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Everything but the kitchen sink: how (not) to give a plenitudinarian solution to the paradox of flexible origin essentialism

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Abstract I explore options for a plenitudinarian solution to the Paradox of Flexible Origin Essentialism, taking as my unlikely starting point the views of Sarah-Jane Leslie, who holds that if plenitudinarianism is true, then there is in fact no paradox to be solved, only the illusion of one. The first three sections are expository: Sect. 1 on plenitudinarianism, Sect. 2 on the paradox, and Sect. 3 on Leslie's views about how plenitudinarianism bears on the paradox. In Sect. 4, I reject the contention that there is no paradox and critically explore three options for a plenitudinarian solution. In Sect. 5, I argue that the plenitudinarian ought to endorse a fourth option. In Sect. 6, I consider an objection. I endorse neither plenitudinarianism nor its denial; the main aim of the paper is to argue that for one who does endorse plenitudinarianism, the best solution to the Paradox of Flexible Origin Essentialism is clear.

Keyword Chandler's paradox · Chisholm's paradox · Essence · Modal logic · Permissivism · Plenitude · Tolerant origin essentialism

1 Plenitudinarianism

The statue in my study is coincident with a lump of clay. Nonetheless, says the *standard pluralist*, the statue is distinct from the lump. The statue could survive the loss of some of its matter—for example, the matter that constitutes its nose—whereas the lump could not. The lump could survive squashing whereas the statue

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could not.[1] Relating possibilities for survival to essential properties: it is not essential to the statue to be constituted by certain matter, but it is essential to the lump to be so constituted;[2] it is not essential to the lump to be statue-shaped, but it is essential to the statue to be so-shaped. Having distinguished between the statue and the lump, one may begin to wonder whether there are still more objects coincident with those two. One may wonder whether there is also an *in-statue*, an object differing from both the statue and the lump by having as an essential property being indoors.[3] One may also wonder whether there is an *out-only-once-statue*, an object sharing with my statue the property of having never been outdoors and hence the property of having not been outdoors more than once. Such an object would have the former property accidentally, but the latter property essentially. It differs from the statue and the lump (which have both properties accidentally) and any coincident in-statues (which have both properties essentially). One need not wonder whether there is a coincident object that has the first property essentially and the second accidentally—for there cannot be such an object.

Why, one may ask, should there be something coincident with the statue that could not survive the loss of any matter, but not also something that could not survive outside? Why not also something else that could not survive being outside twice? Given that we recognize objects of the first sort, do we have any principled reason not to recognize objects of the other sorts? More broadly, do we have any principled reason not to recognize a *plenitude* of objects coincident with the statue but differing from it—and from each other—in their modal properties? According to Sarah-Jane Leslie (2011), the existence of a plenitude of objects coincident with any given material object is "guaranteed" by the anti-Quinean doctrine that there is a legitimate distinction between having a property essentially and having a property accidentally (278). I will call the anti-Quinean doctrine *L-essentialism* (short for '*essentialism*, in Leslie's sense').

Unless I indicate otherwise, the terms 'essential', 'accidental', and their cognates refer in this paper to modal notions. On the standard *modal understanding*, an object *o* has a property *P essentially* iff *o* has *P* and it is necessary that *o* has *P* (if *o* exists). An object *o* has a property *P accidentally* iff *o* has *P* and it is possible that *o* (exists

[1] In many cases, consideration of temporal properties also reveals the distinctness of the statue and the lump. Consider a lump of clay that exists before it is molded into a statue—the lump exists at time *t* but the statue does not. But Gibbard (1975) points out that it is possible for a statue and a lump to be created at the same moment and also to be destroyed at the same moment so that the statue and the lump are coincident at all times at which either exists. In such a case there are not obvious differences in temporal properties to cite in support of distinctness, though it would be surprising if such a statue were identical to its lump (so to speak) whereas ordinary statues are not identical to theirs.

[2] The *lump* is constituted by a *hunk* (portion, quantity, bit) of clay. As Gibbard (1975) puts it, "Take first the piece of clay. Here I do not mean the portion of clay of which the piece consists, which may go on existing after the piece has been broken up or merged with other pieces. I will call this clay of which the piece consists a *portion* of clay; a portion of clay, as I am using the term, can be scattered widely and continue to exist. Here I am asking about a *piece* or *lump* of clay. [¶] A lump sticks together: its parts stick to each other, directly or through other parts, and no part of the lump sticks to any portion of clay which is not part of the lump" (188). I follow Salmón (2005/1986) in using 'hunk' where Gibbard uses 'portion'. See also (Salmón 2005/1981, pp. 225–226, n. 8). Sometimes I say simply 'matter'.

[3] My notion of an in-statue adapts the notion of an *incar* due to Hirsch (1982).

and) lacks *P* (that is, iff *o* has *P* but does not have *P* essentially). I take *o*'s *modal essence* to be the class of all of *o*'s modally essential properties. The modal understanding of these terms, which contrasts with the *definitional understanding* favored by Kit Fine (1994), is congenial to Leslie's purpose of addressing the Paradox of Flexible Origin Essentialism (hereafter the *Paradox of FlexOE*), a modal paradox, with the resources of plenitudinarianism, a doctrine dependent on modal notions. It is evidently the modal understanding that Leslie has in mind when she explains how she sees the relationships between L-essentialism, the Paradox of FlexOE, and plenitudinarianism.

> [T]he apparent paradox depends on essentialism, but essentialism itself gives rise to plenitude. Plenitude is not introduced in response to the seeming paradox; rather both have a common ground in the core notion of essentialism, the idea that there are two ways of having properties …, namely the essential way and the accidental way. (288–289)

The Paradox of FlexOE does depend on L-essentialism—a claim that involves modal, not definitional, notions. Flexible Origin Essentialism is the claim that any material artifact of a given kind is such that it could have (existed and) been (originally) made (entirely) from matter that differs slightly but not greatly from the matter from which it was (originally entirely) made. To find this claim intelligible involves finding intelligible the modal essential/accidental property distinction. It does not involve finding intelligible the definitional essential/accidental property distinction. So too for the other (as yet unspecified) claims involved in the paradox. Nonetheless, I will sometimes consider the potential relevance of the definitional essential/accidental property distinction. A definitional essence, or "nature" is supposed to "define an object, or say what it is"—whatever exactly that means (Fine 1994, 2). Famously, it is supposed to lie in the nature of singleton Socrates to have Socrates as a member, but not lie in the nature of Socrates to belong to the singleton. A thing's definitionally essential properties may be thought of as a special subclass of its modally essential properties whereas a thing's definitionally accidental properties may be modally accidental to it (as the property of being a philosopher is to Socrates) or modally essential to it (as the property of being a member of singleton Socrates is to Socrates).

Leslie emphasizes that even one who holds that necessarily every object has every one of its properties essentially subscribes to L-essentialism, "agree[ing] with the in-principle distinction while arguing that certain metaphysical considerations mean that the [accidental] way of having properties is not, and cannot be, exemplified" (277). L-essentialism is the view that for any property, it is meaningful to say that it is had essentially and it is meaningful to say that it is had accidentally. But how can L-essentialism, which is a doctrine so minimal that hardly a soul but Quine would deny it, entail plenitudinarianism, which is a doctrine so extravagant (though evidently not so extravagant that hardly a soul would embrace it) that it

permits room for little more?[4] The heavy lifting in Leslie's argument is done by anti-arbitrariness considerations.

Stressing that she is "simplifying massively", Leslie imagines having an object with just five properties, all of which are "modally independent" of one another, then considers the distinct ways of assigning the modal values *essential* and *accidental* to those five properties (278).

> [S]uppose the essentialist claims to have discovered that *the* item in question has two of the five properties essentially, and the other three merely accidentally. Now we can ask: why is there not also another entity present that has only one of the properties essentially? Why is there not one that has four of the properties essentially? Why not items that have other combinations of the properties essentially? (278–279)[5]

It is natural to attribute to Leslie a standard plenitudinarian argument: there is no relevant difference between the proposed entities and the entity whose existence is antecedently admitted; thus on pain of arbitrariness—on pain of not treating like cases alike—it must be admitted that the proposed entities also exist.[6]

What happens when we start, less simplistically, with an object that has properties that are modally interdependent? This question is pressing for two reasons. First, L-essentialism says that *any property whatsoever* may be meaningfully said to be essential and may be meaningfully said to be accidental. Second, the entities that Leslie relies on in her diagnosis of the Paradox of FlexOE are "generated" by assignments of essential and accidental to properties are modally interdependent.[7] Leslie says that in this case, some of the questions have acceptable answers. Why is there not an object that is essentially blue but accidentally spatially extended? Because that would violate a "general metaphysical principle", so that an essence that included being blue but not being spatially extended would be "internally problematic" (279).[8] Leslie contrasts general

[4] Why "little" rather than "nothing"? See the remarks later in this section concerning the constraints imposed by mundane facts and coherence.

[5] It seems that we can also ask questions like this: why is there not a *plenitude* of entities present that has only one of the properties essentially? There may well be a reason why not, but Leslie does not provide one. See my speculation in note 8. It is natural to wonder whether this style of argument applies to non-material objects as well. Is there "present" with the number 2, which numbers the Martian moons, something (somewhat like a number) that cannot survive Mars's loss of a moon? Is there "present" with Katniss Everdeen something (somewhat like a fictional character) that cannot survive my daughter's reading *The Hunger Games*?

[6] Fairchild and Hawthorne (2018, note 23) offer numerous references to anti-arbitrariness (or parity) arguments for permissive ontologies.

[7] This becomes clear in Sect. 5.

[8] This is the only example that Leslie gives as a clear case of an internally problematic potential essence. Fairchild (2019, p. 157), following Karen Bennett (2004, p. 357), offers another example of the sort of thing that Leslie presumably has in mind: a potential essence that includes the property of being blue but not the property of being colored.

Leslie does offer another example, but she expresses tentativeness about it (279). And it is difficult to know just what to make of the example. It is a case in which a postulated entity is supposed to lack (and so not have essentially or accidentally) a property that the original entity has. The postulated entity—an

metaphysical principles with "principles of limited variety" (perhaps better named *principles of variety limitation*), which impose substantive limits on the variety of objects that there are. Although general metaphysical principles do impose *some* limits on the variety of objects that there are (prohibiting essentially blue but accidentally extended things), the limits are in some intuitive sense trivial, not substantive. The heart of plenitudinarianism is to reject substantive metaphysical principles.[9]

Let a *modal profile M based on a set of properties S* be a function that assigns to each property in *S* exactly one of the two modal classifications essential and accidental. A modal profile specifies a corresponding property—namely that of having each *M*-argument property in the *M*-value way. I say that *an object has a given modal profile* iff it has this corresponding property. In this terminology, it is reasonable to render plenitudinarianism as follows, using 'coherent' to apply to a modal profile that is "internally unproblematic".

> For any material object *o* and any modal profile *M* based on any set of *o*'s properties, if *M* is coherent, then there is a material object coincident with *o* that has *M*.

This formulation of plenitudinarianism respects two important constraints: a *mundane-fact constraint* and a *coherence constraint*. Since it starts, so to speak, from a given material object, there is no risk that it will wind up postulating objects, like pink elephants in my study, that are at odds with the mundane facts. There is also no risk of the view careening toward a Meinongian plenitudinarianism, full of incoherent objects: things essentially blue and accidentally extended, things accidentally self-identical, and things essentially accidentally *F*.[10] The view as stated does not however respect an *overall compatibility constraint*. Given that there is an object *o* that is blue, the view is committed to an object that is essentially identical to *o* and accidentally blue as well as to an object that is essentially identical to *o* and essentially blue. But only one object can be essentially identical to *o*, and it cannot be both accidentally and essentially blue. So, this view is inconsistent with the fact that there are blue things. This troubling result is par for the course, if Maegan Fairchild (2019) is correct, as I think she is, that straightforward attempts to

Footnote 8 continued
entity that is coincident with Socrates at some time when he is sitting and that is essentially sitting—lacks, according to Leslie, the property of being a self-maintaining living thing because nothing coincident with Socrates is striving to maintain a seated position. So, she judges internally problematic a potential essence that includes the property of being a self-maintaining living thing and the property of sitting. My speculation about this example is that the kind *self-maintaining living thing* is such that it can be instantiated by at most one of a plenitude of coincident objects. The kinds *statue* and *lump* are also plausibly like this. (This would, incidentally, provide a reason why the plenitudianrian does not postulate a *plenitude* of entities having a certain subclass of the relevant properties essentially and the rest accidentally. See note 5.) For properties like this, the plenitudinarian faces a choice as to whether the object that has the property has it essentially or accidentally.

[9] *Cf.* Bennett (2004, 356–357) on the "chaste two-thinger" *vs.* the "wild bazillion-thinger".

[10] Suppose *o* is essentially accidentally *F*. Then *o* is accidentally *F* in every possible world (in which *o* exists). If so, then *o* is *F* in every possible world (in which *o* exists). But then *o* is essentially *F*, hence not accidentally *F*, hence not essentially accidentally *F*.

provide a satisfactory formulation of plenitudinarianism are either inconsistent (with given facts) or fail to capture the target idea.[11]

Fortunately, this result need not detain an investigation of plenitudinarian solutions to the Paradox of FlexOE. Every property base of concern is one for which no incompatibility results from the supposition that there is an object for each of the coherent modal profiles based on it. In what follows, I assume the truth of plenitudinarianism as restricted to the property bases I discuss.

A final preliminary remark is in order. The standard pluralist position assumes a tight connection between *sortal kinds* (for want of a better term) and modal properties: for example, statues but not lumps have their rough shape properties essentially. If the plenitudinarian extrapolates in the natural way, coincident objects with different modal profiles are of different sortal kinds. However, Leslie typically assumes that all coincident objects are of the same sortal kind—for example, that all things coincident with an axe are axes. She occasionally hedges—once saying "ships (or ship-like entities)" (281) and once "axe (or axe-like entity)" (290)—perhaps to remain neutral concerning the sortal kinds to which coincident objects belong. To stave off this issue until more pressing, let *L-axe* cover anything coincident with an axe, including an axe.

2 Paradox

Nathan Salmón (2005/1986), following Hugh Chandler (1976), takes the Paradox of FlexOE to pose a challenge to the claim that whatever is possibly possible is possible (or equivalently to the claim that whatever is necessary is necessarily necessary). Trading the idiom of modal operators for the idiom of worlds (that is, maximal ways for things to be)[12] and the accessibility relation (that is, the relation of being *metaphysically possible according to*), the envisioned challenge is to the claim that the accessibility relation between worlds is transitive. (I take the trade in idioms to yield analytic equivalences.) This claim, ***Trans***, is enshrined as not only true, but true *as a matter of logic*, in the most widely accepted system of modal

[11] Fairchild (2019) offers her own formulation of plenitudinarianism to avoid these results, but it is not clear that it can generate the objects needed for Leslie's diagnosis of the Paradox of FlexOE. The problem arises from the fact that Fairchild's formulation restricts the properties that may serve as arguments to a modal profile function. It thereby evidently fails to acknowledge the distinct objects on which Leslie's diagnosis depends—two objects that differ in how they have a property that Fairchild bans. (There may be ways for Fairchild to respond, but treatment of this topic goes beyond the scope of this paper.) It is worth mentioning that any restriction on the properties that may serve as arguments to a modal profile function is antithetical to the thought that L-essentialism guarantees plenitudinarianism. More importantly, restrictions are antithetical to plenitudinarianism's inclusive spirit, and accordingly any restriction must be exceptionally well motivated from the very limited exclusionary principles that the spirit of plenitudinarianism allows.

[12] One way for things to be, such that nothing can, as a matter of logic, be that way, is for contradictions to be true. (Logically impossible things can be true according to worlds just as they can be according to stories.) Such worlds are logically impossible worlds. For the purposes of metaphysical modality, we can ignore them. So no harm is done in the present context by taking 'world' to mean *logically possible way for things to be*.

logic, *S5*, which is characterized by the claim that it is logically true that the accessibility relation is an equivalence relation. The weaker system *S4*, characterized by the claim that it is logically true that the accessibility relation is reflexive and transitive, also counts Trans true as a matter of logic. But the other contenders for the correct logic for metaphysical modality, *B* and *T*, do not. *B* is characterized by the claim that it is logically true that the accessibility relation is reflexive and symmetric while *T*, the weakest of the four systems, is characterized by the claim that it is logically true that the accessibility relation is reflexive.[13] (I here use the phrases 'true as a matter of logic', 'truth of logic', and the like interchangeably. I will say that it is true as a matter of logic that all bachelors are unmarried even though there are models in which some object is in the extension of both 'is a bachelor' and 'is married': such models are inadmissible since they are inconsistent with the very notion of bachelorhood.)

Since it is clear that it is true as a matter of logic that the accessibility relation is reflexive (that is, it is clear that it is logically true that whatever is true is possible, or equivalently that whatever is necessary is true), the correct logic for metaphysical modality is at least as strong as *T*. It is perfectly intelligible to regard *T* and not *S5* as the correct logic for metaphysical modality while nonetheless holding that the accessibility relation is an equivalence relation. This is analogous to thinking that since classical logic says that it is a *truth of logic* that there is something rather than nothing, it is not the correct first-order logic, while nonetheless holding that it is *true* that there is something rather than nothing.

Salmón (2005/1986, 277) formulates the Paradox of FlexOE using these three principles (though I have changed their names).

> *FlexO*$_S$ (*Salmón's Flexibility of Origin*): If a wooden table x is the only table originally formed from a hunk of matter y according to a certain plan P, and y' is any (possibly scattered) hunk of matter that sufficiently substantially overlaps y and has exactly the same mass, volume, and chemical composition as y, then x is such that it might have been the only table originally formed according to the same plan P from y' instead of from y.
>
> *OE*$_S$ (*Salmón's Origin Essentialism*): If a wooden table x is the only table originally formed from a hunk of matter y, and z is any hunk of matter that does not sufficiently substantially overlap y, then x is such that it could not have been the only table originally formed from z instead of from y.
>
> *NecifTrue*$_S$: FlexO$_S$ is such that if it is true at all, it is necessarily so.

The thrust of the derivation is straightforward. (I suppress a host of details that detract from conveying it.) Suppose t is the only table made from h_0. Then, by FlexO$_S$, it could have been the only table made from h_1, where h_1 differs only slightly from h_0. But had it been the only table made from h_1, as it could have been, then, by the necessity of FlexO$_S$, which follows from FlexO$_S$ and NecifTrue$_S$, it would have been that it could have been the only table made from h_2, where h_2

[13] See (Kripke 1963).

differs only slightly from h_1 (in such a way that h_2 differs slightly more from h_0). That is, it is possibly possible that t is the only table made from h_2. Continuing in this way, we derive that it is possibly^{n-1} possible (with attached superscript indicating the number of repetitions of a word) that t is the only table made from h_n, where h_n has no matter in common with h_0.[14] In *S4* (and hence *S5*) it follows that it is possible that t is the only table made from h_n. This also follows in *T*, if we take Trans as an additional premise. But, by $\mathrm{OE_S}$, it is not possible that t is the only table made from h_n. This is the Paradox of $\mathrm{FlexOE_S}$, where $\mathrm{FlexOE_S}$ is the conjunction of $\mathrm{FlexO_S}$ and $\mathrm{OE_S}$.

According to the Chandler-Salmón solution to the paradox, Trans is not true, let alone true as a matter of logic. Arguably, David Lewis (1986) also advocates this solution. It is uncontroversial that Lewis is committed to the truth of the sentence 'It is possibly^{n-1} possible that t is made from h_n, but it is not possible that t is made from h_n'. This is because he analyzes this sentence in terms of his counterpart relation, which is not transitive. According to Lewis, there is a counterpart-of-a^{n-1} counterpart of t that is made from h_n, but there is no counterpart of t that is made from h_n. (Here I make use of the contextually salient counterpart relation.) Insofar as disquotation is legitimate in Lewis's case, this commits him to the claim that it is possibly^{n-1} possible that t is made from h_n but it is not possible that t is made from h_n.[15]

Although Leslie offers direct comment on Salmón's $\mathrm{FlexO_S}$, she engages primarily with a simplified version of the paradox. Exchanging Salmón's table for an axe, Leslie supposes that axes are made from exactly three basic parts—blades, shafts, and handles. She further supposes that the axe that common sense recognizes could be made from any axe-kit that has at least two of its three parts in common with its original axe-kit and could not be made from any axe-kit that does not. No harm is done here by treating these suppositions as tantamount to the supposition that this axe could be made from any matter that overlaps its original matter by at least 2/3 (where only matter of the same mass etc. count as overlapping at all) and could not be made from anything else.

I now reformulate the paradox in a way that is congenial to Leslie's discussion. Let 'Ax' say that x is an axe, 'Mxh' say that x is originally made from h, and 'Oh'h' say that h' overlaps h by at least 2/3. Consider these variations on $\mathrm{FlexO_S}$ and $\mathrm{OE_S}$. (The variable 'x' ranges over material objects; 'h' and 'h'' over hunks of matter.)

FlexO: $(\forall x)(\forall h)(\forall h')[(\mathrm{A}x \;\&\; \mathrm{M}xh \;\&\; \mathrm{O}h'h) \rightarrow \Diamond \mathrm{M}xh']$
or equivalently,
$(\forall x)[\mathrm{A}x \rightarrow (\forall h)(\mathrm{M}xh \rightarrow (\forall h')[\mathrm{O}h'h \rightarrow \Diamond \mathrm{M}xh'])]$

[14] Continuing in this way requires the necessary^{n-2} necessity of $\mathrm{FlexO_S}$. In *S4* (and hence *S5*) this follows from $\mathrm{FlexO_S}$ and $\mathrm{NecifTrue_S}$. Even one who thinks that Trans is not true (let alone logically true) may hold that $\mathrm{FlexO_S}$ is necessarily^{n-2} necessary. It is plausible that for a restricted class of claims that includes $\mathrm{FlexO_S}$, whatever is necessary is necessarily necessary.

[15] *Cf.* (Robertson Ishii 2014).

OE: $(\forall x)(\forall h)(\forall h')[(\mathrm{A}x\ \&\ \mathrm{M}xh\ \&\ \sim \mathrm{O}h'h) \rightarrow\ \sim\Diamond \mathrm{M}xh']$
or equivalently,
$(\forall x)[\mathrm{A}x \rightarrow (\forall h)(\mathrm{M}xh \rightarrow (\forall h')[\sim \mathrm{O}h'h \rightarrow\ \sim\Diamond \mathrm{M}xh'])]$

A notable way these principles differ from Salmón's is in making mention of the relevant kind only in the antecedent but not the consequent (of the universally quantified claims). Another way they differ is in using an overlap predicate that is clearly not vague.[16] We may therefore use two-valued logic, which allows succinct combination of FlexO and OE.

***FlexOE*: $(\forall \boldsymbol{x})[\mathbf{A}\boldsymbol{x} \rightarrow (\forall \boldsymbol{h})(\mathbf{M}\boldsymbol{xh} \rightarrow (\forall \boldsymbol{h'})[\Diamond \mathbf{M}\boldsymbol{xh'} \leftrightarrow \mathbf{O}\boldsymbol{h'h}])]$**

FlexOE says that any axe has a *modal tethering property* I call Mod_{orig}.

- $\mathrm{Mod}_{\mathrm{orig}}$
  - Being something whose origin (im)possibilities are tethered to its original matter
  - More explicitly, being something that is possibly made from all and only those hunks of matter that overlap its original matter by at least 2/3
  - Symbolically, $(\lambda x)[(\forall h)(\mathrm{M}xh \rightarrow (\forall h')[\Diamond \mathrm{M}xh' \leftrightarrow \mathrm{O}h'h])]$
  - Alternatively, $(\lambda x)[(\forall h)(\Diamond \mathrm{M}xh \leftrightarrow \mathrm{O}hm(x))]$, where $m(x) =_{\mathrm{df}} (\iota h)\mathrm{M}xh$[17]

In place of $\mathrm{NecifTrue_S}$, which says that $\mathrm{FlexO_S}$ is necessary if true, we have an analogous claim concerning FlexOE.

***NecifTrue*: FlexOE $\rightarrow \Box$FlexOE**

Although we derived a contradiction from $\mathrm{FlexO_S}$, $\mathrm{OE_S}$, and $\mathrm{NecifTrue_S}$ using *S4* (and from these plus Trans using *T*), we will hit a roadblock if we try to derive a contradiction from FlexOE and NecifTrue using *S4* or from these plus Trans using *T*. Let's see how, doing the derivation this time around using the idiom of worlds and the relation of one world's being possible according to (or *in*) another. (It may bear emphasizing that to say that something is true *in* a world is just to say that it is true *according to* that maximal way for things to be. So, since one maximal way for things to be may be possible according to another, one world may be possible in another. If I say, '*a*, could, in the actual world, be made from h_1' that means that according to the actual world, *a* could be made from h_1—that is, that according to the actual world, it is possible that *a* is made from h_1. Unless of course, my 'could' was epistemic.)

[16] Salmón's overlap predicate is 'sufficiently substantially overlaps'. Sufficient for what? For making $\mathrm{FlexO_S}$ true. One may think that this is a vague matter. Or one may agree with Salmón (2005/1986, 343–344) that it is not. His formulations were intended to leave this an open matter.

[17] Use of the functor '*m*' is legitimate since '$(\forall x)(\exists h)[\mathrm{M}xh\ \&\ (\forall h')(\mathrm{M}xh' \rightarrow (h' = h))]$' is an analytic truth (given the ranges of '*x*', '*h*', and '*h*''). This alternative λ-abstract is easier to compare to the ones introduced in Sect. 3. Note that '$m(x)$' is non-rigid whereas 'h_0' and the like are rigid.

Suppose that in the actual world, w_0, a is an axe originally made from h_0. Call this the *initial condition*. It follows from this by FlexOE (in the right to left direction of the biconditional), that there is a possible world, w_1, in which a is made from h_1, where h_1 overlaps h_0 by 2/3. But it does not follow that a is an axe in w_1, since the kind axe is not involved in the consequent of FlexOE's immediate subformula. This means that modal universal instantiation of □FlexOE to 'a' cannot take us where we want to go. We can get there if we have that a is an axe in w_1. The kind essentialist claim that any axe is essentially an axe can get us that.

***KindEss*: $(\forall x)(\mathbf{A}x \rightarrow \Box[(\exists y)(y = x) \rightarrow \mathbf{A}x])$**

It follows from the initial condition by KindEss that in all possible worlds, and hence in w_1, a is an axe if it exists. And so, in w_1, a is an axe made from h_1. (Here, we obviously needed that a exists in w_1. An uncontroversial premise gives us that: $(\forall x)(\forall h)\Box[\mathrm{M}xh \rightarrow (\exists y)(y = x)]$. This is one of the details I suppressed in the previous derivation.And for simplicity, I assume that in all relevant worlds the same hunks of matter exist and the same overlap relations obtain. I now resume suppressing finicky details.) Now, appealing to the claim that FlexOE is true in all possible worlds (that is, to □FlexOE) is apt. It delivers that there is a world, w_2, possible in w_1, which is itself possible in w_0, in which a is made from h_2.[18] So by Trans, w_2 is a world that is possible in w_0. But it follows from the initial condition by FlexOE (in the left to right direction of the biconditional) that there is no world such as w_2—that is, that there is no world that is possible in w_0 in which a is made from h_2. Thus we derive a contradiction from FlexOE, KindEss, NecifTrue, and Trans.[19]

3 Diagnosis

According to Leslie, there is no real paradox here but only the illusion of one.

> [A] long-standing paradox of essentialism is easily resolved once the connection between essentialism and plenitude is appreciated. … The

[18] We can get to this point with a principle less robust than KindEss. Go back to the point when we discovered that modal universal instantiation of □FlexOE to 'a' cannot take us where we want to go. We can get there if we have that there is a world, w_1', possible in w_0, in which a is made from h_1 and is an axe. An extremely weak principle of kind retention, one that takes us from w_1 to w_1' so to speak, can get us that: $(\forall x)(\mathrm{A}x \rightarrow (\forall h)[\Diamond \mathrm{M}xh \rightarrow \Diamond(\mathrm{M}xh \,\&\, \mathrm{A}x)])$. Now, appealing to the claim that FlexOE is true in all possible worlds (that is, to □FlexOE) is apt. It delivers that there is a world, w_2, possible in w_1', which is itself possible in w_0, in which a is made from h_2.

[19] By in effect using *T* and Trans rather than *S4* alone at the crucial point, the derivation is one that all parties agree is legitimate. This also makes the menu of options simple: instead of being able to reject an inference or reject a premise, one can only reject a premise. This is fitting, since even one who is convinced, as I am, that Trans is no truth of logic, may nonetheless think that it is true, and so may still find the paradox of interest. I would be remiss however not to point out that the derivation makes use of inferential rules that are not part of *T* propositional modal logic. It uses universal instantiation—in standard and modal versions. Salmón (2005/1986, 297, note 12) points out that two well-known versions of counterpart theory fail to validate at least one instance of modal universal instantiation, the inference from '$\Box(\forall x)\phi_x$' to '$\Box[(\exists x)(x = \alpha) \rightarrow \phi_\alpha]$', which is valid in standard quantified modal logic.

> paradoxical element, I argue, is illusory, and arises only because two or more distinct entities are mistakenly thought to be identical. (278)
>
> [T]hese paradoxes take us in because of an illusion of singularity. … We actually don't have a paradox, however. What needs to be shown … is that there is only one axe [not a plenitude of axes] in [w_1] and this is far from obvious … (287)

Since she thinks that there is no paradox, Leslie does not so much try to offer a solution to it as try to provide a diagnosis of why we mistakenly think that there is one.

In order to set out this diagnosis, it will be helpful to introduce three additional modal tethering properties.

- Mod_0
  - Being something whose origin (im)possibilities are tethered to h_0
  - More explicitly, possibly being made from all and only those hunks of matter that overlap h_0 by at least 2/3
  - Symbolically, $(\lambda x)[(\forall h)(\Diamond \text{M}xh \leftrightarrow \text{O}hh_0)]$
- Mod_1: as for Mod_0 but replacing all occurrences of 'h_0' with occurrences of 'h_1'
- Mod_2: as for Mod_0 but replacing all occurrences of 'h_0' with occurrences of 'h_2'

The difference between the general property Mod_{orig} and these three specific properties (specific with respect to the matter involved) is like the difference between the general property of being the wife of the U.S. president and the three specific properties, being the wife of Donald Trump, being the wife of Barack Obama, and being the wife of George W. Bush. Just as it is a conceptual truth that if Trump (Obama, Bush) is the U.S. president then someone is the wife of Trump (Obama, Bush) iff she is the wife of the U.S. president, so it is a conceptual truth that if something is made from h_0 (h_1, h_2) then it has Mod_0 (Mod_1, Mod_2) iff it has Mod_{orig}.

Leslie does not put her points in terms of these modal tethering properties, but instead in terms of essences.

> Essence0: properties P, Q, R … and the property of being originally made of [at least] 2 out of 3 of BladeA, ShaftA, HandleA plus an appropriate third part as needed. (287, with changes of labeling)

Leslie intends us to understand from her display of "Essence0" that anything having this essence is modally somewhat flexible as to its material origins—in particular, she intends us to understand that anything having this essence could be made from all and only matter that overlaps BladeA, ShaftA, and HandleA by at least 2/3. This is a modal tethering property. These are the kinds of properties that are important to Leslie's diagnosis. For this reason, I forego Leslie's talk of essences, adopting instead talk of modal tethering properties. (Essence0 is not a modal essence. If it were, it would tell us only that anything having it *could not* be made from matter that does not overlap BladeA, ShaftA, and HandleA by at least 2/3. It would not tell

us much about what such an object *could* be made from—not even that it could be made from AAA. Plausibly, Leslie is giving a definitional essence.)

Here is Leslie's take (in my terms) on the Paradox of FlexOE. In the actual world, w_0, h_0 is used to make an axe, a, an object that is but one of a plenitude of coincident L-axes. Among this plenitude, there is a L-axe, a_0, that has Mod_0. Also among the plenitude is a L-axe, a_1 that has Mod_1. Leslie identifies a, the axe with which the Paradox of FlexOE is concerned, with a_0. According to Leslie, both a_0 and a_1 exist in w_1 (that is, in w_1 of the derivation at the end of Sect. 2, so to speak). Although a_0 is made from h_1 in w_1, it nonetheless retains Mod_0 (the specific modal tethering property it has in w_0) and thus could not be made from h_2. In w_1, a_1 is also made from h_1, and it retains Mod_1 (the specific modal tethering property it has in w_0). Our commonsensical belief that there is only one axe in w_1 (and in w_0, for that matter) keeps us from distinguishing L-axes a_0 and a_1. When we rightly see that in w_1, a_1 has Mod_1, we wrongly think that in w_1, a_0 has Mod_1, and so conclude that there is a world, w_2, possible in w_1, in which a_0 (rather than a_1) is made from h_2. So instead of concluding correctly that it is possibly possible that a_1 is made from h_2, we wrongly conclude that it is possibly possible that a_0 is made from h_2. So although the "possibility deletion" sanctioned by Trans is legitimate, instead of concluding rightly that it is possible that a_1 is made from h_2, we wrongly conclude that it is possible that a_0 is made from h_2. And thus, given that FlexOE dictates that it is not possible that a_0 is made from h_2, we wrongly find a contradiction (in the claims that it is possible that a_0 is made from h_2 and it is not possible that a_0 is made from h_2) where there is none (but only the non-contradictory claims that it is possible that a_1 is made from h_2 and it is not possible that a_0 is made from h_2). In this sense then, the "illusion of singularity" (287) causes us to think that there is a paradox when in fact there is none.

Leslie offers no explanation of why she thinks we are prone to confusing a_0 and a_1 in w_1 (rather than being prone to confusing a_0 with another of the many coincident L-axes in w_1). Consideration of the general property Mod_{orig} suggests an explanation. Think first about the actual world, w_0. In w_0, a_0 is a *principled* L-axe: it has Mod_{orig}. In w_0, a_1 is *unprincipled*: it lacks Mod_{orig}. It could be made from some matter that overlaps its original matter (which in w_0 is h_0) by 2/3 (like h_1) but not from other such matter (like h_{-1}, so to speak). And it could be made from some matter that overlaps its original matter by only 1/3 (like h_2), but not from other such matter (like h_{-2}, so to speak). Now think about the possible world, w_1. In w_1, a_0 is an *unprincipled* L-axe: it lacks Mod_{orig}. It could be made from some matter that overlaps its original matter (which in w_1 is h_1) by 2/3 (like h_0) but not from other such matter (like h_2). And it could be made from some matter that overlaps its original matter by only 1/3 (like h_{-1}, so to speak), but not from other such matter (like h_3). In w_1, a_1 is *principled*: it has Mod_{orig}. Whereas a_0 is principled in w_0, it is unprincipled in w_1. Whereas a_1 is unprincipled in w_0, it is principled in w_1. With respect to any world, the only L-axe that common sense recognizes is a principled one—one that has Mod_{orig}. Since common sense takes an interest only in L-axes that have Mod_{orig}, and since a_0 is that L-axe in w_0, and a_1 is that L-axe in w_1, we confuse them.

4 From diagnosis to solutions

Leslie tells us that there is no paradox, yet at the end of Sect. 2, we went through an argument that shows that the principles FlexOE, KindEss, NecifTrue, and Trans (together with some innocuous claims) are jointly inconsistent. I repeat the principles for ease of reference.

FlexOE: $(\forall x)[\mathrm{A}x \rightarrow (\forall h)(\mathrm{M}xh \rightarrow (\forall h')[\Diamond \mathrm{M}xh' \leftrightarrow \mathrm{O}h'h])]$
KindEss: $(\forall x)(\mathrm{A}x \rightarrow \Box[(\exists y)(y = x) \rightarrow \mathrm{A}x])$
NecifTrue: FlexOE $\rightarrow \Box$FlexOE
Trans: any instance of $\ulcorner \Box\phi \rightarrow \Box\Box\phi \urcorner$ or equivalently $\ulcorner \Diamond\Diamond\phi \rightarrow \Diamond\phi \urcorner$

Supplementing an inconsistent set with plenitudinarianism will not result in a consistent set. A paradox does not "disappear" (290) by *adding* a claim. What is going on? Perhaps Leslie has succumbed to the telescope view of possible worlds that Kripke inveighs against in *Naming and Necessity*.

> Don't ask: how can I identify this table in another possible world, except by its properties? I have the table in my hands, I can point to it, and when I ask whether *it* might have been in another room, I am talking, by definition, about *it*. I don't have to identify it after seeing it through a telescope. If I am talking about it, I am talking about *it*. (52–53)

We do not look at w_1 and mistake one L-axe for another. We perform (modal) universal instantiation on (the necessitation of) FlexOE and on KindEss. As long as we do not switch instantiating constants midstream, there is no danger that we will lose track of our axe. As Kripke would say, we have it in our hands! Thus we have the legitimate derivation of a contradiction from the principles involved (together with innocuous premises). *Something must give.* The plenitudinarian has the same menu of options for solving the paradox as anyone else has—reject something as untrue. (See note 19.) Plenitudinarianism per se can do nothing to solve the paradox. At best, the plenitudinarian has additional resources for explaining why a claim that is not true seems true. In this section, I examine three plenitudinarian solutions to the paradox that take their cue from Leslie's diagnosis. (For simplicity, I write as though in each world there are no L-axes other than those that are coincident with the axe of our concern.)

4.1 Reject FlexOE

Leslie typically writes as though anything coincident with an axe is an axe, so it is tempting to think that her solution (were she to acknowledge the paradox) would simply be to reject FlexOE. The idea would be that although among the plenitude of axes in the actual world there is at least one (for example, a_0) that has $\mathrm{Mod}_{\mathrm{orig}}$, there are others (for example, a_1) that do not, so that FlexOE—which makes the general claim that all axes have $\mathrm{Mod}_{\mathrm{orig}}$—is not in fact true. The temptation to think that FlexOE is true is perhaps explainable by the fact that we commonsensically recognize exactly one axe, a_0, from among the plenitude of axes that exist in w_0, so that it is natural that we think that *all* axes—all one of them by the lights of common sense—have $\mathrm{Mod}_{\mathrm{orig}}$.

This solution is unattractive. First, it is at odds with common sense about how many axes there are. Common sense is firm in its judgment that there is exactly one *axe* in w_0. Common sense has no judgment about whether there is a plenitude of *entities* coincident with the axe. There is a significant difference between a view, like plenitudinarianism about coincident *axes*, that denies a pronouncement of common sense and a view, like plenitudinarianism about coincident *entities*, that merely affirms something about which common sense is silent.

Second, it severs the expected connection between kinds and their characteristic modal properties. The standard pluralist rightly distinguishes statues from lumps. So too the plenitudinarian should distinguish between axes and a plenitude of other kinds of things that are coincident with axes. Plenitudinarianism should go hand in hand with a plenitude of kinds—each with its characteristic modal properties. A L-axe that could be made only from its original matter or matter that does not overlap that matter at all is not an axe. Ditto a L-axe like a_1 (in w_0).

Finally, it is completely ineffective against another version of the paradox that concerns *schmaxes* rather than axes, where the kind schmax is partially characterized as a kind all of whose instances have Mod_{orig}—in the way that an in-statue is partially characterized as a kind all of whose instances are essentially indoors. FlexOE for schmaxes is trivially necessarily true. So the present solution is basically a non-starter—at best something that works for axes but not for schmaxes.[20]

4.2 Reject KindEss

The problems for the solution of Sect. 4.1 lead naturally to the solution of Sect. 4.2. This solution agrees with common sense that a_0 is the only axe in w_0. It has the additional virtue of respecting (the necessity of) the standard pluralist thought that being an instance of a certain kind goes hand in hand with having certain modal properties—and of respecting it in the natural way, saying that being an axe goes hand in hand with having Mod_{orig}. In doing this, it in effect acknowledges the force of the schmax objection of Sect. 4.1. According to this solution, a_0 is the only axe/schmax in w_0, but if it had been made from some matter distinct from h_0 that overlaps h_0 by 2/3, then it would not have been an axe/schmax at all but instead something coincident with it (like a_1 in w_1) would have been. Similarly, on this solution, a_1 is the only axe/schmax in w_1, but if it had been made from matter distinct from h_1 that overlaps h_1 by 2/3, then it would not have been an axe/schmax

[20] In case this sort of point has lost familiarity, I offer a quotation from Kripke (1980/1972, 108). He is arguing against the view that identity is a relation between two names that designate the same object rather than a relation between an object and itself. "If anyone ever inclines to this particular account of identity, let's suppose we *gave* him his account. Suppose identity *were* a relation in English between the names. I shall introduce an artificial relation called 'schmidentity' (not a word of English) which I now stipulate to hold only between an object and itself. Now then the question whether Cicero is schmidentical with Tully can arise, and if it does arise the same problems will hold for this statement as were thought in the case of the original identity statement to give the belief that this was a relation between the names. If anyone thinks about this seriously, I think he will see that therefore probably his original account of identity was not necessary, and probably not possible, for the problems it was originally meant to solve, and that therefore it should be dropped, and identity should just be taken to be the relation between a thing and itself. This sort of device can be used for a number of philosophical problems."

at all but instead something coincident with it would have been. The temptation to think that KindEss is true is perhaps explainable by the fact that every axe/schmax is (necessarily) essentially coincident with something that is an axe/schmax.

This solution rejects not just KindEss, but even the weaker principle of kind retention (mentioned in note 18).

KindRet: $(\forall x)(\mathrm{A}x \rightarrow (\forall h)[\Diamond \mathrm{M}xh \rightarrow \Diamond(\mathrm{M}xh \ \& \ \mathrm{A}x)])$

It is a strike against this solution that it denies a principle as weak and as plausible as this. It saves (the necessity of) FlexOE at the expense of an essentialist claim with more intuitive plausibility. This is a heavy cost. But at least this response is a starter: the axe/schmax objection does not apply to it.

4.3 Reject NecifTrue and hence the necessity of FlexOE

On this solution, a_0 is an axe in w_0, and remains so in w_1. Material objects are of their "sortal kinds" essentially. However, although it is true in the actual world that all axes have $\mathrm{Mod}_{\mathrm{orig}}$, this claim is not true in w_1, where a_0 lacks that property. The temptation to think that FlexOE is necessary is perhaps explainable by the fact that with respect to any world, common sense notices only L-axes that have $\mathrm{Mod}_{\mathrm{orig}}$. In w_0, the L-axe we notice, a_0, has $\mathrm{Mod}_{\mathrm{orig}}$ and is indeed an axe, but in w_1, the L-axe we notice, a_1, has $\mathrm{Mod}_{\mathrm{orig}}$ but is not an axe. We mistakenly think that with respect to any world, the L-axe we notice is an axe.

John Hawthorne (2006) is attracted to this solution, and he offers an interesting account of our (alleged) mistake.

> [I]n the vicinity of the statue there are many objects, some [of which are] candidate referents for 'that statue'. Objects that are completely origin indifferent are prohibited, as are objects that could not have been made from a slightly different block. Take a candidate *C* that can, we might suppose, be made of a block that is 5 percent different from the actual one but no more. Consider a world where it is made of a block that is 5 percent different. Won't people at that world make mistakes when judging what changes are possible with regard to the statue's origins? Not if they refer to something else (something less fragile as it were) by 'that statue'. Of course, assuming that statues are necessarily statues and that S4 is correct, it still turns out not to be necessarily true that if a statue *s* is made of some quantity of matter *m*, then, for any variant *v* of *m*, it could have been made of *v*. Yet interestingly, given plenitude (as true and a priori), it may yet be a priori (contingently a priori) that if a statue exists and is made of some quantity of matter *m*, then, for any variant *v* of *m*, it is possible that it be made of *v*. Such a package strikes me as quite attractive. (241, note 8)

Hawthorne is not discussing (a version) FlexOE per se, but rather just (a version of) one of its logical consequences, FlexO. This nominal mismatch in our concerns is an artifact of my having combined FlexO and OE into a single thesis, FlexOE. For continuity with the present paper, in discussing this passage, I switch from

Hawthorne's statue to our axe in the obvious way. (Warning to the reader: The first things I say may annoy, but they lay groundwork for explicating the interesting view that I suspect is Hawthorne's. Please bear with me.) Hawthorne asks us to consider a world in which a particular axe is made from matter that is 1/3 different from the matter that actually originally constituted it, then poses a question. His answer suggests that the people at the merely possible world do not even concern themselves with the axe (and thus have no opportunity to make mistakes about its origin (im)possibilities), but instead concern themselves only with what they call 'that axe', assessing its origin (im)possibilities accurately.

When one says that there is a possible world in which such and such happens, then asks one's audience to consider a world in which it does, one is typically performing existential instantiation—basically, assuming an *instance* of the existentially quantified claim with which one began in the hopes of showing that something follows from it, so that, if the instance was "arbitrarily chosen", one can draw the conclusion from the existentially quantified claim itself. This seems to be what Hawthorne is doing—aiming for the conclusion that in any possible world 'if an axe *a* is made of some quantity of matter *m*, then, for any variant *v* of *m*, it could have been made of *v*' expresses something true. Or, to put the point in the terms of the present paper, he seems to be aiming for the conclusion that in any possible world 'any axe has Mod_{orig}' expresses something true. If so, then when Hawthorne asks us to consider a world in which some particular axe is made from matter that is 1/3 different, he must in effect be asking us to consider *any* such world. But there will be lots of worlds like that in which no language resembling English is spoken and in which no word like 'axe' is used. To put the point in terms of existential instantiation, the world that Hawthorne discusses is not "arbitrarily chosen".

Hawthorne seems to think that if a_0 were made from slightly different matter, as it could have been, then the word 'axe' would have meant something different than it does. And he seems to think that in that case—for example, in w_1—it would not be true that any axe has Mod_{orig} (since a_0 would be an axe that lacks it), but it would be true that 'any axe has Mod_{orig}' expresses something true, thanks to the fact that 'axe' would mean something different (something that applies to a_1, which does have Mod_{orig}). The view that 'axe' would change its meaning if some axe were made from slightly different matter is implausible on its face. It would be very surprising if in the closest possible world in which some axe in Japan is made from slightly different matter, the meaning of the English word 'axe' is different.

Although the passage is primarily concerned with *language*, toward the end this concern has disappeared. The claim there is not the expected one—namely that in any possible world *the sentence* 'any axe has Mod_{orig}' expresses something true. Instead Hawthorne says that *the claim* that any axe has Mod_{orig} is a case of the contingent a priori. The former claim has no plausibility. The latter has more. Language is not irrelevant here—and in particular reflection on the semantics of 'actually' permits a plausible guess as to the view that attracts Hawthorne.

I suspect it is the view that 'axe' is semantically equivalent to something like 'axe-like entity whose origin (im)possibilities are tethered to its *actual* origin'. Let me explain what I have in mind by the puzzling and circularity-threatening phrase 'axe-like entity', which is *not* intended to be synonymous with 'L-axe'. According to plenitudinarianism,

coincident with any axe, there are at least two L-axes that have Mod_{orig}: one of these is the axe itself, which has (say) the property of being owned by Bernie Sanders accidentally; another of these is a L-axe that would cease to exist were Sanders to give his axe to Joe Biden. The linguistic conjecture I am offering on Hawthorne's behalf is that 'axe' is semantically equivalent to a phrase that would apply to the axe (which is accidentally owned by Sanders), but not to a L-axe that is essentially owned by Sanders.

On this view 'axe' is indexical: it has a meaning (in the sense of character, which is a function from contexts of potential utterance to semantic contents) that dictates that it expresses different meanings (in the sense of semantic contents) relative to different contexts. If this is the view that Hawthorne is after, then the first part of the passage—in which Hawthorne is talking about the linguistic facts that obtain in a merely possible world—may turn on confusing a sentence's being true *in (that is, according to) a possible world* and its being true *relative to a context*, where the salient feature of the context is the possible world it includes. The *sentence* 'the first child actually born in the twenty-second century (if such there be) is the first child born in the twenty-second century' is not true *in every possible world*, since, for example, there are worlds, very similar to our own, in which the word 'actually' expresses *erroneously alleged to be*. Nor is it even true *with respect to every possible world*: the sentence in the language that we (actually) speak expresses a proposition that is contingent, not necessary. Nonetheless the sentence is true *relative to every context*—that is, the sentence in the language that we (actually) speak is such that relative to each context, it expresses a truth.[21]

On this view, the sentence 'any axe has Mod_{orig}' expresses relative to a context that includes the actual world, w_0, the proposition that any *axe-like entity whose origin (im)possibilities are tethered to its original matter in* w_0 has Mod_{orig}. Or, as we might say, it expresses the proposition that any axe_0 has Mod_{orig}. Relative to a context that includes w_1, it expresses the proposition that any *axe-like entity whose origin (im)possibilities are tethered to its original matter in* w_1 has Mod_{orig}. Or as we might say, it expresses the proposition that any axe_1 has Mod_{orig}. And so on: relative to a context that includes w_n, 'axe' refers to the kind axe_n.

Using '$\text{Mod}_{@\text{orig}}$' to *abbreviate* (not simply to name the same thing that is named by) '$(\lambda x)[(\forall h)(@\text{M}xh \rightarrow (\forall h')[\Diamond \text{M}xh' \leftrightarrow \text{O}h'h])]$' (where '@' corresponds to the sentential operator 'actually' in the indexical sense in English), the view I am speculatively attributing to Hawthorne says that the following claim is contingent but knowable a priori: any axe-like entity that has $\text{Mod}_{@\text{orig}}$ has Mod_{orig}. (It is contingent because although in w_0 any axe_0 has Mod_{orig}, that is not so in w_1, where instead any axe_1 has Mod_{orig}.) Compare this with a standard (purported) example of the contingent a priori: the first child actually born in the twenty-second century (if such there be) is the first child born in the twenty-second century. If 'axe' is indexical because it is short for 'axe-like entity that has $\text{Mod}_{@\text{orig}}$', then we have an explanation of our tendency to think that FlexOE (that is, the claim that any axe has Mod_{orig}) is necessary. We think this because it is a case of the contingent a priori, and it is so very easy to think that such cases are cases of necessity.

[21] See (Kaplan 1989).

The view is puzzling. It evidently replaces implausible claims about what happens counterfactually to our language with implausible claims about what happens counterfactually to our interests. On this view, we care about the kind axe_0, but had things gone a little differently—had a_0 been made from h_1 as it could have been—we would have cared about the kind axe_1 instead. (If a version of FlexOE is true for *Homo sapiens*, then if (say) David Kaplan had had a slightly different origin, would we have cared about a different kind? Would that kind have been a species?) Yet according to the view, we are not utterly inconstant: we have an abiding interest in *axe-like entities with* Mod_{orig}. Or, I should say, an abiding interest in *schmaxes*.

Even if we concede that 'axe' is indexical and concomitantly allow that the paradox is concerned with the kind axe_0—a kind in which, even by the lights of the solution on offer, we lack an abiding interest—we may nonetheless consider a version of the paradox involving the kind *schmax*. Like the solution of Sect. 4.1, the present solution is completely ineffective against this version of the paradox. FlexOE for schmaxes is trivially true; moreover it is trivially true in all possible worlds. So the proposed solution is basically a non-starter—at best something that works for "axes" (axe_0s) but not for schmaxes. Again, we find ourselves drawn back to the solution of Sect. 4.2.

There is a further problem. In rejecting NecifTrue, this solution makes an arbitrary and unjustifiable distinction between FlexOE and plenitudinarianism. As we have seen, plenitudinarians help themselves to the claim that their view is necessary if true—taking completely for granted that there is in w_1 a plenitude of L-axes. That is as it should be. Typical philosophical claims are necessary if true. FlexOE should be no exception. Like cases should be treated alike. This is not a mere *tu quoque*. The important point is that it is correct to view FlexOE in exactly the way that the plenitudinarian views plenitudinarianism—as necessary if true.

5 A better solution

There is a better plenitudinarian solution to the Paradox of FlexOE. To pave its way, let's take time to consider some of the various modal profiles the plenitudinarian takes to be instantiated in w_0. For simplicity, I use a toy model in which there are exactly two of each of the three types of axe-parts: an A-version and a B-version. This yields exactly eight relevant "hunks of matter" that could be made into an axe: AAA, AAB, ABA, ABB, BAA, BAB, BBA, and BBB, where the name for each specifies in order the constituent version of blade, shaft, and handle. (I assume that all eight hunks of matter exist in every world and that any object that is material in one world is material in all worlds.) Let's say that h_0 is AAA, h_1 is BAA, and h_2 is BBA. The Paradox of FlexOE takes as its initial condition that in the actual world, w_0, h_0 (aka AAA) is used to make an axe, a. This axe has the *positive property* of being made from AAA, and thus has for each of the seven remaining hunks of matter the *negative property* of *not* being made from that hunk. These eight properties provide the base for some modal profiles that are of interest. (See Table 1.)

Table 1 Some modal profiles based on properties of a in w_0

	1 Incoherent	2 Incoherent	3 Weirdly flexible	4 Principled a_0 a_x	5 Unprincipled a_1
Being made from AAA (h_0)	Essential	Accidental	Accidental	Accidental	Accidental
Not being made from AAB	Accidental	Essential	Essential	Accidental	Essential
Not being made from ABA	Essential	Essential	Essential	Accidental	Essential
Not being made from ABB	Essential	Essential	Essential	Essential	Essential
Not being made from BAA (h_1)	Essential	Essential	Essential	Accidental	Accidental
Not being made from BAB	Essential	Essential	Essential	Essential	Accidental
Not being made from BBA (h_2)	Essential	Essential	Essential	Essential	Accidental
Not being made from BBB	Essential	Essential	Accidental	Essential	Essential

The modal profile represented by column (1) (hereafter *MP(1)*) is incoherent. If it is essential to an object to be made from AAA, then it is also essential to the object not to be made from AAB. And similarly for the remaining negative properties. So any object that has the positive property essentially has all seven negative properties essentially as well. Thus MP(1)—along with 2^7–2 others—is incoherent. MP(2) is also incoherent. If an object is accidentally made from AAA, then there is a possible world in which it exists and is not made from AAA and is therefore made from one of the other hunks of matter. Thus, any object that has the positive property accidentally cannot have all seven of the negative properties essentially.

MP(3) is coherent, although any object having it is weirdly flexible with regard to its origin (im)possibilities. Any object having MP(3) has the property of being made from AAA, and thus it could be made from AAA. It has the property of not being made from BBB accidentally, and thus it could lack that property (and yet exist). Thus, it could be made from BBB, which does not overlap its original matter, AAA, at all. For each of the other six hunks of matter, it has the property of not being made from it as an essential property. In short: any object having MP(3) is made from AAA, could be made from AAA or BBB, but could not be made from any of the other hunks of matter. Although MP(3) is of only passing interest to us, it does have a property in common with all the modal profiles (based on these eight properties) that will be of more interest: it assigns accidental to the lone positive property in the base. Any particular modal profile, M, having this property determines that any object having M has the following features: it is made from the matter involved in the lone positive property; it *could* be made from any of the hunks of matter involved in the properties to which M assigns *accidental*; and it *could not* be made from any of the hunks of matter involved in the properties to which M assigns *essential*.

MP(4) is coherent. Any object having it has the following features: it is made from h_0; it could be made from any of AAA, AAB, ABA, and BAA; and it could not be made from any of ABB, BAB, BBA, and BBB. The first four hunks of matter are the ones involved in the properties to which the modal profile assigns accidental. They are also the four hunks of matter that overlap AAA (which is the matter used in w_0 to make an axe) by at least 2/3. The last four hunks of matter are the ones involved in the properties to which the modal profile assigns essential. They are also the four hunks of matter that do not overlap AAA (which is the matter used in w_0 to make an axe) by at least 2/3. All this is to say that any object that has MP(4) is made from h_0 and has Mod_0. It therefore has Mod_{orig}. An object with this modal profile exhibits a principled flexibility with regard to its origin (im)possibilities. The L-axe a_0 has MP(4) in w_0.

MP(5) is also coherent. Any object having it has the following features: it is made from h_0; it could be made from any of the hunks of matter AAA, BAA, BAB and BBA; and it could not be made from any of the hunks of matter AAB, ABA, ABB, and BBB. The first four hunks of matter are the ones involved in the properties to which the modal profile assigns accidental. They are also the four hunks of matter that overlap BAA (which is *not* the matter used in w_0 to make an axe) by at least 2/3. The last four hunks of matter are the ones involved in the properties to which the modal profile assigns essential. They are also the four hunks of matter that do not overlap BAA (which is *not* the matter used in w_0 to make an axe) by at least 2/3. All this is to say that any object that has MP(5) is made from h_0 and has Mod_1. It therefore lacks Mod_{orig}. It could be made from some hunks of matter that overlap its original matter by at least 2/3 (like BAA) but not others (like AAB); and it could be made from some hunks of matter that overlap its original matter by only 1/3 (like BBA), but not others (like ABB). An object with this modal profile is flexible in an unprincipled way. The L-axe a_1 has MP(5) in w_0.

Because axe a has many properties besides the ones involved in the property base considered—recall the discussion in Sect. 4.3 concerning the property of being owned by Sanders—each of the coherent modal profiles of Table 1 will be instantiated by multiple objects in w_0. Multiple objects coincident with a in w_0 will be flexible in a principled way (that is, multiple objects coincident with a in w_0 will have MP(4)—for example, one such L-axe will be essentially owned by Sanders and another accidentally so). Consider now the properties Mod_0 and Mod_{orig}. In w_0, a_0 is made from h_0 and has both Mod_0 and Mod_{orig}. There are four ways of assigning essential and accidental to those two properties, yielding four potential modal profiles for objects that have MP(4). (See Table 2.)

Table 2 Some modal profiles based on properties of a_0 in w_0

	6 Incoherent	7 Coherent a_0	8 Coherent a_x	9 Coherent
Mod_0	Essential	Essential	Accidental	Accidental
Mod_{orig}	Essential	Accidental	Essential	Accidental

The plenitudinarian is not committed to the existence in w_0 of an object that has MP(6), since that modal profile is incoherent. Why? Suppose that o has Mod_0 essentially in some world, w. Because o has Mod_0, there is a world w', possible in w, in which o is made from h_1. Moreover, since o has Mod_0 essentially, it has Mod_0 in any world, possible in w, in which it exists. In any such world, o's origin (im)possibilities are determined by overlap with h_0, regardless of what o is made from. Thus, in w', it is not possible that o is made from h_2. Thus o lacks Mod_{orig} in w'. And hence o does not have Mod_{orig} essentially in w.

But the plenitudinarian *is* committed to the existence in w_0 of an object for each of the remaining modal profiles. Significantly, she is committed not only to the existence in w_0 of an object that has MP(7), as a_0 does, but also to the existence in w_0 of an object that has MP(8). That is, she is committed to the existence in w_0 of an object that has Mod_0 only accidentally and Mod_{orig} essentially.

Leslie does not consider these commitments. This may be due in part to the fact that she would not distinguish Mod_0 and Mod_{orig}, but would instead think of them as a single property that can be specified in two different ways—precisely and imprecisely.[22] Leslie couches her views in terms of essences rather than in terms of modal tethering properties. But since what is important to her diagnosis of the paradox is not the (presumably definitional) essences that she gives but rather the modal tethering properties that she thinks arise from them, it is plausible that Leslie's discussion of precise and imprecise expressions of essences should extend to modal tethering properties.

> [T]here may be something a little suspicious about the description of a_0's essence: we began by supposing that its essence was 'tolerant' in that the axe could have been made with one part different relative to its original composition, and also that its original composition consisted of BladeA, ShaftA, HandleA. Now, it would seem that we might have described this same essence differently. Isn't the 'tolerant' essence just specified equivalent to the following at-one-level *in*tolerant essence? *a_0 is essentially constituted by at least two out of the following three parts: BladeA, ShaftA, and HandleA, plus the appropriate kind of third part if needed.* This would seem to pick out the same essence, only with greater precision, since now the parts are specified in the description of the essence. It is easy to see that any putative tolerant essence will be describable as an essence that is in a certain way *in*tolerant, and surely we should prefer to speak in terms of these more precisely specified

[22] This cannot be the entire reason, since the mere recognition of Mod_0 dictates the default position that something has Mod_0 essentially and something has Mod_0 accidentally. It is natural however to focus on essences rather than on accidents (so to speak), which may explain why although Leslie sees something having Mod_0 essentially, she overlooks something having Mod_0 accidentally. A plenitudinarian who recognizes the distinctness of Mod_0 and Mod_{orig}, and especially one who finds Mod_{orig} more intimately bound up with being an axe, readily recognizes something having Mod_{orig} essentially (hence having Mod_0 accidentally). Leslie thinks of essences of L-axes as given in terms of properties like being made from 2/3 of h_0, rather than in terms of properties like being an axe. (By contrast, Hawthorne, if I have his idea right, thinks of essences of L-axes as given in terms of properties like being an axe_0, which he thinks we express by 'being an axe'.) If one thinks of an axe's essence (modal or definitional) as including being an axe, it is very natural to think that the (modal) essence of any axe includes Mod_{orig}.

> intolerant essences. That is, it would seem that once we are more precise about what a 'tolerant essence' could actually come to, it will in fact be equivalent to a certain *in*tolerant essence. (284–285, with changes of labeling)

In reality, the phrases 'being something that is possibly made from all and only those hunks of matter that overlap its original matter by at least 2/3'—or symbolically, '$(\lambda x)[(\forall h)(\Diamond Mxh \leftrightarrow Ohm(x))]$'—and 'possibly being made from all and only those hunks of matter that overlap h_0 by at least 2/3'—or symbolically '$(\lambda x)[(\forall h)(\Diamond \mathrm{M}xh \leftrightarrow \mathrm{O}hh_0)]$'—are not imprecise and precise ways of picking out the same property. They are equally precise ways of picking out different properties—one general and one specific.

Leslie's precise/imprecise distinction *may* be rooted in the rigid/non-rigid distinction. Her thought may be that the "suspicious" specification (which really rigidly designates a general property) non-rigidly designates the very same property that the "precise" specification (which really rigidly designates a specific property) rigidly designates. It is worth noting that the λ-abstract that designates the general property uses a non-rigid designator, '$m(x)$', where the λ-abstract that designates the specific property uses a rigid designator 'h_0'. (See note 17.)

Once the distinction between the general property $\mathrm{Mod}_{\mathrm{orig}}$ and the specific property Mod_0 is made, the plenitudinarian commitment to a L-axe that has $\mathrm{Mod}_{\mathrm{orig}}$ essentially and Mod_0 accidentally, emerges organically. (See note 22.) It is true that sometimes plenitudinarianism is explained by way of property bases that exclude modal properties like Mod_0 and $\mathrm{Mod}_{\mathrm{orig}}$, but it is intuitively obvious that such limitations go against the spirit of plenitudinarianism. (See note 11.) To her credit, Leslie makes no attempt to limit property bases. Any property is fair game, since L-essentialism guarantees that for any property—modal or otherwise—there is something that it means to say that it is had essentially and there is something that it means to say that it is had accidentally. As long as a property—modal or otherwise—is had by something, the default plenitudinarian assumption is that there is something that has it essentially and something that has it accidentally. The only ground for deviation is that coherence demands it. We saw that coherence does demand deviation from the default assumption in the case of MP(6), but *prima facie* there is no such demand in the case of MP(7), MP(8), or MP(9).

The plenitudinarian commitment to an object with MP(8) entails a commitment to the rejection of Trans. Why? In w_0, any such object is made from h_0 and has $\mathrm{Mod}_{\mathrm{orig}}$ essentially, so there is a world w', possible in w_0, in which it is made from h_1 and has $\mathrm{Mod}_{\mathrm{orig}}$. And so there is a world, w'', possible in w', which is itself possible in w, in which it is made from h_2. But since in w_0, it is made from h_0 and has $\mathrm{Mod}_{\mathrm{orig}}$, there is no world, possible in w_0, in which it is made from h_2. Therefore, w'' is not possible in w_0, even though it is possible in a world that is possible in w_0. (The heavy lifting is being done just by the claim that there is something that has $\mathrm{Mod}_{\mathrm{orig}}$ essentially.)

No doubt, some will protest that if the claim that there is something that has MP(8) entails a violation of Trans, then the plenitudinarian is *not* committed to the existence of such an object. Trans, after all, is considered a truth of logic by those who think that *S5* is the correct logic for metaphysical modality. If Trans is a truth of logic, then MP(8) is incoherent, and the plenitudianrian is not committed to a corresponding object.

But consider this analogy. Suppose I assert the coherence of the view that all humans are mortal even though there are no non-mortals. A devotee of Artistotelian logic's rule of subalternation, which licenses the inference from ⌜All S are P⌝ to ⌜Some S are P⌝, would protest that what I claimed is coherent is in fact incoherent, since it violates logic. If all humans are mortal, then it follows by contraposition that all non-mortals are non-humans. From which it follows by subalternation that some non-mortals are non-humans, and hence that there are non-mortals.[23] Clearly, at this point, the devotee of subalternation must offer more than "This is the way it has always been done" or "You should not go mucking around with logic".

Yet it is common for philosophers to respond in an analogous way to the prospect of rejecting Trans—as though God handed down the logical theory we currently embrace. In fact, the logical systems that we create are our attempts to capture the truth about logic, just as our moral theories are our attempts to capture the truth about morality. It is the norm to fail in our attempts. It is irresponsible to hang onto a theory in the face of *prima facie* evidence against it simply because one feels like Tevye in *Fiddler on the Roof*: Tradition! Clearly, what is called for is *argument*. In Sect. 6, I will consider some arguments that Trans is a truth of logic.

The plenitudinarian is *prima facie* committed to the rejection of Trans. That is enough to solve the Paradox of FlexOE, without rejecting another principle in addition. As I will explain, the temptation that a plenitudinarian may feel to reject some other principle is coming not from plenitudinarianism per se, but rather from the identification of the axe *a*—the one with which the Paradox of FlexOE begins—with a_0. Plenitudinarianism does not force this identification.

Before I explain, let me present a pleasing plenitudinarian picture of two L-axes: a_0 and a_x. The L-axe a_0 has Mod_0 in all transitively possible worlds in which it exists, where a *transitively possible world* is a world that is possible, or possibly possible, or so on. The plenitudinarian's commitment to such a L-axe is obvious: in w_0, something has Mod_0, so something has it essentially (since that is coherent), so something has the property of *having* Mod_0 *essentially*, so something has *that* property essentially (since that is coherent), etc. If we go in for definitional essences, this can be because a_0's definitional essence includes a property—perhaps in a Lesliean spirit, the property of being made from at least 2/3 of h_0, or perhaps in a Hawthornean spirit, the property of being an axe_0— that dictates this. The L-axe a_x has the property of having Mod_{orig} in all transitively possible worlds in which it exists. The plenitudinarian's commitment to such a L-axe is obvious: in w_0, something has Mod_{orig}, so something has it essentially (since that is coherent), so something has the property of *having* Mod_{orig} *essentially*, so something has *that* property essentially (since that is coherent), etc. If we go in for definitional essences, this can be because a_x's definitional essence includes the property of being an axe while the definitional essence of the kind axe includes the property of being a kind such that every instance of it has Mod_{orig}.

It may be useful to consider the modal profiles of a_0, a_1, and a_x based on eight salient properties that a_1 has in w_1. Note that the property bases are different in w_0 and w_1, since different matter is used in them. (See Table 3.)

[23] This observation is due to Saul Kripke.

Table 3 Some modal profiles based on properties of a_1 in w_1

	10 Unprincipled a_0	11 Principled a_1 a_x
Not being made From AAA (h_0)	Accidental	Accidental
Not being made from AAB	Accidental	Essential
Not being made from ABA	Accidental	Essential
Not being made from ABB	Essential	Essential
Being made from BAA (h_1)	Accidental	Accidental
Not being made from BAB	Essential	Accidental
Not being made from BBA (h_2)	Essential	Accidental
Not being made from BBB	Essential	Essential

Recall that on Leslie's diagnosis, a_0 is principled in w_0 and unprincipled in w_1 (that is, a_0 has Mod_{orig} in w_0 but not in w_1) while a_1 is principled in w_1 and unprincipled in w_0 (that is, a_1 has Mod_{orig} in w_1 but not in w_0). The L-axe a_0 has MP(4) in w_0 and MP(10) in w_1. The L-axe a_1 has MP(5) in w_0 MP(11) in w_1. By contrast, the L-axe a_x is principled in all transitively possible worlds (that is, a_x has Mod_{orig} in all transitively possible worlds), and so it is principled in both w_0 and in w_1—sharing MP(4) with a_0 in w_0 and sharing MP(11) with a_1 in w_1.

If we add to this pleasing picture that axe a of the Paradox of FlexOE is a_x, the result is consistent with the truth of FlexOE, KindEss, and NecifTrue. And things will be exactly as the Chandler-Salmón solution says they are: a_x will be an axe in all transitively possible worlds in which it exists, and it will have Mod_{orig} in all transitively possible worlds in which it exists. (It will have Mod_0 (accidentally) in all transitively possible worlds in which is made from h_0, it will have Mod_1 (accidentally) in all transitively possible worlds in which it is made from h_1, and so on.) Thus, the plenitudinarian need not reject any of the principles that underlie the Paradox of FlexOE, beyond Trans, which must in any case be rejected. Assuming that departures from common sense should be kept to a minimum, the best plenitudinarian solution to the Paradox of FlexOE is simply to reject Trans.

If instead we add to the pleasing picture that axe a of the Paradox of FlexOE is a_0, the plenitudinarian must reject something in addition to Trans. Here is why. This identification gives us that a_0 is an axe in w_0, since a is. Either everything coincident with a_0 is an axe or not. If the former, then FlexOE is false, since in that case not all axes have Mod_{orig}. That is the solution of Sect. 4.1. But either way, since in w_0, a_0 is made from h_0 and has Mod_0 essentially, it follows that there is a world, w_1, possible in w_0, in which a_0 is made from h_1 and has Mod_0 and therefore lacks Mod_{orig}. Either

a_0 is an axe in w_1 or not. If the latter, then KindEss/KindRet is false. That is the solution of Sect. 4.2. If the former, then since a_0 lacks Mod_{orig} in w_1, FlexOE is not true in w_1, even if it is in w_0. And that means that FlexOE is not necessary in w_0. And that means NecifTrue is false, given FlexOE. That is the solution of Sect. 4.3. The restricted menu of plenitudinarian solutions that we explored in Sect. 4 arises from the identification of a with a_0, not from plenitudinarianism per se.

Plenitudinarianism should acknowledge entities like a_0 and the kind axe_0 to which it belongs, a_1 and the kind axe_1 to which it belongs, and so on. But not to the exclusion of a_x and the kind axe to which it belongs. While it may be wrong to eschew extraordinary objects, it is pathological to embrace them while overlooking the ordinary ones. No theory is tenable that admits everything *but* the kitchen sink. (The slogan is imperfect, since plenitudinarians who do not countenance the kitchen sink also fail to countenance other objects that they should—for example, a kitchen-sink-like entity that has Mod_{orig} as well as its owner essentially.)

What story should be told about why Trans seems true? The short story is very short: for a restricted—but very wide—range of substituends for ϕ in the characteristic axiom-schema for *S4* ($\ulcorner\Diamond\Diamond\phi \rightarrow \Diamond\phi\urcorner$) every instance of it is true. (See note 14.) And the longer story—saying more about the restricted class of appropriate substituends for ϕ in *S4*'s characteristic axiom-schema—goes well beyond the scope of the present paper.

6 Inconsistency?

My case that the rejection of Trans is the best plenitudinarian solution to the Paradox of FlexOE depends on the thought that MP(8) is coherent. There are *B* (and hence *T*) models that are not *S4* (and hence not *S5*) models according to which there are objects like a_x that have MP(8). If Trans is true as a matter of logic, these models are not admissible. The pressing question is whether Trans is true as a matter of logic.

Let's reflect for a moment on the initial intuitive status of Trans, comparing it to the initial intuitive status of *Reflex*, the claim that the relation of being possible according to is reflexive. We have a clear initial intuition that if something is true it is possible. Or, equivalently that if something is necessary it is true. This is the intuition that the relation of being possible according to is reflexive: every world is possible according to itself. It is just inconsistent with the concept of possibility that something is true but not possible (or equivalently, that something is necessary but not true). Just as it inconsistent with the concept of knowledge that something is known but not true. Just as it is inconsistent with the concept of bachelorhood that someone is a bachelor and not unmarried. Thus it is true as a matter of logic that all truths are possible, that everything known is true, and that all bachelors are unmarried. Our logical theory must limit admissible models to those in which these claims are true. What about Trans? We have no analogous initial intuition that whatever is possibly possible is possible. Nor do we have an analogous initial intuition that this is not the case. (Some do have an initial intuition one way or the other about Trans. That is perfectly compatible with my claim, which is that we do

not have an intuition in this case that is *analogous* to the intuitions in the other three cases.) That does not mean that Trans is not a truth of logic: there are surprising truths of logic, after all. But when something is not clearly true as a matter of logic, to establish it as such requires argument. This is so whether we are plenitudinarians or not. But plenitudinarians are under special obligation to provide an argument, since by their lights to reject a postulated object (like a_x) requires provision of a reason why it does not exist.

In fact, the need for argumentation is even more pressing than this. Our lack of an overwhelming initial intuition concerning the (logical) truth of Trans does not mean that we cannot pump intuition by reflecting on other intuitions. And we do have other relevant intuitions. Intuitively, it is consistent to hold that something has Mod_{orig} essentially. Intuitively, inconsistency results only when one adds Trans into the mix. Our logical intuition thus tells against Trans as a truth of logic. The need for argumentation is dire.

There have been arguments to the effect that *S5* is the correct logic for metaphysical modality. And there have been rebuttals.[24] Leslie does not argue for *S5*, but she does offer an argument against the *Rejection of Trans Solution* (hereafter *RTS*). She argues against RTS on the grounds that it is committed to *a*'s having a "*variable essence*, an essence that varies from possible world to possible world", which she says "conflicts with the very notion of essence" in the sense that it "is just not consistent with the notion of essence"(285). We have by this point in the present paper had a fair bit of experience showing inconsistency with the very notion of essence. We showed that the prospect of something's having any of MP(1), MP(2), or MP(6) is inconsistent in that way. So too the prospect of something's being essentially accidentally *F*. (See note 10.) In each case, a compelling argument was given. Is there a similarly compelling argument that nothing can be accidentally essentially *F*—where the salient property is the negative property of not being made from h_2—as the RTS-theorist contends *a* is?

Leslie's argument may be represented thus.

L1: According to RTS, *a* has a variable essence.
L2: An object's having a variable essence is not consistent with the very notion of essence.
So: RTS is inconsistent (with the very notion of essence).

This amounts to an argument that the correct logic for metaphysical modality is at least as strong as *S4*, since *B* and *T* countenance models that would, if Leslie's argument is sound, be inadmissible because they are inconsistent with the notion of essence.

Given that two notions of 'essence'—modal and definitional—may be at play in Leslie's discussion, how ought we to understand the argument? Let's try the modal notion first. (L1_{modal}) is straightforwardly true. According to RTS, the class of *a*'s modally essential properties in w_0, includes the property of not being made from h_2,

[24] See (Salmón 1989) and (Robertson Ishii 2014).

but the class of a's modally essential properties in w_1, does not. Since w_0 and w_1 are both possible in w_0, this means that according to RTS, one class of properties is a's modal essence in one possible world and another class of properties is a's modal essence in another possible world. (Note that since not being made from h_2 is essential to a in w_0, the concept of a modal essence requires that a has that property in w_1. The RTS-theorist accepts that. It is just that according to RTS, a does not have that property *essentially* in w_1.)

What about ($L2_{modal}$)? There is a seductive, though fallacious, line of argument for ($L2_{modal}$) that Leslie may (or may not) have had in mind. (It is intimately related to the parenthetical remark in the previous paragraph.) First, some terminology: where C is any class of properties, *o has C* iff *o* has every member of C.

(1) For every object x, x's modal essence is such that x has it in every possible world in which x exists.
(2) For every object x, x has the same modal essence in every possible world in which x exists. (From (1))
(3) For every object x, x's modal essence is the same in every possible world in which x exists. (From (2))

(1) follows from "the very notion of essence". This is easy to see if one inserts the definition of 'x has C' explicitly into it: x's modal essence is such that x has every member of it in every possible world in which x exists. (2) is ambiguous between (2′) and (2″).

(2′) For every object x, there is a modal essence E such that x has E in every possible world in which x exists.
(2″) For every object x, there is a modal essence E such that E is x's modal essence in every possible world in which x exists.

(1) does yield (2′), but (2′) does not yield (3). Alternatively, (2″) does yield (3), but (1) does not yield (2″). This will not do as a route to ($L2_{modal}$).

What happens if we try the definitional notion instead? Although I find that notion a bit mysterious, I am happy to grant that it is inconsistent with the notion of a definitional essence that an object's definitional essence varies from possible world to possible world. That is, I am happy to grant ($L2_{def}$). What about ($L1_{def}$)? RTS says nothing about the variability of definitional essences, since it says nothing about definitional essences at all. RTS merely advocates as a solution to the Paradox of FlexOE denying the transitivity of the relation of being metaphysically possible according to. Moreover, assuming that a has a definitional essence, RTS is compatible with its being the same in all possible worlds. As we saw in Sect. 5, the RTS-theorist is free to account for the fact that a (that is, a_x) is an axe and has Mod_{orig} in both w_0 and w_1 by saying that the definitional essence of a includes being an axe and the definitional essence of the kind axe includes being a kind all of whose instances have Mod_{orig}. The RTS-theorist may account for why a has Mod_0

in w_0 (and similarly for Mod_1 in w_1) by appealing to these definitional facts together with the (non-definitional) fact that a is originally made from h_0 in w_0.[25] According to RTS, although a's modal essence varies between w_0 and w_1, a's definitional essence—assuming there is such a thing—may nonetheless be constant.

What if we try a mixed reading? Since (L1_{modal}) is true, it makes good sense to try reading (L2) as (L2_{mixed}): an object's having a variable *modal* essence is inconsistent with the very notion of *definitional* essence. Consider a portion of Leslie's explication of the essential/accidental property distinction.

> An object's essential properties are conditions on what it is to be that object, and this set of conditions fixes just which possibilities or possible worlds the object exists in… (277)

The phrase "what it is to be" is a hallmark of Fine's definitional understanding of 'essence'. The passage suggests then that Leslie may take the following thesis to be part of the very notion of a definitional essence.

DefDetMod: A thing's definitional essence determines its modal essence.

If DefDetMod followed from the very notion of a definitional essence, then I would be happy to grant (L2_{mixed}), since I was happy to grant (L2_{def}). If it were part of the very notion of a definitional essence that a thing's definitional essence is invariable and determines that thing's modal essence, then that thing's modal essence would also be invariable. But DefDetMod is a substantive thesis about the relationship between definitional and modal essences, not something one gets for free from a definition. So it does not provide a route to (L2_{mixed}).

DefDetMod is stronger than Fine's influential but controversial reduction of modality to definitional essence. According to Fine (1994), the definitional essences of all objects determine together the modal essence of each object. Socrates's modal essence includes being (if he exists) a member of singleton Socrates as well as being such that Rover is a dog (if existent). But according to DefDetMod, Socrates's definitional essence alone—without help from the essences of singleton Socrates or Rover—determines that he has both of those properties modally essentially. Perhaps then, Leslie meant simply to be endorsing Fine's weaker thesis. But again, Fine's thesis is substantive, not something that one gets for free from a definition. So Fine's thesis does not provide a route to (L2_{mixed}).

Incidentally, this point makes evident that RTS (with or without plenitudinarianism)—as well as plenitudinarianism (with or without RTS)— provides a forceful consideration against Fine's thesis. Given that definitional essences are invariable, if Fine's thesis is true, then (on reasonable assumptions) no object's modal essence varies from possible world to possible world. But, according to RTS, a's modal essence *does* vary from possible world to possible world—and according to plenitudinarianism, a_x's modal essence does vary from possible world to possible

[25] Note how close this thought is (minus the suspicion) to the one Leslie expresses in the first sentence of the passage on precise/imprecise specifications of essences.

world. So Fine's thesis is false. (Of course one woman's *modus tollens* may be another person's *modus ponens*.)

Leslie fails to show that RTS is inconsistent or that it violates any "general metaphysical principle". So if Trans were true, it would evidently be substantive—a true principle of variety limitation. The heart of plenitudinarianism is to deny that there are such things. So, for the plenitudinarian, the best solution to the Paradox of FlexOE is clear: identify a with a_x, reject Trans but nothing more.

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