

# **The Principles of Rational Choice**

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To my parents

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## Preface

To make *instrumentally rational choices* involves choosing courses of action that best serve one's ends. As a good example, one's chances of having a positive impact on the world are significantly greater if one focuses one's efforts on solving the world's biggest, most urgent problems than if one focuses one's efforts on solving comparatively small, non-urgent problems that will not ultimately matter in the long run if the biggest, most urgent problems are left unsolved. Therefore, if one's goal is to improve the world as much as possible, then it is simply not (instrumentally) rational to focus one's efforts on solving comparatively small, non-urgent problems. It is true that functioning societies do need people working on comparatively small day-to-day problems. But a shortage of such people is itself a big, urgent problem and so, something that one would need to factor into any rational decision about where to focus one's efforts. We face today an unprecedented number of big, urgent problems. With those problems in mind, in this book, I will supply readers with a robust methodology for making (instrumentally) rational decisions. Readers will then be better equipped to solve the world's biggest, most pressing problems.

Specifically, this book aims to address the question of how one ought to choose when one is *uncertain* about what outcomes will result from one's choices, but when one can nevertheless assign probabilities to the different possible outcomes. These choices are commonly referred to as *choices* (or *decisions*) *under risk*. To address this question, in this book, I develop and motivate a radically revisionary normative theory of rational choice under risk, namely *expected comparative utility (ECU) theory*. According to ECU theory, for any choice option, *a*, and for any state of the world, *G*, the measure of the choiceworthiness of *a* in *G* is the *comparative utility* of *a* in *G*—that is, the difference in utility, in *G*, between *a* and whichever alternative(s) to *a* carry the greatest utility in *G*. Roughly speaking, for any agent, *S*, faced with any decision under *risk*, ECU theory says that *S* ought to measure and rank her options (in terms of how choiceworthy they are) according to their ECU and *S* ought to choose whichever option carries the greatest ECU (or one of them in the event that several options are tied). The ECU of an option is a probability-weighted sum of each of its possible comparative utilities. By contrast, according to the standard normative theory of rational choice under risk, namely *expected utility (EU) theory*, for any agent, *S*, faced with any decision under *risk*, *S* ought to rank her options (from least to most choiceworthy) according to their EU and *S* ought to choose whichever option carries the greatest EU (or one of them in the event that several options are tied). The EU of an option is a probability-weighted sum of each of its possible utilities.

ECU theory is superior to EU theory (and other normative theories of rational choice under risk, such as regret-based theories) because ECU theory is based on the novel idea that the appropriate criterion of rational choice under risk is *choiceworthiness* (properly measured) and not *rational preference*, the standard and long-standing criterion of rational choice under risk. To establish this idea and the general theory, I reason explicitly from decision-theoretic assumptions that are all shared by orthodox EU theory, with the exception of one strengthened assumption—that agents ought to make instrumentally rational choices (that is, they ought to adopt suitable means to their ends) just in case their ends are *morally permissible*. Starting from these assumptions, I argue that agents ought to maximize *choiceworthiness* and that choices under *risk* that maximize *choiceworthiness* sometimes differ (in commonplace decision situations) from choices under *risk* that maximize *rational preference*. I conclude that since agents who act in accordance with ECU theory are committed to maximizing *choiceworthiness* rather than *rational preference* and since agents who act in accordance with EU theory are committed to maximizing *rational preference* rather than *choiceworthiness*, ECU theory, and not EU theory, is the correct normative theory of rational choice under risk or, more specifically, the correct theory of instrumental rationality.

This book is a restatement and expansion of my previously published work, Robert, 2018, 2021.

I have revised and restructured the work to present its main points more clearly and convincingly and in greater detail, and to develop new arguments for the proposed principles of rational choice. Section 3.2.3 and Chapters 4, 5, and 6 of this book are completely new.

For very helpful feedback on earlier drafts of my two papers, Robert, 2018, 2021, I am grateful to Marcus Arvan, Paul Bartha, Kenny Easwaran, Elizabeth Jackson, Douglas Lackey, John Quiggin, and several anonymous reviewers and commenters. I would not have been able to carry this book project to fruition without the help and support of my parents, Josephine Moffa and Guy Robert, to which I am deeply grateful.

I would be thankful for any thoughts or comments on this book. They can be sent to me at [jeandavidrobert@hotmail.com](mailto:jeandavidrobert@hotmail.com)

# 1 Introduction

This book aims to break new ground in the articulation and justification of the principles of rational choice. More specifically, this book aims to address the question of how one *ought* to choose when one is *uncertain* about what outcomes will result from one's choices, but when one can nevertheless assign probabilities to the different possible outcomes. These choices are commonly referred to as *choices* (or *decisions*) *under risk*. Along the way, this book will also address the question of how one *ought* to choose when one is *certain* about what outcomes will result from one's choices and when one assigns probability 1 to those outcomes. These choices are commonly referred to as *choices* (or *decisions*) *under certainty*.

Standard decision theory, otherwise known as *expected utility (EU) theory*, requires that when faced with any decision under risk (or any decision under certainty), one ought to rank one's choice options (from least to most choiceworthy) according to their EU and one ought to choose whichever option carries the greatest EU (or one of them in the event that several options are tied). The EU of an option is a probability-weighted sum of each of its possible utilities. EU theory has been the dominant normative theory of rational choice under risk since the 18th century (Bernoulli, 1738), and in more recent times (from the 1920s onwards), has received foundational support from both economists and philosophers (Ramsey, 1931; von Neumann & Morgenstern, 1947; Savage, 1954; Bolker, 1966; Jeffrey, 1983; Joyce, 1999).<sup>1</sup>

In this book, I will argue for a new normative alternative to EU theory.<sup>2</sup> My argument will be predicated on the following assumptions:

I will assume, as EU theory does, that EU is the appropriate criterion of *rational preference* to apply to decisions under certainty and decisions under risk. I will assume that for any agent, *S*, and for any choice option, *a*, for *S*, *a*'s *utility* is a *cardinal* index of preference (which measures the strength of preference between options) and is derived from *S*'s preferences as in standard decision theory, that is, via a *representation theorem*. This requires that *S*'s preferences obey a series of conditions, or “axioms”, of *rational preference*, one of which is the *independence of irrelevant alternatives (IIA)* (Kogelmann, 2018). According to the IIA (for preferences), if an option, *a*, is preferred over some alternative option, *b*, then introducing a third option, *c*, in the choice situation will not change the preference ordering between *a* and *b*.<sup>3</sup> In this book, *rational preferences* are understood as preferences that obey the axioms of rational preference of standard decision theory. Moreover, *preference* and *utility* are understood as ascriptions of *psychological states* not reducible to choice. I therefore reject the behavioral (or revealed) interpretation of *preference* and *utility*. (Hausman, 2011, 2024; Beck, 2024)

I will further assume, as EU theory does, that an option is *choiceworthy* for an agent if and only if it is *maximally choiceworthy* for her over the space of all alternatives in the choice set. In other words, an option is choiceworthy for an agent if and only if that option is at least as choiceworthy for her as each of the other options within the set of alternatives. What's more, I will assume, as EU theory does, that an option is *choiceworthy* for an agent faced with a decision under *certainty* if and only if that option *maximizes utility* over the space of all alternatives in the choice set.

I will lastly assume that agents ought to make *instrumentally rational* choices. More specifically, agents ought to measure and rank their options (for the purpose of choice) in terms of *how*

<sup>1</sup> For overviews of EU theory, see (e.g.) Peterson, 2017; Bradley, 2018; Steele & Stefánsson, 2020; and Briggs, 2023.

<sup>2</sup> For an overview of normative rivals to EU theory, see Buchak, 2022.

<sup>3</sup> The IIA should not be confused with the Von Neumann–Morgenstern independence axiom: where  $XpY$  is “a prospect consisting in a lottery between *X* and *Y* such that *X* occurs with probability *p* and *Y* occurs with probability  $1 - p$ ” and where  $X > Y$  means that *X* is preferred to *Y*, “for all probabilities *p* such that  $0 < p < 1$ , it holds that, if  $X > Y$ , then  $XpZ > YpZ$ .” (Gustafsson, 2022, p. 78)

*instrumentally rational they are*, and agents ought to choose out of *whatever option(s) are instrumentally rational*. Someone makes instrumentally rational choices, according to Kolodny and Brunero (2023), “insofar as she adopts suitable means to her ends.” Orthodox EU theory is a theory of instrumental rationality (Buchak, 2022), where the agent’s *ends* (or rational preferences) are understood as morally permissible or impermissible ends. As such, orthodox EU theory is not a *normative* theory of choice (even though it is intended to be), since it is not the case that an agent ought to choose in accordance with EU theory if some or all of the agent’s ends (or rational preferences) are morally impermissible. I rely here on a unified view of practical reason, which has been recently defended by Brown (2024). Brown (2024) argues that there is such a thing as what one ought *simpliciter* to do, and proposes “an account of our normative concepts according to which only ought *simpliciter* judgments commit one to acting in accordance with those judgments.” (p. 497)

Consequently, in this book, I will depart from orthodoxy by stipulating that to make *instrumentally rational* choices—according to EU theory and any normative alternative to EU theory—is to adopt suitable means to one’s *morally permissible* ends, or (i.e.) one’s morally permissible *rational preferences*, provided that all of one’s ends (or rational preferences) are morally permissible.<sup>4</sup> If not all of one’s ends are morally permissible, then I assume that one ought to adopt morally permissible ends, and one ought to take suitable means to those ends. Thus, for the sake of brevity, in this book, *ends* will be understood as morally permissible ends, or (i.e.) morally permissible rational preferences, and *preferences* (to prefer *a* to *b* or to be indifferent between *a* and *b*) will be understood as morally permissible preferences.

Reasoning from the above assumptions, in this book, I will argue against EU theory, in favor of a new normative theory of rational choice under risk, or, more specifically, a new theory of instrumental rationality, namely *expected comparative utility (ECU) theory*. Arguments against EU theory typically involve identifying decision situations where EU theory gives the intuitively wrong recommendations. For example, the Allais, Ellsberg, and St Petersburg paradoxes do just that (see respectively Allais, 1953; Ellsberg, 1961; and Bernoulli, 1738). This book takes a different approach. Instead of identifying counterexamples to EU theory that strongly suggest a new normative alternative, I will develop conceptual arguments in favor of a new normative alternative (i.e., ECU theory), alternative which I will show to be inconsistent with EU theory. I will show that in some ordinary decisions under risk, ECU theory gives different *verdicts* from those of EU theory (i.e., about what choice options are choiceworthy and about what choice options are more choiceworthy than or just as choiceworthy as alternative choice options) and that EU theory is therefore not the correct normative theory of rational choice under risk, or, more specifically, not the correct theory of instrumental rationality.

ECU theory is superior to EU theory (and other normative theories of rational choice under risk) because ECU theory is based on the novel idea that the appropriate criterion of rational choice under risk is *choiceworthiness* (properly measured) and not *rational preference*, the standard and long-standing criterion of rational choice under risk.<sup>5</sup> In this book, I will argue that agents ought to maximize *choiceworthiness* and that choices under *risk* that maximize *choiceworthiness* sometimes differ (in ordinary decision situations) from choices under *risk* that maximize *rational preference*. I will conclude that since agents who act in accordance with ECU theory are committed to maximizing *choiceworthiness* rather than *rational preference* and since agents who act in accordance with EU theory are committed to maximizing *rational preference* rather than *choiceworthiness*, ECU theory,

<sup>4</sup> I follow here Horton (2023), according to whom “an option is permissible if and only if it could be rationally chosen by someone with permissible preferences.” This view is fully developed in Horton & Ross, 2025.

<sup>5</sup> The idea that the appropriate criterion of rational choice under risk is *choiceworthiness* and not *rational preference* has been previously argued for by Wedgwood (2017). However, he explicitly rejects, whereas I assume, the idea that one should measure the choiceworthiness of options by taking into account their *utility* values. On Wedgwood’s (2017) account, the choiceworthiness of options is a measure of a certain kind of *objective goodness*.

and not EU theory, is the correct normative theory of rational choice under risk or, more specifically, the correct theory of instrumental rationality.

ECU theory aims to improve upon *regret-based decision theories*.<sup>6</sup> These theories are founded on the idea that rational agents strive to minimize regret—the disappointment that occurs when they fail to choose the available option they most prefer (i.e., the available option which carries the greatest utility). The *basic regret-based theory of rational choice under risk* (or *BR theory*) makes use of a *benchmark* (or zero point of choiceworthiness). According to this theory, for any choice option, *a*, and for any state of the world, *G*, the extent of *a*'s “regret”, in *G*, is the extent to which *a*, in *G*, falls short of whichever available option(s) carry the greatest utility in *G*, and the degree of choiceworthiness of *a*, in *G*, is the degree to which the choice of *a* minimizes that “regret”. Thus, the degree of choiceworthiness of *a*, in *G*, is the difference in utility, in *G*, between *a* and *whichever available option(s) carry the greatest utility* in *G* (i.e., the benchmark). Understood in this way, maximizing “expected choiceworthiness”, or (i.e.) the probability-weighted sum of each of an option's possible degrees of choiceworthiness, always coincides with maximizing EU (i.e., EU theory) (see Wedgwood, 2013, p. 2663). BR theory requires that when faced with any decision under risk (or any decision under certainty), one ought to rank one's options (in terms of how choiceworthy they are) according to their “expected choiceworthiness” and one ought to maximize “expected choiceworthiness”, that is, choose whichever option carries the greatest “expected choiceworthiness” or one of them in the event that several options are tied.

ECU theory also makes use of a benchmark (one that is importantly different from the one employed in BR theory): for any choice option, *a*, and for any state of the world, *G*, the degree of choiceworthiness of *a*, in *G*, is the difference in utility, in *G*, between *a* and *whichever alternative(s) to a carry the greatest utility* in *G* (i.e., the benchmark). This difference in utility is what I will call the *comparative utility* of *a*. Roughly speaking, ECU theory requires that when faced with any decision under risk (or any decision under certainty), one ought to rank one's options (in terms of how choiceworthy they are) according to their ECU and one ought to choose whichever option carries the greatest ECU (or one of them in the event that several options are tied). The ECU of an option is the probability-weighted sum of its comparative utilities across the various states of the world.

Beyond regret-based decision theories, the idea of calculating differences between the utility of an option (or outcome) under consideration and the utilities of its alternatives in the choice situation (e.g., for each state of the world) has been explored in the philosophical literature (Bartha, 2007, 2016; Colyvan, 2008; Colyvan & Hájek, 2016) and the economic literature (Kahneman & Tversky, 1979; Zhang, 2015). To give a notable example from the economic literature, Kahneman and Tversky's (1979) descriptive decision theory, namely, *prospect theory*, makes use of a *reference point*—that is, a benchmark, such as the status quo outcome of a decision, from which people actually assess the utility of the decision outcomes, either as gains or losses compared to that benchmark.

This book is divided into six chapters. In the next chapter (Chapter 2), I will contrast EU theory and ECU theory, revealing how and why they differ. I will show that in some commonplace decisions under risk, ECU theory gives different verdicts from those of EU theory. In Chapter 3, I will develop a comprehensive step-by-step argument for ECU theory (and against EU theory). In Chapter 4, I will respond to some possible objections to ECU theory. In Chapter 5, I will suggest an additional constraint on rational preferences. In Chapter 6, I will conclude.

<sup>6</sup> For a helpful discussion of regret-based decision theories, see Yager, 2017. For an early normative decision theory that takes into account anticipated regret, namely, *regret theory*, see Bell, 1982; Fishburn, 1982; and Loomes & Sugden, 1982.



## 2 EU Theory vs. ECU Theory

This chapter will explicate and compare EU theory and ECU theory. I will begin by explaining how EU theory and ECU theory are to be formulated and how they work. I will then explain how and why their verdicts differ in various decision problems.

### 2.1 What Are EU Theory and ECU Theory? How Do They Work?

According to *EU theory*, the EU of an option,  $a$ , in a decision problem with  $n$  states is formally defined as:

$$EU(a) = \sum_{i=1}^n U(a, s_i)P(s_i)$$

where  $U(a, s_i)$  denotes the utility of option  $a$  when state  $s_i$  is actual, and  $P(s_i)$  denotes the probability assigned to state  $s_i$ . In other words, for any number of alternative options,  $a, b, c, d$ , and  $e$ , one calculates the EU of  $a$  as follows: for each state of the world, one calculates  $a$ 's utility and one multiplies the result by the probability that one assigns to that state; finally, one sums the totals for every state.

According to EU theory, for any agent,  $S$ , faced with any decision under *certainty* or any decision under *risk* and for any number of alternative options,  $a, b, c, d$ , and  $e$ , for  $S$ , it is rational for  $S$  to prefer  $a$  to  $b$ , and  $a$  is more choiceworthy than  $b$  for  $S$ , if and only if  $a$ 's EU is greater than  $b$ 's; it is rational for  $S$  to be indifferent between  $a$  and  $b$ , and  $a$  is just as choiceworthy as  $b$  for  $S$ , if and only if  $a$ 's EU is equal to  $b$ 's; the extent to which  $S$  rationally prefers  $a$  to  $b$ , and the extent to which  $a$  is more choiceworthy than  $b$  for  $S$ , is the difference in EU between  $a$  and  $b$ ; finally, it is rational for  $S$  to weakly prefer  $a$  over the alternative options available to  $S$ , and  $a$  is choiceworthy for  $S$ , if and only if  $a$  maximizes EU within the set of alternatives available to  $S$ .<sup>7</sup>

Let a *choice situation* be a decision situation as formalized in a decision matrix. According to what we can call *EU theory's choice situation principle*, for any agent,  $S$ , faced with any decision under *certainty* or any decision under *risk*, for the purpose of determining, in accordance with EU theory, what choice situations are choiceworthy for  $S$  and what choice situations are (for  $S$ ) more choiceworthy than or just as choiceworthy as any alternative choice situations (and to what extent), choice situations are to be treated as *choice options*, and the utility of choice situations is to be treated as the EU of whatever option(s) carry the greatest EU in those choice situations.

Let us now turn to *ECU theory*. To a first approximation, ECU theory says that for any agent,  $S$ , and for any choice option,  $a$ , for  $S$ , the measure of the choiceworthiness of  $a$  for  $S$  (relative to any explicitly given utility unit and zero point of utility) is the ECU of  $a$ . The *ECU* of an option,  $a$ , in a decision problem with  $n$  states is formally defined as:

$$ECU(a) = \sum_{i=1}^n (U(a, s_i) - U(bm(a), s_i))P(s_i)$$

<sup>7</sup> For any agent,  $S$ , and for any two choice options,  $a$  and  $b$ , for  $S$ , if  $S$  *weakly prefers*  $a$  to  $b$ , then  $S$  either prefers  $a$  to  $b$  or is indifferent between  $a$  and  $b$ .

where  $U(a, s_i)$  denotes the utility of option  $a$  when state  $s_i$  is actual,  $U(bm(a), s_i)$  denotes the utility of the benchmark for  $a$  when state  $s_i$  is actual (i.e., the utility in state  $s_i$  of whichever alternative(s) to  $a$  have the greatest utility in state  $s_i$ ), and  $P(s_i)$  denotes the probability assigned to state  $s_i$ . In other words, for any number of alternative options,  $a, b, c, d$ , and  $e$ , one calculates the ECU of  $a$  as follows: for each state of the world, one subtracts from  $a$ 's utility the utility of whichever alternative(s) to  $a$  (i.e.,  $b, c, d$ , or  $e$ ) carry the greatest utility in that state, and one multiplies the result by the probability that one assigns to that state; finally, one sums the totals for every state.

More precisely, ECU theory is the conjunction of the following three principles:

- the comparative utility (CU) principle,
- the comparative expected comparative utility (CECU) principle, and
- ECU theory's choice situation principle.

According to the *CU principle*, for any agent,  $S$ , faced with any decision under *certainty* and for any choice option,  $a$ , for  $S$ , the measure of the choiceworthiness of  $a$  for  $S$  (relative to any explicitly given utility unit and zero point of utility) is the *comparative utility (CU)* of  $a$  in the state of the world to which  $S$  assigns probability 1. For any choice option,  $a$ , and for any state of the world,  $G$ , to which  $S$  assigns probability 1, the CU of  $a$ , in  $G$ , is the difference in utility, in  $G$ , between  $a$  and whichever alternative(s) to  $a$  carry the greatest utility in  $G$ .<sup>8</sup> (Henceforth, *c-utiles* are defined as units of CU.)

According to the *CECU principle*, for any agent,  $S$ , faced with any decision under *risk* and for any choice option,  $a$ , for  $S$ , the measure of the choiceworthiness of  $a$  for  $S$  (relative to any explicitly given utility unit and zero point of utility) is the *comparative expected comparative utility (CECU)* of  $a$ —that is to say, the difference in ECU between  $a$  and whichever alternative(s) to  $a$  carry the greatest ECU. For any two alternative options,  $a$  and  $b$ ,  $a$ 's CECU is greater than  $b$ 's if and only if  $a$ 's ECU is greater than  $b$ 's, and  $a$ 's CECU is equal to  $b$ 's if and only if  $a$ 's ECU is equal to  $b$ 's.

In Chapter 3, I will argue for the *choiceworthiness maximization (CM) principle*, which says that for any agent,  $S$ , and for any option,  $a$ , for  $S$ ,  $a$  is choiceworthy for  $S$  if and only if  $a$  maximizes choiceworthiness for  $S$  over the space of all alternatives in the choice set. According to the conjunction of the CU principle and the CM principle (henceforth, according to the *CU principle*), for any agent,  $S$ , faced with any decision under *certainty*, and for any option  $a$ , for  $S$ ,  $a$  is choiceworthy for  $S$  if and only if  $a$ 's CU is equal to or greater than zero, and according to the conjunction of the CECU principle and the CM principle (henceforth, according to the *CECU principle*), for any agent,  $S$ , faced with any decision under *risk*, and for any option  $a$ , for  $S$ ,  $a$  is choiceworthy for  $S$  if and only if  $a$ 's CECU is equal to or greater than zero.

According to *ECU theory's choice situation principle*, for any agent,  $S$ , faced with any decision under *certainty* or any decision under *risk*, for the purpose of measuring, in accordance with ECU theory, the degree to which choice situations are choiceworthy for  $S$  and for the purpose of determining, in accordance with ECU theory, what choice situations are choiceworthy for  $S$ , choice situations are to be treated as *choice options*, and the utility of choice situations is to be treated as the EU of whatever option(s) carry the greatest EU in those choice situations.

To demonstrate how to apply EU theory and ECU theory to a concrete decision problem involving *objective probabilities*, let us consider the following case: An agent,  $S$ , is faced with a choice between two independent options or gambles: one option,  $a$ , offering a 0.01 probability of winning a prize

<sup>8</sup> *CU* should be distinguished from the purely descriptive economic concept of *opportunity cost*. For any agent,  $S$ , let  $a$  be the highest-valued choice option available to  $S$ . The CU of  $a$ , for  $S$ , is the value of whatever *additional benefit*  $S$  would enjoy by choosing  $a$  over the highest-valued alternative to  $a$ . By contrast, the opportunity cost of  $a$ , for  $S$ , is the value of whatever *cost*  $S$  would incur by choosing  $a$  over the highest-valued alternative to  $a$ , where this includes the *total value* of the highest-valued alternative to  $a$  (Henderson, 2008).

worth 1500 utiles (and nothing otherwise), and one option,  $b$ , offering a 0.02 probability of winning a prize worth 700 utiles (and nothing otherwise). According to ECU theory,  $S$  ought to choose option  $a$ , since its CECU is equal to or greater than zero ( $1 \text{ c-utile} - [-1 \text{ c-utile}] = 2 \text{ units of CECU}$ ). According to EU theory,  $S$  ought to also choose option  $a$ , since its EU (15 utiles) is greater than that of every other option in the decision situation (i.e. the EU of  $b$  is 14 utiles).

The EUs and ECUs of options  $a$  and  $b$  are given by the four equations below. The following notation is used:  $A$  denotes the state “If  $S$  chooses  $a$ , then  $S$  will not win the prize (0 utiles) and if  $S$  chooses  $b$ , then  $S$  will not win the prize (0 utiles)” (probability:  $0.99 \times 0.98 = 0.9702$ ),  $B$  denotes the state “If  $S$  chooses  $a$ , then  $S$  will not win the prize (0 utiles) and if  $S$  chooses  $b$ , then  $S$  will win the prize (700 utiles)” (probability:  $0.99 \times 0.02 = 0.0198$ ),  $C$  denotes the state “If  $S$  chooses  $a$ , then  $S$  will win the prize (1500 utiles) and if  $S$  chooses  $b$ , then  $S$  will not win the prize (0 utiles)” (probability:  $0.01 \times 0.98 = 0.0098$ ),  $D$  denotes the state “If  $S$  chooses  $a$ , then  $S$  will win the prize (1500 utiles) and if  $S$  chooses  $b$ , then  $S$  will win the prize (700 utiles)” (probability:  $0.01 \times 0.02 = 0.0002$ ),<sup>9</sup>  $P(A)$  denotes the probability of state  $A$ , and  $U(a, A)$  denotes the utility of option  $a$  when state  $A$  is actual. (See Table 1).

**TABLE 1** Decision matrix

	<b>A (0.9702)</b>	<b>B (0.0198)</b>	<b>C (0.0098)</b>	<b>D (0.0002)</b>
<b><math>a</math></b>	0	0	1500	1500
<b><math>b</math></b>	0	700	0	700

$$EU(a) = U(a, A) \times P(A) + U(a, B) \times P(B) + U(a, C) \times P(C) + U(a, D) \times P(D) = 15 \text{ utiles}$$

$$EU(b) = U(b, A) \times P(A) + U(b, B) \times P(B) + U(b, C) \times P(C) + U(b, D) \times P(D) = 14 \text{ utiles}$$

$$\begin{aligned} ECU(a) = & (U(a, A) - U(b, A)) \times P(A) + (U(a, B) - U(b, B)) \times P(B) + (U(a, C) \\ & - U(b, C)) \times P(C) + (U(a, D) - U(b, D)) \times P(D) = 1 \text{ c-utile} \end{aligned}$$

$$\begin{aligned} ECU(b) = & (U(b, A) - U(a, A)) \times P(A) + (U(b, B) - U(a, B)) \times P(B) + (U(b, C) \\ & - U(a, C)) \times P(C) + (U(b, D) - U(a, D)) \times P(D) = -1 \text{ c-utiles} \end{aligned}$$

## 2.2 Do EU Theory's and ECU Theory's Verdicts Differ? How and Why?

Mark Colyvan (2008) has argued for a new normative decision theory, namely, *relative expectation theory*, that gives the right verdicts in some (but not all) decision problems where there are an infinite number of states with only finite utilities attached (such as the St. Petersburg game), decision problems where EU theory gives no verdicts whatsoever (see Colyvan and Hájek, 2016, pp. 838–839; Meacham, 2019, pp. 1010–1012). According to Colyvan's new theory, for any agent,  $S$ , and for any two alternative options,  $a$  and  $b$ ,  $S$  rationally prefers  $a$  to  $b$  (and  $a$  is more choiceworthy than  $b$  for  $S$ ) if and only if the probability-weighted sum of the differences in utility between  $a$  and  $b$  for each possible state is positive, and  $S$  is rationally indifferent between  $a$  and  $b$  (and  $a$  is just as choiceworthy as  $b$  for  $S$ ) if and only if the probability-weighted sum of the differences in utility between  $a$  and  $b$  for each possible state is zero. Relative expectation theory gives the same decision advice as EU theory in all

<sup>9</sup> Robert Stalnaker (1972) “suggests that a state of the world be denoted by a conjunction of conditional sentences of the form ‘If action  $A$  were performed then consequence  $C$  would follow; if action  $A'$  were performed then consequence  $C'$  would follow; if ...’.” (Bradley, 2018)

decision problems where there are only a finite number of possible states and where the states are probabilistically independent of all choice options.

Like relative expectation theory, ECU theory requires that one calculate the probability-weighted sum of the differences in utility between the options for each possible state. Therefore, in its handling of decision problems where there are infinitely many states with only finite utilities attached (such as the St. Petersburg game), ECU theory inherits the advantages of relative expectation theory over EU theory. That is to say, in some (but not all) of those *infinite decision* problems, ECU theory delivers the intuitively correct verdicts, whereas EU theory delivers none.

Now, let a *finite decision* problem be a decision problem where there are only finitely many states and no infinite utilities. In all finite decisions under *certainty*, ECU theory delivers the same verdicts as EU theory. And in all finite decisions under *risk* requiring a choice between *only two* alternative options, ECU theory also delivers the same verdicts as EU theory. However, in some finite decisions under *risk* requiring a choice between *more than two* alternative options, ECU theory gives different verdicts from those of EU theory.

Let us consider the following example (Alice's walk): Alice is going for a long walk. She knows that within the next hour, there is a 50% chance of sunny skies (state *A*) and a 50% chance of rain (state *B*). She is faced with a choice between five options: bring a rain poncho and wear rain boots (option *a*), bring an umbrella and wear rain boots (option *b*), bring an umbrella and wear running shoes (option *c*), not bring an umbrella, nor a rain poncho, and wear rain boots (option *d*), and not bring an umbrella, nor a rain poncho, and wear running shoes (option *e*). (From the outset, Alice rules out bringing a rain poncho and wearing running shoes.) Each possible outcome of Alice's choice corresponds to the experience of taking a walk, and the utilities indicate Alice's rational preferences between those possible outcomes. Which option should Alice choose? Should she lug around a poncho or an umbrella and wear heavy rain boots in case it rains, should she forego the poncho and the umbrella and wear running shoes in case the skies are sunny, or should she go for the middle ground: bring an umbrella, but wear running shoes, or not bring an umbrella, nor a rain poncho, but wear rain boots?

The above decision problem can be stated more formally as follows: an agent, *S*, is faced with five choice options: *a*, *b*, *c*, *d*, and *e*. *S* assigns probability 0.5 to a state of the world, *A*, and 0.5 to a state of the world, *B*. If state *A* or state *B* were realized, then *S* would assign the following utilities to the set of options (see Table 2):

**TABLE 2** Decision matrix

	<i>A</i> (0.5)	<i>B</i> (0.5)	EU	CECU
<i>a</i>	2	10	6	0
<i>b</i>	4	8	6	-1
<i>c</i>	6	6	6	-1
<i>d</i>	8	4	6	-1
<i>e</i>	10	2	6	0

According to EU theory, all the options (i.e., *a*, *b*, *c*, *d*, and *e*) are choiceworthy since their EU is the same. By contrast, according to ECU theory, only options *a* and *e* are choiceworthy, since their CECU is equal to or greater than zero. ( $ECU(a) = -3$ ,  $ECU(b) = -4$ ,  $ECU(c) = -4$ ,  $ECU(d) = -4$ ,  $ECU(e) = -3$ ) With respect to Alice's walk, ECU theory therefore recommends that Alice either bring a rain poncho and wear rain boots, or not bring an umbrella, nor a rain poncho, and wear running shoes. This verdict may seem counterintuitive, as ECU theory rules out as unchoiceworthy the middle ground (i.e., bring an umbrella, but wear running shoes, or not bring an umbrella, nor a rain poncho,

but wear rain boots). However, I hope to show that this verdict is plausible in light of the conceptual arguments for ECU theory that I will develop in Chapter 3.

Here is a slightly revised decision matrix (see Table 3):

**TABLE 3** Decision matrix

	<b>A (0.5)</b>	<b>B (0.5)</b>	<b>EU</b>	<b>CECU</b>
<i>a</i>	2	10	6	0
<i>b</i>	5	8	6.5	−0.5
<i>c</i>	6	6	6	−1
<i>d</i>	8	4	6	−1
<i>e</i>	10	2	6	0

According to EU theory, *b* is more choiceworthy than *a*, for *S*, since the EU of *b* (6.5 utiles) is greater than that of *a* (6 utiles). In fact, according to EU theory, *b* is choiceworthy tout court since its EU is greater than that of every other option. By contrast, according to ECU theory, *a* is more choiceworthy than *b*, for *S*, since the CECU of *a* is greater than that of *b*. In fact, according to ECU theory, *a* (and also *e*) is choiceworthy tout court, since its CECU is equal to or greater than zero. ( $ECU(a) = -3$ ,  $ECU(b) = -3.5$ ,  $ECU(c) = -4$ ,  $ECU(d) = -4$ ,  $ECU(e) = -3$ ) Therefore, with respect to Alice's walk, EU theory recommends that Alice bring an umbrella and wear rain boots, whereas ECU theory recommends that Alice either bring a rain poncho and wear rain boots, or not bring an umbrella, nor a rain poncho, and wear running shoes.

ECU theory gives different verdicts from those of EU theory because ECU theory, contrary to EU theory, violates the IIA (for choiceworthiness evaluations). According to this principle, for any decision situation, *T*, and for any choice option, *a*, in *T*, if *a* is choiceworthy in *T*, then *a* is also choiceworthy in *T* if some other option(s) are eliminated from the pool of options in *T*. Likewise, if *a* is not choiceworthy in *T*, then *a* is also not choiceworthy in *T* if some other option(s) are added to the pool of options in *T*.

Let us consider again the decision situation illustrated in Table 3. In that decision situation, ECU theory dictates that *a* is choiceworthy. However, if options *c*, *d*, and *e* are eliminated from the pool of options, then *b* is choiceworthy according to ECU theory (and according to EU theory), as shown below (see Table 4):

**TABLE 4** Decision matrix

	<b>A (0.5)</b>	<b>B (0.5)</b>	<b>EU</b>	<b>CECU</b>
<i>a</i>	2	10	6	−1
<i>b</i>	5	8	6.5	1

According to ECU theory, *b* is choiceworthy tout court since its CECU is equal to or greater than zero. ( $ECU(a) = -0.5$ ,  $ECU(b) = 0.5$ )

Here is another example where ECU theory violates the IIA and gives different verdicts from those of EU theory<sup>10</sup>: An agent, *S*, is faced with two choice options: *a* and *b*. *S* assigns probability 0.001 to a state of the world, *A*, and 0.999 to a state of the world, *B*. If state *A* or state *B* were realized, then *S* would assign the following utilities to the set of options (see Table 5):

<sup>10</sup> Thanks to an anonymous reviewer for giving this example.

**TABLE 5** Decision matrix

	<b>A (0.001)</b>	<b>B (0.999)</b>	<b>EU</b>	<b>CECU</b>
<i>a</i>	1000	1	1.999	0.002
<i>b</i>	0	2	1.998	-0.002

According to ECU theory, *a* is choiceworthy tout court, since its CECU is equal to or greater than zero ( $ECU(a) = 0.001$ ,  $ECU(b) = -0.001$ ). And according to EU theory, *a* is also choiceworthy tout court, since *a* maximizes EU. Let us now introduce a third choice option (*c*) in the decision situation, all else being the same (see Table 6):

**TABLE 6** Decision matrix

	<b>A (0.001)</b>	<b>B (0.999)</b>	<b>EU</b>	<b>CECU</b>
<i>a</i>	1000	1	1.999	-0.898
<i>b</i>	0	2	1.998	0.898
<i>c</i>	900	0	0.9	-2.097

In this new decision situation, *b* is choiceworthy tout court according to ECU theory, since *b*'s CECU is equal to or greater than zero. ( $ECU(a) = -0.899$ ,  $ECU(b) = -0.001$ ,  $ECU(c) = -2.098$ ) By contrast, according to EU theory, *a* is choiceworthy tout court, since *a* maximizes EU. This example is particularly telling because option *c* is statewise dominated by *a*. Whether state A or state B is actual, option *a* is strictly preferred to option *c*. Yet, introducing option *c* in the decision situation changes ECU theory's verdict: *b*, instead of *a*, is uniquely choiceworthy. Thus, ECU theory, contrary to EU theory, violates the *irrelevance of statewise dominated alternatives* (Quiggin, 1994).

Let us now turn to EU theory's and ECU theory's choice situation principles. In decisions under *certainty*, ECU theory always delivers the *same* verdicts as EU theory about what *choice situations* are choiceworthy, and about what *choice situations* are more choiceworthy than or just as choiceworthy as any alternative *choice situations*. Let us consider the following examples:

According to both ECU theory and EU theory, for any agent, *S*, faced with a decision under *certainty* between choice situations 1 and 2 (see Table 7), choice situation 1 is just as choiceworthy for *S* as choice situation 2 since the utility of whatever option(s) in 1 carry the greatest utility is equal to the utility of whatever option(s) in 2 carry the greatest utility. Also, according to both ECU theory and EU theory, for any agent, *S*, faced with a decision under *certainty* between choice situations 1 to 4 (see Table 7), choice situation 4 is uniquely choiceworthy for *S* since the utility of whatever option(s) in 4 carry the greatest utility is greater than the utility of whatever option(s) carry the greatest utility in each of the alternative choice situations (1 to 3).

According to both ECU theory and EU theory, for any agent, *S*, faced with a decision under *certainty* between choice situations 13 and 14 (see Tables 13 and 14 in Section 3.2.2), choice situation 13 is just as choiceworthy for *S* as choice situation 14 since the EU of whatever option(s) in 13 carry the greatest EU is equal to the EU of whatever option(s) in 14 carry the greatest EU. Also, according to both ECU theory and EU theory, for any agent, *S*, faced with a decision under *certainty* between choice situations 13 to 16 (see Tables 13 to 16 in Section 3.2.2), choice situation 16 is uniquely choiceworthy for *S* since the EU of whatever option(s) in 16 carry the greatest EU is greater than the EU of whatever option(s) carry the greatest EU in each of the alternative choice situations (13 to 15).

In these examples, ECU theory gives what seem to be the intuitively correct verdicts. Since I do not have any conceptual arguments to show that these verdicts are implausible, I will take it as given

for the remainder of this book that ECU theory's choice situation principle is correct.

Just as ECU theory delivers some verdicts that are at odds with EU theory, ECU theory also supplies a more discriminating measure of the intervals in rankings of *more than two* choice options. Let us consider four choice situations involving decisions under certainty (see Table 7):

**TABLE 7** Decision matrix

	1	2	3	4
<i>a</i>	5	5	5	5
<i>b</i>	1	1	1	1
<i>c</i>	1	2	2	2
<i>d</i>	1	3	3	3
<i>e</i>	1	3	5	8

The difference in CU between *a* and *b* is greater in situation 1 ( $(5 - 1) - (1 - 5) = 8$  c-utiles) than in situation 2 ( $(5 - 3) - (1 - 5) = 6$  c-utiles), and is greater in situation 2 than in situation 3 ( $(5 - 5) - (1 - 5) = 4$  c-utiles) and situation 4 ( $(5 - 8) - (1 - 8) = 4$  c-utiles). Conversely, the difference in utility between *a* and *b* is the same in all four situations (4 utiles). Therefore, compared to utility, CU is a more discriminating measure of the intervals between *a* and *b* in situations 1 to 4. In other words, compared to utility, CU is a more discriminating measure of the extent to which *a* is more choiceworthy than *b* in situations 1 to 4. And there are not any contrary cases where CU (or CECU) gives a *less* differentiated picture than does utility (or EU).



### 3 The Argument for ECU Theory

This chapter will argue for a new normative theory of rational choice under risk, or, more specifically, a new theory of instrumental rationality, namely ECU theory. The argument can be broken down into 15 steps, which are numbered below.

First, note that, in what follows, *what it is to be choiceworthy* and *choiceworthiness* will be given the following conceptual analyses: For any agent, *S*, faced with any decision under certainty or any decision under risk and for any option, *a*, for *S*, *a* is *choiceworthy* for *S* if and only if *a* is worthy of being chosen by *S* in light of *S*'s rational preferences within each of the various possible states of the world, where *S*'s *rational preferences* are preferences that obey the series of rationality axioms of standard decision theory, and the *degree* to which *a* is *choiceworthy* for *S*, or (i.e.) the *choiceworthiness* of *a* for *S*, is the degree to which *a* is worthy of being chosen by *S* in light of *S*'s rational preferences within each of the various possible states of the world, where *S*'s *rational preferences* are preferences that obey the series of rationality axioms of standard decision theory.

More explicitly, for any agent, *S*, faced with any decision under certainty or any decision under risk and for any option, *a*, for *S*, *a* is *choiceworthy* for *S* if and only if *a* is worthy of being chosen by *S* in light of *S*'s decision matrix, that is, in light of what utilities *S* assigns to each option within each of the various possible states of the world and what probabilities *S* assigns to each of the various possible state of the world, and the *degree* to which *a* is *choiceworthy* for *S*, or (i.e.) the *choiceworthiness* of *a* for *S*, is the degree to which *a* is worthy of being chosen by *S* in light of *S*'s decision matrix, that is, in light of what utilities *S* assigns to each option within each of the various possible states of the world and what probabilities *S* assigns to each of the various possible state of the world.

The *argument for ECU theory* proceeds as follows:

1. For any agent, *S*, and for any option, *a*, for *S*, *a* is choiceworthy for *S* if and only if *a* is *maximally choiceworthy* for *S* over the space of all alternatives in the choice set.
2. For any agent, *S*, and for any option, *a*, for *S*, *a* is *maximally choiceworthy* for *S* over the space of all alternatives in the choice set if and only if *a* *maximizes choiceworthiness* for *S* over the space of all alternatives in the choice set.
3. For any agent, *S*, and for any option, *a*, for *S*, *a* is choiceworthy for *S* if and only if *a* *maximizes choiceworthiness* for *S* over the space of all alternatives in the choice set (i.e., the *choiceworthiness maximization (CM) principle*). (3 follows from 1 and 2.)
4. For any agent, *S*, faced with any decision under *risk* and for any option, *a*, for *S*, the measure of the choiceworthiness of *a* for *S* is its CECU (i.e., the CECU principle).
5. For any agent, *S*, faced with any decision under *risk* and for any option, *a*, for *S*, *a* is choiceworthy for *S* if and only if *a* maximizes CECU. (5 follows from 3 and 4.)
6. For any agent, *S*, faced with any decision under *risk* and for any number of alternative options, *a*, *b*, *c*, *d*, and *e*, for *S*, it is rational for *S* to prefer *a* to *b* if and only if *a*'s EU is greater than *b*'s, it is rational for *S* to be indifferent between *a* and *b* if and only if *a*'s EU is equal to *b*'s, and the extent to which *S* rationally prefers *a* to *b* is the difference in EU between *a* and *b*.
7. For any agent, *S*, faced with any decision under *risk* and for any option, *a*, for *S*, it is rational for *S* to weakly prefer *a* over the alternative options in the choice set if and only if *a* maximizes EU. (7 follows from 6.)
8. In decisions under *risk*, what option(s) maximize CECU sometimes differ from what option(s) maximize EU.
9. In decisions under *risk*, what option(s) are choiceworthy sometimes differ from what option(s) it is rational to weakly prefer over the alternative options in the choice set. (9 follows from 5, 7, and



8.)

10. For any agent, *S*, faced with any decision under *risk*, *S* ought to measure and rank her options (for the purpose of choice) in terms of *how choiceworthy* they are for *S*.
11. It is *not* the case that for any agent, *S*, faced with any decision under *risk*, *S* ought to measure and rank her options (for the purpose of choice) in order of rational preference. (11 follows from 3, 9, and 10.)
12. For any agent, *S*, faced with any decision under *risk*, *S* ought to measure and rank her options (for the purpose of choice) in terms of how choiceworthy they are for *S*, that is, according to their CECU, rather than in order of rational preference, that is, according to their EU. (12 follows from 4, 6, 10, and 11, as well as from 3, 4, 6, 9, and 13.)
13. For any agent, *S*, faced with any decision under *risk*, *S* ought to choose out of what option(s) are *choiceworthy* for *S*.
14. It is *not* the case that for any agent, *S*, faced with any decision under *risk*, *S* ought to choose out of what option(s) it is rational for *S* to weakly prefer over the alternative options in the choice set. (14 follows from 9 and 13.)
15. For any agent, *S*, faced with any decision under *risk*, *S* ought to choose out of what option(s) are choiceworthy for *S* (i.e., what option(s) maximize CECU), even in cases where what option(s) are choiceworthy for *S* differ from what option(s) it is rational for *S* to weakly prefer over the alternative options in the choice set (i.e., what option(s) maximize EU). (15 follows from 5, 7, 9, 13, and 14, as well as from 3, 5, 7, 9, and 10.)

I will now discuss the different steps in the argument:

1. For any agent, *S*, and for any option, *a*, for *S*, *a* is choiceworthy for *S* if and only if *a* is *maximally choiceworthy* for *S* over the space of all alternatives in the choice set.

The question of whether a given option is more (or less) choiceworthy than (or just as choiceworthy as) another option within a set of alternatives is well-formed and meaningful. Therefore, the question of whether a given option is *maximally choiceworthy* within a set of alternatives is also well-formed and meaningful. A given option is *maximally choiceworthy* within a set of alternatives if and only if it is at least as choiceworthy as each of the other options within the set of alternatives. I will assume that Step 1 is true without further argument.

2. For any agent, *S*, and for any option, *a*, for *S*, *a* is *maximally choiceworthy* for *S* over the space of all alternatives in the choice set if and only if *a* *maximizes choiceworthiness* for *S* over the space of all alternatives in the choice set.

For any number of alternative choice options, *a*, *b*, *c*, *d*, and *e*, we want to say that *a* (utility: 100) is more choiceworthy than *b* (utility: 5) even if *a* is not choiceworthy tout court (i.e., *a* does not maximize utility). We also want to say that the *extent* to which *a* is more choiceworthy than *b* is greater than the extent to which *c* (utility: 10) is more choiceworthy than *b*. In order to say that *a* is more choiceworthy than *b* (and to what extent), we cannot rely on a binary measure of choiceworthiness. Whether (and to what extent) *a* is more choiceworthy than *b*, and by implication, whether (and to what extent) any option is more choiceworthy than any other within a set of alternatives is necessarily a function of *how* choiceworthy each of the two options is within the set of alternatives (and not necessarily a function of one being choiceworthy tout court and the other unchoiceworthy tout court). To ask how choiceworthy an option is to ask how worthy of being chosen that option is. Such a question is well-formed and meaningful. In order to answer the question, we require a graded,

quantitative measure of how choiceworthy options are—that is, we require a graded, quantitative measure of *choiceworthiness*.

3. For any agent, *S*, and for any option, *a*, for *S*, *a* is choiceworthy for *S* if and only if *a* *maximizes choiceworthiness* for *S* over the space of all alternatives in the choice set (i.e., the CM principle). (3 follows from 1 and 2.)
4. For any agent, *S*, faced with any decision under *risk* and for any option, *a*, for *S*, the measure of the choiceworthiness of *a* for *S* is its CECU (i.e., the CECU principle).

### 3.1 Step 4: The CU Principle

In order to establish the CECU principle, I first need to argue for a graded, quantitative measure of choiceworthiness for decisions under *certainty* (i.e. the CU principle). According to the CU principle, for any agent, *S*, faced with any decision under *certainty* and for any option, *a*, for *S*, the measure of the choiceworthiness of *a* for *S* is its comparative utility (CU) in the state of the world to which *S* assigns probability 1. For any choice option, *a*, and for any state of the world, *G*, *a*'s CU in *G* is the difference in utility, in *G*, between *a* and whichever alternative(s) to *a* carry the greatest utility in *G*. In what follows, I will develop several different lines of argument in support of the CU principle.

To that end, I will assume that for any agent, *S*, faced with any decision under *certainty* and for any option, *a*, for *S*, *a* is *choiceworthy* for *S* if and only if *a* is (amongst any of) the most (rationally) preferred option(s) in the state of the world to which *S* assigns probability 1—that is to say, *a* is *choiceworthy* for *S* if and only if *a* maximizes *utility* over the space of all alternatives in the state of the world to which *S* assigns probability 1. I will refer to this principle as the *utility maximization (UM) principle*. The UM principle defines a binary measure of choiceworthiness for decisions under certainty (i.e., whether an option is choiceworthy tout court or unchoiceworthy tout court).

According to EU theory, for any agent, *S*, faced with any decision under *certainty* and for any number of alternative options, *a*, *b*, *c*, *d*, and *e*, available to *S*, the extent to which *a* is more choiceworthy than *b*, for *S*, is the extent to which *S* (rationally) prefers *a* to *b*, or equivalently the extent to which *S* (rationally) prefers *a* to *b* *more* than *S* (rationally) prefers *b* to *a*. However, intuitively, that is a mistake. Even though we are comparing *a* to *b*, we want to see how *a* and *b* measure up to the *very best alternative options on offer*, in the following way: the extent to which *a* is more choiceworthy than *b*, for *S*, is the extent to which *S* (rationally) prefers *a* to the most (rationally) preferred alternative(s) to *a* (either *b*, *c*, *d*, or *e*) *more* than *S* (rationally) prefers *b* to the most (rationally) preferred alternative(s) to *b* (either *a*, *c*, *d*, or *e*). It follows that the extent to which *a* is choiceworthy for *S* *tout court* is the extent to which *S* (rationally) prefers *a* to the most (rationally) preferred alternative(s) to *a* (either *b*, *c*, *d*, or *e*). In other words, the extent to which *a* is choiceworthy for *S* (or [i.e.] the measure of *how* choiceworthy *a* is for *S*) is *a*'s CU—that is to say, the difference in utility between *a* and whichever alternative(s) to *a* carry the greatest utility (i.e., *b*, *c*, *d*, or *e*). (The same goes for option *b*.) This is what I have referred to as the CU principle.

The rationale behind the CU principle is simple: If *S* must forgo option *a*, then *S* ought to choose (any of) the most (rationally) preferred alternative(s) to *a* (either *b*, *c*, *d*, or *e*), and not necessarily the option to which *S* is comparing *a* (i.e., option *b*). If *S* *does* choose option *a*, then *S* necessarily forgoes the most (rationally) preferred alternative(s) to *a* (either *b*, *c*, *d*, or *e*). Therefore, the extent to which *a* is choiceworthy for *S* *tout court* is the extent to which *a* is worthy of being chosen over the most (rationally) preferred alternative(s) to *a* (either *b*, *c*, *d*, or *e*). By conceptual analysis, it follows that the extent to which *a* is choiceworthy for *S* is the extent to which *S* (rationally) prefers *a* to the most (rationally) preferred alternative(s) to *a* (either *b*, *c*, *d*, or *e*)—that is, the extent to which *a* is

choiceworthy for  $S$  is  $a$ 's CU.

To express this point differently,  $a$  is choiceworthy for  $S$  if and only if  $a$  is worthy of being chosen by  $S$  over whichever alternative(s) to  $a$  are the most choiceworthy for  $S$ . (This is true by definition.) And  $a$  is choiceworthy for  $S$  if and only if  $a$  maximizes choiceworthiness for  $S$  over the space of all alternatives in the choice set. (This is the CM principle, or step 3 of the argument for ECU theory.) Therefore, *the extent to which  $a$  is choiceworthy for  $S$  is the extent to which  $a$  is worthy of being chosen by  $S$  over whichever alternative(s) to  $a$  are the most choiceworthy for  $S$* . From the conjunction of the CM principle and UM principle, it follows that the extent to which  $a$  is choiceworthy for  $S$  is the extent to which  $a$  is worthy of being chosen by  $S$  over the most (rationally) preferred alternative(s) to  $a$ . And by conceptual analysis, it follows that the extent to which  $a$  is choiceworthy for  $S$  is the extent to which  $S$  (rationally) prefers  $a$  to the most (rationally) preferred alternative(s) to  $a$ —that is, the extent to which  $a$  is choiceworthy for  $S$  is  $a$ 's CU.

A related observation: In determining the extent to which  $a$  is choiceworthy for  $S$  *tout court*,  $S$  ought to take into account the utility cost or benefit to  $S$  of choosing (any of) the most (rationally) preferred alternative(s) to  $a$  over  $a$ . The greater the utility *cost* to  $S$  of choosing (any of) the most (rationally) preferred alternative(s) to  $a$  over  $a$ , the more choiceworthy  $a$  is to  $S$ , and the greater the utility *benefit* to  $S$  of choosing (any of) the most (rationally) preferred alternative(s) to  $a$  over  $a$ , the less choiceworthy  $a$  is to  $S$ . This observation provides further support to the CU principle.

An alternative to the CU principle is a principle which states that the extent to which  $a$  is choiceworthy for  $S$  (or [i.e.] the measure of *how* choiceworthy  $a$  is for  $S$ ) is  $a$ 's  $CU^*$ , that is, the difference in utility between  $a$  and whichever *option(s)* carry the greatest utility (i.e.,  $a$ ,  $b$ ,  $c$ ,  $d$ , or  $e$ ), where this difference in utility is calculated *in the state of the world to which  $S$  assigns probability 1*. I will refer to this principle as the  $CU^*$  principle. According to BR theory, as discussed in the Introduction, the degree of choiceworthiness of  $a$  *in any state of the world*,  $G$ , is the difference in utility, in  $G$ , between  $a$  and whichever *option(s)* carry the greatest utility in  $G$ . Understood in this way, *expected* choiceworthiness—or *choiceworthiness tout court* according to BR theory—is a probability-weighted sum of each of an option's degrees of choiceworthiness across *the various possible states of the world*.

The  $CU^*$  principle (and, by implication, BR theory) is however untenable, not only for the reasons just stated, but also because it results in a double standard: On the one hand, the degrees of choiceworthiness of all the option(s) that do *not* carry the greatest utility *do* depend (beyond the fact of their *not* carrying the greatest utility) on what other options are available in the choice set—those degrees of choiceworthiness may be different negative numbers, but never 0. On the other hand, the degrees of choiceworthiness of all the option(s) that *do* carry the greatest utility *do not* depend (beyond the fact of their carrying the greatest utility) on what other options are available in the choice set—those degrees of choiceworthiness are 0 no matter what the utilities of the other options are. Moreover, the latter standard is implausible. It's as if the degrees of choiceworthiness of all the option(s) that *do not* carry the greatest utility did *not* depend (beyond the fact of their *not* carrying the greatest utility) on what other options are available in the choice set—it's as if those degrees of choiceworthiness were the same negative number, e.g.,  $-1$ , no matter what the utilities of the other options are.

The  $CU^*$  principle (and, by implication, BR theory) faces another problem: Since the  $CU^*$  principle requires that all choiceworthy options carry a  $CU^*$  of 0, it blurs the distinction between options that are merely *optional* (i.e., *non-uniquely* choiceworthy options) and options that are *not* optional, that is, options that one is *required* to choose (i.e., *uniquely* choiceworthy options). The latter options are more choiceworthy than the former options and should therefore carry a greater comparative value. As such, while it makes sense to assign a comparative value of 0 to merely optional choice options (i.e., non-uniquely choiceworthy options), it makes better sense to assign a (graded) *positive* comparative value to options that one is required to choose (i.e., uniquely choiceworthy

options), in accordance with the original CU principle.

Ralph Wedgwood (2013) has objected to a version of BR theory for what is, in his words, an “intuitively obvious reason”:

[A]ccording to the rule of minimizing evidentially expected *regret*, the suboptimal options in every state of nature are “marked down” for the extent to which they are inferior to the optimal options, but the optimal options are in any way not “marked up” for the extent to which they are better than those suboptimal alternatives. For example, consider the following case: on the assumption that one chooses *A*, it is to all intents and purposes conditionally certain that *A* will be optimal, and that *B* will be *very slightly* less good than *A*—whereas on the assumption that one chooses *B*, it is conditionally certain that *B* will be optimal, and that *A* will be *dramatically* less good than *B*. The rule of minimizing evidentially expected regret would treat these two choices as equally rational; but this seems to overlook an intuitively obvious reason for favouring *B* over *A*. (p. 2664)

Contrary to the CU\* principle (and BR theory), the original CU principle does not suffer from any of these problems.

Let us consider four choice situations involving decisions under certainty (see Table 8): Compared to the *difference in utility* and the *difference in CU\**, the *difference in CU* is a more plausible measure of the extent to which *a* is more choiceworthy than *b* in situations 1–4, as explained above. The differences in utility and CU\* between *a* and *b* are the same in all four situations (4 units), whereas the differences in CU between *a* and *b* are as follows (in situations 1–4):

**TABLE 8** Decision matrix<sup>a</sup>

	1	2	3	4
<i>a</i>	5	5	5	5
<i>b</i>	1	1	1	1
<i>c</i>	1	2	2	2
<i>d</i>	1	3	3	3
<i>e</i>	1	3	5	8

<sup>a</sup> Table 8 is identical to Table 7.

1.  $(5 - 1) - (1 - 5) = 8$  c-utiles
2.  $(5 - 3) - (1 - 5) = 6$  c-utiles
3.  $(5 - 5) - (1 - 5) = 4$  c-utiles
4.  $(5 - 8) - (1 - 8) = 4$  c-utiles

Let us now consider a final line of argument for a graded, quantitative choiceworthiness measure for decisions under certainty (i.e., CU). *Measures* of quantities that have an interval scale, for example 20°C for temperature, are meaningful (and *only* meaningful) relative to a given zero point and unit of measurement. (Let us call this the *measurement principle*.) In accordance with the measurement principle, for any agent, *S*, faced with *any* decision situation under certainty and for any option, *a*, for *S*, the measure of the choiceworthiness of *a* for *S* depends on a unit of measurement of choiceworthiness as well as a zero point of choiceworthiness (or *benchmark*) in the following way: the measure of the choiceworthiness of *a* for *S* (relative to *any* explicitly given utility unit and zero point of utility) is the *difference in utility* between *a* and some *benchmark* for *a*, such that (i) *a* is

choiceworthy for  $S$  if and only if the difference in utility between  $a$  and the benchmark for  $a$  is equal to or greater than zero (and not choiceworthy otherwise), and (ii) the degree of choiceworthiness of  $a$  for  $S$  is the difference in utility between  $a$  and the benchmark for  $a$ . In other words, the measure of the choiceworthiness of  $a$  for  $S$  is the degree to which  $a$  is worthy of being chosen over the benchmark for  $a$ . The benchmark for  $a$  can be, for example, some option in the set of available options, such as whichever option has the highest utility, whichever option has the lowest utility, or the status quo, or some average of the utilities of the available options. As will become clear in what follows, the concept of choiceworthiness itself presupposes a given benchmark (or zero point of choiceworthiness).

If there are any alternatives to  $a$  which carry a greater utility than does  $a$ , then (in accordance with BR theory) the benchmark for  $a$  is whichever alternative to  $a$  carries the greatest utility (or one of them in the event that several alternatives are tied). Indeed, if there are any alternatives to  $a$  with a greater utility than  $a$ , then, in accordance with the UM principle,  $a$  is not choiceworthy for  $S$ . But if  $a$  is not choiceworthy for  $S$ , then how choiceworthy  $a$  is for  $S$  is simply how  $a$  compares to whichever alternative(s) are choiceworthy for  $S$  (or, per the UM principle, whichever alternative(s) to  $a$  carry the greatest utility). I will now argue (contra BR theory) that if there are *not* any alternatives to  $a$  which carry a greater utility than does  $a$ , then the benchmark for  $a$  still has to be whichever alternative to  $a$  carries the greatest utility (or one of them in the event that several alternatives are tied).

Let us consider two decision situations: 1 and 2. In each situation,  $S$  is faced with the same three options:  $a$ ,  $b$ , and  $c$ . What's more, in each situation,  $S$  assigns probability 1 to a given state of the world (but not the same state for both situations). If that state of the world were realized, then  $S$  would assign the following utilities to the set of options (see Table 9):

**TABLE 9** Decision matrix

	1	2
$a$	100	100
$b$	-100	99
$c$	-100	99

Per the UM principle,  $a$  is choiceworthy for  $S$  in both situations 1 and 2.  $a$  is also more choiceworthy for  $S$  in 1 than in 2. In 2,  $S$  misses out on only 1 utile by not choosing  $a$ , but instead choosing the best alternative to  $a$  (i.e.,  $b$  or  $c$ ), whereas in 1,  $S$  misses out on 200 utiles by not choosing  $a$ , but instead choosing the best alternative to  $a$  (i.e.,  $b$  or  $c$ ). Another way of putting it is that  $a$  is more choiceworthy in 1 than in 2 because  $a$  is more worthy of being chosen over the best alternatives to  $a$  in 1 than in 2.

Here is a different example (see Table 10):

**TABLE 10** Decision matrix

	1	2
$a$	100	100
$b$	-100	-100
$c$	-100	100

Per the UM principle,  $a$  is choiceworthy for  $S$  in both situations 1 and 2.  $a$  is also more choiceworthy for  $S$  in 1 than in 2. In 2,  $a$  is merely optional— $S$  misses out on *zero* utiles by not choosing  $a$ , but instead choosing the best alternative to  $a$  (i.e.,  $c$ )—whereas in 1,  $a$  is *not* optional— $S$  misses out on 200 utiles by not choosing  $a$ , but instead choosing the best alternative to  $a$  (i.e.,  $b$  or  $c$ ).

Again,  $a$  is more choiceworthy in 1 than in 2 because  $a$  is more worthy of being chosen over the best alternative(s) to  $a$  in 1 than in 2.

Let me now introduce Ralph Wedgwood's (2013) *benchmark theory* (BT).<sup>11</sup> The basic idea of BT is to rank choice options (in terms of how *choiceworthy*<sup>12</sup> they are) according to their *expected comparative value*, where the comparative value of an option is its generic *value* (or goodness) in some state of the world compared to a benchmark for that state of the world. Wedgwood identifies the benchmark as an average of the options' values within a given state of the world. He emphasizes that all statewise dominated options and more generally, "all the options that *do not deserve to be taken seriously*" (p. 2664) should be excluded from consideration at the outset. Wedgwood explicitly does not assume that the value of an option is its utility. Nevertheless, it is interesting to see how BT (henceforth, *BT\**) fairs when the value of an option is understood to be its utility.

Let us consider the following example (see Table 11):

**TABLE 11** Decision matrix

	1	2
$a$	100	100
$b$	-100	-500
$c$	-100	99

Per the UM principle,  $a$  is choiceworthy for  $S$  in both situations 1 and 2.  $a$  is also more choiceworthy for  $S$  in 1 than in 2. In 2,  $S$  misses out on only 1 utile by not choosing  $a$ , but instead choosing the best alternative to  $a$  (i.e.,  $c$ ), whereas in 1,  $S$  misses out on 200 utiles by not choosing  $a$ , but instead choosing the best alternative to  $a$  (i.e.,  $b$  or  $c$ ). Once again,  $a$  is more choiceworthy in 1 than in 2 because  $a$  is more worthy of being chosen over the best alternative(s) to  $a$  in 1 than in 2.

BT\* agrees with the verdict that  $a$  is choiceworthy for  $S$  in 1 and 2, but *not* with the verdict that  $a$  is more choiceworthy for  $S$  in 1 than in 2. According to BT\*,  $a$  is equally choiceworthy for  $S$  in situations 1 and 2 since  $b$  and  $c$  are strictly dominated by  $a$  in both 1 and 2 and are therefore excluded from consideration at the outset. If  $b$  and  $c$  are *not* excluded from consideration and the benchmark is identified as an average of the values (or utilities) of all the options, then  $a$  is more choiceworthy for  $S$  in 2 than in 1. I take this to be a counterexample to BT\*.

These three examples serve to illustrate that if there are *not* any alternatives to  $a$  with a greater utility than  $a$ , then how choiceworthy  $a$  is depends on how much utility  $S$  would miss out on by not choosing  $a$ , but instead choosing the best alternative to  $a$ . The greater the amount of utility  $S$  would miss out on by not choosing  $a$ , but instead choosing the best alternative to  $a$ , the more choiceworthy  $a$  becomes. Thus, the benchmark for  $a$  must be whichever alternative to  $a$  carries the highest utility (or one of them in the event that several alternatives are tied).

What follows is that whether or not there are any alternatives to  $a$  which carry a greater utility than does  $a$ , the benchmark for  $a$  has to be whichever alternative to  $a$  carries the greatest utility (or one of them in the event that several alternatives are tied). This means that there is no unique benchmark for a given choice situation. Instead, the benchmark is defined in relation to a specific choice option. The benchmark for  $a$  may be some alternative,  $b$ , and the benchmark for  $b$  may be  $a$ . Therefore, for any agent,  $S$ , faced with any decision under certainty and for any option,  $a$ , for  $S$ , the measure of the choiceworthiness of  $a$  for  $S$  (relative to any explicitly given utility unit and zero point of utility) is the *CU* of  $a$  (in the state of the world to which  $S$  assigns probability 1). The CU of  $a$  is the difference in utility between  $a$  and whichever alternative(s) to  $a$  carry the greatest utility. As

<sup>11</sup> For critiques of BT, see Briggs, 2010; and Bassett, 2015.

<sup>12</sup> In Wedgwood's (2013) terminology, BT shows how to rank choices in terms of how *rational* they are.



previously indicated, I will refer to this principle as the CU principle. The CU principle implies the UM principle.

### 3.2 Step 4: The CECU Principle

As I argued in discussing Step 2 of the argument for ECU theory, we require a graded, quantitative measure of how choiceworthy options are. It is sometimes assumed that if expected utility theory is the correct normative theory of rational choice under *certainty* and rational choice under *risk*, then for any agent, *S*, faced with any decision under *certainty* or any decision under *risk* and for any option, *a*, for *S*, the measure of the choiceworthiness of *a* for *S* is *a*'s *expected utility*. For example, according to Briggs (2023),

expected utility theory provides a way of ranking the acts according to how *choiceworthy* they are: the higher the expected utility, the better it is to choose the act. (It is therefore best to choose the act with the highest expected utility—or one of them, in the event that several acts are tied.)

Similarly, according to Steele & Stefánsson (2020),

preference between options is a judgment of comparative desirability or choiceworthiness [...] The above analysis presumes that lotteries are evaluated in terms of their expected choice-worthiness or desirability. [...] That is, the desirability of [a] lottery is a probability weighted sum of the utilities of its prizes, where the weight on each prize is determined by the probability that the lottery results in that prize.

Let us suppose that for any agent, *S*, faced with any decision under *certainty* or any decision under *risk* and for any option, *a*, for *S*, the measure of the choiceworthiness of *a* for *S* is *a*'s *expected utility*. Since *a*'s expected utility is *a*'s *utility* in the state of the world to which *S* assigns probability 1, it then follows that for any agent, *S*, faced with any decision under *certainty* and for any option, *a*, for *S*, the measure of the choiceworthiness of *a* for *S* is *a*'s *utility* in the state of the world to which *S* assigns probability 1. I will refer to the latter as the *utility principle*. The UM principle is true if (*but not only if*) the utility principle is true. However, in light of the CU principle, the utility principle can be falsified. It therefore follows that for any agent, *S*, faced with any decision under *certainty* or any decision under *risk* and for any option, *a*, for *S*, the measure of the choiceworthiness of *a* for *S* *cannot* be *a*'s *expected utility*.

If the utility principle were true, then in accordance with the measurement principle, it would be the case that for *any* given decision situation, there is at least one specification of a utility unit and zero point of utility such that it is possible to ascertain how choiceworthy any available option is (for *S*) by solely considering its utility value in relation to that specification of a utility unit and zero point of utility. In other words, it would be the case that for *any* given decision situation, there is at least one specification of a utility unit and zero point of utility such that (a) any available option is choiceworthy (for *S*) if and only if its utility value is equal to or greater than zero (and not choiceworthy otherwise) and (b) *the degree of choiceworthiness of any available option (for S) is its utility value*. As we will now see, that is not the case. Let us consider the following decision setup: *S* is faced with three options: *a*, *b*, and *c*. What's more, *S* assigns probability 1 to a given state of the world. If that state of the world were realized, then *S* would assign the following utilities to the available options: *a* (0), *b* (−100), *c* (−1000). Therefore, no matter what zero point of utility is selected,

$S$  assigns the following utility intervals between the available options: between  $a$  and  $b$ ,  $S$  assigns a positive interval of 100 utiles, between  $b$  and  $c$ ,  $S$  assigns a positive interval of 900 utiles and between  $a$  and  $c$ ,  $S$  assigns a positive interval of 1000 utiles. Per the CU principle, the degrees of choiceworthiness of the available options are as follows:  $a$  (100),  $b$  (−100),  $c$  (−1000). Therefore, the differences between the degrees of choiceworthiness of the available options are as follows: between  $a$  and  $b$ , the difference is 200 c-utiles, between  $b$  and  $c$ , the difference is 900 c-utiles and between  $a$  and  $c$ , the difference is 1100 c-utiles. Since the utility intervals and the differences in degrees of choiceworthiness are at variance, we have a decision situation where no matter what zero point of utility (and what utility unit) is selected, it is *not* the case that the degree of choiceworthiness of any available option is its utility value.

In Section 3.1, I argued that for any agent,  $S$ , faced with any decision under *certainty* and for any number of alternative options,  $a$ ,  $b$ ,  $c$ ,  $d$ , and  $e$ , available to  $S$ , the extent to which  $a$  is more choiceworthy than  $b$ , for  $S$ , is the extent to which  $S$  (rationally) prefers  $a$  to the most (rationally) preferred alternative(s) to  $a$  (either  $b$ ,  $c$ ,  $d$ , or  $e$ ) *more* than  $S$  (rationally) prefers  $b$  to the most (rationally) preferred alternative(s) to  $b$  (either  $a$ ,  $c$ ,  $d$ , or  $e$ ). It could be proposed that the same holds for any agent,  $S$ , faced with any decision under *risk*. Therefore, in decisions under *risk*, the extent to which  $a$  is more choiceworthy than  $b$ , for  $S$ , would be the extent to which {the difference in *expected utility* between  $a$  and whichever alternative(s) to  $a$  carry the greatest *expected utility* (i.e.,  $b$ ,  $c$ ,  $d$ , or  $e$ )} is greater than {the difference in *expected utility* between  $b$  and whichever alternative(s) to  $b$  carry the greatest *expected utility* (i.e.,  $a$ ,  $c$ ,  $d$ , or  $e$ )}. It would follow that the extent to which  $a$  is choiceworthy for  $S$  (or [i.e.] the measure of *how* choiceworthy  $a$  is for  $S$ ) is the difference in *expected utility* between  $a$  and whichever alternative(s) to  $a$  carry the greatest *expected utility* (i.e.,  $b$ ,  $c$ ,  $d$ , or  $e$ ). The same would be the case for option  $b$ .<sup>13</sup> However appealing this theory might seem, it runs afoul of Ralph Wedgwood's (2013) Gandalf's principle, which I will now introduce.

If we accept, as nearly all decision theorists do,<sup>14</sup> that computing the *expected value* (or [i.e.] the *probability-weighted sum* of all *possible* values) of options is the appropriate value-aggregation rule to apply to decisions under risk, then *Gandalf's principle* says that, for any agent,  $S$ , faced with any decision under *risk* and for any option,  $a$ , for  $S$ , how choiceworthy  $a$  is for  $S$  is the probability-weighted sum of  $a$ 's degrees of choiceworthiness *within* each of the various states of the world in which  $S$  might find herself. What's more, *Gandalf's principle* says that, for any agent,  $S$ , faced with any decision under *risk* and for any option,  $a$ , for  $S$ , how choiceworthy  $a$  is for  $S$  *within* each of the various states of the world in which  $S$  might find herself should be calculated as if  $S$  were faced with a *decision under certainty* *within* each of the various states of the world in which she might find herself. (E.g. what should I do if it rains tomorrow? I should decide *as if* I am faced with a *decision under certainty* in the state "it will rain tomorrow" even if I am not certain that it will rain tomorrow.) In other words, Gandalf's principle says that for any agent,  $S$ , faced with any decision under *risk* and for any option,  $a$ , for  $S$ , how choiceworthy  $a$  is for  $S$  *within any given state of the world* in which  $S$  might find herself should only be measured in relation to the values of the other options *in that state of the world*, and not to the values of the options *in the other states of the world* in which  $S$  might find herself.

Now, for any agent,  $S$ , faced with any decision under *certainty* and for any option,  $a$ , for  $S$ , the choiceworthiness of  $a$  for  $S$  depends on both  $a$ 's utility and the utility of the best (i.e., most rationally preferred) option(s) to be forgone if  $S$  chooses  $a$ . The greater the difference in utility between  $a$  and the best option(s) to be forgone if  $S$  chooses  $a$ , the more choiceworthy  $a$  is for  $S$ . Therefore, the choiceworthiness of  $a$  for  $S$  is the difference in utility between  $a$  and the best option(s) to be forgone

<sup>13</sup> The idea of calculating the difference in expected utility between some option and whichever alternative option(s) carry the greatest expected utility is mentioned in Zhu, 2018.

<sup>14</sup> According to Martin Peterson (2017, p. 66), "nearly all decision theorists agree that [...] the principle of maximizing expected value is the appropriate decision rule to apply to decisions under risk. [...] There are no serious contenders."



if  $S$  chooses  $a$  (i.e.,  $a$ 's CU). This is the CU principle.

If Gandalf's principle and the CU principle are correct, then, for any agent,  $S$ , faced with any decision under *risk* and for any option,  $a$ , for  $S$ , how choiceworthy  $a$  is for  $S$  *within* each of the various states of the world in which  $S$  might find herself is  $a$ 's CU within each of those states of the world, and, consequently, how choiceworthy  $a$  is for  $S$  *tout court* is the probability-weighted sum of  $a$ 's CUs within each of the various states of the world in which  $S$  might find herself—that is, how choiceworthy  $a$  is for  $S$  *tout court* is  $a$ 's ECU. That roughly encapsulates *ECU theory*.

ECU theory, as formulated above, is not quite right though. In accordance with the measurement principle, if the measure of the choiceworthiness of options is their ECU, then only options with ECU equal to or greater than zero can be choiceworthy. However, as I illustrated in Section 2.2, there will always be cases (regardless of what utility unit and zero point of utility are specified) where every option in a decision situation *under risk* has negative ECU. Since at least one option in a decision situation must be choiceworthy—the one with the highest degree of choiceworthiness (or one of them in the event that several options are tied)—ECU theory, as defined above, is false in decision situations *under risk*. By the same line of reasoning as employed in Section 3.1 (see my final line of argument), we reach the following conclusion: ECU theory is the conjunction of the CU principle (for decisions under certainty) and the CECU principle (for decisions under risk) (as well as ECU theory's choice situation principle). Let us recall that according to the CECU principle, for any agent,  $S$ , faced with any decision under *risk* and for any choice option,  $a$ , for  $S$ , the measure of the choiceworthiness of  $a$  for  $S$  (relative to any explicitly given utility unit and zero point of utility) is the CECU of  $a$ , that is to say, the difference in ECU between  $a$  and whichever alternative(s) to  $a$  carry the greatest ECU.

In what follows, I will develop three lines of argument in support of the CECU principle. We have axiomatic justifications for the principle that rational preferences maximize expected utility. Not so for the principle that (instrumentally) rational choices under risk maximize choiceworthiness (or CECU). It would be useful to have such a justification, but we can do without. In support of the CECU principle, I will begin by discussing Gandalf's principle at greater length.

### 3.2.1 Argument 1: Gandalf's Principle

According to Wedgwood (2013, p. 2654),

to determine how one should choose when one is certain about what the actual state of nature is, one simply *does not need to know* whether one is in a nice state of nature or a nasty state of nature. [...] I suggest the following way of generalizing Gandalf's point to cases involving uncertainty [i.e., risk]: to make a rational choice in such cases, one *does not need to consider* whether one is in a nice state of nature or a nasty one. All that one needs to consider are the *degrees* to which each of the available options is better (or worse) than the available alternatives *within* each of the relevant states of nature. Admittedly, when one is uncertain which state of nature one is in, one must make *some* comparisons across the states of nature. But since one does not even need to know whether one is in a nice state of nature or a nasty one, it seems that the only relevant comparisons are comparisons of the *differences* in levels of goodness between the various options *within* each state of nature with the *differences* between those options within each of the other states of nature—not any comparisons of *absolute* levels of goodness across different states of nature.

Although Wedgwood uses terms such as “better,” “worse,” and “levels of goodness” in his explication

of Gandalf's principle, the principle can be expressed equally well using replacement terms such as "preferred," "dispreferred," and "levels of utility."

Gandalf's principle is an eminently reasonable principle (see Wedgwood, 2013, pp. 2652–2655). In a paper critiquing Wedgwood's BT, Robert Bassett (2015) concurs:

Gandalf's Principle strikes me as an eminently sensible principle to incorporate into rational decision-making. [...] Whatever state you're actually in now, it does you no good to compare what you're best off actually doing now to how you might have done had things been different. That comparison does not serve to reflect the choice that you actually face. (pp. 244–245)

As discussed above, when combined with an expected-value aggregation rule, the CU principle, and the measurement principle, Gandalf's principle implies the CECU principle (and not EU theory). There is, however, one alternative to the CECU principle that is consistent with the conjunction of Gandalf's principle, the CU principle, and the measurement principle, and that has some *prima facie* plausibility—we can call this new theory *maximum likelihood comparative utility (MLCU) theory*. According to MLCU theory, for any agent, *S*, and for any option, *a*, for *S*, the measure of the choiceworthiness of *a* for *S* (relative to any explicitly given utility unit and zero point of utility) is the *most likely value* of *a*'s choiceworthiness (or CU) in the actual state of the world, and in cases where there is more than one maximally likely value of *a*'s choiceworthiness (or CU) in the actual state of the world, the measure of the choiceworthiness of *a* for *S* (relative to any explicitly given utility unit and zero point of utility) is *a*'s CECU across the maximally likely states of the world. We require a further argument to rule out MLCU theory.

This brings me to the following decision problem: Let us suppose that an agent, *S*, is faced with three choice options: *a*, *b*, and *c*. *S* assigns probability 0.51 to a state of the world, *A*, and 0.49 to a state of the world, *B*. If state *A* or state *B* were realized, then *S* would assign the following utilities to the set of options (see Table 12):

**TABLE 12** Decision matrix

	<b>A (0.51)</b>	<b>B (0.49)</b>
<i>a</i>	110	−1000
<i>b</i>	80	110
<i>c</i>	100	100

According to MLCU theory, *a* is uniquely choiceworthy for *S*, since state *A* is more likely to obtain than state *B* and the CU of option *a* in state *A* is greater than that of any other available option. Yet, it is clear that choosing option *a* is a mistake, since state *B* is almost as likely to obtain as state *A* and the comparative *disutility* of option *a* in state *B* is very high (−1110 c-utiles). Since I do not have any conceptual arguments to show that MLCU theory's verdict in this case is plausible, I take this verdict to be a counterexample to MLCU theory.

### 3.2.2 Argument 2: Differences in Choiceworthiness under Risk

In support of the CECU principle, I can offer a second line of argument, one that is based on the same lines of reasoning as those given in Section 3.1, except that we consider here rational preferences within various possible states of the world (i.e., within a situation in which a decision must be made under *risk*) instead of rational preferences within a *single* state of the world (i.e., within a situation in

which a decision must be made under *certainty*). The argument can be put as follows: Compared to the difference in utility, the difference in CU is a more plausible measure of the extent to which option *a* is more choiceworthy than option *b* in the *single* states 1–4 of Table 8 (see Section 3.1). And the CU functions in Tables 13–16 (below) have respectively as their *expected values* the CUs in the *single* states 1–4 of Table 8. Therefore, compared to the difference in EU, the difference in ECU (or, more precisely, CECU) is a more plausible measure of the extent to which option *a* is more choiceworthy than option *b* in Tables 13–16. The differences in EU between *a* and *b* are the same in all four decision matrices (4 units), whereas the differences in CECU between *a* and *b* are as follows:

**TABLE 13** Decision matrix

	A (0.5)	B (0.5)	EU	CECU
<i>a</i>	4	6	5	8
<i>b</i>	0	2	1	−8
<i>c</i>	0	2	1	−8
<i>d</i>	0	2	1	−8
<i>e</i>	0	2	1	−8

*Note:* The difference in CECU between *a* and *b* = 16 units.

**TABLE 14** Decision matrix

	A (0.5)	B (0.5)	EU	CECU
<i>a</i>	4	6	5	4
<i>b</i>	0	2	1	−6
<i>c</i>	1	3	2	−5
<i>d</i>	2	4	3	−4
<i>e</i>	2	4	3	−4

*Note:* The difference in CECU between *a* and *b* = 10 units.

**TABLE 15** Decision matrix

	A (0.5)	B (0.5)	EU	CECU
<i>a</i>	4	6	5	0
<i>b</i>	0	2	1	−4
<i>c</i>	1	3	2	−3
<i>d</i>	2	4	3	−2
<i>e</i>	4	6	5	0

*Note:* The difference in CECU between *a* and *b* = 4 units.

**TABLE 16** Decision matrix

	<b>A (0.5)</b>	<b>B (0.5)</b>	<b>EU</b>	<b>CECU</b>
<i>a</i>	4	6	5	−6
<i>b</i>	0	2	1	−10
<i>c</i>	1	3	2	−9
<i>d</i>	2	4	3	−8
<i>e</i>	7	9	8	6

*Note:* The difference in CECU between *a* and *b* = 4 units.

### 3.2.3 Argument 3: A Long-Run Argument

A standard argument for maximizing *expected utility* is that agents do best in the long run when they consistently maximize expected utility (see Feller, 1968). This *long-run argument* is based on the assumption that what an agent ought to care about maximizing in the long run is *utility*. In this section, I will argue for maximizing *expected comparative utility* (and, by implication, CECU) under the premise that what an agent ought to care about maximizing in the long run is not utility, but *choiceworthiness*. My argument comprises nine steps, which are numbered as follows:

(1) Let a sequence of choices (or trials) be *independent* if and only if (i) the range of options available in each choice situation is independent from the range of options available in each of the other choice situations in the sequence, and (ii) the probability and utility of the outcomes of each choice are independent from the probability and utility of the outcomes of each of the other choices in the sequence. What's more, let a sequence of choices be *identically distributed* if and only if the outcomes of each choice have the same probability distribution. For any agent, *S*, and for any long sequence of independent and identically distributed (IID) choices under risk,  $\varphi$ , for *S*,  $\varphi$  is maximally choiceworthy for *S* if  $\varphi$  is the sequence of choices which, by *S*'s lights, is almost certain to come the closest to cumulatively maximizing the quantity, *q*, such that for any agent, *S*, faced with a decision under certainty and for any option, *a*, for *S*, the measure of the choiceworthiness of *a* for *S* is *q*, and  $\varphi$  is *not* maximally choiceworthy for *S* if  $\varphi$  is the sequence of choices which, by *S*'s lights, is almost certain to *not* come the closest to cumulatively maximizing the quantity, *q*. (See Step 2 of the argument for ECU theory and the discussion thereof.) (2) For any agent, *S*, and for any choice or sequence of choices,  $\varphi$ , for *S*,  $\varphi$  is choiceworthy for *S* if and only if  $\varphi$  is maximally choiceworthy for *S*—that is to say,  $\varphi$  is at least as choiceworthy as the most choiceworthy alternative to  $\varphi$ .

(3) Given (1) and (2), for any agent, *S*, and for any long sequence of IID choices under risk,  $\varphi$ , for *S*,  $\varphi$  is choiceworthy for *S* if  $\varphi$  is the sequence of choices which, by *S*'s lights, is almost certain to come the closest to cumulatively maximizing the quantity, *q*, such that for any agent, *S*, faced with a decision under certainty and for any option, *a*, for *S*, the measure of the choiceworthiness of *a* for *S* is *q*, and  $\varphi$  is *not* choiceworthy for *S* if  $\varphi$  is the sequence of choices which, by *S*'s lights, is almost certain to *not* come the closest to cumulatively maximizing the quantity, *q*. That is to say, over any long sequence of IID trials, an agent ought to always choose, if she can, according to a decision rule, *r*, such that if she always chooses according to *r* over that sequence of trials, she will almost certainly accumulate a greater amount of whatever quantity, *q*, she ought to care about maximizing in the long run (or lose a smaller amount of *q*) than if she always chooses according to a different rule which delivers different verdicts for some or all of those trials. And the quantity, *q*, that an agent ought to care about maximizing in the long run is choiceworthiness (and not utility).

(4) For any agent, *S*, faced with any decision under certainty and for any option, *a*, for *S*, the measure of the choiceworthiness of *a* for *S* is its *comparative utility*—i.e., the difference in utility

between any option,  $a$ , and whichever alternative(s) to  $a$  carry the greatest utility. (Note that the concept of comparative utility applies only to individual choices. Thus, the (cumulative) comparative utility of a sequence of choices should be understood not as the difference between the utility of that sequence and the utility of the best alternative sequence(s) of choices, but rather as the sum of the comparative utilities of each individual choice in that sequence.)

(5) Given (3) and (4), for any agent,  $S$ , and for any long sequence of IID choices under risk,  $\varphi$ , for  $S$ ,  $\varphi$  is choiceworthy for  $S$  if  $\varphi$  is the sequence of choices which, by  $S$ 's lights, is almost certain to come the closest to cumulatively maximizing comparative utility, and  $\varphi$  is *not* choiceworthy for  $S$  if  $\varphi$  is the sequence of choices which, by  $S$ 's lights, is almost certain to *not* come the closest to cumulatively maximizing comparative utility, where the *comparative utility of a choice* is the difference in utility between whichever option is chosen in a given decision situation and whichever alternative would carry the greatest utility if it were chosen in that situation. In other words, over any long sequence of IID trials, an agent ought to always choose, if she can, according to a decision rule,  $r$ , such that if she always chooses according to  $r$  over that sequence of trials, she will almost certainly accumulate a greater amount of comparative utility (or lose a smaller amount of comparative utility) than if she always chooses according to a different rule which delivers different verdicts for some or all of those trials.

(6) Let a *random variable* be a rule or function that assigns a value to each possible outcome of a random trial or experiment. Moreover, let the *expected value* of a random variable (or decision option) be a probability-weighted average of each of its possible values. The *strong and weak laws of large numbers* state that as the number of IID random variables in a sequence approaches infinity, their sample average converges with overwhelming probability to their expected value. Now, it is straightforward to come up with a rule or function such that the values of a random variable can be expressed as comparative utilities (see variable  $x$  below). For any choice option,  $a$ , the *expected comparative utility* of  $a$  is the expected value of  $a$ 's comparative utility, or (i.e.) a *probability-weighted average* of  $a$ 's comparative utilities across the various states of the world. It follows from the strong and weak laws of large numbers that for any agent,  $S$ , and for any number of alternative options,  $a$ ,  $b$ ,  $c$ ,  $d$ , and  $e$ , the expected comparative utility of  $a$ , for  $S$ , is overwhelmingly likely to be close to the *long-run average value* of  $a$ 's comparative utility. The long-run average value of  $a$ 's comparative utility is given by the following notation:

$$\lim_{n \rightarrow \infty} \frac{1}{n} (x_1 + \dots + x_n)$$

where  $n$  is the ordinal number of a trial in a sequence of IID trials where five agents ( $S_1 \dots S_5$ ) who are in all relevant respects identical to  $S$  respectively and simultaneously perform options  $a$ ,  $b$ ,  $c$ ,  $d$ , and  $e$ , and where  $x$  (the random variable) is the difference between the utility of  $a$  in a given trial and the utility of  $b$ ,  $c$ ,  $d$ , or  $e$  in that trial, whichever has the greatest utility in that trial.

What this means is that over any long sequence of IID trials (or choices under risk), an agent will almost certainly accumulate a greater amount of comparative utility (or lose a smaller amount of comparative utility) if she always chooses whichever option has the greatest expected comparative utility (or one of them in case of a tie) than if she always chooses according to a different rule which delivers different verdicts for some or all of those trials.

(7) Given (5) and (6), for any agent,  $S$ , and for any long sequence of IID choices under risk,  $\varphi$ , for  $S$ ,  $\varphi$  is choiceworthy for  $S$  if and only if  $\varphi$  is such that each choice in the sequence maximizes expected comparative utility over the space of all alternatives. In other words, over any long sequence of IID trials, an agent ought to always choose whichever option has the greatest expected comparative utility (or one of them in case of a tie).

(8) In long sequences of IID trials, the rule of maximizing expected utility always delivers the

same verdicts (for all those trials) as the rule of maximizing expected comparative utility.<sup>15</sup> Let us consider the following case: An agent, *S*, is faced with a choice between three independent options or gambles: one option, *a*, offering a 0.01 probability of winning a prize worth 1500 utiles (and nothing otherwise), one option, *b*, offering a 0.02 probability of winning a prize worth 700 utiles (and nothing otherwise), and one option, *c*, offering a 0.03 probability of winning a prize worth 400 utiles (and nothing otherwise). According to the rule of maximizing *expected utility*, *S* ought to choose option *a*, since its expected utility is greater than that of every other option in the decision situation. According to the rule of maximizing *expected comparative utility* (or CECU), *S* ought to also choose option *a*, since its expected comparative utility (or CECU) is greater than that of every other option in the decision situation. See Table 17.

**TABLE 17** Decision matrix

	<b>A (0.9410)</b>	<b>B (0.0095)</b>	<b>C (0.0001)</b>	<b>D (0.0000)</b>
<i>a</i>	0	1500	1500	1500
<i>b</i>	0	0	700	700
<i>c</i>	0	0	0	400

  

	<b>E (0.0192)</b>	<b>F (0.0005)</b>	<b>G (0.0291)</b>	<b>H (0.0002)</b>
<i>a</i>	0	0	0	1500
<i>b</i>	700	700	0	0
<i>c</i>	0	400	400	400

  

	<b>EU</b>	<b>CECU</b>
<i>a</i>	14.7	1.8
<i>b</i>	13.86	-1.8
<i>c</i>	11.92	-5.69

*Note:* The following notation is used: *A* denotes the state “If *S* chooses *a*, then *S* will not win the prize (0 utiles), if *S* chooses *b*, then *S* will not win the prize (0 utiles), and if *S* chooses *c*, then *S* will not win the prize (0 utiles)” (probability:  $0.99 \times 0.98 \times 0.97 = 0.9410$ ), *B* denotes the state “If *S* chooses *a*, then *S* will win the prize (1500 utiles), if *S* chooses *b*, then *S* will not win the prize (0 utiles), and if *S* chooses *c*, then *S* will not win the prize (0 utiles)” (probability:  $0.01 \times 0.98 \times 0.97 = 0.0095$ ), *C* denotes the state “If *S* chooses *a*, then *S* will win the prize (1500 utiles), if *S* chooses *b*, then *S* will win the prize (700 utiles), and if *S* chooses *c*, then *S* will not win the prize (0 utiles)” (probability:  $0.01 \times 0.02 \times 0.97 = 0.0001$ ), *D* denotes the state “If *S* chooses *a*, then *S* will win the prize (1500 utiles), if *S* chooses *b*, then *S* will win the prize (700 utiles), and if *S* chooses *c*, then *S* will win the prize (400 utiles)” (probability:  $0.01 \times 0.02 \times 0.03 = 0.0000$ ), *E* denotes the state “If *S* chooses *a*, then *S* will not win the prize (0 utiles), if *S* chooses *b*, then *S* will win the prize (700 utiles), and if *S* chooses *c*, then *S* will not win the prize (0 utiles)” (probability:  $0.99 \times 0.02 \times 0.97 = 0.0192$ ), *F* denotes the state “If *S* chooses *a*, then *S* will not win the prize (0 utiles), if *S* chooses *b*, then *S* will win the prize (700 utiles), and if *S* chooses *c*, then *S* will win the prize (400 utiles)” (probability:  $0.99 \times 0.02 \times 0.03 = 0.0005$ ), *G* denotes the state “If *S* chooses *a*, then *S* will not win the prize (0 utiles), if *S* chooses *b*, then *S* will not win the prize (0 utiles), and if *S* chooses *c*, then *S* will win the prize (400 utiles)” (probability:  $0.99 \times 0.98 \times 0.03 = 0.0291$ ), *H* denotes the state “If *S* chooses *a*, then *S* will win the prize (1500 utiles), if *S* chooses *b*, then *S* will not win the prize (0 utiles), and if *S* chooses *c*, then *S* will win the prize (400 utiles)” (probability:  $0.01 \times 0.98 \times 0.03 = 0.0002$ ).

(9) Given (7) and (8), maximizing *expected comparative utility* (or CECU) makes for an optimal decision rule when an agent is faced with any long sequence of IID trials (or choices under risk). Since

<sup>15</sup> As far as I can tell, this holds for all cases. I will not here undertake the task of proving this conjecture.

in long sequences of IID choices, the rule of maximizing expected comparative utility always delivers the same verdicts (for all those choices) as the rule of maximizing expected utility, and since agents typically face long sequences of choices of this sort in games of chance, this vindicates the standard way of gambling when the probabilities involved are known—that is, by maximizing *expected utility*.

### 3.3 The Argument for ECU Theory: Steps 5 to 15

In this section, I will discuss the remaining steps (5 to 15) of the argument for ECU theory:

5. For any agent,  $S$ , faced with any decision under *risk* and for any option,  $a$ , for  $S$ ,  $a$  is choiceworthy for  $S$  if and only if  $a$  maximizes CECU. (5 follows from 3 and 4.)
6. For any agent,  $S$ , faced with any decision under *risk* and for any number of alternative options,  $a$ ,  $b$ ,  $c$ ,  $d$ , and  $e$ , for  $S$ , it is rational for  $S$  to prefer  $a$  to  $b$  if and only if  $a$ 's EU is greater than  $b$ 's, it is rational for  $S$  to be indifferent between  $a$  and  $b$  if and only if  $a$ 's EU is equal to  $b$ 's, and the extent to which  $S$  rationally prefers  $a$  to  $b$  is the difference in EU between  $a$  and  $b$ .

Decision-theoretic representation theorems such as those of von Neumann and Morgenstern (1947), Savage (1954), Bolker (1966) and Jeffrey (1983), and Joyce (1999) show that, for any agent,  $S$ , faced with any decision under certainty or any decision under risk, and for any number of alternative options,  $a$ ,  $b$ ,  $c$ ,  $d$ , and  $e$ , for  $S$ , if  $S$  fails to prefer  $a$  to  $b$  when  $a$ 's EU is greater than  $b$ 's, or if  $S$  fails to be indifferent between  $a$  and  $b$  when  $a$ 's EU is equal to  $b$ 's, then  $S$  violates at least one of a series of axioms of *rational preference*. While different representation theorems include different sets of axioms, all the above theorems (implicitly) include the IIA (Kogelmann, 2018). Besides the intuitive plausibility of the axioms of rational preference, further justifications for Step 6 come from money-pump arguments for those axioms (see Gustafsson, 2022)<sup>16</sup> as well as arguments for those axioms from dynamic consistency (see Hammond, 1988; Machina, 1989; and McClennen, 1990).<sup>17</sup> These justifications for Step 6 represent challenges for several recent normative alternatives to Step 6—for example, Buchak's (2013) *risk-weighted expected utility theory* and Bradley & Stefansson's (2017) *counterfactual desirability theory*.

7. For any agent,  $S$ , faced with any decision under *risk* and for any option,  $a$ , for  $S$ , it is rational for  $S$  to weakly prefer  $a$  over the alternative options in the choice set if and only if  $a$  maximizes EU. (7 follows from 6.)
8. In decisions under *risk*, what option(s) maximize CECU sometimes differ from what option(s) maximize EU.

For examples, see Section 2.2.

9. In decisions under *risk*, what option(s) are choiceworthy sometimes differ from what option(s) it is rational to weakly prefer over the alternative options in the choice set. (9 follows from 5, 7, and 8.)

Rational preference is not a reliable measure of choiceworthiness. That is because whereas the criterion of rational preference (i.e., EU) satisfies the IIA, the criterion of choiceworthiness (i.e., CU/CECU) violates that principle (see Section 2.2 for examples). It is important to emphasize that the

<sup>16</sup> A *money pump* is a combination of choices that leads to a sure loss.

<sup>17</sup> An agent's choices in a given decision situation are *dynamically consistent* if and only if they do *not* evolve over time in that decision situation.



proposed criterion of choice (i.e., choiceworthiness) is independent from the standard choice criterion (i.e., rational preference). The latter is not shown here to violate the conditions, for example, the IIA, which are needed to derive utilities from preferences via a representation theorem.

10. For any agent, *S*, faced with any decision under *risk*, *S* ought to measure and rank her options (for the purpose of choice) in terms of *how choiceworthy* they are for *S*.

Whether (and to what extent) any option is more choiceworthy than any other within a set of alternatives is necessarily a function of *how choiceworthy* each of the two options is within the set of alternatives. (See my discussion of Step 2 of the argument for ECU theory.)

As noted upfront, in this book, *choiceworthiness*, or (i.e.) *how choiceworthy* an option is, is given the following conceptual analysis: For any agent, *S*, faced with any decision under certainty or any decision under risk and for any option, *a*, for *S*, the *degree* to which *a* is *choiceworthy* for *S*, or (i.e.) the *choiceworthiness* of *a* for *S*, is the degree to which *a* is worthy of being chosen by *S* in light of *S*'s rational preferences within each of the various possible states of the world, where *S*'s *rational preferences* are preferences that obey the series of rationality axioms of standard decision theory. Let us now consider the *instrumental rationality argument* (v1):

1. If {for any agent, *S*, faced with any decision under certainty or any decision under risk and for any option, *a*, for *S*, the *degree* to which *a* is *choiceworthy* for *S* is the degree to which *a* is a suitable means to *S*'s ends}, then {the *degree* to which *a* is *choiceworthy* for *S* is the degree to which *a* is worthy of being chosen by *S* in light of *S*'s rational preferences within each of the various possible states of the world, where *S*'s *rational preferences* are preferences that obey the series of rationality axioms of standard decision theory}. (With respect to the consequent, the idea is that the *degree* to which *a* is *choiceworthy* for *S* is the degree to which *a* is worthy of being chosen by *S* *not necessarily* in light of *S*'s rational preferences *tout court* (i.e., *S*'s EU function), but rather in light of *S*'s rational preferences *within each of the various possible states of the world* (i.e., *S*'s alternative possible utility functions). This idea follows from Gandalf's principle.)
2. The *degree* to which *a* is *choiceworthy* for *S* is the degree to which *a* is a suitable means to *S*'s ends. (Assumption)
3. The *degree* to which *a* is *choiceworthy* for *S* is the degree to which *a* is worthy of being chosen by *S* in light of *S*'s rational preferences within each of the various possible states of the world. (3 follows from 1 and 2.)
4. The *degree* to which *a* is a *suitable means* to *S*'s ends is the degree to which *a* is worthy of being chosen by *S* in light of *S*'s rational preferences within each of the various possible states of the world. (4 follows from 2 and 3.)
5. The *degree* to which *a* is a *suitable means* to *S*'s ends is the degree to which *a* is instrumentally rational for *S*.
6. The degree to which *a* is *instrumentally rational* for *S* is the degree to which *a* is choiceworthy for *S*, or (i.e.) the degree to which *a* is worthy of being chosen by *S* in light of *S*'s rational preferences within each of the various possible states of the world. (6 follows from 3 to 5.)

For any agent, *S*, faced with any decision under certainty or any decision under risk, I assume that *S* ought to measure and rank her options (for the purpose of choice) in terms of the *degrees* to which they are *instrumentally rational* for *S*, or (i.e.) the degrees to which they are suitable means to *S*'s ends. Therefore, in accordance with the instrumental rationality argument (v1), *S* ought to measure and rank her options (for the purpose of choice) in terms of the *degrees* to which they are *choiceworthy* for *S*, or (i.e.) the degrees to which they are worthy of being chosen by *S* in light of *S*'s rational



preferences within each of the various possible states of the world.

11. It is *not* the case that for any agent, *S*, faced with any decision under *risk*, *S* ought to measure and rank her options (for the purpose of choice) in order of rational preference. (11 follows from 3, 9, and 10.)

In accordance with my discussion of Step 10, for any agent, *S*, faced with any decision under *risk*, *S* ought to measure and rank her options (for the purpose of choice) in terms of the degrees to which they are worthy of being chosen by *S* in light of *S*'s rational preferences *within each of the various possible states of the world* (i.e., *S*'s alternative possible utility functions). It follows that it is a *theoretical possibility* that for any agent, *S*, faced with any decision under *risk*, *S* ought to measure and rank her options (for the purpose of choice) in order of rational preference, or (i.e.) in order of EU. However, it does *not* follow that it is true *by definition* that for any agent, *S*, faced with any decision under *risk*, *S* ought to measure and rank her options (for the purpose of choice) in order of rational preference, or (i.e.) in order of EU.

12. For any agent, *S*, faced with any decision under *risk*, *S* ought to measure and rank her options (for the purpose of choice) in terms of how choiceworthy they are for *S*, that is, according to their CECU, rather than in order of rational preference, that is, according to their EU. (12 follows from 4, 6, 10, and 11, as well as from 3, 4, 6, 9, and 13.)

I can offer two further arguments for Step 12: First, there is Wedgwood's Gandalf's principle (see Section 3.2.1). Second, compared to the criterion of rational preference (i.e., EU), the criterion of choiceworthiness (i.e., CU/CECU) supplies a more plausible measure of the extent to which any given option is more choiceworthy than any other in any ranking of more than two choice options, both in decision situations involving certainty and decision situations involving risk. For decisions under certainty, see decision matrix 8 in Section 3.1. For decisions under risk, consider the extent to which option *a* is more choiceworthy than option *b* in decision matrices 13–16 (see Section 3.2.2).

13. For any agent, *S*, faced with any decision under *risk*, *S* ought to choose out of what option(s) are *choiceworthy* for *S*.

Step 13 is a tautology. Therefore, it is necessarily true.

Here is another way of arguing for Step 13: As noted upfront, in this book, *what it is to be choiceworthy* is given the following conceptual analysis: For any agent, *S*, faced with any decision under certainty or any decision under risk and for any option, *a*, for *S*, *a* is *choiceworthy* for *S* if and only if *a* is worthy of being chosen by *S* in light of *S*'s rational preferences within each of the various possible states of the world, where *S*'s *rational preferences* are preferences that obey the series of rationality axioms of standard decision theory. Let us now consider the *instrumental rationality argument* (v2):

1. If {for any agent, *S*, faced with any decision under certainty or any decision under risk and for any option, *a*, for *S*, *a* is *choiceworthy* for *S* if and only if *a* is a suitable means to *S*'s ends}, then {*a* is *choiceworthy* for *S* if and only if *a* is worthy of being chosen by *S* in light of *S*'s rational preferences within each of the various possible states of the world, where *S*'s *rational preferences* are preferences that obey the series of rationality axioms of standard decision theory}. (With respect to the consequent, the idea is that *a* is *choiceworthy* for *S* if and only if *a* is worthy of being chosen by *S* *not necessarily* in light of *S*'s rational preferences *tout court* (i.e., *S*'s EU function), but rather

in light of  $S$ 's rational preferences *within each of the various possible states of the world* (i.e.,  $S$ 's alternative possible utility functions). This idea follows from Gandalf's principle and the CM principle.)

2.  $a$  is *choiceworthy* for  $S$  if and only if  $a$  is a suitable means to  $S$ 's ends. (Assumption)
3.  $a$  is *choiceworthy* for  $S$  if and only if  $a$  is worthy of being chosen by  $S$  in light of  $S$ 's rational preferences within each of the various possible states of the world. (3 follows from 1 and 2.)
4.  $a$  is a *suitable means* to  $S$ 's ends if and only if  $a$  is worthy of being chosen by  $S$  in light of  $S$ 's rational preferences within each of the various possible states of the world. (4 follows from 2 and 3.)
5.  $a$  is a *suitable means* to  $S$ 's ends if and only if  $a$  is instrumentally rational for  $S$ .
6.  $a$  is *instrumentally rational* for  $S$  if and only if  $a$  is choiceworthy for  $S$ , or (i.e.)  $a$  is worthy of being chosen by  $S$  in light of  $S$ 's rational preferences within each of the various possible states of the world. (6 follows from 3 to 5.)

For any agent,  $S$ , faced with any decision under certainty or any decision under risk, I assume that  $S$  ought to choose out of what option(s) are *instrumentally rational* for  $S$ , or (i.e.) what option(s) are suitable means to  $S$ 's ends. Therefore, in accordance with the instrumental rationality argument (v2),  $S$  ought to choose out of what option(s) are *choiceworthy* for  $S$ , or (i.e.) what option(s) are worthy of being chosen by  $S$  in light of  $S$ 's rational preferences within each of the various possible states of the world.

14. It is *not* the case that for any agent,  $S$ , faced with any decision under *risk*,  $S$  ought to choose out of what option(s) it is rational for  $S$  to weakly prefer over the alternative options in the choice set. (14 follows from 9 and 13.)

In accordance with my discussion of Step 13, for any agent,  $S$ , faced with any decision under *risk*,  $S$  ought to choose out of what option(s) are worthy of being chosen by  $S$  in light of  $S$ 's rational preferences *within each of the various possible states of the world* (i.e.,  $S$ 's alternative possible utility functions). It follows that it is a *theoretical possibility* that for any agent,  $S$ , faced with any decision under *risk*,  $S$  ought to choose out of what option(s) it is rational for  $S$  to weakly prefer over the alternative options in the choice set, or (i.e.) out of what option(s) maximize EU for  $S$ . However (and contrary to Step 13), it does *not* follow that it is true *by definition* that for any agent,  $S$ , faced with any decision under *risk*,  $S$  ought to choose out of what option(s) it is rational for  $S$  to weakly prefer over the alternative options in the choice set, or (i.e.) out of what option(s) maximize EU for  $S$ .

15. For any agent,  $S$ , faced with any decision under *risk*,  $S$  ought to choose out of what option(s) are choiceworthy for  $S$  (i.e., what option(s) maximize CECU), even in cases where what option(s) are choiceworthy for  $S$  differ from what option(s) it is rational for  $S$  to weakly prefer over the alternative options in the choice set (i.e., what option(s) maximize EU). (15 follows from 5, 7, 9, 13, and 14, as well as from 3, 5, 7, 9, and 10.)

## 4 Potential Objections to ECU Theory

In this chapter, I will respond to several potential objections to ECU theory. In Section 4.1, I will lay out the *problem of act alterations* and the *problem of act versions* as they apply to ECU theory, and I will advance a solution to these problems that takes inspiration from Gustafsson (2014). In Section 4.2, I will respond to objections to the fact that ECU theory violates the IIA condition on choiceworthiness evaluations. In Section 4.3, I will rebut an objection alleging that agents who maximize choiceworthiness are vulnerable to exploitation in money pumps, where a *money pump* is a series of choices that leads to a loss with certainty. In Section 4.4, I will rebut an objection alleging that agents who maximize choiceworthiness are subject to *dynamic inconsistency*, that is to say, their choices change over time within a given decision situation. In Section 4.5, I will address the question of whether a type of risk-averse or risk-seeking preference called *global sensitivity* undermines the CECU principle. Lastly, in Section 4.6, I will deal with a potential counterexample to the CU principle (and therefore to ECU theory).

### 4.1 The Problem of Act Alterations and the Problem of Act Versions

One possible objection to ECU theory is that its ranking of options according to how choiceworthy they are is highly sensitive to the introduction of slight act alterations.<sup>18</sup> We can call this the *problem of act alterations*. Let us consider the following decision problem (see Table 18), where option *a* involves ‘pressing a green button’, and option *b* involves ‘pressing a red button’ (I assume here that not pressing any button is not a feasible option).

**TABLE 18** Decision matrix

	A (0.333)	B (0.333)	C (0.333)	EU	CECU
<i>a</i>	12	6	6	8	−2
<i>b</i>	3	12	12	9	2

Let us now consider the following variation on decision matrix 18 (see Table 19):

**TABLE 19** Decision matrix

	A (0.333)	B (0.333)	C (0.333)	EU	CECU
<i>a</i>	12	6	6	8	1.667
<i>b</i> <sub>1</sub>	3	12	12	9	−1.667
<i>b</i> <sub>2</sub>	3	11	12	8.667	−2.333

The only difference between decision matrices 18 and 19 is that decision matrix 19 includes options *b*<sub>1</sub> and *b*<sub>2</sub>, more specific *versions* of option *b* with option *b*<sub>2</sub> having a slightly different amount of utility (compared to *b*<sub>1</sub>) in only one state—i.e., 11 utiles instead of 12 utiles in state *B*. Option *b*<sub>1</sub> involves pressing the red button with one’s right index finger, and option *b*<sub>2</sub> involves pressing the red button with one’s left index finger.<sup>19</sup> Yet, introducing the more specific options *b*<sub>1</sub> and *b*<sub>2</sub> in decision matrix

<sup>18</sup> Thanks to Paul Bartha for raising this objection and for giving the following examples: Tables 18 and 19, and a variation on the ‘button pressing’ example.

<sup>19</sup> I assume here for the sake of argument that pressing the button with one’s left index finger instead of one’s right index finger can make a difference in the decision situation.

19 reverses ECU theory's choiceworthiness ranking of options  $a$  and  $b$  compared to decision matrix 18. On the other hand, introducing the more specific options  $b_1$  and  $b_2$  in decision matrix 19 does not reverse *EU theory's* ranking of options  $a$  and  $b$  compared to decision matrix 18. Thus, the problem of act alterations would appear to be a problem for ECU theory (but not for EU theory).

How can we solve the problem of act alterations? I believe that we can do so by finding a principled way of individuating the various choice options that are available to an agent in any decision problem. One way of individuating options has been proposed by Gustafsson (2014). He has argued that choice options in a decision problem should be construed as sets of acts such that, for each set of acts, one could jointly intentionally perform, at any time  $t$ , all the acts in the set, but no additional acts (Gustafsson, 2014). One of the reasons given by Gustafsson is that if one construes choice options as individual acts, then one runs into the *problem of act versions* (Bergström, 1966; Castaneda, 1968). Consider the following example:

It is raining outside, but Ann will feel invigorated if she takes a brisk walk around the block (10 utiles), more so than if she stays inside (2 utiles). However, Ann has an injured toenail which causes her a great deal of pain when she tries to walk with her rain boots on. She will therefore experience a great deal of pain if she goes out for a walk wearing her rain boots (−30 utiles), more so than if she stays inside wearing her rain boots (−2 utiles). Luckily, Ann has a very comfortable pair of shoes which do not cause her any pain. However, there is a problem: it is raining very hard and her feet will get soaked if she wears her shoes. Ann will experience considerable discomfort if she goes out for a walk not wearing her rain boots (−15 utiles), more so than if she stays inside not wearing her rain boots (0 utiles).

Let us suppose that Ann assigns probability 1 to the state of the world as described above. Although the utility of the act 'Ann stays inside' is lower than that of the act 'Ann goes out for a walk', the utility of at least one version of the act 'Ann stays inside'—that is, 'Ann stays inside and does not wear her rain boots' ( $2 + 0 = 2$  utiles)—is greater than the utility of all versions of the act 'Ann goes out for a walk'—that is, 'Ann goes out for a walk and wears her rain boots' ( $10 + -30 = -20$  utiles) and 'Ann goes out for a walk and does not wear her rain boots' ( $10 + -15 = -5$  utiles). Thus, intuitively, Ann ought to stay inside. However, if choice options are construed as individual acts, then EU theory counsels Ann *not* to stay inside, but instead to go out for a walk.

Therefore, to be intuitively plausible, ECU theory should be minimally cashed out as follows:<sup>20</sup>

For any agent,  $S$ , faced with any decision under certainty or any decision under risk and for any number of mutually exclusive and jointly exhaustive options, or sets of acts,  $a$ ,  $b$ ,  $c$ ,  $d$ , and  $e$ , such that, for each set,  $S$  could jointly intentionally perform, at any time,  $t$ , all the acts in the set, but no additional acts,

- $a$  is more choiceworthy than  $b$ , for  $S$ , at  $t$ , if and only if the CU/CECU of  $S$  jointly intentionally performing  $a$  at  $t$  is greater than the CU/CECU of  $S$  jointly intentionally performing  $b$  at  $t$ , and
- $a$  is just as choiceworthy as  $b$ , for  $S$ , at  $t$ , if and only if the CU/CECU of  $S$  jointly intentionally performing  $a$  at  $t$  is equal to the CU/CECU of  $S$  jointly intentionally performing  $b$  at  $t$ .

This implies the following derivative decision rule for individual acts:<sup>21</sup>

<sup>20</sup> Inspired by Gustafsson (2014, pp. 593–594).

<sup>21</sup> Inspired by Gustafsson (2014, p. 595).

For any agent,  $S$ , faced with any decision under certainty or any decision under risk and for any two mutually exclusive acts,  $a$  and  $b$ ,

- $a$  is more choiceworthy than  $b$ , for  $S$ , at any time,  $t$ , if and only if  $a$  is logically implied by every set of acts such that, for each set,  $S$  could jointly intentionally perform, at  $t$ , all the acts in the set, but no additional acts and such that, in accordance with ECU theory, the set of acts would be more choiceworthy for  $S$ , at  $t$  than each set of acts such that  $S$  could jointly intentionally perform, at  $t$ , all the acts in the set, but no additional acts and such that the set of acts logically implies  $b$ , and
- $a$  is just as choiceworthy as  $b$ , for  $S$ , at any time,  $t$ , if and only if  $a$  is not more choiceworthy than  $b$ , and  $a$  is logically implied by every set of acts such that, for each set,  $S$  could jointly intentionally perform, at  $t$ , all the acts in the set, but no additional acts and such that, in accordance with ECU theory, the set of acts would not be less choiceworthy for  $S$ , at  $t$  than each set of acts such that  $S$  could jointly intentionally perform, at  $t$ , all the acts in the set, but no additional acts and such that the set of acts logically implies  $b$ .

If we follow Gustafsson (2014) in construing *choice options* as sets of acts such that, for each set of acts, one could jointly intentionally perform, at any time  $t$ , all the acts in the set, but no additional acts, then we have a principled way of resolving the problem of act alterations. That is to say, it is decision matrix 20 (Table 20) that properly formalizes the ‘button pressing’ decision problem, and not decision matrix 18 or 19. In decision matrix 20, option  $a_1$  involves ‘pressing the green button with one’s right index finger’, option  $a_2$  involves ‘pressing the green button with one’s left index finger’, option  $b_1$  involves ‘pressing the red button with one’s right index finger’, and option  $b_2$  involves ‘pressing the red button with one’s left index finger’. In other words, option  $a_1$  is ‘the act of pressing the green button and the act of pressing the button with one’s right index finger’, option  $a_2$  is ‘the act of pressing the green button and the act of pressing the button with one’s left index finger’, option  $b_1$  is ‘the act of pressing the red button and the act of pressing the button with one’s right index finger’, and option  $b_2$  is ‘the act of pressing the red button and the act of pressing the button with one’s left index finger’.

**TABLE 20** Decision matrix

	A (0.333)	B (0.333)	C (0.333)	EU	CECU
$a_1$	12	6	6	8	−1.336
$a_2$	12	6	6	8	−1.336
$b_1$	3	12	12	9	0.666
$b_2$	3	11	12	8.667	−0.666

The only difference between decision matrices 18 and 19, on the one hand, and decision matrix 20, on the other hand, is that decision matrix 20 includes options  $a_1$ ,  $a_2$ ,  $b_1$  and  $b_2$ , versions of options  $a$  and  $b$  with option  $b_2$  having a slightly different amount of utility (compared to  $b_1$ ) in only one state—i.e., 11 utiles instead of 12 utiles in state  $B$ . Introducing the more specific options  $a_1$ ,  $a_2$ ,  $b_1$  and  $b_2$  in decision matrix 20 does *not* reverse ECU theory’s choiceworthiness ranking of options  $a$  and  $b$  compared to decision matrix 18. Thus, the problem of act alterations is *not* a problem for ECU theory.

## 4.2 ECU Theory Violates the Independence of Irrelevant Alternatives

In personal communication, Paul Bartha objects to ECU theory on the grounds that it violates the IIA condition on choiceworthiness evaluations:

Let me turn to the problem of act alterations. Your proposed solution is to reject the decision matrix in Table 19, in favour of Table 20. This is a fair response to my example, since I suggested that the difference between two outcomes might correspond to pressing a button with different fingers. Your response addresses the particular example, but it doesn't seem to me that it solves the fundamental problem. We can change the story so that there really are just three alternatives. With reference to Table 18: the assassin has to choose between poison (*a*) and a dagger (*b*). The three possible states are A, B, C with probability 1/3 each—for example, indicating three possible venues where the target individual might be found. Table 18 gives the utilities: the dagger is significantly better in states B and C and EU Max reflects the assassin's overall preference for the dagger. ECU gives the same verdict. But just as the assassin is reaching for the dagger, he realizes that there is a slightly bigger (and harder to conceal) dagger in his kit—no other tools are available other than the poison (*a*) and the two daggers, now labelled choices *b*<sub>1</sub> and *b*<sub>2</sub>, with utilities as represented in Table 19 because the large dagger is a bit awkward to use if the target is in venue B. The assassin still prefers the dagger *b*<sub>1</sub> to the poison *a* and he certainly prefers it to the awkward dagger *b*<sub>2</sub>. But if he uses ECU reasoning, he now has to switch to the poison because that's the only choiceworthy option. This seems absurd. [This is] an objection to the fact that ECU gives up Independence of Irrelevant Alternatives. (P. Bartha, personal communication, January 15, 2024)

The intuition that ECU theory's verdict in the above decision problem is absurd would represent a *reductio ad absurdum* of the theory and, in particular, of its violation of the IIA condition on choiceworthiness evaluations. I do not deny that ECU theory's verdict in this decision problem is counterintuitive, but I think that its verdict is nevertheless plausible in light of Chapter 3's step-by-step argument for ECU theory. I think that the reason one may consider ECU theory's verdict in this decision problem to be absurd is that one is entirely focused on the question of which option it is rational to *prefer*, when (i) what matters for the purposes of choice is which option is *choiceworthy*, and (ii) the question of which option is *choiceworthy* comes apart from the question of which option it is rational to *prefer*. I have argued for both propositions in Chapter 3. I have argued for a new criterion of choice—i.e., choiceworthiness (or CU/CECU)—in replacement of the standard criterion of choice—i.e., rational preference (or EU). Therefore, I do not consider ECU theory and its violation of the IIA condition on choiceworthiness evaluations to be absurd or untenable.

Nevertheless, if we give up the IIA (and the irrelevance of statewise dominated alternatives), then it is possible to make up alternatives in any choice set and these manufactured alternatives would be altering the degrees of choiceworthiness of reasonable options. This opens the door to strategic manipulation in the decision process.<sup>22</sup> The worry can be overcome, however, if we accept some form of *probability discounting*. According to this view, for any given decision situation, any outcome with probability  $\leq p$ , where *p* is very close to 0, should be excluded from consideration in the decision process. As such, if we accept probability discounting, then the *very improbable* outcomes of manufactured alternatives cannot alter the degrees of choiceworthiness of the other available options in the choice set. Endorsing some form of probability discounting in our case is not ad hoc since there are

<sup>22</sup> Thanks to Douglas Lackey for raising this objection and for wording suggestions.

independent reasons for doing so: probability discounting provides a unified solution to several thorny decision problems, such as the St. Petersburg paradox and Pascal's Mugging,<sup>23</sup> that involve very small chances of enormous utility payoffs.<sup>24</sup>

### 4.3 The Objection from Money Pumps

Kosonen (2024) argues that agents who discount small probabilities are vulnerable to money pumps and that this undermines the plausibility of probability discounting. However, her argument depends on the following crucial step: if an agent's *preferences* should maximize EU (e.g., if her *preferences* should not depend on discounting small probabilities), then she should maximize EU tout court (e.g., she should not discount small probabilities tout court). This step can be shown to be false. Consider the following counter-argument:

1. An agent's *preferences* satisfy all of the 'axioms of rational preference' that jointly imply that rational preferences maximize EU if and only if her *preferences* maximize EU.
2. An agent is *not* vulnerable to exploitation in *money pumps* if and only if her *preferences* satisfy all of those axioms of rational preference. (Gustafsson, 2022)
3. An agent is *not* vulnerable to exploitation in *money pumps* if and only if her *preferences* maximize EU. (3 follows from 1 and 2.)
4. If an agent's *preferences* depend on discounting small probabilities, then her preferences *do not* maximize EU.
5. If an agent's *preferences* depend on discounting small probabilities, then she *is* vulnerable to exploitation in *money pumps*. (5 follows from 3 and 4.)
6. An agent is vulnerable to exploitation in a *money pump* if and only if she is vulnerable to exploitation in a sequence of *preference-based choices* that leads to a sure loss.
7. An agent need not make *preference-based choices*; she can instead choose in accordance with the principle of maximizing *choiceworthiness*.
8. An agent chooses in accordance with the principle of maximizing *choiceworthiness* only if her *preferences* satisfy all of the 'axioms of rational preference' that jointly imply that rational preferences maximize EU.
9. An agent chooses in accordance with the principle of maximizing *choiceworthiness* only if her *preferences* maximize EU. (9 follows from 1 and 8.)
10. An agent chooses in accordance with the principle of maximizing *choiceworthiness* only if her *preferences* *do not* depend on discounting small probabilities. (10 follows from 4 and 9.)
11. If an agent chooses in accordance with the principle of maximizing *choiceworthiness*, then she is *not* vulnerable to exploitation in money pumps. (11 follows from 3, 6, 7, and 9.)
12. If an agent chooses in accordance with the principle of maximizing *choiceworthiness*, then she

<sup>23</sup> The St. Petersburg paradox was introduced by Nicolaus Bernoulli in 1713 (see Peterson, 2023). Pascal's mugging was introduced by Bostrom (2009), who describes a scenario where a mugger is trying to convince a passer-by, named Pascal, to hand his wallet over to him. After some back-and-forth between the mugger and Pascal, the mugger says, "If you hand me your wallet, I will perform magic that will give you an extra 1,000 quadrillion happy days of life" (p. 445). Given Pascal's assessment of the probability that the mugger will deliver on his promise (i.e., one in 10 quadrillion), the mugger argues that, in accordance with EU theory, Pascal ought to give him his wallet. Convinced, Pascal hands his wallet to the mugger. This counterintuitive result suggests that EU theory is undermined.

<sup>24</sup> Contemporary proponents of probability discounting include Jordan (1994), Smith (2014, 2016), Chalmers (2017), Monton (2019), and Hiller & Hasan (2023). Critics of probability discounting include Hájek (2014), Isaacs (2016), Lundgren & Stefánsson (2020), Cibinel (2023), and Kosonen (2024). Kosonen (n.d.) helpfully formulates and assesses various versions of probability discounting.



discounts small probabilities.

13. If an agent discounts small probabilities, then she is *not* vulnerable to exploitation in money pumps if she chooses in accordance with the principle of maximizing *choiceworthiness*. (13 follows from 10, 11, and 12.)
14. An agent ought to choose in accordance with the principle of maximizing *choiceworthiness* and ought to discount small probabilities if by doing both she is *not* vulnerable to exploitation in money pumps.
15. An agent ought to choose in accordance with the principle of maximizing *choiceworthiness* and ought to discount small probabilities. (15 follows from 13 and 14.)

#### 4.4 The Objection from Dynamic Inconsistency

As Kosonen (2024) shows, if an agent's preferences depend on discounting small probabilities, then she violates the independence axiom of EU theory. And if an agent violates the independence axiom (or sure-thing principle), then she is subject to *dynamic inconsistency*, that is, her choices evolve over time within a given decision situation.<sup>25</sup> Therefore, it could be argued that if an agent discounts small probabilities, then she is subject to dynamic inconsistency, and that this undermines the plausibility of probability discounting. According to Lotito (2022),

The problem with dynamically inconsistent shifts in preferences is that they are not compatible with the agent always maximising with respect to his preferences for consequences. The agent may act according to a plan that is strictly dominated by another available plan with respect to preferences for outcomes. (p. 21)

This objection depends, it too, on the following crucial step: if an agent's *preferences* should not depend on discounting small probabilities, then she should not discount small probabilities tout court. This step can be shown to be false. Consider the following counter-argument:

1. An agent is *not* subject to dynamic inconsistency if and only if her *preferences* satisfy the rational-preference axioms of weak ordering (i.e., completeness and transitivity) and independence.
2. If an agent's *preferences* depend on discounting small probabilities, then her *preferences* do *not* satisfy the rational-preference axiom of independence.
3. If an agent's *preferences* depend on discounting small probabilities, then she *is* subject to dynamic inconsistency. (3 follows from 1 and 2.)
4. An agent is subject to *dynamic inconsistency* if and only if her *preference-based choices* change over time within a given decision situation.
5. An agent need not make *preference-based choices*; she can instead choose in accordance with the principle of maximizing *choiceworthiness*.
6. An agent chooses in accordance with the principle of maximizing *choiceworthiness* only if her *preferences* satisfy the rational-preference axioms of weak ordering (i.e., completeness and transitivity) and independence.
7. An agent chooses in accordance with the principle of maximizing *choiceworthiness* only if her *preferences* do *not* depend on discounting small probabilities. (7 follows from 2 and 6.)
8. If an agent chooses in accordance with the principle of maximizing *choiceworthiness*, then she

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<sup>25</sup> For arguments from dynamic consistency in support of the axioms of rational preference of EU theory, see Hammond, 1988; Machina, 1989; and McClennen, 1990.



discounts small probabilities.

9. If an agent chooses in accordance with the principle of maximizing *choiceworthiness*, then she is *not* subject to dynamic inconsistency. (9 follows from 1, 3, 4, 5, 6, 7, and 8.)
10. If an agent discounts small probabilities, then she is *not* subject to dynamic inconsistency if she chooses in accordance with the principle of maximizing *choiceworthiness*. (10 follows from 8 and 9.)
11. An agent ought to choose in accordance with the principle of maximizing *choiceworthiness* and ought to discount small probabilities if by doing both she is *not* subject to dynamic inconsistency.
12. An agent ought to choose in accordance with the principle of maximizing *choiceworthiness* and ought to discount small probabilities. (12 follows from 10 and 11.)

## 4.5 The Objection from Global Sensitivity

To be risk-averse or risk-seeking are properties of *preferences*. To be risk-averse is to disprefer *mean-preserving spreads* and to be risk-seeking is the reverse (Rothschild & Stiglitz, 1972). For example, if an agent is *risk-averse* in money, then she will prefer \$10 for sure over any lottery whose average value is \$10. *Decreasing* or *increasing marginal utility* is, respectively, a kind of risk-averse or risk-seeking preference that can be captured by an agent's standard utility function and that, therefore, can be captured by an agent's CU function. If an agent has a *decreasing marginal utility function*, then her utility function will *decrease* marginally—that is, each additional unit of something that is of value to her will add *less* utility than the previous unit. If, on the other hand, an agent has an *increasing marginal utility function*, then her utility function will *increase* marginally—that is, each additional unit of something of value will add *more* utility than the previous unit. *Global sensitivity* is a different kind of risk-averse or risk-seeking preference that *cannot* be captured by an agent's standard utility function (Buchak, 2013) and that, therefore, *cannot* be captured by an agent's CU function. An example of global sensitivity is the so-called Allais preferences (Allais, 1953). To quote Buchak (2022),

the *Allais Paradox* identifies a set of preferences that are intuitive but cannot be captured by any expectational utility function (Allais 1953). Consider the choice between the following two lotteries:

- $L_1 : \{\$5,000,000, 0.1; \$0, 0.9\}$
- $L_2 : \{\$1,000,000, 0.11; \$0, 0.89\}$

Separately, consider the choice between the following two lotteries:

- $L_3 : \{\$1,000,000, 0.89; \$5,000,000, 0.1; \$0, 0.01\}$
- $L_4 : \{\$1,000,000, 1\}$

Most people (strictly) prefer  $L_1$  to  $L_2$ , and (strictly) prefer  $L_4$  to  $L_3$ , but there are no values  $u(\$0)$ ,  $u(\$1M)$ , and  $u(\$5M)$  such that  $EU(L_1) > EU(L_2)$  and  $EU(L_4) > EU(L_3)$ .

The Allais preferences violate at least one of the axioms of rational preference that jointly imply the principle that rational preferences maximize EU. In this book, I have assumed that very principle. But whether or not global sensitivity is a property of *rational preferences*, we can reason that since global sensitivity is a property of *preferences*, but *not* choiceworthiness (i.e., CU/CECU), and since the CECU principle is a way of aggregating the *degrees of choiceworthiness* (or CUs) of options across the

various states of the world, it follows that global sensitivity does not call into question the CECU principle.

#### 4.6 A Potential Counterexample to the CU Principle

Let us consider two choice situations: 1 and 2. In each situation, an agent,  $S$ , is faced with the same three options:  $a$ ,  $b$ , and  $c$ . What's more, in each situation,  $S$  assigns probability 1 to a given state of the world (but not the same state for both situations). If that state of the world were realized, then  $S$  would assign the following utilities to the set of options (see Table 21):

**TABLE 21** Decision matrix

	1	2
$a$	100	600
$b$	-100	600
$c$	-100	600

According to the UM principle,  $a$  is choiceworthy for  $S$  in both choice situations 1 and 2. But according to the CU principle,  $a$  is more choiceworthy in 1 than it is in 2, since  $a$ 's CU in 1 (200 c-utiles) is greater than  $a$ 's CU in 2 (0 c-utiles). In other words,  $a$  is more choiceworthy in 1 than in 2 because  $a$  is more worthy of being chosen over  $b$  and  $c$  in 1 than in 2. To put it another way, in 2,  $a$  is merely optional— $S$  misses out on 0 utiles by choosing  $b$  or  $c$  over  $a$ —whereas in 1,  $a$  is not optional— $S$  misses out on 200 utiles by choosing  $b$  or  $c$  over  $a$ . And yet, the proposition that  $a$  is more choiceworthy in 1 than it is in 2 may seem counterintuitive, since  $a$ 's utility in 2 (600 utiles) is greater than  $a$ 's utility in 1 (100 utiles). How can this be...? To make sense of this, we need to distinguish between the degree to which an *option* (e.g.,  $a$ ) is choiceworthy *within* a given choice situation (i.e., 1 or 2), and the degree to which *that choice situation* (i.e., 1 or 2) is *itself* choiceworthy (or desirable). Whereas the former is measured in accordance with the CU principle (for decisions under certainty), the latter is measured in accordance with ECU theory's choice situation principle.

According to *ECU theory's choice situation principle* (as explicated in Chapter 2), for any agent,  $S$ , faced with any decision under *certainty* or any decision under *risk*, for the purpose of measuring, in accordance with ECU theory, the degree to which choice situations are choiceworthy for  $S$  and for the purpose of determining, in accordance with ECU theory, what choice situations are choiceworthy for  $S$ , choice situations are to be treated as *choice options*, and the utility of choice situations is to be treated as the EU of whatever option(s) carry the greatest EU in those choice situations.

Now, the utility of whatever option(s) carry the greatest utility in choice situation 2 (i.e., 600 utiles) is greater than the utility of whatever option(s) carry the greatest utility in choice situation 1 (i.e., 100 utiles). It follows that whereas option  $a$  is more choiceworthy in choice situation 1 than it is in choice situation 2 (in accordance with the CU principle), choice situation 2 is *itself* more choiceworthy (or desirable) than choice situation 1 (in accordance with ECU theory's choice situation principle).

## 5 Accommodating Changes in Preferences over Time

The right to choose to live or to die is the most fundamental right there is; conversely, the duty to give others that opportunity to the best of our ability is the most fundamental duty there is.

—Aubrey de Grey

Should you support healthy-life-extension medical research? Aubrey de Grey thinks that you should. De Grey is a biomedical gerontologist and co-founder of SENS Research Foundation, a public charity that funds biomedical research into regenerative medicine to repair the damage underlying age-related diseases and disabilities. For over 25 years, de Grey has been researching and promoting regenerative medicine solutions to the problem of aging. De Grey has argued that investments in regenerative medicine to extend healthy human life are morally imperative (de Grey & Rae, 2007).

One of his arguments can be put as follows: What gives us the right to decide for future generations whether they should or should not live healthily past the age of 120 years? If we refrain from *investing* in healthy life-extension technologies today because of moral concerns, then we are in effect denying future generations (our children and grandchildren) the right to live healthy lives past our current maximum lifespan. What is considered moral today might be considered loathsome tomorrow, either because circumstances have changed or because values have shifted (Schwitzgebel, 2018). So if we decide today not to *implement* those technologies because of overriding moral concerns, then we are rationally entitled not to do so, but what we are *not* rationally entitled to do is to prevent future generations from benefiting from those technologies should they deem them morally obligatory. Therefore, we can have a reasoned debate about whether it would be moral to *implement* healthy life-extension technologies today (and de Grey has forcefully argued that it *is* morally obligatory to do so), but what we cannot rationally refrain from doing is *investing* in healthy life-extension technologies today for the benefit of future generations (our children and grandchildren) should they deem those technologies morally necessary.<sup>26</sup>

This argument suggests an additional constraint on rational preferences (or values): Preferences may change over time. Therefore, to the extent that no current preferences are violated, rational preferences should leave open as many choice opportunities as possible to accommodate future changes in preferences.

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<sup>26</sup> For a helpful introduction to the ethics of ageing, see Wareham, 2018.

## 6 Conclusion

When arguing for a new normative alternative to EU theory, decision theorists typically come up with decision situations in which their new theory, contrary to EU theory, delivers what seems to be the intuitively correct verdicts. In this book, I have taken a different approach. I have shown that ECU theory gives verdicts that are different from those of EU theory, and I have presented several lines of reasoning to show that the verdicts of ECU theory are more plausible than those of EU theory, even in decision situations where EU theory, contrary to ECU theory, delivers what may seem to be the intuitively correct verdicts.

In this book, I have assumed that (i) EU is the appropriate criterion of *rational preference* to apply to decisions under certainty and decisions under risk, where *preference* and *utility* are understood as *mental states* not reducible to choice; (ii) an option is *choiceworthy* for an agent if and only if it is *maximally choiceworthy* for her over the space of all alternatives in the choice set; (iii) an option is *choiceworthy* for an agent faced with a decision under *certainty* if and only if that option *maximizes utility* over the space of all alternatives in the choice set; and (iv) agents ought to measure and rank their options (for the purpose of choice) in terms of *how instrumentally rational they are*, and agents ought to choose out of *whatever option(s) are instrumentally rational*. If all of these assumptions are correct, then, as I have argued in this book, we can derive a notion of *choiceworthiness* according to which (i) agents ought to measure and rank their options (for the purpose of choice) in terms of how choiceworthy they are—that is, according to their CU (in decisions under certainty) or their CECU (in decisions under risk)—and (ii) agents ought to choose out of what option(s) are choiceworthy—that is, what option(s) maximize CU (in decisions under certainty) or what option(s) maximize CECU (in decisions under risk). As I have argued, how comparatively *choiceworthy* a set of options are for an agent is independent from, and sometimes differs from, whatever *preferences* she ought to have with respect to those options.

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