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# In the Beginning was Chiasmus

## On the epistemology of (non-quantified) modal modelling

### * Introduction & References *

Tom Ritchey
Swedish Morphological Society

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**Abstract:**
*Chiastic order* is an ancient expression for *cross-classification*. Cross-classification, in turn, is one of many terms used for the operation of conjoining or cross-mapping one domain, class or set of concepts with another. As such, it is the primordial form of non-quantified modelling and combinatory heuristics. This article presents a brief epistemological history of non-quantified modelling: its prehistory in the form of rhetorical *chiasmus*; its early (pre-symbolic) use by Plato as a cross-order (*paradigmatic*) modelling method; and its "modern" (symbolic) use by Leibniz as a *calculus of concepts*. It will also be shown how classification theory itself is built on a cross-classificatory construct involving two fundamental logical/structural relationships: subordination and conjunction. Finally, examples of modern computer-aided, non-quantified *modal modelling* are presented in the areas of design theory, operational research and decision science.

## Contents

# 1. Introduction

## 1.1 "The science of cross-classification"

In 1906, Alfred North Whitehead (1861-1947) published an article – "The Axioms of Projective Geometry" (**APG**) – in which he made a curious association between geometry, classification theory and combinatoric (heuristic) modelling. Note that by the late 19th century, projective geometry had become a significantly more abstract mathematical discipline than its original conception as the study of perspective and geometric projection *per se*[1]. It had developed into a highly abstract, non-metric form of geometry that deals with inter-dimensional relationships between geometric concepts and their symbolic representations. Whitehead wanted to develop a symbolic system for a geometry without magnitudes and independent of figures.[2] He called this "the science of *cross-classification*".

> "It is well-known that Geometry can be developed without any reference to measurement – and thus without any reference to distance, and without any reference to numerical coordinates for the indication of points. Geometry, developed in this fashion has been termed 'Non-metrical Projective Geometry'. … I have termed it, *'the science of cross-classification'*." [3]

Then he says something very interesting:

> "[The] science of cross-classification ... is not a science with a *determinate subject matter*. It is concerned with *any subject matter to which the formal axioms may apply,*"[4]

So what is Whitehead up to? In what way is APG "the science of cross-classification" and what does it mean when he states that this science does not have any "*determinate subject matter*"?

Two developments were taking place during the second half of the 19th century which engaged Whitehead in his early carrier as a mathematician. One was a revolution in the conception of *space* and a corresponding generalization of geometry.[5] The other concerned the further development of operative symbolic systems[6] as represented in the 19th century by the works of Frege, Boole, DeMorgan, Peacock and others. Whitehead is working at the intersection of these two areas of mathematics. Just as one can use the rules (axioms) of arithmetic to create a symbolic system for quantitative operations (i.e. the algebra of Viète and Descartes), then if one can formulate a rigorous axiomatic for a *non-metric* geometry, one should be able to use such axioms to create a symbolic system for a non-metric (non-quantified) "algebra of concepts".[7] Like its quantitative cousin, such an "algebra" should have the potential to become an *ars inveniendi*, a heuristic tool which is not fixed to any determinate subject matter, but which can "model" anything that is *consistent with its syntactic rules*.[8]

In this endeavor, Whitehead is not only plugging himself into a epistemological tradition emerging out of the 17th century "symbolic revolution"[9], but also into a pre-symbolic tradition stretching back to antiquity. We can see this more clearly if we follow Whitehead's subsequent development from mathematics and logic into the philosophy of science. During the period between 1906 and 1947 (the year of his death), he wrote a number of books and articles in which he further discusses and develops the epistemology of *cross-classification*.[10] His remarks reveal not only something about the concept *per se*, but also about its *epistemological history* and its association with *conceptual modelling* in general. Indeed, he finds its ancient roots in the Platonic dialogues, and its modern roots in the works of Gottfried Leibniz (1646-1716) and Hermann Grassmann (1809-1877).

Along the way, he has some interesting things to say about the distinction between two fundamental, perennial classificatory systems: linear hierarchal (*hypotactic*) classification and conjunctive coordinate (*paratactic*) classification[11] – including the different types of reasoning involved in these two epistemic structures. (It is notable that, over the four decades of Whitehead's work in this area, a simultaneous development was taking place in bibliographic classification theory, where this very same distinction was being explored and applied.[12] Somewhat later, we also find the same distinction being applied in archaeological methodology.[13])

- ***"Classical" (Platonic pre-symbolic) roots***

In "Adventures of Ideas" (1933), Whitehead contrasts the "science of ***cross***-classification" with the "science of ***mutually exclusive*** classification": i.e. the strict hierarchal "classification into genera, and species, and sub-species" which he attributes to Aristotle.[14] At first, he simply states that this (historically dominating) linear hierarchal classification approach was taken from Plato's "method of divisions" (*diaeresis*). However, later on, in "Essays in Science and Philosophy" (1947), he points out that Platonic *diaeresis* also involved another type of classificatory structure – called *parallel-division* – based not on hierarchal *sub-ordination*, but on conjunctive *co-ordination* (or the *co-mingling of classes*). He refers to this as:

> "... a wider conception of the scope of Logic which was obscured by the dominant Aristotelian [linear-hierarchal] theory. The [wider] concept was adumbrated by Plato, when in *Sophist* he points out the importance of a *science of the mingling of forms* [by which] to obtain *more complex forms*."[15]

He then cites the following lines from *Sophist* (253d-e):

> *"Visitor.* Now since we have agreed that the classes also commingle with one another or do not commingle, in the same way, must not he [the dialectician] possess some science [*epistemes*] and proceed by the processes of reason [logos] ... to show correctly which of the classes harmonize with which, and which reject one another, and also if he is to show whether there are some elements extending through all and holding them together so that they ***can comingle***... .[16]

We shall return to the Platonic notion of the "comingling" or the "intertwining of concepts" (*symploke eidon*) in §4.

- ***"Modern" (Leibnizian symbolic) roots***

As will be discussed in §5, what Whitehead means by non-metric, axiomatic-synthetic geometry was essentially what Gottfried Leibniz (1646-1716) was developing in the 1670's and 80's with his work on *Characteristica Geometrica* ("symbolic geometry") and in a later article given the title *Generales Inquisitiones de Analysi Notionum et Veritatum* (General Inquiries on the Analysis of Concepts and Facts) from c. 1686. Leibniz was attempting to develop a "geometry of relations without magnitudes, independent of figures, embedded in a complete axiomatic and endowed with an expressive symbolism".[17]

Furthermore, Leibniz presents many of the basic axioms put forward by Whitehead and makes the self-same remark concerning the status of the symbolic system thus developed – i.e. that it is not fixed to any *determinate subject area*, but that it is a formal heuristic that can be applied to any area that conforms to its assertions. Whitehead was fully aware of this:

> "The founder of the *modern point of view* … was Leibniz, who, however, was so far in advance of contemporary thought that his ideas remained neglected and undeveloped until recently; … especially "*Generales Inquisitiones de Analysi Notionum et Veritatum*" (written in 1686)."[18]

The crux of this "modern point of view" involved the realization that symbolic systems can be employed as cognitive tools for exploring the deep (unseen) structure of their empirical targets. In this way, the symbolic system – or model – *itself* becomes an object of "empirical study".[19] This is what Leibniz called *cognitio symbolica*, and which today goes by such various names as *surrogate reasoning*[20], *generative symbolism*[21] and *operative symbolic systems*.[22] As Serfati (2010) puts it:

> "After the introduction of the symbolic writing system, nothing in mathematics was anymore like before. The outcome was, strictly speaking, a (symbolic) revolution, one of the major components of the scientific revolution."[23]

Whitehead was also heavily influenced by – and made a thorough study of – Hermann Grassmann who, in turn, was well acquainted with Leibniz's earlier work on non-metric geometry (*analysis situs*) and was engaged in the very same "project". Indeed, he considered his *Ausdehnungslehre* (1844) to be "the natural development of Leibniz's ideas" [24]. Their common aim was to develop a heuristic symbolic system that went beyond mathematics as such:

> "The more general aims of both Leibniz and Grassmann were the same – the setting up of a convenient calculus or art of manipulating signs by fixed rules, and of deducing therefrom true propositions for the things represented by the signs... In each case their geometrical calculus was a particular application to geometry of a ***wider calculus*** for which each desired *more than mere applicability to mathematics*."[25]

The "wider" epistemological issue involved here is the realization that science develops not only in the light of new empirical discoveries or technical innovations; not only through the accumulation of empirical anomalies; nor necessarily on a so-called Tech-Last basis of involving "major unsolved practical problems".[26] Scientific revolutions are also facilitated by way of "rational discoveries": anomalies and surprises emerging *from within* the very heuristic symbolic systems – or models – developed in order to represent empirical phenomena. This is the core idea behind what is today termed Model-Based Reasoning (MBR) and model-based science.[27]

Thus the development of a general *science of symbols* (*Formenlehre / Morphologie*), or a universal calculus, goes beyond both mathematics and formal logic into pure concepts, further facilitating heuristic reasoning and supporting the art of discovery. However, what I wish to show is that *ars inveniendi* is not only supported via abstract symbolic systems, but is also found in pre-symbolic, conceptual modelling invented in ancient Greek science. Although non-symbolic conceptual modelling cannot match operative symbolic systems in either demonstration or discovery in the areas of mathematics and natural science, it can be applied to important areas of study that operative symbolic systems cannot (yet?) reliably represent in any "depth": matters of human volition, including policy analysis, organisational design, strategic management and statecraft. These "deliberative" arts are part of what today is called soft-systems analysis in decision science – systems which are self-reflexively complex and involve what have been termed *wicked problems*.[28]

## 1.2 Epistemological history

I have chosen to begin this article with Whitehead's axiomatic treatment of non-metric geometry in order to emphasise how late 19th and early 20th century mathematics and philosophy of science were still in the process of rediscovering and further developing fundamental concepts emerging out of the 17th Century symbolic revolution, and which, ultimately, date back to antiquity. The study of this type of historical conceptual development belongs to what is called *Épistémologie Historique* – usually inverted in English as *Epistemological History*.[29]

The continental tradition of Epistemological History developed in the early half of the 20th century in France.[30] It is not aimed simply at an "objective" chronological description of the history of theories of knowledge, but as an evaluative study of the history of scientific *concepts* developing within evolving epistemological-ontological frameworks – i.e. the *reciprocal development* of empirical research and rational frameworks. Among other things, this involves what has been called *regulated* or *controlled anachronism*: a self-conscious approach to historically relating the essential aspects of *concepts* or *schemata* from different time periods[31]. In this, it aims at:

> "… the establishment of a dynamic relationship between the past and the present…"[32] by which to study "… the history of the *production of scientific concepts* [where] new theories integrated old theories in new paradigms, *changing the sense of concepts*."[33]

Thus the fundamental EH *antimetabole*: W*e use the past to better understand the present, and the present to better understand the past*. This is expressed admirably by one of the great theoretical innovators and philosophers of science of the 20th century, Erwin Schrödinger (1887-1961) in "Nature and the Greeks" (1954). In studying ancient Greek science:

> "There is not only ... the hope of unearthing obliterated wisdom, but also of discovering inveterate error at the source, where it is easier to recognize. By the serious attempt to put ourselves back into the intellectual situation of the ancient thinkers, far less experienced as regards the actual behaviour of nature, but also very often much less biased, we may regain from them their freedom of thought."[34]

A stellar example of this productive relationship between the present and the past is exemplified by three of the central figures in the 20th century "quantum revolution" – Schrödinger, Werner Heisenberg and Wolfgang Pauli – all of whom deliberately (and by their own admission) looked "back" to Plato and the pre-Socratic philosophers for inspiration in formulating a theoretical framework for the interpretation of quantum physics.[35]

## 1.3 Non-quantified modal modelling

At this point it is appropriate to provide at least a minimal definition of what is meant – in this article – by both the notion of a (scientific) *model* in general, and a *non-quantified model* in particular. As has been observed elsewhere[36], the notion of a (scientific) model, like the notion of a "system" or "theory", belongs to a class of concepts which essentially have unbounded domains. The open-ended nature of these concepts makes it difficult to give them both an all inclusive *and* a precise definition. However, we can at least supply a minimal, operative definition which suffices for the purposes of this study. We define a *model* (and we now drop the "scientific" qualifier and ask that it be understood) by way of the so-called E-R ("Entity-Relationship") schema:

A model consists of:

A) Two or more (mental) constructs that can serve as variables which can support a range of states or values – otherwise called the variable's *domain*. We shall call these variables the *parameters* of the model.[37]

B) The establishment of relationships (e.g. mathematical/functional, statistical, logical, modal, normative) between the domains of the different parameters, such that each parameter is "connected to" (i.e. constrained or influenced by) at least one of the other parameters.

The development of these two components (variables and connective links) into a model is essentially an iterative process of (compositional) analysis and synthesis. In the analysis phase, variables and their respective domains are formulated which represent the model's initial *problem space*. In the synthesis phase, connective relationships between parameters are defined and applied, thus *binding* the modelling space, determining its topological properties and constraining it in order to produce a solution or outcome space.

Thus the basic framework for a model is an internally connected, n-dimensional conceptual space which goes under a number of different names depending on the nature of the model, its area of application and the *properties of the space to be emphasised*: for instance, a coordinate space, parameter space, configuration space, state space, phase space, or morphospace.

A *non-quantified* model consists of:

1) A set of category (a.k.a. categorical) variables, the *discrete domains* of which consist of elements which are either *non-ordinal* or can be *rank-ordered*. (In statistical analysis these are somewhat misleadingly called *nominal* and *ordinal* scales respectively).

2) Relationships (or links) between the domains of the given variables can be established on the basis of non-metric (e.g. *modal* alethic and deontic) constraints.

According to these "definitions", a *cross-classification* or *cross-mapping* that coordinates two or more concepts or domains is the basic formal structure – and the essence – of a *model*. Also, when alethic (possibility, necessity and impossibility) and/or deontic (prescription and proscription) constraints are employed, this is an example of *modal modelling*, a species of modelling which has gained considerable interest during the past few decades.[38]

The epistemological issues involved in *modal modelling* are complex and still variously contested – as is modal epistemology in general.[39] In the present work, we are mainly concerned with "scientific modal inference" based on "objective modality".[40] This means that modal claims must be built on both logical consistency and empirical evidence concerning well defined targets. The targets can be 1) *actual* objects or systems to be evaluated or "tested" under different specified conditions, and 2) *hypothetical* (e.g. projected) systems which need to be explored and tested for compossibility beforehand, in order to weed out alethic and/or deontic inconsistencies or incompatibilities.[41]

Note that while we treat alethic modality as empirically "objective" in scientific investigation generally (cf. Williamson, 2016), we also consider deontic modality as "circumstantially objective" in the context of social science, in modelling e.g. policy spaces and legal/ethical issues. This is what Hirvonen et. al. (2021) call “relative modality”. These issues will be treated in more detail in §6.

One of the more interesting issues is how *modal* modelling relates to the reciprocal contexts of *justification* (proper inference) and *discovery* (heuristics). As will be explored in more detail below, combinatoric *modal* models – like model-based reasoning in general – serve both of these purposes: 1) they support constraint-based *modal inference*[42] (in the sense of inference by the exclusion of impossibilities), and 2) they provide a *modal heuristic* (cross-order) search-space for conceptual integration and combinatorial creativity.[43]

* * *

In the following sections we will present a brief epistemological history of the development of what Whitehead meant by “cross-classification”, and show how this, together with combinatorial heuristics, further developed into non-metric, multivariate cross-order modelling. This will be presented in its two natural, historical phases: classical pre-symbolic and modern symbolic systems.

The *ars inveniendi* of “modern” operative symbolic systems have been employed in science for at least 300 years and so thoroughly permeate modern science that we seem to have lost our wonder of them.[44] What we wish to show is that *pre-symbolic* modelling and combinatory synthesis – in the form of cross-classificatory verbal-conceptual modelling – is also an *ars inveniendi*, and is being applied today in this manner, as exemplified by contemporary discrete-variable morphological modelling. The article will conclude by giving examples of contemporary computer-aided applications of this modelling method, in the areas of engineering design, policy analysis and decision science in general.

However, as a contribution to Epistemological History, the first question to be asked at this point is: When did the idea of conjoining or cross-ordering concepts – in order to produce or discover new knowledge – begin, and why? All that can be said with any certainty is that it began, like human language itself, in the “fog of prehistory”.

## 2. In the Beginning was *X* (*Chi*)

***X*** (χεῖ, Latin: *chi*) is the twenty-second letter of the ancient Greek alphabet. Besides being a standard *consonant*, because of its X-form it has also come to be associated with – or be a generalised symbol for – a cross-order pattern of inverted parallelism. For this reason, such cross-order patterns were given the name *chiasmus*, which is the Latinised form of the classical Greek *khiasmos* and the verb *khiazein*, which literally means “to mark with the letter X”. The Latinized term, however, meant more like “to arrange or order in the shape of X”. In this signification, what *chiasmus* actually denotes predates the letter X, predates the Greek alphabet and predates written language altogether. Chiasmus is the name for any “cross-order” arrangement of objects, lines, symbols, diagrams, concepts, or any type of locution: words, expressions, phrases or whole narrative accounts. More generally, *chiasmus* is the essence of cross-classification and – as will be discussed (§3) – of cross-order modelling in general.

Chiastic structure can be divided into three broad, overlapping stages: iconic, rhetorical/linguistic, and epistemic. Here we are principally concerned with epistemic chiasma, but it is interesting to begin by briefly considering earlier iconic and rhetorical forms. [To be continued...]

## References and further reading

Ackrill, J. L. (1997) "ΣΥΜΠΛΟΚΗ ΕΙΔΩΝ" in J. L. Ackrill, Essays on Plato and Aristotle, New York: Oxford University Press, Chapter 4, pp. 72-79.

Alvarez, J. (2018) Models and Analogy, Homologies, Representation, Embodiments and Reproductions, Seripta Phil. Naturalis 14, pp. 37-63.

Balzer, W., Moulines, C.U., Sneed, J.D.: An Architectonic for Science, The Structuralist Program. Reidel, Dordrecht (1987).

Banks, E. (2013) Extension and measurement: A constructivist program from Leibniz to Grassmann, Studies in History and Philosophy of Science 44, pp. 20-31

Benis-Sinaceur (1989/2023) “ARS INVENIENDI" TODAY, Acta Morphologica Generalis, 9(1), pp. 1-11. Original article: "ARS INVENIENDI" AUJOURD’HUI, Les Études philosophiques, No. 2, LEIBNIZ (APRIL-JUNE 1989) pp. 201-214. Translated from the French by Asunción Álvarez.

Bobzien, S. (2020) “Ancient Logic”, in The Stanford Encyclopaedia of Philosophy (Summer 2020), Edward Zalta (ed.).(SEP)

Boden, M. (1999). Computer models of creativity. In Sternberg, R. (Ed.), Handbook of Creativity, pag. 351–373. Cambridge University Press.

Bolt, J., Coecke, B., Genovese, F., Lewis M., Marsden D., Piedeleu, R.: Interacting Conceptual Spaces: Grammatical Composition of Concepts. In: 2016 Workshop on Semantic Spaces at the Intersection of NLP, Physics and Cognitive Science (SLPCS’16), EPTCS 221, pp. 11–19 (2017).

Breck, J. (1994) The Shape of Biblical Language, St. Vladimir’s Seminary Press.

Brown: L. (2010) Definition and Division in Plato’s Sophist, in Charles D. (ed.) Definition in Greek Philosophy, Oxford: Oxford University Press, pp. 151-171.

Brumbaugh, R. (1954) Plato's Mathematical Imagination, Bloomington: Indiana University Press.

Brumbaugh, R. (1961) Logical and Mathematical Symbolism in the Platonic Scholia, Journal of the Warburg and Courtauld Institutes, 24(1/2), pp. 45-58.

Brumbaugh, R. (1989). Platonic Studies of Greek Philosophy: Form, Arts, Gadgets, and Hemlock, State University of New York Press.

Couturat, L., 1902. The Logic of Leibniz (Translated by Donald Rutherford and Timothy Monroe, 1997-2003). [Online] Available at: http://web.archive.org/web/20030308235650. [Last assessed 2-9-2016].

Crivelli. P (2019) Division and Classification: Philebus 14c–20a. In: Plato’s Philebus: A Philosophical Discussion. Edited by: Panos Dimas, Russell E. Jones, and Gabriel R. Lear, Oxford University Press

Dahlberg, I. (1976). Classification theory, yesterday and today. Knowledge Organization, 3(2), 85-90

Dawson, A. (2008) Whitehead’s Universal Algebra, in Weber, M. & Desmond, W. (Eds.) Handbook of Whiteheadian Process Thought, [Vol. 2], Frankfurt: Ontos Verlag, pp.67-86.

De Beer, S. (2023) The power of chiasmus: Exploring the prayer of Esther in LXX Esther, HTS Teologiese Studies/Theological Studies 79(1), pp. 1-8.

Debuiche, V. (2013) Perspective in Leibniz's invention of Characteristica Geometrica: The problem of Desargues' influence, Historia Mathematica 40, pp.359-385.

de Grolier, E. (1962) A study of general categories applicable to classification and coding in documentation, United Nations Educational,Scientscand Cultural Organization (UNESCO).

Dunnell, R. (1971) Systematics in Prehistory, The Blackburn Press

Esquisabel, O. (2011). Visualization and Symbolic Knowledge in Leibniz and Lambert. Internationaler Leibniz-Kongress Natur und Subjekt, Actas del IX, Bände 1-3, Vol. 1 (pp. 312-323). Hannover: Gottfried-Wilhelm-Leibniz-Gesellschaft.

Esquisabel, O. (2012) Representing and abstracting: An Analysis of Leibniz's Concept of Symbolic Knowledge, Studies in Logic, Vol. 41: Symbolic Knowledge from Leibniz to Husserl, London: College Publication, Kings College.

Fauconnier, G., and Turner, M. (1998). Conceptual integration networks. *Cognitive Science, 22*(2), 133-187.

Fauconnier, G. & Turner, M. (2002). *The way we think: Conceptual blending and the mind's hidden complexities*. New York, NY: Basic Books.

Feest, U. & Sturm, T. (2011). What (Good) is Historical Epistemology? Editors' Introduction. Erkenntnis 75, pp. 285-302.

Fielding, A. & Goldstein, H. (2006) Cross-classified and Multiple Membership Structures in Multilevel Models, Research Report RR 791, University of Birmingham.

Fischer, B. & Leon, F. (Eds.) (2017) Modal Epistemology after Rationalism, Synthese Library:Springer.

Gärdenfors, P. (2004) Conceptual Spaces: The Geometry of Thought. The MIT Press.

Gerhardt, C. I. (Ed.) (1875-90) Die Philosophische Schriften von Gottfried Wilhelm Leibniz, 7 vols., Berlin: Wiedmann

Gill, M.-L. (2006) Models In Plato's Sophist And Statesman Plato: The Internet Journal of the International Plato Society **6.**

Gill, M.-L. (2010) "Division and Definition in Plato's Sophist and Statesman." in: Definition in Greek Philosophy. Ed. David Charles. Oxford: Oxford University Press.

Gillies, D. (2015) Why do Scientific Revolutions begin, in Ippoliti, E. (ed.) *Heuristic Reasoning*, Studies in Applied Philosophy, Epistemology and Rational Ethics 16, Springer.

Gingras, Y. (2010) Naming without necessity: On the genealogy and uses of the label "historical epistemology", CIRST – Note de recherche.

Goldfarb, L., Gay, D., Golubitsky, O & Korkin, D. (2004). What is a Structural Representation? Technical Report TR04-165, Faculty of Computer Science, University of New Brunswick.

Grassmann H. (1995, orig. 1844, 1862) A New Branch of Mathematics, trans Lloyd Kannenberg, LaSalle: Open Court.

Johns, C.. (2014) Leibniz and the Square: A Deontic Logic for the Vir Bonus, History and Philosophy Of Logic 35(4), pp. 369–376.

Harman, P. M. 1998. The Natural Philosophy of James Clerk Maxwell, Cambridge University Press.

Heath, A. E: (1917) The Geometrical Analysis of Grassmann and its Connection with Leibniz's Characteristic, The Monist 27(1), pp. 36-56.

Heeffer, A. (2008) On The Nature and Origin of Algebraic Symbolism, New Perspectives on Mathematical Practices, pp. 1-27.

Hirvonen, I. , Koskinen, R. & Pättiniemi, I. (2021) Modal inferences in science: a tale of two epistemologies, *Synthese* 199, pp. 13823–13843

Hjørland, B. (2013) Facet analysis: The logical approach to knowledge organization, *Information Processing & Management* 49(2), pp. 545-557.

Ippoliti, E. & Nickles, T. (2020) Scientific Discovery and Inference, Topoi 39, pp. 835-839**.**

Kadison, L. & Kromann, M. (1996) Projective Geometry and Modern Algebra, Birkhäuser.

Kanamori, A. (2009) The Higher Infinite (Second Edition), Heidelberg: Springer.

Khemlani, S., Byrne, R, & Johnson-Laird, P. (2018) Facts and Possibilities: A Model-Based Theory of Sentential Reasoning, Cognitive Science (2018), pp. 1–38.

Knobloch, E. (2010) “Leibniz between ars characteristica and ars inveniendi: Unknown news about Cajori’s ‘master-builder of mathematical notations’”, in Heeffer, A. & Van Dyck, M. (Eds) (2010) Philosophical Aspects of Symbolic Reasoning in Early Modern Mathematics, College Publications: King’s College London.

Kolaitis, P. & Vardi, M. (2008) A Logical Approach to Constraint Satisfaction, in N. Creignou et al. (Eds.) Complexity of Constraints, LNCS 5250, Springer-Verlag, pp. 125–15.

Koperski, J. (2016). Models in Science. Internet Encylopedia of Philosophy (IEP). [Online] Available at: http://www.iep.utm.edu/models. Last accessed: 2016-02-01.

Körner, S. (1976) Classification Theory, *Intern. Classificat* 3(1), pp. 3-6.

Krämer, S. (1996) Mind, symbolism, formalism: Is Leibniz a precursor of Artificial Intelligence Knowl.Org. 23(2), pp.84-87

Krämer, S. (1997). “Kalkiile als Reprasentation. Zur Genese des operativen Symbolismus in der Neuzeit”. In: H.-J. Rheinberger, M. Hagner & B. Wahring-Schmidt (eds.), Raume des Wissens: Reprasentation,Codierung, Spur. Berlin, Akademier Verlag, pp. 113-122.

Krämer, S. (2003): Writing, Notational Iconicity, Calculus: On Writing as a Cultural Technique, in Sybille Kramer, Anita McChesney MLN, 118(3), pp. 518—537

Krämer, S. (2016) Leibniz on Symbolism as a Cognitive Instrument, in Juliet Floyd, and James E.

Katz (eds), *Philosophy of Emerging Media: Understanding, Appreciation, Application* (New York: Oxford Academic.

Leibniz, G.W., 1673. Characteristica geometrica. De lineis et angulis. In: Leibniz (1923–, vol. VII/1, N. 9, 109–119).

Leibniz, G.W., 1677. Characteristica Geometrica. In: Leibniz (1995, 50–71)

Toadvine, T. (2012) The Chiasm, in The Routledge Companion To Phenomenology.

Lui, Hi (2021) On Diairesis, Parallel Division, and Chiasmus: Plato's and Aristotle's Methods of Division, *PLATO JOURNAL*, *22*, 27-52.

Lissner, P. (2007) Chi-Thinking: Chiasmus and Cognition, Dissertation submitted to the Faculty of the Graduate School of the University of Maryland at College Park in partial fulfilment of the requirements for the degree of Doctor of Philosophy, (Accessed 2022-12.25 from: https://drum.lib.umd.edu/bitstream/handle/1903/7687.)

Lowe, V. (2020) Understanding Whitehead. Johns Hopkins University Press, Project MUSE. doi:10.1353/book.71824

Malink, M. & Vasudevan, A. (2016) The Logic of Leibniz's Generales Inquisitiones de Analysi Notionum et Veritatum,The Review of Symbolic Logic. doi:10.1017/S1755020316000137

Magnani, L. (2015) Are Heuristics Knowledge–Enhancing? Abduction, Models, and Fictions in Science, in E. Ippoliti (ed.), *Heuristic Reasoning*, Studies in Applied Philosophy, Epistemology and Rational Ethics 16, Springer International.

Mazzola, G. (2002) "Paradigmatic Classification", in The Topos of Music, Basel: Birkhäuser.

McCoy, B. (2003) Chiasmus: An Important Structural Device Commonly Found in Biblical Literature, Chafer Theological Seminary Journal 9(2), pp. 18-34.

Mouján, R. (2020). Greek philosophy for quantum physics. The return to the Greeks in the works of Heisenberg, Pauli and Schrödinger. In: Probing the Meaning of Quantum Mechanics, D. Aerts, J. Arenhart, C. de Ronde and G. Sergioli (Eds.), World Scientific, Singapore.

Nänny, M. (1987) Chiastic structures in literature: some forms and functions. in SPELL: Swiss papers in English language and literature Vol. 3, p. 75-97.

Merleau-Ponty, M. (1968) The Visible and the Invisible, Northwestern University Press.

Loraux N. (2005) Éloge de l'anachronisme en histoire. In: Espaces Temps, 87-88, 2005. Les voies traversières de Nicole Loraux. Une helléniste à la croisée des sciences sociales. pp. 127-139

Lehrer, R. & Schauble, L. (2000) Developmental Model-based reasoning in Mathematics and Science, Journal of Applied Developmental Psychology 21(1), pp. 39-48.

Lehrer, R. & Schauble, L. (2007) A Developmental Approach for Supporting the Epistemology of Modeling, in Blum, W. et. al. (Eds.) (2007) Modelling and Applications in Mathematics Education, New ICMI Study Series, Springer Verlag, pp 153-160.

Loemker, L. (Ed.) (1969). Leibniz - Philosophical Papers and Letters, Vol. I & II, Second Edition. Dordrecht: D.Reidel Publishing Company.

Malink, M. & Vasudevan, A. (2016) The Logic Of Leibniz's *Generales Inquisitiones De Analysi Notionum Et Veritatum*, The Review Of Symbolic Logic, Pp. 1- 66.

Minsky, M. (1965): Matter, Mind and Models. Proceedings of the IFIP Congress 65, 1: 45-49

Montelle, C. (2011). A 'Symbolic' History of the Derivative. In D. Jardine & A. Shell-Gellasch (Eds.), Mathematical Time Capsules: Historical Modules for the Mathematics Classroom, (pp. 151-158). Mathematical Association of America.

Mugnai, M. (2010) Logic and Mathematics in the Seventeenth Century, *History and Philosophy of Logic* **31**, pp. 297-314.

Müller-Merbach H. (1976). The Use of Morphological Techniques for OR-Approaches to Problems, Operations Research 75, 27-139.

Parrochia, D. & Neuville, P. (2013) *Towards a General Theory of Classifications*, Studies in Universal Logic, Basel: Springer.

Pelkey, J. (2016) Symbiotic modeling Linguistic anthropology and the promise of chiasmus, *Reviews in Anthropology* 45(1), pp. 22–50.

Pelkey, J. (1917) The Semiotics of X, London: Bloombury Academic.

Podnieks, K. (2017) Philosophy of Modeling: Some Neglected Pages of History. [Preprint]PhilSci Archive: http://philsci-archive.pitt.edu/id/eprint/13538.

Pombo, O. (1987). Leibniz and the Problem of a Universal Language, Münster: Nodus Publikationen.

Pombo, O. (2010) Three Roots for Leibniz's Contribution to the Computational Conception of Reason, in Programs, Proofs, Processes, 6th Conference on Computability in Europe, CiE 2010, Ponta Delgada, Azores, Proceedings (pp.352-361)

Portides, D. (2014). Models, in M. Curd, & S. Psillos, The Routledge Companion to Philosophy of Science (s. 429). London: Routhledge

Riemann, B. (1953)."On the Hypotheses which lie at the Foundations of Geometry", in Riemann, Gesammelte Mathematische Werke. Dover, New York.

Ritchey, T. (1991, rev. 1996). "Analysis and Synthesis - On Scientific Method based on a Study by Bernhard Riemann". Systems Research 8(4), 21-41 (1991). (Available at: http://www.swemorph.com/pdf/anaeng-r.pdf.)

Ritchey, T. (1997). "Scenario Development and Risk Management using Morphological Field Analysis". Proceedings of the 5th European Conference on Information Systems (Cork: Cork Publishing Company) Vol. 3:1053-1059.

Ritchey, T. (2002). "General Morphological Analysis - A general method for non-quantified modelling". Adapted from a paper presented at the 16th Euro Conference on Operational Analysis, Brussels, July 1998. (Available at: http://www.swemorph.com/pdf/gma.pdf. Last accessed 2012-03-09.)

Ritchey, T. (2002) "Modelling Complex Socio-Technical Systems using Morphological Analysis", Adapted from an address to the Swedish Parliamentary IT Commission, Stockholm, December 2002. (Available for download at: www.swemorph.com/downloads.html.)

Ritchey, T. (2003) "MA/Carma – Advanced Computer Support for Morphological Analysis". (Available for download at: www.swemorph.com/macarma.html.)

Ritchey, T. (2005b) "Futures Studies using Morphological Analysis". Adapted from an article for the UN University Millennium Project: Futures Research Methodology Series (Available for download at: www.swemorph.com/downloads.html.)

Ritchey, T. (2006) "Problem Structuring using Computer-Aided Morphological Analysis". Journal of the Operational Research Society, 57, 792–801. (Available at: http://www.swemorph.com/pdf/psm-gma.pdf.)

Ritchey, T. (2011). Wicked Problems/Social Messes: Decision support Modelling with Morphological Analysis. Springer, Berlin.

Ritchey, T. (1012). "Outline for a Morphology of Modelling Methods: Contribution to a General Theory of Modelling". Acta Morphologica Generalis, Vol 1, No. 1 ). (Available at: http://www.amg.swemorph.com/pdf/amg-1-1-2012.pdf.)

Ritchey, T. (2013) Wicked Problems: Modelling Social Messes with Morphological Analysis, Acta Morphologica Generalis, 2(1), pp. 1-8.

Ritchey, T. (2018). "General morphological analysis as a basic scientific modelling method", in Ritchey, T. & Arciszewski, T. (Eds.) (2018) "General Morphological Analysis: Modelling, Forecasting, Innovation", Special Issue of Technological Forecasting and Social Change, Vol. 126.

Ritchey, T. (2022) Conceptual Modelling, Combinatorial Heuristics and Ars Inveniendi - An Epistemological History, Acta Morphologica Generalis (2022) Chapters 1 & 2.)

Rivet, A., et. al. (2016) What Are Crosscutting Concepts in Science? Four Metaphorical Perspectives, ICLS 2016 Proceedings.

Schrödinger, E, (1954) Nature and the Greeks, Cambridge University Press.

Serfati, M. (2010) Symbolic revolution, scientific revolution: mathematical and philosophical aspects, in Heeffer, A., & Van Dyck, M. (Red.). (2010). Philosophical Aspects of Symbolic Reasoning in Early Modern Mathematics. London: College Publications, King’s College London.

Serfati, M. (2020) The rise of symbolic notation in scientific communication: the case of mathematics, in Leßmollmann, A., Dascal, M. & Gloning, T. (Eds) (2020) Science Communication, De Gruyter Mouton.

Serres, M. (1972) Hermès II, L"interférence. Paris: Éditions de Minuit.

Sjölin Wirling, Y., & Grüne-Yanoff, T. (2021). The epistemology of modal modeling. *Philosophy Compass*, 16(10).

Sjölin Wirling, Y., Grüne-Yanoff, T. (2023) Introduction to the Synthese topical collection ‘modal modeling in science: modal epistemology meets philosophy of science’. Synthese 201, 208.

Sutherland, T. (2021) The Philosophy of Transport: Michel Serres' Recursive Epistemology in the Hermes Pentalogy, Media theory, 5(1), pp. 201-218.

Swoyer, C. (1991) Structural Representation and Surrogative Reasoning Synthese 87, pp. 449-508.

Swoyer, C. (1995a) Leibnizian Expression, Journal of the History of Philosophy 33(1), pp. 65-99.

Swoyer, C. (1995b) Leibniz on Intension and Extension, NOÛS 291, pp. 96-114.

Tyler, S. & Strecker, I. (2009) The Rhetoric Culture Project. In Culture and Rhetoric. Ivo Strecker and Stephen Tyler (Eds.) Pp. 1–18. Studies in Rhetoric and Culture, Oxford: Berghahn Books.

van den Heuvel, C. (2011) Multidimensional classifications: Past and future conceptualizations and visualizations. In Smiraglia, Richard P., ed. Proceedings from North American Symposium on Knowledge Organization, Vol. 3. Toronto, Canada, pp. 105-121.

Whitehead, A. (1898) A Treatise on Universal Algebra. With applications. Cambridge: Cambridge University Press.

Whitehead, A. (1906) The Axioms of Projective Geometry. Cambridge: Cambridge University Press

Whitehead, A. (1911) Axioms of Geometry/Definition of Abstract Geometry, in Encyclopaedia Britannica, Vol. 11, p. 675-736.

Whitehead, A. (1933) Adventures of Ideas. New York: New American

Whitehead, A. (1948) Essays in Science and Philosophy. London: Philosophical Library.

Wigner, E. (1960). The Unreasonable Effectiveness of Mathematics in the Natural Sciences, Communications in Pure and Applied Mathematics 13, John Wiley & Sons, pp. 1-14.

Williamson, T. (2016) Modal science. Canadian Journal of Philosophy, 46(4-5), 453–492.

Wiseman, B. & Paul, A. (eds.) (2014) Chiasmus and culture (Studies in Rhetoric and culture 6), Berghahn Books, Oxford.

## Notes

1 For background, see e.g. Kadison & Kromann (1996), "Historical Foreword"

2 Whitehead, 1906, p. 4.

3 Whitehead, 1933, p. 137f. I have chosen to cite this later article because it is expressed more succinctly. In APG (1906) he writes: "Geometry, in the widest sense in which it is used by modern mathematicians, is a department of what in a certain sense may be called the general science of classification." (p. 4.) … "In Projective Geometry the subject viewed simply as a study of classification has great interest. Thus in the foundations of the subject [of projective geometry] this conception is emphasized, while the introduction of 'order ' is deferred. (p. 6).

In his last work, "Essays In Science and Philosophy" (1948), Whitehead writes:

"*Definition of Abstract Geometry*. – Existent space is the subject matter of only one of the applications of the modern science of abstract geometry, viewed as a branch of pure mathematics. Geometry has been defined00201 as "the study of series of two or more dimensions." It has also been defined as "the science of cross classification." These definitions are founded upon the actual practice of mathematicians in respect to their use of the term "Geometry." Either of them brings out the fact that geometry is not a science with a determinate subject matter. It is concerned with any subject matter to which the formal axioms may apply. Geometry is not peculiar in this respect. All branches of pure mathematics deal merely with types of relations. Thus the fundamental ideas of geometry (e.g., those of points and of straight lines) are not ideas of determinate entities, but of any entities for which the axioms are true. And a set of formal geometrical axioms cannot in themselves be true or false, since they are not determinate propositions, in that they do not refer to a determinate subject matter. The axioms are propositional functions." (p. 178.)

4 Whitehead, 1911, p 730. Written for an article on Geometry for Encyclopaedia Britannica (Emphasis added). He goes on to write here:

"… these formal geometrical axioms cannot in themselves be true or false, since they are not determinate propositions, in that they do not refer to a determinate subject matter."

Earlier, in **APG** (1906) he writes:

> "… the points mentioned in the axioms are not a special determinate class of entities; but they are in fact *any entities whatever*, which happen to be inter-related in such a manner, that the axioms are true when they are considered as referring to those entities and their inter-relations. Accordingly—since the class of points is undetermined—the axioms are not propositions at all: they are propositional functions." (p. 2, emphasis added)

5 This was a continuation of the earlier work of e.g. Karl Gauss, Bernhard Riemann and James Maxwell. In this context, Whitehead was specifically influenced by two important figures who had carried this tradition forward: the German mathematician Felix Klein (who was a frequent visitor to Cambridge) and the British non-Newtonian physicist William Clifford (who produced the first English translation of Bernhard Riemann's famous Habilitationsvortrag "On the Hypothesis which lie at the Foundations of Geometry", conceived of a general theory of relativity a generation before Einstein, and then died at the age of 34 in 1879.). Note that Whitehead's concept of projective geometry built on what Maxwell had called the "geometry of position", which he (Maxwell) had taken from Gauss. Gauss', in turn, had been inspired by Leibniz's notion of *analysis situs* (the analysis of situation) and *Characteristica Geometrica* (1679).

> "Maxwell, drawing upon Gauss, had used the term 'geometry of position' in 1871 for the projective geometry of Lazare Carnot and Michel Chasles. Gauss had been inspired by "Leibniz"s remarks on the need to formulate geometric algorithms to express geometric location (analysis situs)." Dawson (2008), p. 17, Note 57, in reference to Harman (1998), p. 154. Cf. Debuiche (2013).

For a more detailed account of Whitehead's relationship with Klein and Clifford, see Dawson (2008).

6 I.e. the move from rhetorical and abbreviate mathematical representation to generative, embedded symbolic representation initiated in the 17th century. (See e.g. Serfati 2010, 2020; Ritchey 2022.)

7 At this point, the details of Whitehead's actual axioms are not important. They will, however, be taken up in §5.

8 Note the association between this and modern mathematical "model theory", which was being developing during the last decades of Whitehead's life. All of this is a central aspect of the so-called Leibniz Program. See e.g. Benis-Sinaceur (1989/2023) for an account of the relationship between Leibniz, Kurt Gödel, *ars inveniendi* and the development of model theory.

9 Actually that are (at least) three "symbolic revolutions" in the development of human cognition:

- The emergence of human spoken language (orality)
- The development of written language (literacy)
- The development of operative symbolic systems (extended symbolic cognition)

A truly "rational" AI may be the next stage – whatever that may entail.

10 Whitehead 1906; 1911; 1933; 1947. The nominal definition of *cross-classification* is: "The placing of items into classes based on the features of two or more variables." [OR] "a system of classification used in which each item is assigned to the intersection of a row category and a column category. [APA Dictionary of Psychology]. This is technically correct, but conceptually limited. Whitehead is using the term in the wider epistemological sense of multivariate cross-order mapping and heuristic symbolic modelling.

11 The terms *hypotaxis* and *parataxis* essentially mean subordinate and coordinate arrangements respectively. Although they are used – most notably – in linguistics and the study of grammar and syntax, they are far too important to be associated only with specific disciplines. They are fundamental logical and epistemological-classificatory concepts.

12 I.e. between linear hierarchal classification (e.g. Dewey Decimal system) and multivariate cross-classification (e.g. Facet-analysis, and as in contemporary, non-pre-categorized "tagging" systems used in digital search engines). As noted by Hjørland (2013), facet-analysis was developed in the early 20th as a complementary knowledge organization approach within Library and Information Science.

> "The facet-analytic paradigm is probably the most distinct approach to knowledge organization within Library and Information Science: and in many ways it has dominated what has been termed "modem classification theory". It was mainly developed by S. R. Ranganathan and the

British Classification Research Group, but it is mostly based on principles of logical division developed more than two millennia ago." (Hjørland, 2013, Abstract.)

For a quick overview see e.g. Dahlberg (1976); van den Heuvel (2011). Cf. de Grolier (1962).

[13] See influential Dunnell (1971).

[14] Whitehead (1933, p. 137f). Whitehead's reference to (strict) linear hierarchal classification (nowadays called "proper taxonomies") as "Aristotle's method" is somewhat misleading. Aristotle – like Plato (see § 4, infra) – also clearly recognized the usefulness of cross-ordering as a method of classification (e.g. in The Categories). However, it is nonetheless true that 1) Aristotle considered strict hierarchal classification and its use in deductive inference or "demonstration" (apódeixis) as the only true "scientific" method, and 2) that this became doctrine in the classical (Aristotelian) scientific tradition. I understand Whitehead as referring to this "tradition". (Cf. Ritchey, 2022).

[15] Whitehead, 1948, p. 237f. Emphasis added. The contemporary expression of this blending of forms is Conceptual Integration Theory (CIT) put forward by Fauconnier &Turner (2002).

[16] Whitehead, 1948, p. 98. Emphasis and brackets added.

[17] This is Debuiche's (2013, pp, 359f) description of Leibniz's similar work in developing a *Geometrica characteristica* (symbolic geometry) and a logical calculus, which has been called "The Leibniz Project". Cf. Loemker (1969), pp. 371-382. See §5, *infra*.

"During his whole life, Leibniz attempted to elaborate a new kind of geometry devoted to relations and not to magnitudes, based on space and situation, independent of shapes and quantities, and endowed with a symbolic calculus. Such a "geometric characteristic" shares some elements with the perspective geometry: they both are geometries of situational relations, founded in a transformation preserving some invariants, using infinity, and *constituting a general method of knowledge*." [Debuiche, 2013. Abstract.)

[18] Whitehead, 1948, p. 207. (Emphasis added.)

[19] Leibniz presented the first manifest of this notion in a paper published in Acta Eruditorum (1684) titled "Meditationes de Cognitione, Veritate et Ideis [Reflectons on Knowledge, Truth and Ideas]. What was first accomplished in mathematics (Viete & Descartes) Leibniz wanted to further develop, on a purely conceptual bases, with a calculus (or algebra) of concepts. The realization of his Universal characteristica (Universal symbolic system) what is today called the *Leibniz program*.

[20] Swoyer (1991).

[21] Kanamori (2009) p. 474.

[22] Krämer (1997, 2003, 2016); cf. Esquisabel (2012). See Ritchey (2022) p. 29f for further examples with sources.

Krämer also calls it "operative writing" and describes it as:

"... inspired by three attributes: (1) Symbolic language can be used as a technique for problem solving. (2) The rules of manipulating symbols can be independent of their interpretation. (3) Symbols do not only depict, but constitute knowledge." (Krämer 2016, Abstract.)

Cf. Ritchey (2022), with the following similar "attributes":

1. A medium for *representing* cognitive phenomena,
2. A tool for *operating* with this medium in order to solve problems or to prove theories,
3. A *formal (syntactic) structure* that has an *inherent generative potential* for the development of new entities and relationships.

[23] See e.g. Serfati (2010; 2020). "After the introduction of the symbolic writing system, nothing in mathematics was anymore like before. The outcome was, strictly speaking, a (symbolic) revolution, one of the major components of the scientific revolution." (Serfati, 2010, p. 120.)

[24] "Grassmann looked back to Leibniz and claimed that that the geometric algebra developed in his groundbreaking 1844 *Ausdehnungslehre* was the natural development of Leibniz's ideas on the 'geometric characteristic', announced in the letter to Huygens in 1679. Grassmann's prize essay of 1847 was written in response to a challenge to complete Leibniz's project for the characteristic." (Banks 2013, p. 21).

Writing about his own work in the third person, Grassmann states: "[Leibniz presents] yet another remarkable point where he quite clearly expresses the applicability of this [geometrical] analysis to objects that are *not of a spatial nature*, but adds that it is not possible to give a clear concept of this in a few words. Now, in fact, as is demonstrated throughout Grassmann's *Ausdehnungslehre*, all concepts and laws of the new analysis can be developed *completely independently of spatial intuitions*, since they can be tied to the abstract concept of a continuous transformation; and, once one has grasped this idea of a pure, conceptually interpreted continuous transformation, it is easy to see that the laws developed in this essay are also capable of this interpretation, stripped of spatial intuitions. Grassmann 1995, p. 384; cited in Banks (2013), p. 21.

[25] Heath (1917) p. 38. (Emphasis added.)

[26] See e.g. Gilles (2015).

[27] See e.g. Magnani, Nersessian & Thagard (1999). Thanks to Lorenzo Magnani et. al. there is a whole library of books and articles on this subject.

[28] See e.g. Ritchey (2013) for a review of this concept.

[29] For a detailed review see e.g. Feest & Sturm (2011); Sturm (2011); Nickles (2017); Vagelli (2019). Note that the original French concept has been variously translated into English a "Historical Epistemology", "History of Epistemology" and "Epistemological History". There has been some discussion as to possible differences in the meanings of these terms, which will not be addressed here. Following Georges Canguilhem, we choose "Epistemological History" (See e.g. Gingras, 2010; cf. Tiles 1987).

[30] For a brief overview, see e.g. Ritchey (2022).

[31] For a more detailed discussion see e.g. Vagelli (2019); Loraux (2005).

[32] Vagelli (2019), p. 106.

[33] Renn & Gutfreund (2020) P. 115. (My emphasis).

[34] Schrödinger (1954) p. 18f.

[35] For a detailed account see Mauján (2020).

[36] See Ritchey (2012, 2018) for a more detailed discussion.

[37] The term parameter is being used here in its broader "systems science" sense, as being one of a set of factors that defines a system and determines its behaviour, and which can be varied in an experiment – including a Gedankenexperiment.

[38] There has been some disagreement as to what it is that should be included under the rubric of "model logic".

Narrowly construed, modal logic studies reasoning that involves the use of the expressions 'necessarily' and 'possibly'. However, the term 'modal logic' is used *more broadly* to cover a family of logics with similar rules and a variety of different symbols. (Garson (2003).)

Thus – using the "narrow" sense of the term – *alethic* reasoning is "modal", whereas *deontic*, or the normative logic of obligation, prescription and proscription, is to be considered a separate family. However here we use the broader designation which considers both alethic and deontic logic – as well as e.g. doxastic (what one believes) and epistemic (what one knows) reasoning – as part of the family of "modal" logics.

[39] See e.g. Sjölin & Grüne-Yanoff (2021; 2023) for a discussion of recent developments.

[40] See e.g. Williamson (2016); cf. Hirvonen *et. al.* (2021).

[41] Note that we are treating *deontic modality* as "objective" to the extent that such constraints reflect "social facts" that must to be considered in e.g. policy analysis and social planning.

[42] Khemlani et. al. (2018). Modal inference differs from (logical) inference *stricru sensu, i.e.* as deduction from premises to necessary conclusions. The different notions of *inference* used in relationship to strict deduction, abduction and heuristics in general will not be addressed here. See e.g. Ippoliti, E. & Nickles, T. (2020); Magnani (2015).

[43] See e.g. Boden (1999); Fauconnier & Turner (2002). Cf. Ritchey (2021) for overview. The status of the heuristic/discovery process described by Magnani (below) for model-base reasoning in general applies just as well (mutatis mutandis) to combinatoric modal modelling:

> "… the 'fundamental' role played by models in science is the one we find in the core abductive discovery processes … [which] are *constitutive* of new scientific frameworks and new empirical domains. The abduction of these models in science is epistemically productive; … they are knowledge-enhancing devices, which play an important role in reaching empirically fecund knowledge." (**Magnani (2015),** p. 43).

[44] This is what Wigner's (1960) article "The unreasonable effectiveness of Mathematics in the Natural Sciences" is about – although I think that he approached the issue from the wrong perspective and with a strangely pessimistic attitude.)

* * *

**The author:** Tom Ritchey is a former Research Director for the *Institution for Technology Foresight and Assessment* at the Swedish National Defence Research Agency in Stockholm. He is a modelling theorist and facilitator who works primarily with non-quantified decision support modelling – especially with General Morphological Analysis (GMA), Bayesian Networks and Multi-Criteria Decision support. Since 1995 he has directed more than 100 projects involving computer aided GMA for Swedish and other national government agencies, international NGO:s and private companies. He is the founder of the Swedish Morphological Society and Director of Morphologics (Stockholm).