Foundationalism with infinite regresses of probabilistic support

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ABSTRACT: There is a long-standing debate in epistemology on the structure of justification. Some recent work in formal epistemology promises to shed some new light on that debate. I have in mind here some recent work by David Atkinson and Jeanne Peijnenburg, hereafter "A&P", on infinite regresses of probabilistic support. A&P show that there are probability distributions defined over an infinite set of propositions $\{p_1, p_2, p_3, \ldots, p_n, \ldots\}$ such that (i) p_i is probabilistically supported by p_{i+1} for all i and (ii) p_1 has a high probability. Let this result be "APR" (short for "A&P's Result"). A&P oftentimes write as though they believe that APR runs counter to foundationalism. This makes sense, since there is some prima facie plausibility in the idea that APR runs counter to foundationalism, and since some prominent foundationalists argue for theses inconsistent with APR. I argue, though, that in fact APR does not run counter to foundationalism. I further argue that there is a place in foundationalism for infinite regresses of probabilistic support.

KEYWORDS: Atkinson, foundationalism, infinite regresses of probabilistic support, justification, Peijnenburg

1 Introduction

There is a long-standing debate in epistemology on the structure of justification. Some recent work in formal epistemology promises to shed new light on that debate. I have in mind here some recent work by David Atkinson and Jeanne Peijnenburg, hereafter "A&P", on infinite regresses of probabilistic support.[1] They show that:

[1] See Atkinson and Peijnenburg (2006, 2009, 2012), Peijnenburg (2007), and Peijnenburg and Atkinson (2008, 2011, 2013, 2014).

A&P's Result (APR): There are probability distributions defined over an infinite set of propositions $\{p_1, p_2, p_3, \ldots, p_n, \ldots\}$ such that (i) p_i is probabilistically supported by p_{i+1} for all i and (ii) p_1 has a high probability.

This can be glossed: there can be high probability by an infinite regress of probabilistic support.[2,3]

Does APR run counter to foundationalism? A&P oftentimes write as though they believe that the answer is affirmative. Consider, for example, the following passage (where notation has been slightly modified):

> It can be readily understood that this constitutes the answer to the first conceptual objection, viz. the complaint that in an infinite regress there is no point at which the justification can start. For in a probabilistic regress a starting point turns out to be superfluous: $\Pr(p_1)$ can have a well-defined and unique value, well beyond a given threshold of acceptance, without the need of a starting point. It will also be clear that this finding is at odds with a foundationalist outlook in which justification has to start from some grounding proposition of belief. (Peijnenburg and Atkinson 2014, p. 170)

The second to last sentence here is in effect APR. The last sentence, then, is in effect the claim that APR runs counter to foundationalism.

There is some prima facie plausibility in the idea that APR runs counter to foundationalism. Some prominent foundationalists even argue for theses inconsistent with APR. I have in mind here, for example, C. I. Lewis (1952). He argues that if a regress of probabilistic support does not bottom out in certainties and instead continues on ad infinitum, then its target proposition has a probability of zero.[4] I aim to show,

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[2] APR concerns *non-circular* infinite regresses of probabilistic support. A&P also show that there can be high probability by *circular* infinite regresses of probabilistic support (or "justification by infinite loops"). See Atkinson and Peijnenburg (2010a).

[3] APR concerns *one*-dimensional infinite regresses of probabilistic support, that is, infinite regresses of probabilistic support where each node is a single proposition. A&P also argue that there can be high probability by *many*-dimensional infinite regresses of probabilistic support (or "many-dimensional probabilistic networks"). See Atkinson and Peijnenburg (2012).

[4] See Atkinson and Peijnenburg (2006) and Peijnenburg and Atkinson (2011) for helpful discussion of Lewis, along with Bertrand Russell and Hans Reichenbach, on infinite regresses of probabilistic support.

though, that in fact APR does not run counter to foundationalism. I further aim to show that there is a place in foundationalism for infinite regresses of probabilistic support.[5,6]

These arguments, even if they succeeded, would be trivial if they relied on a radically non-standard construal of foundationalism, for example, a construal on which foundationalism implies that justification has no foundation. But they do not. By "foundationalism" I mean, roughly, the view that justification has a foundation in that all inferential justification is ultimately grounded in or based on non-inferential justification, where the justification at issue is propositional as opposed to doxastic and is relative to a subject *S*.[7] This is a standard (though somewhat imprecise) construal of foundationalism.

I want to stress that my overall evaluation of A&P's work on infinite regresses of probabilistic support is positive. APR is an ingenious result and opens up a new and surprising route to foundationalist justification. It is thus an important contribution to the extant literature on foundationalism and the regress problem more generally.[8]

The remainder of the paper is organized as follows. In Section 2, I explain APR. In Section 3, I give a more precise construal of foundationalism. In Section 4, I argue that APR does not run counter to foundationalism. In Section 5, I argue that there is a place in foundationalism for infinite regresses of probabilistic support. I do this in part by modifying the construal of foundationalism given in Section 3. In Section 6, I argue that there is also a place in foundationalism for finite regresses of probabilistic support (of a non-standard sort), infinite regresses of probabilistic anti-support, and finite regresses of probabilistic anti-support. In Section 7, I conclude.

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[5] Turri (2009, pp. 162-163) argues for a similar thesis. He argues in particular that there is a place in foundationalism for an infinite and non-repeating series of reasons (available to a subject). He does not address APR or A&P's work more generally, though, and the infinite regresses he has in mind differ in important respects from the infinite regresses I, following A&P, have in mind. See Peijnenburg and Atkinson (2011, secs. 5 and 6) for discussion of Turri's argument. See also Herzberg (2013, sec. 2).

[6] I am not the first to discuss A&P's work on infinite regresses of probabilistic support. See Gwiazda (2010), Herzberg (2010), and Podlaskowski and Smith (2014). But my discussion is very different than Gwiazda's, Herzberg's, and Podlaskowski and Smith's discussions. See Peijnenburg (2010) for a response to Gwiazda (2010). See Atkinson and Peijnenburg (2010b) for a response to Herzberg (2010).

[7] For a recent discussion of the distinction between propositional and doxastic justification, and for references, see Silva (2015). See also Korcz (2015) on the basing relation.

[8] The extant literature on the regress problem is vast. See Cling (2008) for references (and for helpful discussion).

2 A&P's Result (APR)

The task in this section is to explain APR and the reasoning behind it. I shall do this in three main steps. In Section 2.1, I shall address the notion of probabilistic support. In Section 2.2, I shall address finite regresses of probabilistic support. In Section 2.3, building on the discussion in Section 2.2, I shall address infinite regresses of probabilistic support. All the main points in Section 2.2 and Section 2.3 are due essentially to A&P, but, for reasons of presentation, I shall refrain from using expressions such as "A&P argue that", "A&P show that", and "A&P note that".

2.1 Probabilistic support

There are multiple ways of understanding the notion of probabilistic support.[9] Below are three (where **t** is the threshold for high probability and might be context-sensitive):

> *High Probability (HP)*: For any propositions p and q, p is probabilistically supported by q if and only if $\Pr(p \mid q) > \mathbf{t}$.
>
> *Increase in Probability (IP)*: For any propositions p and q, p is probabilistically supported by q if and only if $\Pr(p \mid q) > \Pr(p)$.
>
> *High Probability and Increase in Probability (HIP)*: For any propositions p and q, p is probabilistically supported by q if and only if (a) $\Pr(p \mid q) > \mathbf{t}$ and (b) $\Pr(p \mid q) > \Pr(p)$.

Some cases of probabilistic support in the sense of HP are not cases of probabilistic support in the sense of IP or, thus, in the sense of HIP. And some cases of probabilistic support in the sense of IP are not cases of probabilistic support in the sense of HP or, thus, in the sense of HIP. So HP, IP, and HIP are all distinct from each other.

IP and HIP can be reformulated as follows:

> *Increase in Probability* (IP*)*: For any propositions p and q, p is probabilistically supported by q if and only if $\Pr(p \mid q) > \Pr(p \mid \neg q)$.

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[9] See Douven (2011), Roche (2012a, 2015), and Roche and Shogenji (2014).

High Probability and Increase in Probability (HIP*)*: For any propositions p and q, p is probabilistically supported by q if and only if (a) $\Pr(p \mid q) > \mathbf{t}$ and (b) $\Pr(p \mid q) > \Pr(p \mid \neg q)$.

This is because $\Pr(p \mid q) > \Pr(p)$ if and only if $\Pr(p \mid q) > \Pr(p \mid \neg q)$.

Hereafter, for definiteness, I shall assume that probabilistic support is to be understood in terms of HIP/HIP* where $\mathbf{t} = 0.5$. Nothing of substance hinges on this assumption, however, since all the main points in the remainder of the paper about probabilistic support hold regardless of which of HP, IP/IP*, and HIP/HIP* is assumed.[10]

2.2 Finite regresses of probabilistic support

Let $\{p_1, p_2, p_3, \ldots, p_n\}$ be a finite set of propositions. Then, for any $i \leq n$-1, it follows by the rule of total probability that:

(1) $$\Pr(p_i) = \Pr(p_i \mid p_{i+1})\Pr(p_{i+1}) + \Pr(p_i \mid \neg p_{i+1})\Pr(\neg p_{i+1})$$

This is equivalent to:

(2) $$\Pr(p_i) = \Pr(p_i \mid \neg p_{i+1}) + \left[\Pr(p_i \mid p_{i+1}) - \Pr(p_i \mid \neg p_{i+1})\right]\Pr(p_{i+1})$$

Suppose, for ease of expression, that $\alpha_i = \Pr(p_i \mid p_{i+1})$, $\beta_i = \Pr(p_i \mid \neg p_{i+1})$, and $\gamma_i = \alpha_i - \beta_i$. Then (2) can be rewritten as:

(3) $$\Pr(p_i) = \beta_i + \gamma_i \Pr(p_{i+1})$$

It follows from (3) that:

(4) $$\Pr(p_1) = \beta_1 + \gamma_1 \Pr(p_2)$$

(5) $$\Pr(p_2) = \beta_2 + \gamma_2 \Pr(p_3)$$

Thus:

[10] A&P typically assume IP/IP* in their work on infinite regresses of probabilistic support. Peijnenburg and Atkinson (2014) is an exception. There they assume HIP/HIP*.

$$(6) \qquad \Pr(p_1) = \beta_1 + \gamma_1\beta_2 + \gamma_1\gamma_2 \Pr(p_3)$$

This generalizes to:

$$(7) \qquad \Pr(p_1) = \beta_1 + \gamma_1\beta_2 + \gamma_1\gamma_2\beta_3 + \ldots + \gamma_1\gamma_2 \ldots \gamma_{n-2}\beta_{n-1} + \gamma_1\gamma_2 \ldots \gamma_{n-1} \Pr(p_n)$$

$\Pr(p_1)$ is thus fully determined by $(n - 1)(2)$ conditional probabilities and 1 unconditional probability.

It will help to assign some values to α_i and β_i. Suppose that $\alpha_i = 0.99$ and $\beta_i = 0.04$ for all $i \leq n$-1. Consider three cases:

Case 1: $\Pr(p_n) = 0.1$
Case 2: $\Pr(p_n) = 0.5$
Case 3: $\Pr(p_n) = 0.9$

If $n = 3$, then:

Case 1: $\Pr(p_1) \approx 0.168$
Case 2: $\Pr(p_1) \approx 0.529$
Case 3: $\Pr(p_1) \approx 0.890$

If, instead, $n = 100$, then:

Case 1: $\Pr(p_1) \approx 0.796$
Case 2: $\Pr(p_1) \approx 0.798$
Case 3: $\Pr(p_1) \approx 0.801$

This means that $\Pr(p_n)$ always has an impact on the value of $\Pr(p_1)$ but the impact gets less and less as n gets bigger and bigger.

It is easy to see why this is the case. The only addend in (7) involving $\Pr(p_n)$ is the last one. Given that $\alpha_i = 0.99$ and $\beta_i = 0.04$ for all $i \leq n$-1, it follows that $\gamma_i = 0.95$ for all $i \leq n$-1 and so the last addend in (7) is given by:

$$(8) \qquad \gamma_1\gamma_2 \ldots \gamma_{n-1} \Pr(p_n) = (0.95)^{n-1} \Pr(p_n)$$

But $(0.95)^{n-1}$ gets less and less as n gets bigger and bigger. If, say, $n = 100$, then $(0.95)^{n-1}$ is roughly equal to 0.006 and so the right side of (8)—and thus the only addend in (7) involving $\Pr(p_n)$—is extremely small regardless of the value of $\Pr(p_n)$.

2.3 Infinite regresses of probabilistic support

Let $p_1, p_2, p_3, \ldots, p_n, \ldots$ be an infinite regress of probabilistic support such that $\Pr(p_i \mid p_{i+1}) = \alpha$ and $\Pr(p_i \mid \neg p_{i+1}) = \beta$ for all i. Then, it turns out, (7) gives way to:

$$(9) \qquad \Pr(p_1) = \frac{\beta}{1-\alpha+\beta}$$

If $\alpha = 0.99$ and $\beta = 0.04$, then $\Pr(p_1) = 0.8$. If $\alpha = 0.999$ and $\beta = 0.04$, then $\Pr(p_1) \approx 0.976$. If $\alpha = 0.9999$ and $\beta = 0.04$, then $\Pr(p_1) \approx 0.998$. And so on. $\Pr(p_1)$ tends to 1 as α tends to 1 with β fixed at 0.04. Thus APR.[11]

A&P give a helpful example. They write:

> Consider an inheritable trait, T, conducive to survival in a particular environment, which a girl is sure to have if her mother had it. Suppose though that the child might also carry T if her father had it, whether or not her mother did so. Thus the probability that the child has T, given that mother has T, is 1; but the probability that she has T if mother lacks T is not zero, since there is a chance after all that father has T (assuming that there is no other way that the child can get T). (Peijnenburg and Atkinson 2011, p. 123)

Let F_1 be some female and F_i be F_{i-1}'s mother for all i. Let $p_1, p_2, p_3, \ldots, p_n, \ldots$ be an infinite regress of probabilistic support such that p_i is the proposition that F_i carries T for all i. Then, given the details of the case, $\alpha = 1$ and $\beta > 0$. It follows from (9) that $\Pr(p_1) = 1$.

Where exactly does p_1's probability come from? The answer, of course, is not that it comes from the last member of $p_1, p_2, p_3, \ldots, p_n, \ldots$, for this regress has no last member. But then what is the answer?

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[11] The assumption that $\Pr(p_i \mid p_{i+1}) = \alpha$ and $\Pr(p_i \mid \neg p_{i+1}) = \beta$ for all i is inessential. A&P show that the target proposition in an infinite regress of probabilistic support can have a high probability even if it is not the case that $\Pr(p_i \mid p_{i+1}) = \alpha$ and $\Pr(p_i \mid \neg p_{i+1}) = \beta$ for all i. See, for example, Atkinson and Peijnenburg (2009), Peijnenburg (2007), and Peijnenburg and Atkinson (2008, 2014).

A&P answer as follows:

> It is intuitively clear—reaching further and further back into the family tree—that the genetic condition of a great-great grandmother in the *n*th ancestral generation contributes less and less, as n increases, to the probability that the girl has T. In the formal limit of an infinite number of generations, *all* the contributions to the probability that the child has T are coming from the conditional probabilities: the contribution of a remote ancestress diminishes more and more as the ancestress is further and further away, until she is hidden in the mists of time and her influence has vanished completely. (Peijnenburg and Atkinson 2011, p. 123, emphasis original)

Their answer, then, is that p_1's probability comes from the various conditional probabilities involved in the regress: $\Pr(p_1 \mid p_2)$, $\Pr(p_1 \mid \neg p_2)$, $\Pr(p_2 \mid p_3)$, $\Pr(p_2 \mid \neg p_3)$, …, $\Pr(p_{n-1} \mid p_n)$, $\Pr(p_{n-1} \mid \neg p_n)$, ….

This is an interesting result to say the least. p_1 can have a high probability even though there is no last member of the regress and even though, thus, there is no last member of the regress the probability of which is high.

Does APR pose a problem for foundationalism? And how exactly is foundationalism to be understood? I address the latter question in the next section and the former in the section after the next.

3 Foundationalism

I noted above in Section 1 that by "foundationalism" I mean, roughly, the view that justification has a foundation in that all inferential justification is ultimately grounded in or based on non-inferential justification, where the justification at issue is propositional as opposed to doxastic and is relative to a subject *S*. This can be made more precise as follows:

> *Foundationalism on Justification (FJ)*: *p* is justified for *S* if and only if (i) *p* is non-inferentially justified for *S* (that is, justified for *S* independently of any probabilistic support from one or more other propositions justified for *S*) or (ii) *p* is the target proposition in a regress of probabilistic support *RPS* such that (a) *RPS* has a finite number of nodes, (b) none of *RPS*'s nodes is an ancestor of itself, and (c) *RPS*'s root nodes are non-inferentially justified for *S*.

Some terminology is in need of clarification. Consider the following (graphical representation of a) regress of probabilistic support (where the subscript in "$RPS_{F\&NC}$" is short for "Finite and Non-Circular"):

$$\begin{array}{ll} RPS_{F\&NC}: & p_1 \\ & \uparrow \\ & p_2 \\ & \uparrow \\ & p_3 \end{array}$$

This regress has three "nodes": p_1, p_2, and p_3. Thus $RPS_{F\&NC}$ meets (a) in FJ. There is an arrow from p_3 to p_2 and from p_2 to p_1. This indicates that p_3 probabilistically supports p_2 and that p_2 probabilistically supports p_1. This can be put in terms of parent/ancestor relations. p_3 is p_2's parent (and thus p_2 is p_3's child). p_2, in turn, is p_1's parent (and thus p_1 is p_2's child). p_1 is not an ancestor of p_2, p_3, or itself. p_2 is an ancestor of p_1 but not of p_3 or itself. And p_3 is an ancestor of p_1 and p_2 but not of itself. It follows from all this that $RPS_{F\&NC}$ meets (b) in FJ. p_3 is a root node since there is no arrow from another node to it. And there are no other root nodes. Thus if p_3 is non-inferentially justified for S, then $RPS_{F\&NC}$ meets (c) in FJ and so (ii) in FJ is met and, by FJ, p_1 is justified for S.[12]

Consider, by contrast, the following (where the subscript in "$RPS_{F\&C}$" is short for "Finite and Circular" and the subscript in "$RPS_{I\&NC}$" is short for "Infinite and Non-Circular"):

$$\begin{array}{ll} RPS_{F\&C}: & p_1 \\ & \uparrow \\ & p_2 \\ & \uparrow \\ & p_3 \\ & \uparrow \\ & p_1 \end{array}$$

[12] FJ is similar to some extant construals of foundationalism. See, for example, the construals in Cornman (1977) and Zalabardo (2008).

$$
\begin{array}{ll}
RPS_{I\&NC}: & p_1 \\
& \uparrow \\
& p_2 \\
& \uparrow \\
& p_3 \\
& \uparrow \\
& \vdots
\end{array}
$$

$RPS_{F\&C}$ has a finite number of nodes but p_1 is an ancestor of itself. So $RPS_{F\&C}$ fails to meet (b) in FJ and thus (ii) in FJ is not met. Suppose that $RPS_{I\&NC}$ meets (b) in FJ. Then, still, (ii) in FJ is not met since $RPS_{I\&NC}$ has an infinite number of nodes.

There is a clear sense in which FJ is underspecified. When is it that *p* is non-inferentially justified for *S*? Is it when, for example, *S* has a perceptual experience as if *p*? Or when something else is the case? This is okay for my purposes however. The main points below hold regardless of when exactly *p* is non-inferentially justified for *S*.[13]

It should be noted that FJ is similar to, though a bit more detailed than, how A&P construe foundationalism. Consider, for example, the following passages:

> After all, what could be more central to foundationalism than the claim that there has to be a foundation, a last member that serves as the basis of the entire edifice, the source from which the whole justification springs? This foundation may be a certainty, as it is in Lewis's philosophy, or a fixed probability, as it is for moderate foundationalists, or a probability that is not fixed, as it is for so-called weak foundationalists …. But however he twists and turns, a genuine foundationalist seems attached to a foundation, a last link in the chain. (Peijnenburg and Atkinson 2011, p. 119)

> Foundationalism and coherentism come in various sorts and sizes, but the difference between the two is clear: foundationalists hold that basic beliefs justify nonbasic beliefs while coherentists maintain that beliefs justify one another and that basic beliefs do not exist. (Atkinson and Peijnenburg 2009, p. 183)

[13] See Fumerton (2010) for relevant discussion.

They say in effect in the first passage that foundationalism is the view that justification has a foundation. Their focus in the second passage is on beliefs as opposed to propositions and the expression "basic" is used instead of the expression "non-inferentially justified". But suppose that belief-talk is replaced by proposition-talk and that the expression "basic" is replaced by the expression "non-inferentially justified". Then the claim is that foundationalism is the view that non-inferentially justified propositions justify inferentially justified propositions. This claim, I take it, is tantamount to the claim that foundationalism is the view that all inferential justification is ultimately grounded in or based on non-inferential justification, and the latter claim is simply a gloss on the claim that foundationalism is FJ. I take it, then, that A&P should be happy with FJ as a construal of foundationalism.

4 Does A&P's Result (APR) run counter to Foundationalism on Justification (FJ)?

There are passages in A&P's work on infinite regresses of probabilistic support suggesting that they hold that it follows from APR that:

> *Infinitism on Justification 1 (IJ1)*: p is justified for S if p is the target proposition in a regress of probabilistic support *RPS* such that (a) $\Pr(p) > \mathbf{t}$, (b) *RPS* has an infinite number of nodes, and (c) none of *RPS*'s nodes is an ancestor of itself.

The passage given above in Section 1 is a case in point. The idea there, it seems, is that APR runs counter to foundationalism by leading to IJ1.

I aim to show below that there is reason to believe that the passages in question are misleading in that A&P do not hold that it follows from APR that IJ1 is correct. The issue for now, though, is whether in fact it follows from APR that IJ1 is correct. If yes, then APR runs counter to FJ.

It is straightforward to show that the answer is negative. Let p_1 be the proposition that S has exactly ten fingers. There are probability distributions defined over an infinite set of propositions $\{p_1, p_2, p_3, \ldots, p_n, \ldots\}$ such that (i) p_i is probabilistically supported by p_{i+1} for all i and (ii) $\Pr(p_1)$ is high. But there are also probability distributions defined over an infinite set of propositions $\{\neg p_1, p_2, p_3, \ldots, p_n, \ldots\}$ such that (i) $\neg p_1$ is probabilistically supported by p_2, (ii) p_i is probabilistically supported by p_{i+1} for all $i \geq 2$, and (iii) $\Pr(\neg p_1)$ is high. Suppose, for reductio, that IJ1 is correct. Then it follows that each of p_1 and $\neg p_1$

is justified for S. Surely, though, it is not the case that each of p_1 and $\neg p_1$ is justified for S.[14] Hence IJ1 is not correct. Hence IJ1 does not follow from APR (which is correct).[15,16]

The worry, notice, is *not* that if IJ1 is correct, then there are probability distributions on which both a proposition and its negation have a probability greater than $\mathbf{t} = 0.5$. There are, of course, no such probability distributions. The worry, rather, is that if IJ1 is correct, then there can be cases where both a proposition and its negation are justified for a subject.

It might seem puzzling that APR is correct while IJ1 is incorrect. But note that APR is framed in terms of probability distributions as opposed to justification. This is significant. It follows by APR that there exists a function Pr defined over an infinite set of propositions $\{p_1, p_2, p_3, \ldots, p_n, \ldots\}$ such that (i) $\Pr(p_i \mid p_{i+1}) > \mathbf{t}$ and $\Pr(p_i \mid p_{i+1}) > \Pr(p_i \mid \neg p_{i+1})$ for all i and (ii) $\Pr(p_1) > \mathbf{t}$. Pr, though, is a mere function. Its existence per se leaves it open, contra IJ1, that FJ is right that p_1 is justified for S only if p_1 is non-inferentially justified for S or p_1 is the target proposition in a regress of probabilistic support *RPS* such that *RPS* has a finite number of nodes, none of *RPS*'s nodes is an ancestor of itself, and *RPS*'s root nodes are non-inferentially justified for S. Its existence per se thus leaves it open that IJ1 is false.

This point generalizes. Consider, for example, the following alternative to IJ1:

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[14] Cornman (1977, p. 291) defends a claim to this effect by appeal to the thesis that a proposition is justified for a subject only if it is more reasonable for that subject than is its denial.

[15] See Cornman (1977) and Post (1980) for similar worries with theses similar to IJ1. See Aikin (2011, Ch. 2, sec. 2.3) for relevant discussion.

[16] IJ1 stands in contrast to:

> *Infinitism on Justification 1* (IJ1*)*: p is justified for S only if p is the target proposition in a regress of probabilistic support *RPS* such that (a) $\Pr(p) > \mathbf{t}$, (b) *RPS* has an infinite number of nodes, and (c) none of *RPS*'s nodes is an ancestor of itself.

IJ1 gives a putative *sufficient* condition for justification whereas IJ1* gives a putative *necessary* condition. The latter, unlike the former, is correct. In fact, it is *trivially* correct. Any p such that p is justified for S is also such that there is regress of probabilistic support *RPS* where p is the target proposition, $\Pr(p) > \mathbf{t}$, the number of nodes is infinite, and none of the nodes is an ancestor of itself. See Cling (2004, sec. 2, p. 103) for a closely related point.

Infinitism on Justification 2 (IJ2): p is justified for S if p is the target proposition in a regress of probabilistic support *RPS* such that (a) $\Pr(p) > \mathbf{t}$, (b) *RPS* has an infinite number of nodes, (c) none of *RPS*'s nodes is an ancestor of itself, and (d) S believes all and only the nodes in *RPS*.

This theory, unlike IJ1, implies that there can be no cases where both a proposition and its negation are justified for a subject. This is good for IJ2. But, again, the existence of a function Pr defined over an infinite set of propositions $\{p_1, p_2, p_3, \ldots, p_n, \ldots\}$ such that (i) $\Pr(p_i \mid p_{i+1}) > \mathbf{t}$ and $\Pr(p_i \mid p_{i+1}) > \Pr(p_i \mid \neg p_{i+1})$ for all i and (ii) $\Pr(p_1) > \mathbf{t}$ leaves it open, contra IJ2, that FJ is right that p_1 is justified for S only if p is non-inferentially justified for S or p is the target proposition in a regress of probabilistic support *RPS* such that *RPS* has a finite number of nodes, none of *RPS*'s nodes is an ancestor of itself, and *RPS*'s root nodes are non-inferentially justified for S.[17]

These points can be reinforced by turning from *infinite* regresses of probabilistic support to *finite* regresses of probabilistic support. Consider the following variant of APR:

A&P's Result (APR*)*: There are probability distributions defined over an finite set of propositions $\{p_1, p_2, p_3, \ldots, p_n\}$ such that such that (i) $\Pr(p_i \mid p_{i+1}) > \mathbf{t}$ and $\Pr(p_i \mid p_{i+1}) > \Pr(p_i \mid \neg p_{i+1})$ for all $i \leq n$-1 and (ii) $\Pr(p_1) > \mathbf{t}$.

There is no questioning APR*. It does not follow from APR*, however, that FJ is correct and that infinitist theories such as IJ1 and IJ2 are incorrect. If infinitist theories such as IJ1 and IJ2 are incorrect, then this is not because of a trivial result in probability theory such as APR*.

The lesson, then, is that APR by itself does not run counter to FJ. If A&P's work on infinite regresses of probabilistic support poses a problem for FJ, then this is not because of APR.

I noted above that there are passages in A&P's work on infinite regresses of probabilistic support suggesting that A&P hold that it follows from APR that IJ1 is correct. There is reason to believe, though, that all such passages are misleading.

A&P raise and respond to in effect the worry above with IJ1 (the worry that if IJ1 is correct, then there can be cases where both a proposition and its negation are justified for a subject). They respond by stressing that only *empirically credible* (or empirically based) conditional probabilities will do. They write:

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[17] See Roche (2012b) for discussion of a coherentist theory similar in relevant respects to IJ2.

> But notice the difference. Since justification now is not postponed, a rivaling regress for the target's negation is not so easily made, and the threat of a reductio attack is thereby diminished. For in order to set up a reductio in the probabilistic case, we have to produce alternative conditional probabilities that possess a similar *empirical credibility* as the original probabilistic regress. In the classical regress, by contrast, we do not have to produce anything at all. There it is enough to point to a fathomless borehole whence all the justification is supposed to originate, or to a bank teller light-years away who is supposed to administer my fortune. In a probabilistic regress we have to deliver justification (albeit piecemeal), but in a classical regress we broker a never-ending mortgage. Precisely because justification is constantly anticipated but never attained, it is easier to concoct a rivaling regress in that case than it is in the probabilistic scenario. Hence a reductio-like attack is easier to set up for a classical than for a probabilistic regress. (Peijnenburg and Atkinson 2014, pp. 175-176, emphasis added)

> We have seen that the probability of the target proposition is a function of the conditional probabilities alone, and that the absence of a starting proposition is no impediment. For the justification is now provided by the conditional probabilities. If they are *empirically based* in some way, then each link of the chain will contribute an increment of justification to the target proposition. Thus the no starting point objection has been sidestepped; and the force of the reductio has been weakened. Rather than that justification is forever postponed, it seeps through the conditional probabilities to the target proposition, and it is not at all clear that one could fabricate an alternative series of conditional probabilities that does the same thing for the target's negation. (Peijnenburg and Atkinson 2014, p. 176, emphasis added)

So they reject IJ1 and, thus, assuming consistency, do not hold that it follows from APR (which they accept) that IJ1 is correct.

What, then, is A&P's view? Their view, it seems, is:

> *Infinitism on Justification 3 (IJ3)*: p is justified for S if p is the target proposition in a regress of probabilistic support *RPS* such that (a) $\Pr(p) > \mathbf{t}$, (b) *RPS* has an infinite number of nodes, (c) none of *RPS*'s nodes is an ancestor of itself, and (d) the conditional probabilities involved are empirically credible for S.

This theory is like IJ1 except that it includes a condition of empirical credibility.[18,19]

It might be wondered whether A&P initially accepted IJ1 and then later rejected it in favor of IJ3. The answer is no. This is clear from a passage in Atkinson and Peijnenburg (2006). Consider (where notation has been slightly modified):

> Let $\Pr(e)$ … stand for the probability that a man will suffer from prostatic cancer, and $\alpha = \Pr(e \mid g_1)$ for the conditional probability that he will have the complaint, given that his father did so. Since not all prostatic cancer patients have fathers with a similar affliction, $\beta = \Pr(e \mid \neg g_1)$ is not zero. On the other hand, a man is more likely to contract prostatic cancer if his father had it than if he did not. Thus $\alpha > \beta > 0$, and *empirical values of α and β have been estimated from the study of large populations*. (Atkinson and Peijnenburg 2006, pp. 447-448, emphasis added)

There is no explicit mention here of empirical credibility. But the idea is clear: $\Pr(e \mid g_1)$ and $\Pr(e \mid \neg g_1)$ are backed by, or are based on, adequate sample frequency data and thus are empirically credible.[20]

The situation is this. APR by itself does not run counter to FJ. And A&P do not believe otherwise.

This is not the end of the story however. IJ3 is suggested by various cases given by A&P involving infinite regresses of probabilistic support. But IJ3 runs counter to FJ, for the latter implies that a regress of probabilistic support *RPS* with p as the target proposition is justification-yielding for *S* only if it has a *finite* number of nodes and thus only if it does not have an infinite number of nodes. Is IJ3, though, really an infinitist theory? If not, and if this is because it is really a foundationalist theory, is there a way to modify FJ so that it allows for A&P's cases? I turn now to these questions.

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[18] It is important to note that (a) in IJ3 is not redundant. Let $p_1, p_2, p_3, \ldots, p_n, \ldots$ be a regress of probabilistic support *RPS* such that *RPS* has an infinite number of nodes, none of *RPS*'s nodes is an ancestor of itself, and the conditional probabilities involved are empirically credible for *S*. Suppose, consistent with this, that $\Pr(p_i \mid p_{i+1}) = 2/100$ and $\Pr(p_i \mid \neg p_{i+1}) = 1/100$ for all i. Then by (9) it follows that $\Pr(p_1)$ is roughly equal to 0.010 and thus is less than **t**. See Herzberg (2014, sec. 7) for related discussion.

[19] IJ3, understood as IJ3* below, is similar to but importantly different than "(PBPIJ)" in Herzberg (2014, p. 714). IJ3 gives a putative *sufficient* condition for justification whereas (PBPIJ) gives a putative *necessary and sufficient* condition for justification.

[20] See also Peijnenburg and Atkinson (2011, p. 124).

5 From Infinitism on Justification 3 (IJ3) to Foundationalism on Justification* (FJ*)

How exactly is the expression "empirically credible" in IJ3 to be understood? The answer, presumably, is that it is to be understood as "empirically justified" or more simply as "justified". IJ3 can thus be reformulated as follows:

> *Infinitism on Justification 3* (IJ3*)*: p is justified for S if p is the target proposition in a regress of probabilistic support *RPS* such that (a) $\Pr(p) > \mathbf{t}$, (b) *RPS* has an infinite number of nodes, (c) none of *RPS*'s nodes is an ancestor of itself, and (d) the conditional probabilities involved are justified for S.

Recall the case from Section 2.3 where F_1 is some female, F_i is F_{i-1}'s mother for all i, and p_i is the proposition that F_i carries T for all i. Let e be a conjunction of the following *population* frequency claims (where "P" in "Freq_P" is short for "Population"):

Freq_P(females who carry T | females whose mothers carry T) = $\alpha = 1$

Freq_P(females who carry T | females whose mothers do not carry T) = $\beta = 0.04$

Let e*, in turn, be a conjunction of the following *sample* frequency claims (where "S" in "Freq_S" is short for "Sample"):

Freq_S(females who carry T | females whose mothers carry T) = $\alpha = 1$

Freq_S(females who carry T | females whose mothers do not carry T) = $\beta^* > 0$

Suppose that e* is justified for S and that in part because of this e is justified for S. Then, it seems, each of $\Pr(p_1 \mid p_2) = 1$, $\Pr(p_1 \mid \neg p_2) = 0.04$, $\Pr(p_2 \mid p_3) = 1$, $\Pr(p_2 \mid \neg p_3) = 0.04$, … is justified for S.

It is important to note that conditional probabilities are not themselves propositions and thus are not themselves relata in relations of probabilistic support. The idea, thus, is not the each of $\Pr(p_1 \mid p_2) = 1$, $\Pr(p_1 \mid \neg p_2) = 0.04$, $\Pr(p_2 \mid p_3) = 1$, $\Pr(p_2 \mid \neg p_3) = 0.04$, … is probabilistically supported by e. The idea, rather, is that each of $\Pr(p_1 \mid p_2) = 1$, $\Pr(p_1 \mid \neg p_2) = 0.04$, $\Pr(p_2 \mid p_3) = 1$, $\Pr(p_2 \mid \neg p_3) = 0.04$, … is *rationally determined* by e. Each of $\Pr(p_1 \mid p_2) = 1$, $\Pr(p_1 \mid \neg p_2) = 0.04$, $\Pr(p_2 \mid p_3) = 1$, $\Pr(p_2 \mid \neg p_3) = 0.04$, …, that is, is rational given e.

The question now is how e^* is justified for *S*. A&P could answer that e^* is justified for *S* if e^* is the target proposition in a regress of probabilistic support *RPS* such that (a) *RPS* has an infinite number of nodes and (b) none of *RPS*'s nodes is an ancestor of itself. But then the worry with IJ1 would carry over to IJ3* in that the latter would imply that there can be cases where both a proposition and its negation are justified for a subject.[21] A&P could instead answer that e^* is justified for *S* if e^* is the target proposition in a regress of probabilistic support *RPS* such that (a) *RPS* has an infinite number of nodes, (b) none of *RPS*'s nodes is an ancestor of itself, and (c) the conditional probabilities involved are justified for *S*. Then, though, A&P would need to answer the question of how those additional conditional probabilities are justified for *S*. And they would need to do so in such a way that IJ3* so fleshed out does not imply that there can be cases where both a proposition and its negation are justified for a subject.

There is a foundationalist answer open to A&P. They can claim that e^* is justified for *S* because (i) e^* is non-inferentially justified for *S* or (ii) e^* is the target proposition in a regress of probabilistic support *RPS* such that (a) *RPS* has a finite number of nodes, (b) none of *RPS*'s nodes is an ancestor of itself, and (c) *RPS*'s root nodes are non-inferentially justified for *S*. This would help since, it seems, the worry with IJ1 does not carry over to FJ and so does not carry over to IJ3* thus understood.

Suppose that A&P give the foundationalist answer just set out. Then, though IJ3* runs counter to FJ (for the same reason that IJ3 runs counter to FJ), there is a clear sense in which the justification at issue in A&P's cases involving infinite regresses of probabilistic support is foundationalist. This suggests that FJ should be modified so as to allow for A&P's cases involving infinite regresses of probabilistic support. Is there a way to do that?

A&P describe their cases in terms of *propositions* and *relations of probabilistic support*. But there is no necessity in this. Their cases can instead be understood in terms of *credences* and *relations of rational determination*. Take the case from Section 2.3 where $p_1, p_2, p_3, \ldots, p_n, \ldots$ is an infinite regress of probabilistic support such that $\Pr(p_i \mid p_{i+1}) = \alpha$ and $\Pr(p_i \mid \neg p_{i+1}) = \beta$ for all i. Suppose that $\alpha = 0.99$ and $\beta = 0.04$. Let $\mathrm{Cr}(p_1)$ be *S*'s actual or potential credence in p_1, $\mathrm{Cr}(p_1 \mid p_2)$ be *S*'s actual or potential credence in p_1 given p_2, and so on for $\mathrm{Cr}(p_1 \mid \neg p_2)$, $\mathrm{Cr}(p_2 \mid p_3)$, $\mathrm{Cr}(p_2 \mid \neg p_3)$, …, $\mathrm{Cr}(p_{n-1} \mid p_n)$, $\mathrm{Cr}(p_{n-1} \mid \neg p_n)$, …. Then the case can be described as follows:

[21] Cling (2008, sec. 3, p. 407, and sec. 8) and Moser (1989, Ch. 2, sec. 2.2.2) argue along these lines. See Cling (2004) for related discussion.

(a) $Cr(p_1) = 0.8$ is rationally determined by $Cr(p_1 \mid p_2) = 0.99$, $Cr(p_1 \mid \neg p_2) = 0.04$, $Cr(p_2 \mid p_3) = 0.99$, $Cr(p_2 \mid \neg p_3) = 0.04$, …, $Cr(p_{n-1} \mid p_n) = 0.99$, $Cr(p_{n-1} \mid \neg p_n) = 0.04$, ….
(b) $Cr(p_1 \mid p_2) = 0.99$, $Cr(p_1 \mid \neg p_2) = 0.04$, $Cr(p_2 \mid p_3) = 0.99$, $Cr(p_2 \mid \neg p_3) = 0.04$, …, $Cr(p_{n-1} \mid p_n) = 0.99$, $Cr(p_{n-1} \mid \neg p_n) = 0.04$, … are rationally determined by $Cr(e)$.
(c) $Cr(e)$ is rationally determined by $Cr(e^*)$.
(d) $Cr(e^*)$ is rationally determined by $Cr(o_1)$, $Cr(o_2)$, …, and $Cr(o_n)$, where o_i is an observational proposition for all i, and where each of $Cr(o_1)$, $Cr(o_2)$, …, and $Cr(o_n)$ is non-inferentially justified for S.

This description, in turn, can be represented as follows (where "RRD" is short for "Regress of Rational Determination" and "F&NC" is short for "Finite + Non-Circular"):

$RRD_{F\&NC}$:

$$\text{N1: } \left[\mathrm{Cr}(p_1) = 0.8\right]$$

$$\Uparrow$$

$$\text{N2: } \begin{bmatrix} \mathrm{Cr}(p_1 \mid p_2) = 0.99, \mathrm{Cr}(p_1 \mid \neg p_2) = 0.04, \\ \mathrm{Cr}(p_2 \mid p_3) = 0.99, \mathrm{Cr}(p_2 \mid \neg p_3) = 0.04, ..., \\ \mathrm{Cr}(p_{n-1} \mid p_n) = 0.99, \mathrm{Cr}(p_{n-1} \mid \neg p_n) = 0.04, ... \end{bmatrix}$$

$$\Uparrow$$

$$\text{N3: } \left[\mathrm{Cr}(e)\right]$$

$$\Uparrow$$

$$\text{N4: } \left[\mathrm{Cr}(e^*)\right]$$

$$\Uparrow$$

$$\text{N5: } \left[\mathrm{Cr}(o_1), \mathrm{Cr}(o_2), ..., \mathrm{Cr}(o_n)\right]$$

Here the arrows indicate relations of rational determination as opposed to relations of probabilistic support.

It is important to note that the number of nodes in $RRD_{F\&NC}$ is finite and that each credence in N5—the root node—is non-inferentially justified. There is thus a clear sense in which $RRD_{F\&NC}$ is foundationalist.

FJ can be modified so as to allow for this. Consider:

Foundationalism on Justification (FJ*)*: Cr(p) is justified for S if and only if (i) Cr(p) is non-inferentially justified for S or (ii) Cr(p) is the target credence in a regress of rational determination *RRD* such that (a) *RRD* has a finite number of nodes, (b) none of *RRD*'s nodes is an ancestor of itself, and (c) each member of *RRD*'s root node is non-inferentially justified for *S*.

The key here is that nothing in FJ* allows for a node with an infinite number of credences. So, though N2 in $RRD_{F\&NC}$ has an infinite number of credences, this is okay because the regress itself has just a finite number of nodes.

Can it be said that there is a place in FJ* for infinite regresses of probabilistic support? Yes. Take N2 in $RRD_{F\&NC}$. It follows from the credences therein that p_1 is probabilistically supported by p_2, p_2 is probabilistically supported by p_3, and so on ad infinitum. This is an infinite regress of probabilistic support.

It might be wondered whether the credences in N5 are doing any work in terms of making it the case that $\mathrm{Cr}(p_1) = 0.8$ is justified for *S*. The credences in N2 fully determine $\mathrm{Cr}(p_1) = 0.8$ in that any credence distribution, understood as a probability distribution, on which $\mathrm{Cr}(p_1 \mid p_2) = 0.99$, $\mathrm{Cr}(p_1 \mid \neg p_2) = 0.04$, $\mathrm{Cr}(p_2 \mid p_3) = 0.99$, $\mathrm{Cr}(p_2 \mid \neg p_3) = 0.04$, …, $\mathrm{Cr}(p_{n-1} \mid p_n) = 0.99$, $\mathrm{Cr}(p_{n-1} \mid \neg p_n) = 0.04$, … is a credence distribution on which $\mathrm{Cr}(p_1) = 0.8$. What work, then, is left for the credences in N5?

The answer is simple: the credences in N5 ultimately serve to make the credences in N2 justified for *S*. This is essential work if FJ* is assumed. If the credences in N5 were not non-inferentially justified for *S*, and if the case were otherwise the same, then the credences in N2 would not be justified for *S* and so, despite the fact the credences in N2 fully determine $\mathrm{Cr}(p_1) = 0.8$, $\mathrm{Cr}(p_1) = 0.8$ would not be justified for *S*.[22,23]

6 Three variants of $RRD_{F\&NC}$

$\mathrm{Cr}(p_1)$ in $RRD_{F\&NC}$ is rationally determined by a node with an infinite number of credences $\mathrm{Cr}(p_1 \mid p_2)$, $\mathrm{Cr}(p_1 \mid \neg p_2)$, $\mathrm{Cr}(p_2 \mid p_3)$, $\mathrm{Cr}(p_2 \mid \neg p_3)$, …, $\mathrm{Cr}(p_{n-1} \mid p_n)$, $\mathrm{Cr}(p_{n-1} \mid \neg p_n)$,

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[22] There would be a similar point if N2 were broken up into an infinite number of nodes: N2.1: [$\mathrm{Cr}(p_1 \mid p_2) = 0.99$], N2.2: [$\mathrm{Cr}(p_1 \mid \neg p_2) = 0.04$], N2.3: [$\mathrm{Cr}(p_2 \mid p_3) = 0.99$], N2.4: [$\mathrm{Cr}(p_2 \mid \neg p_3) = 0.04$], …, N2.2$n$-1: [$\mathrm{Cr}(p_{n-1} \mid p_n) = 0.99$], N2.2$n$-2: [$\mathrm{Cr}(p_{n-1} \mid \neg p_n) = 0.04$], …. FJ* would need to be modified accordingly. Then the point would be that if FJ* so modified were assumed, if the credences in N5 were not non-inferentially justified for *S*, and if the case were otherwise the same, then the credences in N2.1, N2.2, … would not be justified for *S* and so, despite the fact those credences together fully determine $\mathrm{Cr}(p_1) = 0.8$, $\mathrm{Cr}(p_1) = 0.8$ would not be justified for *S*.

[23] There is a place in *foundationalism* for infinite regresses of probabilistic support. The same is true with respect to *coherentism* and *infinitism*.

… such that, given those credences, p_1 is probabilistically supported by p_2, p_2 is probabilistically supported by p_3, and so on ad infinitum. Is it essential that $\mathrm{Cr}(p_1)$ is rationally determined by a node with an *infinite* number of credences? In other words, can $\mathrm{Cr}(p_1)$ be high even if it is rationally determined by a node with a *finite* number of credences? Is it essential that the credences in question are such that p_1 is *probabilistically supported* by p_2, p_2 is *probabilistically supported* by p_3, and so on ad infinitum? In other words, can $\mathrm{Cr}(p_1)$ be high even if there is an i such that p_i is *not* probabilistically supported by p_{i+1}?

First, consider:

RRD$_{F\&NC}$*:

$$\mathrm{N1}:\ \left[\mathrm{Cr}(p_1)\approx 0.796\right]$$
$$\Uparrow$$
$$\mathrm{N2}:\ \begin{bmatrix}\mathrm{Cr}(p_1\mid p_2)=0.99,\mathrm{Cr}(p_1\mid\neg p_2)=0.04,\\ \mathrm{Cr}(p_2\mid p_3)=0.99,\mathrm{Cr}(p_2\mid\neg p_3)=0.04,...,\\ \mathrm{Cr}(p_{99}\mid p_{100})=0.99,\mathrm{Cr}(p_{99}\mid\neg p_{100})=0.04,\\ \mathrm{Cr}(p_{100})=0.1\end{bmatrix}$$
$$\Uparrow$$
$$\mathrm{N3}:\ \left[\mathrm{Cr}(e)\right]$$
$$\Uparrow$$
$$\mathrm{N4}:\ \left[\mathrm{Cr}(e^*)\right]$$
$$\Uparrow$$
$$\mathrm{N5}:\ \left[\mathrm{Cr}(o_1),\mathrm{Cr}(o_2),...,\mathrm{Cr}(o_n)\right]$$

This regress is like *RRD$_{F\&NC}$* in that $\mathrm{Cr}(p_1)$ is high (greater than $\mathbf{t}$ = 0.5) and in that, given the credences in N2, p_1 is probabilistically supported by p_2, p_2 is probabilistically supported by p_3, …, and p_{99} is probabilistically supported by p_{100}. But note that, unlike *RRD$_{F\&NC}$*, *RRD*$_{F\&NC}$* is such that the number of credences in N2 is finite. Note too that $\mathrm{Cr}(p_{100})$ is low (at 0.1).

Second, consider:

$RRD^{**}_{F\&NC}$:

$$\text{N1}: \left[\text{Cr}(p_1) \approx 0.508\right]$$
$$\Uparrow$$
$$\text{N2}: \begin{bmatrix} \text{Cr}(p_1 \mid p_2) = 0.04, \text{Cr}(p_1 \mid \neg p_2) = 0.99, \\ \text{Cr}(p_2 \mid p_3) = 0.04, \text{Cr}(p_2 \mid \neg p_3) = 0.99, \ldots, \\ \text{Cr}(p_{n-1} \mid p_n) = 0.04, \text{Cr}(p_{n-1} \mid \neg p_n) = 0.99, \ldots \end{bmatrix}$$
$$\Uparrow$$
$$\text{N3}: \left[\text{Cr}(e)\right]$$
$$\Uparrow$$
$$\text{N4}: \left[\text{Cr}(e^*)\right]$$
$$\Uparrow$$
$$\text{N5}: \left[\text{Cr}(o_1), \text{Cr}(o_2), \ldots, \text{Cr}(o_n)\right]$$

This regress is like $RRD_{F\&NC}$ in that Cr(p_1) is high and in that the number of credences in N2 is infinite. But with $RRD^{**}_{F\&NC}$, unlike with $RRD_{F\&NC}$, the credences in N2 are *not* such that p_1 is probabilistically supported by p_2, p_2 is probabilistically supported by p_3, and so on ad infinitum. In fact, they are such that p_1 is probabilistically *anti*-supported (or incrementally disconfirmed) by p_2, p_2 is probabilistically *anti*-supported (or incrementally disconfirmed) by p_3, and so on ad infinitum.[24]

Third, and finally, consider:

[24] If, instead, Cr(p_1 | p_2) = 0.98, Cr(p_1 | $\neg p_2$) = 0.99, Cr(p_2 | p_3) = 0.98, Cr(p_2 | $\neg p_3$) = 0.99, …, Cr(p_{n-1} | p_n) = 0.98, Cr(p_{n-1} | $\neg p_n$) = 0.99, …, then Cr(p_1) ≈ 0.980.

$RRD^{***}_{F\&NC}$:

$$\text{N1}: \left[\mathrm{Cr}(p_1) \approx 0.510\right]$$

$$\Uparrow$$

$$\text{N2}: \begin{bmatrix} \mathrm{Cr}(p_1 \mid p_2) = 0.04, \mathrm{Cr}(p_1 \mid \neg p_2) = 0.99, \\ \mathrm{Cr}(p_2 \mid p_3) = 0.04, \mathrm{Cr}(p_2 \mid \neg p_3) = 0.99, ..., \\ \mathrm{Cr}(p_{99} \mid p_{100}) = 0.04, \mathrm{Cr}(p_{99} \mid \neg p_{100}) = 0.99, \\ \mathrm{Cr}(p_{100}) = 0.1 \end{bmatrix}$$

$$\Uparrow$$

$$\text{N3}: \left[\mathrm{Cr}(e)\right]$$

$$\Uparrow$$

$$\text{N4}: \left[\mathrm{Cr}(e^*)\right]$$

$$\Uparrow$$

$$\text{N5}: \left[\mathrm{Cr}(o_1), \mathrm{Cr}(o_2), ..., \mathrm{Cr}(o_n)\right]$$

This regress is like *RRD*$_{F\&NC}$* in some respects and like *RRD**$_{F\&NC}$* in others. It is like *RRD*$_{F\&NC}$* in that Cr(p_1) is rationally determined by a node with a *finite* number of credences. It is like *RRD**$_{F\&NC}$* in that those credences are such that p_1 is probabilistically *anti*-supported by p_2, p_2 is probabilistically *anti*-supported by p_3, …, and p_{99} is probabilistically *anti*-supported by p_{100}. It is further like *RRD**$_{F\&NC}$* in that Cr(p_{100}) is low (at 0.1). But it is like both *RRD*$_{F\&NC}$* and *RRD**$_{F\&NC}$* in that Cr(p_1) is high.[25]

So each of the questions raised at the beginning of this section is to be answered in the affirmative. Cr(p_1) can be high even if it is rationally determined by a node with a finite number of credences. And Cr(p_1) can be high even if there is an i such that p_i is not probabilistically supported by p_{i+1}. In fact, Cr(p_1) can be high even if it is rationally determined by a node with a finite number of credences and there is an i such that p_i is not probabilistically supported by p_{i+1}.

It follows from *RRD$_{F\&NC}$* that there is a place in FJ* for infinite regresses of probabilistic support. It follows from *RRD*$_{F\&NC}$*, *RRD**$_{F\&NC}$*, and *RRD***$_{F\&NC}$*, in turn, that there is a place in FJ* for finite regresses of probabilistic support (of a non-standard sort), infinite regresses of probabilistic anti-support, and finite regresses of probabilistic anti-support.

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[25] If, instead, Cr($p_1 \mid p_2$) = 0.98, Cr($p_1 \mid \neg p_2$) = 0.99, Cr($p_2 \mid p_3$) = 0.98, Cr($p_2 \mid \neg p_3$) = 0.99, …, Cr($p_{99} \mid p_{100}$) = 0.98, Cr($p_{99} \mid \neg p_{100}$) = 0.99, and Cr(p_{100}) = 0.1, then Cr(p_1) ≈ 0.980.

It might be worried that $RRD_{F\&NC}$ and $RRD^{**}_{F\&NC}$ are unrealistic given that in each of them the number of credences in N2 is infinite.[26] This worry, though, does not carry over to $RRD^{*}_{F\&NC}$ or $RRD^{***}_{F\&NC}$.

7 Conclusion

A&P show that there can be high probability by an infinite regress of probabilistic support in that:

> *A&P's Result (APR)*: There are probability distributions defined over an infinite set of propositions $\{p_1, p_2, p_3, \ldots, p_n, \ldots\}$ such that (i) p_i is probabilistically supported by p_{i+1} for all i and (ii) p_1 has a high probability.

It might seem that this result—this ingenious result—runs counter to foundationalism. I have argued, though, that in fact it does not and that, indeed, it opens up a new and surprising route to foundationalist justification by way of an infinite regress of probabilistic support. I have also argued that there are similar routes to foundationalist justification by way of, respectively, a finite regress of probabilistic support (of a non-standard sort), an infinite regress of probabilistic anti-support, and a finite regress of probabilistic anti-support.

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[26] See Herzberg (2013), Podlaskowski and Smith (2011, 2014), Smith and Podlaskowski (2013), and Turri (2013) for relevant discussion.

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