EFQ and the Unexpected Examination Paradox

GABRIELE RASTELLO
to Kiririn-shi
June 4, 2017

Abstract

Analysis of the surprise examination paradox supporting the idea that the students' conclusion is not wrong but simply contradictory.

Keywords: Ex Falso Quodlibet, Principle of explosion, Scoto's law, Surprise Examination Paradox

Introduction

Let's start by introducing the paradox; there are many variants of this paradox and of course the core of the paradox is the same in all of them. I personally love the Surprise Examination version so we're gonna work on that. Let's first introducing the paradox. Here's the paradox exactly as it is presented in [1]:

> A teacher announces in class that an examination will be held on some day during the following week, and more over that the examination will be a surprise. The students argue that a surprise exam cannot occur. For suppose the exam were on the last day of the week. Then on the previous night, the students would be able to predict that the exam would occur on the following day, and the exam would not be a surprise. So it is impossible for a surprise exam to occur on the last day. But then a surprise exam cannot occur on the penultimate day, either, for in that case the students, knowing that the last day is an impossible day for a surprise exam, would be able to predict on the night before the exam that the exam would occur on the following

day. Similarly, the students argue that a surprise exam cannot occur on any other day of the week either. Confident in this conclusion, they are of course totally surprised when the exam occurs (on Wednesday, say). The announcement is vindicated after all. Where did the students' reasoning go wrong?

The tool I'm gonna use to explain why the students' reasoning is wrong it's a law of classical logic called *Ex Falso Quodlibet*[1] according to witch we can prove *any* statement from a single contradiction. If ϕ and ψ are statements we can write *EFQ* formally as: $(\phi)(\psi) : \phi . \sim \phi \supset \psi$. The reader can easily convince itself that *EFQ* hold for every possible statement ϕ and ψ by reading the proof you can find at [2].

Paradox breakdown

Let's then start by considering a very simple case: let's suppose that instead of "an examination will be held on some day during the following week" the teacher had said "an examination will be held on some day during the following *two* days". Let's also suppose, as many others have done, that *surprise* mean "not predictable". Let's try to apply the students' reasoning to this simple case.

First let's suppose the exam were on the *second* day, then we are able to *predict* it the previous night and it cannot be a surprise. So we can infer that no examination can be hold on the second day. But now, since there are just two possible days for the surprise examination to occur and we already proved that it can't be on the second day the examination *must* be held on the first one and again it cannot be a surprise. We've just proved that:

α: No surprise examination can occur on the second day.

β: No surprise examination can occur on the first day.

Since we're considering just two days we can now conjugate these two proposition to obtain the same result the students obtained:

ϕ: $\alpha . \beta$: no surprise examination can occur[2].

We're now in the same situation the students in the paradox are: we are sure that no surprise examination will occur but let's say that the second day the teacher decides to hold an examination: we would be really surprised.

[1] Sometimes called Principle of explosion or Principle of Pseudo-Scotus. From now on *EFQ*.

[2] In the following two days.

This seems to be against our conclusion ϕ that no surprise examination will occur. Where did our reasoning go wrong?

Many thinkers focus on the process that bring us to ϕ, arguing that there's something fallacious there, but I think that the real problem is more ϕ rather than how we obtain it. We have to recognize that we're assuming the teacher statement "a surprise examination will be held on some day during the following *two* days" as an axiom, let' call it σ. Without assuming σ as an axiom we can't obtain ϕ. Let's see why.

σ is basically the conjunction of two statements: (σ_1) that state "an examination will occur" and (σ_2) that state "the examination will be a surprise". When we supposed the exam will be held on the *second* day we argued that it would be impossible because the examination must be a surprise; in other words the exam can't occur on the second day because this contradict σ (particularly σ_2). Then we said that since an examination *must* occur (this is σ_1) and it can't be hold on the second day (via α, that follows from σ_2) then the examination *must* occur on the first day; but this is against σ_2. Now it's not hard to see that ϕ is the *negation* of σ since one state "a surprise examination will occur" and the other "no surprise examination will occur". This mean that we can actually construct, from σ, a statement like $\sigma. \sim \sigma$ and now, via *EFQ* we can prove *everything*.

Of course every line of this section still apply if we pick 3, 4, 5, n days instead of 2 in account.

Conclusions

Since we can prove almost *everything* from the teacher's statement we now have to ask ourselfs: is σ (the teacher's statement) a valid statement? The answer can be yes or not and I kinda support both.

If we say σ is worthless then we're basically saying there is some problem with it. The problem can be hard to find but since every word in σ seems pretty clear except *surprise* I think the problem is there. We had assumed *surprise* to mean "not predictable" but since this lead us to contradictions maybe we should have gone with a different meaning; like "at random" or "the day I like but I'm not going to tell you". Statements like "an examination will occur during the following week at a day chosen at random" or "an examination will occur during the following week at a day I like and I'm not going to tell you" does not permit to infer statement like α and β and are safe from contradictions.

But if we chose to maintain *surprise* to mean "not predictable" and accept σ as invalid statement we still don't have very much problems. We know that from σ we can infer anything thanks to *EFQ* so we know that no examination will occur, that an examination will occur on the first day, that an examination will not occur on the first day, that an examination will occur on the second day and that it will not occur on the second day and so on. We know *everything but nothing at all* so on any day[3] an examination would be a surprise.

To conclude: there's nothing wrong in the students' reasoning; they just did not confronted they're conclusion with the statement they derived it from.

References

1. Timothy Y. Chow. The surprise examination or unexpected hanging paradox. *The American Mathematical Monthly*, 105:41–51, 1998.

2. Wikipedia. Principle of explosion — wikipedia, the free encyclopedia, 2017. [Online; accessed 3-June-2017].

[3]except maybe the very last.