# The Ptolemy-Copernicus transition: Intertheoretic Context

Almagest

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**Abstract**

The Ptolemy-Copernicus transition is analyzed in the intertheoretic context provided by an analogy between the Copernican revolution and the Einsteinian one. In the case of Einstein's revolution it was argued that its cause consisted in the clash between the main research programmes of classical physics: Newtonian mechanics, Maxwellian electrodynamics, thermodynamics and statistical mechanics. In the present paper it is demonstrated that the Ptolemy-Copernicus transition is similar to the Lorentz-Einstein one with Lorentz playing the Ptolemy role and Einstein – the role of Copernicus. Just as Lorentz's "theory of electrons" of the second half of the 19th century can be evaluated as a dual theoretical scheme mixing Newtonian mechanics with maxwellian electrodynamics, one can evaluate Ptolemaic cosmology as a dual theoretical scheme mixing the principles of "Platonic mathematics" with those of "Aristotelian physics". Analogously to the Einsteinian revolution, it is argued that the Copernican one can be considered as a realization of the dualism between mathematical astronomy and Aristotelian qualitative physics and the corresponding gradual efforts to eliminate it. Hence the works of Copernicus, Galileo, Kepler and Newton can be comprehended as the stages of the mathematics descendance from skies to earth and reciprocal extrapolation of earth physics on divine phenomena.

# Introduction

Why did Copernicus' research programme supersede Ptolemy's? – The main epistemological theories answering the question belong to the following groups: (I) inductivist account; (II) falsificationist account; (III) conventionalist account; (IV) account of T.S. Kuhn; (V) account of I. Lakatos and E. Zahar. However, all the proposed explanations of the reasons of Copernicus's victory over Ptolemy seem to be insufficient due to the following arguments.
(I)*Inductivist* explanation seems to be especially vulnerable since theories from both rival programmes –that of Copernicus and Ptolemy– were incompatible with available observation data.

> "Ptolemy's theory was not very accurate. The positions for Mars, for example, were sometimes wrong by nearly 5 degrees. But […] the planetary positions predicted by Copernicus […] were nearly as bad". (Gingerich 2004)

(II) *Falsificationist* account of Ptolemy-Copernicus transition can be reduced to the following two options that seem to be unsatisfactory also.
(a) According to the first of them, Ptolemy's theory was irrefutable and hence unscientific while Copernicus's theory was just the opposite. The Ptolemy programme heuristic was *ad hoc*; any fact could be explained in retrospect by multiplying the number of epicycles, equants and deferents (Duhem [1906] 1991).

However, proliferation of epicycles in Ptolemaic astronomy is a "historical myth" (see, for instance, Gingerich 1997, chapters 11-13). In reality Copernicus's programme could assimilate no less epicycles. To compensate the equant elimination, Copernicus had to introduce a new series of epicycles. As a result, Ptolemy's model contained fewer epicycles than that proposed by the Copernican one, and was more accurate.

(b) According to the second option (Popper [1935] 2002), both rival theories were equally refutable for a long time: however in the long run the crucial experiment refuted Ptolemy to support Copernicus. Yet when does it happen? - In 1616, when Galileo had discovered the cycles of Venus?

But fundamental historical error is the claim that Galileo anticipated the phases of Venus (Ariew 1987). One can understand Galileo's critics among the Aristotelians refusing to believe the observational data obtained with a help of recently invented telescope with dubious principles of action (Kuhn 1957, Feyerabend [1975] 2010, Chaunu 1984).
(III) According to the *conventionalist* account, one cannot make the choice between the rival theories remaining on purely empirical grounds. One theory is better than another because it is more "simple", "coherent", "economic", etc. However, it is well known that

> "the myth of superior simplicity was dispelled by the careful and professional work

> of modern historians [...] The Copernican system is certainly simpler since it dispenses with equants and with some eccentrics; but each equant and eccentric removed has to be replaced by new epicycles and epicyclets. The system is simpler in so far as it leaves the eighth sphere of fixed stars immobile and removes its two Ptolemaic motions; but Copernicus has to pay for the immobile eighth sphere by transferring its irregular Ptolemaic movements to the already corrupt earth which Copernicus sets spinning with a rather complicated wobble; he also has to put the center of the universe, not at the Sun, as he originally intended, but at an empty point fairly neat to it". (Lakatos & Zahar 1974, 362)

(IV) According to Thomas Kuhn, Ptolemaic astronomy was by 1543 in a state of "paradigm-crisis" which is the inevitable prelude to any scientific revolution:

> "The state of Ptolemaic astronomy was a recognized scandal before Copernicus proposed a basic change in astronomical theory, and the preface in which Copernicus described his reasons for innovation provides a classic description of this crisis state". (Kuhn 1963, 367; see also Kuhn 1957, 177)

However, as Imre Lakatos and Elie Zahar put it, the "crisis" concept is too vague to operate with. For how many people (except Copernicus) *really* had felt this "community crisis"? In Kuhn's paradigm-shift model a "*crisis*" must always precede a "*revolution*". But even in the transition from classical radiation theory to quantum one ($19^{th}$-$20^{th}$ centuries) "paradigm-crisis" notion appears to be rather dubious (Nugayev 2000a). Who had fixed it? Poincare? Lorentz? Michelson & Morley? And where? However, a respectable patriarch Lord Kelvin (William Thomson) in his prophetic speech, for instance, wrote only about the "two clouds" on serene sky of theoretical physics but not about "two tornado" or "two hurricanes" (Kelvin 1901). At the same time Albert Michelson was awarded in 1907 the Copley Medal not for 1881/1887 refutation of the ether scientific programme but for "experimental investigations in optics".

Hence it is not surprising that in the Copernicus case Gingerich (2004) insists that "Kuhn conjures up a scandal where there was none". At that time scientific community was rather small. And if Kuhn insisted that his 'structure of scientific revolutions' analysis is completely applicable to the Copernican one why so few researchers had joined Copernicus before Kepler and Galileo ?

(V) According to Imre Lakatos' methodology of scientific research programmes (SRP methodology), the basic problem of philosophy of science is to provide *normative* appraisal of scientific theories. *Appraisal* of the change is a normative problem and belongs to philosophy. But *explanation* of the change –of real reasons of theories' acceptance and rejection– is a "psychological problem". Scientific Research Programme is a unit of appraisal of the growth of knowledge. It consists of a developing series of theories that has a subtle structure: a tenacious hard core + heuristic (a set of problem-solving techniques)

+ protecting belt (that is constantly modified to protect the hard core from experimental refutations).

Copernican programme was a "theoretically progressive" one. It did anticipate some new facts not observed before. But *actually* Copernican programme empirically progressed only beginning from Newton (Lakatos & Zahar 1974, 374).

As a result, Lakatos-Zahar account

> "explains Copernicus' achievement as constituting genuine progress compared with Ptolemy. The Copernican Revolution became a great scientific revolution not because it changed the European Weltanschauung, not –as Paul Feyerabend would have it– because it became also a revolutionary change in man's vision of his place in the Universe, but simply because it was scientifically superior. It also shows that there were good objective reasons for Kepler and Galileo to adopt the heliostatic assumption, for already Copernicus' (and indeed Aristarchus') rough model had excess predictive power over its Ptolemaic rival". (Lakatos & Zahar 1974, 380)

This is the "Ptolemy-Copernicus" transition given by the methodology of SRP. I think that one should not be bothered with small deficiencies of the account considered that can always can be discovered in any treatise (see Thomason 1992, for example). The question is a *matter of principle*. The aim of Lakatos's methodology is to give an objective appraisal of scientific transition but not the explanation of its actual reasons.

> "The whole development is narrowly internal; its progressive part could have taken place at any time, given a Copernicus genius, between Aristotle and Ptolemy or in any year, say, after the 1175 translation of the *Almagest* into Latin". (Lakatos & Zahar 1974, 381)

Within the frame of Lakatos-Zahar account it is natural to suppose that all the content of the Copernican programme could be produced even by Aristarchus. But why it did not take place? And why these considerations did not have any influence on Ptolemy and his pupils?

The aim of the present paper is to supplement the Lakatos-Zahar account and to make further step on a path of theoretical explanation for the reasons of the genesis and victory of Copernican programme. In what follows I'll try to demonstrate that the abovementioned accounts (I)-(V) miss an important point connected with the intertheoretic contextignorance: *Copernican and Ptolemaic programmes were realizing the opposite ways of astronomy and physics unification.*

From the point of view of the general model considering the growth of knowledge as interaction, interpenetration and unification of the research programmes, springing out of different cultural traditions (Nugayev 1998; 1999), the Ptolemy-Copernicus transition is similar to the Lorentz-Einstein one with Lorentz playing the Ptolemy role and Einstein – the role of Copernicus (Nugayev 2000).

Therefore to give a more accurate theoretical explanation of the "Ptolemy-Copernicus transition" one has to take into account the *intertheoretic facet* of the Copernican revolution. The present paper tries to demonstrate that the Copernican Revolution can be comprehended as a result of revelation and (partial) resolution of the dualism, of the gap between Ptolemy's mathematical astronomy and Aristotelian qualitative physics. From this point of view the works of Copernicus, Galileo, Tycho, Kepler, Descartes and Newton can be described as the stages of mathematics descendance from skies to earth and reciprocal extrapolation of earth physics on skies.

Hence the second part of the present paper is devoted to the Ptolemaic astronomy, while the third one deals with the origins of the Copernican Revolution seen in the intertheoretic context and with some of the reasons of Copernicus' victory over Ptolemy.

# Rise and Fall of Ptolemaic Research Programme

As Imre Lakatos and Elie Zahar put it, both Ptolemy and Copernicus worked on *research programmes*. They did not simply test conjectures or try to put in order a vast conjunction of observational results, nor did they entirely devote themselves to community based "paradigms". Both programmes branched off from the Pythagorean-Platonic programme. Its basic principle was that since heavenly bodies are perfect, all astronomical appearances should be "saved" by a combination of as few uniform circular motions as possible. This principle remained the cornerstone of the *heuristic* of both programmes. Pythagorean-Platonic proto-programme contained no directives as to where the center of the universe lies.

In this case the "*heuristic*" was primary, while the "*hard core*" only secondary. The geocentric hypothesis "hardened" into a hard core assumption due to the development of Aristotelian physics, with its natural and violent motions and its strict separation of the terrestrial (sublunary) and celestial (eternal) worlds (Aristotle 2007a).

According to our common experience, clearly fixed by the notions of Aristotelian metaphysics, common reality in which we live and act (the "*lebenswelt*" of Edmund Husserl) is not a mathematical one nor can it be "mathematized". Nature is free of circles, straight lines and well swept streets.

Hence the Greek thought could hardly imagine an opportunity that exactness can successfully exist in sublunary world and that "the matter of our sublunary world can represent mathematical entities" (Koyre 1957). Aristotle himself claimed that "mathematical exactness should be demanded only for the objects lacking matter. Thus this way is not appropriate for natural science since nature in all the cases is connected with matter" (Aristotle 2007b).

However the situation at the divine world is quite the opposite. The heavens are made up of an entirely different immutable substance, the ether (fifth element). Heavenly bodies are part of spherical shells of ether. The shells fit tightly around each other in the following order: the shell of the Moon, Mercury, Venus, Sun, Mars, Jupiter, Saturn, fixed stars. Each shell has its specific rotation, that accounts for the motion of the heavenly body contained in it. Outside the sphere of the fixed stars, there exists the *prime mover*; it imports motion from the outside inward. The natural motions of heavenly bodies and their shells are perfectly circular and neither speeding nor slowing down.

The perfect and absolutely ordered motions of the stars take place in full harmony with strict and constant geometrical laws. Hence, due to Aristotle, *mathematical astronomy is possible and mathematical physics is not.* It is not surprising that the Greek astronomers not only effectively applied mathematics but with amazing patience and exactness observed the sky with a help of measuring devices. But they did not mathematize terrestrial motions and had not applied the devices on the Earth surface.

The ancient science summit was reached by Claudius Ptolemy (87-150) – Hellenistic astronomer, astrologer, mathematician, geographer and poet. Ptolemy's opus magnus –the *Almagest*– dominated European thought for more than 14 centuries (the book's name is derived from an Arabic "al-majasti" – the greatest; it' s real title is *Syntaxis Mathematica*). Ptolemy was a resident of Alexandria, the capital of Hellenized Egypt. Hellenistic civilization represented a fusion of the Ancient Greek culture with that of Egypt and Babylon and a departure from earlier Greek attitudes towards "barbarian" cultures. The extent to which genuinely hybrid Greco-Asian cultures emerged is contentious, but it is not open to doubt that the three cultures meeting captured the domain of mathematical astronomy (Neugebauer 1975).

Unlike the Babylonians and Egyptians, who observed the heavens to keep track of the seasons, the Greeks approached astronomy from a purely theoretical viewpoint: they wanted to know the basic nature and makeup of the Universe. The Babylonian and Egyptian astronomy was contrasted with the Greek: the former were merely arithmetical and geometrical, but incomplete because lacking *φυσιολογία. As a genuine Hellenist, Ptolemy tried to balance himself between the three cultures.*

On the one hand, he was skeptical to Aristarchus' heliocentric hypothesis due to rational considerations connected with Aristotelian physics principles. Ptolemy admitted that it may perhaps be simpler, from a strictly celestial viewpoint, to have the Earth spinning daily on its axis rather than the entire heavens rotating about the Earth. But this, he said, fails to take into account the terrestrial physics. For if the Earth moved,

> "animals and other weights would be left hanging in the air, and the Earth would very quickly fall out of the heavens. Merely to conceive such things makes them appear ridiculous". (quoted from Kline 1986)

On the other hand, since Aristotle was the only philosopher to whom Ptolemy referred explicitly, the author of the *Almagest* is often accused of slavish adherence to the tenets of Aristotelian philosophy. However, this is an oversimplification both in physical and epistemological aspects. First of all, the Ptolemaic model of the solar system did deviate from the orthodox Aristotelianism in a number of important respects.
(i) Aristotle argued that heavenly bodies should move in *single* uniform circles. Yet, in the Ptolemaic system, the motion of the planets is a *combination* of two circular motions. In addition, at least one of them is non-uniform.
(ii) Aristotle also argued –again from purely philosophical grounds– that the Earth is located at the *exact* center of the Universe. But in the Ptolemaic system the Earth is slightly *displaced* from the center of the Universe. In Ptolemy's cosmology there is no unique center of the Universe, since the orbit of the Sun and the planetary deferents all have slightly different geometric centers, none of which coincide with the Earth. And in the *Almagest* Ptolemy had frequently pointed out that the non-orthodox (from the Aristotelian point of view) aspects of his models all were directly dictated by observations.

Modern astronomy states, due to Kepler, that planetary orbits are actually ellipses. They possess 2 main properties : (1) they are *eccentric*, i.e. the Sun is displaced from the geometric center of the orbit; (2) they are *elliptical*, i.e. the orbit is "elongated" along a certain axis. In astronomy Keplerian orbits are described by a quantity e, called *the eccentricity*. It measures their deviations from circularity. It can be demonstrated that the eccentricity of a Keplerian orbit scales as e, while the corresponding degree of elongation scales as $e^2$. Since the orbits of the solar system planets possess relatively small value of e ($e < 0{,}21$), it follows that, *to a very good approximation*, these orbits can be treated as eccentric circles. One can neglect the ellipticities of planetary orbits compared to their eccentricities. This is what is done in the *Almagest*. "It follows that Ptolemy's assumption that heavenly bodies move in circles is actually one of the main strengths of his model" (Fitzpatrick 2010).

Ptolemy's second outstanding achievement was the famous "*equant*". The latter is a point near the center of the planet's orbit which, if you were to stand there and watch, the center of the planet's epicycle would always appear to move at the same speed. "In fact, *this discovery is one of Ptolemy's claims to fame*" (Fitzpatrick 2010, 6).

According to Imre Lakatos and Elie Zahar (1974, 371), after Eudoxus model (the system of rotating spheres) was abandoned, every move in the geostatic programme ran counter to the Platonic heuristic. The eccentric displaced the earth from the center of the circle; the Apollonian and Hipparchan epicycles meant that the real path of the planets about the earth was not circular; and finally the Ptolemaic equants entailed that even the motion of the epicycle's empty center was not simultaneously uniform and circular. The equant introduction was the heaviest blow upon the Platonic heuristic: it was equal to its full elimination. Thus, in the Ptolemaic programme mathematical exactness that demanded the introduction of noncircular orbits and the centers of rotation not coinciding with the earth center began to diverge more and more from the principles of Aristotelian physics well-grounded empirically. Hence in the long run one can evaluate *Ptolemaic cosmology as a dual theoretical scheme mixing the principles of "Platonic mathematics" with those of "Aristotelian physics"* (just as Lorentz's "theory of electrons" of the second half of the 19th century can be evaluated as a dual theoretical scheme mixing newtonian mechanics with maxwellian electrodynamics – see Nugayev 2000b).

(a) The region of their obvious cross-contradiction was the "planet theory" since it was namely the planet motions that needed for their description especially significant violations of Aristotelian physics principles. For instance, Proclus interpreted Plato in the *Republic* to be saying that while the fixed stars are both regular and orderly, and sublunary things are both irregular and disorderly, the planets are intermediate between them (Lloyd 1999).
(b) The second cross-contradictions domain was the astrology espoused in Ptolemy's *Tetrabiblos* (a work written after the *Almagest* but probably before the *Planetary Hypotheses*). In this book Ptolemy considers the interaction of divine and mundane worlds. He turns from pure contemplation of the celestial realm of ether to an investigation of the action of the heavens on the sub-lunar world of the four elements.

> "A very few considerations would make it apparent to all that certain power emanating from the eternal ethereal substance is dispersed through and permeates the whole region about the earth, which throughout is subject to change, since, of the primary sublunar elements, fire and air are encompassed and changed by the motions in the ether, and in turn encompass and change all else, earth and water and the plants and animals therein". (Ptolemy, *Tetrabiblos*, 1.2)

There were, however, very important differences between the basic hypotheses of the *Almagest* and the *Tetrabiblos*. In both treatises, the ethereal matter is simply a given. But the property of uniform circular motion in the *Almagest* is justified on a priori grounds (circular rotation being the only kind of eternally unchanging motion that can be conceived), whereas the property of exerting a power of change on their elements is argued directly from empirical facts. More significantly, uniform circular motion is a mathematical behavior, which leads immediately to the modeling of the *Almagest*, while power to change the elements is by nature not mathematical. It operates with qualities such as "hot" and "cold", "wet" and "dry".

> "And this created a problem: how can cause-and-effect relations operating at the qualitative level, and largely within the 'irregular' sublunary part of the cosmos, be well described by the predictive mathematical models of astrology?" (Jones 2005, 35)

Yet Ptolemy evades the problem. Essentially he relies on the orderliness of the heavens to justify the mathematical structure of the predictive schemes of his astrology. However, he appeals to the disorderliness and complexity of the mundane environment to explain why astrological predictions, even made according to the most correct principles, are not certain to be borne out.

Hence with the translation of the *Tetrabiblos* into Latin in the 12th century Ptolemy's astrological scheme met severe criticisms that arise when the practice of predictions was seen to suggest "fatal necessity". The point was crucial to later integration into medieval Christian doctrine (Albertus Magnus and Thomas Aquinas), since the religious doctrine dictates that the individual sole must possess free will, in order to be responsible for its own choices and the consequences flowing from them. In particular, Gerard of Feltre's 13th century text *Summa on the Stars* describes the problem that astrological determinism created for the theological considerations:

> "If the stars make a man a murderer or a thief, then all the more it is the first cause, God, who does this, which it is shameful to suggest".

One can admire the core of Pico della Mirandola's (1463-1499) argument against astrology posed in Disputationes adversus astrologiam divinatricem:

> "At the same time as the astrologers say that every motion below depends on the motion of the heavens [motum omnem inferiorem a caeli motu dependere], they immediately contradict their teaching, since that commonplace among the philosophers follows from this, that the caelum is a universal cause of lower effects. Moreover, a universal cause does not distinguish effects, nor is why this comes-to-be or that sought from it, but from proximate causes, which are varied and different, to account for the difference and variety of the effects; and since something makes different things from these [proximate causes], a universal cause makes everything with all [the proximate causes]". (quoted from Rutkin 2010, 138)

From the modern point of view, Ptolemy's equant is Kepler ellipses precursor. But for such stern critics as the Islamic/Arabic astronomers and Copernicus the equant introduction was a typical "ad hoc hypothesis" that contradicted the spirit of Aristotle-Ptolemy programme – the principle of uniformity of motion in respect to the center of the Universe.

Ptolemy chose the rational way of constructing sophisticated ideal models to mimic the peculiarities of planet rotations. The "equant" introduction to geocentric programme positive heuristic was typical for all his subtle constructions. It seems to me that carefully combining epicycles and deferents, the author of the *Almagest*, in accord with Eastern instrumentalist tradition, was (partly) inspired by "economy of thought" principle and initially did not intend to bother himself with reflections on the essence of things.

Sometimes things went so that for the description of some planet motions Ptolemy had constructed several alternative empirically-equivalent theoretical schemes. Afterwards, as a rule, he chose the mathematically simplest one. The author of the *Almagest* was especially skeptical to philosophical discussions and claimed that in astronomy one should choose the simplest mathematical model among those that are at his disposal. For instance, in one of the well-known *Almagest* passages (III, ch.4) Ptolemy comments, in connection with his theory of the Sun, that the appearances may be saved on either an epicyclic or on eccentric hypothesis. What Ptolemy actually says, after noting that both models can be used to account for the appearances in relation to the Sun, is that the eccentric hypothesis is to be preferred because "*it is simpler and effected by one, not two movements*". The passage enabled Pierre Duhem to place Ptolemy among the ancient instrumentalists (Geminus, Ptolemy, Proclus, Simplicius; see Goldstein 1997 for details). Shortly, the method of astronomers was "instrumentalist", and the method of the physicists "realist".

But it seems an obvious over-simplification for me. In the *Almagest* one can find realist pieces as well, and in the *Planetary Hypotheses* Ptolemy maintains an obvious *naïve realist* position. For instance, even in XIII, ch.2 of the *Almagest*, when he asks the reader not to be dismayed by the complexity of the hypotheses that he has to use, his standpoint is not one of indifferences to the question of whether his devices represent the true system.

> "It is not fitting to compare human things with divine ones, nor to form beliefs concerning such great things from examples that are so unlike them. For what could be more unlike than those things that are eternal and unchanging and those that are never unchanging, or those that can be hindered by anything and those that cannot be hindered even by themselves?" (*Almagest*, XIII, ch. 2, quoted from Lloyd 1999, 216)

For Ptolemy the mathematical agreement of the "hypotheses" (the models) with the observational data was evidence for the *physical reality* of the hypotheses, even if they seemed unduly complicated.
According to the *Almagest* (XIII.2),

> "one should try, as far as possible, to fit the simpler hypotheses to the heavenly motions, but if this does not succeed, [one should apply such hypotheses] which do fit. For provided that each of the phenomena is duly saved by the hypotheses, why should anyone think it strange that such complications can characterize the motions of the heavens when their nature is such as to afford no hindrance, but of a kind to yield and give way to the natural motions of each part, even if [the motions] are opposed to one another". (quoted from Goldstein 1997, 8)

Thus, in the *Almagest* Ptolemy holds an *eclectic* epistemological position, combining instrumentalist and naïve realist standpoints in the spirit of Hellenistic culture. However, to grasp his dominating epistemological inclinations one has to listen to the other outstanding eclectic, Albert Einstein, who tried to combine Mach's positivism with Planck's and Boltzmann's realism. At the very beginning of the paper "On the Method of Theoretical Physics" he confessed:

> "If you want to find out anything from the theoretical physicists about the methods they use, I advice you to stick closely to one principle: don't listen to their words, fix your attention on their deeds". (Einstein 1935, 30)

So, if one looks at the *Almagest* from this standpoint, one finds out that the most important results were obtained there on the basis of the "principle of economy of thought"; *in his research practice Ptolemy acted as a committed instrumentalist.* Hence, one should put apart $Ptolemy_1$ – the author of *Almagest*, and $Ptolemy_2$ – the author of *Planetary Hypotheses*, *Tetrabiblos* and *Optics*. It is $Ptolemy_1$ who is usually associated with the slogan "saving the phenomena".

*This is not to ignore Ptolemy's realist standpoint,* since, after the *Almagest* he had to make the ends meet with the Zeitgeist, with the realist paradigm. To retain the achievements of the Aristotelian natural philosophy and to retain the achievements of his own theoretical astronomy, Ptolemy had to make the next step in elaborating the Aristotelian dualism of mundane and celestial phenomena in his *Planetary Hypotheses*. Since the *Almagest* confines itself to mathematical models, Ptolemy had to try to provide preliminary physical models for the same constructions.

The *Planetary Hypotheses* book, only parts of which have become well known in the West recently, consists of two volumes. In the first one Ptolemy corrects some of the parameters of the *Almagest* and suggests an improved model to explain planetary latitude. However, the second volume, dealing with ether hypotheses, produced nothing new for the empirical success of the Ptolemaic programme, and can be considered as a pile of ad hoc hypothesis with respect to the *Almagest* content.

Ptolemy intended to develop a celestial mechanism based upon the mathematical theories that he set out in the *Almagest*. And his models should "adhere to the principles that correspond to the continuous substance", that is, the ether. So, he lists the characteristics of etherial bodies: each is invariable and finite, receives no agitation, moves uniformly and circularly, and performs only a single, "voluntary" motion. It is crucial that Ptolemy also claims that celestial bodies maintain a faculty which may be compared to the human faculties of vision and intelligence.

In his description of the motion of celestial bodies, Ptolemy draws a sharp distinction between the Earth and the heavens. His cosmology requires every body, both celestial and terrestrial, to participate in its own natural motion. Whereas an earthly body can be propelled violently with a motion not proper to it, a celestial one enjoys only uniform and circular motion. And unlike sub-lunar motion, the motion of the heavens cannot be determined by outside forces. Ptolemy criticizes Aristotle for using terrestrial examples to generate an image of the nature of the celestial spheres. The philosophers fail to understand planetary motion, he writes, because they "begin their analogy from the spherical motions that are near us", by using the motion of mundane bodies to understand celestial rotation. But Ptolemy's celestial spheres follow laws different from those that govern terrestrial motion.

He also applies terrestrial examples to explain the structure of the heavens. He treats the celestial spheres as parts of "universal animal". Birds and clouds may be compared to the planets since they are composed of the same elements that surround them, although their colours and densities are different from those of the air.

Ptolemy doubts about the Aristotelian idea of the "prime mover". Like Aristotle, he admits that motion is transmitted by contact, yet the prime mover touches only the outermost of all the celestial bodies. Only the outermost sphere would rotate with its proper motion, because it could hardly transmit its own particular motion to the other celestial bodies.

To explain the order of the Universe, Ptolemy turns to another metaphysical concept, *the soul*. Anticipating Johannes Kepler, he believes that "the planets are ensouled and are moved with a voluntary motion". It is this soul that eventually provides the faculty for producing brightness and motion. Such a faculty is natural to ether. The wandering stars command a substantially larger amount of the faculty than the other etherial bodies due to the following reason.

Each planet has the power to direct not only its own motion, but the motions of the neighboring celestial bodies as well. To illustrate this faculty, Ptolemy turns to the terrestrial example of a very peculiar kind: the motion of birds in the layer of air above the Earth. In sub-lunar world the psychic faculty of the bird sends an emission to its nerves and then its wings; it allows the bird to perform its movement. Similarly, the planet's psychic faculty sends emissions to the epicycle, then to the deferent, then to the outermost of that planet's celestial bodies. The emission functions as a "green light" (Murschel) to which each body responds with its own, "voluntary" motion, just as the motion of the nerves differs from the motion of the wings.

The ideal model described should help to answer another basic question: how celestial bodies that are spherical by necessity connect to one another; by what means do they transmit motion to one another? Citing the poles that Aristotle retained in his system, Ptolemy wants to know

> "Why isn't anything destroyed at the contact between the inner sphere and the first outer sphere? And why aren't all the motion[s] of the spheres equal in speed?"(quoted from Murschel 1995, 39)

Since the ether is by nature frictionless and nothing can slow down the uniformity of its movement, any supra-lunar, physical, fixed poles must be unable to impose motion on the surrounding ether. Moreover, no sphere would need a mechanism to cause its motion since ether by nature will rotate uniformly.

Yet poles need not be fixed, physical bodies. Hence in the spirit of the *Almagest*, Ptolemy defines this concept *mathematically*, as the points on a rotating sphere that do not trace out a circular path. But this definition fails to satisfy him. Ptolemy finds it unrealistic that a mathematical object such as a point would be able to move a physical object like an ethereal sphere. But he argues that the now-physical points, also made of ether like the spheres they connect, could now function as movers for the spheres, since they would be nothing more than "knots" or "warts". And if they are different from the surrounding etherial surfaces, they would "sink to the center of the world".

In constructing the objects of his cosmology Ptolemy applies now the "principle of economy of nature" instead of the "principle of economy of thought": "it is appropriate that we do [not] assume that anything be made in nature that has no meaning and that is of no use". Since an entire sphere is not necessary for the celestial machine, Ptolemy cuts some of the spheres into segments. These sawn-off pieces may be hollow, like bracelets or whorls, or they may be solid disks, like tambourines. The tambourine-like sawn-off pieces are the solid epicycles in which the planets are located, and all other spheres are necessarily hollow. Yet spheres such as the eccentric deferents have large amounts of surface area that fulfill no real function; hence, they are truncated.

Sawn-off pieces have no surface area at either end of their axes and so dismiss any type of physical or non-physical pole. The outermost sphere of the universe generates a diurnal rotation that affects every planetary system as well as the sphere of the fixed stars. How, then, do the various sawn-off pieces and spheres impose their motions upon other celestial bodies, all of which have their own motion ? Ptolemy's answer takes the form of the following basic principles.

(i) A spherical body will move another spherical body lying within it if their mathematical axes are not collinear.
(ii) A spherical body may be moved by another spherical body if it is entirely contained within the latter.

Yet there are too many questions unanswered. Why don't the emissions of planets affect each other? How can a sphere have its voluntary motion and share in the diurnal rotation? And if some spheres are cut away into sawn-off pieces, what fills up the remaining volume of the sphere?

The question then remained as to whether Ptolemy's models represented *true*physical hypotheses for the cosmos. But almost all the *Almagest* geometrical models were repro-

duced in a comparable physical form. Moreover, the principles governing the heavens that Ptolemy had accepted as true and necessary were not compromised by his physical models. For instance, sawn-off pieces seemed no more unlikely than the ethereal shells first assumed to fill the heavens. These structures may seem mathematically trivial, but Ptolemy's object was to provide a suitable and consistent physical representation of the heavens, and not to invent new planetary theories.
However,

> "the pronouncements in the *Planetary Hypotheses* are more equivocal than those of the *Almagest*. Ptolemy is quite sure of the ethereal composition of the heavens, and also quite sure of the fundamental geometrical structures of the celestial motions; but the specific way that these geometrical structures are embodied in the ethereal matter is *open to alternatives* (complete spheres or equatorial slices, spinning driven by cosmic souls or by planetary rays or by the axes)". (Jones 2005, 35)

Moreover, no rigid links connecting the mathematical and the physical models were provided by Ptolemy, so that each new mathematical result would support the physical hypothesis and vice versa.

Thus, after the *Planetary Hypotheses* the gap between the physical and mathematical principles became deeper, so Ptolemy's cosmology came under repeated attack during the Middle Ages (Linton 2010). It obviously contradicted the principles of monotheism not admitting the profound gap between the celestial and mundane worlds. In the second book of *Planetary Hypotheses*, describing a physical actualization of the mathematical models of the planets in the *Almagest*, the conflict with Aristotelian physics became especially sharp. It was in attempting to remove the discrepancies that the "School of Maragha" and also Ibn-Shatir in the 13th and 14th centuries devised new planetary models (Saliba 2007). The Middle Age Arabic astronomers aimed at harmonizing their science to become more scientifically coherent.

The new science of Nay'a (theoretical astronomy) was created due to the flourishing activities at the time of the early Abbasides that created an unprecedented recovery of the sciences of antiquity with a deep desire to deploy them for the purposes of the time, a phenomenon that was not to be repeated until the time of the late European renaissance. It is important that once they could shun the discipline of astrology, then the importers of Greek astronomy, as well as the composers of Hay'a texts could pose as allies of the religious establishment. Most of the Arabic scientists were religious functionaries as well. Religion's main impact on astronomy was to force its separation from astrology and to compel attempts to harmonize Ptolemaic cosmology and Aristotelean physics (Ragep 2007). Theology compelled astronomers to deny the universe's objects divine attributes, and thus their motions had to be explained according to the mathematical laws of their models.

The re-examination of Ptolemaic astronomy reached its full maturity during the 11th century. The most extensive and sophisticated attack on it was leveled by Ibn al-Haytham (965-1040), also known as Alhazen. His main objective was to "*harmonize the mathematical and physical aspects of astronomy*".

In *Doubts concerning Ptolemy*, published between 1025 and 1028, Alhazen criticized the *Almagest*, *Planetary Hypotheses* and *Optics*, pointing out "innumerous contradictions" he found there. For instance, Ibn-al-Haitham wrote that Ptolemy had set himself the task of accounting for the phenomena using uniform circular motions and that, since he had introduced the equant mechanism, he could not be considered to have succeeded. Ibn-al-Haitham objected to Ptolemy's lunar theory because it involved an imaginary point opposite the center of the deferent controlling the motion of the lunar apogee. This seemed physically impossible. Above all, he argued, astronomy should deal with *real* bodies, and not imaginary ones. As he put it,

> "Thus the duty of the man who investigates the writings of scientists, if learning the truth is his goal, is to make himself an enemy of all that he reads, and, applying his mind to the core and margins of its content, attack it from every side [...] I constantly sought knowledge and truth, and it became my belief that for gaining access to the effulgence and closeness to God, there is no better way that of searching for truth and knowledge". (quoted from Ragep 2007, 72)

The Persian Abu Ubayad al-Juzjani constructed his own non-Ptolemaic models. Al-Juzjani's efforts were ultimately unsuccessful, but the search for alternate models continued during the 12th century in the Islamic west. Astronomers of Andalusian Spain rejected geometric models that failed to satisfy Aristotle's axiom of uniform circular motion about a fixed point.

Finally astronomers from Damascus and Samarkand had initiated the process later called the "Maragha Revolution" (the Maragha school's revolution against Ptolemaic astronomy, 13th-14th centuries). The Maragha school astronomers were more successful than their Andalusian predecessors in producing non-Ptolemaic configurations which eliminated the equant and eccentrics.

An important facet of Maragha revolution included the realization that astronomy should aim to describe the behavior of physical bodies in mathematical language. But it should not remain a mathematical hypothesis, which would only "save the phenomena".

Unlike the ancient Greek and Hellenistic astronomers (like Eudoxus and Ptolemy) who were not concerned with the coherence between the mathematical and physical principles of a planetary theory, Islamic astronomers insisted on the need to match the mathematics with the real world surrounding them. The Maragha School gradually evolved from a reality based on Aristotelian physics to one based on empirical and mathematical physics after the work of Ibn al-Shatir. The "Maragha Revolution" was thus characterized by a shift away both from the philosophical foundations of Aristotelian physics and Ptolemaic astronomy and towards a greater emphasis on the empirical observation, as exemplified in the works of Ibn al-Shatir, Qushji, al-Birjandi and al-Khafri.

Thus, the main purpose of all the Arabic theorists was to try to reconcile the requirements of Aristotelian natural philosophy with the mathematical models of Ptolemaic cosmology. Only in very few cases were objections to Greek astronomy made on the basis of its failure to account for the observed facts.

The Golden Age of Islamic astronomy had extended from the mid-thirteenth to mid-fourteenth century. During this time, attempts were made to reform Ptolemaic astronomy by Nasir al-Din al-Tusi (1201-1274), Mu'ajjad al-Din al-Urdi (died in 1266), Qutb al-Din al-Shirazi (1236-1311), Ibn al-Shatir (1305-1375), and others.

It seems to me that the works of Islamic astronomers are useful for Ptolemy-Copernicus transition rational reconstruction since they were a part of general bias – to "*harmonize the mathematical and physical aspects of astronomy*".

# Copernican Programme Genesis and Development

The contradiction realization and (partial) elimination of which belongs to intertheoretic context of the Copernican revolution was many years ago (in the 20th century fifties) revealed by French philosopher and historian of science Alexander Koyre. He spoke of "tremendous gap" between mathematical astronomy and Aristotelian qualitative physics. Hence the heliocentric programme construction motive consisted not in eliminating the discrepancies between the Ptolemaic cosmology and experience. On the contrary, Copernicus was inspired by aesthetic and metaphysical –*and theological in essence*– considerations directly connected with elimination of the abovementioned dualism.

In the famous introduction to his *De revolutionibus orbium coelestium*, published in 1543 in Nuremberg and dedicated to "his holiness Pope Paul III", Nicolaus Copernicus, a canon at Frombork Cathedral (who's maternal uncle was Lucas Watzenrode, the bishop of Warnia) directly points out that

> "I was impelled to consider a different system of deducing the motions of the universe's spheres for no other reason than the realization that *astronomers do not agree among themselves in their investigations of this subject.* For, in the first place, they are so uncertain about the motion of the sun and moon

> that they cannot establish and observe a constant length even for the tropical year. Secondly, in determining the motions not only of these bodies but also of the other five planets, they do not use the same principles, assumptions, and explanations of the apparent revolutions and motions. For while some employ only homocentrics, others utilize eccentrics and epicycles, and yet they do not quite reach their goal. For although those who put their faith in homocentrics showed that some nonuniform motions could be compounded in this way, nevertheless by this means they were unable to obtain any incontrovertible result in absolute agreement with the phenomena. On the other hand, those who devised the eccentrics seem thereby in large measure to have solved the problem of the apparent motions with appropriate calculations. But meanwhile they introduced a good many ideas which apparently *contradict the first principles of uniform motion.* Nor could they elicit or deduce from the eccentrics the principal consideration, that is, the structure of the universe and the true symmetry of its parts". (Copernicus [1453] 1972)

As a result,

> "I began to be annoyed that the movements of the world machine, *created for our sake by the best and most systematic Artisan of all,* were not understood with greater certainty by the philosophers, who otherwise examined so precisely the most insignificant trifles of this world".

The paradoxes' core, as seen by Copernicus, consists in non-ideal movement of the planets. But they belong to ideal spheres and *should* be engaged in uniform motions along the circles or along their combinations. Like Aristotle, Copernicus was convinced that the supposed perfection of heavens requires celestial bodies to execute uniform circular motion only. Hence he was spurred to reject Ptolemy's equant model.

Thus, inspired by the "best intentions to demonstrate the Divine Order of the Heavens", Copernicus proposes to place the center of the Universe on Sun. However namely this causes the deep paradoxes within the Aristotelian physics caused with the notions of natural and violent movements.

As a matter of fact Copernicus had constructed a genuine *crossbred* theory (analogous to Maxwell's displacement current or to first Planck's 1900 semiclassical theory – see Nugayev 2000a for details) that paved the way to divine mathematics and mundane physics *interpenetration*. As a modern French historian put it

> "Copernicus in an insinuating manner and maybe unconsciously had introduced into Aristotelian fortress two small assumptions with the help of which Kepler, Galileo and Descartes blowed it up". (Chaunu 1984, 430)

As a matter of fact Copernicus appeals to a favorable clergy consisting of Pope Paul III (to whom his *De revolutionibus* was dedicated), Pope Climentus VII (who adopted his opus magnus and even insisted on its publication), Nicholas Schonberg, cardinal of Capua, his maternal uncle, bishop of Warmia, Tiedemann Giese, bishop of Chelmno, etc. and condemns Ptolemy for *paganism*. He criticizes Ptolemy for that his sophisticated system containing tens and hundreds of epicycles and equants is nevertheless lacking single, monotheistic God, "the best and most systematic Artisan"; so the various parts of Ptolemaic system reflect the various plans of different pagan "artisans". Thus,

> "on the contrary, their [Ptolemy's proponents] experience was just like someone taking from various places hands, feet, a head, and other pieces, very well depicted, it may be, but not for the representation of a single person; since these fragments would not belong to one another at all, a monster rather than a man would be put together from them". (introduction to Copernicus [1453] 1972)

Namely that way Copernicus even unconsciously paved the way to Galileo.
If there is no strict demarcation line between divine and mundane worlds and Earth is an ordinary planet, the mathematical notions and principles should be applicable to its rotations around its own axis and around the Sun and to the bodies moving along its surface. It was Copernicus who had begun

> "getting rid of the celestial machinery with which generations of astronomers had encumbered the heavens, and thus 'sweeping cobwebs off' the sky". (Maxwell [1872] 1890, 311)

For more complete and systematic rational reconstruction of Copernican programme "hard core", "heuristic" and "protecting belt" one has to turn to the works of Nicolas of Cusa (1401-1464) who was a cardinal of the Catholic church from Germany. He was a philosopher, theologian, jurist, mathematician and an astronomer, one of the greatest polymaths of the 15th century. Nicolaus was widely read; in particular, Nicolaus Copernicus, Johannes Kepler and Galileo Galilei were all aware of his writings. In *De Concordantia Catholica*, *De Docta Ignorantia* and *De Visione Dei* the metaphysical intuitions constituting the "Zeitgeist" were ordered by Cusanus systematically and consequently. Nicolas's "monotheistic creationism" was directed against the cosmos of Aristotle and Ptolemy: as a result of divine creation skies did not differ significantly from the earth. On the other hand, Man was treated as a "second God" and hence he was an architect of ideal (mathematical) theoretical objects. This laid the foundations of mathematical natural science.

Galileo descends mathematics from skies to Earth being inspired by Copernicus and Plato (especially his *Timaeus* dialogue) and his own astronomical observations as well. In the especially pithy programmatic work *The Assayer* (1623) Galileo claims that

> "philosophy is written in that great book which ever lies before our eyes –I mean the universe– but we cannot understand it if we do not first learn the language and grasp the symbols, in which it is written This book is written in the mathematical language, and the symbols are triangles, circles and other geometrical figures, without whose help it is impossible to comprehend a single word of it; without which one wanders in vain through a dark labyrinth". (Galilei [1623] 1957, quoted from Burtt 2003, 75; see also Galilei [1632] 2001)

For Galileo "mathematics is the language with which God has written the Universe" (quoted from Lial, Miller and Hornsby 1995, 2). It is well-known that Galileo's interpretation of Christian theology was inspired by Plato and his *Timaeus*. The central figure of the dialogue –the Demiurge, a divine Craftsman– imposes mathematical order on a preexistent chaos to generate the ordered universe ("cosmos"). For that purpose he cuts out small triangles to construct four regular solids; and then he uses these solids to architect real bodies, plants and animals out of them (Plato [360 BC] 2000).

At the sake of mathematization Galileo transforms the natural science methodology elevating the idealization and giving to real and thought experiments the ranks of leading methods of science (Husserl [1936] 1970). All that made it possible for Galileo to come close to "the principle of inertia" and to Newton's second law of dynamics (Mach [1883] 2007).

Analogous platonic (and neo-platonic) motives and especially "delightful accordance between the Cosmos and the Holy Trinity" (Kepler [1596] 1981) where among those that brought Kepler to the search for mathematical laws governing the planet motions. Educated in the Lutheran faith, Kepler devoted himself to the *Book of Nature*, looking on his work in the field as the service of a priest. "We astronomers are priests of the highest God in regard to the book of Nature" (quoted from Barker and Goldstein 2001).

Kepler considered the sun as the geometric and dynamical center of the cosmos on the grounds of Trinitarian theology. The coequality of Father, Son, and Spirit required the continuity of the center, periphery and space of the cosmos. Sun was God the Father, the Stars God the Son and the planets were Holy Ghost. Moreover, Kepler's repeated attempts to find a universal law that would account for the motion of both earth and the planets were also inspired by his analogy between the created cosmos and the uncreated Trinity.

Kepler took the next step towards the unification of mathematical astronomy and physics and discovered the laws roughly breaking the Aristotle-Ptolemy principle of divine bodies uniform rotation. Skies began to destroy the qualitative physics (Kepler [1609] 2005).

According to Sir Roger Cotes, Sir Isaac Newton's main purpose was to discover the laws that govern the motion of both terrestrial and divine motions. The first thing to be done consisted in demonstration with the help of Copernicus and Galileo heuristic that it was the same force that attracted all the bodies to earth that forced the moon to orbit the Earth. Namely that was done in *Philosophiae Naturalis Principia Mathematica* (1687). As is stated in Sir Roger Cotes's preface

> "Therefore it is plain that the centripetal force, by which the Moon is perpetually , either impelled or attracted out of the tangent and retained in its orbit, is the very force of terrestrial gravity reaching up to the Moon". (Newton [1687] 1846, 31)

Newton created the "hard core" of his programme by fusing and remaking the theoretical schemes of Copernicus, Kepler, Hooke and Galileo and coming up to the conjunction of three laws of dynamics with the gravitation law that provided an empirically progressive problemshift of the Copernican programme.

Thus, elucidation of the Copernican revolution intertheoretic context enables to reveal the deep analogy between the Ptolemy-Copernicus and Lorentz-Einstein transitions. In both cases rather important role was played by the considerations connected with unification problems. Copernicus, as well as Einstein, proposed the hypotheses that led to interpenetration and unification of those research programmes that previously had existed separately from each other.

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# References


- Ariew, R. (1989), "The phases of Venus before 1610", *Studies in History and Philosophy of Physical Science* 18: 81-92.
- Aristotle (2007a), *Physics*. Translated by R.P. Hardy and R.K. Goye. eBooks@Adelaide.
- Aristotle (2007b), *Metaphysics*. Translated by W.D.Ross. eBooks@Adelaide.
- Barker, P., Goldstein, B.R. (2001), "Theological Foundations of Kepler's Astronomy", *Osiris* 16: 88-113.
- Burtt, E.A. (2003), *The Metaphysical Foundations of Modern Science*. New York: Dover Publications.
- Chaunu, P. (1984), *La Civilisation de l'Europe Classique*. Paris: Les Editions Arthaud.
- Copernicus, N. ([1543] 1972), *On the Revolutions of the Heavenly Bodies*. Translated by Edward Rosen. London: Macmillan.
- Duhem, P. ([1906] 1991), *The Aim and Structure of Physical Theory*. Princeton University Press.
- Einstein, A. (1934), *The World As I See It*. New York: Covici Free Publishers.
- Feyerabend, P. ([1975] 2010), *Against Method*, $4^{th}$ ed. New York: Verso Books.
- Fitzpatrick, R. (2010), *A Modern Almagest. An Updated Version of Ptolemy's Model of the Solar System*. Lulu Enterprises.
- Galileo, G. ([1623] 1957), *The Assayer*. Translated by Stillman Drake. New York: Doubleday.
- Galileo, G. ([1632] 2001), *Dialogue Concerning the Two Chief Systems – Ptolemaic and Copernican*. Translated by Stillman Drake. New York: The Modern Library.
- Gingerich, O. (1997), *The Eye of Heaven*. Berlin: Springer.
- Gingerich, O.J. (2004), *The Book Nobody Read*. New York: Walker.
- Goldstein, B.R. (1997), "Saving the phenomena: the background to Ptolemy's planetary theory", *Journal for the History of Astronomy* 28: 3-22.
- Husserl, E. ([1936] 1970), *The Crisis of European Sciences and Transcendental Phenomenology*. Translated by David Carr. Northwestern University Press.
- Jones, A. (2005), "Ptolemy's Mathematical Models and their Meaning", in Kinyon M., van Brummelen G.(eds), *Mathematics and the Historian's Craft*. Springer, 23-42.
- Kelvin, W.T., Lord (1901), "19th Century Clouds over the Dynamical Theory of Heat and Light", *Philosophical Magazine* 2: 1-39.
- Kepler, J. ([1596] 1981), *The Secret of the* Universe. Translated by F.M. Duncan. Originally published as *Mysterium Cosmographicum*. New York: Abaris.
- Kepler, J. ([1609] 2005), "Astronomia Nova", in *Selections from Kepler's Astronomia Nova: A Science Classics Module for Humanities Studies*. Translated by W.H. Donahue. Santa Fe, NM: Green Lion Press.
- Kline, M. (1986), *Mathematics and the Search for Knowledge*. Oxford University Press.
- Koyre, A. (1957), *From the Closed World to the Infinite Universe*. Baltimore: John Hopkins Press.
- Kuhn, T.S. (1957), *The Copernican Revolution: Planetary Astronomy in the Development of Western Though*. Cambridge: Harvard University Press.

- Kuhn, T.S. (1963), "The Function of Dogma in Scientific Research", in Crombie, A.C. (ed.), *Scientific Change*. London: Heinemann.
- Lakatos, I., Zahar, E. (1974), "Why did Copernicus's Research Program Supersede Ptolemy's?", in *The Copernican Achievement*. University of California, Los Angeles, ch. X, 168-192.
- Lakatos, I. (1970), "Falsification and the Methodology of Scientific Research Programmes", in Lakatos, I., Musgrave, A. (eds), *Criticism and the Growth of Knowledge.* Cambridge University Press, 91-195.
- Lial, M.L., Miller, C.D., Hornsby, J.E. (1995), *Beginning Algebra.* Harper Collins College Publishers.
- Linton, C.M. (2010), *From Eudoxes to Einstein: a history of mathematical astronomy.* Cambridge: Cambridge University Press.
- Lloyd, G.E.R. (1999), *Methods and Problems in Greek Science*. Cambridge: Cambridge University Press.
- Lorentz, H.A. (1909), *The theory of electrons and its applications to the phenomena of light and radiant heat*. New York: G.F. Stechart & Co.
- Mach, E. ([1883] 2007), *The Science of Mechanics: A Critical and Historical Account of its Developments*. Lightning Source Inc.
- Maxwell, J.C. ([1872] 1890), *On Action at a Distance*, in Maxwell, J.C. , *The scientific papers of J.C. Maxwell,* vol. 1. Ed. by W.D. Niven. New York: Dover Publications, 311-320.
- Murschel, A. (1985), "The structure and function of Ptolemy's physical hypotheses of planetary motion", *Journal for the History of Astronomy* 26: 33-61.
- Nasir, al-Din al-Tusi (1993), *Memoir on Astronomy*. New York: Springer Verlag.
- Neugebauer, O. (1975), *A History of Ancient Mathematical Astronomy*, 3 vols. Berlin: Springer.
- Newton, I. ([1687]1846), *The Mathematical Principles of Natural Philosophy* (translated into English by Andrew Motte), with preface of Mr. Roger Cotes. New York: Daniel Ades.
- Nugayev, R.M. (1998), "A Simple Theory-Change Model", *Epistemologia* 21: 225-260.
- Nugayev, R.M. (1999), *Reconstruction of Mature Theory Change: A Theory –Change Model*. Frankfurt am Main: Peter Lang.
- Nugayev, R.M. (2000), "Einstein's Revolution: Reconciliation of Mechanics, Electrodynamics and Thermodynamics", *Physis* (Nuova Serie) 37: 181-208.
- Plato ([386 BC] 2000), *Timaeus*. Translated by D.J. Zeyl. Indianapolis and Cambridge, Massachusetts: Hackett Publishing Co.
- Popper, K. ([1935] 2002), *The Logic of Scientific Discovery*. London and New York : Routledge.
- Ptolemy, C. (1998), *The Almagest*. Translated by G.J.Toomer with a foreword by Owen Gingetich. Princeton University Press.
- Ptolemy, C. ( 2005), *Tetrabiblos*. Translated by J.M. Ashmand. Astrology Center of America.
- Ragep, F.J. ( 2007), "Copernicus and his Islamic predecessors", *History of Science* 45: 65-81.
- Rutkin, H.D. ( 2010), "The Use and Abuse of Ptolemy's *Tetrabiblos* in Renaissance and Early Modern Europe: Two Case Studies (Giovanni Pico della Mirandola and Filippo Fantoni)", in Jones, A. (ed.) *Ptolemy in perspectives*. Springer.
- Saliba, G. (1999), "Seaking the Origins of Modern Science?", *Bulletin of the Royal Institute for Inter-Faith Studies* 1: 139-152.
- Saliba, G. (2007), *Islamic Science and the Making of the European Renaissance*. Cam-

bridge, MA: MIT Press.

- Thomason, N. (1992), "Could Lakatos even with Zahar's Criterion for novel fact, evaluate the Copernican Research Programme?", *The British Journal for the Philosophy of Science* 43(2): 161-2.