication between elementary propositions. On the one hand, as mentioned above, possible world semantics does not seem to render the notion of modality at stake in ancient logic; on the other hand, the legal/ethical effects of performing an action are to be conceived as acts of predication. In brief, quantifying over events and actions yields, so we claim, a framework that makes the insights on modalities contained in the textual sources under consideration more apparent.⁴⁹

Note, too, that our formal reconstruction, though grounded in an inferentialist approach to ancient logic,⁵⁰ does not commit to intuitionistic logic. Within the CTT approach, third-excluded is added as an explicit assumption, not unlike standard proof-theoretical presentations of classical logic within the framework of natural

⁴⁶ The notation for propositional identity, namely " $x = \{D\}y$ ", meaning " x is identical to y within the set D ," is more similar to what is employed in first-order logic. In fact, within the CTT-framework, the usual notation is $\text{Id}(D, x, y)$.

⁴⁷ Cf. Ranta (1994) chapter 7.

⁴⁸ Cf. Chisholm (1963a). For an overview, see Hilpinen and McNamara (2013, section 6.2).

⁴⁹ Our integrating quantification over events and actions within a CTT framework is based on Ranta's (1994) discussions. Indeed, Ranta (1994, p. 54) combines CTT with Davidson's (1980, essays 6–10) idea that an individual action makes an action-proposition *true*. Accordingly, the proposition (that) *al-Fārābī read Aristotle's Analytica Posteriora* is made *true* by individual readings of al-Fārābī performing actions of that type. Notice that this not only fits nicely with Ibn Ḥazm's original text, where he uses the term *al-ashyā'*, "things," to include actions and events, but, as mentioned in the preface and discussed below, it is a consequence of the insight that deontic and modal concepts qualify both actions and events.

⁵⁰ As do the reconstructions by Ebbinghaus (1964), Corcoran (1972), Thom (1981), Marion and Rückert (2016), and Crubellier et al. (2019); for syllogistic in general see Malink and Rosen (2013), and for modalities see Strobino and Thom (2016).

deduction.⁵¹ In our case, the universal binding of the hypothesis implements the classical assumption. Accordingly, obligation now receives the notation

$$(\forall x : A_1 \vee \neg A_1) \{ [(\forall y : A_1) \mathbf{left}^\vee(y) = \{H1\}x \supset R_1(y)] \wedge [(\forall z : \neg A_1) \mathbf{right}^\vee(z) = \{H1\}x \supset S_1(z)] \} \mathbf{true}$$

8.4.2 Logical Analysis of Ibn Ḥazm's Heteronomous Imperatives

As already mentioned, deontic qualifications of actions presuppose that the performer is legally accountable and has been given the liberty to choose (*takhyīr*) between two alternatives. The CTT-framework for hypotheticals provides the formal means to express (i) that the deontic qualifications assume such a choice, and (ii) that *sanction* and *reward* are predicates that apply to performances of the action at stake. Each of the deontic concepts determines a **subset** of a general set of actions in a straightforward manner:

- **wājib, farḍ, lāzim**: Doing A_1 is rewarded. Omitting A_1 is sanctioned.
 $b_1(x) : [(\forall y : A_1) \mathbf{left}^\vee(y) = \{H1\}x \supset R_1(y)] \wedge [(\forall z : \neg A_1) \mathbf{right}^\vee(z) = \{H1\}x \supset S_1(z)] (x : A_1 \vee \neg A_1)$.
- **ḥarām, maḥzūr**: Doing A_2 is sanctioned. Omitting A_2 is rewarded.
 $b_2(x) : [(\forall y : A_2) \mathbf{left}^\vee(y) = \{H2\}x \supset S_2(y)] \wedge [(\forall z : \neg A_2) \mathbf{right}^\vee(z) = \{H2\}x \supset R_2(z)] (x : A_2 \vee \neg A_2)$.
- **mubāḥ mustaḥabb**: Doing A_3 is rewarded. Omitting A_3 is neither sanctioned nor rewarded.
 $b_3(x) : [(\forall y : A_3) \mathbf{left}^\vee(y) = \{H3\}x \supset R_3(y)] \wedge [(\forall z : \neg A_3) \mathbf{right}^\vee(z) = \{H3\}x \supset (\neg S_3(z) \wedge \neg R_3(z))] (x : A_3 \vee \neg A_3)$.
- **mubāḥ makrūh**: Omitting A_4 is rewarded. Doing A_4 is neither sanctioned nor rewarded.
 $b_4(x) : [(\forall y : A_4) \mathbf{left}^\vee(y) = \{H4\}x \supset (\neg S_4(y) \wedge \neg R_4(y))] \wedge [(\forall z : \neg A_4) \mathbf{right}^\vee(z) = \{H4\}x \supset R_4(z)] (x : A_4 \vee \neg A_4)$.
- **mubāḥ mustawin**: Doing A_5 is neither sanctioned nor rewarded. Omitting A_5 is neither sanctioned nor rewarded.
 $b_5(x) : [(\forall y : A_5) \mathbf{left}^\vee(y) = \{H5\}x \supset (\neg S_5(y) \wedge \neg R_5(y))] \wedge [(\forall z : \neg A_5) \mathbf{right}^\vee(z) = \{H5\}x \supset (\neg S_5(z) \wedge \neg R_5(z))] (x : A_5 \vee \neg A_5)$.

In some contexts, it might be desirable to define deontic qualifications as expressions building propositions. In fact, it is quite straightforward, since a hypothetical is one inference away from a universal:

$$(\forall x : A_1 \vee \neg A_1) \{ [(\forall y : A_1) \mathbf{left}^\vee(y) = \{H1\}x \supset R_1(y)] \wedge [(\forall z : \neg A_1) \mathbf{right}^\vee(z) = \{H1\}x \supset S_1(z)] \} \mathbf{true}$$

Thus, the whole expression can form new propositions in the usual way; for example, as the consequent of some implication, and so on. The like applies to *Forbidden* and *Permissible*.

⁵¹ In fact, as pointed by Sundholm (2004), the proof-theoretical meaning explanations of CTT are logically neutral.

8.5 A Landmark in the History of the Logical Analysis of Norms. Natural and Deontic Modalities

As discussed above, Ibn Ḥazm's examples of modalities and the denominations he employs make apparent one of the most innovative insights of the Arabic perspective on deontic concepts, namely, the correspondence between deontic and alethic modalities. In the following paragraphs we will propose a logical analysis that renders the causal structure of natural modalities and makes the parallelism apparent. This suggests that heteronomous imperatives and natural modalities share a similar hypothetical structure. In the realm of alethic modalities as applied to natural events the relevant hypothesis/presupposition is the contingency of the causal conditions. Thus, the hypothesis $x : C_1 \vee \neg C_1$, indicates that the causal condition may or may not happen. If, again, we bind this with a universal quantifier, we will express the classical assumption that one of the two will necessarily obtain.

The main idea of our reconstruction of alethic modalities is that the different categories result from comparing the cardinality of the occurrence of causal conditions with the occurrence of the correspondent effect. A primary motivation animating our approach is that the logical analysis of alethic modalities should be compatible with the several forms of occasionalism developed by Muslim thinkers.

From the perspective of contemporary logic, generalized quantifiers (such as *Most*, *As-Many*, etc.) are the instruments for rendering propositions involving cardinality. Let us now introduce the logical analysis of the generalized quantifiers *More* and *As-Many*, recalling briefly their notation in the context of CTT (Sundholm (1989) provides a thorough study on the constructive formation of such quantifiers).

More($D, (x)A, (x)B$) There are more A in D than B . Here $(x)A$ indicates that the variable x is bound in the predicate A —the same applies to $(x)B$.
(The definition amounts to verifying that there is no surjection from B to A .)

This can easily be generalized for two domains:

More($D_1, (x)A; D_2, (x)B$) There are more A in D_1 than there are B in D_2 .

As-Many($D, (x)A, (x)B$) There are as many A in D as B .

As-Many($D_1, (x)A; D_2, (x)B$) There are as many A in D_1 as there are B in D_2 .

(The definition amounts to verifying that there is a bijection.)

We apply these generalized quantifiers to *Cause* and *Effect*. In the present context, we obtain the propositional function: ⁵²

$x = \{C\}x(x) : prop (x : C)$ If we take an arbitrary x from the set C , then we gather the proposition that this x is identical to itself.

⁵² Recall that our notation for propositional identity is " $x = \{D\}y$ ", standing for " x is identical to y within the set D " rather than " $Id(D, x, y)$ ".

Or, in a notation uniform to the one employed for generalized quantifiers:

$(C, (x)x = \{C\}x) : prop$ “ $(x)\varphi$ ” indicates that x is (existentially) bound in C (in our case φ is $x = \{C\}x$).

Let us illustrate by way of two examples:

- *As-Many*($C, (v)E; C, (s)s = C_1s$).
***E* happens as often as *C* does.**
More precisely, there are as many cases of presences of *E* as there are instances of *C*.
- *More*($C_3, (v)E_3, (v)\neg E_3$).
***E* is present more often than not (*Most C* are *E*).**

Now we are ready to deal with the natural modalities. For example, the logical structure of *near possibility*, in both the causal and the simpler formulation, is the following:

mumkin qarīb.

If, *E* is present when *C* is; then *E* is present more often than not (*Most C* are *E*).

$\{ (\forall y : C_3) [\text{left}^\vee(y) = \{H\} x \supset (\forall w : E_3 \vee \neg E_3) \{ (\forall u : E_3) [\text{left}^\vee(u) = \{H\} w \supset \text{More}(C_3, (v)E_3, (v)\neg E_3)] \}] \} \}$ **true** ($x : C_3 \vee \neg C_3$).

Notation: Bear in mind the convention that the subscripts “ $\{H\}$ ” and “ $\{H'\}$ ” stand as abbreviations for the set (in our case a hypothesis) upon which the corresponding propositional identity has been defined, such as $\{A_1 \vee \neg A_1\}$, $\{C_1 \vee \neg C_1\}$, $\{E_1 \vee \neg E_1\}$.

The Logical Structure of Natural and Deontic Modalities

<p>wājib, farḍ, lāzim. If, given A or not-A, A is performed, then this performance is rewarded; but if not-A is performed, then sanction follows. $[(\forall y : A_1) \text{left}^\vee(y) = \{H\}x \supset R_1(y)] \wedge [(\forall z : \neg A_1) \text{right}^\vee(z) = \{H\}x \supset S_1(z)]$ true $(x : A_1 \vee \neg A_1)$.</p>	Deontic
<p>wājib. If, given C or not-C, E is present when C is, then E happens as often as C does. $\{ (\forall y : C_1) [\text{left}^\vee(y) = \{H\}x \supset (\forall w : E_1 \vee \neg E_1) \{ (\forall u : E_1) [\text{left}^\vee(u) = \{H\}w \supset \text{As-Many}(C_1, (v)E_1; C_1, (s)s = C_1s)] \}] \} \}$ true ($x : C_1 \vee \neg C_1$).</p>	Modal
<p>ḥarām. If, given A or not-A, A is performed, then this performance is sanctioned; but if not -A is performed, then reward follows. $[(\forall y : A_2) \text{left}^\vee(y) = \{H\}x \supset S_2(y)] \wedge [(\forall z : \neg A_2) \text{right}^\vee(z) = \{H\}x \supset R_2(z)]$ true $(x : A_2 \vee \neg A_2)$.</p>	Deontic
<p>mumtani . If, given C or not-C, not-E is present when C is, then, the number of absences of E equals the number of presences of C. $\{ (\forall y : C_2) [\text{left}^\vee(y) = \{H\}x \supset (\forall w : E_2 \vee \neg E_2) \{ (\forall u : \neg E_2) [\text{right}^\vee(u) = \{H\}w \supset \text{As-Many}(C_2, (v)\neg E_2; C_2, (s)s = C_2s)] \}] \} \}$ true ($x : C_2 \vee \neg C_2$).</p>	Modal

<p><i>mubāh mustahabb</i>. If, given <i>A</i> or <i>not-A</i>, <i>A</i> is performed, then this performance is rewarded; but if <i>not-A</i> is performed, then neither reward nor sanction follows.</p> $[(\forall y : A_3) \text{left}^\vee(y) = \{H\}x \supset R_3(y)] \wedge [(\forall z : \neg A_3) \text{right}^\vee(z) = \{H_3\}x \supset (\neg S_3(z) \wedge \neg R_3(z))] \text{true} (x : A_3 \vee \neg A_3).$	Deontic
<p><i>mumkin qarīb</i>. If, given <i>C</i> or <i>not-C</i>, <i>E</i> is present when <i>C</i> is, then <i>E</i> is present more often than not (<i>Most C</i> are <i>E</i>).</p> $\{ (\forall y : C_3) [\text{left}^\vee(y) = \{H\}x \supset (\forall w : E_3 \vee \neg E_3) \{ (\forall u : E_3) [\text{left}^\vee(u) = \{H\}w \supset \text{More}(C_3, (v)E_3, (v)\neg E_3)] \}] \} \text{true} (x : C_3 \vee \neg C_3).$	Modal
<p><i>mubāh makrūh</i>. If, given <i>A</i> or <i>not-A</i>, <i>A</i> is performed, then this performance is neither rewarded nor sanctioned; but if <i>not-A</i> is performed, then reward follows.</p> $[(\forall y : A_4) \text{left}^\vee(y) = \{H\}x \supset (\neg S_4(y) \wedge \neg R_4(y))] \wedge [(\forall z : \neg A_4) \text{right}^\vee(z) = \{H\}x \supset R_4(z)] \text{true} (x : A_4 \vee \neg A_4).$	Deontic
<p><i>mumkin ba id</i>. If, given <i>C</i> or <i>not-C</i>, <i>E</i> is present when <i>C</i> is, then <i>not-E</i> is more often present than not present (<i>Most C</i> are <i>not-E</i>).</p> $\{ (\forall y : C_4) [\text{left}^\vee(y) = \{H\}x \supset (\forall w : E_4 \vee \neg E_4) \{ (\forall u : E_4) [\text{left}^\vee(u) = \{H\}w \supset \text{More}(C_4, (v)\neg E_4, (v)E_4)] \}] \} \text{true} (x : C_4 \vee \neg C_4).$	Modal
<p><i>mubāh mustawin</i>. If, given <i>A</i> or <i>not-A</i>, <i>A</i> is performed, then this performance is neither rewarded nor sanctioned; the same holds for performances of <i>not-A</i>.</p> $[(\forall y : A_5) \text{left}^\vee(y) = \{H\}x \supset (\neg S_5(y) \wedge \neg R_5(y))] \wedge [(\forall z : \neg A_5) \text{right}^\vee(z) = \{H\}x \supset (\neg S_5(z) \wedge \neg R_5(z))] \text{true} (x : A_5 \vee \neg A_5).$	Deontic
<p><i>mumkin maḥḍ</i>. If, given <i>C</i> or <i>not-C</i>, <i>E</i> or <i>not-E</i> are present when <i>C</i> is, then, when <i>E</i> is present, this presence equals the number of absences.</p> $\{ (\forall y : C_5) [\text{left}^\vee(y) = \{H\}x \supset (\forall w : E_5 \vee \neg E_5) \{ (\forall u : E_5) [\text{left}^\vee(u) = \{H\}w \supset \text{As-Many}(C_5, (v)E_5, (v)\neg E_5)] \}] \} \text{true} (x : C_5 \vee \neg C_5).$	Modal

At this point, the reader might wonder why we need this heavy hypothetical structuring, given that the entire logical operation seems to be delivered by the generalized-quantifier-expressions. In fact, it is true that the generalized quantifiers perform the logical operation properly; however, if we retain only them, we lose the epistemological features of the relevant texts. Take the example of *distant possibility*. The point is not only to indicate that it is more often the case that the event does not happen than that it does, but also to express explicitly that, *given some specific contingent conditions, if the event is present, then it is unlikely so*.

8.6 Ibn Ḥazm's Parallelism, *Qiyās*, and the Inferential Structure of Imperatives

It is interesting that Ibn Ḥazm's parallelism fits almost perfectly with the general structure of inferences via parallel reasoning and analogy subsumed by the term *qiyās*, and developed with skill and sophistication by the *Shāfi* *īs* and other schools

of Islamic jurisprudence.⁵³ It is particularly significant because, despite having adhered to the Shāfi *ī madhhab* earlier in his career, Ibn Ḥazm eventually converted to the Zāhirī *madhhab*—in fact becoming its most famous proponent—and rejected with vehemence any form of reasoning by parallelism or analogy, a stance in keeping with the radically literalist Zāhirī approach to the interpretation of the sacred sources.

Nevertheless, let us recall that his main motivation for the parallelism was to translate into Islamic juridical terms the logic of the Greek tradition. He did not mean to employ this parallelism as a form of reasoning to interpret the sacred texts, but as a way to bring together the approaches to necessity and normativity in the Greek tradition of logic and Islamic jurisprudence. It might also be argued that the parallelism which is the subject of our paper occupies a completely different level than the sort of parallelism (i.e., *qiyās*) which he rejects.

Still, when Ibn Ḥazm defends the study of logic, he defends it for its fruitful application in jurisprudence. Moreover, Ibn Ḥazm elsewhere explicitly defends a crucial principle of pure permissibility; namely: *All actions are permissible unless proscribed by Law*.⁵⁴

As mentioned earlier, this principle negotiated with that set of correlational modes of argumentation called *qiyās*, developed and applied by most Sunni schools, which integrated “updated” deontic qualifications for novel and still-contended cases into expanding bodies of substantive law. In fact, in conjunction with other factors, this principle opens a vast arena for juristic contention and disagreement (*ikhtilāf*). This, coupled with—and governed by—systematized if constantly evolving rules for juristic dialectic (*jadallmunāzara*), made possible a set of systems capable of continual updating and refinement, in a dynamic cycle Young (2017) calls the *dialectical forge*. Without such principles, methods, and disputational dynamics, Islamic legal systems would have remained closed and static.

Moreover, in this context, iterations such as *It ought to be that A is obligatory*, should not be read as the sheer iteration of the deontic operator, but as the call to integrate the obligation of *A* into the legal system. In other words, *It ought to be that A is obligatory* reads

The norm “A is obligatory” should be integrated into the legal system.

And this again calls for a legal argument justifying the claim. The same holds for *It ought to be that A is reprehended*, and *It ought to be that A is recommended*.

On the other hand, Ibn Ḥazm’s literalist approach is developed within a framework where, as already mentioned, language is conceived quite statically, and where

⁵³ For a comprehensive study of the *qiyās* theory of the Shāfi *ī* Abū Ishāq al-Shīrāzī, see Young (2017). Rahman and Iqbal (2018) developed a formalism based on Young’s study, already employing the structure of hypothetical imperatives. See also Rahman et al. (2019b).

⁵⁴ That is, the principle that “the original state of things is permissibility except for what the Law proscribed” (*al-aṣl fi’l-ashyā’ al-ibāḥa illā mā ḥazarahu al-Shar*). See Ibn Ḥazm (1928–1933, vol. 1, p. 177); (1988, vol. 1, p. 176); (2010, p. 81); (1926–1930, vol. 3 pp. 76–77), vol. 6, p. 161.

epistemological fictionalism—and in particular the use of *conjecture* for attaining knowledge—is rejected.⁵⁵ This might seem to disqualify our suggested approach to understanding the parallelism between natural and deontic necessity as achieved by some manner of parallel reasoning. Moreover, the reader might at this point wonder if the hypothetical structure of natural modalities developed in the present paper does not betray Ibn Ḥazm’s rejection of conjectures. As mentioned, however, we can substitute hypotheticals with universal quantifications; these latter, logically speaking, are only one inference away from hypotheticals. Still, this might not be of help in understanding the epistemological mechanism at work when grasping the more general parallelism at issue in our study.

If we seek to locate an understanding of the parallelism within Ibn Ḥazm’s own epistemology, we should recall his notion of *rational perception* (*idrāk al-aql*), an immediate intellectual act of knowledge, which is closer to the concept of *immediate inference* of contemporary constructivists than it is to Cartesian intuition. This immediate act of knowledge produces rational understanding (*fahm*).

Now, despite its rational and logical nature, *rational perception* is not devoid of sense perception. Arnaldez (1956, p. 128) has pointed out that within epistemology there is always reasoning at the level of the senses. Rational perception is, accordingly, a kind of immediate act of abstraction.⁵⁶ And in such an epistemological framework, the correspondence between the world of events and the world of actions seems to have a natural place.

8.7 Beyond Ibn Ḥazm: Conclusions and the Work Ahead

8.7.1 Brief Remarks on Ibn Ḥazm’s Heteronomous Imperatives and Deontic Logic

We share von Wright’s (1981, p. 34) qualification of the traditional logical analysis of norms concerning

[...] *structures resembling what Kant called hypothetical imperatives*

although we certainly take exception to the remark that such approaches do not constitute a *genuine deontic logic*—if indeed that remark cannot be reduced to the trivial assertion that the traditional logic of norms is different from the analysis delivered by contemporary formal semantics.

Our preliminary exploration, based on Ibn Ḥazm’s analysis of legal norms, does not yet deliver a logic of norms; it delivers a logical analysis of deontic notions, where obligation can be defined both as a particular kind of inference (namely

⁵⁵ See Puerta Vílchez (2013, pp. 298–300).

⁵⁶ See Puerta Vílchez (2013, p. 301).

a hypothetical judgement), and as an operator (namely as a universally quantified expression).⁵⁷ The point of the logical analysis of deontic notions in such a context lies in shaping the argumentation in favour of, or against, transferring the juridical decision from an established case to a new one. Thus, according to our approach, the main focus of deontic qualifications is in contributing to both the content and implementation of legal norms, rather than in studying the logical validity of arguments involving deontic qualifications.

To formulate the point a bit differently, our logical analysis of the deontic content of legal norms takes the side of those who prioritize **ought to do** (*Tunsollen*) over the Leibnizian concept of **ought to be** (*Seinsollen*).⁵⁸ While it seems natural to endorse the assertion that *it ought to be the case that contradictions are false*, it is not at all clear how this notion of obligation is linked to *what agents ought to do or bring about*.⁵⁹

A direct consequence of this perspective is that, in such a context, it does not make sense to include tautologies within the set of obligatory legal norms—a standard problematic issue in standard deontic logic. On the other hand, certain other known paradoxes of current standard deontic logic, such as paradoxes triggered by conditional obligations, do not seem to arise (Rahman et al. (2019a) elaborate this point in the context of legal reasoning).

For example, Ross's (1941) paradox becomes harmless in the logic of deontic imperatives. The paradigmatic example of this paradox is based on the fact that we can infer it is obligatory to send a letter, or to burn it, from the premise that it is, say, legally obligatory to send it. But in the framework of heteronomous imperatives this paradox does not arise.

For if action of type *A* (sending the letter) has been chosen to be performed, and we know that this performance is *law-abiding* (and also that omitting to perform it is *law-breaking*), then weakening the antecedent is harmless.⁶⁰ And if carrying out some action *A* is law-abiding, adding the performance of an action of type *B* (burning the letter) does not support the inference that performing *B* will also be law-abiding. More precisely $R(y)$ will still apply to the performance of *A*. Notice that weakening the consequent is harmless, too: extending the consequent with a

⁵⁷ Contemporary deontic logicians often point out that a deontic sentence *p* can be interpreted either *prescriptively* as expressing a mandatory norm, or *descriptively* as a statement that it is obligatory that *p*, according to some unspecified system of norms or law. See von Wright (1963, viii, pp. 104–5); Stenius (1963, pp. 250–1); Alchourrón (1969, pp. 243–5); Alchourrón and Bulygin (1971, p. 121). Our approach might appear closer to the descriptive interpretation, though perhaps our distinction between the type of action and its performance presents a middle path.

⁵⁸ Cf. Castañeda (1970). For a recent discussion on the issue see Hilpinen and McNamara (2013, pp. 59–69, 97–110).

⁵⁹ Rahman et al. (2019a) discuss Chisholm's (1963b) puzzle on *contrary to duty imperatives* in the context of Islamic deontic categories.

⁶⁰ NB: In the context of contemporary law we interpret the expressions $R(x)$, $S(x)$, as the qualifications *Law-abiding*, and *Law-breaking*, rather than *Reward* and *Sanction*.

disjunction does not change the fact that the scope of $R(y)$ is still some performance of A .

In short, Ross's puzzle does not apply; whatever performance it is that makes *Sending the letter* true, it is of a type different from the one that makes *Burning the letter* true. Something similar applies to its dual, based on conjunction. The point is that the apparent oddity of applying the introduction rule for disjunction in Ross's example stems from the fact that standard semantics leaves the proof-object for the disjunction in the metalanguage.

8.7.2 *On Normativity the Other Way Round and the Internalization of Nature*

It is the aim of the present study to suggest that an alternative perspective to the logic of norms has been achieved; and, as already mentioned, our proposal has not yet been developed into a logical system. We will now briefly mention the philosophical concepts grounding that project.

The authors of the present paper are ready to claim that the logic of norms should be built on a conception where logic itself is understood as normatively constituted. This is the reason why traditional logic involving deontic concepts is so close to inference itself. The point is that traditional logic—from Plato and Aristotle, through to the Stoics and the Islamic tradition, to Leibniz, Kant, Bolzano and even Frege—is about inference; and inference is understood as being normative. Let us briefly elaborate on this point that should certainly, we believe, be the focus of future research.

Logical approaches to meaning subsequent to the work of Frege and Tarski are based on what me might call, in a general manner, the *semantization of pragmatics* (SP): contexts are formalized with the help of some specific indexes, usually in the metalanguage, upon which the truth-value is made dependent. According to this view, a propositional kernel is complemented by “modalities” expressing necessity, commands, temporality and knowledge. These modalities are defined then as truth-functional operators conceived as certain kinds of logical connectives.

As pointed out by Per Martin-Löf (2017, p. 9), however, standard approaches to the layers underlying logic have gotten the order of priority between deontic and epistemic notions the wrong way round. Martin-Löf's remark is motivated by his fine analysis of the dialogical conception of logic, particularly that which is proposed by Paul Lorenzen (1958), his point being that—different to other standard logical approaches—dialogical logic got the order right.⁶¹

[...] *deontic logic deals only with assertions, just like in standard logic, it's only assertions whose content contains the deontic operations. This puts an opposite order between the deontic notions and the epistemic ones than the one that I have been advocating here,*

⁶¹ For a comprehensive presentation of the dialogical conception of logic, see Rahman et al. (2018).

namely that the deontic notions make their appearance in the analysis of what it means to know how to do something: that's where the deontic notions have their place. So, the deontic notions are at a more basic level than the epistemic notion of knowledge-how [. . .]. Maybe this is the explanation for the difficulties of deontic logic: [. . .] maybe it can be explained in this way, that one has got the order of priority between the deontic notions and the epistemic notions the wrong way around. Per Martin-Löf (2017, p. 9)

According to the dialogical approach, knowledge and meaning emerge as a fine intertwining of having the *right to ask for reasons* and the *duty to give them*, to put it in the words of Brandom (2000). From this perspective, not only are inferences understood as the duty to give reasons for supporting a conclusion, the interface right-duty is constitutive of the judgements that structure an inference.

Thus, the traditional view on the logic of norms as related to hypothetical imperatives is only a consequence of this overall normative view on knowledge and reasoning. The lesson of the parallelism between natural and deontic necessity is that in order to achieve knowledge both have to be internalized by the same act: they must be *placed in the space of reasons*.⁶² This internalization is one way to understand the inverse parallelism that ascribes moral features to nature, and is part and parcel of Ibn Ḥazm's notion of *rational perception* (*idrāk al- aql*) mentioned above.

To know about our world requires to know *why* the events in the world are linked the way they are. But *knowing why* comes always together with *knowing how*: we need to bring to the fore, to the tribunal of public criticism, how those reasons fulfil our obligation to justify our assertions. And this is no different to the way we provide reasons for the guiding of our actions.

Still, the underlying logical building blocks of Ibn Ḥazm's heteronomous imperatives is hypothetical. Moreover, the parallelism is based on admitting several levels of knowledge rather than only on certainty. This is a puzzle within Ibn Ḥazm's own epistemology; and our paper does not provide an answer beyond the modest observation that if rational perception is targeted at the parallelism Ibn Ḥazm has bequeathed us, we might grasp in a sole act of consciousness that epistemology requires a gamut of perspectives after all.⁶³

⁶² The notion of the *space of reasons* stems from Sellars (1991, pp. 129–194).

⁶³ **Acknowledgments:** The authors would like to thank M. Brown (Syracuse U.), A. Jones (King's College London) and P. McNamara (U. New Hampshire, Durham) who invited us to contribute to the fascinating volume they are editing; the Laboratory STL: UMR-CNRS 8163 and to Leone Gazziero (STL), Laurent Cesalli (Genève), and Tony Street (Cambridge), leaders of the ERC-Generator project "Logic in Reverse: Fallacies in the Latin and the Islamic traditions," for fostering the research leading to the present study. Many thanks to T. Bénatouïl (U. Lille, UMR-8163: STL), M. Crubellier (U. Lille, UMR-8163: STL), A. Hasnawi (Paris, CNRS, SPHERE), K. Hülser (U. Konstanz), A. Klev (Prague, Czech Academy of Sciences), F. Mariani (U. Lille), C. Natali (U. Venezia), T. Street (U. Cambridge), R. Rashed (Paris, CNRS, SPHERE), and R. Wisnovsky (McGill U.), for their wise comments and advice during fruitful exchanges with S. Rahman with regard to the origins of the inception of the parallelism and its analysis. We would also extend our gratitude to J. Lameer, M. Iqbal (Lille, UMR-8163: STL), Z. McConaughy (Lille, UMR-8163: STL), and M. Shafiei (Shahid Beheshti University, Teheran), for contributing with inspiring

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