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Generalized plithogenic whole hypersoft set, PFHSS-Matrix, operators and applications as COVID-19 data structures

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Abstract. This article is a preliminary draft for initiating and commencing a new pioneer dimension of expression. To deal with higher-dimensional data or information flowing in this modern era of information technology and artificial intelligence, some innovative super algebraic structures are essential to be formulated. In this paper, we have introduced such matrices that have multiple layers and clusters of layers to portray multi-dimensional data or massively dispersed information of the plithogenic universe made up of numerous subjects their attributes, and sub-attributes. For grasping that field of parallel information, events, and realities flowing from the micro to the macro level of universes, we have constructed hypersoft and hyper-super-soft matrices in a Plithogenic Fuzzy environment. These Matrices classify the non-physical attributes by accumulating the physical subjects and further sort the physical subjects by accumulating their non-physical attributes. We presented them as Plithogenic Attributive Subjectively Whole Hyper-Super-Soft-Matrix (PASWHSS-Matrix) and Plithogenic Subjective Attributively Whole-Hyper-Super-Soft-Matrix (PSAWHSS-Matrix). Several types of views and level-layers of these matrices are described. In addition, some local aggregation operators for Plithogenic Fuzzy Hypersoft Set (PPFHS-Set) are developed. Finally, few applications of these matrices and operators are used as numerical examples of COVID-19 data structures.

Keywords: Plithogenic, Hyper-Super-Soft-Matrix, matrix layers, parallel-universes, events, realities, aggregation operators, non-physical classifications, as COVID-19, data structures

1. Introduction

Classical mathematics and its applications are based on certain laws and results, while they can be observed in everyday life. When the human mind makes decisions about scientific, philosophical, or economic facts, it is not 100% sure of its results. The natural human brain exhibits a certain uncertainty factor and precariousness in its judgments and conclusions due to different opinions about attributes, events, and information. Therefore, to manage this vagueness in the study of mathematics In 1965 Zadeh [1] introduced fuzzy mathematics. In the theory of fuzzy mathematics, all laws and results are discussed by considering some degree of certainty or truth (membership) and some degree of uncertainty or the opposite of truth (non-membership), so that the combined effect of membership and non-membership is considered complete. That means if membership value of any element $x \in X$ with

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respect some attribute A is represented by, $\mu_A(x) \in [0, 1], \forall x \in X$ and non-membership is represented by $\upsilon_A(x) \in [0, 1], \forall x \in X$ such that $\mu_A(x) + \upsilon_A(x) = 1, \forall x \in X$, then the general representation of a fuzzy set is $\{x : \mu_A(x)\}$.

In 1986 Atanassov developed the Intuitionistic Fuzzy Set Theory (IFS) [2, 3]. In IFS theory, Atanassov expanded the vagueness of the human mind by addressing the doubt which arises in assigning membership and non-membership. The doubt which is evoked in decision making was quantified by using linguistic scales and expressed by assigning a numeric value between "0" and "1". Atanassov named this doubt the level of hesitation that was measured by assigning a degree to the hesitation denoted by $\iota_A(x) \in [0, 1] \forall x \in X$. The elements of IFS are expressed as $\{x : (\mu_A(x), \upsilon_A(x))\}$ with a modified condition $\mu_A(x) + \upsilon_A(x) + \iota_A(x) = 1, \forall x \in X$. It is observed that the degree of hesitation of IFS is a dependent factor. Smarandache further expanded the Cloud of vagueness by introducing Neutrosophic Set [4–6]. He introduced indeterminacy and considered the degrees of membership, non-membership, and hesitation/indeterminacy as independent factors. These three factors are represented in a unit cube with the non-standard unit interval $\left]0^-, 1^+\right[$. The neutrosophic set was represented as $\{x : (\mu_A(x), \upsilon_A(x), \iota_A(x))\}$ with modified condition $0 \leq (\mu_A(x) + \upsilon_A(x) + \iota_A(x) \leq 3$. Some further latest dilation and modernization of neutrosophic set are portrayed in [7–12]. In 1999, Molodtsov introduced the soft set [13], where he represented the elements of this set as a parameterized family of the subset of the universal set and thereafter some of the further extensions of the soft set were discussed in, [14–16].

Later, in 2018, Smarandache [17, 18] introduced Hypersoft set and plithogenic hypersoft set. In these sets he transformed the function of a single attribute into a multi-attribute/sub-attribute function and assigned a combined membership $\mu_{A_1 \times A_2 \times \ldots \times A_N}(x)$, non-membership $\upsilon_{A_1 \times A_2 \times \ldots \times A_N}(x)$, and Indeterminacy $\iota_{A_1 \times A_2 \times \ldots \times A_N}(x), \forall x \in X$ with condition $A_i \cap A_j = \phi$ for the case of hypersoft set. Whereas individual memberships non-memberships and indeterminacies were assigned for each given attribute for the case of the plithogenic Hypersoft-Set. And introduced hybrids of Crisp/Fuzzy/Intuitionistic Fuzzy and Neutrosophic Hypersoft-Set and Plithogenic Hypersoft-Set. By introducing these sets he raised many open problems for the development of new literature, such as the development of appropriate modern algebraic structures for expressing such widely dispersed higher-dimensional information and the formulation of MADM techniques.

In 2019 Rana et al. [19] extended the Plithogenic Hyper-Soft Set to Plithogenic Whole-Hyper-Soft Set and introduced a new path of organized expression as hypersoft-matrix and Hyper-super-soft-matrix and formulated some local aggregation operators. When these local operators were applied to the Plithogenic Fuzzy Hyper-Soft Set (PFHSS), a new type of soft set emerged called the Plithogenic Fuzzy Whole Hyper-Soft Set (PFWHSS). In addition, the Plithogenic Fuzzy Hyper-Soft Set and the Plithogenic Fuzzy Whole Hyper-Soft Set were presented as a modern extended matrix in the fuzzy environment called Plithogenic Fuzzy Hyper-Soft Matrix (PFHS-Matrix) and Plithogenic Fuzzy Whole Hyper-Soft Matrix. These Hypersoft Matrices are made up of the parallel layers of ordinary matrices that represent parallel universes, parallel realities, parallel events, or parallel information.

In the next phase, Rana et al. [20] further generalized the Plithogenic Whole Hyper-Soft Set to Plithogenic Crisp/Fuzzy/Intuitionistic/Neutrosophic Subjective Hyper-Soft Set and represented it in the expanded version of Soft-Matrix in the fuzzy environment known as the plithogenic subjective hyper-super-soft matrix. Furthermore, these modern matrix expressions were used to develop a new higher dimensional ranking model called the Local-Global Universal Subjective Ranking Model. It is also revealed that the Plithogenic Whole Hyper-Soft Matrix (PWHS Matrix) is a special case of the Plithogenic Subjective Hyper-Super-Soft Matrix (PSHSS Matrix). Additional literature on Hyper-Soft Set and Plithogency can be found in [21–26].

This article, structured as, Section 2, describes materials such as some new concepts of the expanded plithogenic universe, such as universal collective consciousness, clustered data of the accelerated universe, and some Preliminaries. Section 3 contains a brief description of the Generalized Plithogenic Fuzzy Whole HyperSoft and its novelties with some basic concepts and new definitions of Plithogenic Hypersoft Set/Matrix, index-based views of parallel matrix layers. Section 4 describes the development of local aggregation operators for PFHSS matrix. Section 5 presents the construction of hyper-supersoft matrices and their application as COVID-19 data structures. Section 6 mentions conclusions, discussions, and some open questions for future research.

2. Materials, methods, and preliminaries

In this article, on the first level, we have expanded the previously presented Plithogenic Fuzzy Whole HyperSoft Set and Plithogenic Subjective HyperSoft Set [19, 20] to Plithogenic Attributive Subjectively Whole HyperSoft Set (PASWHS Set) and Plithogenic Subjective Attributively Whole HyperSoft Set (PSAWHS Set). To represent these newly introduced hypersoft sets, a new type of matrix structure of connected matrix layers (hypersoft matrix) and connected clusters of matrix layers (hyper-super-soft matrix) is designed. These super-algebraic matrix structures are initially formulated in a fuzzy environment named the Plithogenic Attributive Subjectively Whole Hyper-Super-Soft-Matrix (PASWHSS-Matrix) and the Plithogenic Subjective Attributively Whole-Hyper-Super-Soft-Matrix (PSAWHSS-Matrix). These advanced types of matrices are generated by the hybridization of hyper matrices, super matrices, and soft matrices [13–20, 24–27]. These hypersoft matrices are sets/clusters of connected parallel matrix layers that represent clusters of parallel universes, parallel realities, Parallel events, or parallel information (a combination of attributes and sub-attributes relating to subjects). Furthermore, these modern matrices are rank three and four tensors with three and four variation indices, respectively. Later on, various types of cross-sectional sights are described as parallel layers of hypersoft matrices. These cross-sectional sights are formulated by taking multiple variations in numerous ways and exhibit some symmetries in their structures regarding their number of rows and columns. Furthermore, some aggregation operators are structured. Application of these aggregation operators and Hyper-Super-Soft Matrices are provided as COVID-19 Data structures.

Now comes the query, "Why do we use HyperSoft and Hyper-Super-Soft matrices specifically for the expression of Plithogenic HyperSoft Set, Plithogenic Subjective HyperSoft Set, and Plithogenic Attributive HyperSoft Set?" The answer described in (2.1–2.3), and (3.1.1–3.1.5) could certainly be convincing.

2.1. Expanded plithogenic universe

This Plithogenic Universe is so huge and expanded in its interior (like having Fuzzy, Intuitionistic Fuzzy, Neutrosophic environments with Memberships, Non-Memberships, and Indeterminacies) and its exterior (manages many attributes, sub-attributes and might be sub-sub-attributes concerning to its subjects). To organize and analyze the dispersed information of such an expanded plithogenic universe we are in great need to formulate some super algebraic structures like these hyper-super-soft matrices.

2.2. Universal collective consciousness

Imagine you are a part of a lot of information, events, realities that are constantly flowing all around you. Like everything you see, everything you look at, everything you observe, is in a parallel view. By the way, we are all observing from a different angle so we all are getting a different piece of information, event, or a reality. Because of this connectedness with this universe and its universal attributes, we are actually collectively producing diversity in this flow of information, events, thoughts, and even realities (i.e., Universal Data, Universal Space, Universal Events, Universal Consciousness). That is why this universe is giving different sets of information and numerous interpretations of the same thing (**Subject, Attributes, Sub-Attributes, Sub-Sub-Attributes Collectively All as Consciousness**). We are gathering this information in terms of observing different events, realities, universes as one reality and not as parallel realities, and thus, we are feeding them back to space through our brain structure into the atomic structure and into the vacuum state though they are underlying numerous angles, thoughts, events, visions, etc. In the end, this universal feedback returns and changes those attributes, events a little bit different from each other and makes parallel realities for which we have no expressions. Therefore, these hypersoft and hyper-super-soft matrices are initiatives towards a way of expression for the Universal Collective Consciousness.

2.3. Clustered Data of accelerated universe

In order to find and pave the way for the expression of those parallel realities for such a massively accelerated huge universe. This is, of course, not viable through using regular algebra or matrices therefore, for the handling of this expanded universe along with its clustered and littered data, we are in dire need to construct like super-algebra or hypersoft and hyper-super-soft matrices and likely many more.

2.4. Preliminaries

This section describes some basic definitions of Soft-Sets, Fuzzy-Soft-Sets, Hypersoft-Sets, Fuzzy-Hypersoft-Sets, Plithogen-Hypersoft-Sets, and Plithogen-Fuzzy-Hypersoft-Sets, etc. These definitions are useful in developing literature and widely dispersed data modeling structures.

Definition 2.4.1 [13](Soft Set)
Let U be the initial universe of discourse, and E be a set of parameters or attributes with respect to U let $P(U)$ denote the power set of U, and $A \subseteq E$ is a set of attributes. Then pair (F, A), where $F : A \to P(U)$ is called Soft Set over U. Fore $e \in A$, $F(e)$ may be considered as a set of e elements or e approximate elements

$$(F, A) = \left\{ \begin{array}{c} (F(e) \in P(U) : \\ e \in E, F(e) = \varphi if e \notin A \end{array} \right\} \quad (2.1)$$

Definition 2.4.2 [1] (Fuzzy set)
Let U be the universe. A fuzzy set X over U is a set defined by a membership function μ_X representing a mapping $\mu_X : U \to [0, 1]$

The value of $\mu_X(x)$ for the fuzzy set X is called the membership value of the grade of membership of $x \in U$. The membership value represents the degree of belonging to fuzzy set X. A fuzzy set X on U can be expressed as follows.

$$X = \{(\mu_X(x)/x) : x \in U, \mu_X(x) \in [0, 1]\} \quad (2.2)$$

Definition 2.4.3 [14] (Fuzzy soft set)
Let U be the initial universe of discourse, $F(U)$ be all fuzzy sets over U. E be the set of all parameters or attributes with respect to U and $A \subseteq E$ is a set of attributes. A fuzzy soft set Γ_A on the universe U is defined by the set of ordered pairs as follows,

$$\Gamma_A = \{x, \gamma_A(x) : x \in E, \gamma_A(x) \in F(U)\} \quad (2.3)$$

where $\gamma_A : E \to F(U)$ such that $\gamma_A(x) = \phi$ if $x \notin A$

$$\gamma_A(x) = \left\{ \mu_{\gamma_A(x)}(u)/u : u \in U, \mu_{\gamma_A(x)}(u) \in [0, 1] \right\}$$

Definition 2.4.4 [17] (HyperSoft Set)
Let U be the initial universe of discourse $P(U)$ the power set of U let $a_1, a_2, \ldots, a_n$ for $n \geq 1$ be n distinct attributes, whose corresponding attributes values are respectively the sets $A_1, A_2, \ldots, A_n$ with $A_i \cap A_j = \varphi$ for $i \neq j$ and $i, j \in \{1, 2, \ldots, n\}$.

Then the pair $(F, A_1 \times A \times \cdots \times A_n)$ where,

$$F : A_1 \times A \times \cdots \times A_n \to P(U), \quad (2.4)$$

is called a Hypersoft set over U

Definition 2.4.5 [17] (Crisp Universe of Discourse)
A Universe of Discourse U_C is called Crisp if $\forall x \in U_c$ $x \in 100\%$ to U_C or membership of x $T(x)$ with respect to A in M is 1 denoted as $x(1)$.

Definition 2.4.6 [17] (Fuzzy Universe of Discourse)
A Universe of Discourse U_F is called Fuzzy if $\forall x \in U_C$ x partially belongs to U_F or membership of x $T(x) \subseteq [0, 1]$ where $T(x)$ may be a subset, an interval, a hesitant set, a single value set, denoted as x $T(x)$.

Definition 2.4.7 [17] (Crisp Hypersoft-Set set)
Let U_c be the initial universe of discourse $P(U_c)$ the power set of U.

let $a_1, a_2, \ldots, a_n$ for $n \geq 1$ be n distinct attributes, whose corresponding attributes values are respectively the sets $A_1, A_2, \ldots, A_n$ with $A_i \cap A_j = \varphi$ for $i \neq j$ and $i, j \in \{1, 2, \ldots, n\}$, Then the pair, $(F_c, A_1 \times A \times \ldots \times A_n)$ s.t

$$F_c : A_1 \times A \times \cdots \times A_n \to P(U_c)\}, \quad (2.5)$$

is called Crisp Hypersoft set over U_c

Definition 2.4.8 [17] (Fuzzy Hypersoft set)
Let U_F be the initial universe of discourse $P(U_F)$ the power set of U_F.

let $a_1, a_2, \ldots, a_n$ for $n \geq 1$ be n distinct attributes, whose corresponding attributes values are respectively the sets $A_1, A_2, \ldots, A_n$ with $A_i \cap A_j = \varphi$ for $i \neq j$ and $i, j \in \{1, 2, \ldots, n\}$.

Then the pair
$(F_F, A_1, A_2, \ldots, A_n)$, s.t

$$F_F : A_1 \times A \times \cdots \times A_n \to P(U_F)\} \quad (2.6)$$

is called Fuzzy Hypersoft set over U_c.

Special cases of Hypersoft set: Crisp, Fuzzy, Intuitionistic Fuzzy and Neutrosophic sets are special cases of Hypersoft set by taking $N = 1$ in the Combination of N attributes $A_1 \times A_2 \times \cdots \times A_N$.

Definition 2.4.9 [17] (Plithogenic, Crisp, Fuzzy, Intuitionistic Fuzzy, and Neutrosophic Hypersoft Set)
Now instead of assigning combined membership $\mu_{A_1 \times A_2 \times \cdots \times A_N}(x)$ $\forall x \in X$ for Hypersoft sets if each attribute A_j is assigned an individual membership $\mu_{A_j}(x)$, non-membership $\upsilon_{A_j}(x)$ and Indeterminacy $\iota_{A_j}(x)$ $\forall x \in X$ $j = 1, 2, \ldots, n$ in Crisp, Fuzzy, Intuitionistic and neutrosophic Hypersoft set then these generalized Crisp, Fuzzy, Intuitionistic and Neutrosophic Hypersoft sets are called Plithogenic, Crisp,

Fuzzy, Intuitionistic Fuzzy and Neutrosophic Hypersoft Set.

Definition 2.4.10 [27, 28] (Super Matrices)

A Square or rectangular arrangements of numbers in rows and columns are matrices we shall call them simple matrices while a Super-Matrix is one whose elements are themselves matrices with elements that can be either scalars or other matrices.

$a = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$, where

$a_{11} = \begin{bmatrix} 2 & -4 \\ 0 & 1 \end{bmatrix}, a_{12} = \begin{bmatrix} 0 & 40 \\ 21 & -12 \end{bmatrix},$

$a_{21} = \begin{bmatrix} 3 & -1 \\ 5 & 7 \\ -2 & 9 \end{bmatrix}, a_{22} = \begin{bmatrix} 4 & 12 \\ -17 & 6 \\ 3 & 7 \end{bmatrix}$ a is a super-matrix.

Note: The elements of super-matrices are called sub-matrices i.e. $a_{11}, a_{12}, a_{21}, a_{22}$ are submatrices of the super-matrix a in this example, the order of super-matrix a is 2×2, and the order of sub-matrices a_{11} is 2×2, a_{12} is 2×2 a_{21} is 3×2 and order of sub-matrix a_{22} is 3×2, we can see that the order of super-matrix doesn't tell us about the order of its sub-matrices.

Definition 2.4.11 [29, 30] (Hyper-matrices)

For $n_1, \ldots, n_d \in N$, a function,

$$f : (n_1) \times \cdots \times (n_d) \to F \qquad (2.7)$$

is a hyper-matrix, also called an order-d hyper-matrix or d-hyper-matrix. We often just write $a_{k_1 \ldots k_d}$

to denote the value $f(k_1 \ldots k_d)$ of f at $(k_1 \ldots k_d)$ and think of f (renamed as A) as specified by a d-dimensional

table of values, writing $A = \left[a_{k_1 \ldots k_d}\right]_{k_1 \ldots k_d}^{n_1, \ldots, n_d}$

A 3-hypermatrix would be written down on a (2-dimensional) piece of paper as a list of usual matrices, called slices. For example,

$$A = \begin{bmatrix} a_{111} & a_{121} & a_{131} & \cdot & a_{112} & a_{122} & a_{132} \\ a_{211} & a_{221} & a_{231} & \cdot & a_{212} & a_{222} & a_{232} \\ a_{311} & a_{321} & a_{331} & \cdot & a_{312} & a_{322} & a_{332} \end{bmatrix} \qquad (2.8)$$

3. Generalized plithogenic fuzzy whole hypersoft set

In this section, some new concepts and definitions are described that would be utilized for the development of Generalized Plithogenic Fuzzy Whole Hyper-soft Set as PFASWHSS-Matrix and. PFSAHSS-Matrix. This literature is organized in three subsections in the following manner.

3.1 A brief description of the Generalized Plithogenic Fuzzy Whole Hyper-soft Set is presented and its novelties are discussed.
3.2 Some new concepts/definitions relevant to the Plithogenic hypersoft set are developed.
3.3 The index-based views of PFHS-Matrix are described through variation indices

3.1. Generalized Plithogenic Fuzzy Whole Hyper-Soft Set and its Novelties

3.1.1. Matrix of connected matrix-layers

The clusters of connected matrix layers that we will introduce in this article are the expanded versions of the previously presented [19] Plithogenic Fuzzy Whole HyperSoft Set (PFWHS Set). In PFWHS-Set the aggregation operators and the Plithogenic Frequency Matrix Multi-Attribute Decision Making Technique have been developed for a single required combination of attributes/sub-attributes related to some given subjects. While in the new generalized versions of connected matrix layers, the aggregation operators and ranking models are not only developed for a single combination of attributes/sub-attributes rather, these operators and ranking models are extended to several parallel-connected layers of combinations of attributes/sub-attributes related to the given subjects.

For the development of this ranking model (FMMADM-Model), a novel type of Hyper-Soft and Hyper- Super-Soft Matrices are introduced. These special types of matrices would organize and utilize the widely expanded higher dimensional dispersed information. This information consists of several attributes, sub-attributes, and subjects that are connected by the cluster of matrix layers. These clusters of matrix layers are formulated as Hypersoft and Hyper-Super-Soft matrices. To preserve the expansion of the article inside the required limits of this journal, all of the expanded literature is first presented in the fuzzy environment.

In this version, some new definitions and concepts of the Generalized Plithogenic Fuzzy whole hypersoft set are developed. In addition, their representation is organized as hypersoft, and hyper-super-soft matrix. After the formulation of these hypersoft matrices, some aggregation operators are developed as the set laws of operations for these matrices. The application of these aggregation operators and the Hyper-Super-Matrix as a ranking model for the organization and classification of the conditions of COVID-19 patients will be presented later. This ranking model is called the classification model for COVID-19 states.

3.1.2. Multi-dimensional mathematical ways of expression

One novelty of this model is modern multi-dimensional mathematical form of expression. It is observed that from the fuzzy set to its extended versions such as intuitionistic, neutrosophic, and other extended fuzzy sets. The membership, non-membership, and indeterminacy were assigned to a specific element or subject concerning to its specific attribute. While in generalized plithogenic Whole-Hypersoft-Set, the fuzzy memberships were assigned to several subjects (elements of the universe) concerning their numerous attributes /sub-attributes separately as individual fuzzy memberships and at the same time collectively as whole fuzzy memberships. If these whole fuzzy memberships are constructed to accumulate information subject-wise for several subjects and being displayed for each attribute, then it is subjectively a whole hype-super-soft matrix. And if these whole fuzzy memberships are constructed to accumulate information attribute-wise for several given attributes while being displayed for each subject it is the case of attributively whole hyper-super-soft-matrix. In addition, this individual and collective information is organized in certain required time levels. Therefore, these hyper-super-soft matrices offer the viewer multiple inspecting expression options by observing many subjects with their numerous attributes/sub-attributes on multiple time levels and expressing their individual and collective states in many environments with different levels of ambiguity. For example, one possibility is to express individual and collective states/information in a crisp environment. This means expressing two opposing states of mind, either true (yes) or false (no). Another expression is the fuzzy environment, which contains some doubts about two opposite states, or an intuitionistic environment an expression with expanded doubts, or a neutrosophic environment that includes indeterminacy. Or some kind of combined environment that expresses different states of mind in one expression. Therefore, this novel form of expression and its mathematical structures are introduced to deal with higher-dimensional data/information/states of subjects by observing them through several angles of vision. The model of this article will first be expressed in the fuzzy environment in order to maintain the length restrictions of the article, further extended expressions will be introduced in future articles. To illustrate these multi-vision expressions, consider that a field is visited with several types of flowers, and it is observed that each type of flower has some specific beneficial properties (attributes) and property levels (sub-attributes) When, for a certain type of flower, its collective level of utility is expressed in a statement taking into account all attributes and expressed as a numerical value using fuzzy linguistic scales. It is a form of the whole expression (case of an attributively whole Hyper-Supersoft-Matrix). When, for a particular type of flower, the utility for each property (attribute) is expressed as a single statement or as a numerical value using fuzzy linguistic scales. It is another form of expression called individual expression (in the case of the Hypersoft Matrix) and the choice of environment will represent the state of mind of the observer or decision-maker.

3.1.3. Whole dilated and compact vision of the shattered information

This mathematical model describes such novel expressions (originally constructed in the plithogenic fuzzy environment) through the formulation and use of accumulated fuzzy memberships. This means that the entire vision of the scattered information/states/universe could be represented as a single numerical value (whole fuzzy membership) that offers a compact view of information/events/universe. Whereas the detailed insight view of the information/events/universe is indicated by certain numerical values that are called individual fuzzy memberships

3.1.4. Higher-dimensional Matrix with multi-layers and clusters of layers

To organize and analyze all relations between several subjects, their attributes (subject states), and sub-attributes (levels of states of subjects) by considering their individual and collective states and then expressing them in a single mathematical structure was not possible by using the classical algebra of ordinary matrices. To represent such higher dimensional information, some super algebraic structures like these hyper super matrices had to be constructed. These hyper-super-matrices can absorb such multi-dimensional information by using their multilayered structures. These higher dimensional matrices with multiple layers and clusters of layers are another novelty that is being introduced into matrix theory for the first time.

3.1.5. New framework of modeling

In this article, we have developed a new modeling framework that can not only organize the high dimensional numerical data/information, but rather would accept information in any form and transform it into the required mathematical form through linguistic scales. It would also expand or contract the information through the use of established operators. The elements of classical matrices are usually numbers and they follow the classical rules of addition, multiplication, or inversion, rather these modern high-dimensional matrix structures would interact through the modern flexible laws of Set theory such as soft set theory or extended fuzzy set theory, etc. These matrices would interact within the matrix through Local Operators which will be developed in this article. They would also interact outside the matrix through global operators that would be introduced in the next upcoming versions.

3.2. Definitions and concepts of GPFWHSS

In order to develop a better understanding of the literature, some new definitions and concepts are presented below.

Let's consider a brief description of the mathematical terms and expressions used to develop the model.

Definition 3.2.1 (Universe of discourse):

$U_F\ (X) = \{x_i\}$ is the fuzzy universe of discourse where x_i , $i = 1, 2, \ldots, M$ represents the number of subjects (elements of the universe). The subject can be perceived as a physical entity that is being discussed.

Definition 3.2.2 (Attributes): A_j $j = 1, 2, \ldots, N$ are N number of attributes under consideration. These are the states of subjects i.e. characteristics or behaviors associated with the elements of the set (physical subjects).

Definition 3.2.3 Sub-Attributes: A_j^k $j = 1, 2, \ldots, N$ $k = 1, 2, \ldots, L$ where j represents the given number of attributes (states of matter-bodies or subjects) and each attribute is categorized into a number of levels, which are referred to as sub-attributes (substates of subjects). Attributes or sub-attributes are considered to be non-physical aspects of the physical bodies (subjects) under consideration.

Definition 3.2.4 (Plithogenic Fuzzy HyperSoft-Matrix (PFHS-Matrix)):

Let $\mathbf{U_F}$ be the Fuzzy universe of discourse, $P(U_F)$ be the power set of $\mathbf{U_F}$, $\mathbf{A_{jk}}$ is a combination of attributes/Sub-Attributes for some $j = 1, 2, 3, \ldots, N$ Attributes, $k = 1, 2, 3, \ldots, L$ Sub-Attributes and $x_i i = 1, 2, 3, \ldots, M$ subjects under consideration then PFHS-Matrix, is a mapping F from the cross product of attributes/sub-attributes on the power set $P(U_F)$ represented in matrix form. This mapping with its matrix form in the plithogenic fuzzy environment is described in Equation 3.1 or Equation 3.2 respectively.

$$F : A_1^k \times A_2^k \times A_3^k \times \cdots \times A_N^k \rightarrow P(U_F) \quad (3.1)$$

$$F = \left[\mu_{A_j^k}(x_i)\right] \quad (3.2)$$

s.t $\mu_{A_j^k}(x_i) + \upsilon_{A_j^k}(x_i) = 1,$

$$\mu_{A_j^k}(x_i) \in [0, 1]\,, \upsilon_{A_j^k}(x_i) \in [0, 1]$$

$\mu_{A_j^k}(x_i)$ and $\upsilon_{A_j^k}(x_i)$ are fuzzy memberships and non-memberships for the given x_i subjects regarding each given A_j^k attributes/sub-attributes where, A_j^k is a combination of attributes/sub-attributes for some $j = 1, 2, 3, \ldots, N$ attributes, $k = 1, 2, 3, \ldots, L$ sub-attributes associated to $x_i i = 1, 2, 3, \ldots, M$ subjects under consideration. These fuzzy memberships are assigned by the relevant committees for the piece of universe/reality/event/information.

The expanded form of F is described, in Equation 3.3

$$F = \begin{array}{c} \\ x_1 \\ x_2 \\ . \\ . \\ . \\ x_M \end{array} \begin{array}{c} \begin{array}{ccc} A_1^k & A_2^k \ldots & A_N^k \end{array} \\ \begin{bmatrix} \mu_{A_1^k}(x_1) & \mu_{A_2^k}(x_1) & \ldots \mu_{A_N^k}(x_1) \\ \mu_{A_1^k}(x_2) & \mu_{A_2^k}(x_2) & \ldots \mu_{A_N}(x_2) \\ . & . & \ldots . \\ . & . & \ldots . \\ . & . & \ldots . \\ \mu_{A_1^k}(x_M) & \mu_{A_2^k}(x_M) & \ldots \mu_{A_N^k}(x_M) \end{bmatrix} \end{array} \quad (3.3)$$

Example 1 (PFHS-Set and PFHS-Matrix expressions)

Let mapping F be defined as

$$F : A_1^k \times A_2^k \times A_3^k \times \cdots \times A_N^k \rightarrow, P(U_F)$$

(Taking some specific numeric values of A_j^k)

Consider $T = \{x_1, x_2, x_3\}$, is a subset of $P(U_F)$ where x_1, x_2, x_3 represent x_i subjects under consideration with A_j^k attributes/sub-Attributes for $j = 1, 2, 3, 4$ and $k = 3, 1, 1, 2$. The fuzzy memberships would be assigned to x_1, x_2, x_3 for the $A_1^3, A_2^1, A_3^1, A_4^2$, attributes/sub-attributes i.e. a particular α-combination of attributes by the concerned bodies by using the linguistic five-point scale method see Ref. [31–34]. These plithogenic fuzzy memberships are the elements PFHS-Set represented as below,

$$F_\alpha\left(A_1^3, A_2^1, A_3^1, A_4^2\right) = F(\alpha)$$

$$= \left\{\begin{array}{c} x_1(0.3, 0.6, 0.5, 0.5), \\ x_2(0.4, 0.4, 0.3, 0.1), \\ x_3(0.6, 0.3, 0.4, 0.7) \end{array}\right\} \quad (3.4)$$

A more organized form of this PFHS-Set is expressed as a single layer of the PFHS-Matrix F_{ij}^α,

$$\left[A_1^3, A_2^1, A_3^1, A_4^2\right]$$

$$F_{ij}^\alpha = \begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array}\begin{bmatrix} 0.3 & 0.6 & 0.5 & 0.5 \\ 0.4 & 0.4 & 0.3 & 0.1 \\ 0.6 & 0.3 & 0.4 & 0.7 \end{bmatrix} \quad (3.5)$$

Where $A_1^3 A_2^1\ A_3^1 A_4^2$ is a specific combination of Attributes/Sub-Attributes associated with subjects x_1, x_2, x_3 and F_{ij}^α represents a single layer out of several possible layers of PFHS-Matrix. For further detailed descriptions and applications, see ref [19].

Example 2. Consider the example of Plithogenic Fuzzy HyperSoft matrix $F = \left[\mu_{A_j^k}(x_i)\right]$ for $= 1, 2, 3, j = 1, 2, 3, 4$ and, $k = 1$ (the first level layer) $k = 2$, (the second level layer). The fuzzy memberships that are assigned by the concerned bodies for the Part of universe/reality/event for Hypersoft Set are described as, let $T = \{x_1, x_2, x_3\}$ be the set of three Subjects under consideration in PFHS-Set with respect to the given attribute. The fuzzy memberships are assigned by the concerned bodies using the five-point linguistic scale method [19–22]. These fuzzy memberships are assigned regarding each attribute A_j^k at each level and are described underneath as PFHS-Set,

$$F\left(A_1^1, A_2^1, A_3^1, A_4^1\right) = \left\{\begin{array}{c} x_1(0.3, 0.6, 0.3, 0.5), \\ x_2(0.4, 0.5, 0.2, 0.1), \\ x_3(0.6, 0.2, 0.3, 0.7) \end{array}\right\} \quad (3.6)$$

$$F\left(A_1^2, A_2^2, A_3^2, A_4^2\right) = \left\{\begin{array}{c} x_1(0.5, 0.4, 0.2, 0.6) \\ , x_2(0.5, 0.7, 0.8, 0.4), \\ x_3(0.7, 0.6, 0.5, 0.9) \end{array}\right\} \quad (3.7)$$

This information from the PFHS-Set is organized in the form of PFHS-Matrix F as,

$$\mathbf{F} = \begin{bmatrix} \begin{bmatrix} 0.3 & 0.6 & 0.3 & 0.5 \\ 0.4 & 0.5 & 0.2 & 0.1 \\ 0.6 & 0.2 & 0.3 & 0.7 \end{bmatrix} \\ \begin{bmatrix} 0.5 & 0.4 & 0.2 & 0.6 \\ 0.5 & 0.7 & 0.8 & 0.4 \\ 0.7 & 0.6 & 0.5 & 0.9 \end{bmatrix} \end{bmatrix} \quad (3.8)$$

Detailed descriptions and applications can be found in Ref [20].

Definition 3.2.5 (Individual fuzzy memberships): Elements of PFHS-Matrix $\mu_{A_j^k}(x_i)$ represents Individual fuzzy memberships these memberships are individual fuzzy states of subjects given for each attribute i.e. fuzzy membership of a certain x_i-subject say for $i =$ m, concerning to each A_j^k attribute/sub-attribute. The elements of the Matrix F_{ij}^α and Matrix F in Equations (3.9) and (3.12) represent the individual fuzzy memberships. These Individual fuzzy memberships $\mu_{A_j^k}(x_i)$ are used to represent the inside view of the universe/reality/event/information as individual memberships.

Definition 3.2.6 (Whole fuzzy membership): Whole fuzzy memberships are accumulated states of subjects. Whether accumulated by subject or attribute. For example, if the individual fuzzy memberships $\mu_{A_j^k}(x_i)$ are accumulated for a given combination of attributes A_j^k $j = 1, 2, \ldots, N$ and represented with respect to a certain subject x_i for some $i =$ m, then they are called Whole Fuzzy Membership. The individual fuzzy memberships are accumulated to see the combined effect (e.g. average degree of membership) of the elements (subjects) with respect to a particular combination of attributes/sub-attributes. These are two types of whole fuzzy membership i.e.

(a) *Attributively-Whole Fuzzy Memberships.*
(b) *Subjectively-Whole Fuzzy Memberships.*

Definition 3.2.7 (Attributively-Whole Fuzzy Membership): Attributively-Whole Fuzzy Membership represents such fuzzy memberships that are accumulated (attribute-wise) along the rows of PFHS-Matrix by using any aggregation operator (t) represented by $\Omega_{A^l}(x_i)$. These memberships are obtained by accumulating all given states (attributes) of a certain subject for a fixed sub-attribute level l. and represented with respect for a certain subject. The Whole Fuzzy Memberships $\Omega^t_{\mathbf{A}^k}(x_\mathrm{i})$ used to represent the subject-wise exterior view of the universe/reality/event/information revealing the attributive wholeness. For example the whole memberships $\Omega^t_{\mathbf{A}^1}(x_1)$ given in Equation (3.9) represents subjectively-whole fuzzy memberships for the first subject with respect to all given attributes of first level.

$$\left[\left[\mu_{A^1_j}(x_1)\right]\ \left[\Omega^t_{\mathbf{A}^1}(x_1)\right]\right] \tag{3.9}$$

Definition 3.2.8 (Subjectively-Whole Fuzzy Membership): Subjectively-Whole Fuzzy Memberships representssuch fuzzy memberships that are accumulated (subject-wise) along with the columns of PFHS-Matrix by using any aggregation operator operator (t) represented by $\Omega_{A^l_j}(X)$. The Subjectively-Whole Fuzzy Membership $\Omega^t_{\mathrm{A}^k_j}(X)$ used to represent the attribute-wise exterior view of the universe/reality/event/information revealing the subjective wholeness. For example the whole fuzzy memberships $\Omega^t_{A^1_j}(X)$ given in Equation (3.10) represents attributively-whole fuzzy memberships for first subject with respect to all given attributes of first level.

$$\begin{bmatrix}\left[\mu_{A^1_1}(x_i)\right]\\ \left[\Omega^t_{\mathbf{A}^1_1}(X)\right]\end{bmatrix} \tag{3.10}$$

Definition 3.2.9 (Individual Fuzzy Non-Membership: $\upsilon_{A^k_j}(x_i)$ represents fuzzy non-membership (level of non-belongingness) of a subject (x_i) concerning a certain attribute/sub-attribute given in Equation (3.11)

$$\upsilon_{A^k_j}(x_i) = 1 - \mu_{A^k_j}(x_i) \tag{3.11}$$

Definition 3.2.10 (Attributively-Whole Fuzzy Non-Membership): Attributively-Whole Fuzzy Non-Membership is described as the accumulated level of non-belongingness of a particular subject with respect to a combination of attributes given in Equation (3.12)

$$\Phi_{A^l}(x_i) = 1 - \Omega_{A^l}(x_i) \tag{3.12}$$

If one accumulates individual fuzzy non-memberships along rows of the PFHS matrix, one obtains a combined degree of non-membership with respect to all given attributes for a particular subject, called the attributively-whole fuzzy non-membership.

Definition 3.2.11 (Subjectively-Whole Fuzzy Non-Membership): Subjectively-Whole Fuzzy Non-Membership is described as the accumulated level of non-belongingness of any given A^k_j attribute for fixed $j = \mathrm{n}$, $k = 1$ regarding a specific combination of subjects. Subjectively whole fuzzy non-memberships are mathematically described as,

$$\Phi_{A^l_j}(X) = 1 - \Omega_{A^l_j}(X) \tag{3.13}$$

If one cumulate these fuzzy non-memberships column-wise for PFHS-Matrix, he gets a combined degree of non-membership with respect to all given subjects for a particular attribute/sub-attribute called Attributively-Whole Non-Membership for the fuzzy environment

Note: For a precise description and application of these terms and mathematical expression a detailed numerical example is constructed as COVID19 data structures in the section-5.

Definition 3.2.12 (Plithogenic Fuzzy Whole HyperSoft-Set)

Let U_F be the initial universe of discourse, in the Fuzzy environment and $P(U_F)$ be the power set of U_F. Let $A^k_1, A^k_2, \ldots, A^k_N$ for $N \geq 1$ be N distinct attributes, $k = 1, 2, \ldots L$ represents attribute values and F is a mapping from cross product of sub-attributes to the power set of U_F ($F : A^k_1 \times A^k_2 \times \cdots \times A^k_\mathrm{N} \to P(U_F)$) Then $\left(F, A^k_1 \times A^k_2 \times \cdots \times A^k_\mathrm{N}\right)$ represented in the set form is called a Plithogenic Fuzzy Whole-Hypersoft-Set (PFWHS-Set) over U_F if its elements are expressed by both individual fuzzy memberships $\mu_{A^k_j}(x_i)$ and Combined fuzzy memberships $\Omega_{\mathbf{A}_\alpha}(x_i)$, $\forall x_i \in U_F$ for the given α-combination of attributes sub-attributes regarding each subject. In PFWHS-Set F deals with a single specific combination of attributes/sub-attributes out of the complex cross products. It is observed that the individual fuzzy memberships ($\mu_{A^k_j}(x_i)$) of subjects are expressed with respect to each given attribute/sub-attribute and the cumulative fuzzy memberships $\Omega_{\mathrm{A}_\alpha}(x_i)$ are aggregated by some operators and expressed for a specific α-combination of attributes/sub-attributes concerning the given subjects (case of an α universe, that expresses one of the possible reality/event/information in a fuzzy environment).

Definition 3.2.13 (PFSAWHSS-Matrix)

Let U_F be the initial universe of discourse, in the Fuzzy environment and $P(U_F)$ be the power set of U_F. Let $A_1^k, A_2^k, \ldots, A_N^k$ are N distinct attributes sub-attributes for $j = 1, 2, \ldots N$, $k = 1, 2, \ldots L$ then the mapping F, from the cross product of the attributes sub-attributes to the powerset of U_F ($F : A_1^k \times A_2^k \times \cdots \times A_{\text{N}}^k \to P(U_F)$) represented in the matrix form in such a way that its elements consist of individual fuzzy memberships $\mu_{A_j^k}(x_i)$ as well as attributively aggregated fuzzy memberships $\Omega^t_{\text{A}^k}(x_{\text{i}})$, $\forall x_i \in U_F$, then this matrix is called a Plithogenic Subjective Attributively Whole Hyper-Super-soft-Matrix (PFSAWHSS-Matrix).

The PFSAWHSS-Matrix F_{S_t} is described underneath in Equation (3.14) and its expanded form is presented in Equation (3.15)

$$F_{\mathbf{S}_t} = \left[\left[\mu_{A_j^k}(x_i) \right] \left[\Omega^t_{\text{A}^k}(x_{\text{i}}) \right] \right] \tag{3.14}$$

$$F_{\text{S}_t} = \left[\begin{bmatrix} \mu_{A_1^k}(x_1) & \mu_{A_2^k}(x_1) & \ldots & \mu_{A_N^k}(x_1) \\ \mu_{A_1^k}(x_2) & \mu_{A_2^k}(x_2) & \ldots & \mu_{A_N^k}(x_2) \\ . & . & \ldots. \\ . & . & \ldots. \\ . & . & \ldots. \\ \mu_{A_1^k}(x_M) & \mu_{A_2^k}(x_M) & \ldots & \mu_{A_N^k}(x_M) \end{bmatrix} \begin{bmatrix} \Omega^t_{\text{A}^k}(x_1) \\ \Omega^t_{\text{A}^k}(x_2) \\ . \\ . \\ . \\ \Omega^t_{\text{A}^k}(x_M) \end{bmatrix} \right] \tag{3.15}$$

F_{s_t} is Plithogenic subjective attributively whole hyper-super-soft-matrix in the fuzzy environment, for further details see ref [20].

The elements of the last column of this matrix represented in (Equation 3.16) represent Attributively-Whole Fuzzy Memberships $\Omega^t_{\text{A}^l}(x_{\text{i}})$,

$$\begin{bmatrix} \Omega^t_{\text{A}^k}(x_1) \\ \Omega^t_{\text{A}^k}(x_2) \\ . \\ . \\ . \\ \Omega^t_{\text{A}^k}(x_M) \end{bmatrix} \tag{3.16}$$

t represents an aggregation operator that is used to accumulate the fuzzy memberships i.e. $t = 1$, (Disjunction operator), $t = 2$, (conjunction operator), $t = 3$ (averaging operator). This PFSAWHS-Matrix shows both an inner and outer interpretation of the universe. The inside state of the universe, event, or reality is reflected by individual memberships $\mu_{A_j^k}(x_i)$ of subjects while the outside state of the universe, event, or reality represented by attributively aggregated fuzzy memberships $\Omega_{A^k}(x_i)$. Therefore, the PFSAWHS-Matrix would allow a subjective classification through attributive accumulation. Attributive aggregation is obtained by applying several suitable aggregation operators to $\mu_{A_j^k}(x_i)$ at the index j.

Definition 3.2.14 (PFASWHSS-Matrix)

Let U_F be the initial universe of discourse, in the Fuzzy environment and $P(U_F)$ be the power set of U_F. Let $A_1^k, A_2^k, \ldots, A_N^k$ are N distinct attributes/subattributes for $j = 1, 2, \ldots N$, $k = 1, 2, \ldots L$ then the mapping F, from the cross product of the attributes sub-attributes to the powerset of the fuzzy universe of discourse $F : A_1^k \times A_2^k \times \cdots \times A_{\text{N}}^k \to P(U_F)$ represented in the matrix form in such a way that its elements consist of individual fuzzy memberships $\mu_{A_j^k}(x_i)$ as well as subjectively aggregated fuzzy memberships $\Omega_{A_{\text{j}}^k}(X)$, $\forall x_i \in U_F$, then this matrix is called a Plithogenic Fuzzy Attributive Subjectively Whole Hyper-Super-soft-Matrix (PFASWHSS-Matrix).

The PFSAWHSS-Matrix F_{A_t} is described underneath in (Equation 3.17)

$$F_{\mathbf{A}_t} = \begin{bmatrix} \left[\mu_{A_j^k}(x_i) \right] \\ \left[\Omega^t_{\mathbf{A}_j^k}(X) \right] \end{bmatrix} \tag{3.17}$$

It is observed that his matrix is represented by both individual fuzzy memberships $\mu_{A_j^k}(x_i)$ (individual fuzzy states of subjects regarding each attribute) and the aggregated fuzzy memberships $\Omega_{A_{\text{j}}^k}(X)$ (subject-wise aggregated states). The expanded form of this matrix is described below.

$$F_{A_t} = \begin{bmatrix} \begin{bmatrix} \mu_{A_1^k}(x_1) & \mu_{A_2^k}(x_1) & \ldots & \mu_{A_N^k}(x_1) \\ \mu_{A_1^k}(x_2) & \mu_{A_2^k}(x_2) & \ldots & \mu_{A_N^k}(x_2) \\ . & . & \ldots. \\ . & . & \ldots. \\ . & . & \ldots. \\ \mu_{A_1^k}(x_M) & \mu_{A_2^k}(x_M) & \ldots & \mu_{A_N^k}(x_M) \end{bmatrix} \\ \\ \begin{bmatrix} \Omega^t_{A_1^k}(X) \; \Omega^t_{A_2^k}(X) \ldots \Omega^t_{A_N^k}(X) \end{bmatrix} \end{bmatrix} \quad (3.18)$$

The elements of the last column-matrix as given below, in Equation 3.19 are subjectively-Whole Fuzzy Memberships ($\Omega^t_{A_N^k}(X)$) of PFASWHSS-Matrix.

$$\begin{bmatrix} \Omega^t_{A_1^k}(X) \; \Omega^t_{A_2^k}(X) \ldots \Omega^t_{A_N^k}(X) \end{bmatrix} (3.19) \quad (3.19)$$

Note: The whole fuzzy memberships $\Omega_{\mathbf{A}_\alpha}(x_i)$ and $\Omega^k_{A_j}(X)$ that are aggregated attribute-wise and subject-wise respectively and whole fuzzy non-memberships $\Phi_{\mathbf{A}_\alpha}(x_i)$ and $\Phi^k_{A_j}(X)$ are dependent on individual fuzzy memberships $\mu_{A_j^k}(x_i)$, and non-membership $\upsilon_{A_j^k}(x_i)$.

Definition 3.2.15 (Hyper-Super-Matrices): Hyper Super Matrices are clusters of super-matrix-layers. These are such Hyper-matrices that have several layers of super-matrices represented by more than two variation indices ($d \succ 2$) and further their elements are matrices or scalars.

As we know, all ordinary $M \times N$ Matrices on real vector space are tensors of rank 2, i.e., these ordinary matrices are represented by the use of two variation indices. For example $[a_{ij}]$ is an ordinary matrix having represents one variation by index i as rows of the Matrix and the second variation by index j represents columns of the Matrix. While the hyper-super-matrices are constructed using four variation indices which represent four types of variations that occur together. The hyper-super-matrices are rank-4 tensors. While the hypermatrices represented by three variation indices that describe three types of variations at a time are rank-3 tensors.

These Hyper Super matrices are formulated by hybridizing hyper-matrices and super-matrices.

$A = [a_{ijk}]$ is a Hyper-Matrix. This hypermatrix represents three kinds of variations, described by the use of three indices i, j and k where the i, j, k are positive integers.

$B = \begin{bmatrix} [b_{ijk}] \\ [a_{jkt}] \end{bmatrix}$ represents a Hyper-Super-Matrix that represents four types of variations described by the use of four indices i, j, k and t.

Definition 3.2.16 (HyperSoft-Matrix): Let U be the initial universe of discourse, and $P(U)$ be the power set of U. Let $A_1^k, A_2^k, \ldots, A_n^k$ for $n \geq 1$ be n distinct attributes, $k = 1, 2, \ldots L$ represents attributes values, a function $F : A_1^k \times A_2^k \times \cdots \times A_n^k \to P(U)$ is a hypersoft matrix, also called an order-$M \times N \times L$ hypersoft matrix or d-hypersoft Matrix ($d = 3$) i.e., a matrix that represents a hypersoft-set is a hypersoft-matrix (HS-Matrix). An HS-Matrix with $d \succ 3$ (More than three variations) and whose elements are matrices is called a Hyper-Super-Soft-Matrices (HSS-Matrix). This HSS-Matrix consisted of clusters of several parallel layers of ordinary matrices, and elements of these matrix layers are matrices or scalars.

Note: All simple $M \times N$ Matrices on real vector space are tensors of rank 2. The new HyperSoft-Matrix (HS- Matrix) with three variation indices is a rank three tensor and the Hyper-Super-Soft matrix (HSS matrix) with four variation indices is a Fourth rank tensor. The HS-Matrix (third rank tensors) and HSS-Matrix (fourth rank tensors) are a generalized version of the ordinary matrices (second rank tensors).

3.3. PFHS-Matrix and index-based views

The Plithogenic Fuzzy HyperSoft Set in matrix form is expressed as, $F = [\mu_{A_j^k}(x_i)]$

Plithogenic Fuzzy HyperSoft-Matrix represents three types of variation indices used to portray the fuzzy memberships $\mu_{A_j^k}(x_i)$. The first variation on index i is with respect to subjects representing M rows of $M \times N \times L$ Matrix while the second variation on index j is concerning to attributes representing N columns of $M \times N \times L$ Matrix, i and j represent arrangements of rows and columns in the form of simple $M \times N$ Matrix-layer.

However, when we consider sub-attributes of respective attributes, the third type of variation arises. This third variation is on the index k that is used to portray sub-attributes (attribute levels). These sub-attributes are displayed in the form of L level layers of an $M \times N$ Matrix. The hyper-matrix is generated by these parallel layers of $M \times N$ ordinary matrices.

These level layers of hyper-matrix are categorized into three types:

1 The front to back and inner Vertical level layers are generated by$L$ number of matrix layers. Where each layer is an ordinary matrix of order $M \times N$.
2 The left to right and inner vertical level layers are generated by N number of matrix layers such that each layer is a Matrix of order $M \times L$.
3 The top to bottom horizontal and inner level layers are generated by M number of matrix layers of order, $L \times N$.

Consider The PFHS-Matrix $F = \left[\mu_{A_j^k}(x_i)\right]$ and its dilated version described in Equation 3.3, the three types of level layers of this PFHS-Matrix F are described as under,

3.3.1. Type-1 level-layers of PFHS-Matrix

If the Equation 3.3 is further expanded with respect to k by varying k from 1 to L, then L number of front to back, and inner level layers of PFHS-Matrix, would be constructed where each layer is a matrix of order $M \times N$. We can describe numerous layers of PFHS-Matrix in three different ways, i.e., parallel layers of type-1 are obtained by vertically cutting a box matrix of order $M \times N \times L$ from front to back. These level cuts are called level-layers of type 1. They can be expressed on a two-dimensional page by giving step-by-step variation to the index k in Equation (3.3) and represented as described below in Equation (3.20).

$$F = \begin{bmatrix} \begin{bmatrix} \mu_{A_1^1}(x_1) & \mu_{A_2^1}(x_1) & \cdots & \mu_{A_N^1}(x_1) \\ \mu_{A_1^1}(x_2) & \mu_{A_2^1}(x_2) & \cdots & \mu_{A_N^1}(x_2) \\ . & . & \cdots & . \\ . & . & \cdots & . \\ . & . & \cdots & . \\ \mu_{A_1^1}(x_M) & \mu_{A_2^1}(x_M) & \cdots & \mu_{A_N^1}(x_M) \end{bmatrix} \\ \begin{bmatrix} \mu_{A_1^2}(x_1) & \mu_{A_2^2}(x_1) & \cdots & \mu_{A_N^2}(x_1) \\ \mu_{A_1^2}(x_2) & \mu_{A_2^2}(x_2) & \cdots & \mu_{A_N^2}(x_2) \\ . & . & \cdots & . \\ . & . & \cdots & . \\ . & . & \cdots & . \\ \mu_{A_1^2}(x_M) & \mu_{A_2^2}(x_M) & \cdots & \mu_{A_N^2}(x_M) \end{bmatrix} \\ \vdots \\ \begin{bmatrix} \mu_{A_1^L}(x_1) & \mu_{A_2^L}(x_1) & \cdots & \mu_{A_N^k}(x_1) \\ \mu_{A_1^L}(x_2) & \mu_{A_2^L}(x_2) & \cdots & \mu_{A_N^L}(x_2) \\ . & . & \cdots & . \\ . & . & \cdots & . \\ . & . & \cdots & . \\ \mu_{A_1^L}(x_M) & \mu_{A_2^L}(x_M) & \cdots & \mu_{A_N^L}(x_M) \end{bmatrix} \end{bmatrix} \tag{3.20}$$

Example 3. Consider the PFHS-Matrix F described in Equation (3.8)

$$F = \begin{bmatrix} \begin{bmatrix} 0.3\ 0.6\ 0.3\ 0.5 \\ 0.4\ 0.5\ 0.2\ 0.1 \\ 0.6\ 0.2\ 0.3\ 0.7 \end{bmatrix} \\ \begin{bmatrix} 0.5\ 0.4\ 0.2\ 0.6 \\ 0.5\ 0.7\ 0.8\ 0.4 \\ 0.7\ 0.6\ 0.5\ 0.9 \end{bmatrix} \end{bmatrix}$$

F represents the front view of PFHS-Matrix and its front to back two Matrix-layers are given bellow,

$$\text{First layer, } \begin{bmatrix} 0.3\ 0.6\ 0.3\ 0.5 \\ 0.4\ 0.5\ 0.2\ 0.1 \\ 0.6\ 0.2\ 0.3\ 0.7 \end{bmatrix} \tag{3.21}$$

for $k = 1$

$$\text{Second layer, } \begin{bmatrix} 0.5\ 0.4\ 0.2\ 0.6 \\ 0.5\ 0.7\ 0.8\ 0.4 \\ 0.7\ 0.6\ 0.5\ 0.9 \end{bmatrix} \tag{3.22}$$

for $k = 2$

3.3.2. Type-2 level layers of PFHS-Matrix level layers of PFHS-Matrix

Similarly, on the other hand, each column of the $M \times N$ matrix when expanded at its rear side creates a matrix of order $M \times L$ which expands N (attribute) into vertical layers (attribute leves) i.e., like creating vertical sections from left to right of the box matrix of the dimension $M \times N \times L$.

These vertical slices from left to right are level layers of type 2 and are expressed on the two-dimensional page by varying the index j as represented in Equation (3.23).

$$F = \begin{bmatrix} \begin{bmatrix} \mu_{A_1^1}(x_1) & \mu_{A_1^2}(x_1) & \cdots & \mu_{A_1^L}(x_1) \\ \mu_{A_1^1}(x_2) & \mu_{A_1^2}(x_2) & \cdots & \mu_{A_1^L}(x_2) \\ \vdots & \vdots & \cdots & \vdots \\ \mu_{A_1^1}(x_M) & \mu_{A_1^2}(x_M) & \cdots & \mu_{A_1^L}(x_M) \end{bmatrix} \\ \begin{bmatrix} \mu_{A_2^1}(x_1) & \mu_{A_2^2}(x_1) & \cdots & \mu_{A_2^L}(x_1) \\ \mu_{A_2^1}(x_2) & \mu_{A_2^2}(x_2) & \cdots & \mu_{A_2^L}(x_2) \\ \vdots & \vdots & \cdots & \vdots \\ \mu_{A_2^1}(x_M) & \mu_{A_2^2}(x_M) & \cdots & \mu_{A_2^L}(x_M) \end{bmatrix} \\ \vdots \\ \begin{bmatrix} \mu_{A_N^1}(x_1) & \mu_{A_N^2}(x_1) & \cdots & \mu_{A_N^L}(x_1) \\ \mu_{A_N^1}(x_2) & \mu_{A_N^2}(x_2) & \cdots & \mu_{A_N^L}(x_2) \\ \vdots & \vdots & \cdots & \vdots \\ \mu_{A_N^1}(x_M) & \mu_{A_N^2}(x_M) & \cdots & \mu_{A_N^L}(x_M) \end{bmatrix} \end{bmatrix} \tag{3.23}$$

3.3.3. Type-3 level layers of PFHS-Matrix

In a similar manner, level layers of type-3 are top to bottom and middle inner layers. These parallel layers are combined to create M layers of $L \times N$ Matrix.

These top to bottom M number of slices are level layers of type-3 and can be expressed on the two-dimensional page by gradually varying the index i as described below in Equation (3.24).

$$F = \begin{bmatrix} \begin{bmatrix} \mu_{A_1^1}(x_1) & \mu_{A_1^2}(x_1) & \cdots & \mu_{A_1^L}(x_1) \\ \mu_{A_2^1}(x_1) & \mu_{A_2^2}(x_1) & \cdots & \mu_{A_2^L}(x_1) \\ \vdots & \vdots & \cdots & \vdots \\ \mu_{A_N^1}(x_1) & \mu_{A_N^2}(x_1) & \cdots & \mu_{A_N^L}(x_1) \end{bmatrix} \\ \begin{bmatrix} \mu_{A_1^1}(x_2) & \mu_{A_1^2}(x_2) & \cdots & \mu_{A_1^L}(x_2) \\ \mu_{A_2^1}(x_2) & \mu_{A_2^2}(x_2) & \cdots & \mu_{A_N^L}(x_2) \\ \vdots & \vdots & \cdots & \vdots \\ \mu_{A_N^1}(x_2) & \mu_{A_N^2}(x_2) & \cdots & \mu_{A_N^L}(x_2) \end{bmatrix} \\ \vdots \\ \begin{bmatrix} \mu_{A_1^1}(x_M) & \mu_{A_1^2}(x_M) & \cdots & \mu_{A_1^L}(x_M) \\ \mu_{A_2^1}(x_M) & \mu_{A_2^2}(x_M) & \cdots & \mu_{A_2^L}(x_M) \\ \vdots & \vdots & \cdots & \vdots \\ \mu_{A_N^1}(x_M) & \mu_{A_N^2}(x_M) & \cdots & \mu_{A_N^L}(x_M) \end{bmatrix} \end{bmatrix} \tag{3.24}$$

4. Local aggregation operators for PFHSS-Matrix

In this section, local aggregation operators such as disjunction operators, conjunction operators, averaging operators, and complements for the plithogenic fuzzy hypersoft set/matrix are constructed. These operators would be used to formulate PFASWHSS matrix and PFSAWHSS matrix. The Whole (combined) memberships $\Omega^t_{\mathbf{A}_j^k}(X)$ for Plithogenic HyperSoft-Matrix would be developed by using the following operators.

In PFASWHSS-Matrix $F_{\mathbf{A}_t} = \begin{bmatrix} \left[\mu_{A_j^k}(x_i)\right] \\ \left[\Omega^t_{\mathbf{A}_j^k}(X)\right] \end{bmatrix}$ the last row of collective memberships $\Omega^t_{\mathbf{A}_j^k}(X)$ is accomplished by using three local aggregation operators i.e $t = 1$ represents Max-operator $t = 2$ represents the Min-operator, and $t = 3$ represents $Averaging$-operator. In addition, $t = 4$ stands for Complement.

In PFSAWHS-Matrix, $F_{\mathbf{S}_t} = \left[\left[\mu_{A_j^k}(x_i)\right] \left[\Omega^t_{\mathrm{A}^k}(x_\mathrm{i})\right]\right]$ the last column of cumulative memberships $\Omega_{A^k}(x_i)$ is acquired by using three local aggregation operators, $t = 1$, the Max-operator $t = 2$, the Min-operator, and $t = 3$ the $Averaging$-operator. And $t = 4$ represents the Complement.

These four operators are structured as follows,

4.1. Local Disjunction Operator for construction of PFASWHSS-Matrix

$$\cup_i \left(\mu_{A_j^k}(x_i)\right) = \underset{i}{Max}\left(\mu_{A_j^k}(x_i)\right) = \Omega^1_{\mathbf{A}_j^k}(X), \tag{4.1}$$

for some $k = l$

Choose maximum membership from j_{th}col of PFHS-Matrix.

4.2. Local Disjunction Operator for construction of PFSAWHSS-Matrix

$$\cup_j \left(\mu_{A_j^k}(x_i)\right) = \underset{j}{Max}\left(\mu_{A_j^k}(x_i)\right) = \Omega^1_{\mathrm{A}^k}(x_\mathrm{i}), \tag{4.2}$$

for some $k = l$

(Choose maximum membership from i_{th} row of PFHS-Matrix)

4.3. Local Conjunction Operator for construction of PFASWHSS-Matrix

$$\cap_i \left(\mu_{A_j^k}(x_i)\right) = \underset{i}{Min}\left(\mu_{A_j^k}(x_i)\right) = \Omega^2_{\mathbf{A}_j^k}(X), \quad (4.3)$$

for some $k = l$

Choose minimum membership from j_{th} col of PFHS-Matrix.

4.4. Local Conjunction Operator for construction of PFSAWHSS-Matrix

$$\cap_j \left(\mu_{A_j^k}(x_i)\right) = \underset{j}{Min}\left(\mu_{A_j^k}(x_i)\right) = \Omega^2_{\mathrm{A}^k}(x_\mathrm{i}), \quad (4.4)$$

for some $k = l$

Choose minimum membership from i_{th} row of PFHS-Matrix.

4.5. Local averaging Operator for construction of PFASWHSS-Matrix

$$\Gamma_i \left(\mu_{A_j^k}(x_i)\right) = \frac{\sum_i \left(\mu_{A_j^k}(x_i)\right)}{M} = \Omega^3_{\mathbf{A}_j^k}(X), \quad (4.5)$$

for some $k = l$

Take the average of memberships of j_{th} col) of given specific k_{th}-layer.

4.6. Local averaging Operator for construction of PFSAWHSS-Matrix

$$\Gamma_j \left(\mu_{A_j^k}(x_i)\right) = \frac{\sum_j \left(\mu_{A_j^k}(x_i)\right)}{N} = \Omega^3_{\mathrm{A}^k}(x_\mathrm{i}), \quad (4.6)$$

for some $k = l$

Take the average of memberships of i_{th} row of PFHS-Matrix of given specific k_{th}-layer.

In $\mu_{A_j^k}(x_i)$ i offers a row-wise variation of states of subjects, j gives a column-wise variation of states and k produces variations of states as sub-attributes in the form of matrix layers.

4.7. Local Complement for construction of PFASWHSS-Matrix

$$C_{loc}(F) = \begin{Bmatrix} Max_i \left(1 - \mu_{A_j^k}(x_i)\right) \\ Min_i \left(1 - \mu_{A_j^k}(x_i)\right) \\ \sum_{i=1}^{M} \frac{\left(1-\mu_{A_j^k}(x_i)\right)}{M} \end{Bmatrix} \quad (4.7)$$

for some $k = l$

4.8. Local Complement for construction of PFSAWHSS-Matrix

$$C_{loc}(F) = \begin{Bmatrix} Max_j \left(1 - \mu_{A_j^k}(x_i)\right) \\ Min_j \left(1 - \mu_{A_j^k}(x_i)\right) \\ \sum_{j=1}^{N} \frac{\left(1-\mu_{A_j^k}(x_i)\right)}{N} \end{Bmatrix} \quad (4.8)$$

for some $k = l$

Here C_{loc} represent the local *Complement* of PFHS-Matrix F for a certain level of attributes $k = l$. This Complement is applied across PFHS-Matrix F by taking the Complement of each fuzzy membership $\mu_{A_j^k}(x_i)$ and then choosing either maximum or minimum or an average membership from the established column or row.

Further description and application of these local aggregation operators are presented in Sec-5.

5. Construction of PFASWHSS-Matrix, PFSAWHSS-Matrix, and Application of local aggregation operators as COVID-19 Data Structures

5.1. (Plithogenic Fuzzy Attributive Subjectively-Whole Hyper-Super-Soft-Matrix)

The matrix representation of Plithogenic Attributive Subjectively Whole HyperSoft Set in the fuzzy environment named, PFASWHSS-Matrix. The elements of PFASWHSS-Matrix are matrix-layers, matrices, or scalars, so a PFASWHSS-Matrix is a hybridization of HS-Matrix and HSS-Matrix. The HSS-Matrix has such layers of matrices whose elements are further matrices or scalars. If one considers the HS-Matrix in a fuzzy environment then the subject-wise combined fuzzy memberships ($\Omega^{t}_{\mathrm{A}^{k}_{j}}$ (X)) are constructed by using the aggregation operator, t such that $t = 1, 2, 3, 4$, represents local operators constructed in the previous section see Equation 4.1, to 4.8. The compact form of PFASWHSS-Matrix $F_{\mathbf{A}_t}$ can be described as

$$F_{\mathbf{A}_t} = \left[\begin{bmatrix} \mu_{A^k_{\mathrm{j}}}(x_{\mathrm{i}}) \\ \Omega^{t}_{\mathbf{A}^{k}_{\mathrm{j}}}(X) \end{bmatrix} \right]$$

In this PFASWHSS-Matrix, four types of diversification are described. These diversification /variations are described as under, the first Variation of $i = 1, 2, \ldots M$ would produce M rows of $F_{\mathbf{A}_t}$. The second variations on $j = 1, 2, \ldots N$ produce N columns, and the third variations on $k = 1, 2, \ldots L$ would generate L combinations of rows and columns as parallel matrix-layers of $M \times N$ matrix. The fourth variation on $t = 1, 2, \ldots P$ would provide P sets, i.e., P numbers of Clusters of L numbers of parallel layers of $M \times N$ matrices. These sets of parallel layers can represent clusters of parallel universes or parallel realities. These clusters in the form of expanded form of PFASWHSS-Matrix $F_{\mathbf{A}_t}$ are described below in Equation 5.1.

$$\mathbf{F}_{\mathbf{A_t}} = \begin{bmatrix}
\begin{bmatrix}
\begin{bmatrix} \mu_{A^1_1}(x_1) & \mu_{A^1_2}(x_1) & \cdots & \mu_{A^1_N}(x_1) \\ \mu_{A^1_1}(x_2) & \mu_{A^1_2}(x_2) & \cdots & \mu_{A^1_N}(x_2) \\ . & . & \cdots & . \\ . & . & \cdots & . \\ . & . & \cdots & . \\ \mu_{A^1_1}(x_M) & \mu_{A^1_2}(x_M) & \cdots & \mu_{A^1_N}(x_M) \end{bmatrix} \\
\begin{bmatrix} \Omega^1_{\mathbf{A}^1_1}(\boldsymbol{X}) & \Omega^1_{\mathbf{A}^1_2}(\boldsymbol{X}) & \cdots & \Omega^1_{\mathbf{A}^1_N}(\boldsymbol{X}) \end{bmatrix}
\end{bmatrix} \\
\begin{bmatrix}
\begin{bmatrix} \mu_{A^2_1}(x_1) & \mu_{A^2_2}(x_1) & \cdots & \mu_{A^2_N}(x_1) \\ \mu_{A^2_1}(x_2) & \mu_{A^2_2}(x_2) & \cdots & \mu_{A^2_N}(x_2) \\ . & . & \cdots & . \\ . & . & \cdots & . \\ . & . & \cdots & . \\ \mu_{A^2_1}(x_M) & \mu_{A^2_2}(x_M) & \cdots & \mu_{A^2_N}(x_M) \end{bmatrix} \\
\begin{bmatrix} \Omega^1_{\mathbf{A}^1_2}(\boldsymbol{X}) & \Omega^1_{\mathbf{A}^2_2}(\boldsymbol{X}) & \cdots & \Omega^1_{\mathbf{A}^2_N}(\boldsymbol{X}) \end{bmatrix}
\end{bmatrix} \\
\vdots \\
\begin{bmatrix}
\begin{bmatrix} \mu_{A^L_1}(x_1) & \mu_{A^L_2}(x_1) & \cdots & \mu_{A^k_N}(x_1) \\ \mu_{A^L_1}(x_2) & \mu_{A^L_2}(x_2) & \cdots & \mu_{A^L_N}(x_2) \\ . & . & \cdots & . \\ . & . & \cdots & . \\ . & . & \cdots & . \\ \mu_{A^L_1}(x_M) & \mu_{A^L_2}(x_M) & \cdots & \mu_{A^L_N}(x_M) \end{bmatrix} \\
\begin{bmatrix} \Omega^1_{\mathbf{A}^L_1}(\boldsymbol{X}) & \Omega^1_{\mathbf{A}^L_2}(\boldsymbol{X}) & \cdots & \Omega^1_{\mathbf{A}^1_N}(\boldsymbol{X}) \end{bmatrix}
\end{bmatrix} \\
\vdots \\
\begin{bmatrix}
\begin{bmatrix} \mu_{A^1_1}(x_1) & \mu_{A^1_2}(x_1) & \cdots & \mu_{A^1_N}(x_1) \\ \mu_{A^1_1}(x_2) & \mu_{A^1_2}(x_2) & \cdots & \mu_{A^1_N}(x_2) \\ . & . & \cdots & . \\ . & . & \cdots & . \\ . & . & \cdots & . \\ \mu_{A^1_1}(x_M) & \mu_{A^1_2}(x_M) & \cdots & \mu_{A^1_N}(x_M) \end{bmatrix} \\
\begin{bmatrix} \Omega^2_{\mathbf{A}^1_1}(\boldsymbol{X}) & \Omega^2_{\mathbf{A}^1_2}(\boldsymbol{X}) & \cdots & \Omega^2_{\mathbf{A}^1_N}(\boldsymbol{X}) \end{bmatrix}
\end{bmatrix} \\
\begin{bmatrix}
\begin{bmatrix} \mu_{A^2_1}(x_1) & \mu_{A^2_2}(x_1) & \cdots & \mu_{A^2_N}(x_1) \\ \mu_{A^2_1}(x_2) & \mu_{A^2_2}(x_2) & \cdots & \mu_{A^2_N}(x_2) \\ \vdots & \vdots & \cdots & \vdots \\ . & . & \cdots & . \\ . & . & \cdots & . \\ . & . & \cdots & . \\ \mu_{A^2_1}(x_M) & \mu_{A^2_2}(x_M) & \cdots & \mu_{A^2_N}(x_M) \end{bmatrix} \\
\begin{bmatrix} \Omega^2_{\mathbf{A}^2_1}(\boldsymbol{X}) & \Omega^2_{\mathbf{A}^2_2}(\boldsymbol{X}) & \cdots & \Omega^2_{\mathbf{A}^2_N}(\boldsymbol{X}) \end{bmatrix}
\end{bmatrix} \\
\vdots \\
\begin{bmatrix}
\begin{bmatrix} \mu_{A^L_1}(x_1) & \mu_{A^L_2}(x_1) & \cdots & \mu_{A^k_N}(x_1) \\ \mu_{A^L_1}(x_2) & \mu_{A^L_2}(x_2) & \cdots & \mu_{A^L_N}(x_2) \\ . & . & \cdots & . \\ . & . & \cdots & . \\ . & . & \cdots & . \\ \mu_{A^L_1}(x_M) & \mu_{A^L_2}(x_M) & \cdots & \mu_{A^L_N}(x_M) \end{bmatrix} \\
\begin{bmatrix} \Omega^2_{\mathbf{A}^L_1}(\boldsymbol{X}) & \Omega^2_{\mathbf{A}^L_2}(\boldsymbol{X}) & \cdots & \Omega^2_{\mathbf{A}^L_N}(\boldsymbol{X}) \end{bmatrix}
\end{bmatrix}
\end{bmatrix} \quad (5.1)$$

The Hyper-Super-Soft Matrix consists of four clusters associated with four aggregation operators $t = 1, 2, 3, 4$. Every cluster has two matrix-layers associated with attribute-levels. Each matrix-layer has M rows and N columns.

These multiple clusters of parallel layers (parallel universes/realities/events/information) are accomplished by gradually varying the fourth index, which is used to represent multiple aggregation operators to accumulate the memberships of all subjects. These aggregation operators are called local operators.

5.2. *(Plithogenic Fuzzy Subjective Attributive-Whole Hyper-Super-Soft-Matrix)*

The Plithogenic Fuzzy Subjective Attributively-Whole Hyper-Super-Soft-Matrix is shown in (Equation 5.1a) and is described on a similar basis for further details see Ref. [20]

$$F_{S_t} = \left[\left[\mu_{A_j^k}(x_i) \right] \left[\Omega^t_{A^k}(x_i) \right] \right] \tag{5.1a}$$

5.3. *Application as COVID-19 Data Structure*

Example 4. Let $U = \{x_1, x_2, x_3, x_4, x_5, x_6\}$ be the set of six patients who have visited the hospital with symptoms of COVID-19. They were examined by some doctors. Consider a medical exam case of a doctor who examined three of them by asking some questions about four symptoms (attributes), and each symptom is categorized into two sub-symptoms (sub-attributes). These three patients are considered to be test subjects. The information from their visits was recorded and organized as a plithogenic fuzzy hypersoft matrix and further represented and analyzed with the aid of the plithogenic fuzzy hyper super soft matrix.

Let $T = \{x_1, x_2, x_3\} \subset U$ be the set of these three patients considered by a doctor for examination.

Let the attributes be A_j^k; $j = 1, 2, 3, 4$ with sub-attributes k=1, 2 are described as,

A_1^k = Fever with numeric values, $k = 1, 2$,
A_1^1 = Mild fever, A_1^2 = High-grade fever
A_2^k = Dry cough, with numeric values, $k = 1, 2$
A_2^1 = Dry cough, A_2^2 = Acute dry cough
A_3^k = Breathing difficulty with numeric values, $k = 1, 2$
A_3^1 = Minor breathing difficulty,
A_3^2 = Swear breathing difficulty needs oxygen
A_4^k = Nausea, with numeric values $k = 1, 2$
A_4^1 = Minor nausea A_4^2 = Strong nausea

PFHS-Set representation:

Let the Function F indicate these given attributes/sub-attributes as described below,

$$F : A_1^k \times A_2^k \times A_3^k \times A_4^k \to P(U) \tag{5.1}$$

$$\text{S.t } F\left(A_1^1, A_2^1, A_3^1, A_4^1\right) = \{x_1, x_2, x_3\}$$

be a hypersoft set. let we consider $A_1^1, A_2^1, A_3^1, A_4^1$ a combination (α combination) represents the first level of disease on the first visit)

$F\left(A_1^2, A_2^2, A_3^2, A_4^2\right) = \{x_1, x_2, x_3\}$, $A_1^2, A_2^2, A_3^2, A_4^2$ a combination (β combination) represents the second level of disease. The patient with the second level, if necessary, would be admitted to the hospital for special care after a week through observation and analysis of conditions. The Individual fuzzy memberships are assigned using a linguistic scale and then represented in PFHS-Set. Consider the fuzzy memberships of elements of T=$\{x_1, x_2, x_3\}$ as $\mu_{A_j^k}(x_i)$ for $i = 1, 2, 3$ and $j = 1$ to 4. The Plithogenic Fuzzy Memberships that reflect the health-states of x_i subjects associated with α combination of attributes are organized as PFHS-Set shown in Equation (5.2).

$$F(\alpha) = F\left(A_1^1, A_2^1, A_3^1, A_4^1\right) = \left\{ \begin{array}{c} x_1(0.3, 0.6, 0.4, 0.5), \\ x_2(0.4, 0.5, 0.3, 0.1), \\ x_3(0.6, 0.3, 0.3, 0.7) \end{array} \right\} \tag{5.2}$$

The information from the first visit (organized as a PFHS-Set) (shown in Equation (5.2) would generate the first level of the PFHS-Matrix.

Now regarding the states of patients for the second visit for β combination of attributes. Information is portrayed as PFHS-Set that is given in Equation (5.3).

$$F(\beta) = F\left(A_1^1, A_2^1, A_3^1, A_4^1\right) = \left\{ \begin{array}{c} x_1(0.5, 0.7, 0.5, 0.8), \\ x_2(0.7, 0.5, 0.5, 0.8), \\ x_3(0.6, 0.5, 0.6, 0.7) \end{array} \right\} \tag{5.3}$$

$F(\beta)$ is a Plithogenic fuzzy hypersoft set represents the second visit information as the second level of Matrix

PFHS-Matrix representation: Let F be the matrix form of PFHS-Set. Here the elements of rows represent states of subjects x_1, x_2, x_3 as fuzzy memberships. And elements of columns represent (the non-Physical aspect of subjects) A_1^k, A_2^k, A_3^k, A_4^kAttributes.

The information of hypersoft set shown in Equations (5.2) and (5.3) are organized in the form of PFHS-Matrix F as follows,

$$F = \begin{bmatrix} \begin{bmatrix} 0.3\ 0.6\ 0.4\ 0.5 \\ 0.4\ 0.5\ 0.3\ 0.1 \\ 0.6\ 0.3\ 0.3\ 0.7 \end{bmatrix} \\ \begin{bmatrix} 0.5\ 0.7\ 0.5\ 0.8 \\ 0.7\ 0.5\ 0.5\ 0.8 \\ 0.6\ 0.5\ 0.6\ 0.7 \end{bmatrix} \end{bmatrix} \tag{5.4}$$

This PFHS matrix consists of two layers and represents an inner view of the universe

** Online mode **

$$\begin{bmatrix} 0.3\ 0.6\ 0.4\ 0.5 \\ 0.4\ 0.5\ 0.3\ 0.1 \\ 0.6\ 0.3\ 0.3\ 0.7 \end{bmatrix}, \begin{bmatrix} 0.5\ 0.7\ 0.5\ 0.8 \\ 0.7\ 0.5\ 0.5\ 0.8 \\ 0.6\ 0.5\ 0.6\ 0.7 \end{bmatrix} \tag{5.5}$$

Equation (5.5) represents the First and Second level layers of PFHS-Matrix.

Individual fuzzy memberships $\mu_{A_j^k}(x_i)$: The elements of This PFHS-Matrix represent individual fuzzy memberships s $\mu_{A_j^k}(x_i)$.

5.3.1. Local Disjunction Operator for construction of PFASWHSS-Matrix

By applying local Disjunction operators i.e. the Max-operator ($t = 1$) described in Equation 4.1 on columns of F, we will get Subject-wise whole Fuzzy Memberships $\Omega^1_{\mathbf{A}_j^k}(X)$. These Subject-wise whole Fuzzy Memberships are represented in the last row of PFASWHS-Matrix F_{A_1} as shown below in Equation 5.6

$$F_{A_1} = \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} 0.3\ 0.6\ 0.4\ 0.5 \\ 0.4\ 0.5\ 0.3\ 0.1 \\ 0.6\ 0.3\ 0.3\ 0.7 \end{bmatrix} \\ \begin{bmatrix} \mathbf{0.6\ 0.6\ 0.4\ 0.7} \end{bmatrix} \end{bmatrix} \\ \begin{bmatrix} \begin{bmatrix} 0.5\ 0.7\ 0.5\ 0.8 \\ 0.7\ 0.5\ 0.5\ 0.8 \\ 0.6\ 0.5\ 0.6\ 0.7 \end{bmatrix} \\ \begin{bmatrix} \mathbf{0.7\ 0.7\ 0.6\ 0.8} \end{bmatrix} \end{bmatrix} \end{bmatrix} \tag{5.6}$$

$\begin{bmatrix} 0.6\ 0.6\ 0.4\ 0.7 \end{bmatrix}$ and $\begin{bmatrix} 0.7\ 0.7\ 0.6\ 0.8 \end{bmatrix}$ are Subject-wise whole Fuzzy Memberships$\Omega_{A_j^k}(X)$ are represented for each attribute of the first and second level-layer of the given matrix and represent the optimist outer view of the universe through the last row of F_{A_1}.

$\begin{bmatrix} 0.4\ 0.4\ 0.6\ 0.3 \end{bmatrix}$ and $\begin{bmatrix} 0.3\ 0.3\ 0.4\ 0.2 \end{bmatrix}$ are Subject-wise whole fuzzy non-memberships $\Phi_{A_j^l}(X)$ for attributes of the first and second levels.

5.3.2. Local Disjunction Operator for construction of PFSAWHSS-Matrix:

When the aggregation operator, $t = 1$ (max-operator described in Equation 4.2) is applied across the rows of PFHS-Matrix, F (Equation 5.4) we will get attribute-wise whole fuzzy Memberships $\Omega^1_{\mathrm{A}^k}(x_\mathrm{i})$ that are represented in the last column of PFSAWHS-Matrix F_{S_1} given bellow in Equation 5.7,

$$F_{S_1} = \begin{bmatrix} \begin{bmatrix} 0.3\ 0.6\ 0.4\ 0.5 \\ 0.4\ 0.5\ 0.3\ 0.1 \\ 0.6\ 0.3\ 0.3\ 0.7 \end{bmatrix} & \begin{bmatrix} \mathbf{0.6} \\ \mathbf{0.5} \\ \mathbf{0.7} \end{bmatrix} \\ \begin{bmatrix} 0.5\ 0.7\ 0.5\ 0.8 \\ 0.7\ 0.5\ 0.5\ 0.8 \\ 0.6\ 0.5\ 0.6\ 0.7 \end{bmatrix} & \begin{bmatrix} \mathbf{0.8} \\ \mathbf{0.8} \\ \mathbf{0.7} \end{bmatrix} \end{bmatrix} \tag{5.7}$$

F_{S_1} is the Plithogenic Subjective Attributively Whole Hyper-Super Soft-Matrix.

$\begin{bmatrix} 0.6 \\ 0.5 \\ 0.7 \end{bmatrix}, \begin{bmatrix} 0.8 \\ 0.8 \\ 0.7 \end{bmatrix}$ are Attributively Whole Fuzzy Memberships constructed for combination attributes of first and second levels of PFSAWHSS-Matrix F. These Attributively Whole Fuzzy Memberships represent the Subjectively optimist exterior manifestation of the universe.

Attributively Whole Fuzzy Non-Memberships $\Phi_{A^l}(x_i) = (1 - \Omega_{A^l}(x_i))$ for attributes of first and second levels of PFSAWHSS-Matrix are $\begin{bmatrix} \mathbf{0.4} \\ \mathbf{0.5} \\ \mathbf{0.3} \end{bmatrix}, \begin{bmatrix} \mathbf{0.2} \\ \mathbf{0.2} \\ \mathbf{0.3} \end{bmatrix}$. These are attributively whole Fuzzy Non-Memberships for attributes of first and second level of PFSAWHSS-Matrix F_{S_1} (Equation 5.7).

5.3.3. Local Conjunction Operator for construction of PFASWHSS-Matrix

The PFASWHSS-Matrix constructed by applying the local conjunction operator described in Equation 4.3 column-wise ($t = 2$) on each specific layer of PFHS-Matrix F (equation 5.4). By applying this aggregation operator on columns of F, we will get Subjectively whole Fuzzy Memberships $\Omega^2_{\mathbf{A}^k_j}(X)$ that are represented in the last row of PFASWHS-Matrix F_{A_2} see Equation 5.8

$$F_{\mathrm{A}_2} = \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} 0.3 & 0.6 & 0.4 & 0.5 \\ 0.4 & 0.5 & 0.3 & 0.1 \\ 0.6 & 0.3 & 0.3 & 0.7 \end{bmatrix} \\ \begin{bmatrix} \mathbf{0.3} & \mathbf{0.3} & \mathbf{0.3} & \mathbf{0.1} \end{bmatrix} \end{bmatrix} \\ \begin{bmatrix} \begin{bmatrix} 0.5 & 0.7 & 0.5 & 0.8 \\ 0.7 & 0.5 & 0.5 & 0.8 \\ 0.6 & 0.5 & 0.6 & 0.7 \end{bmatrix} \\ \begin{bmatrix} \mathbf{0.5} & \mathbf{0.5} & \mathbf{0.5} & 0.7 \end{bmatrix} \end{bmatrix} \end{bmatrix} \quad (5.8)$$

Where,

$$\begin{bmatrix} \begin{bmatrix} 0.3 & 0.6 & 0.4 & 0.5 \\ 0.4 & 0.5 & 0.3 & 0.1 \\ 0.6 & 0.3 & 0.3 & 0.7 \end{bmatrix} \\ \begin{bmatrix} 0.3 & 0.3 & 0.3 & 0.1 \end{bmatrix} \end{bmatrix}$$

and

$$\begin{bmatrix} \begin{bmatrix} 0.5 & 0.7 & 0.5 & 0.8 \\ 0.7 & 0.5 & 0.5 & 0.8 \\ 0.6 & 0.5 & 0.6 & 0.7 \end{bmatrix} \\ \begin{bmatrix} 0.5 & 0.5 & 0.5 & 0.7 \end{bmatrix} \end{bmatrix}$$

are two front and back vertical level-layers of type 1 of the given matrix F_{A_2} for $k = 1$ and $k = 2$ and in these layers the following matrices,

$$\begin{bmatrix} 0.3 & 0.6 & 0.3 & 0.5 \\ 0.4 & 0.5 & 0.2 & 0.1 \\ 0.6 & 0.2 & 0.3 & 0.7 \end{bmatrix}, \begin{bmatrix} 0.5 & 0.4 & 0.2 & 0.6 \\ 0.5 & 0.7 & 0.8 & 0.4 \\ 0.7 & 0.6 & 0.5 & 0.9 \end{bmatrix}$$

The two matrix-layers given above represent the interior view of the universe by individual memberships while the last rows $[0.3\ 0.3\ 0.3\ 0.1]$ and $[0.5\ 0.5\ 0.5\ 0.7]$ of each layer represent the pessimist attributive exterior perception of the universe. The elements of these given rows represent the Whole memberships $\Omega_{A^k_j}(X)$.

5.3.4. Local Conjunction Operator for construction of PFSAWHSS-Matrix

The PFSAWHS-Matrix is constructed using the local conjunction operator ($t = 2$) row-wise as described in Equation 4.4 on each specific layer of PFHS-Matrix F (Equation 5.4).

The attributively whole memberships $\Omega^2_{\mathrm{A}^k}(x_\mathrm{i})$, represented by the last column of F_{S_2} as shown below in Equation (5.9),

$$F_{\mathrm{S}_2} = \begin{bmatrix} \begin{bmatrix} 0.3 & 0.6 & 0.4 & 0.5 \\ 0.4 & 0.5 & 0.3 & 0.1 \\ 0.6 & 0.3 & 0.3 & 0.7 \end{bmatrix} & \begin{bmatrix} \mathbf{0.3} \\ \mathbf{0.1} \\ \mathbf{0.3} \end{bmatrix} \\ \begin{bmatrix} 0.5 & 0.7 & 0.5 & 0.8 \\ 0.7 & 0.5 & 0.5 & 0.8 \\ 0.6 & 0.5 & 0.6 & 0.7 \end{bmatrix} & \begin{bmatrix} \mathbf{0.5} \\ \mathbf{0.5} \\ \mathbf{0.5} \end{bmatrix} \end{bmatrix} \quad (5.9)$$

F_{S_2} is PFSAWHS-Matrix constructed by using the Local Conjunction Operator.

The elements of the last columns of the matrix are Attributively Whole Fuzzy Memberships for attributes of the first and second level of PFSAWHSS-Matrix F. Furthermore, these two columns are representing the pessimist Subjective exterior perception of the universe whaich is obtained by using the conjunction operator.

5.3.5. Local averaging Operator for construction of PFASWHSS-Matrix

The PFASWHSS-Matrix is constructed using local averaging operators ($t = 3$) (described in equation 4.5) for each specific layer of PFHS-Matrix F (Equation 5.4). This aggregation operator (Averaging-operator) is applied along the columns of PFHS-Matrix to get attributively whole Fuzzy Memberships $\Omega^3_{\mathbf{A}^k_j}(X)$. These Memberships are represented In the last row of the PFASWHSS-Matrix F_{A_3} as described below in Equation 5.10,

$$F_{\mathrm{A}_3} = \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} 0.3 & 0.6 & 0.4 & 0.5 \\ 0.4 & 0.5 & 0.3 & 0.1 \\ 0.6 & 0.3 & 0.3 & 0.7 \end{bmatrix} \\ \begin{bmatrix} \mathbf{0.43} & \mathbf{0.46} & \mathbf{0.33} & \mathbf{0.43} \end{bmatrix} \end{bmatrix} \\ \begin{bmatrix} \begin{bmatrix} 0.5 & 0.7 & 0.5 & 0.8 \\ 0.7 & 0.5 & 0.5 & 0.8 \\ 0.6 & 0.5 & 0.6 & 0.7 \end{bmatrix} \\ \begin{bmatrix} \mathbf{0.6} & \mathbf{0.56} & \mathbf{0.53} & \mathbf{0.76} \end{bmatrix} \end{bmatrix} \end{bmatrix} \quad (5.10)$$

Where,

$$\begin{bmatrix} \begin{bmatrix} 0.3 & 0.6 & 0.4 & 0.5 \\ 0.4 & 0.5 & 0.3 & 0.1 \\ 0.6 & 0.3 & 0.3 & 0.7 \end{bmatrix} \\ \begin{bmatrix} \mathbf{0.43} & \mathbf{0.46} & \mathbf{0.33} & \mathbf{0.43} \end{bmatrix} \end{bmatrix},$$

$$\begin{bmatrix} \begin{bmatrix} 0.5 & 0.7 & 0.5 & 0.8 \\ 0.7 & 0.5 & 0.5 & 0.8 \\ 0.6 & 0.5 & 0.6 & 0.7 \end{bmatrix} \\ \begin{bmatrix} \mathbf{0.6} & \mathbf{0.56} & \mathbf{0.53} & \mathbf{0.76} \end{bmatrix} \end{bmatrix}$$

are two front and back vertical level-layers of $F_{\mathbf{A}_3}$ for $k = 1$ and $k = 2$ and in these layers the rows $\begin{bmatrix} \mathbf{0.43} & \mathbf{0.46} & \mathbf{0.33} & \mathbf{0.43} \end{bmatrix}$ and $\begin{bmatrix} \mathbf{0.6} & \mathbf{0.56} & \mathbf{0.53} & \mathbf{0.76} \end{bmatrix}$ are representing the neutral attributive exterior view of the universe through the averaging operator.

5.3.6. Local averaging Operator for construction of PFSAWHSS-Matrix

The PFSAWHSS-Matrix is constructed by applying local averaging operators described in Equation (4.6) ($t = 3$) on each specific layer of PFHS-Matrix. The aggregation operator (Averaging-operator) is applied along the rows of PFHS-Matrix to get subjectively whole Memberships $\Omega^3_{A^k}(x_i)$

$$F_{S_3} = \begin{bmatrix} \begin{bmatrix} 0.3 & 0.6 & 0.4 & 0.5 \\ 0.4 & 0.5 & 0.3 & 0.1 \\ 0.6 & 0.3 & 0.3 & 0.7 \end{bmatrix} & \begin{bmatrix} \mathbf{0.45} \\ \mathbf{0.32} \\ \mathbf{0.47} \end{bmatrix} \\ \begin{bmatrix} 0.5 & 0.7 & 0.5 & 0.8 \\ 0.7 & 0.5 & 0.5 & 0.8 \\ 0.6 & 0.5 & 0.6 & 0.7 \end{bmatrix} & \begin{bmatrix} \mathbf{0.62} \\ \mathbf{0.75} \\ \mathbf{0.6} \end{bmatrix} \end{bmatrix} \quad (5.11)$$

F_{S_3} is Plithogenic Subjective attributively Whole Hyper-Super Soft-Matrix. And the elements of the given last columns of F_{S_3} are Attributively Whole Fuzzy Memberships for attributes of first and second levels of PFSAWHSS-Matrix. These Whole fuzzy memberships represent the neutral Subjective outer view of the universe through the averaging operator.

5.3.7. Final PFASWHSS-Matrix

The final PFASWHSS-Matrix that is described in Equation 5.1 is obtained by combining the F_{A_1} (Equation 5.6), F_{A_2} (Equation 5.8) and F_{A_2} (Equation 5.10) in a single PFASWHSS-Matrix which is consisted of three Clusters of Matrices associated to the aggregation operators $t = 1$ (Max-operator) $t = 2$ (Min-operator), and $t = 3$ (*averaging*-operator). This Final PFASWHSS-Matrix is described bellow in Equation 5.12,

$$F_{A_t} = \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} 0.3 & 0.6 & 0.4 & 0.5 \\ 0.4 & 0.5 & 0.3 & 0.1 \\ 0.6 & 0.3 & 0.3 & 0.7 \end{bmatrix} \\ \begin{bmatrix} \mathbf{0.6} & \mathbf{0.6} & \mathbf{0.4} & \mathbf{0.7} \end{bmatrix} \end{bmatrix} \\ \begin{bmatrix} \begin{bmatrix} 0.5 & 0.7 & 0.5 & 0.8 \\ 0.7 & 0.5 & 0.5 & 0.8 \\ 0.6 & 0.5 & 0.6 & 0.7 \end{bmatrix} \\ \begin{bmatrix} 0.7 & 0.7 & 0.6 & 0.8 \end{bmatrix} \end{bmatrix} \end{bmatrix} \\ \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} 0.3 & 0.6 & 0.4 & 0.5 \\ 0.4 & 0.5 & 0.3 & 0.1 \\ 0.6 & 0.3 & 0.3 & 0.7 \end{bmatrix} \\ \begin{bmatrix} \mathbf{0.3} & \mathbf{0.3} & \mathbf{0.3} & \mathbf{0.1} \end{bmatrix} \end{bmatrix} \\ \begin{bmatrix} \begin{bmatrix} 0.5 & 0.7 & 0.5 & 0.8 \\ 0.7 & 0.5 & 0.5 & 0.8 \\ 0.6 & 0.5 & 0.6 & 0.7 \end{bmatrix} \\ \begin{bmatrix} \mathbf{0.5} & \mathbf{0.5} & \mathbf{0.5} & \mathbf{0.7} \end{bmatrix} \end{bmatrix} \end{bmatrix} \\ \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} 0.3 & 0.6 & 0.4 & 0.5 \\ 0.4 & 0.5 & 0.3 & 0.1 \\ 0.6 & 0.3 & 0.3 & 0.7 \end{bmatrix} \\ \begin{bmatrix} \mathbf{0.43} & \mathbf{0.46} & \mathbf{0.33} & \mathbf{0.43} \end{bmatrix} \end{bmatrix} \\ \begin{bmatrix} \begin{bmatrix} 0.5 & 0.7 & 0.5 & 0.8 \\ 0.7 & 0.5 & 0.5 & 0.8 \\ 0.6 & 0.5 & 0.6 & 0.7 \end{bmatrix} \\ \begin{bmatrix} \mathbf{0.6} & \mathbf{0.56} & \mathbf{0.53} & \mathbf{0.76} \end{bmatrix} \end{bmatrix} \end{bmatrix} \end{bmatrix} \quad (5.12)$$

5.3.8. Final PFSAWHSS-Matrix

The final PFSAWHSS-Matrix that is described in (Equation 5.1a) is constructed by combining the F_{S_1}(Equation 5.7), F_{S_2}(Equation 5.9) and F_{S_2}

(Equation 5.11) in a single PFSAWHSS-Matrix which is consisted of three Clusters of Matrices associated to the aggregation operators $t = 1$ (Max-operator) $t = 2$ (Min-operator), and $t = 3$ (*averaging*-operator). This Final PFSAWHSS-Matrix is described in Equation 5.13

$$F_{S_t} = \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} 0.3\ 0.6\ 0.4\ 0.5 \\ 0.4\ 0.5\ 0.3\ 0.1 \\ 0.6\ 0.3\ 0.3\ 0.7 \end{bmatrix} & \begin{bmatrix} \mathbf{0.6} \\ \mathbf{0.5} \\ \mathbf{0.7} \end{bmatrix} \\ \begin{bmatrix} 0.5\ 0.7\ 0.5\ 0.8 \\ 0.7\ 0.5\ 0.5\ 0.8 \\ 0.6\ 0.5\ 0.6\ 0.7 \end{bmatrix} & \begin{bmatrix} \mathbf{0.8} \\ \mathbf{0.8} \\ \mathbf{0.7} \end{bmatrix} \end{bmatrix} \\ \begin{bmatrix} \begin{bmatrix} 0.3\ 0.6\ 0.4\ 0.5 \\ 0.4\ 0.5\ 0.3\ 0.1 \\ 0.6\ 0.3\ 0.3\ 0.7 \end{bmatrix} & \begin{bmatrix} \mathbf{0.3} \\ \mathbf{0.1} \\ \mathbf{0.3} \end{bmatrix} \\ \begin{bmatrix} 0.5\ 0.7\ 0.5\ 0.8 \\ 0.7\ 0.5\ 0.5\ 0.8 \\ 0.6\ 0.5\ 0.6\ 0.7 \end{bmatrix} & \begin{bmatrix} \mathbf{0.5} \\ \mathbf{0.5} \\ \mathbf{0.5} \end{bmatrix} \end{bmatrix} \end{bmatrix} \\ \begin{bmatrix} \begin{bmatrix} 0.3\ 0.6\ 0.4\ 0.5 \\ 0.4\ 0.5\ 0.3\ 0.1 \\ 0.6\ 0.3\ 0.3\ 0.7 \end{bmatrix} & \begin{bmatrix} \mathbf{0.45} \\ \mathbf{0.32} \\ \mathbf{0.47} \end{bmatrix} \\ \begin{bmatrix} 0.5\ 0.7\ 0.5\ 0.8 \\ 0.7\ 0.5\ 0.5\ 0.8 \\ 0.6\ 0.5\ 0.6\ 0.7 \end{bmatrix} & \begin{bmatrix} \mathbf{0.62} \\ \mathbf{0.75} \\ \mathbf{0.6} \end{bmatrix} \end{bmatrix} \end{bmatrix} \tag{5.13}$$

5.3.9. *The Global and Local Complement of PFSAWHSS-Matrix*

The Global complement PFASWHSS-Matrix is constructed by applying Complement-operator ($t = 4$, described in Equation 4.7) on each specific layer of PFHS-Matrix $F_{\mathbf{A}_t}$ (Equation 5.12).

$$\mathbf{F}_{\mathbf{A}_{4_t}} = \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} 0.7\ 0.4\ 0.6\ 0.5 \\ 0.6\ 0.5\ 0.7\ 0.9 \\ 0.4\ 0.7\ 0.7\ 0.3 \end{bmatrix} \\ \begin{bmatrix} \mathbf{0.7\ 0.7\ 0.7\ 0.9} \end{bmatrix} \end{bmatrix} \\ \begin{bmatrix} \begin{bmatrix} 0.5\ 0.3\ 0.5\ 0.2 \\ 0.3\ 0.5\ 0.5\ 0.2 \\ 0.4\ 0.5\ 0.4\ 0.3 \end{bmatrix} \\ \begin{bmatrix} \mathbf{0.5\ 0.5\ 0.5\ 0.3} \end{bmatrix} \end{bmatrix} \end{bmatrix} \\ \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} 0.7\ 0.4\ 0.6\ 0.5 \\ 0.6\ 0.5\ 0.7\ 0.9 \\ 0.4\ 0.7\ 0.7\ 0.3 \end{bmatrix} \\ \begin{bmatrix} \mathbf{0.4\ 0.4\ 0.6\ 0.3} \end{bmatrix} \end{bmatrix} \\ \begin{bmatrix} \begin{bmatrix} 0.5\ 0.3\ 0.5\ 0.2 \\ 0.3\ 0.5\ 0.5\ 0.2 \\ 0.4\ 0.5\ 0.4\ 0.3 \end{bmatrix} \\ \begin{bmatrix} \mathbf{0.3\ 0.3\ 0.4\ 0.2} \end{bmatrix} \end{bmatrix} \end{bmatrix} \\ \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} 0.7\ 0.4\ 0.6\ 0.5 \\ 0.6\ 0.5\ 0.7\ 0.9 \\ 0.4\ 0.7\ 0.7\ 0.3 \end{bmatrix} \\ \begin{bmatrix} \mathbf{0.4\ 0.4\ 0.6\ 0.3} \end{bmatrix} \end{bmatrix} \\ \begin{bmatrix} \begin{bmatrix} 0.5\ 0.3\ 0.5\ 0.2 \\ 0.3\ 0.5\ 0.5\ 0.2 \\ 0.4\ 0.5\ 0.4\ 0.3 \end{bmatrix} \\ \begin{bmatrix} \mathbf{0.3\ 0.3\ 0.4\ 0.2} \end{bmatrix} \end{bmatrix} \end{bmatrix} \end{bmatrix} \tag{5.14}$$

The Local Attributive Subjectively Whole Complement Matrices are achieved by applying three specific aggregation operators, $t = 1$ (Max-operator) $t = 2$ (Min-operator), and $t = 3$ (*averaging*-operator).

$$F_{A_{4_1}} = \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} 0.7\ 0.4\ 0.6\ 0.5 \\ 0.6\ 0.5\ 0.7\ 0.9 \\ 0.4\ 0.7\ 0.7\ 0.3 \end{bmatrix} \\ \begin{bmatrix} \mathbf{0.7\ 0.7\ 0.7\ 0.9} \end{bmatrix} \end{bmatrix} \\ \begin{bmatrix} \begin{bmatrix} 0.5\ 0.3\ 0.5\ 0.2 \\ 0.3\ 0.5\ 0.5\ 0.2 \\ 0.4\ 0.5\ 0.4\ 0.3 \end{bmatrix} \\ \begin{bmatrix} \mathbf{0.5\ 0.5\ 0.5\ 0.3} \end{bmatrix} \end{bmatrix} \end{bmatrix} \tag{5.14a}$$

$$F_{A4_2} = \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} 0.7 & 0.4 & 0.6 & 0.5 \\ 0.6 & 0.5 & 0.7 & 0.9 \\ 0.4 & 0.7 & 0.7 & 0.3 \end{bmatrix} \\ \begin{bmatrix} \mathbf{0.4} & \mathbf{0.4} & \mathbf{0.6} & \mathbf{0.3} \end{bmatrix} \end{bmatrix} \\ \begin{bmatrix} \begin{bmatrix} 0.5 & 0.3 & 0.5 & 0.2 \\ 0.3 & 0.5 & 0.5 & 0.2 \\ 0.4 & 0.5 & 0.4 & 0.3 \end{bmatrix} \\ \begin{bmatrix} \mathbf{0.3} & \mathbf{0.3} & \mathbf{0.4} & \mathbf{0.2} \end{bmatrix} \end{bmatrix} \end{bmatrix} \tag{5.14b}$$

$$F_{A4_3} = \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} 0.7 & 0.4 & 0.6 & 0.5 \\ 0.6 & 0.5 & 0.7 & 0.9 \\ 0.4 & 0.7 & 0.7 & 0.3 \end{bmatrix} \\ \begin{bmatrix} \mathbf{0.56} & \mathbf{0.53} & \mathbf{0.67} & \mathbf{0.56} \end{bmatrix} \end{bmatrix} \\ \begin{bmatrix} \begin{bmatrix} 0.5 & 0.3 & 0.5 & 0.2 \\ 0.3 & 0.5 & 0.5 & 0.2 \\ 0.4 & 0.5 & 0.4 & 0.3 \end{bmatrix} \\ \begin{bmatrix} \mathbf{0.4} & \mathbf{0.43} & \mathbf{0.46} & \mathbf{0.23} \end{bmatrix} \end{bmatrix} \end{bmatrix} \tag{5.14c}$$

in F_{A4_1} index $t = 4$ represents the complement operator of F, and its subscript 1,2,3 represents the subjective accumulation of complements of fuzzy memberships through the following given three operators. Here the Attributive Complement Hyper-Super-Soft Matrix F_{A4_t} represented by equation 5.14 is a cluster of HSS-matrices F_{A4_1}, F_{A4_2}, F_{A4_3} represented by Equation 5.14a, Equation 5.14b, Equation 5.14c respectively. these are the three attributive subjectively whole complement matrix-layers of this cluster. These matrices are constructed using the complement operator described in Equation 4.7. The last row of each matrix layer represents the whole complement fuzzy memberships.

5.3.10. The global and local complement of PFSAWHSS-Matrix

The Global Complement PFSAWHSS-Matrix is constructed by applying Complement-operator as described in Equation. 4.8 ($t = 4$) on each specific layer of PFHS-Matrix $F_{\mathbf{S}_t}$ (Equation 5.13) and represented as under,

$$F_{S4_t} = \begin{bmatrix} \begin{bmatrix} \begin{bmatrix} 0.7 & 0.4 & 0.6 & 0.5 \\ 0.6 & 0.5 & 0.7 & 0.9 \\ 0.4 & 0.7 & 0.7 & 0.3 \end{bmatrix} & \begin{bmatrix} 0.7 \\ 0.9 \\ 0.7 \end{bmatrix} \\ \begin{bmatrix} 0.5 & 0.3 & 0.5 & 0.2 \\ 0.3 & 0.5 & 0.5 & 0.2 \\ 0.4 & 0.5 & 0.4 & 0.3 \end{bmatrix} & \begin{bmatrix} 0.5 \\ 0.5 \\ 0.5 \end{bmatrix} \end{bmatrix} \\ \begin{bmatrix} \begin{bmatrix} 0.7 & 0.4 & 0.6 & 0.5 \\ 0.6 & 0.5 & 0.7 & 0.9 \\ 0.4 & 0.7 & 0.7 & 0.3 \end{bmatrix} & \begin{bmatrix} 0.4 \\ 0.5 \\ 0.3 \end{bmatrix} \\ \begin{bmatrix} 0.5 & 0.3 & 0.5 & 0.2 \\ 0.3 & 0.5 & 0.5 & 0.2 \\ 0.4 & 0.5 & 0.4 & 0.3 \end{bmatrix} & \begin{bmatrix} 0.2 \\ 0.2 \\ 0.3 \end{bmatrix} \end{bmatrix} \\ \begin{bmatrix} \begin{bmatrix} 0.7 & 0.4 & 0.6 & 0.5 \\ 0.6 & 0.5 & 0.7 & 0.9 \\ 0.4 & 0.7 & 0.7 & 0.3 \end{bmatrix} & \begin{bmatrix} 0.55 \\ 0.67 \\ 0.52 \end{bmatrix} \\ \begin{bmatrix} 0.5 & 0.3 & 0.5 & 0.2 \\ 0.3 & 0.5 & 0.5 & 0.2 \\ 0.4 & 0.5 & 0.4 & 0.3 \end{bmatrix} & \begin{bmatrix} 0.62 \\ 0.37 \\ 0.4 \end{bmatrix} \end{bmatrix} \end{bmatrix} \tag{5.15}$$

The Local Subjective Attributively-Whole Complement Matrices are achieved by applying three specific aggregation operators, $t = 1$ (Max-operator) $t = 2$ (Min-operator), and $t = 3$ (*averaging*-operator). Are described as under

$$F_{S4_1} = \begin{bmatrix} \begin{bmatrix} 0.7 & 0.4 & 0.6 & 0.5 \\ 0.6 & 0.5 & 0.7 & 0.9 \\ 0.4 & 0.7 & 0.7 & 0.3 \end{bmatrix} & \begin{bmatrix} 0.7 \\ 0.9 \\ 0.7 \end{bmatrix} \\ \begin{bmatrix} 0.5 & 0.3 & 0.5 & 0.2 \\ 0.3 & 0.5 & 0.5 & 0.2 \\ 0.4 & 0.5 & 0.4 & 0.3 \end{bmatrix} & \begin{bmatrix} 0.5 \\ 0.5 \\ 0.5 \end{bmatrix} \end{bmatrix} \tag{5.15a}$$

$$F_{S4_2} = \begin{bmatrix} \begin{bmatrix} 0.7 & 0.4 & 0.6 & 0.5 \\ 0.6 & 0.5 & 0.7 & 0.9 \\ 0.4 & 0.7 & 0.7 & 0.3 \end{bmatrix} & \begin{bmatrix} 0.4 \\ 0.5 \\ 0.3 \end{bmatrix} \\ \begin{bmatrix} 0.5 & 0.3 & 0.5 & 0.2 \\ 0.3 & 0.5 & 0.5 & 0.2 \\ 0.4 & 0.5 & 0.4 & 0.3 \end{bmatrix} & \begin{bmatrix} 0.2 \\ 0.2 \\ 0.3 \end{bmatrix} \end{bmatrix} \tag{5.15b}$$

$$F_{S_{4_3}} = \left[\begin{array}{cc} \begin{bmatrix} 0.7 & 0.4 & 0.6 & 0.5 \\ 0.6 & 0.5 & 0.7 & 0.9 \\ 0.4 & 0.7 & 0.7 & 0.3 \end{bmatrix} & \begin{bmatrix} 0.55 \\ 0.67 \\ 0.52 \end{bmatrix} \\ \begin{bmatrix} 0.5 & 0.3 & 0.5 & 0.2 \\ 0.3 & 0.5 & 0.5 & 0.2 \\ 0.4 & 0.5 & 0.4 & 0.3 \end{bmatrix} & \begin{bmatrix} 0.62 \\ 0.37 \\ 0.4 \end{bmatrix} \end{array}\right] \quad (5.15c)$$

6. Conclusions and discussions

1. The classical matrices would connect several equations and variables by rows and columns which is a limited approach to organize the higher-dimensional data consists of scattered information in numerous forms and vagueness levels therefore to broaden the approach of organizing the higher-dimensional data this unique Model of hyper-super-soft matrix is constructed. The set operations can be performed at each position (i, j) with a localized computation that is applied within the Matrix layer. This is a huge constructed matrix that first connects rows and columns like the classical matrix and then connects the sets of rows and columns as matrix layers. At this stage, it is called the hyper matrix. In each hyper-matrix layer, more matrices would be connected known as super-matrix and these super-matrices would be further connected as hyper-super-matrix, as described in the presented model. Equations 5.12, 5.13, 5.14, and 5.15 present examples of this Connected Hyper-Super-Matrix.
2. In this article, we developed the Max, Min, Averaging, and Complement operators for the PFASWHSS-Matrix and PFSAWHSS-Matrix. Applying the aggregation operator to a particular layer of the PFHS-Matrix would compress the entire layer of rows and columns into a single column (case of constructing PFSAWHSS matrix) or a single row (in constructing PFASWHSS matrix)
3. The final PFASWHSS-Matrix and the PFSAWHSS-Matrix are designed to organize the COVID-19 data structure and are presented in Equation 5.12. or 5.13. The matrix is made up of three clusters associated with three aggregation operators, and each cluster has two layers, with each layer representing the information from each visit.
4. The final PFSAWHSS-Matrix created for the COVID-19 data structure is shown in Equation 5.12 and would be utilized to classify the non-physical attributes (symptoms of COVID-19). It is observed from subjectively whole fuzzy memberships, which are shown in PFASWHSS-Matrix $F_{\mathbf{A}_1}$, the sub-attributes A_4^1, A_4^2 which represent minor nausea and acute nausea, respectively, being the dominant attributes on the first and second visit levels
5. The final PFSAWHSS-Matrix that is constructed for the COVID-19 data Structure is shown in Equation 5.13 and would be utilized to classify the physical subjects (patients of COVID-19)
The patient whose membership of the cumulative attributes as fuzzy membership is 80% or more than 80% should be hospitalized for intensive care. Therefore, subjects x_1, x_2 would be admitted on their second visit as their attributively whole Fuzzy memberships are 80%.
6. It is observed that the Global Complement PFASWHSS-Matrix $\mathbf{F}_{A_{4_t}}$ described by Equation 5.14 is a higher-dimensional matrix, consisting of three clusters of matrices each cluster being described in Equation 5.14a, Equation 5.14b. equation 5.14c i.e. represents a local complement. While each cluster described has two matrix layers and each layer has two sub-matrices, the first sub-matrix of the layer represents a detailed attributive view of the local complement. And the last row-matrix of the matrix-layer reflects the compact view of the attributive complement. The Global Complement of PFSAWHSS-Matrix $\mathbf{F}_{S_{4_t}}$ is described by Equation 5.14 and its Clusters are given in (Equation 5.15a, Equation 5.15b. Equation 5.15c)
7. This modern hyper-super-matrix has shown the individual and cumulative effect of universe/event/reality/information through several angles of visions in a fuzzy environment. These angles of vision are described through the use of Max, Min, Averaging, and complement operators. These operators represent optimistic, pessimistic, neutral, and antithetical behavior of the universe in the fuzzy environment.

6.1. Future open problems

1. In this article, we have developed the Max, Min, Averaging, and Complement operators

for PFASW HSS-Matrix and PFSAWHSS-Matrix which represent a Hyper-Neutrosophic approach of considering Optimist, Pessimist, Neutral and Antithetical behavior of the Universe/Event/Reality/Information. In addition, some other local operators can be developed to express a broader approach.

2. The choice of a suitable environment like Crisp, Fuzzy, Intuitionistic, or Neutrosophic would reflect the state of mind of the observer. Moreover, this Model of expression (Hyper-Super-Matrix) has the capability of the adoption of any suitable environment to reflect the worth style nature with its physical and non-physical aspects.
3. The aggregation operators are applied row-wise to compact a certain Matrix-layer into a single column and applied column-wise to compact a certain layer of the Matrix into a single row. This procedure of development and application of aggregation operators was carried out by considering only the front view of the dilated Matrix i.e. Type-1 level-layers of PFHS-Matrix described In equation 3.14. This development and Application would be extended to *Type*-2 *level layers of PFHS-Matrix* described in equation 3.17. This unique construction model is presented, as an example in the Fuzzy environments by considering the front view of the dilated Matrix i.e *Type*-1 *level-layers of PFHS-Matrix* described In Equation 3.14 to preserve the duration of the article within the required limits of this general. later, the proposed unique built examples would be constructed in various suitable environments taking into account other described views of the matrix.

By considering *Type-3 level layers of PFHS-Matrix* described in Equation 3.18. and using the proposed extensions Extended Attributive and Subjective Models would be constructed.

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