Evidential support, transitivity, and screening-off

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ABSTRACT: Is evidential support transitive? The answer is negative when evidential support is understood as confirmation so that X evidentially supports Y if and only if $p(Y \mid X) > p(Y)$. I call evidential support so understood "support" (for short) and set out three alternative ways of understanding evidential support: support-**t** (support plus a sufficiently high probability), support-**t*** (support plus a substantial degree of support), and support-**tt*** (support plus both a sufficiently high probability and a substantial degree of support). I also set out two screening-off conditions (under which support is transitive): SOC1 and SOC2. It has already been shown that support-**t** is non-transitive in the general case (where it is not required that SOC1 holds and it is not required that SOC2 holds), in the special case where SOC1 holds, and in the special case where SOC2 holds. I introduce two rather weak adequacy conditions on support measures and argue that on any support measure meeting those conditions it follows that neither support-**t*** nor support-**tt*** is transitive in the general case, in the special case where SOC1 holds, or in the special case where SOC2 holds. I then relate some of the results to Douven's evidential support theory of conditionals along with a few rival theories.

1 Introduction

Is evidential support transitive? The answer is negative when evidential support is understood as confirmation so that X evidentially supports Y if and only if $p(Y \mid X) > p(Y)$.[1] Suppose, to illustrate, a card is randomly drawn from a standard (and well-shuffled) deck of cards. Let X be the proposition that the card drawn is a Heart, Y be the proposition that the card drawn is a Red, and Z be the proposition that the

[1] This is "incremental" confirmation as opposed to "absolute" confirmation. See Carnap (1962, Preface to 2nd ed.) on "concepts of increase in firmness" and "concepts of firmness".

card drawn is a Diamond. Then $p(Y \mid X) = 1 > 1/2 = p(Y)$, $p(Z \mid Y) = 1/2 > 1/4 = p(Z)$, and yet $p(Z \mid X) = 0 < 1/4 = p(Z)$. There are at least three alternative ways of understanding evidential support however.[2] First, evidential support can be understood as confirmation together with a sufficiently high probability so that X evidentially supports Y if and only if $p(Y \mid X) > p(Y)$ and $p(Y \mid X)$ is sufficiently high (close to 1). Second, evidential support can be understood as confirmation together with a substantial degree of confirmation so that X evidentially supports Y if and only if $p(Y \mid X) > p(Y)$ and the degree of confirmation $c(Y, X)$ is substantial. Third, evidential support can be understood as confirmation together with both a sufficiently high probability and a substantial degree of confirmation so that X evidentially supports Y if and only if $p(Y \mid X) > p(Y)$, $p(Y \mid X)$ is sufficiently high, and the degree of confirmation $c(Y, X)$ is substantial. Perhaps, though evidential support understood as confirmation is non-transitive, things are different when evidential support is understood as confirmation together with a sufficiently high probability, or when evidential support is understood as confirmation together with a substantial degree of confirmation, or when evidential support is understood as confirmation together with both a sufficiently high probability and a substantial degree of confirmation.

It will help to introduce some terminology. By "X supports Y" I mean "$p(Y \mid X) > p(Y)$". By "X supports-**t** Y" I mean "$p(Y \mid X) > p(Y)$ and $p(Y \mid X) > \mathbf{t}$" where **t** is the threshhold for sufficiently high probability. I assume that $0.5 \leq \mathbf{t} < 1$ but do not assume any particular value for **t**. By "X supports-**t*** Y" I mean "$p(Y \mid X) > p(Y)$ and $c(Y, X) > \mathbf{t^*}$" where **t*** is the threshhold for substantial support. I assume that **t*** is greater than the neutral value for $c(Y, X)$—the value for $c(Y, X)$ in cases where X neither increases nor decreases the probability of Y—and less than the maximum value for $c(Y, X)$ (which may be ∞). By "X supports-**tt*** Y" I mean "$p(Y \mid X) > p(Y)$, $p(Y \mid X) > \mathbf{t}$, and $c(Y, X) > \mathbf{t^*}$". The main points from the previous paragraph can then be put as follows: support is non-transitive but perhaps things are different with support-**t**, or with support-**t***, or with support-**tt***.

It turns out that support is transitive under each of the following screening-off conditions:[3]

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[2] See Douven (2011). See also Roche and Shogenji (2014a) on "confirmation-TSF" for yet another alternative way of understanding evidential support.

[3] That support is transitive under SOC1 is shown in Shogenji (2003). See Sober (2009, p. 76) for an equivalent result. That support is transitive under SOC2 is

Screening-Off Condition 1 (SOC1): $p(Z \mid Y \wedge X) = p(Z \mid Y)$ and $p(Z \mid \neg Y \wedge X) = p(Z \mid \neg Y)$

Screening-Off Condition 2 (SOC2): $p(Z \mid Y \wedge X) > p(Z \mid Y)$ and $p(Z \mid \neg Y \wedge X) > p(Z \mid \neg Y)$

These conditions hold in many cases and so are helpful in establishing relations of support.[4,5]

That support is transitive under SOC1 and SOC2 raises the possibility that even if each of support-**t**, support-**t***, and support-**tt*** is non-transitive, things are different under SOC1 and SOC2. Perhaps, that is, even if each of support-**t**, support-**t***, and support-**tt*** is non-transitive in the general case (where it is not required that SOC1 holds and it is not required that SOC2 holds), each of support-**t**, support-**t***, and support-**tt*** is nonetheless transitive both in the special case where SOC1 holds and in the special case where SOC2 holds.

So there are two questions to consider regarding support-**t**, two questions to consider regarding support-**t***, and two questions to consider regarding support-**tt***. Is support-**t** transitive in the general case? Is support-**t** transitive in the special case where SOC1 holds or in the special case where SOC2 holds? Is support-**t*** transitive in the general case? Is support-**t*** transitive in the special case where SOC1 holds or in the special case where SOC2 holds? Is support-**tt*** transitive in the general case? Is support-**tt*** transitive in the special case where SOC1 holds or in the special case where SOC2 holds?

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shown in effect in Roche (2012a). There it is shown that support is transitive under the condition:

Screening-Off Condition 3 (SOC3): $p(Z \mid Y \wedge X) \geq p(Z \mid Y)$ and $p(Z \mid \neg Y \wedge X) \geq p(Z \mid \neg Y)$.

It follows that support is transitive under SOC2.

[4] See Roche (2012a, 2014a, 2014b), Roche and Shogenji (2014a), and Shogenji (2003) for relevant discussion.

[5] It is worth noting that "anti-support" in the sense of disconfirmation is intransitive under each of SOC1 and SOC2 in that if X anti-supports Y, Y anti-supports Z, and SOC1 or SOC2 holds, then X supports Z. See Atkinson and Peijnenburg (2013) for relevant discussion.

Some progress in answering these questions has already been made. First, it has been shown that support-**t** is non-transitive in the general case, in the special case where SOC1 holds, and in the special case where SOC2 holds.[6] Second, it has been shown that support-**t*** and support-**tt*** are non-transitive both in the general case and in the special case where SOC1 holds if any of the following support measures is assumed:[7]

$$c_d(Y,X) = p(Y \mid X) - p(Y)$$

$$c_{ll}(Y,X) = \log\left[\frac{p(X \mid Y)}{p(X \mid \neg Y)}\right]$$

$$c_{KO}(Y,X) = \frac{p(X \mid Y) - p(X \mid \neg Y)}{p(X \mid Y) + p(X \mid \neg Y)}$$

The first of these measures, c_d, is the "difference" measure. The second, c_{ll}, is the "log likelihood" measure. The third, c_{KO}, is the Kemeny-Oppenheim measure. Each measure is among the most popular measures in the literature.

There is a clear sense in which each of the results just mentioned is robust. No particular values are specified for **t** or **t***. But, at the same time, there is also a clear sense in which some of the results are not robust (or at least not as robust as is desirable). The class of extant and merely possible support measures is vast (in fact, infinite) in size. Perhaps some fourth measure is preferable to c_d, c_{ll}, and c_{KO}. And perhaps support-**t*** and support-**tt*** are transitive (in the general case and thus also in the special case where SOC1 holds and the special case where SOC2 holds) if that fourth measure is assumed. It would be welcome, then, if it could be determined whether the results mentioned above regarding support-**t*** and support-**tt*** can be generalized so that they hold for *any* adequate support measure.

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[6] That support-**t** is non-transitive in the general case is shown in Douven (2011). That support-**t** is non-transitive in the special case where SOC1 holds is shown in Douven (2011). That support-**t** is non-transitive in the special case where SOC2 holds is shown in Roche (2012b).

[7] See Douven (2011). Douven (2011) notes (but leaves it to readers to verify) that these results regarding support-**t*** and support-**tt*** in the *general* case carry over to several alternative support measures in the literature.

I aim to show that in fact the results in question can be so generalized. So there is no "problem of measure sensitivity" here.[8]

The remainder of the paper is organized as follows. In Section 2, I set out two rather weak adequacy conditions on support measures. I call them (the conditions) "AC1" and "AC2". In Section 3, I argue that on any support measure meeting AC1 and AC2 it follows that (i) neither support-**t*** nor support-**tt*** is transitive in the general case, (ii) neither support-**t*** nor support-**tt*** is transitive in the special case where SOC1 holds, and (iii) neither support-**t*** nor support-**tt*** is transitive in the special case where SOC2 holds. In Section 4, I relate some of the results from Section 3 to the idea that the assertability/acceptability of a conditional is a matter of whether the antecedent evidentially supports the consequent. Here I discuss Douven's evidential support theory of conditionals (2008, forthcoming) along with a few rival theories. In Section 5, I conclude.

2 Two adequacy conditions on support measures

I want to remain relatively neutral on the thorny issue of how exactly support is to be measured. I assume, as is standard, that any adequate support measure c should meet the condition:

> *Adequacy Condition 1 (AC1)*: There is a value n such that $c(Y, X) > / = / < n$ if and only if $p(Y \mid X) > / = / < p(Y)$.

Beyond AC1 I assume just that any adequate support measure c should meet the condition:

> *Adequacy Condition 2 (AC2)*: (c1) $c(Y, X)$ is fully determined by $p(Y \mid X)$ and $p(Y)$, is an increasing function of the former, and is a decreasing function of the latter, or (c2) $c(Y, X)$ is fully determined by $p(Y \mid X)$ and $p(Y \mid \neg X)$, is an increasing function of the former, and is a decreasing function of the latter, or (c3) $c(Y, X)$ is fully determined by $p(X \mid Y)$ and $p(X)$, is an increasing function of the former, and is a decreasing function of the latter, or (c4) $c(Y, X)$ is fully

[8] See Brossel (2013) and Fitelson (1999, 2001) for helpful discussion of measure sensitivity.

determined by $p(X \mid Y)$ and $p(X \mid \neg Y)$, is an increasing function of the former, and is a decreasing function of the latter.

This condition is meant to be restricted to cases where the various probabilities are non-extreme so that $0 < p(Y \mid X) < 1$, $0 < p(Y) < 1$, $0 < p(X \mid Y) < 1$, and so on. This means that a support measure c can meet AC2 even if, say, $c(Y, X)$ is constant at 1 when $p(Y \mid X) = 1$ regardless of $p(Y)$.

There is no questioning AC1 as an adequacy condition on support measures. But why accept AC2?

AC2 is suggested in part by the fact that it is met by each of the following support measures (three of which are repeated from above though with different subscripts):

$$c_{1.0}(Y,X) = p(Y \mid X) - p(Y)$$

$$c_{1.1}(Y,X) = \frac{p(Y \mid X)}{p(Y)}$$

$$c_{1.2}(Y,X) = \log\left[\frac{p(Y \mid X)}{p(Y)}\right]$$

$$c_{1.3}(Y,X) = \frac{p(Y \mid X) - p(Y)}{p(Y \mid X) + p(Y)}$$

$$c_{1.4}(Y,X) = \frac{p(\neg Y)}{p(\neg Y \mid X)} = \frac{1 - p(Y)}{1 - p(Y \mid X)}$$

$$c_{1.5}(Y,X) = \frac{\log[p(Y \mid X)] - \log[p(Y)]}{-\log[p(Y)]}$$

$$c_{1.6}(Y,X) = \frac{p(X \mid Y) - p(X)}{p(X \mid Y) + p(X) - p(Y \wedge X)} = \frac{p(Y \mid X) - p(Y)}{p(Y \mid X) + p(Y) - p(Y \mid X)p(Y)}$$

$$c_{1.7}(Y,X) = \begin{cases} \dfrac{p(Y \mid X) - p(Y)}{1 - p(Y)} & \text{if } p(Y \mid X) \geq p(Y) \\[2ex] \dfrac{p(Y \mid X) - p(Y)}{p(Y)} & \text{if } p(Y \mid X) < p(Y) \end{cases}$$

$$c_{1.8}(Y,X) = \frac{p(Y \mid X) - p(Y)}{p(Y \mid X) + p(Y) + p(Y \mid X)p(Y)}$$

$$c_{1.9}(Y,X) = \frac{p(Y \mid X) - p(Y)}{p(Y \mid X) + p(Y) + \pi p(Y \mid X) p(Y)} \text{ where } \pi > 0$$

$$c_{1.10}(Y,X) = \frac{p(Y \mid X) - p(Y)}{p(Y \mid X) p(Y)}$$

$$= \frac{1}{p(Y)} - \frac{1}{p(Y \mid X)}$$

$$c_{1.11}(Y,X) = \frac{p(Y \mid X) + \alpha p(Y) p(Y \mid X)}{p(Y) + \alpha p(Y) p(Y \mid X)} \text{ where } \alpha > 0$$

$$c_{2.0}(Y,X) = p(Y \mid X) - p(Y \mid \neg X)$$

$$c_{2.1}(Y,X) = \frac{p(Y \mid X)}{p(Y \mid \neg X)}$$

$$c_{2.2}(Y,X) = \log\left[\frac{p(Y \mid X)}{p(Y \mid \neg X)}\right]$$

$$c_{2.3}(Y,X) = \frac{p(Y \mid X) - p(Y \mid \neg X)}{p(Y \mid X) + p(Y \mid \neg X)}$$

$$c_{3.0}(Y,X) = p(X \mid Y) - p(X)$$

$$c_{3.1}(Y,X) = \frac{p(X \mid Y)}{p(X)}$$

$$c_{3.2}(Y,X) = \log\left[\frac{p(X \mid Y)}{p(X)}\right]$$

$$c_{4.0}(Y,X) = p(X \mid Y) - p(X \mid \neg Y)$$

$$c_{4.1}(Y,X) = \frac{p(X \mid Y)}{p(X \mid \neg Y)}$$

$$c_{4.2}(Y,X) = \log\left[\frac{p(X \mid Y)}{p(X \mid \neg Y)}\right]$$

$$c_{4.3}(Y,X) = \frac{p(X \mid Y) - p(X \mid \neg Y)}{p(X \mid Y) + p(X \mid \neg Y)}$$

Measures $c_{1.0}$-$c_{1.11}$ meet AC2 by meeting (c1).[9] Measures $c_{2.0}$-$c_{2.3}$ meet AC2 by meeting (c2). Measures $c_{3.0}$-$c_{3.2}$ meet AC2 by meeting (c3).[10] Measures $c_{4.0}$-$c_{4.3}$

[9] Measures $c_{1.9}$ and $c_{1.11}$ meet (c1) in AC2 assuming, as we may, that (c1) is understood as: $c(Y, X)$ is fully determined by $p(Y \mid X)$ and $p(Y)$ *(and perhaps a*

meet AC2 by meeting (c4). Each of the measures in the list is taken from the literature. And each of the most popular measures in the literature is included in the list.[11]

Some of the measures in the list are ordinally equivalent to each other (i.e., they impose the same ordering on any two ordered pairs of propositions): $c_{1.1}$, $c_{1.2}$, $c_{1.3}$, $c_{3.1}$, and $c_{3.2}$ and are pairwise ordinally equivalent to each other; $c_{2.1}$, $c_{2.2}$, and $c_{2.3}$ are pairwise ordinally equivalent to each other; $c_{4.1}$, $c_{4.2}$, and $c_{4.3}$ are pairwise ordinally equivalent to each other.[12] But taken as a group the measures are motley: no two of $c_{1.0}$, $c_{1.1}$, $c_{1.4}$, $c_{1.5}$, $c_{1.6}$, $c_{1.7}$, $c_{1.8}$, $c_{1.9}$, $c_{1.10}$, $c_{1.11}$, $c_{2.0}$, $c_{2.1}$, $c_{3.0}$, $c_{4.0}$, and $c_{4.1}$ are ordinally equivalent to each other. This, together with the fact that each such measure meets AC2, speaks to the fact that AC2 is a rather weak adequacy condition on support measures.

A further consideration in support of AC2 (as an adequacy condition on support measures)—a consideration not unrelated to the fact that each of $c_{1.0}$-$c_{4.3}$ meets AC2—is that each of (c1)-(c4) has some intuitive plausibility as an adequacy condition on support measures. Consider the proposals:

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constant), is an increasing function of the former, and is a decreasing function of the latter.

[10] Given that $c_{1.1}(Y,X)=\dfrac{p(Y\mid X)}{p(Y)}=\dfrac{p(X\mid Y)}{p(X)}=c_{3.1}(Y,X)$, assuming all the relevant probabilities are defined, and given that $c_{3.1}$ meets (c3), it follows that $c_{1.1}$ meets not just (c1) but also (c3). This shows that a given support measure can meet AC2 without meeting just one of AC2's disjuncts.

[11] For discussion of, and references regarding, the various measures in the list (and/or measures ordinally equivalent to those measures), see Atkinson, Peijnenburg, and Kuipers (2009), Brossel (2013), Crupi, Chater, and Tentori (2013), Crupi, Tentori, and Gonzalez (2007), Eells and Fitelson (2002), Festa (2012), Fitelson (1999, 2001), Hawthorne and Fitelson (2004), Joyce (2008), and Roche and Shogenji (2014b).

[12] Let c and c^* be support measures. Then c and c^* are ordinally equivalent to each other just in case, for any two ordered pairs of propositions $<Y1, X1>$ and $<Y2, X2>$, the following holds: $c(Y1, X1) > / = / < c(Y2, X2)$ iff $c^*(Y1, X1) > / = / < c^*(Y2, X2)$.

Degree of Increase in Probability (DIP): $c(Y, X)$ gives the degree to which the evidence X increases the probability of the hypothesis Y.

Degree of Predictive Success (DPS): $c(Y, X)$ gives the degree to which the hypothesis Y is predictively successful with respect to the evidence X, where this is a matter of the degree to which the hypothesis Y increases the probability of the evidence X.

Each of these proposals has some intuitive plausibility. And each can be understood in two main ways. DIP can be understood either (i) in terms of the degree to which $p(Y \mid X)$ is greater than $p(Y)$ or (ii) in terms of the degree to which $p(Y \mid X)$ is greater than $p(Y \mid \neg X)$. (c1) in AC2 follows from (i) whereas (c2) follows from (ii).[13] DPS, in turn, can be understood either (iii) in terms of the degree to which $p(X \mid Y)$ is greater than $p(X)$ or (iv) in terms of the degree to which $p(X \mid Y)$ is greater than $p(X \mid \neg Y)$. (c3) in AC2 follows from (iii) whereas (c4) follows from (iv).[14]

Each of (c1)-(c4) has some intuitive plausibility as an adequacy condition on support measures. But then, as AC2 is weaker than each of (c1)-(c4), it follows that AC2 is at least as plausible as an adequacy condition on support measures as are (c1)-(c4) taken individually.

It should be noted that not all extant measures meet AC2. Consider, for example, the following:

$$c_C(Y, X) = p(Y \wedge X) - p(Y)p(X)$$

[13] See Christensen (1999), Climenhaga (2013), Eells and Fitelson (2000), Hajek and Joyce (2008), and Joyce (1999, Ch. 6) for discussion of (in effect) DIP understood in terms of the degree to which $p(Y \mid X)$ is greater than $p(Y)$ versus DIP understood in terms of the degree to which $p(Y \mid X)$ is greater than $p(Y \mid \neg X)$.

[14] See Crupi, Chater, and Tentori (2013), Huber (2008), Joyce (2008), Kuipers (2000), Mackie (1969), Mortimer (1988), Nozick (1981), Roche (forthcoming b), Roush (2005), and Zalabardo (2009) for relevant discussion.

This measure, which is due to Carnap (1962), meets none of (c1)-(c4) and thus does not meet AC2.[15] It follows, by the assumption that any adequate support measure should meet AC2, that c_C is inadequate.

This result strikes me as quite acceptable. c_C is put forward as a "relevance" measure (or function), where a relevance measure is a measure of the degree to which the evidence increases the probability of the hypothesis.[16] No measure meeting neither (c1) nor (c2) is adequate as a measure of the degree to which the evidence increases the probability of the hypothesis.

3 Main results

3.1 The general case

Consider the following schema where $\beta \in \mathbb{R}^+$ and
$\tau = 1 + (1/10)^\beta + (9/100)^\beta + (9/10)^\beta + (9/10)^\beta + (9/100)^\beta + (1/5)^\beta + (2)^\beta$:

[15] This can be verified by appeal to Schema A below. Any measure c meeting AC2 is such that $c(Z, X)$ is greater when $\beta = 5$ than when $\beta = 1$. But $c_C(Z, X)$ is roughly equal to 0.042 when $\beta = 1$ and is roughly equal to 0.027 when $\beta = 5$.
[16] Carnap writes:

> It is the purpose of relevance functions in general to represent the change in the confirmation of *h* on *e* by the addition of a new evidence *i*. (1962, p. 361)

Here "*h*" is the hypothesis, "*e*" is the background evidence, "*i*" is the evidence, and "confirmation" is to be understood as "absolute confirmation" which is simply the probability of *h*.

Schema A

X	Y	Z	p	X	Y	Z	p
T	T	T	$1/\tau$	F	T	T	$(9/10)^{\beta}/\tau$
T	T	F	$(1/10)^{\beta}/\tau$	F	T	F	$(9/100)^{\beta}/\tau$
T	F	T	$(9/100)^{\beta}/\tau$	F	F	T	$(1/5)^{\beta}/\tau$
T	F	F	$(9/10)^{\beta}/\tau$	F	F	F	$(2)^{\beta}/\tau$

Each of (a)-(e) holds on all instances of Schema A (see Appendix A.1 for details):

(a) $p(Y \mid X) > p(Y)$
$p(Z \mid Y) > p(Z)$
$p(Z \mid X) > p(Z)$

(b) $p(Y \mid X) > p(Z \mid X)$ and $p(Y) < p(Z)$
$p(Z \mid Y) > p(Z \mid X)$ and $p(Z) = p(Z)$

(c) $p(Y \mid X) > p(Z \mid X)$ and $p(Y \mid \neg X) < p(Z \mid \neg X)$
$p(Z \mid Y) > p(Z \mid X)$ and $p(Z \mid \neg Y) < p(Z \mid \neg X)$

(d) $p(X \mid Y) > p(X \mid Z)$ and $p(X) = p(X)$
$p(Y \mid Z) > p(X \mid Z)$ and $p(Y) = p(X)$

(e) $p(X \mid Y) > p(X \mid Z)$ and $p(X \mid \neg Y) < p(X \mid \neg Z)$
$p(Y \mid Z) > p(X \mid Z)$ and $p(Y \mid \neg Z) < p(X \mid \neg Z)$

Let c be a support measure meeting AC1 and AC2. Given (a), it follows that X supports Y, Y supports Z, and X supports Z. Given (b)-(e), and given that c meets AC2, it follows that $c(Y, X) > c(Z, X)$ and $c(Z, Y) > c(Z, X)$. If, say, c meets AC2 by meeting (c1), then by (b) it follows that $c(Y, X) > c(Z, X)$ and $c(Z, Y) > c(Z, X)$.

If, then, regardless of the value specified for **t***, there is an instance of Schema A on which $c(Z, X) =$ **t***, it follows immediately that, regardless of the value specified for **t***, there is an instance of Schema A on which $c(Y, X) > c(Z, X) =$ **t*** and $c(Z, Y) > c(Z, X) = \mathbf{t^*}$. The task now is to show that, in fact, regardless of the value specified for $\mathbf{t^*}$, there is an instance of Schema A on which $c(Z, X) = \mathbf{t^*}$.

Take some value for β and some value for $\mathbf{t^*}$. There are three possibilities: (i) $c(Z, X) = \mathbf{t^*}$, (ii) $c(Z, X) < \mathbf{t^*}$, and (iii) $c(Z, X) > \mathbf{t^*}$. If (i) holds, then let β remain at its current value. If (ii) holds, then let β get closer and closer to ∞ until $c(Z, X) = \mathbf{t^*}$. That there is such a value for β is guaranteed by the fact that each of $p(Z \mid X)$ and $p(X \mid Z)$ is an increasing function of β, each of $p(Z)$, $p(Z \mid \neg X)$, $p(X)$, and $p(X \mid \neg Z)$ is a decreasing function of β, and the following (see Appendix A.2 for details):

(f)
$$\lim_{\beta \to \infty} p(Z \mid X) = 1 \text{ and } \lim_{\beta \to \infty} p(Z) = 0$$
$$\lim_{\beta \to \infty} p(Z \mid X) = 1 \text{ and } \lim_{\beta \to \infty} p(Z \mid \neg X) = 0$$
$$\lim_{\beta \to \infty} p(X \mid Z) = 1 \text{ and } \lim_{\beta \to \infty} p(X) = 0$$
$$\lim_{\beta \to \infty} p(X \mid Z) = 1 \text{ and } \lim_{\beta \to \infty} p(X \mid \neg Z) = 0$$

Note that, since c meets AC2, $c(Z, X)$ approaches the maximum value for c as $p(Z \mid X)$ and $p(X \mid Z)$ approach 1 while $p(Z)$, $p(Z \mid \neg X)$, $p(X)$, and $p(X \mid \neg Z)$ approach 0. If, instead, (iii) holds, then let β get closer and closer to 0 until $c(Z, X) = \mathbf{t^*}$. That there is such a value for β is guaranteed by the fact that each of $p(Z \mid X)$ and $p(X \mid Z)$ is an increasing function of β, each of $p(Z)$, $p(Z \mid \neg X)$, $p(X)$, and $p(X \mid \neg Z)$ is a decreasing function of β, and the following (see Appendix A.3 for details):

(g)
$$\lim_{\beta \to 0} p(Z \mid X) = 1/2 \text{ and } \lim_{\beta \to 0} p(Z) = 1/2$$
$$\lim_{\beta \to 0} p(Z \mid X) = 1/2 \text{ and } \lim_{\beta \to 0} p(Z \mid \neg X) = 1/2$$
$$\lim_{\beta \to 0} p(X \mid Z) = 1/2 \text{ and } \lim_{\beta \to 0} p(X) = 1/2$$
$$\lim_{\beta \to 0} p(X \mid Z) = 1/2 \text{ and } \lim_{\beta \to 0} p(X \mid \neg Z) = 1/2$$

Note that, since c meets AC2, $c(Z, X)$ approaches the neutral point for c as $p(Z \mid X)$ and $p(X \mid Z)$ approach 1/2 while $p(Z)$, $p(Z \mid \neg X)$, $p(X)$, and $p(X \mid \neg Z)$ approach 1/2. So, regardless of the value specified for $\mathbf{t^*}$, there is an instance of Schema A on which $c(Z, X) = \mathbf{t^*}$.

Thus:

Theorem 1 (T1): Suppose c meets AC1 and AC2. Then, regardless of the value specified for **t***, there are probability distributions on which (a) $p(Y \mid X) > p(Y)$ and $c(Y, X) >$ **t***, (b) $p(Z \mid Y) > p(Z)$ and $c(Z, Y) >$ **t***, and (c) $p(Z \mid X) > p(Z)$ but $c(Z, X) =$ **t***.

So support-**t*** is non-transitive in the general case.

It is now straightforward to show that:

Theorem 2 (T2): Suppose c meets AC1 and AC2. Then, regardless of the values specified for **t** and **t***, there are probability distributions on which (a) $p(Y \mid X) > p(Y)$, $p(Y \mid X) >$ **t**, and $c(Y, X) >$ **t***, (b) $p(Z \mid Y) > p(Z)$, $p(Z \mid Y) >$ **t**, and $c(Z, Y) >$ **t***, and (c) $p(Z \mid X) > p(Z)$ but $p(Z \mid X) =$ **t** or $c(Z, X) =$ **t***.

The key (at this point) is that, given that each of $p(Z \mid X)$ and $p(X \mid Z)$ is an increasing function of β, given that each of $p(Z)$, $p(Z \mid \neg X)$, $p(X)$, and $p(X \mid \neg Z)$ is a decreasing function of β, and given (f) and (g) from above, regardless of the values specified for **t** and **t***, there is an instance of Schema A on which (a) $p(Z \mid X) =$ **t** and $c(Z, X) \geq$ **t*** or (b) $p(Z \mid X) \geq$ **t** and $c(Z, X) =$ **t***. Any such probability distribution is such that $p(Y \mid X) > p(Y)$, $p(Z \mid Y) > p(Z)$, $p(Z \mid X) > p(Z)$, $p(Y \mid X) > p(Z \mid X)$, $p(Z \mid Y) > p(Z \mid X)$, $c(Y, X) > c(Z, X)$, and $c(Z, Y) > c(Z, X)$. So T2. So support-**tt***, as with support-**t***, is non-transitive in the general case.

3.2 The special case where SOC1 holds

It turns out that (h), below, holds on all instances of Schema A (see Appendix B for details):

(h) $p(Z \mid Y \wedge X) = p(Z \mid Y)$
$p(Z \mid \neg Y \wedge X) = p(Z \mid \neg Y)$

It follows immediately from T1 and T2 that:

Theorem 3 (T3): Suppose c meets AC1 and AC2. Then, regardless of the value specified for **t***, there are probability distributions on which (a) $p(Y \mid X) > p(Y)$ and $c(Y, X) >$ **t***, (b) $p(Z \mid Y) > p(Z)$ and $c(Z, Y) >$ **t***, (c) SOC1 holds, and (d) $p(Z \mid X) > p(Z)$ but $c(Z, X) =$ **t***.

Theorem 4 (T4): Suppose c meets AC1 and AC2. Then, regardless of the values specified for **t** and **t***, there are probability distributions on which (a) $p(Y \mid X) > p(Y)$, $p(Y \mid X) > \mathbf{t}$, and $c(Y, X) > \mathbf{t^*}$, (b) $p(Z \mid Y) > p(Z)$, $p(Z \mid Y) > \mathbf{t}$, and $c(Z, Y) > \mathbf{t^*}$, (c) SOC1 holds, and (d) $p(Z \mid X) > p(Z)$ but $p(Z \mid X) = \mathbf{t}$ or $c(Z, X) = \mathbf{t^*}$.

So support-**t*** and support-**tt*** are non-transitive in the special case where SOC1 holds.

T3 is similar in certain respects to the theorem of "dwindling confirmation" (Roche and Shogenji 2014b). This theorem (when restricted to a three-member series X, Y, Z) can be put as follows:

Dwindling Confirmation (DC): Suppose X supports Y which in turn supports Z. Suppose c meets the Weak Law of Likelihood (WLL) and so $c(Y, X) > c(Z, X)$ if $p(X \mid Y) > p(X \mid Z)$ while $p(X \mid \neg Y) < p(X \mid \neg Z)$. Suppose SOC1 holds. Then $c(Z, X) < c(Y, X)$.

There are some important differences between T3 and DC however. DC is restricted to measures meeting WLL. T3 is not so restricted. Some measures meeting AC1 and AC2 fail to meet WLL, for example, $c_{1.8}$, $c_{1.9}$, $c_{1.10}$, and $c_{1.11}$ (see Appendix C for details).[17]

3.3 The special case where SOC2 holds

The aim now is to show that:

Theorem 5 (T5): Suppose c meets AC1 and AC2. Then, regardless of the value specified for **t***, there are probability distributions on which (a) $p(Y \mid X) > p(Y)$ and $c(Y, X) > \mathbf{t^*}$, (b) $p(Z \mid Y) > p(Z)$ and $c(Z, Y) > \mathbf{t^*}$, (c) SOC2 holds, and (d) $p(Z \mid X) > p(Z)$ but $c(Z, X) = \mathbf{t^*}$.

[17] It might seem that $c_{1.8}$, $c_{1.9}$, $c_{1.10}$, and $c_{1.11}$ should be rejected for failing to meet WLL. Perhaps, though, there is a sense of support on which any adequate support measure should *fail* to meet WLL. See Festa (2012) and Roche (forthcoming a) for discussion of support and the "reverse Matthew effect".

Theorem 6 (T6): Suppose c meets AC1 and AC2. Then, regardless of the values specified for **t** and **t***, there are probability distributions on which (a) $p(Y \mid X) > p(Y)$, $p(Y \mid X) > \mathbf{t}$, and $c(Y, X) > \mathbf{t}^*$, (b) $p(Z \mid Y) > p(Z)$, $p(Z \mid Y) > \mathbf{t}$, and $c(Z, Y) > \mathbf{t}^*$, (c) SOC2 holds, and (d) $p(Z \mid X) > p(Z)$ but $p(Z \mid X) = \mathbf{t}$ or $c(Z, X) = \mathbf{t}^*$.

Since SOC1 holds on all instances of Schema A, and since SOC1 holds only if SOC2 does not, a different schema is needed to establish T5 and T6.

Consider, then, a slight variant of Schema A where $\beta \in \mathbb{R}^+$ and $\tau = 1 + (1/10)^\beta + (9/100)^\beta + (9/10)^\beta + (89/100)^\beta + (9/100)^\beta + (19/100)^\beta + (2)^\beta$:

Schema B

X	Y	Z	p	X	Y	Z	p
T	T	T	$1/\tau$	F	T	T	$(89/100)^\beta/\tau$
T	T	F	$(1/10)^\beta/\tau$	F	T	F	$(9/100)^\beta/\tau$
T	F	T	$(9/100)^\beta/\tau$	F	F	T	$(19/100)^\beta/\tau$
T	F	F	$(9/10)^\beta/\tau$	F	F	F	$(2)^\beta/\tau$

Each of (i)-(m) holds on all instances of Schema B (see Appendix D.1 for details):

(i) $p(Y \mid X) > p(Y)$
$p(Z \mid Y) > p(Z)$
$p(Z \mid Y \wedge X) > p(Z \mid Y)$
$p(Z \mid \neg Y \wedge X) > p(Z \mid \neg Y)$

(j) $p(Y \mid X) > p(Z \mid X)$ and $p(Y) < p(Z)$
$p(Z \mid Y) > p(Z \mid X)$ and $p(Z) = p(Z)$

(k) $p(Y \mid X) > p(Z \mid X)$ and $p(Y \mid \neg X) < p(Z \mid \neg X)$
$p(Z \mid Y) > p(Z \mid X)$ and $p(Z \mid \neg Y) < p(Z \mid \neg X)$

(l) $p(X \mid Y) > p(X \mid Z)$ and $p(X) = p(X)$
$p(Y \mid Z) > p(X \mid Z)$ and $p(Y) < p(X)$

(m) $p(X \mid Y) > p(X \mid Z)$ and $p(X \mid \neg Y) < p(X \mid \neg Z)$
$p(Y \mid Z) > p(X \mid Z)$ and $p(Y \mid \neg Z) < p(X \mid \neg Z)$

Let c be a support measure meeting AC1 and AC2. Given (i), it follows that X supports Y, Y supports Z, and SOC2 holds. Given this, and given that support is transitive under SOC2, it follows that X supports Z. Given (j)-(m), and given that c meets AC2, it follows that $p(Y \mid X) > p(Z \mid X)$, $p(Z \mid Y) > p(Z \mid X)$, $c(Y, X) > c(Z, X)$, and $c(Z, Y) > c(Z, X)$.

The argument then continues in parallel to the arguments above for T1 and T2. The crucial point vis-à-vis T5 is that, regardless of the value specified for **t***, there is an instance of Schema B on which $c(Z, X) =$ **t***. The crucial point vis-à-vis T6 is that, regardless of the values specified for **t** and **t***, there is an instance of Schema B on which (a) $p(Z \mid X) =$ **t** and $c(Z, X) \geq$ **t*** or (b) $p(Z \mid X) \geq$ **t** and $c(Z, X) =$ **t***. These points follow from the fact that each of $p(Z \mid X)$ and $p(X \mid Z)$ is an increasing function of β, each of $p(Z)$, $p(Z \mid \neg X)$, $p(X)$, and $p(X \mid \neg Z)$ is a decreasing function of β, and the following (see Appendix D.2 and Appendix D.3 for details):

(n) $\lim_{\beta\to\infty} p(Z \mid X) = 1$ and $\lim_{\beta\to\infty} p(Z) = 0$
$\lim_{\beta\to\infty} p(Z \mid X) = 1$ and $\lim_{\beta\to\infty} p(Z \mid \neg X) = 0$
$\lim_{\beta\to\infty} p(X \mid Z) = 1$ and $\lim_{\beta\to\infty} p(X) = 0$
$\lim_{\beta\to\infty} p(X \mid Z) = 1$ and $\lim_{\beta\to\infty} p(X \mid \neg Z) = 0$

(o) $\lim_{\beta\to 0} p(Z \mid X) = 1/2$ and $\lim_{\beta\to 0} p(Z) = 1/2$
$\lim_{\beta\to 0} p(Z \mid X) = 1/2$ and $\lim_{\beta\to 0} p(Z \mid \neg X) = 1/2$
$\lim_{\beta\to 0} p(X \mid Z) = 1/2$ and $\lim_{\beta\to 0} p(X) = 1/2$
$\lim_{\beta\to 0} p(X \mid Z) = 1/2$ and $\lim_{\beta\to 0} p(X \mid \neg Z) = 1/2$

So, first, regardless of the value specified for **t***, there is an instance of Schema B on which X supports Y, Y supports Z, X supports Z, $c(Y, X) >$ **t***, $c(Z, Y) >$ **t***, SOC2 holds, and $c(Z, X) =$ **t***, and, second, regardless of the values specified for **t** and **t***, there is an instance of Schema B on which X supports Y, Y supports Z, X supports Z, $p(Y \mid X) >$ **t**, $p(Z \mid Y) >$ **t**, $c(Y, X) >$ **t***, $c(Z, Y) >$ **t***, and $p(Z \mid X) =$ **t** or $c(Z, X) =$ **t***.

This establishes T5 and T6. Therefore support-**t*** and support-**tt*** are non-transitive in the special case where SOC2 holds.

4 Discussion

Douven's evidential support theory of conditionals (2008, forthcoming) can be put as follows:[18,19]

> *Evidential Support Theory of Conditionals 1 (ESTC1)*: "If X, Y" is assertable/acceptable if and only if X supports-$\mathbf{t}$ Y.

X supports-$\mathbf{t}$ Y if and only if X supports Y and $p(Y \mid X) > \mathbf{t}$. So, by ESTC1, "If X, Y" is assertable/acceptable if and only if X supports Y and $p(Y \mid X) > \mathbf{t}$.

ESTC1 has some intuitive plausibility. This can be seen by considering two rival theories:[20]

> *Evidential Support Theory of Conditionals 2 (ESTC2)*: "If X, Y" is assertable/acceptable if and only if X supports Y.

> *Evidential Support Theory of Conditionals 3 (ESTC3)*: "If X, Y" is assertable/acceptable if and only if $p(Y \mid X) > \mathbf{t}$.

ESTC2 is problematic because of cases where X supports Y but $p(Y \mid X)$ is negligible. Douven writes:

> My colleague Henry's quitting his job is evidence that [i.e., supports the proposition that] I shall teach next year's introductory course in social

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[18] Douven has in mind *simple* (indicative) conditionals, i.e., conditionals such that neither the antecedent nor the consequent contains a conditional.
[19] The version of ESTC1 proposed in Douven (2008) involves a defeater condition. But that condition can be ignored for my purposes. See Douven (forthcoming, Ch. 4, sec. 4.7) for relevant discussion.
[20] ESTC3 is in effect the theory called "Qualitative Adams' Thesis" in Douven (forthcoming, p. 87) and Douven and Verbrugge (2012, p. 483).

> philosophy, because conditional on the former the latter is a bit more probable than it is unconditionally. But even the conditional probability is still exceedingly low, given that I simply lack the requisite background for teaching such a course. … Thus, "If Henry quits his job, I shall teach next year's introductory course in social philosophy" is not assertable for me/acceptable to me, notwithstanding that its antecedent is evidence for [i.e., supports] its consequent. (2008, pp. 27-28)

ESTC3 is problematic for a different reason. Suppose (adapting a case from Douven 2008) a fair coin is tossed 1,000,000 times. Let X be the proposition that the coin comes up heads on the first toss, Y be the proposition that Chelsea wins the Champions League, and Z be the proposition that the coin comes up heads on at least 1 of the 1,000,000 tosses. Then, at least on certain ways of filling in the details, $1 = p(Z \mid X) > p(Z \mid Y) = p(Z) > \mathbf{t}$. By ESTC3 it follows that each of "If X, Z" and "If Y, Z" is assertable/acceptable. But, intuitively, just the first of the two conditionals is assertable/acceptable. The lesson, it seems, is that—just as ESTC1 implies—a conditional is assertable/acceptable only if the antecedent supports the consequent and the probability of the consequent given the antecedent is sufficiently high.[21]

Now consider the following theses:

> Trans 1: Whenever "If X, Y" is assertable/acceptable and "If Y, Z" is assertable/acceptable, then "If X, Z" is assertable/acceptable.
>
> Trans 2: Whenever "If X, Y" is assertable/acceptable, "If Y, Z" is assertable/acceptable, and SOC1 holds, then "If X, Z" is assertable/acceptable.
>
> Trans 3: Whenever "If X, Y" is assertable/acceptable, "If Y, Z" is assertable/acceptable, and SOC2 holds, then "If X, Z" is assertable/acceptable.

Each of these theses is false on ESTC1. This follows from the fact that support-**t** is non-transitive in the general case, in the special case where SOC1 holds, and in the special case where SOC2 holds.

[21] See Douven and Verbrugge (2012) for an empirical argument in favor of ESTC1 and against ESTC3.

It is helpful to know that Trans 1, Trans 2, and Trans 3 are false on ESTC1. This is a step in the direction of having an adequate understanding of ESTC1 and its implications (for the purpose of assessing its overall plausibility).[22]

ESTC1 is superior to ESTC2 and ESTC3. But there are at least two alternative evidential support theories worth considering:[23]

Evidential Support Theory of Conditionals 4 (ESTC4): "If X, Y" is assertable/acceptable if and only if X supports-**t*** Y.

Evidential Support Theory of Conditionals 5 (ESTC5): "If X, Y" is assertable/acceptable if and only if X supports-**tt*** Y.

It might seem that ESTC5 is clearly preferable to ESTC4. The former but not the latter explicitly requires that $p(Y \mid X)$ be sufficiently high. Suppose, though, support-**t** is understood so that **t** = 0.9, support is measured by $c_{1.0}$ (the difference measure), and support-**t*** is understood so that **t*** = 0.9. Then, given that $p(Y \mid X) - p(Y) > 0.9 =$ **t*** only if $p(Y \mid X) > 0.9$, it follows that X supports-**t*** Y only if $p(Y \mid X) > 0.9 =$ **t**. This means that there are ways of understanding ESTC4 and ESTC5 on which they are logically equivalent to each other and thus on which ESTC4 implicitly requires that $p(Y \mid X)$ be sufficiently high.

There is a potential problem, though, with ESTC4 and ESTC5 understood in terms of $c_{1.0}$. Recall the coin case from above. The intuition is supposed to be that "If X, Z" is assertable/acceptable. But since $p(Z)$ is very high, $c_{1.0}(Z, X)$ is very low. If **t*** is set at 0.9 (or at any other non-negligible value), then X does not support-**t*** Z and thus by ESTC4 and ESTC5 it follows that "If X, Z" is not assertable/acceptable.

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[22] See Douven (forthcoming, Ch. 5) for discussion of more such implications. It might seem that ESTC1 should be rejected for the reason that Trans 1, Trans 2, and Trans 3 are false on ESTC1. But see Douven (2008, sec. 4).

[23] Douven (2008, p. 32, n. 35) notes the possibility of measuring the degree to which "If X, Y" is assertable/acceptable in terms of both $p(Y \mid X)$ and the degree to which X supports Y. This idea is similar in some respects to ESTC5. But bear in mind that ESTC5 is a *qualitative* theory as opposed to a *quantitative* theory. If, say, X supports-**tt*** Y, then it follows by ESTC5 that "If X, Y" is assertable/acceptable but does not follow that it is assertable/acceptable to this or that particular degree.

I want to set aside ESTC4 and focus on ESTC5. Is there a way of understanding ESTC5 such that in the coin case "If *X*, *Z*" is assertable/acceptable?

The answer is affirmative. The key point is that on many support measures it follows that any case where the evidence entails the hypothesis is a case where the degree of support is maximal. One such measure is $c_{1.7}$ (the Z measure). If the evidence entails the hypothesis, then the degree of support is maximal at 1. If, then, ESTC5 is understood in terms of a support measure such as $c_{1.7}$ on which any case where the evidence entails the hypothesis is a case where the degree of support is maximal, it follows that ESTC5 implies that in the coin case "If *X*, *Z*" is assertable/acceptable.

A careful investigation of ESTC5 and its implications is beyond the scope of this paper. But one thing is clear at this point: each of Trans 1, Trans 2, and Trans 3 is false on ESTC5. This follows from T2, T4, and T6.[24] If there is adequate reason to prefer ESTC1 over ESTC5, then this is not because of any differences in what ESTC1 and ESTC5 imply with respect to Trans 1, Trans 2, and Trans 3.

5 Conclusion

Each of support-**t**, support-**t***, and support-**tt*** is non-transitive in the general case, in the special case where SOC1 holds, and in the special case where SOC2 holds. One question for future investigation is whether there are any non-trivial conditions under which support-**t**, support-**t***, and support-**tt*** are transitive. If the answer is affirmative, then, depending on the fates of ESTC1, ESTC4, and ESTC5, it might be that those conditions are also conditions under which Trans 1 is true.

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[24] Similarly, given T1, T3, and T5, it follows that Trans 1, Trans 2, and Trans 3 are false on ESTC4.

Appendix A

A.1

Each of (1)-(13) holds on all instances of Schema A:

(1) $$p(Y \mid X) = \frac{1+\left(\frac{1}{10}\right)^{\beta}}{1+\left(\frac{1}{10}\right)^{\beta}+\left(\frac{9}{100}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}} > \frac{1+\left(\frac{1}{10}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}+\left(\frac{9}{100}\right)^{\beta}}{\tau} = p(Y)$$

(2) $$p(Z \mid Y) = \frac{1+\left(\frac{9}{10}\right)^{\beta}}{1+\left(\frac{1}{10}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}+\left(\frac{9}{100}\right)^{\beta}} > \frac{1+\left(\frac{9}{100}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}+\left(\frac{1}{5}\right)^{\beta}}{\tau} = p(Z)$$

(3) $$p(Z \mid X) = \frac{1+\left(\frac{9}{100}\right)^{\beta}}{1+\left(\frac{1}{10}\right)^{\beta}+\left(\frac{9}{100}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}} > \frac{1+\left(\frac{9}{100}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}+\left(\frac{1}{5}\right)^{\beta}}{\tau} = p(Z)$$

(4) $$p(Y \mid X) = \frac{1+\left(\frac{1}{10}\right)^{\beta}}{1+\left(\frac{1}{10}\right)^{\beta}+\left(\frac{9}{100}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}} > \frac{1+\left(\frac{9}{100}\right)^{\beta}}{1+\left(\frac{1}{10}\right)^{\beta}+\left(\frac{9}{100}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}} = p(Z \mid X)$$

(5) $$p(Z \mid Y) = \frac{1+\left(\frac{9}{10}\right)^{\beta}}{1+\left(\frac{1}{10}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}+\left(\frac{9}{100}\right)^{\beta}} > \frac{1+\left(\frac{9}{100}\right)^{\beta}}{1+\left(\frac{1}{10}\right)^{\beta}+\left(\frac{9}{100}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}} = p(Z \mid X)$$

(6) $$p(Y) = \frac{1+\left(\frac{1}{10}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}+\left(\frac{9}{100}\right)^{\beta}}{\tau} < \frac{1+\left(\frac{9}{100}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}+\left(\frac{1}{5}\right)^{\beta}}{\tau} = p(Z)$$

(7) $$p(Y \mid \neg X) = \frac{\left(\frac{9}{10}\right)^{\beta}+\left(\frac{9}{100}\right)^{\beta}}{\left(\frac{9}{10}\right)^{\beta}+\left(\frac{9}{100}\right)^{\beta}+\left(\frac{1}{5}\right)^{\beta}+\left(2\right)^{\beta}} < \frac{\left(\frac{9}{10}\right)^{\beta}+\left(\frac{1}{5}\right)^{\beta}}{\left(\frac{9}{10}\right)^{\beta}+\left(\frac{9}{100}\right)^{\beta}+\left(\frac{1}{5}\right)^{\beta}+\left(2\right)^{\beta}} = p(Z \mid \neg X)$$

(8) $$p(Z \mid \neg Y) = \frac{\left(\frac{9}{100}\right)^{\beta}+\left(\frac{1}{5}\right)^{\beta}}{\left(\frac{9}{100}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}+\left(\frac{1}{5}\right)^{\beta}+\left(2\right)^{\beta}} < \frac{\left(\frac{9}{10}\right)^{\beta}+\left(\frac{1}{5}\right)^{\beta}}{\left(\frac{9}{10}\right)^{\beta}+\left(\frac{9}{100}\right)^{\beta}+\left(\frac{1}{5}\right)^{\beta}+\left(2\right)^{\beta}} = p(Z \mid \neg X)$$

(9) $$p(X \mid Y) = \frac{1+\left(\frac{1}{10}\right)^{\beta}}{1+\left(\frac{1}{10}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}+\left(\frac{9}{100}\right)^{\beta}} > \frac{1+\left(\frac{9}{100}\right)^{\beta}}{1+\left(\frac{9}{100}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}+\left(\frac{1}{5}\right)^{\beta}} = p(X \mid Z)$$

$$(10)\quad p(Y \mid Z) = \frac{1+\left(\frac{9}{10}\right)^{\beta}}{1+\left(\frac{9}{100}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}+\left(\frac{1}{5}\right)^{\beta}} > \frac{1+\left(\frac{9}{100}\right)^{\beta}}{1+\left(\frac{9}{100}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}+\left(\frac{1}{5}\right)^{\beta}} = p(X \mid Z)$$

$$(11)\quad p(Y) = \frac{1+\left(\frac{1}{10}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}+\left(\frac{9}{100}\right)^{\beta}}{\tau} = \frac{1+\left(\frac{1}{10}\right)^{\beta}+\left(\frac{9}{100}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}}{\tau} = p(X)$$

$$(12)\quad p(X \mid \neg Y) = \frac{\left(\frac{9}{100}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}}{\left(\frac{9}{100}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}+\left(\frac{1}{5}\right)^{\beta}+(2)^{\beta}} < \frac{\left(\frac{1}{10}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}}{\left(\frac{1}{10}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}+\left(\frac{9}{100}\right)^{\beta}+(2)^{\beta}} = p(X \mid \neg Z)$$

$$(13)\quad p(Y \mid \neg Z) = \frac{\left(\frac{1}{10}\right)^{\beta}+\left(\frac{9}{100}\right)^{\beta}}{\left(\frac{1}{10}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}+\left(\frac{9}{100}\right)^{\beta}+(2)^{\beta}} < \frac{\left(\frac{1}{10}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}}{\left(\frac{1}{10}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}+\left(\frac{9}{100}\right)^{\beta}+(2)^{\beta}} = p(X \mid \neg Z)$$

Given that each of (1)-(3) holds on all instances of Schema A, it follows that (a) holds on all instances of Schema A. Given that each of (4)-(6) holds on all instances of Schema A, it follows that (b) holds on all instances of Schema A. Given that each of (4), (5), (7), and (8) holds on all instances of Schema A, it follows that (c) holds on all instances of Schema A. Given that each of (9)-(11) holds on all instances of Schema A, it follows that (d) holds on all instances of Schema A. Given that each of (9), (10), (12), and (13) holds on all instances of Schema A, it follows that (e) holds on all instances of Schema A. QED

A.2

Observe that:

$$(14)\quad \lim_{\beta\to\infty} \frac{1+\left(\frac{9}{100}\right)^{\beta}}{1+\left(\frac{1}{10}\right)^{\beta}+\left(\frac{9}{100}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}} = 1 \text{ and } \lim_{\beta\to\infty} \frac{1+\left(\frac{9}{100}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}+\left(\frac{1}{5}\right)^{\beta}}{\tau} = 0$$

$$(15)\quad \lim_{\beta\to\infty} \frac{1+\left(\frac{9}{100}\right)^{\beta}}{1+\left(\frac{1}{10}\right)^{\beta}+\left(\frac{9}{100}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}} = 1 \text{ and } \lim_{\beta\to\infty} \frac{\left(\frac{9}{10}\right)^{\beta}+\left(\frac{1}{5}\right)^{\beta}}{\left(\frac{9}{10}\right)^{\beta}+\left(\frac{9}{100}\right)^{\beta}+\left(\frac{1}{5}\right)^{\beta}+(2)^{\beta}} = 0$$

$$(16)\quad \lim_{\beta\to\infty} \frac{1+\left(\frac{9}{100}\right)^{\beta}}{1+\left(\frac{9}{100}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}+\left(\frac{1}{5}\right)^{\beta}} = 1 \text{ and } \lim_{\beta\to\infty} \frac{1+\left(\frac{1}{10}\right)^{\beta}+\left(\frac{9}{100}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}}{\tau} = 0$$

$$(17)\quad \lim_{\beta\to\infty} \frac{1+\left(\frac{9}{100}\right)^{\beta}}{1+\left(\frac{9}{100}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}+\left(\frac{1}{5}\right)^{\beta}} = 1 \text{ and } \lim_{\beta\to\infty} \frac{\left(\frac{1}{10}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}}{\left(\frac{1}{10}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}+\left(\frac{9}{100}\right)^{\beta}+\left(2\right)^{\beta}} = 0$$

It follows that (f). QED

A.3

Observe that:

$$(18)\quad \lim_{\beta\to 0} \frac{1+\left(\frac{9}{100}\right)^{\beta}}{1+\left(\frac{1}{10}\right)^{\beta}+\left(\frac{9}{100}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}} = 1/2 \text{ and } \lim_{\beta\to 0} \frac{1+\left(\frac{9}{100}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}+\left(\frac{1}{5}\right)^{\beta}}{\tau} = 1/2$$

$$(19)\quad \lim_{\beta\to 0} \frac{1+\left(\frac{9}{100}\right)^{\beta}}{1+\left(\frac{1}{10}\right)^{\beta}+\left(\frac{9}{100}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}} = 1/2 \text{ and } \lim_{\beta\to 0} \frac{\left(\frac{9}{10}\right)^{\beta}+\left(\frac{1}{5}\right)^{\beta}}{\left(\frac{9}{10}\right)^{\beta}+\left(\frac{9}{100}\right)^{\beta}+\left(\frac{1}{5}\right)^{\beta}+\left(2\right)^{\beta}} = 1/2$$

$$(20)\quad \lim_{\beta\to 0} \frac{1+\left(\frac{9}{100}\right)^{\beta}}{1+\left(\frac{9}{100}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}+\left(\frac{1}{5}\right)^{\beta}} = 1/2 \text{ and } \lim_{\beta\to 0} \frac{1+\left(\frac{1}{10}\right)^{\beta}+\left(\frac{9}{100}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}}{\tau} = 1/2$$

$$(21)\quad \lim_{\beta\to 0} \frac{1+\left(\frac{9}{100}\right)^{\beta}}{1+\left(\frac{9}{100}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}+\left(\frac{1}{5}\right)^{\beta}} = 1/2 \text{ and } \lim_{\beta\to 0} \frac{\left(\frac{1}{10}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}}{\left(\frac{1}{10}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}+\left(\frac{9}{100}\right)^{\beta}+\left(2\right)^{\beta}} = 1/2$$

It follows that (g). QED

Appendix B

Each of (22) and (23) holds on all instances of Schema A:

$$(22)\quad p(Z \mid Y \wedge X) = \frac{1}{1+\left(\frac{1}{10}\right)^{\beta}} = \frac{1+\left(\frac{9}{10}\right)^{\beta}}{1+\left(\frac{1}{10}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}+\left(\frac{9}{100}\right)^{\beta}} = p(Z \mid Y)$$

(23) $$p(Z \mid \neg Y \wedge X) = \frac{\left(\frac{9}{100}\right)^{\beta}}{\left(\frac{9}{100}\right)^{\beta} + \left(\frac{9}{10}\right)^{\beta}} = \frac{\left(\frac{9}{100}\right)^{\beta} + \left(\frac{1}{5}\right)^{\beta}}{\left(\frac{9}{100}\right)^{\beta} + \left(\frac{9}{10}\right)^{\beta} + \left(\frac{1}{5}\right)^{\beta} + (2)^{\beta}} = p(Z \mid \neg Y)$$

It follows that (h) holds on all instances of Schema A. QED

Appendix C

Consider the following probability distribution:

X	Y	Z	p	X	Y	Z	p
T	T	T	$\frac{1}{999}$	F	T	T	$\frac{1}{600}$
T	T	F	$\frac{5}{27}$	F	T	F	$\frac{2}{17}$
T	F	T	$\frac{1}{18}$	F	F	T	$\frac{1}{22}$
T	F	F	$\frac{11}{1332}$	F	F	F	$\frac{65663}{112200}$

On this distribution it follows that:[25]

$0.609 \approx p(X \mid Y) > p(X \mid Z) \approx 0.546$
$0.092 \approx p(X \mid \neg Y) < p(X \mid \neg Z) \approx 0.216$
$0.344 \approx c_{1.8}(Y, X) < c_{1.8}(Z, X) \approx 0.347$
$0.292 \approx c_{1.9}(Y, X) < c_{1.9}(Z, X) \approx 0.325$
$1.931 \approx c_{1.10}(Y, X) < c_{1.10}(Z, X) \approx 5.225$
$1.578 \approx c_{1.11}(Y, X) < c_{1.11}(Z, X) \approx 1.814$

So each of $c_{1.8}$, $c_{1.9}$, $c_{1.10}$, and $c_{1.11}$ fails to meet WLL. QED

[25] It is being assumed that π in $c_{1.9}$ equals 2 and α in $c_{1.11}$ equals 2.

Appendix D

D.1

Each of (24)-(37) holds on all instances of Schema B:

(24) $$p(Y \mid X) = \frac{1+\left(\frac{1}{10}\right)^{\beta}}{1+\left(\frac{1}{10}\right)^{\beta}+\left(\frac{9}{100}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}} > \frac{1+\left(\frac{1}{10}\right)^{\beta}+\left(\frac{89}{100}\right)^{\beta}+\left(\frac{9}{100}\right)^{\beta}}{\tau} = p(Y)$$

(25) $$p(Z \mid Y) = \frac{1+\left(\frac{89}{100}\right)^{\beta}}{1+\left(\frac{1}{10}\right)^{\beta}+\left(\frac{89}{100}\right)^{\beta}+\left(\frac{9}{100}\right)^{\beta}} > \frac{1+\left(\frac{9}{100}\right)^{\beta}+\left(\frac{89}{100}\right)^{\beta}+\left(\frac{19}{100}\right)^{\beta}}{\tau} = p(Z)$$

(26) $$p(Z \mid Y \wedge X) = \frac{1}{1+\left(\frac{1}{10}\right)^{\beta}} > \frac{1+\left(\frac{89}{100}\right)^{\beta}}{1+\left(\frac{1}{10}\right)^{\beta}+\left(\frac{89}{100}\right)^{\beta}+\left(\frac{9}{100}\right)^{\beta}} = p(Z \mid Y)$$

(27) $$p(Z \mid \neg Y \wedge X) = \frac{\left(\frac{9}{100}\right)^{\beta}}{\left(\frac{9}{100}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}} > \frac{\left(\frac{9}{100}\right)^{\beta}+\left(\frac{19}{100}\right)^{\beta}}{\left(\frac{9}{100}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}+\left(\frac{19}{100}\right)^{\beta}+(2)^{\beta}} = p(Z \mid \neg Y)$$

(28) $$p(Y \mid X) = \frac{1+\left(\frac{1}{10}\right)^{\beta}}{1+\left(\frac{1}{10}\right)^{\beta}+\left(\frac{9}{100}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}} > \frac{1+\left(\frac{9}{100}\right)^{\beta}}{1+\left(\frac{1}{10}\right)^{\beta}+\left(\frac{9}{100}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}} = p(Z \mid X)$$

(29) $$p(Z \mid Y) = \frac{1+\left(\frac{89}{100}\right)^{\beta}}{1+\left(\frac{1}{10}\right)^{\beta}+\left(\frac{89}{100}\right)^{\beta}+\left(\frac{9}{100}\right)^{\beta}} > \frac{1+\left(\frac{9}{100}\right)^{\beta}}{1+\left(\frac{1}{10}\right)^{\beta}+\left(\frac{9}{100}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}} = p(Z \mid X)$$

(30) $$p(Y) = \frac{1+\left(\frac{1}{10}\right)^{\beta}+\left(\frac{89}{100}\right)^{\beta}+\left(\frac{9}{100}\right)^{\beta}}{\tau} < \frac{1+\left(\frac{9}{100}\right)^{\beta}+\left(\frac{89}{100}\right)^{\beta}+\left(\frac{19}{100}\right)^{\beta}}{\tau} = p(Z)$$

(31) $$p(Y \mid \neg X) = \frac{\left(\frac{89}{100}\right)^{\beta}+\left(\frac{9}{100}\right)^{\beta}}{\left(\frac{89}{100}\right)^{\beta}+\left(\frac{9}{100}\right)^{\beta}+\left(\frac{19}{100}\right)^{\beta}+(2)^{\beta}} < \frac{\left(\frac{89}{100}\right)^{\beta}+\left(\frac{19}{100}\right)^{\beta}}{\left(\frac{89}{100}\right)^{\beta}+\left(\frac{9}{100}\right)^{\beta}+\left(\frac{19}{100}\right)^{\beta}+(2)^{\beta}} = p(Z \mid \neg X)$$

(32) $$p(Z \mid \neg Y) = \frac{\left(\frac{9}{100}\right)^{\beta}+\left(\frac{19}{100}\right)^{\beta}}{\left(\frac{9}{100}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}+\left(\frac{19}{100}\right)^{\beta}+(2)^{\beta}} < \frac{\left(\frac{89}{100}\right)^{\beta}+\left(\frac{19}{100}\right)^{\beta}}{\left(\frac{89}{100}\right)^{\beta}+\left(\frac{9}{100}\right)^{\beta}+\left(\frac{19}{100}\right)^{\beta}+(2)^{\beta}} = p(Z \mid \neg X)$$

(33) $$p(X \mid Y) = \frac{1+\left(\frac{1}{10}\right)^{\beta}}{1+\left(\frac{1}{10}\right)^{\beta}+\left(\frac{89}{100}\right)^{\beta}+\left(\frac{9}{100}\right)^{\beta}} > \frac{1+\left(\frac{9}{100}\right)^{\beta}}{1+\left(\frac{9}{100}\right)^{\beta}+\left(\frac{89}{100}\right)^{\beta}+\left(\frac{19}{100}\right)^{\beta}} = p(X \mid Z)$$

(34) $$p(Y \mid Z) = \frac{1+\left(\frac{89}{100}\right)^{\beta}}{1+\left(\frac{9}{100}\right)^{\beta}+\left(\frac{89}{100}\right)^{\beta}+\left(\frac{19}{100}\right)^{\beta}} > \frac{1+\left(\frac{9}{100}\right)^{\beta}}{1+\left(\frac{9}{100}\right)^{\beta}+\left(\frac{89}{100}\right)^{\beta}+\left(\frac{19}{100}\right)^{\beta}} = p(X \mid Z)$$

(35) $$p(Y) = \frac{1+\left(\frac{1}{10}\right)^{\beta}+\left(\frac{89}{100}\right)^{\beta}+\left(\frac{9}{100}\right)^{\beta}}{\tau} < \frac{1+\left(\frac{1}{10}\right)^{\beta}+\left(\frac{9}{100}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}}{\tau} = p(X)$$

(36) $$p(X \mid \neg Y) = \frac{\left(\frac{9}{100}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}}{\left(\frac{9}{100}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}+\left(\frac{19}{100}\right)^{\beta}+(2)^{\beta}} < \frac{\left(\frac{1}{10}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}}{\left(\frac{1}{10}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}+\left(\frac{9}{100}\right)^{\beta}+(2)^{\beta}} = p(X \mid \neg Z)$$

(37) $$p(Y \mid \neg Z) = \frac{\left(\frac{1}{10}\right)^{\beta}+\left(\frac{9}{100}\right)^{\beta}}{\left(\frac{1}{10}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}+\left(\frac{9}{100}\right)^{\beta}+(2)^{\beta}} < \frac{\left(\frac{1}{10}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}}{\left(\frac{1}{10}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}+\left(\frac{9}{100}\right)^{\beta}+(2)^{\beta}} = p(X \mid \neg Z)$$

Given that each of (24)-(27) holds on all instances of Schema B, it follows that (i) holds on all instances of Schema B. Given that each of (28)-(30) holds on all instances of Schema B, it follows that (j) holds on all instances of Schema B. Given that each of (28), (29), (31), and (32) holds on all instances of Schema B, it follows that (k) holds on all instances of Schema B. Given that each of (33)-(35) holds on all instances of Schema B, it follows that (l) holds on all instances of Schema B. Given that each of (33), (34), (36), and (37) holds on all instances of Schema B, it follows that (m) holds on all instances of Schema B. QED

D.2

Observe that:

(38) $$\lim_{\beta\to\infty} \frac{1+\left(\frac{9}{100}\right)^{\beta}}{1+\left(\frac{1}{10}\right)^{\beta}+\left(\frac{9}{100}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}} = 1 \text{ and } \lim_{\beta\to\infty} \frac{1+\left(\frac{9}{100}\right)^{\beta}+\left(\frac{89}{100}\right)^{\beta}+\left(\frac{19}{100}\right)^{\beta}}{\tau} = 0$$

(39) $$\lim_{\beta\to\infty} \frac{1+\left(\frac{9}{100}\right)^{\beta}}{1+\left(\frac{1}{10}\right)^{\beta}+\left(\frac{9}{100}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}} = 1 \text{ and } \lim_{\beta\to\infty} \frac{\left(\frac{89}{100}\right)^{\beta}+\left(\frac{19}{100}\right)^{\beta}}{\left(\frac{89}{100}\right)^{\beta}+\left(\frac{9}{100}\right)^{\beta}+\left(\frac{19}{100}\right)^{\beta}+(2)^{\beta}} = 0$$

(40) $$\lim_{\beta\to\infty}\frac{1+\left(\frac{9}{100}\right)^{\beta}}{1+\left(\frac{9}{100}\right)^{\beta}+\left(\frac{89}{100}\right)^{\beta}+\left(\frac{19}{100}\right)^{\beta}}=1 \text{ and } \lim_{\beta\to\infty}\frac{1+\left(\frac{1}{10}\right)^{\beta}+\left(\frac{9}{100}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}}{\tau}=0$$

(41) $$\lim_{\beta\to\infty}\frac{1+\left(\frac{9}{100}\right)^{\beta}}{1+\left(\frac{9}{100}\right)^{\beta}+\left(\frac{89}{100}\right)^{\beta}+\left(\frac{19}{100}\right)^{\beta}}=1 \text{ and } \lim_{\beta\to\infty}\frac{\left(\frac{1}{10}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}}{\left(\frac{1}{10}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}+\left(\frac{9}{100}\right)^{\beta}+(2)^{\beta}}=0$$

It follows that (n). QED

D.3

Observe that:

(42) $$\lim_{\beta\to 0}\frac{1+\left(\frac{9}{100}\right)^{\beta}}{1+\left(\frac{1}{10}\right)^{\beta}+\left(\frac{9}{100}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}}=1/2 \text{ and } \lim_{\beta\to 0}\frac{1+\left(\frac{9}{100}\right)^{\beta}+\left(\frac{89}{100}\right)^{\beta}+\left(\frac{19}{100}\right)^{\beta}}{\tau}=1/2$$

(43) $$\lim_{\beta\to 0}\frac{1+\left(\frac{9}{100}\right)^{\beta}}{1+\left(\frac{1}{10}\right)^{\beta}+\left(\frac{9}{100}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}}=1/2 \text{ and } \lim_{\beta\to 0}\frac{\left(\frac{89}{100}\right)^{\beta}+\left(\frac{19}{100}\right)^{\beta}}{\left(\frac{89}{100}\right)^{\beta}+\left(\frac{9}{100}\right)^{\beta}+\left(\frac{19}{100}\right)^{\beta}+(2)^{\beta}}=1/2$$

(44) $$\lim_{\beta\to 0}\frac{1+\left(\frac{9}{100}\right)^{\beta}}{1+\left(\frac{9}{100}\right)^{\beta}+\left(\frac{89}{100}\right)^{\beta}+\left(\frac{19}{100}\right)^{\beta}}=1/2 \text{ and } \lim_{\beta\to 0}\frac{1+\left(\frac{1}{10}\right)^{\beta}+\left(\frac{9}{100}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}}{\tau}=1/2$$

(45) $$\lim_{\beta\to 0}\frac{1+\left(\frac{9}{100}\right)^{\beta}}{1+\left(\frac{9}{100}\right)^{\beta}+\left(\frac{89}{100}\right)^{\beta}+\left(\frac{19}{100}\right)^{\beta}}=1/2 \text{ and } \lim_{\beta\to 0}\frac{\left(\frac{1}{10}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}}{\left(\frac{1}{10}\right)^{\beta}+\left(\frac{9}{10}\right)^{\beta}+\left(\frac{9}{100}\right)^{\beta}+(2)^{\beta}}=1/2$$

It follows that (o). QED

References

Atkinson, D., & Peijnenburg, J. (2013). Transitivity and partial screening off. *Theoria*, **79**, 294-308.

Atkinson, D., Peijnenburg, J., & Kuipers, T. (2009). How to confirm the conjunction of disconfirmed hypotheses. *Philosophy of Science*, **76**, 1-21.

Brossel, P. (2013). The problem of measure sensitivity redux. *Philosophy of Science*, **80**, 378-397.

Carnap, R. (1962). *Logical Foundations of Probability* (2nd ed.). Chicago: University of Chicago Press.

Christensen, D. (1999). Measuring confirmation. *Journal of Philosophy*, **96**, 437-461.

Climenhaga, N. (2013). A problem for the alternative difference measure of confirmation. *Philosophical Studies*, **164**, 643-651.

Crupi, V., Chater, N., & Tentori, K. (2013). New axioms for probability and likelihood ratio measures. *British Journal for the Philosophy of Science*, **64**, 189-204.

Crupi, V., Tentori, K., & Gonzalez, M. (2007). On Bayesian measures of evidential support: Theoretical and empirical issues. *Philosophy of Science*, **74**, 229-252.

Douven, I. (2008). The evidential support theory of conditionals. *Synthese*, **164**, 19-44.

Douven, I. (2011). Further results on the intransitivity of evidential support. *Review of Symbolic Logic*, **4**, 487-497.

Douven, I. (forthcoming). *The Epistemology of Indicative Conditionals: Formal and Empirical Approaches*. Cambridge University Press.

Douven, I., & Verbrugge, S. (2012). Indicatives, concessives, and evidential support. *Thinking & Reasoning*, **18**, 480-499.

Eells, E., & Fitelson, B. (2000). Measuring confirmation and evidence. *Journal of Philosophy*, **97**, 663-672.

Eells, E., & Fitelson, B. (2002). Symmetries and asymmetries in evidential support. *Philosophical Studies*, **107**, 129-142.

Festa, R. (2012). "*For unto every one that hath shall be given*". Matthew properties for incremental confirmation. *Synthese*, **184**, 89-100.

Fitelson, B. (1999). The plurality of Bayesian measures of confirmation and the problem of measure sensitivity. *Philosophy of Science*, **66**, S362-S378.

Fitelson, B. (2001). A Bayesian account of independent evidence with applications. *Philosophy of Science*, **68**, S123-S140.

Hajek, A., & Joyce, J. (2008). Confirmation. In Psillos, S., & Curd, M., editors. *The Routledge Companion to Philosophy of Science*. London: Routledge, pp. 115-128.

Hawthorne, J., & Fitelson, B. (2004). Re-solving irrelevant conjunction with probabilistic independence. *Philosophy of Science*, **71**, 505-514.

Huber, F. (2008). Milne's argument for the log-ratio measure. *Philosophy of Science*, **75**, 413-420.

Joyce, J. (1999). *The Foundations of Causal Decision Theory*. Cambridge: Cambridge University Press.

Joyce, J. (2008). Bayes' theorem. In Zalta, E., editor. *The Stanford Encyclopedia of Philosophy* (Fall 2008 ed.). URL = <http://plato.stanford.edu/archives/fall2008/entries/bayes-theorem/>.

Kuipers, T. (2000). *From Instrumentalism to Constructive Realism: On Some Relations Between Confirmation, Empirical Progress, and Truth Approximation*. Dordrecht: Kluwer.

Mackie, J. (1969). The relevance criterion of confirmation. *British Journal for the Philosophy of Science*, **20**, 27-40.

Mortimer, H. (1988). *The Logic of Induction*. Chichester: Ellis Horwood.

Nozick, R. (1981). *Philosophical Explanations*. Cambridge MA: Belknap.

Roche, W. (2012a). A weaker condition for transitivity in probabilistic support. *European Journal for Philosophy of Science*, **2**, 111-118.

Roche, W. (2012b). Transitivity and intransitivity in evidential support: Some further results. *Review of Symbolic Logic*, **5**, 259-268.

Roche, W. (2014a). Evidence of evidence is evidence under screening-off. *Episteme*, **11**, 119-124.

Roche, W. (2014b). On the truth-conduciveness of coherence. *Erkenntnis*, **79**, 647-665.

Roche, W. (forthcoming a). A note on confirmation and Matthew properties. *Logic and Philosophy of Science*.

Roche, W. (forthcoming b). Confirmation, increase in probability, and partial discrimination: A reply to Zalabardo. *European Journal for Philosophy of Science*.

Roche, W., & Shogenji, T. (2014a). Confirmation, transitivity, and Moore: The Screening-Off Approach. *Philosophical Studies*, **168**, 797-817.

Roche, W., & Shogenji, T. (2014b). Dwindling confirmation. *Philosophy of Science*, **81**, 114-137.

Roush, S. (2005). *Tracking Truth: Knowledge, Evidence, and Science*. Oxford: Oxford University Press.

Shogenji, T. (2003). A condition for transitivity in probabilistic support. *British Journal for the Philosophy of Science*, **54**, 613-616.

Sober, E. (2009). Absence of evidence and evidence of absence: Evidential transitivity in connection with fossils, fishing, fine-tuning, and firing squads. *Philosophical Studies*, **143**, 63-90.

Zalabardo, J. (2009). An argument for the likelihood-ratio measure of confirmation. *Analysis*, **69**, 630-635.