A condition for transitivity in high probability

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ABSTRACT: There are many scientific and everyday cases where (a) each of $\Pr(H_1 \mid E)$ and $\Pr(H_2 \mid H_1)$ is high and (b) *it seems that* $\Pr(H_2 \mid E)$ is high. But high probability (or absolute confirmation) is not transitive and so it might be in such cases that (a) each of $\Pr(H_1 \mid E)$ and $\Pr(H_2 \mid H_1)$ is high and (c) *in fact* $\Pr(H_2 \mid E)$ is not high. There is no issue in the special case where the following condition, which I call "C1", holds: H_1 entails H_2. This condition is sufficient for transitivity in high probability. But many of the scientific and everyday cases referred to above are cases where it is not the case that H_1 entails H_2. I consider whether there are additional (non-trivial) conditions sufficient for transitivity in high probability. I consider three candidate conditions. I call them "C2", "C3", and "C2&3". I argue that C2&3, but neither C2 nor C3, is sufficient for transitivity in high probability. I then set out some further results and relate the discussion to the Bayesian requirement of coherence.

KEYWORDS: high probability, Krauss, multiverse hypothesis, transitivity

1 Introduction

It might seem that we could never have empirical evidence on which the "multiverse" hypothesis (understood as the hypothesis that there is an infinite number of causally separated universes) is highly probable. Some scientists, though, disagree. Lawrence Krauss is a case in point. Here, in a relatively recent interview (Andersen 2012), he explains how we *could* have such evidence:

> How do you tell that there's a multiverse if the rest of the universes are outside your causal horizon? It sounds like philosophy. At best. But imagine that we had a fundamental particle theory that explained why there are three generations of fundamental particles, and why the proton is two thousand times heavier than the electron, and why there are four forces of nature, etc. And it also predicted a period of inflation in the early universe, and it predicts everything that we see and you can follow it through the entire evolution of the early universe to see how we got here. Such a theory might, in addition to predicting everything we see, also predict a host of universes

> that we don't see. If we had such a theory, the accurate predictions it makes about what we can see would also make its predictions about what we can't see extremely likely. And so I could see empirical evidence internal to this universe validating the existence of a multiverse, even if we could never see it directly.

Krauss holds that this is more than a mere possibility. Here, in that same relatively recent interview (Andersen 2012), he explains how we *in fact* have empirical evidence on which the multiverse hypothesis is highly probable:

> There are a variety of multiverses that people in physics talk about. The most convincing one derives from something called inflation, which we're pretty certain happened because it produces effects that agree with almost everything we can observe. From what we know about particle physics, it seems quite likely that the universe underwent a period of exponential expansion early on. But inflation, insofar as we understand it, never ends—it only ends in certain regions and then those regions become a universe like ours. You can show that in an inflationary universe, you produce a multiverse, you produce an infinite number of causally separated universes over time, and the laws of physics are different in each one. … There's a calculable multiverse; it's almost required for inflation—it's very hard to get around it. All the evidence suggests that our universe resulted from a period of inflation, and it's strongly suggestive that well beyond our horizon there are other universes that are being created out of inflation, and that most of the multiverse is still expanding exponentially.

Let E be our empirical evidence, I be the inflation hypothesis, and M be the multiverse hypothesis. The idea, it seems, is this:

(1) $\Pr(I \mid E)$ is high.
(2) $\Pr(M \mid I)$ is high.
Thus
(3) $\Pr(M \mid E)$ is high.

Some theorists question whether there is a version of I such that each of (1) and (2) is true.[1] But I want to grant for the sake of argument that Krauss has in mind a version of I such that each of (1) and (2) is true. The question is: Is Krauss's argument valid?

It might seem that the answer is affirmative, for it might seem that high probability (or absolute confirmation) is transitive in that:

[1] See, for example, Ellis (2008, p. 2.34).

Transitivity of High Probability (THP): For any propositions E, H_1, and H_2, if (a) $\Pr(H_1 \mid E) > t$ and (b) $\Pr(H_2 \mid H_1) > t$, then $\Pr(H_2 \mid E) > t$.

Here and throughout t is the threshold for high probability, I am assuming that $0.5 \leq t < 1$, and I am not assuming any particular value for t (or even that t is invariant across contexts). Clearly, if THP is correct, then any case where (1) and (2) in Krauss's argument are true is a case where (3) in Krauss's argument is true and thus Krauss's argument is valid. Is it the case, though, that THP is correct?

It is straightforward to show that THP is incorrect given at least *some* values for t. Suppose, for instance, that the value for t is 0.5 and that a card is randomly drawn from a standard and well-shuffled deck of cards. Let E be the proposition that the card drawn is a Two, Three, Four, Five, or Six, H_1 be the proposition that the card drawn is a Four, Five, Six, Seven, or Eight, and H_2 be the proposition that the card drawn is a Six, Seven, Eight, Nine, or Ten. It follows that $\Pr(H_1 \mid E) = 0.6 > t$, $\Pr(H_2 \mid H_1) = 0.6 > t$, and $\Pr(H_2 \mid E) = 0.2 < t$. Hence THP is incorrect when the value for t is 0.5.

Is the same true for *all* values for t? It turns out that the answer is affirmative. Consider the thesis:

Transitivity of High Probability & Increase in Probability (THP&IP): For any propositions E, H_1, and H_2, if (a) $\Pr(H_1 \mid E) > t$, (b) $\Pr(H_1 \mid E) > \Pr(H_1)$, (c) $\Pr(H_2 \mid H_1) > t$, and (d) $\Pr(H_2 \mid H_1) > \Pr(H_2)$, then $\Pr(H_2 \mid E) > t$ and $\Pr(H_2 \mid E) > \Pr(H_2)$.

Douven (2011) shows that this thesis is incorrect given any value for t. He shows this by showing that for any value for t there is a probability distribution on which $\Pr(H_1 \mid E) > t$, $\Pr(H_1 \mid E) > \Pr(H_1)$, $\Pr(H_2 \mid H_1) > t$, $\Pr(H_2 \mid H_1) > \Pr(H_2)$, $\Pr(H_2 \mid E) < t$, and $\Pr(H_2 \mid E) < \Pr(H_2)$. It follows immediately that for any value for t there is a probability distribution on which $\Pr(H_1 \mid E) > t$, $\Pr(H_2 \mid H_1) > t$, and $\Pr(H_2 \mid E) \leq t$. Hence THP, as with THP&IP, is incorrect given any value for t.

Return now to Krauss's argument. Since THP is false, it follows that Krauss's argument is invalid.

This does not mean, of course, that Krauss is wrong that $\Pr(M \mid E)$ is high. But, at the same time, caution is needed. It might be the case, for all Krauss shows, that $\Pr(I \mid E)$ is high, $\Pr(M \mid I)$ is high, and yet $\Pr(M \mid E)$ is not high.

The case of Krauss and the multiverse hypothesis is just an example. There are many scientific and everyday cases where (a) each of $\Pr(H_1 \mid E)$ and $\Pr(H_2 \mid H_1)$ is high and (b) *it seems that* $\Pr(H_2 \mid E)$ is high. But since THP is incorrect given any value for t, it might be in such cases that (a) each of $\Pr(H_1 \mid E)$ and $\Pr(H_2 \mid H_1)$ is high and (c) *in fact* $\Pr(H_2 \mid E)$ is not high.

There is no issue in the special case where the following condition holds:

Condition 1 (C1): H_1 entails H_2

It is straightforward to show that:

Transitivity of High Probability under C1 (THP_{C1}): For any propositions E, H_1, and H_2, if (a) $\Pr(H_1 \mid E) > t$, (b) $\Pr(H_2 \mid H_1) > t$, and (c) C1 holds, then $\Pr(H_2 \mid E) > t$.

Suppose that $\Pr(H_1 \mid E) > t$, $\Pr(H_2 \mid H_1) > t$, and C1 holds. Given that C1 holds, it follows that $\Pr(H_2 \mid E) \geq \Pr(H_1 \mid E)$. Given this, and given that $\Pr(H_1 \mid E) > t$, it follows that $\Pr(H_2 \mid E) > t$. Hence THP_{C1}.

It should be noted that (b) in THP_{C1} is redundant given (c) and so THP_{C1} could be reformulated as:

Transitivity of High Probability under C1 (THP_{C1}): For any propositions E, H_1, and H_2, if (a) $\Pr(H_1 \mid E) > t$ and (b) C1 holds, then $\Pr(H_2 \mid E) > t$.

I prefer the initial formulation because the task at hand is to add a condition to THP's antecedent so that the resulting thesis holds without exception. But nothing of importance, for my purposes, hinges on which formulation is used.[2]

But many of the scientific and everyday cases referred to above—cases where (a) each of $\Pr(H_1 \mid E)$ and $\Pr(H_2 \mid H_1)$ is high and (b) it seems that $\Pr(H_2 \mid E)$ is high—are cases where C1 does not hold.[3] Are there additional (non-trivial) conditions sufficient for transitivity in high probability?[4]

This is the main question of the paper. I consider three candidate conditions. I call them "C2", "C3", and "C2&3". I argue that C2&3, but neither C2 nor C3, is sufficient for transitivity in high probability. I address C2 in Section 2, C3 in Section 3, and C2&3 in Section 4. I then set out some further results in Section 5 and relate the discussion to the Bayesian requirement of coherence in Section 6.

2 Condition 2 (C2)

The first candidate condition is this:

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[2] These remarks carry over to THP_{C2} in Section 2 and $\text{THP}_{\text{C2\&3}}$ in Section 4.

[3] Krauss seems to hold that $\Pr(M \mid I)$ is high but less than 1.

[4] An example of a trivial condition sufficient for transitivity in high probability is this: $\Pr(H_2 \mid E) > t$.

Condition 2 (C2): $\Pr(H_1 \mid E) > t_{\mathrm{SH}}$ and $\Pr(H_2 \mid H_1) > t_{\mathrm{SH}}$.

Here and throughout "t_{SH}" is the threshold for super-high probability and I am assuming that $t_{\mathrm{SH}} = \sqrt[2]{t}$.[5] If, say, $t = 0.81$, then $t_{\mathrm{SH}} = 0.9$.

There can be cases where each of $\Pr(H_1 \mid E)$ and $\Pr(H_2 \mid H_1)$ is high but $\Pr(H_2 \mid E)$ is not high. Perhaps, though, there can be no cases where each of $\Pr(H_1 \mid E)$ and $\Pr(H_2 \mid H_1)$ is super-high but $\Pr(H_2 \mid E)$ is not high. Perhaps, that is, the following is correct:

Transitivity of High Probability under C2 (THP_{C2}): For any propositions E, H_1, and H_2, if (a) $\Pr(H_1 \mid E) > t$, (b) $\Pr(H_2 \mid H_1) > t$, and (c) C2 holds, then $\Pr(H_2 \mid E) > t$.

Note that the card case from above, which is problematic for THP when the value for t is 0.5, is not problematic for $\mathrm{THP}_{\mathrm{C2}}$ when the value for t is 0.5 or any other value. Neither $\Pr(H_1 \mid E)$ nor $\Pr(H_2 \mid H_1)$ is greater than $\sqrt[2]{0.5} \approx 0.707$ and thus C2 fails to hold.

Recall that Douven (2011) shows that for any value for t there is a probability distribution on which $\Pr(H_1 \mid E) > t$, $\Pr(H_1 \mid E) > \Pr(H_1)$, $\Pr(H_2 \mid H_1) > t$, $\Pr(H_2 \mid H_1) > \Pr(H_2)$, $\Pr(H_2 \mid E) < t$, and $\Pr(H_2 \mid E) < \Pr(H_2)$. Does it follow that $\mathrm{THP}_{\mathrm{C2}}$ is incorrect given any value for t?

It might seem that the answer is affirmative. Take some value for t. Suppose, say, that $t = 0.81$ so that $t_{\mathrm{SH}} = 0.9$. Then, given that THP is incorrect given any value for t, it follows that there are probability distributions on which $\Pr(H_1 \mid E) > 0.9 = t_{\mathrm{SH}}$ and $\Pr(H_2 \mid H_1) > 0.9 = t_{\mathrm{SH}}$. It might seem that any such probability distribution is a probability distribution on which $\mathrm{THP}_{\mathrm{C2}}$'s antecedent holds but its consequent does not. There is a mistake here however. It is true that there are probability distributions on which $\Pr(H_1 \mid E) > 0.9$, $\Pr(H_2 \mid H_1) > 0.9$, and $\Pr(H_2 \mid E) \leq 0.9$. This leaves it open, though, that on each such probability distribution $\Pr(H_2 \mid E) > 0.81 = t$. Perhaps, then, THP is open to counterexample given any value for t and yet each of the cases in question where $\Pr(H_1 \mid E) > t_{\mathrm{SH}}$ and $\Pr(H_2 \mid H_1) > t_{\mathrm{SH}}$ is a case where $\Pr(H_2 \mid E) > t$.

It will help to consider Douven's argument in more detail. He gives a schema for probability distributions and shows in part that the variables therein can be specified so that

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[5] Nothing of importance in this section hinges on the choice of $\sqrt[2]{t}$ as the threshold for super-high probability. $\mathrm{THP}_{\mathrm{C2}}$ below is incorrect given any value for t and any alternative threshold for super-high probability (greater than t and less than 1). Things are different in Section 4. If the threshold for super-high probability were greater than $\sqrt[2]{t}$, then $\mathrm{THP}_{\mathrm{C2\&3}}$ would still hold without exception but its antecedent would be stronger than it needs to be (in order for it to hold without exception). If the threshold for super-high probability were less than $\sqrt[2]{t}$, then $\mathrm{THP}_{\mathrm{C2\&3}}$ would fail to hold without exception.

$\Pr(H_1 \mid E)$ and $\Pr(H_2 \mid H_1)$ are arbitrarily close to 1 while $\Pr(H_2 \mid E)$ is arbitrarily close to 0. It follows that given any value for t there is an instance of Douven's schema such that $\Pr(H_1 \mid E) > t_{SH}$, $\Pr(H_2 \mid H_1) > t_{SH}$, and $t \geq 0.5 > \Pr(H_2 \mid E)$. It thus follows that THP_{C2} is incorrect given any value for t.

It is not the case, then, that C2 is sufficient for transitivity in high probability. THP_{C2} is open to counterexample.

3 Condition 3 (C3)

It is easy to see why THP is open to counterexample. First, note that:

$$(4) \qquad \Pr(H_2 \mid E) = \Pr(H_2 \,\&\, H_1 \mid E) + \Pr(H_2 \,\&\, \neg H_1 \mid E)$$

Second, note that the right side of (4) is equal to:

$$(5) \qquad \Pr(H_1 \mid E)\Pr(H_2 \mid H_1 \,\&\, E) + \Pr(\neg H_1 \mid E)\Pr(H_2 \mid \neg H_1 \,\&\, E)$$

Suppose that each of $\Pr(H_1 \mid E)$ and $\Pr(H_2 \mid H_1)$ is high. Then, since $\Pr(H_1 \mid E)$ is high, the first multiplicand in the second addend in (5) is low and thus, as $\Pr(H_2 \mid \neg H_1 \,\&\, E) \leq 1$, the second addend in (5) is low. None of this puts any constraints on the value for $\Pr(H_2 \mid H_1 \,\&\, E)$. If $\Pr(H_2 \mid H_1 \,\&\, E)$ is sufficiently low, then (5) is not high and thus $\Pr(H_2 \mid E)$ is not high.

Consider now the condition:

Condition 3 (C3): $\Pr(H_2 \mid H_1 \,\&\, E) \geq \Pr(H_2 \mid H_1)$.

C3 holds if and only if H_1 screens off E from H_2 in that H_1 makes it such that E has no negative impact on the probability of H_2. Any case where $\Pr(H_1 \mid E)$ is high, $\Pr(H_2 \mid H_1)$ is high, and C3 holds is a case where each multiplicand in the first addend in (5) is high. It might seem, then, that the following is correct:

Transitivity of High Probability under C3 (THP_{C3}): For any propositions E, H_1, and H_2, if (a) $\Pr(H_1 \mid E) > t$, (b) $\Pr(H_2 \mid H_1) > t$, and (c) C3 holds, then $\Pr(H_2 \mid E) > t$.

Note that the card case from above, which is problematic for THP when the value for t is 0.5, is not problematic for THP_{C3} when the value for t is 0.5 or any other value. This is because $\Pr(H_2 \mid H_1 \,\&\, E) = 1/3 < 0.6 = \Pr(H_2 \mid H_1)$ and so C3 fails to hold.

It turns out, though, that THP_{C3} is incorrect given any value for t. This can be seen by considering the following:

Condition 4 (C4): $\Pr(H_2 \mid H_1 \,\&\, E) = \Pr(H_2 \mid H_1)$ and $\Pr(H_2 \mid \neg H_1 \,\&\, E) = \Pr(H_2 \mid \neg H_1)$.

Transitivity of High Probability & Increase in Probability under C4 (THP&IP$_{C4}$): For any propositions E, H_1, and H_2, if (a) $\Pr(H_1 \mid E) > t$, (b) $\Pr(H_1 \mid E) > \Pr(H_1)$, (c) $\Pr(H_2 \mid H_1) > t$, (d) $\Pr(H_2 \mid H_1) > \Pr(H_2)$, and (e) C4 holds, then $\Pr(H_2 \mid E) > t$ and $\Pr(H_2 \mid E) > \Pr(H_2)$.

Douven (2011) gives a schema for probability distributions such that the variables therein can be specified so that for any value for t there is a probability distribution on which $\Pr(H_1 \mid E) > t$, $\Pr(H_1 \mid E) > \Pr(H_1)$, $\Pr(H_2 \mid H_1) > t$, $\Pr(H_2 \mid H_1) > \Pr(H_2)$, C4 holds, and $\Pr(H_2 \mid E) \leq t$. Hence THP&IP$_{C4}$ is incorrect given any value for t. It follows that the same is true of:

Transitivity of High Probability under C4 (THP$_{C4}$): For any propositions E, H_1, and H_2, if (a) $\Pr(H_1 \mid E) > t$, (b) $\Pr(H_2 \mid H_1) > t$, and (c) C4 holds, then $\Pr(H_2 \mid E) > t$.

But C4 is stronger than C3 and so any case where C4 holds is a case where C3 holds. It follows that for any value for t there is a probability distribution on which $\Pr(H_1 \mid E) > t$, $\Pr(H_2 \mid H_1) > t$, C3 holds, and $\Pr(H_2 \mid E) \leq t$. Hence THP$_{C3}$ is incorrect given any value for t.

It is not the case, then, that C3 is sufficient for transitivity in high probability. THP$_{C3}$ is open to counterexample.[6]

4 Condition 2&3 (C2&3)

Neither C2 nor C3 by itself is sufficient for transitivity in high probability. But consider the condition:

[6] Consider the conditions:

Condition 5 (C5): $\Pr(H_2 \mid H_1 \,\&\, E) = \Pr(H_2 \mid H_1)$.

Condition 6 (C6): $\Pr(H_2 \mid H_1 \,\&\, E) \geq \Pr(H_2 \mid H_1)$ and $\Pr(H_2 \mid \neg H_1 \,\&\, E) \geq \Pr(H_2 \mid \neg H_1)$.

C4 is stronger than each of C5 and C6. Hence neither C5 nor C6 is sufficient for transitivity in high probability. Hence none of C3, C4, C5, and C6 is sufficient for transitivity in high probability. The situation is a bit different in the context of *increase in* probability: each of C4 and C6 but neither C3 nor C5 is sufficient for transitivity in increase in probability. See Atkinson and Peijnenburg (2013), Roche (2012a, b, 2014, 2015), Roche and Shogenji (2014), Shogenji (2003, forthcoming), and Sober (2015, Ch. 5) for relevant discussion.

Condition 2&3 (C2&3): $\Pr(H_1 \mid E) > t_{SH}$, $\Pr(H_2 \mid H_1) > t_{SH}$, and $\Pr(H_2 \mid H_1 \,\&\, E) \geq \Pr(H_2 \mid H_1)$.

Suppose that $\Pr(H_1 \mid E) > t$, $\Pr(H_2 \mid H_1) > t$, and C2&3 holds. It follows from (4) and (5) that:

(6) $$\Pr(H_2 \mid E) \geq \Pr(H_1 \mid E)\Pr(H_2 \mid H_1 \,\&\, E)$$

Given this, and given that $\Pr(H_2 \mid H_1 \,\&\, E) \geq \Pr(H_2 \mid H_1)$, it follows that:

(7) $$\Pr(H_2 \mid E) \geq \Pr(H_1 \mid E)\Pr(H_2 \mid H_1)$$

Since $\Pr(H_1 \mid E) > t_{SH}$ and $\Pr(H_2 \mid H_1) > t_{SH}$, it follows that:

(8) $$\Pr(H_2 \mid E) \geq \Pr(H_1 \mid E)\Pr(H_2 \mid H_1) > \sqrt[2]{t}\sqrt[2]{t} = t$$

The following, then, holds without exception:

Transitivity of High Probability under C2&3 ($THP_{C2\&3}$): For any propositions E, H_1, and H_2, if (a) $\Pr(H_1 \mid E) > t$, (b) $\Pr(H_2 \mid H_1) > t$, and (c) C2&3 holds, then $\Pr(H_2 \mid E) > t$.

C2&3 is thus sufficient for transitivity in high probability.

Return to the case of Krauss and the multiverse hypothesis. Krauss's argument from (1) and (2) to (3) is invalid. The following, by contrast, is valid:

(1) $\Pr(I \mid E)$ is high.
(2) $\Pr(M \mid I)$ is high.
(9) C2&3 holds in that $\Pr(I \mid E) > t_{SH}$, $\Pr(M \mid I) > t_{SH}$, and $\Pr(M \mid I \,\&\, E) \geq \Pr(M \mid I)$.
Thus
(3) $\Pr(M \mid E)$ is high.

If Krauss is right that each of (1) and (2) is true, and if in addition (9) is true, it follows that Krauss is right that (3) is true.

I want to remain neutral on whether in fact (9) is true. The important point for my purposes is the conditional point that *if* (9) is true, *then*, granting for the sake of argument that Krauss is right that each of (1) and (2) is true, Krauss is right that (3) is true.

5 Further results

5.1 Cases where $\Pr(H_1 \mid E)$ and $\Pr(H_2 \mid H_1)$ are high but not super-high

Suppose that $\Pr(H_1 \mid E)$ and $\Pr(H_2 \mid H_1)$ are high but not super-high. Then C2&3 fails to hold and so there is no guarantee that $\Pr(H_2 \mid E)$ is high. What then?

Here the following variant of THP_{C3} can be useful:

Transitivity of High Probability under C3 (THP*_{C3})*: For any propositions E, H_1, and H_2, if (i) $\Pr(H_1 \mid E) > t$, (ii) $\Pr(H_2 \mid H_1) > t$, and (iii) C3 holds, then $\Pr(H_2 \mid E) > t^2$.

This thesis differs from THP_{C3} in that its consequent is weaker than THP_{C3}'s consequent.[7] If, say, $t = 0.9$, $\Pr(H_1 \mid E) > t$, $\Pr(H_2 \mid H_1) > t$, and C3 holds, then by THP*_{C3} it follows not that $\Pr(H_2 \mid E) > 0.9$ but that $\Pr(H_2 \mid E) > (0.9)(0.9) = 0.81$.

5.2 Cases involving four or more propositions

$\text{THP}_{\text{C2\&3}}$ can be generalized to cases involving four or more propositions. Take, for example, cases involving four propositions such that $\Pr(H_1 \mid E) > t$, $\Pr(H_2 \mid H_1) > t$, and $\Pr(H_3 \mid H_2) > t$. Suppose that $\Pr(H_2 \mid H_1 \,\&\, E) \geq \Pr(H_2 \mid H_1)$ and $\Pr(H_3 \mid H_2 \,\&\, H_1 \,\&\, E) \geq \Pr(H_3 \mid H_2)$. Then it follows that:

$$(10) \qquad \Pr(H_3 \mid E) \geq \Pr(H_1 \mid E)\Pr(H_2 \mid H_1)\Pr(H_3 \mid H_2)$$

Let "t_{SSH}" be the threshold for super-super-high probability where $t_{\text{SSH}} = \sqrt[3]{t}$. Suppose, further, that $\Pr(H_1 \mid E) > t_{\text{SSH}}$, $\Pr(H_2 \mid H_1) > t_{\text{SSH}}$, and $\Pr(H_3 \mid H_2) > t_{\text{SSH}}$. Then it follows that:

$$(11) \qquad \Pr(H_3 \mid E) \geq \Pr(H_1 \mid E)\Pr(H_2 \mid H_1)\Pr(H_3 \mid H_2) > \sqrt[3]{t}\sqrt[3]{t}\sqrt[3]{t} = t$$

Hence $\Pr(H_3 \mid E) > t$.

The same is true with respect to THP*_{C3}. If, say, $t = 0.9$, $\Pr(H_1 \mid E) > t$, $\Pr(H_2 \mid H_1) > t$, $\Pr(H_3 \mid H_2) > t$, $\Pr(H_2 \mid H_1 \,\&\, E) \geq \Pr(H_2 \mid H_1)$, and $\Pr(H_3 \mid H_2 \,\&\, H_1 \,\&\, E) \geq \Pr(H_3 \mid H_2)$, then it follows that $\Pr(H_3 \mid E) > (0.9)(0.9)(0.9) = 0.729$.

[7] That THP_{C3} holds without exception can be seen by appeal to (7) above.

6 Coherence

A central component of Bayesianism is the requirement of coherence. This is the requirement that a subject's degree of belief function f should be a probability function and thus should be such that (for any propositions E and H over which f is defined) (a) $f(E) \geq 0$, (b) $f(E) = 1$ if E is a logical truth, (c) $f(E \vee H) = f(E) + f(H)$ if E and H are mutually exclusive, and (d) $f(E \mid H) = f(E \,\&\, H)/f(H)$.[8]

Given the requirement of coherence, and given $THP_{C2\&3}$, it follows that a subject's degree of belief function f should be such that if (a)-(c) in $THP_{C2\&3}$ all hold, then her degree of belief in H_2 given E is greater than her degree of belief in H_2. So if f is such that (a)-(c) in $THP_{C2\&3}$ all hold and yet her degree of belief in H_2 given E is less than or equal to her degree of belief in H_2, then her degree of belief function fails to meet the requirement of coherence and thus she is less than ideally rational (assuming Bayesianism).

It might be objected that $THP_{C2\&3}$ is redundant given the requirement of coherence. It follows from the latter by itself that a subject's degree of belief function f should be such that if (a)-(c) in $THP_{C2\&3}$ all hold, then her degree of belief in H_2 given E is greater than her degree of belief in H_2. This is because $THP_{C2\&3}$ is a theorem of the probability calculus.

It is true that $THP_{C2\&3}$ is redundant given the requirement of coherence. It is far from trivial, though, for a *realistic* subject to have a coherent degree of belief function. Few, if any, realistic subjects have a coherent degree of belief function and so few, if any, realistic subjects are ideally rational. But some are closer to being ideally rational than are others. $THP_{C2\&3}$ can be helpful on this front. Suppose that you are aware that your degree of belief function is such that (a) and (b) in $THP_{C2\&3}$ hold. Suppose that you are also aware that your degree of belief in H_2 given E is less than your degree of belief in H_2. You then note that your degree of belief function is such that (c) in $THP_{C2\&3}$ holds. Here it would be helpful to you if you knew that $THP_{C2\&3}$ holds without exception and that, thus, you should adjust, say, your degree of belief in H_2 given E so that it is greater than your degree of belief in H_2. This would move you closer to being ideally rational than you were before.

This is the point behind results such as $THP_{C2\&3}$. They serve to make the requirement of coherence more transparent. This is helpful for realistic subjects lacking in logical omniscience.

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[8] There is also a requirement to the effect that upon the receipt of new information a subject's degree of belief function should be updated by conditionalization (strict conditionalization, Jeffrey conditionalization, or Field conditionalization. For discussion of Bayesianism, and for references, see Talbott (2016).

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