Is there a place in Bayesian confirmation theory for the Reverse Matthew Effect?

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ABSTRACT: Bayesian confirmation theory is rife with confirmation measures. Many of them differ from each other in important respects. It turns out, though, that all the standard confirmation measures in the literature run counter to the so-called "Reverse Matthew Effect" ("RME" for short). Suppose, to illustrate, that H_1 and H_2 are equally successful in predicting E in that $p(E \mid H_1)/p(E) = p(E \mid H_2)/p(E) > 1$. Suppose, further, that initially H_1 is less probable than H_2 in that $p(H_1) < p(H_2)$. Then by RME it follows that the degree to which E confirms H_1 is greater than the degree to which it confirms H_2. But by all the standard confirmation measures in the literature, in contrast, it follows that the degree to which E confirms H_1 is less than or equal to the degree to which it confirms H_2. It might seem, then, that RME should be rejected as implausible. Festa (2012), however, argues that there are scientific contexts in which RME holds. If Festa's argument is sound, it follows that there are scientific contexts in which none of the standard confirmation measures in the literature is adequate. Festa's argument is thus interesting, important, and deserving of careful examination. I consider five distinct respects in which E can be related to H, use them to construct five distinct ways of understanding confirmation measures, which I call "Increase in Probability", "Partial Dependence", "Partial Entailment", "Partial Discrimination", and "Popper Corroboration", and argue that each such way runs counter to RME. The result is that it is not at all clear that there is a place in Bayesian confirmation theory for RME.

KEYWORDS: Bayesian confirmation theory; confirmation; Festa; increase in probability; partial dependence; partial discrimination; partial entailment; Popper corroboration; Reverse Matthew Effect

1   Introduction

Bayesian confirmation theory is rife with confirmation measures (a small sampling of which is given below in Section 2).[1] Many of them differ from each other in important respects. It turns out, though, that all the standard confirmation measures in the literature run counter to the so-called "Reverse Matthew Effect" ("RME" for short). Suppose, to illustrate, that H_1 and H_2 are equally successful in predicting E in that:

$$p(E \mid H_1)/p(E) = p(E \mid H_2)/p(E) > 1$$

(I am assuming here and throughout the paper that all lettered propositions have non-extreme unconditional probabilities, i.e., unconditional probabilities greater than 0 and less than 1.) Suppose, further, that initially H_1 is less probable than H_2 in that:

$$p(H_1) < p(H_2)$$

Then by RME it follows that the degree to which E confirms H_1 is greater than the degree to which it confirms H_2. But by all the standard confirmation measures in the literature, in contrast, it follows that the degree to which E confirms H_1 is less than or equal to the degree to which it confirms H_2. Is this a problem for RME?

Festa (2012) answers in the negative. He argues that there are scientific contexts in which RME holds. First, he describes a case from the history of science:

> In 1695 the English astronomer Edmond Halley (1656-1742) applied Newton's theory N to the observed characteristics of a comet observed in 1682 and deduced from N that the comet would return every 76 years. In particular, he deduced the prediction E that the comet would return in December 1758. Although Halley himself did not live to see the comet's return, having died in 1742, his prediction proved to be correct, since the comet—afterwards baptized as Halley's comet—appeared again on 25 December 1758. (Festa 2012, p. 99)

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[1] Here and throughout the sense of confirmation at issue is such that E confirms H if and only if $p(H \mid E) > p(H)$, or equivalently $p(H \mid E) > p(H \mid \neg E)$, or equivalently $p(E \mid H) > p(E)$, or equivalently $p(E \mid H) > p(E \mid \neg H)$. This sense of confirmation stands in contrast to so-called "absolute confirmation". The latter, unlike the former, is a matter of high probability. See Douven (2011), Roche (2012, 2015), and Roche and Shogenji (2014) for further discussion of different senses of confirmation (or evidential support).

Second, he notes that E was commonly perceived to be a great success of N (where success here is a matter of degree of confirmation):

> The return of the comet, i.e., the verification of prediction E, was commonly perceived as a *great success* for the Newton's theory. In our terminology, E is a deductive success of N, while the widespread perception that E was a great success of N can be seen as the attribution to N of a very high degree of confirmation $c(N, E)$. (Festa 2012, p. 99, emphasis original)

Third, he asks readers to imagine some fictional additions to the case. He writes:

> Now let us imagine that a scientist X claimed that N was a too bold hypothesis—since it applies to any body in the universe—and suggested the more modest "restricted" theory R according to which Newton's laws apply to the objects near to our solar system, i.e., to the sun, the planets and their satellites, and the comets orbiting around the sun. (Festa 2012, p. 99)

Fourth, and finally, he notes in effect that his intuition runs counter to X's (intuition that E was a greater success for R than for N):

> We feel that X's intuition is wrong while the widespread perception that the return of the comet was a great success for Newton's theory—but not for the plenty of all the possible weakened versions of such theory—was correct. (Festa 2012, p. 100)

The conclusion is that, as RME implies, the degree to which E confirms N is greater than the degrees to which it confirms weaker theories such as R.

It should be noted that Festa's argument is not meant to be decisive (see Festa 2012, p. 100). It is best seen as a step in the direction of a more decisive discussion of RME and related conditions.

By "related conditions" here I have in mind the so-called "Matthew Effect" ("ME" for short) and "Matthew Independence" ("MI" for short). Return to the illustration above where:

$$p(E \mid H_1)/p(E) = p(E \mid H_2)/p(E) > 1$$

$$p(H_1) < p(H_2)$$

By ME it follows that the degree to which E confirms H_1 is *less than* the degree to which it confirms H_2. By MI, in turn, it follows that the degree to which E confirms H_1 is *equal to* the degree to which it confirms H_2. So, in cases where two hypotheses are equally successful in predicting a given piece of evidence, RME favors the hypothesis with the lower initial probability, whereas ME favors the hypothesis with the higher initial probability and MI favors neither hypothesis.

Where do the names "Matthew Effect", "Matthew Independence", and "Reverse Matthew Effect" come from? Festa (referencing Kuipers 2000) writes:

> Kuipers … introduces the concept of *Matthew effect for confirmation* just w.r.t. [ME restricted to cases where the hypothesis logically implies the piece of evidence]. In fact, [ME thus restricted] "may be seen as a methodological version of the so-called Matthew effect, according to which the rich profit more than the poor" …, in agreement with the sentence—made famous by the Gospel according to St. Matthew—that "unto every one that hath shall be given". (Festa 2012, p. 95, emphasis original)

So, in cases of equal predictive success, ME favors the rich (the hypothesis with the higher initial probability) over the poor (the hypothesis with the lower initial probability), MI favors neither the rich (the hypothesis with the higher initial probability) nor the poor (the hypothesis with the lower initial probability), and RME favors the poor (the hypothesis with the lower initial probability) over the rich (the hypothesis with the higher initial probability). Thus the names "Matthew Effect", "Matthew Independence", and "Reverse Matthew Effect".

If Festa's argument is sound, it follows that there are scientific contexts in which none of the standard confirmation measures in the literature is adequate. Festa's argument is thus interesting, important, and deserving of careful examination.

I want to assume a pluralistic approach to confirmation on which there are different respects in which E can be related (evidentially) to H and on which different confirmation measures are appropriate for different such respects.[2] This allows that there are respects in which E can be related to H such that certain of the standard confirmation measures in the literature are appropriate while, at the same time, there are additional respects in which E can be related to H such that certain measures meeting RME are appropriate. I aim to show that, even on this pluralistic approach, it is not at all clear that there is a place in Bayesian confirmation theory for RME.

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[2] This approach is on display in Hajek and Joyce (2008) and Joyce (1999, Ch. 6, sec. 6.4, 2008). A very different approach—a monistic approach—is on display in Milne (1996).

The remainder of the paper is organized as follows. In Section 2, I set out ME, MI, and RME more precisely. I also set out a small sampling of confirmation measures—some standard and some non-standard—and relate them to ME, MI, and RME. In the next five sections, I consider five distinct respects in which E can be related to H, use them to construct five distinct ways of understanding confirmation measures, which I call "Increase in Probability", "Partial Dependence", "Partial Entailment", "Partial Discrimination", and "Popper Corroboration", and argue that each such way runs counter to RME. I address Increase in Probability in Section 3, Partial Dependence in Section 4, Partial Entailment in Section 5, Partial Discrimination in Section 6, and Popper Corroboration in Section 7. In Section 8, I conclude.

2   ME, MI, and RME

ME, MI, and RME can be put as follows:

> Matthew Effect (ME): If (i) $p(E_1 \mid H_1)/p(E_1) = p(E_2 \mid H_2)/p(E_2) > 1$ and (ii) $p(H_1) < p(H_2)$, then $c(H_1, E_1) < c(H_2, E_2)$.

> Matthew Independence (MI): If (i) $p(E_1 \mid H_1)/p(E_1) = p(E_2 \mid H_2)/p(E_2) > 1$ and (ii) $p(H_1) < p(H_2)$, then $c(H_1, E_1) = c(H_2, E_2)$.

> Reverse Matthew Effect (RME): If (i) $p(E_1 \mid H_1)/p(E_1) = p(E_2 \mid H_2)/p(E_2) > 1$ and (ii) $p(H_1) < p(H_2)$, then $c(H_1, E_1) > c(H_2, E_2)$.

These conditions are identical in their antecedents and differ just in their consequents. ME's consequent should be read as "the degree to which E_1 confirms H_1 is less than the degree to which E_2 confirms H_2". MI's and RME's consequents should be read similarly.[3]

I noted above in Section 1 that Bayesian confirmation theory is rife with confirmation measures. Here is a small sampling:

$$c_1(H,E) = p(H \mid E) - p(H)$$

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[3] ME, MI, and RME can be reformulated in terms of a single hypothesis and a single piece of evidence (see Festa 2012 and Roche 2014). ME, for example, can be reformulated like this: If (i) $p(E \mid H)/p(E) > 1$ and (ii) $p(E \mid H)/p(E)$ is held fixed, then $c(H, E)$ is an increasing function of $p(H)$.

$$c_2(H,E) = \frac{p(H \mid E)}{p(H)}$$

$$c_3(H,E) = p(H \mid E) - p(H \mid \neg E)$$

$$c_4(H,E) = \frac{p(H \mid E)}{p(H \mid \neg E)}$$

$$c_5(H,E) = p(E \mid H) - p(E)$$

$$c_6(H,E) = \frac{p(E \mid H)}{p(E)}$$

$$c_7(H,E) = p(E \mid H) - p(E \mid \neg H)$$

$$c_8(H,E) = \frac{p(E \mid H)}{p(E \mid \neg H)}$$

c_2 and c_6 are equivalent to each other in that $c_2(H, E) = c_6(H, E)$ for all E and H. But no two of c_1, c_2/c_6, c_3, c_4, c_5, c_7, and c_8 are ordinally equivalent to each other.[4,5] It turns out, though, that none of c_1-c_8 meets RME.[6]

Are there confirmation measures meeting RME? Consider:

$$c_9(H,E) = \frac{p(H \mid E) - p(H)}{p(H \mid E) + p(H) + (2)p(H \mid E)p(H)}$$

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[4] Let c and $c*$ be confirmation measures. Then c and $c*$ are ordinally equivalent to each other if and only if the following holds for any ordered pairs of propositions $<H_1, E_1>$ and $<H_2, E_2>$: $c(H_1, E_1) > / = / < c(H_2, E_2)$ if and only if $c*(H_1, E_1) > / = / < c*(H_2, E_2)$.

[5] This is prima facie problematic. Many, if not all of, c_1, c_2/c_6, c_3, c_4, c_5, c_7, and c_8 have some intuitive plausibility. But certain results in Bayesian confirmation theory involving some such measures fail to carry over to at least some of the others. This is "the problem of measure sensitivity". See Brössel (2013) and Fitelson (1999) for helpful discussion.

[6] Each of c_1 and c_8 meets ME whereas each of c_2 and c_6 meets MI. This is noted in Festa (2012) and Roche (2014). None of c_3, c_4, c_5, and c_7 meets ME, MI, or RME. This can be verified on Mathematica using PrSAT (developed by Branden Fitelson in collaboration with Jason Alexander and Ben Blum). See Fitelson (2008) for discussion of PrSAT.

$$c_{10}(H,E) = \frac{p(H \mid E) + (2)p(H)p(H \mid E)}{p(H) + (2)p(H)p(H \mid E)}$$

$$c_{11}(H,E) = \frac{p(H \mid E) - p(H)}{p(H \mid E)p(H)}$$

These measures are like c_1-c_8 in that there is a neutral value n such that $c(H, E) > / = / < n$ if and only if $p(H \mid E) > / = / < p(H)$. The neutral value for c_9 and c_{11} is 0. The neutral value for c_{10} is 1. But c_9-c_{11}, unlike c_1-c_8, all meet RME (see Festa 2012 and Roche 2014).

I turn now to Increase in Probability (the first of five distinct ways of understanding confirmation measures to be discussed in the next five sections) and the issue of whether it runs counter to RME.

3   Increase in Probability

A first respect in which E can be related to H is this:

E increases H's probability.

This leads to a first way of understanding $c(H, E)$:

Increase in Probability (IP): $c(H, E)$ measures the degree to which E increases H's probability.

How is IP to be understood? What conditions, that is, should be met by any adequate measure of increase in probability?

I take it that, at a minimum, any adequate measure of increase in probability should meet each of the following conditions:

IP1 $c(H, E)$ is a function of $p(H \mid E)$ and $p(H)$.

IP2   If (i) $p(H_1 \mid E_1) > p(H_2 \mid E_2)$ and (ii) $p(H_1) \leq p(H_2)$, then $c(H_1, E_1) > c(H_2, E_2)$.

IP3   If (i) $p(H_1 \mid E_1) \geq p(H_2 \mid E_2)$ and (ii) $p(H_1) < p(H_2)$, then $c(H_1, E_1) > c(H_2, E_2)$.

Henceforth when I speak of "IP" I have in mind IP understood at least in part in terms of IP1-IP3.

It might seem that IP3 is too strong. Suppose that:

$$p(H_1 \mid E_1) = 1 = p(H_2 \mid E_2)$$

$$p(H_1) = 0.01 < 0.99 = p(H_2)$$

It follows by IP3 that $c(H_1, E_1) > c(H_2, E_2)$. It might seem that this is the wrong result, for it might seem that since $p(H_1 \mid E_1) = 1 = p(H_2 \mid E_2)$, each of $c(H_1, E_1)$ and $c(H_2, E_2)$ should be maximal in which case $c(H_1, E_1) = c(H_2, E_2)$. I would agree if the context were partial entailment (as explained below in Section 5). But here the context is increase in probability and in this context, it seems, prior probabilities always matter.

The expressions "at a minimum" and "at least in part" in the paragraph immediately before the prior paragraph are crucial. I am not claiming that IP1, IP2, and IP3 together provide an adequate formalization of IP. I am claiming, rather, that IP1, IP2, and IP3 should be included in any adequate formalization of IP. I thus leave it open that there are conditions in addition to IP1, IP2, and IP3 such that they too should be included in any adequate formalization of IP.

Is it the case, though, that IP1, IP2, and IP3 run counter to RME? Consider c_{11}. This measure meets RME (as noted above in Section 2). It also meets each of IP1, IP2, and IP3. This can be seen by reformulating it as follows:

$$c_{11}(H,E) = \frac{1}{p(H)} - \frac{1}{p(H \mid E)}$$

Hence, as at least some measures meeting RME also meet each of IP1, IP2, and IP3, it is not the case that IP1, IP2, and IP3 run counter to RME.[7]

This is good news for RME in the context of IP. I have some concerns however. It is far from clear to me, and in fact seems implausible to me, that RME should be accepted in the context of IP.

My first concern has to do with the following probability distribution:

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[7] This is also true with respect to (a) RME and conditions (P0), (P1), and (P3) in Crupi (2016, sec. 3), (b) RME and conditions (IFPD), (FPI), (IPI), and (E) in Festa (2012, sec. 2), and (c) RME and conditions P1, P2, and P3 in Festa and Cevolani (forthcoming, sec. 2).

Distribution D

E	H_1	H_2	p
T	T	T	$\frac{1}{10000000000}$
T	T	F	$\frac{99999}{10000000000}$
T	F	T	0
T	F	F	0
F	T	T	0
F	T	F	0
F	F	T	0
F	F	F	$\frac{99999}{100000}$

It can be readily verified that on this distribution:

$$p(H_1 \mid E) = 1 > 0.00001 = p(H_1)$$

$$p(H_2 \mid E) = 0.00001 > 0.0000000001 = p(H_2)$$

$$p(E \mid H_1)/p(E) = 100000 = p(E \mid H_2)/p(E)$$

It follows by RME that $c(H_1, E) < c(H_2, E)$.[8] This strikes me as the wrong result (or at least not obviously the right result). H_1's increase in probability is from a value very close to the minimum value of 0 to the maximum value of 1. H_2's increase in probability, in contrast, is from a value very close to the minimum value of 0 to another value very close to the minimum value of 0. Hence H_1's increase in probability, it seems, is closer than

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[8] It is worth noting in this regard that:

$$c_9(H_1, E) \approx 0.99996 < 0.99998 \approx c_9(H_2, E)$$

$$c_{10}(H_1, E) = 33334 < 99998 \approx c_{10}(H_2, E)$$

$$c_{11}(H_1, E) = 99999 < 9999900000 = c_{11}(H_2, E)$$

So by c_9 the degree to which E confirms H_1 is *slightly less* than the degree to which E confirms H_2 whereas both by c_{10} and by c_{11} the degree to which E confirms H_1 is *much less* than the degree to which E confirms H_2.

H_2's increase in probability to an increase in probability from the minimum value of 0 to the maximum value of 1 (which if possible would be an increase in probability of the highest degree). Hence H_1's increase in probability, it seems, is greater than H_2's increase in probability.[9]

I do not mean for this argument to be decisive. I am simply registering a concern. I turn now to a second concern.

Suppose that the background information codified in p includes the information that Tweety is a raven and the information that Smith is highly but not perfectly reliable at color detection in the relevant conditions. Let E be the proposition that Tweety is black, H_1 be the proposition that all ravens are black, and H_2 be the proposition that Smith testified that Tweety is dark brown. Then, given the background information codified in p, it follows that H_1 entails E and thus so does H_1 & H_2. It further follows (given certain rather natural ways of filling in the details) that:

$$p(E \mid H_1)/p(E) = p(E \mid H_1 \& H_2)/p(E) > 1$$

$$p(H_1) > p(H_1 \& H_2)$$

By RME the result is that $c(H_1, E) < c(H_1 \& H_2, E)$. This, it seems, is not the right result. Yes, E increases H_1 & H_2's probability. But since, in part, E decreases H_2's probability, the degree to which E increases H_1 & H_2's probability is less than the degree to which it increases H_1's probability.[10]

This point can be generalized as follows:

IP4   If (i) H_1 entails E and (ii) $p(H_2 \mid E) < p(H_2)$, then $c(H_1, E) > c(H_1 \& H_2, E)$.

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[9] It is not essential that $p(H_1 \mid E)$ equals unity on Distribution D. Any value very close to unity would suffice.

[10] Festa (2012, sec. 3.3) considers the so-called "Problem of Irrelevant Conjunction" in the context of evaluating ME, MI, and RME. His main point can be put as follows: If H_1 is a "genuine" hypothesis and H_2 is an "irrelevant" or "nonsensical" hypothesis such as the hypothesis that the moon is made of green cheese, then H_1 & H_2 is not a genuine hypothesis and thus should be set aside when evaluating ME, MI, and RME. I take it that this point has no application in the case above (where H_1 is the proposition that all ravens are black and H_2 is the proposition that Smith testified that Tweety is dark brown). For, in that case, H_2 is neither irrelevant nor nonsensical.

If IP is understood at least in part in terms of IP4, then IP runs counter to RME.[11]

I find IP4 to be highly plausible in the context of increase in probability. If you do as well (or would if ">" in IP4's consequent were changed to "≥"), then you should steer clear of RME in the context of increase in probability.

Recall that Festa argues that there are scientific contexts in which RME holds. He describes a case from the history of science and then, commenting on that case, writes:

> We feel that X's intuition is wrong while the widespread perception that the return of the comet was a great success for Newton's theory—but not for the plenty of all the possible weakened versions of such theory—was correct. (Festa 2012, p. 100)

Nothing in what I have argued implies that Festa is mistaken here. It could be that RME should be rejected in the context of IP but not in the context of some alternative way of understanding confirmation measures.

4   Partial Dependence

A second respect in which E can be related to H is this:

H's probability partially depends on whether E is true.

This leads to a second way of understanding $c(H, E)$:

Partial Dependence (PDe): $c(H, E)$ measures the degree to which H's probability partially depends on whether E is true.

How is PDe to be understood? What conditions, that is, should be met by any adequate measure of partial dependence?

I am taking the idea of partial dependence from Joyce (1999, Ch. 6, sec. 6.4, 2008, sec. 3) and Hajek and Joyce (2008, p. 122).[12] They distinguish (in effect) between the idea of increase in probability and the idea of partial dependence. They take increase in probability to be a matter of the degree to which $p(H \mid E)$ is greater than $p(H)$. They take

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[11] The same is true with respect to MI.

[12] The expression "partial dependence", though, is mine. Joyce (2008, sec. 3) speaks in terms of "effective evidence". Hajek and Joyce (2008, p. 122) speak in terms of "probative evidence".

partial dependence, in turn, to be a matter of the degree to which $p(H \mid E)$ is greater than $p(H \mid \neg E)$. Suppose that:

$$p(H \mid E) = 0.99 > 0.01 = p(H \mid \neg E)$$

Here it is rather significant for H's probability whether E is true and so the degree of partial dependence is high. Suppose instead that:

$$p(H \mid E) = 0.99 > 0.98 = p(H \mid \neg E)$$

Here it is rather insignificant for H's probability whether E is true and so the degree of partial dependence is low.

I take it that, at a minimum, any adequate measure of partial dependence should meet each of the following conditions:

PDe1 $c(H, E)$ is a function of $p(H \mid E)$ and $p(H \mid \neg E)$.

PDe2   If (i) $p(H_1 \mid E_1) > p(H_2 \mid E_2)$ and (ii) $p(H_1 \mid \neg E_1) \leq p(H_2 \mid \neg E_2)$, then $c(H_1, E_1) > c(H_2, E_2)$.

PDe3   If (i) $p(H_1 \mid E_1) \geq p(H_2 \mid E_2)$ and (iv) $p(H_1 \mid \neg E_1) < p(H_2 \mid \neg E_2)$, then $c(H_1, E_1) > c(H_2, E_2)$.

Henceforth when I speak of "PDe" I have in mind PDe understood at least in part in terms of PDe1-PDe3. (That PDe is distinct from IP is shown in Appendix A.)

Does PDe run counter to RME? It follows on Distribution D (from the prior section) that:

$$p(H_1 \mid E) = 1 > 0.00001 = p(H_2 \mid E)$$

$$p(H_1 \mid \neg E) = 0 = p(H_2 \mid \neg E)$$

By PDe2 it follows that $c(H_1, E) > c(H_2, E)$. This runs counter to RME, since the latter entails that $c(H_1, E) < c(H_2, E)$.[13]

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[13] MI entails that $c(H_1, E) = c(H_2, E)$. So PDe also runs counter to MI.

5 Partial Entailment

A third respect in which E can be related to H is this:

  E partially entails H.

This leads to a third way of understanding $c(H, E)$:

  Partial Entailment (PE): $c(H, E)$ measures the degree to which E partially entails H.

How is PE to be understood? What conditions, that is, should be met by any adequate measure of partial entailment?
  It will help here to appeal to a recent discussion of the following confirmation measure:

$$c_{12}(H,E)=\begin{cases} \dfrac{p(H\mid E)-p(H)}{1-p(H)} & \text{if } p(H\mid E)\geq p(H) \\[2ex] \dfrac{p(H\mid E)-p(H)}{p(H)} & \text{if } p(H\mid E)< p(H) \end{cases}$$

Crupi and Tentori (2013, 2014) put forward c_{12} as a measure of confirmation in the sense of partial entailment. Consider the following passage (where notation has been modified):

  To appreciate this conceptual unity, note that in case of positive inductive support or confirmation $c_{12}(H, E)$ expresses the relative reduction of the initial distance from certainty of H being true as yielded by E, i.e., it measures how far upward the posterior $p(H \mid E)$ has gone in covering the distance between the prior $p(H)$ and 1. Similarly, in the case of negative inductive support or disconfirmation, $c_{12}(H, E)$ reflects the relative reduction of the initial distance from certainty of H being false as yielded by E, i.e., it measures how far downward the posterior $p(H \mid E)$ has gone in covering the distance between the prior $p(H)$ and 0. Accordingly, $c_{12}(H, E)$ measures the extent to which the initial probability distance from certainty concerning the truth (falsehood) of H is reduced by the confirming (disconfirming) statement E. Or, put otherwise, how much of such distance is "covered" by the upward (downward) jump from $p(H)$ to $p(H \mid E)$. Thus, $c_{12}(H, E)$ is a measure of the relative reduction of the distance from certainty that a conclusion/hypothesis of interest is true or false—or, with a slight abuse of language, a *relative distance* measure. (Crupi and Tentori 2013, p. 366, emphasis original)

There is no explicit mention in this passage of partial entailment. But, focusing on the case where E confirms H, the idea of "the relative reduction of the initial distance from certainty of H being true as yielded by E" is meant to express the idea of partial entailment.[14] The degree to which E partially entails H is simply the degree to which E reduces the distance between H's probability and 1 relative to the case where E entails H and thus reduces the distance between H's probability and 1 to nil.

It will help to consider an example. Suppose that:

$$p(H_1 \mid E_1) = 1 > 0.1 = p(H_1)$$

$$p(H_2 \mid E_2) = 1 > 0.9 = p(H_2)$$

The *absolute* reduction in distance between H_1's probability and 1 is greater than the *absolute* reduction in distance between H_2's probability and 1. But the *relative* reductions are the same. This is because in each case the degree to which the evidence reduces the distance between the hypothesis's probability and 1 relative to the case where that distance is reduced to nil is maximal. Hence by PE it follows that $c(H_1, E_1) = c(H_2, E_2)$.

I take it that, at a minimum, any adequate measure of partial entailment should meet each of the following conditions:

PE1 $c(H, E)$ is a function of $p(H \mid E)$ and $p(H)$.

PE2   If (i) $p(H_1 \mid E_1) > p(H_2 \mid E_2)$ and (ii) $p(H_1) \leq p(H_2)$, then $c(H_1, E_1) > c(H_2, E_2)$.

PE3   If (i) $1 > p(H_1 \mid E_1) \geq p(H_2 \mid E_2)$ and (ii) $p(H_1) < p(H_2)$, then $c(H_1, E_1) > c(H_2, E_2)$.

PE4 $c(H, E)$ is maximal if and only if $p(H \mid E) = 1 > p(H)$.

Henceforth when I speak of "PE" I have in mind PE understood at least in part in terms of PE1-PE4. (That PE is distinct from IP and PDe is shown in Appendix B.)

Does PE run counter to RME? Recall that on Distribution D (from Section 3) it follows that:

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[14] The title of Crupi and Tentori (2013) is "Confirmation as partial entailment: A representation theorem in inductive logic".

$$p(H_1 \mid E) = 1 > 0.00001 = p(H_1)$$

$$p(H_2 \mid E) = 0.00001 > 0.0000000001 = p(H_2)$$

Given that $p(H_1 \mid E) = 1 > p(H_1)$ whereas $p(H_2 \mid E) < 1$, it follows by PE4 that $c(H_1, E) > c(H_2, E)$. This runs counter to RME, since the latter entails that $c(H_1, E) < c(H_2, E)$.[15]

6   Partial Discrimination

A fourth respect in which E can be related to H is this:

> E partially discriminates between H and $\neg H$.

This leads to a fourth way of understanding $c(H, E)$:

> Partial Discrimination (PDi): $c(H, E)$ measures the degree to which E partially discriminates between H and $\neg H$.

How is PDi to be understood? What conditions, that is, should be met by any adequate measure of partial discrimination?

Imagine a case where there are diseases D1 and D2. Suppose that there is a test T1 for D1 and a test T2 for D2. Take some subject S. Let H_1 be the proposition that S has D1, E_1 be the proposition that T1 says that S has D1, H_2 be the proposition that S has D2, and E_2 be the proposition that T2 says that S has D2. Suppose that T1 and T2 are such that:

$$p(E_1 \mid H_1) = 1 > 0 = p(E_1 \mid \neg H_1)$$

$$p(E_2 \mid H_2) = 0.99 > 0.01 = p(E_2 \mid \neg H_2)$$

T1 is maximally good qua test of D1 in that no test of D1 could do better than T1 in terms of discriminating between the presence and absence of D1. T1 thus makes it such that E_1 fully discriminates between H_1 and $\neg H_1$. T2, in contrast, is good but not maximally good qua test of D3. T2 partially but not fully discriminates between the presence and absence

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[15] MI entails that $c(H_1, E) = c(H_2, E)$. So PE also runs counter to MI.

of D2. T2 thus makes it such that E_2 partially but not fully discriminates between H_2 and $\neg H_2$.[16]

I take it that, at a minimum, any adequate measure of partial discrimination should meet each of the following conditions:

PDi1 $c(H, E)$ is a function of $p(E \mid H)$ and $p(E \mid \neg H)$.

PDi2   If (i) $p(E_1 \mid H_1) > p(E_2 \mid H_2)$ and (ii) $p(E_1 \mid \neg H_1) \leq p(E_2 \mid \neg H_2)$, then $c(H_1, E_1) > c(H_2, E_2)$.

PDi3   If (i) $p(E_1 \mid H_1) \geq p(E_2 \mid H_2)$ and (ii) $p(E_1 \mid \neg H_1) < p(E_2 \mid \neg H_2)$, then $c(H_1, E_1) > c(H_2, E_2)$.

Henceforth when I speak of "PDi" I have in mind PDi understood at least in part in terms of PDi1-PDi3. (That PDi is distinct from IP, PDe, and PE is shown in Appendix C.)

Does PDi run counter to RME? It follows on Distribution D (from Section 3) that:

$$p(E \mid H_1) = 1 = p(E \mid H_2)$$

$$p(E \mid \neg H_1) = 0 < 0.00001 \approx p(E \mid \neg H_2)$$

By PDi3 it follows that $c(H_1, E) > c(H_2, E)$. This runs counter to RME, since the latter entails that $c(H_1, E) < c(H_2, E)$.[17]

7   Popper Corroboration

A fifth respect in which E can be related to H is this:

E Popper corroborates H.

This leads to a fifth way of understanding $c(H, E)$:

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[16] See Roche (2016) for an argument to the effect that c_8 (the so-called "likelihood measure") is perhaps best understood as a measure of confirmation in the sense of partial discrimination. See also Roush (2005, Ch. 5).

[17] MI entails that $c(H_1, E) = c(H_2, E)$. So PDi also runs counter to MI.

Popper Corroboration (PC): $c(H, E)$ measures the degree to which E Popper corroborates H.

How is PC to be understood? What conditions, that is, should be met by any adequate measure of Popper corroboration?

It will help to begin with some of Popper's informal remarks on corroboration. Here, first, he links degree of corroboration to testability and notes in effect that degree of corroboration is a matter of the degree to which a hypothesis has been tested and has stood up to its tests:

Degree of corroboration is closely related to the *testability* of a theory. The fact itself is fairly obvious: a more testable theory can be better tested; and what we are looking for is a mark, or degree, expressing how severely the theory was tested, and how well it has stood up to its tests. (Popper 1983, pp. 230-231, emphasis original)

Here, second, he links testability to content and improbability:

Since testability in its turn can be measured by the *content* of the theory, and since content, in its turn, can be measured by the absolute logical *improbability* of the theory, content and improbability stand in the same close relation to degree of corroboration as does testability itself. (Popper 1983, p. 231, emphasis original)

Here, third, he notes in effect that stronger hypotheses are greater in testability than weaker hypotheses (where the former entail the latter but not vice versa):

Corroboration cannot possibly be a probability, since it is more closely related to the improbability of a theory than to its probability: a strong theory (such as Maxwell's electromagnetic wave theory of light) can be tested more widely and more severely than a weaker theory entailed by it (such as Fresnel's wave theory of light). Every test of the latter theory is also a test of the former, but not vice versa. (Popper 1983, p. 231)

There is a clear sense in which Popper, like RME, favors the less probable over the more probable. Suppose that H_1 is stronger than H_2 in that H_1 entails H_2 but not vice versa. Then, though $p(H_1) < p(H_2)$, H_1 is greater in testability than H_2 and so can be greater in corroboration than H_2.

Popper (1983, Ch. 4, sec. 32) goes on to set out a number of adequacy conditions on corroboration measures. They can be put as follows:

PC1 $-1 \leq c(H, E) \leq p(\neg H) \leq +1.$

PC2 $-1 = c(H \& \neg H, E) = c(H, \neg H) \leq c(H, E) \leq c(H, H) = p(\neg H) \leq +1.$

PC3 $c(H \vee \neg H, E) = 0.$

PC4   If H entails E, then $c(H, E)$ is an increasing function of $p(\neg E)$.

PC5   If H_1 entails H_2 but not vice versa, then there is an E such that $c(H_1, E) >$
           $c(H_2, E)$.

PC6   If (i) H_1 entails H_2 but not vice versa and (ii) $p(E \mid H_1) \leq p(E \mid H_2)$, then
           $c(H_1, E) < c(H_2, E)$.

Some clarification is in order here. First, PC2 should be understood so that $c(H, E) = -1$ if
E entails $\neg H$ and so that $c(H, E) = p(\neg H)$ if E entails H. Second, PC3 should be
understood so that $c(H, E) = 0$ if $p(H) = 1$. Third, PC6 should be understood so that it is
tacit in its antecedent that E confirms each of H_1 and H_2.[18] Henceforth when I speak of
"PC" I have in mind PC understood at least in part in terms of PC1-PC6.[19] (That PC is
distinct from IP, PDe, PE, and PDi is shown in Appendix D.)

    Are PC1-PC6 consistent as a set? Consider the following measure:

$$c_{13}(H, E) = \frac{p(E \mid H) - p(E)}{p(E \mid H) - p(H \& E) + \Pr(E)}$$

Popper (1983, p. 251) notes that this measure meets each of PC1-PC6 and that, thus,
PC1-PC6 are consistent as a set.[20]

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[18] If PC6 were not so understood, then Popper would be wrong that his preferred
corroboration measure (c_{13} below) meets PC6. Further, it is clear from the surrounding
discussion that Popper has in mind cases where E confirms each of H_1 and H_2.
[19] See Popper (1954) for a different but similar set of adequacy conditions on
corroboration measures (though there Popper speaks in terms of "confirmation" as
opposed to "corroboration"). One notable difference is that PC5 is not included in the
earlier set of adequacy conditions. See Díez (2011) and Sprenger (forthcoming) for
discussion of the earlier set of adequacy conditions.
[20] Popper (1954) initially suggests a different measure. It can be put as follows:

Does PC run counter to RME? Suppose that H_1 entails H_2 but not vice versa. Suppose that H_2 entails E and thus so too does H_1. Then it follows that:

$$p(E \mid H_1) = 1 = p(E \mid H_2)$$

$$p(H_1) < p(H_2)$$

Hence:

$$p(E \mid H_1)/p(E) = p(E \mid H_2)/p(E) > 1$$

$$p(H_1) < p(H_2)$$

By PC6 it follows that $c(H_1, E) < c(H_2, E)$. This runs counter to RME, since the latter entails that $c(H_1, E) > c(H_2, E)$.[21]

It is worth noting that PC6 and ME are logically distinct from each other and that, therefore, the mere fact that c_{13} meets the former leaves it open that it does not meet the latter. It turns out, though, that c_{13} meets ME as well. This can be seen by noting that:

$$c_{13}(H_1, E_1) = \frac{p(E_1 \mid H_1) - p(E_1)}{p(E_1 \mid H_1) - p(H_1 \& E_1) + p(E_1)}$$

$$= \frac{\dfrac{p(E_1 \mid H_1)}{p(E_1)} - 1}{\dfrac{p(E_1 \mid H_1)}{p(E_1)}(1 - p(H_1)) + 1}$$

$$c_{14}(H, E) = \left[\frac{p(E \mid H) - p(E)}{p(E \mid H) + \Pr(E)} \right] [1 + p(H)p(H \mid E)]$$

This measure is not ordinally equivalent to c_{13} in that there are cases where $c_{13}(H_1, E_1) > c_{13}(H_2, E_2)$ but $c_{14}(H_1, E_1) \leq c_{14}(H_2, E_2)$. It can be shown, though, and is noted by Popper (1983, p. 251), that c_{14}, as with c_{13}, meets each of PC1-PC6.

[21] MI entails that $c(H_1, E) = c(H_2, E)$. So PC also runs counter to MI.

$$c_{13}(H_2, E_2) = \frac{p(E_2 \mid H_2) - p(E_2)}{p(E_2 \mid H_2) - p(H_2 \& E_2) + p(E_2)}$$

$$= \frac{\dfrac{p(E_2 \mid H_2)}{p(E_2)} - 1}{\dfrac{p(E_2 \mid H_2)}{p(E_2)}\big(1 - p(H_2)\big) + 1}$$

It follows that if $p(E_1 \mid H_1)/p(E_1) = p(E_2 \mid H_2)/p(E_2) > 1$ and $p(H_1) < p(H_2)$, then $c_{13}(H_1, E_1) < c_{13}(H_2, E_2)$. Hence c_{13} meets ME.[22]

It might seem puzzling that PC runs counter to RME. What about Popper's preference for the less probable over the more probable?

Consider the following passage (where notation has been modified):

> Take … a hypothesis H_1 from which a weaker hypothesis H_2 is deducible. (We may again take H_1 to be Maxwell's and H_2 to be Fresnel's theory.) Now assume that we have tested H_2 by the test E (Fizeau's experiment, or double refraction) but not that part of H_1 which goes beyond H_2. In this case we shall say that $c(H_1, E) < c(H_2, E)$, in spite of the greater corroborability of H_1. For H_1 contains a part, not yet tested, which may be refuted at the first test we undertake. But if E tests this latter part also (as does Herz's experiment) then we have $c(H_1, E) > c(H_2, E)$, provided the test was successful, and sufficiently severe. (Popper 1983, p. 250)

It is true that there are cases where Popper prefers the stronger and thus less probable hypothesis over the weaker and thus more probable hypothesis. But, as is suggested by

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[22] Sober (2015, p. 96) notes in effect that this is true in the special case where (a) H_1 entails H_2 but not vice versa and (b) each of H_1 and H_2 entails E. Festa (2012, sec. 3, p. 97) goes farther and notes that it is true in general and thus not just in special cases. His argument, though, contains a minor mistake (which is perhaps merely typographical). He (2012, sec. 2, p. 93) claims that:

$$\frac{p(H \mid E) - p(H)}{p(H \mid E) + p(H) - p(H \mid E)p(H)} = \frac{\dfrac{p(E \mid H)}{p(E)} - 1}{\dfrac{p(E \mid H)}{p(E)} - p(H) + 1}$$

This is wrong. The denominator on the right should be $\dfrac{p(E \mid H)}{p(E)}\big(1 - p(H)\big) + 1$.

the passage above, the cases in question are *not* cases where the evidence is entailed by both the stronger hypothesis and the weaker hypothesis. The cases in question, rather, are cases where the evidence is entailed by the stronger hypothesis but not by the weaker hypothesis (where, that is, the evidence tests the part of the stronger hypothesis that goes beyond the weaker hypothesis). Let H_1 be the stronger hypothesis and H_2 be the weaker hypothesis. Popper has in mind cases where $p(E \mid H_1) = 1 > p(E \mid H_2)$ and thus where $p(E \mid H_1)/p(E) > p(E \mid H_2)/p(E)$. But no such case is a case where the antecedents of ME, MI, and RME hold. This means that Popper's preference for the less probable over the more probable, when properly understood, in no way requires RME.[23]

8   Conclusion

Is RME plausible as an adequacy condition on confirmation measures when $c(H, E)$ is understood as measuring the degree to which E increases H's probability, or when $c(H, E)$ is understood as measuring the degree to which H's probability partially depends on whether E is true, or when $c(H, E)$ is understood as measuring the degree to which E partially entails H, or when $c(H, E)$ is understood as measuring the degree to which E partially discriminates between H and $\neg H$, or when $c(H, E)$ is understood as measuring the degree to which E Popper corroborates H? I have argued that the answer is negative. Perhaps there is still a place in Bayesian confirmation theory for RME. But, if so, what is it?

Acknowledgments

Appendix A

Is PDe distinct from IP? Suppose that:

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[23] The same is true with respect to "Weak Informativity" and "Strong Informativity" in Sprenger (2016, p. 10). For further discussion of PC, and for references, see Rowbottom (2011).

$$p(H_1 \mid E_1) = 0.99 > p(H_1) = 0.98 > p(H_1 \mid \neg E_1) = 0.01$$

$$p(H_2 \mid E_2) = 0.99 > p(H_2) = 0.03 > p(H_2 \mid \neg E_2) = 0.02$$

It follows by IP3 that $c(H_1, E_1) < c(H_2, E_2)$. It follows by PDe3, in contrast, that $c(H_1, E_1) > c(H_2, E_2)$. Hence PDe is distinct from IP. QED

Appendix B

Is PE distinct from IP and PDe? Suppose that:

$$p(H_1 \mid E_1) = 1 > p(H_1) = 0.98 > p(H_1 \mid \neg E_1) = 0.01$$

$$p(H_2 \mid E_2) = 1 > p(H_2) = 0.03 > p(H_2 \mid \neg E_2) = 0.02$$

It follows both by IP3 and by PDe3 that $c(H_1, E_1) \neq c(H_2, E_2)$. But since $p(H_1 \mid E_1) = 1 > p(H_1)$ and $p(H_2 \mid E_2) = 1\ p(H_2)$, it follows by PE4, in contrast, that $c(H_1, E_1) = c(H_2, E_2)$. Hence PE is distinct from IP and PDe. QED

Appendix C

Is PDi distinct from IP, PDe, and PE? Suppose, first, that:

$$p(H_1 \mid E_1) = 1 = p(H_2 \mid E_2)$$

$$p(H_1) = 0.5 > 0.49 = p(H_2)$$

$$p(E_1 \mid H_1) = 1 > 0.25 = p(E_2 \mid H_2)$$

$$p(E_1 \mid \neg H_1) = 0 = p(E_2 \mid \neg H_2)$$

It follows by IP3 that $c(H_1, E_1) < c(H_2, E_2)$ and by PE4 that $c(H_1, E_1) = c(H_2, E_2)$. It follows by PDi2, in contrast, that $c(H_1, E_1) > c(H_2, E_2)$. Suppose, second, that:

$$p(H_1 \mid E_1) = 0.01 < 0.99 = p(H_2 \mid E_2)$$

$$p(H_1 \mid \neg E_1) = 0 = p(H_2 \mid \neg E_2)$$

$$p(E_1 \mid H_1) = 1 = p(E_2 \mid H_2)$$

$$p(E_1 \mid \neg H_1) = 0.001 < 0.002 = p(E_2 \mid \neg H_2)$$

It follows by PDe2 that $c(H_1, E_1) < c(H_2, E_2)$. It follows by PDi3, in contrast, that $c(H_1, E_1) > c(H_2, E_2)$. Hence PDi is distinct from IP, PDe, and PE. QED.

Appendix D

Is PC distinct from IP, PDe, PE, and PDi? Suppose, first, that E_1 entails $\neg H_1$, that E_2 entails $\neg H_2$, and that:

$$p(H_1) = 1/101 < 49/2599 = p(H_2)$$

It follows by IP3 that $c(H_1, E_1) > c(H_2, E_2)$. It follows by PC2, in contrast, that $c(H_1, E_1) = -1 = c(H_2, E_2)$. Suppose, second, that:

$$p(H_1 \mid E_1) = 1 = p(H_2 \mid E_2)$$

$$p(H_1) = 0.5 > 0.49 = p(H_2)$$

$$p(H_1 \mid \neg E_1) = 0 = p(H_2 \mid \neg E_2)$$

$$p(E_1 \mid H_1) = 1 = p(E_2 \mid H_2)$$

$$p(E_1 \mid \neg H_1) = 0 = p(E_2 \mid \neg H_2)$$

It follows by PDe1 that $c(H_1, E_1) = c(H_2, E_2)$, by PE4 that $c(H_1, E_1) = c(H_2, E_2)$, and by PDi1 that $c(H_1, E_1) = c(H_2, E_2)$. It follows by PC2, in contrast, that $c(H_1, E_1) = 0.5 < 0.51 = c(H_2, E_2)$. Hence PC is distinct from IP, PDe, PE, and PDi. QED

References

Brössel, P. (2013). The problem of measure sensitivity redux. *Philosophy of Science*, 80, 378-397.

Crupi, V. (2016). Confirmation. In E. Zalta (Ed.), *The Stanford Encyclopedia of Philosophy* (Fall 2016 ed.). URL = <http://plato.stanford.edu/archives/fall2016/entries/confirmation/>.

Crupi, V., and Tentori, K. (2013). Confirmation as partial entailment: A representation theorem in inductive logic. *Journal of Applied Logic*, 11, 364-372.

Crupi, V., and Tentori, K. (2014). Erratum to "Confirmation as partial entailment". *Journal of Applied Logic*, 12, 230-231.

Díez, J. (2011). On Popper's strong inductivism (or strongly inconsistent anti-inductivism). *Studies in History and Philosophy of Science*, 42, 105-116.

Douven, I. (2011). Further results on the intransitivity of evidential support. *Review of Symbolic Logic*, 4, 487-497.

Festa, R. (2012). "*For unto every one that hath shall be given*". Matthew properties for incremental confirmation. *Synthese*, 184, 89-100.

Festa, R., and Cevolani, G. (forthcoming). Unfolding the grammar of Bayesian confirmation: Likelihood and anti-likelihood principles. *Philosophy of Science*.

Fitelson, B. (1999). The plurality of Bayesian measures of confirmation and the problem of measure sensitivity. *Philosophy of Science*, 66, S362-S378.

Fitelson, B. (2008). A decision procedure for probability calculus with applications. *Review of Symbolic Logic*, 1, 111-125.

Hajek, A., and Joyce, J. (2008). Confirmation. In S. Psillos and M. Curd (Eds.), *The Routledge companion to philosophy of science* (pp. 115-128). London: Routledge.

Joyce, J. (1999). *The foundations of causal decision theory*. Cambridge: Cambridge University Press.

Joyce, J. (2008). Bayes' theorem. In E. Zalta (Ed.), *The Stanford Encyclopedia of Philosophy* (Fall 2008 ed.). URL = <http://plato.stanford.edu/archives/fall2008/entries/bayes-theorem/>.

Kuipers, T. (2000). *From instrumentalism to constructive realism: On some relations between confirmation, empirical progress, and truth approximation*. Dordrecht: Kluwer.

Milne, P. (1996). log[P(h/eb)/P(h/b)] is the one true measure of confirmation. *Philosophy of Science*, 63, 21-26.

Popper, K. (1954). Degree of confirmation. *British Journal for the Philosophy of Science*, 5, 143-149.

Popper, K. (1983). *Realism and the aim of science*. London: Routledge.

Roche, W. (2012). Transitivity and intransitivity in evidential support: Some further results. *Review of Symbolic Logic*, 5, 259-268.

Roche, W. (2014). A note on confirmation and Matthew properties. *Logic & Philosophy of Science*, XII, 91-101.

Roche, W. (2015). Evidential support, transitivity, and screening-off. *Review of Symbolic Logic*, 8, 785-806.

Roche, W. (2016). Confirmation, increase in probability, and partial discrimination: A reply to Zalabardo. *European Journal for Philosophy of Science*, 6, 1-7.

Roche, W., and Shogenji, T. (2014). Confirmation, transitivity, and Moore: The Screening-Off Approach. *Philosophical Studies*, 168, 797-817.

Roush, S. (2005). *Tracking truth: Knowledge, evidence, and science*. Oxford: Oxford University Press.

Rowbottom, D. (2011). *Popper's critical rationalism: A philosophical investigation*. New York: Routledge.

Sober, E. (2015). *Ockham's razors: A user's manual*. Cambridge: Cambridge University Press.

Sprenger, J. (forthcoming). Two impossibility results for measures of corroboration. *British Journal for the Philosophy of Science*.