João Daniel Dantas, Evelyn Erickson
and Sanderson Molick (eds.)

Proceedings of the 3rd Filomena Workshop

PROGRAMA DE PÓS-GRADUAÇÃO ppgfil EM FILOSOFIA UFRN

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Introduction

Analytic philosophy became the dominant way of thinking philosophy in the XXI century by providing problems and conceptual developments that reach across many diverse fields such as linguistics, neurosciences, mathematics and computer science. However, analytic philosophy is still a neglected subject in many philosophy departments of Brazil, especially to those located in Universities far from the main research centers. Although this reality started to change a few years ago, analytic philosophy is yet restricted to little workshop within big and broad conferences. Thus, the FILOMENA workshop (on philosophy, logic and analytic metaphysics) was created as one initiative of change to this landscape. Our goal is to promote not only academic discussions in the field of analytic philosophy, but mainly to enlarge the scope of the philosophy pursued in the Northeast region of Brazil.

Getting ready to host FILOMENA's fourth edition, I'm happy to say that the workshop has been successful in accomplishing its goal by providing a fertile environment to gather researchers working in different fields of analytic philosophy, with special attention to those working in topics that hinge problems from logic and metaphysics. The intellectual exchange between foreign and local researchers proved to be a good strategy to stimulate and raise the overall level of the philosophy followed by both groups.

The first edition of FILOMENA took place between the 3rd and 7th of November of 2014 together with the XXIV Semana de Filosofia of the Philosophy Department from UFRN. It had the participation of 11 researchers and a roundtable called "On the philosophy of contradictions". One year later, the 2nd edition took place between the 31st of August and 4th of September of 2015 together with the NAT@Logic event, having jointly gathered 140 participants, with 24 talks given at Filomena. The main contributions presented were published on a special issue of Revista Saberes (ISSN 1984-3879).

The 3rd edition took place between the 21st and 23rd of August of 2017, again in Natal. It had the participation of 23 researchers and a round table called "On logic and metaphysics". The event also counted with the presence of the following invited speakers: Tuomas Tahko (University of Bristol), Jonas Arenhart (UFSC), Giorgio Venturi (UNICAMP) and Elaine Pimentel (UFRN). Many topics ranging from metaphysical grounding to plural description were discussed. The third edition was marked by the vivid discussion of many important themes to current philosophy of logic and metaphysics. The papers organized in this volume are the result of some of the best contributions presented at the 3rd edition of FILOMENA.

The book opens with the paper *Bridging the Gap Between Science and Metaphysics, with a Little Help from Quantum Mechanics*, by Jonas R. Becker Arenhart (UFSC), which addresses relevant issues to contemporary metaphysics, especially after the publication of Ladymans Ross's book *Everything must Go*, where the authors raises several criticism to the role of intuitions into metaphysics discourse and to how metaphysics can be prove useful to science. Arenhart then discusses how science can be relevant to metaphysics, arguing that compatibility with quantum mechanics can be a criteria to choose between competing ontologies.

The second paper, *On Quine's Ontology: quantification, extensionality and naturalism (or from commitment to indifference)*, by Daniel Durante (UFRN), tries to understand the source of many controversial claims related to Quine's proposal for ontology, showing that they are all consequences of his main methodological and metaphilisophical theses. The third paper, *What logical pluralism can be about*, by Evelyn Erickson (UFRN), analyses different theses on logical pluralism, and proposes that the way to explore pluralism in logic is to challenge the Tarskian notion of consequence. The fourth paper, *Topics on Philosophy of Information*, by Samir Gorsky (UFRN) gives an overview of the field of philosophy of information and discusses how information theory can be relevant to philosophy.

Also on the topic of logical pluralism, the fifthh paper, *A note on Beall Restall's pluralism and non-Tarskian logics*, by Sanderson Molick (UFRN and RUB) argues for the legitimacy of non-Tarskian entailment relations against Beall Restall's criticism to these forms of logical consequence. The sixth paper, *The special composition question and natural fusion*, by Renato Rocha (UFC), is a defense of how natural fusion can be combined with special composition, evading Lewis's attack on the role of vagueness in restricted composition. The last paper, *Completeness in Quantified Modal Logic and the Interpretations of the Quantifiers*, by Luis Urtubey (University of Córdoba), shows how different interpretations of the quantifiers within quantified modal logic can affect proofs of completeness for QML.

Enjoy!

The editors

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Bridging the Gap Between Science and Metaphysics, with a Little Help from Quantum Mechanics

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1. Introduction

It is a truism to say that the relation between science and metaphysics is one of the most debated topics in current analytic philosophy. There is much discussion on how to appropriately bridge the gap between science and metaphysics, and how to properly understand the relation between science on the one side, and metaphysics on the other (for a recent contribution to the debate, covering much of what is typically discussed, see Guay and Pradeu [Guay and Pradeu, 2017]). However, despite the collective effort, the issue deals mostly with metaphysical methodology, and being so, it is still a *metaphysical* issue, and as such, there is still no agreement on the most appropriate approach to the main problem. Worst yet, there is still much confusion on terminology and on what is being aimed at when we say that science and metaphysics should profitably relate somehow.

Traditionally, the trouble arises from the following opposition: while it is thought that science is an enterprise with epistemic respectable credentials, which has made incredible progress, it is also thought that metaphysics provides no similar epistemic warrant, with its main debates going on forever without any end in sight, exemplifying what some call as sterile disputes, and what others deem as irrelevant for our objective knowledge of the world. In any case, in order to get a good grasp of features of the world, we are recommended to look at science, and if metaphysics wishes to be relevant for human knowledge, it is required that it somehow relates in an appropriate way with our major epistemic achievement, which is science (this is the main claim in Ladyman and Ross [Ladyman and Ross, 2007], for instance).

The charge of irrelevance has become critical after the work of Ladyman and Ross [Ladyman and Ross, 2007], who have sparkled major controversy, after all, with many others following along similar lines. In a much quoted passage, they famously claim that

> standard analytic metaphysics (or 'neo-scholastic' metaphysics as we call it) contributes nothing to human knowledge and, where it has any impact at all, systematically misrepresents the relative significance of what we do know on the basis of science. (Ladyman and Ross [Ladyman and Ross, 2007], p.vii)

Others have advanced similar claims, pointing to the relative ignorance of analytic metaphysicians of the current state of science (for a nice characterization of the main tenets of the "science first" team, see Guay and Pradeu [Guay and Pradeu, 2017]).

The main diagnosis for such a derailed production of metaphysics is that current analytic metaphysics is being developed in complete isolation from science, so that science is irrelevant to metaphysics, and that metaphysics is irrelevant to science, given that it is concerned with problems alien to current science. While some proposals to engage in cooperative work have been advanced, most of the authors still focus on what went wrong with metaphysics, not on how to do the positive work properly (if that is thought to be possible, anyway).

It seems that the main challenge concerns establishing a relation between metaphysics that does not misrepresent science, and more, that takes into account the following desiderata:

- Science should be able to confer part of its epistemic warrant to metaphysical claims, so that when the metaphysics is properly done, it is kept within the boundaries of the epistemic safety given to it by science (talk of "bridging the gap between science and metaphysics" is common here);
- Metaphysics should fit scientific theories, not the other way around (the idea that we should "tailor the metaphysics to suit the physics" is relevant here);
- Science is responsible, somehow, for the "metaphysical profile" of the entities it posits, not the other way around (and here much trouble is to be expected, as we shall see).

The problem in most of the debate on an appropriate relation between science and metaphysics seems to be that, except for the sloganeering, which seems to point to the need of a closer relation between the two, not much is agreed upon on how those ends are supposed to be achieved. Furthermore, and this will be part of our topic, there is some ambiguity about what is understood by *metaphysics* in much of this debate and to what degree one is committed to it.

In this paper, we shall add to the discussion. We approach the problem by first bringing to light a distinction that is commonly ignored in those discussions: a distinction between *metaphysics* on the one hand, and *ontology* on the other. Roughly, while ontology deals with the catalog of what exists, metaphysics, as a general area of philosophy, studies, among other things, the categories into which those things that exist should be classified into.[1] After the distinction is made between the two, it should be easier to establish the field of competence of each, as well as their limits. Furthermore, with that distinction we will be able to locate a source of further confusion on attempts to relate science and metaphysics, a confusion which is related with the problem of scientific realism. In a nutshell, while scientific realism is concerned with ontological matters, it is not necessarily related with metaphysics, in the specific sense we are conceiving it. Finally, we shall also be able to make a clear case that the most extravagant versions of "scientific metaphysics" are hopeless in their attempt to somehow "extract" metaphysics from science. Metaphysics is a specific kind of investigation, which must be compatible with science, but cannot be derived from it. In this search for compatibility lies a key to a possible (modest) relation between metaphysics and science, and it is here that quantum mechanics may help us, making a nice case for how difficult it is to develop metaphysical theories compatible with our findings about the quantum world.

The structure of this paper is as follows. In section 2 we bring to light one of the sources of what seems to be a great confusion on the basis of the challenges to analytical metaphysics: we argue that there is a conflation of "ontology" and "metaphysics" which is detrimental to the debate. We illustrate the confusion with some examples. In section 3, we keep pressing the same point and argue that such confusion infests mainly the discussions on scientific realism. While realism about scientific theories may be responsible for providing answers to the ontological questions, it cannot fully cope with the more abstract and general metaphysical investigations. Metaphysical underdetermination, as typically understood, is a result of such a distinction. In section 4 a distinct proposal to the challenge is advanced, where the notion of compatibility between science and metaphysics plays a prominent role. Science cannot provide us with metaphysical theories, but it may confer epistemic authority on us to declare some meta-

[1] Metaphysics also deals with other problems, such as theory of causality, the nature of modalities, personal identity, free will, the nature of space and time, and others.

physical theories unable to do the job. We conclude in section 5.

2. Between metaphysics and ontology

Our contention in this section is that a conflation of metaphysics and ontology is constantly being made in part of the so-called scientific metaphysics literature. This confusion is one of the sources of the main difficulties when it comes to try to make clearer the relation between metaphysics, ontology, and science.

As is well known, the story of ontology and metaphysics from the twentieth century on is completely related with the debate between Carnap and Quine. Now, this is not the place to review the dispute, but it is usually thought that while Carnap attempted to somehow deflate ontological disputes, Quine has managed to somehow show they are substantial, and that they may be pursued in a satisfactory manner. In other words, it is usually assumed that Quine was pro-metaphysics, while Carnap was anti-metaphysics, and, luckily, Quine won the dispute.

Here is a famous passage of Putnam [Putnam, 2004] pp.78-79, where Quine is again responsible for the revival of *ontology*:

> If we ask *when* Ontology became a respectable subject for an analytic philosopher to pursue, the mystery disappears. It became respectable in 1948, when Quine published a famous paper titled "On What There Is". It was Quine who single handedly made Ontology a respectable subject

However, by saving the debates on ontology, people started to feel that *metaphysics as a whole* was also safe. With Quine's approach, that is, it was felt that metaphysics was saved from the logical empiricists' attack. As Loux and Zimmerman [Loux and Zimmerman, 2003] p.2 put it — and their diagnosis is common wisdom among analytic philosophers —, after the success of Quine's approach to *ontology*, "philosophers no longer felt the need to conceal their interest in metaphysical issues, and there was something like a revival of traditional metaphysics". Most probably against Quine's own will, his work on ontology was seen as saving *traditional metaphysics*.

The problem is that most of the times, ontology and metaphysics are treated as being exactly the same thing, so that Quine's method for ontology is seen as applying to the broader area of metaphysics too. Maudlin puts this point in a very clear fashion, when determining the program for a naturalistic metaphysics (which is sometimes a synonym for scientific metaphysics):

> Evidence for what exists, at least in the physical world, is provided solely by empirical research. Hence the proper object of most metaphysics is the careful analysis of our best scientific theories (and especially of fundamental physical theories) with the goal of determining what they imply about the constitution of the physical world. (Maudlin [Maudlin, 2007], p.104)

As a result, metaphysics deals with an analysis of scientific theories, in search for their *ontological* commitments! That is, according to the naturalistic project, ontology should be regarded as the study of what there is according to our best scientific theories; it should be pursued with the Quinean criterion for ontological commitment as the main tool, and should be employed in the proper study of metaphysics.

Of course, there is no problem if those two labels, "ontology" and "metaphysics" are re-defined to mean the same thing, provided that this is clearly stipulated to begin with. The problem is that this is not usually done, and people are willing to relate metaphysics, as traditionally understood, to science, not only the field of ontology, as an activity that nowadays employs Quinean methodology to scientific theories.

Once metaphysics and ontology are identified like that, metaphysics is reduced to the task of providing a catalog of what there is (which is, after all, the task of ontology). If we think that ontology and metaphysics are just the same thing, then, clearly there seems to be a way, advanced by Quine, to be scientific in metaphysics: just endorse naturalistic realism. There exists precisely what our best scientific theories say there is (see Quine [Quine, 1981] p.21). There may be some room for disagreement here and there, but the world consists, basically, of the posits of contemporary science.

Before we proceed, some remarks are in order. The idea that ontology seeks for a catalog of what there is may seem too impoverished for some, with ontology having more glorious ambitions. This is not the place to address this question, but notice that understanding ontology as a catalog of what exists makes the best sense of what is typically thought of as 'the ontological question': what there is? Also, it makes nice sense of the debate between Carnap and Quine, whose main disagreements seemed to lie on whether some kinds of entities could be posited and still be scientifically respectable. Finally, this leads to some interesting questions concerning explanatory power of the posits and their relation to the available entities, which should also concern ontology. This makes the task of ontology clearly challenging, and, we hope, sill covers what most people are talking about when they talk about ontology. We shall keep speaking of the task of ontology as developing a catalog of what there is, but keeping in mind that there are related tasks involved.

But elaborating this catalog, interesting as it is, is certainly not what is being now typically called the "neo-medieval" approach to *metaphysics*! It seems that no metaphysician, in any tradition, would deny that our current scientific theories provide evidence to the claim that there are electrons, protons, spacetime, and so on. It is not incumbent on metaphysicians to make such a claim. Rather, neo-medieval metaphysics (if we are to keep that label) deals with issues that are more general, among other things, issues in what are the categories according to which these items should be classified. Neo-medieval metaphysics, in this sense, cannot be considered a kind of investigation that was re-habilitated by Quine; as we have been mentioning, it works on a much more general level of abstraction, dealing not directly with the posits of scientific theories, but with the general categories with which to classify those posits (see, for instance, the discussion in Lowe [Lowe, 2008]).

To exemplify the difference in questions of ontology and metaphysics, as we understand them, consider quantum mechanics. According to the naturalistic realism advanced by Quine and others, the theory commits us with, among other things, electrons. So, there are electrons. But that does not exhaust the metaphysical questions that must be addressed. What are they, on metaphysical terms? Are they individuals in some sense? Are they even objects, in some sense of the term? (Even the notion of "object" is subject to metaphysical investigation, and cannot be directly understood from science alone). How should we understand their properties and relations? Are they universals, tropes? And so on.

Once the distinction between metaphysics and ontology is presented, we may go on to address the issue of how conflating them leads to proposals that attempt to bridge the gap between science and metaphysics, but are really relating science and ontology, in the way that was basically preconized by Quine in his realistic naturalism. Indeed, once one assumes a Quinean approach to ontology and allows that existential quantifiers help us express our commitments, we may be said to be realists about the class of entities we quantify over.

The commitment with the posits of a scientific theory may be provided in alternative ways too, of course (for further discussions, see Azzouni [Azzouni, 1998]; how ontological commitments are engaged in is also a major problem to be discussed in ontology and metaontology, so that the 'catalog idea' encompasses these discussions too). Anyway, what is relevant for our purposes is that while those attempts may be addressed as the problem of providing the catalog of what there is, it still leaves the typical metaphysical issues open.

For another prominent example of attempt at bridging the gap between metaphysics and science, consider Alyssa Ney's [Ney, 2012] suggestion of a 'neo-positivistic' metaphysics. Basically, her claim is that metaphysics may relate to science through the adoption of a kind of 'indispensability of formulation' argument:

> Perhaps certain representational elements are found in every formulation of fundamental physics that meet criteria of theory choice accepted by the physics community. There might be some representational features that are as a matter of fact indispensable to our best physical theories as they are actually understood. If one could show that, to state our fundamental physics clearly, precisely, and accurately, one must use certain kinds of representing devices, then perhaps this would show something that had genuine significance and justification, something that went beyond merely expressing one's preferences for a particular kind of conceptual scheme or linguistic framework. (Ney [Ney, 2012], pp.60-61)

She advances the argument as follows (Ney [Ney, 2012] p.61):

(P1) We ought to have metaphysical commitment to all and only the entities, structures, or principles that are indispensable to our best scientific theories.

(P2) X is indispensable to our best scientific theories.
Therefore,

(C) We ought to have metaphysical commitment to X.

However, as she recognizes, this is not the end of the task. There may be further structures, entities, or other metaphysical posits required in order to make the whole theory work. Anyway, those additions notwithstanding, according to her approach this is just as far as science may have an impact on metaphysics. We may safely say that science requires the metaphysical posits that are indispensable to the formulation of our best fundamental physical theories. The rest is added by us, in order to keep the theory working and as simple as possible; the idea is that instead of rejecting those "extras", we are able to be completely honest about assuming them and about their status.

Ney then goes on to apply the above argument to the case of *wave function realism* in quantum mechanics. Suppose that quantum mechanics cannot be formulated without the use of the mathematical apparatus of a wave function. Quantum mechanics being a fundamental theory, according to the argument just presented, we should be committed to the existence of wave functions (and this is, of course, *wave function realism*; see Albert and Ney [Ney and Albert, 2013] for a volume with further presentations of the view and critical discussion).

However, even if the indispensability of the formulation argument works as a reliable tool for establishing ontological commitment (and this is not the place to discuss *this*), and if it is successful in establishing a form of wave function realism, this is not the end. As we have been discussing, doing this would only mean that we are having success in somehow extracting the posits of science with which we should be committed (the ontology), but there is still the issue of the metaphysics involved in articulating the nature of the wave function. Also, among other things, one must still articulate, for instance, the relation of wave function to scientific laws, to the other entities we typically perceive that are not wave functions, and so on. This is something done mostly by armchair speculation. The most important point is to recognize that whatever goes beyond science is not warranted by science, and keeps, as it were, floating free from it.[2]

French [French, 2013], in his discussion of wave function realism, addresses precisely this point, by connecting the ontological aspect (realism about wave function) with the metaphysical aspect. He proposes that the connection may go as follows (here we are not claiming that French would endorse wave function realism; in fact, he assumes it for the sake of argument, in order to show that metaphysical difficulties will arise that make the position untenable): let us concede that we are wave function realists (this is the *ontological part*). A simple consequence of the position is that, in physical terms, the wave function must be a field. Now comes the *metaphysical part* of the job: how should we "metaphysically dress" such a field? In terms of metaphysical categories or, perhaps metaphysical tools, what is a field? As a physical entity, the field is described by quantum mechanics. But that still does not settle the metaphysical questions as to the metaphysical nature of the field.

[2] In this paper, the idea that some elements of a metaphysical theory are 'floating free' from science indicates that they do not get any direct justification from the scientific theory. That means, in particular, that distinct incompatible such elements may be available, generating what we will call metaphysical underdetermination. The plan is that distinct metaphysical elements are compatible with the same scientific theory, which, on its turn, is silent on which metaphysical theory should be preferred. The point is discussed in the paper. Thanks for an anonymous referee for pressing on this point.

French advances two possible candidates for us to metaphysically understand the wave function:

- The field may be understood as a global particular (a form of blobjectivism);
- The field may be understood as involving a version of configuration space substantivalism.

While both versions have their merits and difficulties, it seems that both are compatible with realism about the wave function. The main problem, as French sees it, is that quantum mechanics by itself does not decide the issue of which view should be adopted, and we have underdetermination of the metaphysics by the physics (see French [French, 2013] p.84, and also French [French, 2011] for further discussion on metaphysical underdetermination): the metaphysics cannot be decided on the basis of physics.

Of course, Ney [Ney, 2012] could claim that the metaphysical dressing is precisely that part not afforded by science. It just floats free from science, being added from a metaphysical perspective, not from a scientific perspective. But then we are back to the start! We wanted a scientific metaphysics, not only a scientific ontology. Notice that the realist component of the ontology (the reality of the wave function) was dressed by metaphysics that was not, by itself, extracted with the use of the indispensability of the formulation argument. Rather, it was inserted there from outside of the scientific theory, as it were, with physics having nothing to say about it, not even according to the proposed link through the 'indispensability of formulation' argument.

Now, this is something to be expected, given the distinction between ontology and metaphysics we have proposed. The ontological analysis of scientific theories, proceeding by different strategies, may achieve a catalog of the items that are real according to some scientific theory (and notice that we are not claiming that this "extracting" may be easily achieved; discussing those strategies to somehow extract an ontology from a scientific theory, and how well they fare in this task, is out of the scope of this paper, of course; again, this is also part of the idea that the task of ontology is making a catalog of reality). Once an ontology is provided, there is always a further task, of providing for the metaphysics that suits those items, and this task cannot be performed through an analysis of a scientific theory, given that it proceeds on a more general level and is not in the same business as ontology. One cannot go from realism about the posits of a scientific theory to the metaphysics describing those posits. Some have tried to do so, as Ney, perhaps, and some are still trying. Let us discuss these more recent attempts in the next section.

3. Metaphysics and scientific realism

As we have seen, even when we confine ourselves to a scientific setting, dealing only with more or less well-determined scientific theories, while ontology deals with the posits of the scientific theory, metaphysics seems to be an extra layer of theoretical content, added (perhaps) from above, but not derived (in any sense) from the theory itself through the same means that the ontology is thought to be derived (or obtained, extracted). So, while some attempts to relate metaphysics and science are indeed ways to somehow attempt to extract a definite ontology from science, metaphysics is still "floating free" from science, as it were. As the argument by Steven French mentioned above, in the case of the wave function, makes clear (and as Ney also recognized for those parts of metaphysics not falling in the scope of her indispensability of formulation argument), we are certainly to expect that more than one metaphysical dressing will be available to do the job of dealing with a given ontology, and the risk is that we may have to face metaphysical underdetermination more frequently than expected, challenging direct attempts to somehow obtain the metaphysics of a scientific theory as a kind of by-product that comes with the ontology.

Given this situation, we could perhaps expect that the distinction between the realist's posits of a scientific theory and the metaphysical treatment given to those posits are understood on different grounds, and that attempts to bridge the gap between metaphysics and science really address the relation between science and the metaphysical components, not only the ontological ones, right? But that is not what happens in these discussions. Here, we shall explore another most prominent case where the ontology, understood as encapsulated in the *realistic component* of a scientific theory (and we shall focus on quantum mechanics, again), is supposed to account also for the *metaphysical component*. If that were possible, of course, we would reach a productive relation between science and metaphysics (science would deliver the metaphysics!), but, so far, the conflation between metaphysics and ontology puts some obstacle on the way.

Let us consider the problem of identical particles in quantum mechanics, which gives rise to much of the problem we wish to illustrate (for the standard discussion on the issue, see French and Krause [French and Krause, 2006]; a similar case is made concerning relativistic spacetime by using the so-called "hole argument", but we shall not discuss that, see Pooley [Pooley, 2006]). Roughly speaking, due to permutation symmetry, quantum particles are indiscernible by quantum observables: no property is able to discern any two quantum particles of the same kind. Not even relations are able to discern such particles by the order in which the *relata* are related; that is, one cannot find a quantum mechanical relation R such that for particles x and y, xRy holds, but yRx does not. This happens because, due to permutation symmetry, relations over quanta must be symmetric: whenever xRy holds, yRx must hold too. Recently, it was discovered that some relations obtain in quantum mechanics that, it is said, may discern the particles: those relations are symmetric and irreflexive. That is, if xRy holds, then also yRx, but it is not the case that xRx or yRy (see Muller and Saunders [Muller and Saunders, 2008], Muller [Muller, 2011]). Such relations are called *weakly discerning relations*. It is contentious, to say the least, that such relations really do discern the entities in question; perhaps the best they do is grant numerical diversity (see the discussion in French and Krause [French and Krause, 2006] chap.4; Lowe [Lowe, 2015]).

What is relevant for us is that this indiscernibility by quantum properties, and even by spatio-temporal location, prompted some to think that quantum entities are no longer individuals. Clearly, they cannot be individuated by their properties, as it happens in typical bundle theories of individuality (more on this soon, see also French and Krause [French and Krause, 2006] chap.4). This, together with the fact that identity over time seems to make no sense for quantum entities, caused some to claim, following Schrödinger, that quantum entities are of such a strange kind that identity makes no sense for them (Schrödinger [Schrödinger, 1996] pp.121-122; see also Arenhart [Arenhart, 2017b]). In this sense, they were thought to be non-individuals, entities indiscernible from others of the same kind, for which identity makes no sense.

That would count as a metaphysical discovery coming from quantum mechanics. However, doing that would be going too fast. It is possible to understand quantum entities also as individuals, provided that the individuality principle is of a kind allowing for qualitative indiscernibility, such as Lockean substrata, primitive thisness, or haecceities (see French and Krause [French and Krause, 2006] chap. 4 for such a claim). In that sense, while the qualities of quantum particles do not account for their individuality, some further principle, over and above their qualitative properties, may do so. In this sense, even numerical identity may be retained.

As a result of those possibilities, quantum entities (as posits of the theory) may be metaphysically dressed in at least two very distinct ways: as non-individuals, entities for which identity makes no sense (there are less radical options for non-individuality too, of course, see Arenhart [Arenhart, 2017b]), or else as individuals according to a venerable tradition, which employs as principle of individuality something transcending the qualitative properties of an item. That is the same pattern we have checked in the previous section: there is an *ontology*, provided by a theory, and then there is a *metaphysical* approach (in this case, two families of metaphysical approaches), incompatible among themselves, but compatible with the theory. In this sense, the metaphysics is not derived from the theory, and the theory does not help us choosing one of the metaphysics.

Metaphysical underdetermination appears again precisely here. Given that quantum mechanics does not give us any clue as to which option we should choose, and typical debates between the two options on what concerns their virtues and vices seems to fall on the typical argumentation present in metaphysical debates, and not to rely on the scientific theory in question, a scientific metaphysician could derive the following conclusion here: that metaphysics simply cannot be derived or extracted like that from physics. However, that, once again, was not the conclusion derived in the literature. Rather, philosophers tried once again to look for ontology and attempted to derive their metaphysics from the ontology. That, of course, would be done in order to keep the possibility of relating science and metaphysics in some respectable way, by somehow determining the metaphysics through the ontology.

Ladyman puts the issue in the following remarkable terms:

> We need to recognize the failure of our best theories to determine even the most fundamental ontological characteristic of the purported entities they feature. It is an *ersatz* form of realism that recommends belief in the existence of entities that have such ambiguous metaphysical status. What is required is a shift to a different ontological basis altogether, one for which questions of individuality simply do not arise. (Ladyman [Ladyman, 1998], pp. 419-20)

Here, the "ambiguous metaphysical status" refers to the fact that it is impossible to determine whether quantum entities are individuals or non-individuals (determine, that is, in purely quantum mechanical terms, of course). Now, instead of seeing the problem as the fact that such a determination cannot be made, Ladyman puts the blame on the specific version of realism being adopted in the case, a realism where it is accepted that quantum mechanics deals with objects (what came to be called "object-oriented" realism). The recommendation is that we forget about objects and choose another ontological basis, one where, it is supposed, metaphysical troubles with underdetermination do not arise, presumably because the ontological basis is able to determine the "fundamental ontological characteristic" of its posits.

The same remarks are made by French and Krause [French and Krause, 2006] p.244. Speaking of the theoretical content of quantum mechanics, they put the dilemma raised by metaphysical underdetermination in this way:

> if that theoretical content is taken to have a metaphysical component, in the sense that the realist's commitment to a particular ontology needs to be articulated in metaphysical terms, and in particular with regard to the individuality or non-individuality of the particles, then the realist appears to face a situation in which there are two, metaphysically inequivalent, approaches between which no choice can be made based on the physics itself. [...] The choice for the realist is stark: either fall into some form of antirealism or drop the aforementioned metaphysical component and adopt an ontologically less problematic position.

Notice: as we have been discussing, it is suggested that the theoretical component of a theory, the one the realist is being realist about, should be articulated in metaphysical terms too, so that questions about individuality or non-individuality should be settled by such a theoretical (or realistic) content. One cannot be realist and leave those issues unsettled! However, as we have argued, it is not the task of the articulation of the realistic content to determine those issues, given that the realistic content only determines the entities one is being realist about (the ontology), not their metaphysical nature.

Anyway, given metaphysical underdetermination on the metaphysical nature of quantum objects, the solution advanced, which should bridge the gap between science and metaphysics, with the posits having no such "ambiguous metaphysical status" concerns the adoption of "an ontologically less problematic position", which means, a shift to ontic structural realism (henceforth, OSR): objects are dropped from the realistic content of the theory, with relations taking their place. Objects are re-conceptualized in terms of relations (see Ladyman [Ladyman, 1998] and French [French, 2010, French, 2011, French, 2014] for further discussion). So, one frames the theoretical content of quantum mechanics in terms of relations, and is realist about those relations. This is supposed to solve the problem of the metaphysical ambiguity of quantum mechanics.

In fact, OSR is advertised as a natural requirement of science, when it comes to a preferred metaphysics. According to Muller [Muller, 2011], science determines at least part of its metaphysical content. This is the *Determination Thesis* (Muller [Muller, 2011], p.230):

> *Determination thesis:* scientific theories are incompatible with some specific metaphysical views; in other words, they do have some definite ontological content.

The determined content, as it should be no surprise now, is that what is real is structure. However, that accounts for the *ontology* of the theory (in a slogan, OSR is usually taken to mean "structure is all there is"). But again, that is still not enough to account for the metaphysical nature of the structures. It is typically said that structures are composed of (or even identical to) a family of relations, so that, in the end, OSR is the view that relations are real. But then comes the same move that French so clearly made before, in the case of wave function realism: how should we metaphysically dress the relations? How do they relate to objects? How to account for the priority of relations over objects?

To illustrate how metaphysical issues enter the stage after the preferred ontological basis is chosen, there is a distinction between eliminative versus non-eliminative versions of OSR. Roughly, eliminative versions of OSR consider that only relations are fundamental, and objects are somehow re-conceptualized in terms of relations, having no independent existence (see French [French, 2014]). According to non-eliminative versions of OSR, objects should not be eliminated: although it must be recognized that they do depend on relations for all of their features, their existence does not depend on the relations (the *Moderate structural realism* of Esfeld and Lam [Esfeld and Lam, 2011] is a version of non-eliminative OSR). Weakly discerning relations are also commonly employed to ground a version of non-eliminative OSR, according to which objects are determined by such relations (see Muller [Muller, 2011]).

Once that is given, there is a common problem for all the versions of structural realism to determine the nature of relations. Are they universals, tropes, or what? Of course, one will wish to have a determinate answer, given that this preferred ontological basis is supposed not to produce any ambiguity as to the metaphysics. Unsurprisingly, there is no clear answer to that question, at least not one that is unambiguously derived from the theoretical content of quantum mechanics when it is conceived in terms of relations. This, of course, should be expected, given that quantum mechanics is silent about that question. So, metaphysical underdetermination enters the picture again (to be fair, Esfeld and Lam [Esfeld and Lam, 2011] address that issue, arguing that relations should be understood as tropes. However, their arguments do not derive from quantum mechanics, but rather from traditional metaphysics!).

Another problem concerns the metaphysical profile of the objects. Contrarily to what was advertised, the problem of identity and individuality was not solved by the shift to another ontological basis. For non-eliminative versions, the problem is clear: given that there are objects, one may wish to know their metaphysical status, and the problem was simply not solved by shifting to OSR. For eliminative versions, the problem also appears once the objects are re-conceptualized in terms of relations. Given that objects must be a kind of by-product of the web of relations, the "nodes" where relations meet, they are there; they are no longer independent entities, but it seems legitimate to ask for the metaphysical profile of such derived objects, and the problem is still alive. In any case, quantum mechanics is as impotent to solve the problem as it was in the previous, object-oriented realism; underdetermination is still present, we have just changed our ontological commitments from objects to relations. In this case, if we are to be coherent with the kind of questions raised for object oriented realism, inquiring for the metaphysical profile of the posits, then we must do the same for relations, which are the posits now, and underdetermination on the metaphysical nature of relations arises as well (see Arenhart and Bueno [Arenhart and Bueno, 2015]).

But the problem does not stop there. Even the nature of OSR that should be adopted is not determined by quantum mechanics. Both sides claim that it is science itself that requires elimination or non-elimination of objects, but it is clear that quantum mechanics alone does no such thing. Let us have a look at two typical arguments, each for one of the sides of the debate.

Esfeld and Lam ([Esfeld and Lam, 2011], p.148), defending the permanence of objects in a structuralist ontology, argue that science itself requires such objects. Among other purely logical and metaphysical objections against eliminativism, they argue that eliminativism is just *too revisionary*, in the specific sense that it is revisionary beyond what is really required by current physics. There is no evidence coming from current physics requiring the elimination of objects as the eliminativist versions of OSR advance, so, the argument goes, we should retain objects in our metaphysics. The objects, of course, are in the context of a structuralist ontology, so they have all their features determined by the structures they are part of. In this sense, these objects end up being mere placeholders for relations, featureless objects. It is precisely against this kind of objects that the eliminativist argues against.

Indeed, French ([French, 2010], pp.105-106) claims that for these objects, these "bare relation bearers", "to have any worth in this context, it needs a physical correlate and there is no physical correlate to this aspect of putative objects". That is, objects having no feature of their own, playing a role of mere placeholders of relations, have no physical role, given that they depend on the relations and structures and are nothing by themselves. In this sense, without a physical counterpart, they should be eliminated, and only relations remain.

These debates (the nature of relations, eliminative versus no-eliminative structuralism), of course, cannot be solved by an appeal to quantum mechanics. The answers are not, as it was thought, obtained from the theoretical content of the theory, or, in other terms, if the theoretical content of the theory is to be articulated in metaphysical terms, then, the realist is again on shaky grounds, given that quantum mechanics does no such thing, even if we shift to a structuralist basis. As Ladyman [Ladyman, 1998] and French and Krause [French and Krause, 2006] have recommended, perhaps we should shift to another ontological basis, or, perhaps, we should not expect that the metaphysical content of a theory should be somehow determined by the theoretical content of the scientific theory. In this sense, those expectations were set too high, and as we have been arguing, metaphysics will not be extracted from science (for further discussions on the kinds of underdetermination plaguing even OSR, see Arenhart [Arenhart, 2017a]).

4. Science as a test field for metaphysical doctrines

So, as we have been discussing, it seems that most of the typical attempts to make metaphysics respectable with the use of science seems to labor on the idea that the realistic content of the theories (which is also called their theoretical content, in this context) should also determine the metaphysical profile of the entities that are posited by that content. As we have seen, even though some authors as Steven French seem to recognize that the metaphysical profile of the entities may be picked from the outside of science, that does not prevent such projects of attempting to directly relate the theoretical content with the metaphysical profile. In fact, those attempts reflect the hope that we could avoid that metaphysics keeps floating free from science: the proposed solution consists in trying to look for another ontological basis, one which could deliver the metaphysics along with the ontology provided by the theoretical content. However, it should be clear by now that this is not feasible: just by changing the ontological basis we cannot grant that the *metaphysical profile* gets determined and sort of "pops out" of the theoretical content. In fact, given a fixed ontological basis, it seems that we are always able to advance further metaphysical options and engender further situations of metaphysical underdetermination. Metaphysics keeps floating free from science. Scientific theories are compatible with different metaphysical dressings, due to the fact that the metaphysics is not derived from science or completely determined by it in the way expected.

But then, it seems, metaphysical underdetermination is an obstacle in the attempts to bring science and metaphysics closer, with the result that there may always be another metaphysical theory that may be plugged into a scientific theory, and no hope for a scientific choice of which to prefer. Isn't such a proliferation of metaphysical theories a danger to metaphysics, just as the critics were claiming, as we presented in the beginning of the paper? The main point of the discussion so far is that we cannot expect to satisfy such a strong demand as to somehow derive metaphysics from science (from the realistic content, anyway), just as we do not expect to derive science from experience. So, along the same lines, just as science is a creative activity, one that must account for experience, metaphysics too is a creative activity, and if we want current science to lend some of its epistemic respectability to metaphysics, we may demand from metaphysics that it should be compatible with science, and allows us to make better sense of it.

Our proposal consists in this recognition that, while we are not able to derive the metaphysics from science, our best metaphysical theories must account for the metaphysical profile of the posits of our best science, being at least compatible with its restrictions. Recall Muller [Muller, 2011], who advanced a *metaphysical determination thesis*, which, as stated, was ambiguous between science determining metaphysics, in the positive sense of providing for it, or in the negative sense of being incompatible with some approaches to metaphysics, ruling them out (see the discussion in Arenhart [Arenhart, 2017a]). Here, given that the positive part seems to be hopeless, we propose to keep only the negative aspect of the thesis, trying to see the benefits of confronting metaphysics with our best science. Given that scientific theories are incompatible with some metaphysical views, we will be able to cut down the number of contenders, and to adjust the metaphysics to suit the theories as we check where the problems lie (for a proposal along those lines, see also Arenhart [Arenhart, 2012]; Morganti and Tahko [Morganti and Tahko, 2017] also propose to use science as a test field for metaphysics).

One may wonder whether this approach is not too small of a progress. Not really. In fact, while at first it seems that metaphysics, floating free from science, is always compatible with science, serious work on the articulation of scientific theories with current metaphysical theories will reveal that incompatibility is more widespread than we thought, with compatibility obtaining in fewer cases than we could anticipate. Indeed, a theory like quantum mechanics is incompatible with most of our well-known articulations of metaphysical theories, and here we shall exemplify the problem with the metaphysical attempts to articulate the notion of a particular entity and some of the difficulties it faces in trying to meet the demands of quantum mechanics. As we shall argue, typical approaches to particular entities have a hard time facing the demands of quantum mechanics, so that they may well be discarded by the theory (to avoid terminological monotony, we shall also call the "particular entities" by "particular objects" or "objects", with "object" synonymous with "item" here, not implying any theory about the notion of object in this paper).

Perhaps one of the most common theories of the nature of particulars concerns the constitution of those entities by their properties and, sometimes, a further ingredient (in most cases a bare particular, as we shall discuss soon). Theories about the constitution of an object answer questions as to how are those entities somehow made up of more fundamental ingredients and which those ingredients are that go into the constitution of the objects. As Demirli [Demirli, 2010] p.2 have put:

> In answering the internal constitution question, we may begin an inquiry about the various categories that go into the composition of individual substances and hope that at the end of this inquiry we will come up with a list of ingredients that constitute various individuals. Just as a certain recipe in a cook book provides us with a list of ingredients and instructions for mixing these ingredients together, we may maintain that the list or the recipe of individual substances — God's recipe book, so to say — will tell us what items from various categories are used, and how these items are combined.

Typical theories doing just that are the bundle theory and the theory of substratum, understood as theories accounting for the constitution of objects. Roughly speaking, according to the bundle theory of constitution, particular objects are bundles of properties and relations (depending on the version of the theory), which are kept together by a compresence relation. Typically, the individuality of the items is granted by the adoption of a version of the Principle of the Identity of Indiscernibles (PII), which grants that two items sharing every property are one and the same (in other words: numerically distinct objects must differ by at least one property). Sometimes, the bundle metaphor is unpacked not in terms of compresence, but in terms of sets of properties (bundles being sets), or even with properties taken as being *parts* of the individuals, in a specific mereological sense. These theories are also theories about the constitution of the particulars, and the argument applies to them too.

The theory of substratum, on its turn, admits — along with the properties and relations — a further ingredient, a bare particular, as a kind of peg on which the properties hang, and which also accounts for the individuality of the object; each object having its own bare particular primitively individuated. The bare particular is responsible for the unity of the entity, for uniting in a single entity the diverse properties instantiated by the entity. Of course, a kind of mystery surrounds the nature of the bare particular, but we shall not enter into that dispute here. What is relevant is that the bare particular is a further ingredient in the composition of a particular object, an ingredient that is not a property, but which is said to bear the properties (again, there are tricky issues concerning instantiation and bearing a property in this case, but we shall not be concerned with them, given that they are irrelevant for our purposes). In this case, the PII may fail: it is possible that numerically distinct items may have all their properties in common, while still being different individuals due to their having distinct bare particulars as ingredients.

In both cases, properties may be understood as universals or as tropes, giving rise to distinct versions of the theories. Also, relations may be admitted to constitute the object or not, depending on the version of the theory. Instantiation of a property is explained by the property being one of the ingredients of the bundle. Absence of the property, of course, means that it is not instantiated by the item in question. This is a rather rough description of those two theories, which are acting both as theories of the constitution of the particulars as well as theories about the individuality of the objects (for an introductory exposition of those two theories, see Loux [Loux, 1998]).

Now, what is wrong with those theories, as they were roughly delineated? Notice, they are theories about the constitution of objects, they somehow describe objects as being built of their properties and, in the case of the substratum theory, of a bare particular. Any property not in the list comprising the ingredients of the object is a property not instantiated by the object. It is precisely this that makes the theories just too strong to suit quantum mechanics. In fact, *every property must either be in the list of ingredients composing the object, or else it is not*. In this sense, for every property, it is determined whether the object in question either has or does not have that property.

Quantum theory, however, is clearly not like that, ruling such approach to properties (which, all in all, seems to work fine in classical physics, it seems). Indeed, quantum mechanics obeys the so-called *Kochen-Specker theorem*: basically, one of the consequences of the theorem is that one cannot coherently think of properties in quantum mechanics as pre-existing objectively. For any object of quantum theory, it is not possible to go through all the possible properties it could have in quantum mechanics and assign them as being had or not had by the object, completely determining the object by its properties. Only state independent properties are somehow "objective", independent of the context of measurement (or of a selected basis to represent the state-vector).

As a result of the Kochen-Specker theorem, a state vector cannot really represent an entity, if an entity is to be somehow comprised of all of its dynamical properties, as the theories of composition seem to require. That puts severe limitations on any account of particulars which requires that an entity either has or does not have any specific property that could be attributed to it (see de Ronde and Massri [de Ronde and Massri, 2016] for a full account of the impact of the Kochen-Specker argument on independently existing properties). Objectively, once a context is selected, only the properties that are compatible with the property being measured may be said to have an independently existing reality. All others are in a kind of limbo of uncertainty. However, according to the constitution theory, as delineated above, all other properties, not being part of the constitution of the object, are not instantiated by it. Anyway, the theories of composition examined require a positive assignment of properties to objects or lack of them that is just incompatible with quantum theory.

So, quantum theory seems to be incompatible with a large class of metaphysical theories accounting for the composition of objects. None of those theories could be derived from quantum mechanics, of course; but more may be said, because they are shown untenable with a help of quantum mechanics (reasoning with macroscopic objects would not, we presume, allow us to rule them with such a safety, at least not if we use classical mechanics to describe the behavior of such objects). Once we stick with quantum mechanics, those theories of constitution should go.

Now, there is the option of keeping those theories for the task of granting individuality, but abandoning some of its aspects (perhaps the very idea that properties constitute the objects, or that one must have a complete list of properties constituting the object). In this case, we may also be sure that the traditional version of the bundle theory also fails (see French and Krause [French and Krause, 2006] chap. 4). Recall that the theory requires that a version of the PII should obtain. Now, quantum mechanics violates some versions of the PII, as we have already commented, by requiring that quantum entities be indiscernible by both their properties as well as relations, *except for weakly discerning relations*. Anyway, those weakly discerning relations will not save the bundle approach to individuality. Typically, relations are not good candidates for accounting for the individuality of objects; it seems that relations already require the relata individuated. That is even more serious with weakly discerning relations, so that the version of the PII that may hold in quantum mechanics is just too weak to account for the individuality of the particles (see Lowe [Lowe, 2015], Arenhart [Arenhart, 2017b]).

The theory of substratum may still be applied to individuate quantum entities, provided that it is not understood in the context of a theory of constitution, which was already seen as implausible due to the Kochen-Specker. One possible obstacle furnished by quantum mechanics to the substratum theory would appear if it could be argued that a substratum always requires a well-determined position in space time. In fact, that clearly seems to be the case, given that, according to the theories of substratum, any other property, even the state-independent ones, should be tied to the substratum. So, the substratum must always be there to instantiate those properties. If the substratum could exist without being anywhere in space and time, then it would be a mystery how it may work as the entity that accounts for the unity of the particular objects, as it is supposed to do, and how the state-independent properties are said to be objective. It would be senseless to speak of properties being hanged together by the substratum, it seems. If that is the case, then even substratum theories are ruled out by typical understanding of quantum entities, due to the fact that position is not always well-defined for those entities. Perhaps only a Bohmian-like approach would be compatible with the existence of a substratum.

What is relevant for us is that quantum mechanics seems to refuse much of the typical approaches to objects as offered in traditional metaphysics. A careful articulation of the theories with quantum mechanics promises to deliver incompatibilities which may teach us interesting lessons as to how to better pursue those metaphysical theories, or theories replacing those that failed. And, what is more relevant, this establishes a link between the science and the metaphysics, without false hopes of attempting to extract the metaphysics from science.

5. Conclusion

For a conclusion, let us recover the major steps and conclusions of the paper. We have presented here a two-step diagnosis of what is typically treated as the current naturalistic approach to metaphysics (as a warning note, naturalism in metaphysics is also taken to mean a scientific way to develop metaphysics). Both steps of the diagnosis are completely related, and we have distinguished them only for the sake of exposition.

The first step consists in identifying the conflation of metaphysics and ontology. As a result, typical methodology employed to study ontology is thought to work for metaphysics *per se* too. After Quine's major contribution to ontology, it is usually thought that the analysis of scientific theories may deliver the ontological commitments of those theories, and given that no one wishes to ignore science, this is the place to look for the fundamental ontology of the world. So, that's the link between science and metaphysics.

We have argued that this is not enough, given that metaphysics comprises a much broader area of investigation, whose subject matter is much more general. Basically, we have resorted to the traditional definition of ontology as a field of metaphysics, with metaphysics dealing with far more issues than ontology. So, even if we could solve ontological problems (that is, determine the ontology of our currently best scientific theories), metaphysical problems would not be all settled. Following Steven French's examples on wave function realism and his discussions on identity and individuality in quantum mechanics, we have argued that after the ontology is settled, there is an extra layer of metaphysics that must be added; the metaphysical profile of the entities is still to be determined.

There comes the second step in the diagnosis. After determining the ontological commitments of a theory, one has somehow managed to identify its posits, some of which are eminently theoretical (even though the distinction is a complicated one, but that is not relevant here). The theoretical content of a theory, as part of the realistic content, the content about which one is being realist, is just part of the ontological commitments. As a result of the first diagnosis, one should not expect that this is all that there is to metaphysics. But as we have seen, the major attempts are to derive the metaphysical profile of the posited entities from the realistic content. The posits should somehow be characterized metaphysically, and, as friends of structural realism have emphasized, if the theory is unable to determine the metaphysics, one should change the ontological basis, that is, one should shift the realistic content to another kind of content. It is expected, of course, that a distinct ontological basis could deliver the metaphysical profile, making a case for the extraction of a metaphysical profile from the theory itself, as the theory is responsible for the ontological basis.

Again, this is a continuation of the first strategy to obtain metaphysical profile from ontology. By the same kind of argument it is not feasible. Even in the case of a change of ontological basis, as suggested by OSR, it is always possible to provide for distinct metaphysical profiles for the entities posited. That only makes a stronger case for the argument that metaphysics cannot be derived from science, but is an added layer of determination of the entities.

At first sight, it may be the case that the impression left from this discussion is that metaphysics is a rather arbitrary endeavor, with no relation to science. Metaphysical underdetermination could proliferate at will. In order to avoid this pessimistic conclusion, we have suggested that metaphysicians should work closer to science, attempting to articulate their views having scientific theories in mind. The idea is that when it comes to elaborate a relation between science and metaphysics, it is possible, sometimes, to determine how *not* to articulate the metaphysical nature of the posited entities. We have illustrated the issue with quantum mechanics, and it seems that this theory will rule as implausible a great number of metaphysical theories. Quantum mechanics acts as a test ground for metaphysical theories, and we have argued, by way of example, that so-called constitution theories seem to fail to account for quantum entities.

However, underdetermination still obtains for those views that are not eliminated. That will call for even closer articulation and metaphysical study. Underdetermination is not a problem *per se*. It is a misguided belief that science settles all of its questions undoubtedly that prompted some to demand that metaphysics do the same, or else to leave the battle field of the fight for objective knowledge. But that is asking too much. Science too is full of problems and controversies. What is left for us is to continue the investigation.

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2

On Quine's Ontology: quantification, extensionality and naturalism (or from commitment to indifference)*

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1. Introduction

Two of the Quinean metaontology's most basic theses which are so well accepted that, according to van Inwagen (2009), became the methodological foundations of a whole ontological tradition, say:

Existence is univocal. (1)

The single sense of existence is adequately captured by the existential quantifier. (2)

If existence is univocal, then there are no different modes of being. Any existing thing exists precisely in the same sense as anything else. If besides, this unique sense of existence is captured by the logical quantifiers, then there is only one unrestricted range of quantification covering everything that there is. If numbers, stones and attributes exist, then the same variable 'x' can take values among numbers, stones and attributes. There are no multiple types of variables, but a single one. So, theses (1) and (2) lead Quine to the following thesis:

* This article is a further development of a material published in Session 6 of a previous paper of mine on ontological commitment. (Durante, 2014). Despite some minor overlaps, the content here presented develops different arguments in more detail.

There is a single and unrestricted domain of quantification that covers all there is. (3)

The admission of a single and unrestricted domain of quantification helps explain the simplicity of the famous slogan "*to be is to be the value of a variable*" (Quine, 1963c, 15), but it also requires Quine to be careful with what may be among these values, i.e., with what may exist. I want to relate this idea of *ontological caution* to some of the most controversial theses of Quine, such as:

(*a*) His insistence on accepting only first-order quantification and his rejection of higher-order logic.

(*b*) His resistance on accepting the intensionality of ontological commitments.

(*c*) His rejection of first-order modal logic.

(*d*) His rejection of the distinction between analytic and synthetic statements.

I intend to argue that the ontological caution required by principles (1), (2) and (3) imposes to all theoretical discourses a requirement of extensionality as protection and that these controversial theses are just interconnected consequences of this requirement. Moreover, I will also argue that extensionality and even principle (1) of the univocity of existence are not self-justified in Quine's approach. They are consequences of a more basic metaphilosophical principle: Quine's naturalism. So, the metaphilosophical thesis of naturalism and the methodological thesis (2) of the binding between existence and logical quantification, which took in isolation don't seem so harmful, are the main responsible factors for all controversial Quinean theses listed above.

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One of the primary sources of criticism in Quine's work is the strictness of the canonical notation in which theories must be regimented to have their ontological commitments evaluated. The ontological commitments of a theory are the entities it assumes exist and, according to Quine,

> entities of a given sort are assumed by a theory if and only if some of them must be counted among the values of the variables in order that the statements affirmed in the theory be true. (Quine, 1963a, 103)

However, talking about the values of variables of a theory presupposes it to be regimented in a formal language since there are no variables in natural language. Moreover, this formal language where a theory must be regimented before any evaluation of its ontological commitments is, for Quine, the language of first-order classical logic. He argues that we must be able to regiment in a first-order language whatever deserves to be called a theory. But why not using second-order logic with its richer language? Why not allowing quantification over predicates and also relations and thereby to expand the possibilities of the being through an expansion of what can be a value of a variable?

This alternative, however, is not open to Quine. It is incompatible with thesis (3), the admission of a single and unrestricted domain of quantification, because if he admits second order, i.e., if he admits quantification over predicates and relations, then he immediately gets Russell's paradox. As Potter (2004, 300) has well pointed, there are only two ways to avoid Russell's paradox in second-order logic: either abandoning the universality of the domain of quantification, thesis (3), or abandoning extensionality, i.e., considering that there can be predicates 'G' and 'F' that are true precisely of the same individuals, but are (conceptually) distinct: $\exists G \, \exists F \, (\forall x \, (F(x) \leftrightarrow G(x)) \wedge (G \neq F))$.

These, however, are two concessions Quine is not willing to do. As we have seen, the absolute generality of the domain of quantification (3) is a consequence of the thesis of the univocity of being (1) and of its translatability into existential quantification (2). So, rejecting (3) requires a rejection of either (1) or (2). Yet (1) and (2) are among the most basic principles that underlie his entire philosophical project. (1) is his most basic mataontological thesis and (2) is his most basic methodological thesis.

The other possibility of avoiding Russell's paradox in second-order logic is also not an option for Quine. Giving up extensionality is to abandon what he considers the minimum requirement for acceptance of any entity, his standard of ontological admissibility, which is founded on the principles of identity, mainly on the indiscernibility of identicals. The most obvious clue for Quine that a supposed entity doesn't exist presents itself when the principle of substitutability *salva veritate* is violated in statements containing terms intended to refer to these supposed entities. In other words, if the assumption of some supposed entities in our ontology requires an intensional semantical context, then this is the best evidence we can have of the non-existence of those entities (Durante, 2011, 35–36). As his other famous slogan says, "*no entity without identity*" (Quine, 1981a, 102).

> [W]hat sense can be found in talking of entities which cannot meaningfully be said to be identical with themselves and distinct from one another? (Quine, 1963c, 4)

> Extensionalism is a policy I have clung to through thick, thin, and nearly

> seventy years of logicizing and philosophizing (Quine, 2008, 215)

The only choice left to Quine to avoid paradoxes without giving up his most basic theses is, therefore, to restrict theory regimentation to first-order classical logic, abandoning both, second-order quantification and intensional contexts. Nonetheless, that does not mean assuming a nominalistic position that simply denies the existence of any abstract entity. Quine accepts and even argues for abstract entities, provided they are extensional. (Quine, 1963a, 115).

> [M]y extensionalist scruples decidedly outweigh my nominalistic ones. (Quine, 1986, 397)

However, the only legitimate abstract entities Quine accepts are obtained by transforming an equivalence relation into identity. In this case, the class of indiscernible individuals according to the equivalence relation (the equivalence class) is individuated as a single abstract object whose nature is different from the nature of the indistinguishable individuals that compose it (Quine, 1963a, 117). If, for example, our individuals are three-dimensional physical objects and we aggregate them in equivalence classes composed of objects with the same volume, then to treat this volumetric equivalence as identity represents a change of ontology. It means stop talking about concrete three-dimensional physical objects and start talking about abstract volumes. Whatever property such an equivalence class has, or relation it engages in, it will be a property or relation of an abstract volume, not of concrete physical objects. The individuation movement that reifies this abstract entity is obviously driven by the principle of identity of indiscernibles, and it is allowed by Quine only because it respects the law of indiscernibility of identicals, which is a criterion for extensionality.

So, to ensure that our theories can deal with these abstract entities which, in turn, comply with the restrictions of extensionality, Quine enriches the language of canonical notation with a binary predicate for membership, '$\in$', and adds the axioms of his set theory **NF** (Quine, 1963b) to the ones of first-order classical logic.[1] This formal system is, according to him, the only one we need for all theoretical discourses.

Our theories then ontologically commit to abstract entities only when they are explicitly regimented in **NF** as classes. When we say, for instance, that some dogs are white through the following regimented sentence,

[1] Moreover, Quine removes individual constants from canonical notation, adopting Russell's theory of descriptions as a substitute for singular terms. Thus, variables become the only vehicle for reference, which assures formal correction to its famous motto about being: "*to be is to be the value of a variable*".

$$\exists x \, (\mathsf{Dog}(x) \land \mathsf{White}(x))$$

we do not commit ourselves to abstract entities such as dogness or whiteness. To assume such commitments, according to Quine (1963a, 113), we have to interpret dogness and whiteness as classes and state that something is a member of both of them:

$$\exists x \, (x \in \mathsf{dogness} \land x \in \mathsf{whiteness})$$ [2]

So regimented, however, abstract entities are constrained by theory **NF**, which while keeping them both extensional and protected from paradoxes,[3] it accepts the universal set, and it is, therefore, compatible with thesis (3) of unrestricted quantification, which is so fundamental to Quine.[4]

This helps us understand another heavily criticized issue of Quine's philosophy: his insistence on the extensionality of ontological commitments. If the formal methods Quine accepts for theory regimentation avoid intensional contexts, and if the ontological commitments of a theory are certain values of its variables, those who are required for the truth of the theory's statements, then, as much as the values of variables are protected from intensionality, the ontological commitments should also be so protected. It was probably this trivial reasoning that led Quine to say without further arguments that "the question of the ontology of a theory is a question purely of the theory of reference" (Quine, 1951,

[2] I use 'dogness' and 'whiteness' as names for the classes of dogs and white stuff respectively, just to shorten the formalization in canonical notation, which by requiring the removal of names and their replacement with descriptions, would have the following longer and less elegant form:

$$\exists x \, (\exists y \, \forall z \, ((\mathsf{Dogness}(z) \leftrightarrow y = z) \land x \in y) \land \exists u \, \forall w \, ((\mathsf{Whiteness}(w) \leftrightarrow u = w) \land x \in u))$$

There are ontological commitments to the classes of dogs and white things (dogness and whiteness) because these classes are the values of the variables 'y' and 'u' required to make the sentence true. It is worth noticing that 'Dog' is the predicate satisfied by only and all dogs and 'Dogness' is the predicate satisfied by only one entity, the class of all dogs. I.e., $\exists y \, \forall z \, ((\mathsf{Dogness}(z) \leftrightarrow y = z) \land \forall x \, (\mathsf{Dog}(x) \leftrightarrow x \in y))$. The same holds for 'White' and 'Whiteness'.

[3] Up to now, nobody has found any contradiction in **NF**, but as a consequence of Godel's theorem, we know that without compromising suppositions there is no way to ensure that any formal set theory is consistent (free from contradictions and paradoxes). So, the protection against paradoxes that **NF** or any other set theory provides isn't perfect. But it is as good as it can be.

[4] The formalized set theory most commonly used is Zermelo-Fraenkel set theory which, however, would not serve Quine's purposes because its methods to avoid Russell's and other paradoxes preclude the admission of a universal set and therefore impose restrictions on the domain of quantification.

15).

However, discourses on values of a theory's variables are local; they are concerned only to the theory and need not be relativized. They are internal, and its intelligibility requires no metatheory. To this extent, Quine's precaution protects them from intensionality. On the other hand, discourses on ontological commitments are external to the theory. They occur in ontological debates whose reasoning demands a metatheory with more sophisticated formal tools than Quine allows for theories themselves.

It is not difficult to see that the very same phenomenon of referential opacity Quine uses to reject first-order modal logic (Quine, 1963d) occurs with the notion of ontological commitment. If we try to theoretically accommodate the notion of ontological commitment under Quine's extensional canonical notation we immediately get in trouble. Suppose, for instance, an ontological debate between the holders of these two theories:

$\mathbf{T_1}$	$\exists x\, \mathsf{Angel}(x)$	—	(*There are angels*)
$\mathbf{T_2}$	$\neg\exists x\, \mathsf{Angel}(x)$	—	(*There are no angels*)

If we try to state this debate in an extensional theory of ontological commitment formalized on Quine's canonical notation, the views of these two theories' proponents could be stated respectively as: [5]

$$\exists y\, \exists x\, (\mathsf{Angel}(x) \land \mathsf{ComT}_1(y) \land x \in y) \tag{4}$$

$$\exists z\, \forall x\, ((\mathsf{Angel}(x) \land \mathsf{ComT}_2(z)) \to x \notin z) \tag{5}$$ [6]

Yet as a simple logical consequence of (4) and (5) we have:

$$\exists z\, \exists x\, (\mathsf{Angel}(x) \land \mathsf{ComT}_2(z) \land x \notin z) \tag{6}$$

[5] There are many different forms to express in an extensional way the relation of a theory $\mathbf{T_i}$ to its ontological commitments. I just choose to use predication $\mathsf{ComT_i}(y)$ to characterize that y is the class of the ontological commitments of $\mathsf{T_i}$, and to use **NF**'s membership relation to state that the members of the class y such that $\mathsf{ComT_i}(y)$ are exactly the ontological commitments of $\mathsf{T_i}$.

[6] We can see that (5) is a fair way to state $\mathbf{T_2}$'s holders view because besides assuring that there are no angels among the ontological commitments of $\mathbf{T_2}$, (5) itself doesn't assume any commitment with angels while saying that. It says that truly even when the extension of 'Angel' is empty.

However, (6) does not do justice to the position of the holders of $\mathbf{T_2}$. They would immediately disagree that there are angelical things they don't assume as ontological commitments. It was precisely this disadvantageous situation of claims of non-existence in an ontological debate that motivated Quine to propose the notion of ontological commitment at first place (Quine, 1963c). [7]

This situation illustrates that in ontological debates we cannot treat ontological commitments the same extensional way we treat the values of interpreted theory's variables. Values of variables can be thought extensionally as the things themselves, but ontological commitments can't. Otherwise, the metatheory would ontologically commit itself with all commitments of the debating theories, and this would not help to solve the debate in a neutral way. Such an extensional notion of ontological commitment would be completely useless. It would let Quine in the same situation from where he started: hung on Plato's beard.

When I say that theory $\mathbf{T_1}$ is ontologically committed with angels, I'm not talking about angels, after all, they may not exist. Instead, I'm talking about concepts, intensions, meanings of angels. And as Cartwright (1954), Scheffler and Chomsky (1958), Parsons (1967), Jubien (1972), Chateaubriand (2003) and others have shown, there is no way to make clear this difference in an extensional theory like **NF**.

Therefore, although regimentation resources allowed by Quine ensure that ontologies of all regimented theories are extensional, still the notion of ontological commitment will not be, because it requires us to put theories and the entities they assume into perspective, in a situation where they can be compared in the same way we do in an intensional semantics of possible worlds. Wherever we have formal resources strong enough to describe different universes and to compare them and the way they affect truth and falsity of sentences, we will be out of extensionality. These resources are not only required by first-order modal logic, but they are also a minimum requirement needed to make any discourse

[7] Someone could protest, saying that we should have to have two predicates for angels, 'Angel_1' in $\mathbf{T_1}$ and 'Angel_2' in $\mathbf{T_2}$. If so, sentence (6) would turn into (6'): $\exists z\, \exists x\, (\mathsf{Angel}_1(x) \wedge \mathsf{ComT}_2(z) \wedge x \notin z)$, which is committed with the existence of angels in $\mathbf{T_1}$'s sense, but not in $\mathbf{T_2}$'s sense. In such a situation the ontological foes have two options: they can agree on the equivocality of their debate. They are not talking about the same thing. But such agreement implies (through (6')) that $\mathbf{T_2}$'s holders concede that angels in $\mathbf{T_1}$'s sense do exist. Then, the debate has changed nothing on the view of $\mathbf{T_1}$'s holders. It is a victory for them. The other option is denying the equivocality of the debate. There are no two different senses of angel. Then, we don't need two predicates 'Angel_1' and 'Angel_2', but only one, 'Angel', which take us back to the situation where (6) ensures again the victory for $\mathbf{T_1}$'s holders.

about ontological commitments intelligible. Thus, even in a situation where all admissible ontologies are extensional, there is no way to present a clear notion of ontological commitment that is also extensional. So, the very same reason Quine has to reject first-order modal logic is a reason to reject the notion of ontological commitment.[8]

Perhaps because he realized this fact, after the end of the sixties Quine stopped talking about ontological commitments. Instead of trying to answer his critics, he focused his attention on proposing a minimalist and extensional ontology consisting only of classes, which he thought was suitable for all science. He didn't explicitly reject the notion of ontological commitment or change his view on the subject, but, as suggested by Chateaubriand (2003), he regarded the discourse on ontological commitment as just a way of talking. If discourses about ontological commitment require intensional contexts, then these discourses can't be regimented in his canonical notation, and therefore they are not theoretical. They are, at best, just a loose way of talking.

But this is a serious issue for Quine because his notion of ontological commitment is the primary methodological tool he designed to assure rationality to ontological debates. Giving up ontological commitment is to abandon the possibility of presenting a rational and conclusive philosophical argument that decides on alternative ontologies. No wonder that the ostracism Quine put the notion of ontological commitment began in the same period in which he proposed his views on the inscrutability of reference and ontological relativity. As his famous proxy function argument has settled, in many cases "there can be no evidence for one ontology over against another" (Quine, 1992, 8).

Quine's entrenched "extensionalist scruples" are responsible not only for his rejection of first-order modal logic and of higher-order logic. As we just have seen, they are also the reason why he lost interest in the notion of ontological commitment and adopted the thesis of ontological relativity and inscrutability of reference. Not only that, but extensionalism is also a major motivation for Quine to reject the distinction between analytic and synthetic judgments. According to his famous arguments from "Two Dogmas of Empiricism" (Quine, 1963e), such a distinction would depend on the establishment of a theory of meaning founded on a notion of synonymy whose intelligibility would be extensionally unscrupulous by requiring an intensional context.

[8] We, of course, don't need to do that, as Church (1958) showed, even being intensional the idea of ontological commitment still useful and fundamental to ontology. Quine's extensionalism, however, leaves him no other option but abandon his own notion of ontological commitment.

Why Quine advocates so fundamentally this demanding thesis of extensionality, rejecting any notion that contravenes it? We have already seen that Quine uses extensionality as a standard of ontological admissibility, excluding any supposed entity whose intelligibility requires intensional contexts. Our question now is why does he do it? Being extensionality such a demanding thesis that leads to all these controversial rejections, what kind of gain it offers? What justifies Quine's extensional scruples?

I'll answer this question in two steps. First, Quine's defense of extensionality and consequent rejection of the notions of meaning theory and of everything that requires intensional contexts can be understood as a more fundamental compromise with thesis (1) of the univocity of existence. Any formal treatment of intensional contexts requires one of the following alternatives: (*a*) a logic explicitly of higher order (second or greater) and, along with it, the typifying of variables and the irreconcilable separation of distinct domains of quantification that is necessary to prevent Russell's paradox; or (*b*) the use of non-truth functional operators whose occurrence in formulas introduces contexts of referential opacity, whose quantification into, as in '$\exists x \Box P(x)$', demands the admission of more subtle modes of existence that are distinct from the mode of existence of actual entities. Alternative (a) is a direct violation of thesis (3) therefore it violates at least one of its premises: theses (1) or (2). However, the own statement of alternative (a) is according to thesis (2), after all, we are talking there of a formal treatment whose quantifiers and variables are our ontological resources. Then, it is clear that alternative (a) violates the thesis (1) of the univocity of existence. It is also clear that alternative (b) violates (1) too, because if we live in a world where there are no angels, but there could have been, then the status of angels as non-actual but possible existent beings is a sense of existence clearly different from the sense of my or your existence, which is actual.

This leads us to the second step of our response, which starts with another question: why Quine advocates in such a fundamental way the thesis (1) of the univocity of being? Why not admitting alternative modes of existence catchable by distinct domains of quantification? It would suffice to accept this possibility to be allowed accommodating both intensional contexts and the use of higher-order logic. This not only could soften a little the austerity of his narrow canonical notation, as well as it could free him from several criticisms, objections and prohibitions.

The answer lies in the naturalism of his conception of philosophy. Quine's naturalism mainly means that he sees no essential distinction between philosophy, mathematics, and science. Philosophy does not legislate or regulate science or mathematics but collaborates with them.

> [I]t is within science itself, and not in some prior philosophy, that reality is to be identified and described. (Quine, 1981b, 21)

> The philosopher's task differs from the others' then in detail, but in no such drastic way as those suppose who imagine for the philosopher a vantage point outside the conceptual scheme he takes in charge. There is no such cosmic exile. (Quine, 1960, 275)

We can interpret that violation of thesis (1) contradicts naturalism. The admission of distinct ways of being that could be addressed by different sorts of variables confined to distinct types of quantifiers, which therefore would not be absolutely generic, opens space for a fundamental separation between philosophy and the rest of science. While to science would correspond the sense of being connected to individuals, actual beings and extensional abstractions, to philosophy would fit the sense of being connected to intensional contexts, meanings, and non-actual universes. The incommunicability between the domains of quantification could protect and insulate philosophy in an inadmissible "cosmic exile".

For Quine, there is no place for philosophy outside the same conceptual scheme we use to do science. As much as (and for the same reasons that) we can change our scientific theories and paradigms, we can also change our philosophical claims. There is no analyticity nor *a prioriness* protecting philosophical claims from possible revision. However, if we allow multiple modes of being, if we give up the univocity of existence, then we open room for this kind of separation. The realm of intensional beings, for instance, would be untouchable by recalcitrant empirical observations. It would demand another way of thinking, another conceptual scheme, which is forbidden by Quine's naturalism.

In short, the two steps of our answer to the question of why Quine defends extensionalism so firmly is that (i) extensionalism is required to ensure the thesis of the univocity of being and (ii) the univocity of being, in turn, is required to ensure naturalism, which is his most fundamental metaphilosophical thesis.

Although, as we have seen in the case of ontological commitments, it is not that easy to constrain our philosophical discourses to the same conceptual scheme Quine has devised for science and knowledge in general. Even under his austere regimentation requirements, intensionality and the need for separation of modes of being shows up in most of the philosophical discourses we engage.

Quine has never responded directly to the critics who pointed out that intelligibility of his notion of ontological commitment requires an intensional context, and after the end of the 1960s, he seldom furthered the subject. Instead of solving the problems caused by extravagant ontological commitments of artificially constructed outlandish theories, he has chosen to focus his attention on the substantive proposition of a minimalist and extensional ontology consisting only of classes, which he thought was suitable for all science. He also proposed that only scientific theories should have its ontological commitments evaluated.

We could, therefore, to accuse him of trying to remove from the scope of philosophical considerations legitimate questions that not only should be there, as have been there throughout history. After all, we *conceptualize* and we *mean*. Not only that, but we also think, consider, believe, forbid, doubt, theorize, allow, conceive, admit, *assume* and so many other activities that lead us to intensional contexts.

When confronted with such charges, Quine's answer is radical, almost impolite. It is a stark commitment to naturalism, which justifies us to regard it as his most fundamental thesis and reminds us that, despite having exceeded the ideas of logical positivists in many ways, he kept for himself the same project of philosophy that inspired the investigations of his teacher Carnap and other philosophers of the Vienna Circle:

> If certain problems of ontology, say, or modality, or causality, or contrary-to-fact-conditionals, which arise in ordinary language, turn out not to arise in science as reconstituted with the help of formal logic, then those problems have in an important sense been solved: they have been shown not to be implicated in any necessary foundation of science. [...] Philosophy of science is philosophy enough. (Quine, 1953, 446)

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3

What Logical Pluralism Can be About

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1. Introduction

There is a general trend when talking about logical pluralism to do so in terms of the application a logic might have and how more than one logic can be correct for that application. In this paper, this view is found wanting and a more fundamental approach to pluralism in logic is proposed, one which focuses on formal and consequence-related aspects of the study of logic.

Generally, a logic may be defined as a relation among sets of formulas of a certain language. The (propositional) language is constituted of atomic propositions, which are the smallest components of the language, and logical connectives, which glue together atomic propositions into more complex formulas in a recursive way. Given a language $\mathcal{L}$ and a consequence relation $\Rightarrow$, a logic is a structure $\langle \mathcal{L}, \Rightarrow \rangle$. A consequence relation is a relation between a set Γ of formulas of the language $\mathcal{L}$ and a formula δ (called "conclusion"), such that $\Gamma \models \delta$ (δ follows from Γ) if given a set of truth values ($\mathcal{V}$) such that each formula is assigned a truth value, there is a distinguished set of truth values ($\mathcal{D}$, where $\mathcal{D} \subseteq \mathcal{V}$), whenever every formula in Γ is assigned a value in $\mathcal{D}$, then δ is also assigned a value in $\mathcal{D}$, that is, there is a preservation of this distinguished set of truth values from the premises to the conclusion[1].

In recent decades, there has been a great deal of work done on the topic of non-classical logics, which are logics that reject some core aspect of so-called "classical" logic. These logics have different consequence relations than classical logic, and sometimes a different language. Work on non-classical logics led

[1] For more generality take this definition to be multiple-conclusion or over multi-sets, as desired.

to a recent interest within the philosophy of logic on the question of logical pluralism. Logical pluralism would be the opposite of logical monism, the view which claims that there is only one correct logic. It is then the view that there is more than one correct logic. Yet what might the multiplication of correct logics involve? This view may be further divided into logical pluralism proper, or simpliciter, and logical relativism. Logical relativism amounts to taking a logic to be correct relative to some parameter, while logical pluralism amounts to accepting more than one logic as correct for the same parameter [Shapiro, 2014]. There are plenty of pluralist theses in the literature, and to make some sense of them, the taxonomy of pluralist theses offered by Roy Cook [Cook, 2010] and Graham Priest [Priest, 2005] prove useful.

Both authors present taxonomies of kinds of pluralism based on different ways of making sense of what logical pluralism could mean. The starting point is simply to admit that there are many pure logics, that is, there is simply more than one formal system of logic. Both authors agree that this claim makes for a trivial and uninteresting kind of pluralism. Another kind of pluralism comes in when an application is fixed and a pure logic is applied by interpreting it in some way, and in this respect more than one logic may be appropriate. An example of a similar enterprise is how the different kinds of pure geometry (Euclidean, hyperbolic and elliptical, for instance [Priest, 2005]) become applied geometries.

While Cook admits such pluralism, Priest dismisses it as a kind of relativism, claiming that, following some criteria of evaluation, one logic will turn out best for that application. For Priest, there are ways to evaluate which logic is best suited for a specific application, following criteria such as adequacy to the data, simplicity, consistency, power, and avoidance of ad hoc elements. Priest indicates that the next level of pluralism comes in when one thinks of the "canonical application" of a pure logic, namely, its application to the analysis of reasoning. In this form of pluralism, there is a translation from the vernacular language to a formal language, and "a vernacular inference is valid iff its translation into the formal language is valid in the pure logic" [Priest, 2005, p. 196]. Yet according to him, there is but one logic which is best suited for such an application and no actual cases of pluralism.

Cook goes further than Priest, articulating a kind of pluralism in which there is more than one logic, $\langle \mathcal{L}, \Rightarrow \rangle$, for which, there being a mapping from natural language into the formal language $\mathcal{L}$, an argument is valid if and only if its premises logically entail its conclusion according to $\langle \mathcal{L}, \Rightarrow \rangle$. In this case, there are many ways to make such a translation, pluralism simply having more than one language for which this translation works. Cook judges this pluralism frivolous, inasmuch as the dispute is among translations, not in the form of reasoning. The sole serious form of pluralism for Cook is that in which, when there is "one fixed formal language and logical/non-logical divide, there is more than one logic that correctly codifies logical consequence in natural language" [Cook, 2010, p. 497], which is to have at least two logics, with different consequence relations, for the same formal language.

2. Pluralism dissected

From this brief survey, it seems there are three key notions in play when discussing the topic of logical pluralism: language, validity and application. Language is the most straightforward way in which two logics can be said to differ, but a simple syntactic difference seems insufficient to satisfactory encompass the way in which there is plurality in logic. The second way in which logics can differ, with regard to validity finds this difference in the notion of consequence ($\Rightarrow$) which each logic employs. A third way comes in when there is talk about how a logic becomes applied to some discourse or purpose. These approaches to pluralism will be examined in order to determine the most fruitful and interesting from a logical point of view.

First, one may attempt to formulate a logical pluralist thesis based on a multitude of languages. It may be asked whether the languages in question are formal or ordinary. The answer must be the former, because logic can at best serve as a model of ordinary language. If the debate is one about models, then the most reasonable position regarding logics is relativism, for there is a way in which a certain language can be best relative to a given model. The approach through language must be then made through a plurality of formal languages. Rightly, both Cook and Priest reject this approach as a serious form of pluralism. The kinds of pluralism which emanate from a difference of languages suffer from what has been called "the meaning variance problem", proposed by Rudolph Carnap [Carnap, 1937] and W. V. O. Quine [Quine, 1986]. Meaning variance occurs when the difference between logics can be traced back to a difference in languages, since there is no way to guarantee that both logics deal with the same subject, there is no real dispute between the logics in question. If two logics are not in dispute, they are not apt for being a case of pluralism. A pluralist approached through language is mere verbal dispute, not genuine disagreement, not the kind of pluralism of interest to a logician anyway[2]. Studies in this area

[2] This kind of pluralism is akin to "global pluralism" [Haack, 1978, p. 223].

can be fruitful for other purposes, but plurality of language does not seem to entail logical pluralism.

Second, there is pluralism approached through the plurality of ways in which an argument can be valid. Here the difference is due to the logical consequence relation, and not the formal language, thus avoiding the meaning variance problem. Since a logic has been defined as a structure $\langle \mathcal{L}, \Rightarrow \rangle$, if the difference is not in $\mathcal{L}$, it must be in $\Rightarrow$. Pluralist proposals which fit into this group include Restall's pluralism based on proofs [Restall, 2014], Hjortland's 3-sided sequent system [Hjortland, 2013], Paoli's "genuine rivalry criteria" [Paoli, 2003], and Beall and Restall's GTT pluralism[3] [Beall and Restall, 2006]. These proposals do not make any claims about application prior to the presentation of the pluralist framework. Frameworks are "neutral" in some sense; and questions of application to other areas are addressed later, as with Restall's interest in different propositional attitudes speakers might hold [Restall, 2005]. This approach to pluralism offers its own questions and challenges: the meta-Quinean challenge [Hjortland, 2014], the issue of framework-dependence (particularly on sequent calculus), issues about the meaning of logical connectives, and discussion on structural properties of logics and the nature of logical consequence.

The proposal of pluralism which relies on a multitude of logical consequence relations is the most fruitful one from the standpoint of logic, as it reflects better the notion of plurality in logic (as opposed to mere plurality of logics). The main lesson to be learned from the different frameworks mentioned above is that there is a plurality of notions of validity not only among different logics, but within a single system. Taking this approach forward, there is still lots of work to be done by logicians and philosophers alike.

Third and lastly, there is the approach to logical pluralism which focuses on application. On this account, there is a case of pluralism when more than one logic share the same application. A first couple of questions this position raises is: How are applications specified? How fined-grained do they need to be? It seems that before there is a discussion about applied logics, one needs to be clear about what is an application. If talking about mathematics, is one talking about all that may count as mathematics, or just a part of it? These questions are never made quite clear by most proponents of this view, and along with a lack of examples, it seems that the phrase "application" is supposed to be self-explanatory, which does not actually seems to be the case. Of a similiar opinion is Matti Eklund

[3] GTT pluralism received criticism for being more of the kind of pluralism based on language and not on consequence relation. See [Priest, 2005, Chapter 12].

[Eklund, 2017]. He holds a skeptic view on the philosophical meaningfulness of logical pluralism, and argues that most examples of pluralist theses in the literature are trivial, either because they are uncontroversially accepted or because they offer no significant theoretical substance. For him, one big problem for logical pluralism is to be able to explain what is the "purpose", "use", "domain on inquiry", "discourse", and "task" a logic might have.

Another question related to the discussion of fruitfully applying two different logics to the same purpose is: What are the criteria of adequacy? That is, how precisely is it determined that a logic fits a purpose and, moreover, how can two logics have the same applicability? One might reply that this is done by choosing some criteria of adequacy. It seems, however, that once these criteria are fixed, it is difficult that two logics will show exactly the same adequacy, with the consequence that this view seems to lead at best to a kind of relativism, one in which a logic is chosen as best for a purpose relative to some criteria. This follows from Priest's view above. Priest raises other problems with this view. Even if one could find two distinct logics (two distinct notions of validity) for the same application ("some situation about which we are reasoning" [Priest, 2005, p. 203]), does this mean that if one were to ask if an argument $\Gamma \vdash \delta$ was valid, there would always be two answers, one according to each logic? Certainly not, so for Priest one would fall back into monism (having to choose a logic after all). It seems this view would then lead back also to some kind of relativism (from the vocabulary used here), in which the choice of the right answer would be conditioned to some further choice. What is more, why then not say that the best logic for that application is the intersection of the other two logics? This option is also akin to monism.

One could also inquire as to when exactly there needs to be a choice about the correct/best logic for an application. If the answer is expected to be given immediately, then this is a form of time-based relativism [4], with the right answer being relative to the present time. If this answer is to be given at some future time, it seems that the question of application might lead to endless revision (a form of relativism), or even worse, for the putative pluralist, it could turn out that there is one true logic after all (back to monism).

On the topic of revision of logics as applied to some topic, this seems to be the view of anti-exceptionalists about logic [Hjortland, 2017]. It is due notice that some pluralist view of logic is taken to be a supplementary position towards

[4] Moreover, it seems that this kind of view is counter-intuitive to the field of research of logic, as tying up loose ends like this hinders future research.

their bigger goal of giving an account of how one logic might come to replace another in some application [Arenhart and da Costa, 2018]. The kind of pluralism required for this, however, is one which accepts that there are more than one logical system, and it not what is meant here by logical pluralism proper. Since the revision is done on basis of some criteria [Priest, 2016], the anti-exceptionalist view is here taken to be a form of relativism based on the criteria chosen.

Still not everything is lost for the approach to pluralism through application. One way to answer the question of when to decide on a best logic is "never". This view seems to be Shapiro's [Shapiro, 2014], when he remarks that "legislating for the future, from a philosophical armchair, is risky" [p. 95]. In Shapiro's approach, there should always be room for investigation and pluralism should be treated as a methodological approach to investigating how logics can be applied (as a bonus, he also presents examples). In his work, there is also focus on the issue of meaning variance and how this influences what pluralism might look like. This discussion of the meaning of logical connectives seems to be at home in the discussion of logical pluralism at the level of reasoning, rather than of application. So the limits of the approach to pluralism through application is another indication that the interesting developments for logic will come from investigating things at the formal level. As stated above, this train of thought has it's own interest but should be somewhat unrelated to the investigation of pluralism in logic from the logician's point of view. For Eklund one way to save logical pluralism would be in relating logic to some canonical purpose associated to some normative project, but this might as well be an impossible task, given the difficulties he presents [Eklund, 2017].

3. Going forward

In sum, discussion about logical pluralism can be of three different kinds: about language, about validity, and about application. For logical pluralism to be minimally interesting, it must present a genuine disagreement between logics, so pluralism based on language differences must been discarded. And it is at the formal level that this disagreement can be explained (by way of different notions of validity for a same language), so any decent for of pluralism in logic must past through this crucial step. Without being able to establish a dispute between logics, the first and third proposals for pluralism seem to lead back to some form or another of relativism (with the exception of a pluralism of Shapiro's kind), and as Shapiro's brand of pluralism presents interesting challenges to be solved at the level of validity, it seems it is there that logical pluralism is to be more fruitfully investigated.

Embracing the perspective of plurality of ways in which an argument can be valid, studies on non-Tarskian consequence relations ([C. Blasio and Wansing, 2017, C. Blasio and Marcos, 2018, Molick, 2018] are specially significant in when it comes to show in which direction the study of logic can go. In these works, the authors present perfectly reasonable notions of consequence that challenge the Tarskian properties which consequence relations should display (reflexivity, transitivity and monotonicity). And so, logical pluralism shows itself to be an interesting view on philosophy of logic, one that expands the boundaries of the space carved out by the formal methods on hand. It should not, however, be entangled with other views about how the many logical systems available might be applied, this task being better off left for the metaphysicians.

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4

Topics on Philosophy of Information

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1. Introduction

The concept of information is directly related to various content of philosophical character. In [Capurro and Hjorland, 2003] there is a study on etymology and different conceptions of information throughout history. These studies indicate that this concept has a very important philosophical role. In [Floridi, 2011] a philosophy of information is presented. This philosohy is based on the various areas that have the notion information as central (information science, information technology, computer science, etc). However, there is not yet a more systematic and broad organization of the possibilities of philosophical analysis of the term in order to show how to identify and situate metaphysical, logical, political, ethical, aesthetic, etc. issues (about information).

2. Information Theory

Suppose you were invited to dinner at a friend's apartment. When you get to the building, you realize that you forgot what the apartment number is. The building has 4 floors and each floor has 8 apartments. If a neighbor of your friend passes by and reports that your friend lives on the top floor, then the uncertainty in relation to the apartment number will go to 1/8 (before it was 1/32). By reducing your uncertainty, we say the neighbor provided you with *information*.

"Information theory is a mathematical theory defining the limits and possibilities of communication. It provides a quantitative measure of the information content of a message, which is independent of the meaning of the message, in terms of the reduction of uncertainty resulting from receiving the message." [1].

Like Keith Devlin in [Devlin, 1995], we will start the present study with the assumption that in the world there are things such as *information* and *cognitive*

[1] [Nadel, 2003] *Encyclopedia of cognitive science*, p 3696

agents (people, types of organisms, mechanical, electronic or virtual devices, etc.). Agents are, in principle, able to access information[2].

Traditionally, information theory is understood from the statistical point of view. That happens because important works and founders of this discipline were developed from such perspective (cf. [Shannon, 1948] and [Shannon and Weaver, 1974]). The conception of communication situation in these texts is given as the one in which a signal is chosen by a specific class to be transmitted by a channel, but the channel output is not determined by the input. Instead, the channel is statistically described by a given probability distribution over a set of all possible outputs for each input allowed. At the output of the channel, the received signal is observed and a decision must be made, where the main objective of decision is to identify a property of the input signal as close as possible to the original (cf. [Ash, 2000] preface).

According to [Cover and Thomas, 1991], Information theory responds to two fundamental issues of communication theory: what limits the compression of data? What limits the average amount of information transmission in the communication? For the first question, the answer provided by the communication theory is the entropy, for the second is *the capacity of the channel* C.

The concept of information has played an important role in several areas of knowledge, such as in computer theory, communication, mathematics, logic and philosophy. It's study takes place mainly with the objective of being a tool to obtain probabilistic results, in addition information acts in the structure of learning, of the semantic, deductive inference and, in general, the scientific method.

The concept of information also became more technical in nature from the publication of mathematical results in communication theory, and the transmission of information (see [Shannon and Weaver, 1974]). All these studies and results came to be known as *Information Theory*, in some authors we can also find the denomination *Statistical Theory of Information* [Fano, 1959, Usher, 2001, Burgin, 2003, Perry and Moffat, 2004]. An important question to be posed here is the following: Why a philosopher or a logician would be interested in statistical theory of information or bring it closer to a new point of view? The reason for his interest is that the statistical theory of information seems to have something important to say about the behavior of the information (in sintatical term). In the usual sense, the estatistical theory of information characterizes significant sentences as results of comparisons of sets of symbols. However, there has been

[2] [Devlin, 1995], cap 2, p 14.

constant comment that much of the statistical applications of information theory has no relation with the concept of information in its basic sense. It is also argued that the the use of the term 'information' in such contexts can produce false expectations about what the statistical theory of information can (in fact) offer to logicians and epistemic philosophers [Hintikka, 1970]. This position partly explains why few early philosophers have dealt with this subject. Basically we can mention Carnap and Bar-Hillel as precursors of the study of information in philosophy (cf. [Bar-Hillel, 1964], [Carnap, 1963], [Carnap and Bar-Hillel, 1952] [Carnap, 1966] and [Carnap and Bar-Hillel, 1953]).

A very striking feature of the concept of information is its polysemy. Hence, the existence of numerous theories about information, or even many interpretations of existing theories (cf. [Floridi, 2004]).

According to Luciano Floridi, even Claude Shannon was cautious in considering the possibility of a unified theory of information, as can be easily seen in the following passage.

> The word "information" has been given different meanings by various writers in the general field of information theory. It is likely that at least a number of these will prove sufficiently useful in certain applications to deserve further study and permanent recognition. It is hardly to be expected that a single concept of information would satisfactorily account for the numerous possible applications of this general field. ([Shannon, 1993] p. 80)

Therefore, it is not the purpose of this article to investigate a single meaning for the term information, but rather to present an initial approach to the various cases already known of information and some relations of this concept with philosophy.

These issues can be addressed from what computers do with the information. About a search for new information, it is notorious that active (autonomous) acquisition of information is not characteristic of the kind of reasoning simulated by computers. Despite the importance of already contemplated by these machines as managing data or information, and calculations; it would be very interesting and it is important for computers to ask relevant questions as well. Questions are the main means of actively obtaining information (and in some cases knowledge). Data managements are resources already implemented satisfactorily on these machines. It is known that the base of these processes is Boolean algebra (and, by correspondence, propositional logic) over the 0 and 1 elements. What computers do in summary, is to organize and calculate the information autonomously, or better, the simulation of this organization and calculation in such a way that it turns out to be a valuable instrument for the most diverse activities of the human being.

In many cases, frequent failures in computers comes not only from the lack of some kind of data but also from of not possibility to find the data in a certain place. The formalization of a (logical) algebra of questions could be a simpler way to solve these problems. The possibility of formalizing the universe of questions and answers in order to have a good base for the implementation of this approach in the machines would be, quickly, given the possibility of the existence of machines that would be able to formulate interesting and relevant questions on countless situations. Riddles, puzzles and philosophical problems, and scientific problems could be optimally analyzed by such machines. We can not expect such machines to be able to answer any question but you can find quick solutions starting from a large and initially confusing database of uncertain information.

2.1. Redutionists, Antireductionists and Non-Reductionist

According to Luciano Floridi, scholars of information theory can be classified into three distinct classes[3]:

1 - The redutionists;

2 - antireductionists; and

3 - non-reductionist.

1 - The reductionists hold the possibility of a "unified theory of information". Such a unified theory would capture the information concept of the most general way possible. The reductionists try show that all kinds of information can be reduced conceptually, genetically or genealogically to some generic concept.

2 - Antireductionists take on the multifaceted character of the concept of information and its phenomenon. They radically argue the irreducibility of the different types of information in a single type. Consequently, the antireductionists argue that there is no possibility of a unified theory of information, but the separate study of the various meanings of this term. A milder reductionist could still try to argue that the the polysemy of the term information originates from a single family of concepts, but the replica of the antireductionists consists in saying that not even a family of concepts would be coherent and that the source of such diversity lies in multiple roots independent from one another.

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[3] see [Floridi, 2004]

3 - Non-reductionists, however, try to escape of this dichotomy between reductionists and antireductionists. One of the strategies of non-reductionists to achieve this goal is replace the hierarchical model of reductionists with a network of interconnected concepts where connections are defined by different influences and dynamics are not necessarily genetic or genealogical. There is, therefore, a displacement of the core of the model, becoming possible the existence of multiple nuclei at the same time.

2.2. The Semantic of Information

The use of the term "information" is often linked to the study of the phenomenon of communication. In this context there is a strong interest in systematizing what may be called content semantic goal of communication. These contents may have sizes, shapes, and values and is considered an important reference in the studies about the concept of information. Encoding and transmission of information may also have different types of implementation physics, but it can be argued that the existence of information regardless of its encoding or transmission. For example, the Philosophy Cambridge Dictionary defines information in the following way:

> "an objective (mind independent) entity. It can be generated or carried by messages (words, sentences) or by other products of cognizers (interpreters) Information can be encoded and transmitted, but the information would exist independently of its encoding or transmission." [Floridi, 2004]

Some analysis seem to agree that it is possible to produce a general definition of information (DGI) in terms of semantic content and having as parameters the date and the meaning.

The DGI indicates that there can be no information without data, but this does not show what types of data δ are required to constitute the information. Suppose that when consulting a database or when asking a question, the absence of answers or feedback of the research occurs. In this case there are two possibilities: failure to search the data, and thus, no specific information σ will be available, or some δ data can be provided with the objective of showing that the process has entered a loop. In the same way, the silence as a response can indicate both: assent as well as denial. One can still derive from this non-primary information of type μ "The person did not hear the question." This "typological neutrality" (TN) is justified by the following fact: when an apparent absence of data can not be reduced to an occurrence of primary negative data, then what becomes available as information is some non-primary information of type μ over σ constituted by some non-primary data of the types δ_2 and δ_4. In the simplest cases the information may be of only one datum (Dd). A datum is defined as the lack of uniformity between two signals (see [Floridi, 2004] for more explanation about this point).

3. Philosophy under a informational point of view

There are at least two ways of understanding the relationship between philosophy and information:

a. Using elements of the information sciences to understand philosophical problems (Philosophy under a informational (science and technology) point of view)

b. Philosophical questions about information concepts (Philosophy of Information)

The item b., in turn, can be considered from two ways:

b.1. Information as a primordial element of reality (Jinping Dai and Hongbing He) [Dai and He, 2017]

b.2. Information as a constitutive element of reality (along with energy and matter)

The **b.1.** position has been defended by Jinping Dai and Hongbing He and can be considered as being a Chinese strand of Philosophy of Information. According to these authors, the research done on the philosophy of information refers to a research done by some Chinese philosophers in the last 40 years on the philosophy of information (cf. [Dai and He, 2017]).

The main purpose of these surveys (among other important issues) is to determine the nature of the information. In this case, this may be ontological or epistemological.

"How could the information be possible to exist in the ontological sense, or how to establish the ontological status of information? Is "Ontological information" a kind of "ontological informationalism" or "Pan-informationalism"? etc. It is of great significance for the study on PI to sort out and reconsider these arguments." (Jinping Dai and Hongbing He, 2017), Abstract.

In view of these possibilities of approaches on the relationship between philosophy and information, which will be described below, in more detail some possible topics in information philosophy.

It can be considered that PI is a mature discipline for three reasons[4]:

- it represents an autonomous field of research;
- it provides an innovative approach to both traditional and new philosophical topics;
- it can stand beside other branches of philosophy, offering a systematic treatment of the conceptual foundations of the world of information and the information society.

According to Floridi, Philosophy of Information is defined as the philosophical field concerned with the critical investigation of the conceptual nature and basic principles of information, including its dynamics, utilisation, and sciences, and the elaboration and application of information-theoretic and computational methodologies to philosophical problems (see [Floridi, 2002]).

This approach indicates a precise and necessary investigation into the the concept of information. In fact, determining the nature and basic principles of information is a way of answering the question "what is information?" and this is the first topic to be considered in a agenda concerning the philosophy of information. The second part of the research, which refers to the use of computational methodologies, is not a consensus and should be considered as a position on the methodology.

Therefore, following the computational methodology, the Philosophy of Information concerns:

- The critical investigation of the conceptual nature and basic principles of information, including its dynamics (especially computation), utilization (especially computer ethics) and sciences
- The elaboration and application of computational and information-theoretic methodologies to philosophical problems.

Applications of computational methods to philosophical issues may be approached in three main ways[5]:

1. Conceptual experiments in computers;
2. Pancomputationalism (given the right Level of Abstraction, anything could be presented as a computational system);
3. Regulae ad directionem ingenii (Rules for the Direction of the Mind);

[4] see (Floridi, 2002)
[5] [Greco et al., 2005], p. 623

The obvious criticism of these three points is that a large number of philosophical problems are not amenable to simplification. The abstraction suggested in the second item does not have defined objective criteria. Finally, it is not easy to determine the ideal level of abstraction for the simulation of philosophical problems by computational means.

4. Conclusion

As already mentioned at the beginning of above section, some Chinese authors have reflected on the ontological questions of information: Jinping Dai, Hongbing He, Mingfang Feng, Yixin Zhong, Liang Feng e Wu Kun. It is important to note that Chinese researchers on the philosophy of information has tried to approach Chinese philosophy and western philosophy.

It can be argued that this type of treatment for philosophical problems is only part of the problems and this is enough to consider this type of approach. In this sense, it's necessary a search for some methods of reduction of philosophical problems to computable or computational problems. In addition, other approaches can be tested. The fact that philosophy is a very broad area can indicate that one should consider the study of information from its sub-areas. Probably there will be a different method for each of these sub-areas. In this sense, information is considered as the basic element of analysis and the philosophy and its sub-areas as constituents of the methodology.

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5

*A Note on Beall & Restall's Pluralism and Non-Tarskian Logics**

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1. Introduction

In logic textbooks it is not rare to find the description of logic as the study of "what logically follows from what" [1]. Let us call this position the *folk view* on logic. According to the folk view, the fundamental role of logic is to demarcate what conclusions we are allowed to accept from a given set of premises. Logic is thus concerned with argument *validity*, i.e. with being able to distinguish valid arguments from invalid ones. According to Hofweber (2014) this feature of logic is linked to one particular conception of the word 'logic': as the study of formally valid inferences and logical consequence. From such a point of view the notion of logical consequence plays a normative role since it is possible to determine what conclusions *necessarily* follow from a given set of premises.

The normative character of logical consequence rely on another common feature related to the idea of logic as the study of formally valid inference: the often cited property that logic is about "the structure of arguments, not their content". This thesis has been addressed as the *form versus matter distinction*. It has its roots in Aristotle and in modern logic has been understood in different ways. To put it short, when we say that an argument is valid, we mean that its validity comes from the *form* of the argument, not from its *content*. Therefore it is often said that the task of logic is not to investigate whether the premises or the conclusion are *materially* true, rather the logician must comply to investigate whether

* To Carol Blasio, *in memoriam*

[1] Cf. Barwise et al. (2000), Gensler (2002).

the structure of the argument is logically coherent. As described by C. Dutilh-Novaes in Novaes (2012), "the form versus matter distinction is to be applied to objects such as arguments so as to outline what is distinctively *logical* about them – which is associated to their *formal* aspects – as opposed to their merely material aspects."

In modern logic a logical system is understood as a mathematical structure $L = \langle Fr, \vdash \rangle$, where Fr is a set of formulas and $\vdash \subseteq \wp(Fr) \times Fr$ is called a consequence relation over the set of formulas . When we restrict our look to the consequence relation of the logic, to explore the form means to explore the so-called *formal* features of logic. Formality is then taken to be a criteria to what deserve to be called logic, but this formality can mean different things ranging from the abstraction of the content of sentences to its topic-neutrality or its structurality (invariance under substitutions)[2]. As to the "matter" of arguments, to explore it means to explore the very content of the propositions involved in an argument and to investigate to what extent it is possible to find structural patterns of reasoning. The challenge of carrying this task led, among other things, to the field of *defeasible reasoning*. Reasoning is called defeasible when the corresponding argument is rationally compelling but not deductively valid[3]. So logics of defeasible reasoning challenged the assumption that logic is only about "what (deductively) follows from what" and proposed that they were also about "what (defeasibly) follows from what". It is well-known that the formal study of defeasible reasoning led to the development of the class of non-monotonic logics.

Different criteria to determine what is formal led to different views on what should count as logical. In the same manner, different views on what should be taken as the standards of deduction led to the investigation of new conceptions of validity. Logical systems then proliferated in different directions. Whereas some of them challenged the norms of deduction imposed by classical logic, thereby creating non-classical logics; others challenged the norms of deduction imposed by the Tarskian conception of entailment[4], giving rise to non-Tarskian systems. Logical pluralism emerged as the view that there is more than one correct consequence relation. In the form proposed by J.C. Beall and G. Restall in Beall and Restall (2006), logical pluralism accepts rival logical systems to

[2] The reader might check Dutilh Novaes (2011).

[3] Cf. Koons (2014).

[4] By a "Tarskian notion of consequence" we shall mean any non-trivial consequence relation that respects the properties of Reflexivity, Monotonicity and Transitivity. By a "Tarskian notion of entailment" we shall mean a notion of entailment that preserves truth from the premises to the conclusion. In the absence of ambiguities, we shall use the expressions interchangeably.

live under the properties of the Tarskian consequence relation, i.e under a forward truth-preserving notion of entailment. However, the authors maintain that in spite of the Tarskian notion of entailment be able to include rival logical systems, no other forms of consequence relation should be accepted, for the usual requirements for a legitimate notion of consequence relation are normativity and necessity. One is allowed to accept non-classical systems, but she should not accept non-Tarskian logical systems. Beall & Restall's pluralism accepts the plurality of logical systems arisen from non-classical logics, but not the plurality of logical systems arisen from non-Tarskian logics. The form of pluralism defended in Beall and Restall (2006) is tied to the normative character of logic. In their understanding of what is entailment, logical consequence is not only a normative relation, but it must be understood only as preservation of truth from the premises to the conclusion. According to the authors, any non truth-preserving notion of entailment has to forsake the normative character of logical consequence.

The present paper departs from Beall & Restall's approach to logical consequence as dependent upon normativity and necessity. Their view that logical consequence has a normative component insofar as it is useful to separate inferences in which the conclusions follows from the premises from those that do not, will be shared. However, it will be defended that even though every notion of consequence is the expression of a conception of normativity, it does not follow that all of them must be tied to a forward truth-preserving notion of entailment. Under this view we defuse Beall & Restall's argument in order to leave room for non-Tarskian notions of entailment. In the following paper the folk view on logic shall be refined by assuming that *logic is about what follows from what (**in a given dimension/perspective**)*. It will be argued that under this conception of logic, one is allowed to obtain a variety of notions of entailment without giving up on the normative aspects tied to logical consequence.

The present approach provides good motivation for a logical pluralism that not only considers the plurality emerging from different logical systems and its underlying language, but also the plurality of logical systems that emerge from different notions of entailment. This form of pluralism provide a good foundation for the study of non-Tarskian logics in the sense of Malinowski (1990)'s non-reflexive logics and Frankowski (2004)'s non-transitive logics.

The paper has the following structure: section 2 is a presentation of Beall & Restall's pluralism and the main reasons for their defense that any legitimate form of logical consequence must be forward truth-preserving; in section 3 the framework of b-entailment is presented. It will be argued that b-logics can be seen as a natural generalizations of the Tarskian notion of entailment and are able to include classes of non-Tarskian logical systems. At last, in section 4 it is presented how to understand normativity in the B-entailment framework.

2. Beall & Restall's pluralism

Logical pluralism is described as the thesis that there is more than one correct logic. As defended by R. Cook in Cook (2010), logical pluralism has many facets. It can arise as the result of varying many of the characterizing properties of a logical system, be they its language or its associated consequence relation. A famous defense of a form of pluralism relative to logical consequence was presented in Beall and Restall (2006). Pluralism about logical consequence holds that there is more than one correct relation of logical consequence. One of the reasons for this is the fact that terms like 'valid' or 'follows from' may be defined in more than one way, an argument that is taken to be valid in one logic, could be rejected into another. One example is the principle of explosion in classical logic and in paraconsistent logics. The upshot of Beall & Restall's view is to defend that expressions like 'valid' or 'follows from' are made precise in more than one way. Therefore the logical pluralist is allowed to claim that in the dispute between the classical logician and the paraconsistent logician both are correct.

According to Beall & Restall's pluralism there is a plurality of logical systems as long as their associated conception of logical consequence can be expressed/fit into the following schema:

Generalized Tarski Thesis (GTT):
An argument is valid$_x$ if and only if in every case$_x$ in which the premises are true, so is the conclusion.

where each expression case$_x$ in the (GTT) schema can be made precise in at least two ways which result in different extensions for 'valid'. For example, by 'case' one might mean a first-order interpretation of a Tarskian model or even a situation or a possible-world, but inconsistent or incomplete interpretations are also allowed. Different choices for the interpretation of 'case' will result in different precisifications of the (GTT) analysis of logical consequence which may result in different conceptions of logical consequence. Beall & Restall's defense of (GTT) rests on the idea that logical consequence is determined by three main features: *necessity* (the truth of the premises in a valid argument necessitates the truth of the conclusion), *formality* (valid arguments are so in virtue of their logical form), and *normativity* (it is irrational to reject the norms imposed by a valid argument). According to them, those features combined imply the idea that logical consequence should be truth-preserving and therefore it is a reflexive, monotonic and transitive relation.

From the view exposed above, the form of logical pluralism proposed by Beall & Restall does not include any non-reflexive, non-transitive or non-monotonic logic since these logics cannot fit the (GTT) schema. Although some of these logics have been investigated by many authors, such as Malinowski (1990), Frankowski (2004) and their investigation of non-reflexive and non-transitive logics, when asked whether non-reflexive or non-transitive accounts of logical consequence should be taken as legitimate forms of entailment, Beall & Restall replied:

> "(...) The given kinds of non-transitive or irreflexive systems of 'logical consequence' are logics by courtesy and by family resemblance, where the courtesy is granted via analogy with logics *properly* so called. Non-transitive or non-reflexive systems of 'entailment' may well model interesting phenomena, but they are not accounts of *logical consequence*. One must draw the line somewhere and, pending further argument, we (defeasibly) draw it where we have. We require transitivity and reflexivity in logical consequence. We are pluralists. It does not follow that absolutely *anything goes*." (Beall and Restall, 2006, p.91)

For the authors, if we assume that any account of logical consequence must be determined by necessity, formality, and normativity, then the only legitimate conception of logical consequence is the Tarskian one. A straight consequence of this fact, according to Beall & Restall, is the defense that any legitimate form of logical consequence must be truth-preserving. It is based on this reason that they argue further that non-reflexive and non-transitive systems are not *accounts* of logical consequence. Non-reflexive and non-transitive systems receive the label 'logic' only by courtesy, but they are not exactly logics in the strict sense of the word. In Estrada-González (2014), the author introduces Beall & Restall's arguments for logical consequence as truth-preserving in the following way:

- P1 "A logic is an account of the relation of logical consequence (or validity).
- P2 An account of logical consequence must be a tool useful for the analysis of the inferential relationships between premises and conclusions expressed in arguments we actually employ.
- P3 Logical consequence has at least the features of necessity (the truth of the premises in a valid argument are so in virtue of their logical form), and normativity (rejecting a valid argument is irrational).

P4 Logical consequence is forward truth-preserving.
P5 Forwards truth-preserving is a (non-empty) reflexive and transitive relation.
P6 Hence, logical consequence is a reflexive and transitive relation.
P7 Then, if a relation between the elements of a structure is either non-reflexive or non-transitive, such a relation is not one of logical consequence.
P8 Therefore, if no relation on a structure is a logical consequence, such a structure is not a logic." Estrada-González (2014)[p. 4]

In the present paper it shall be discussed premises P3 and P4 in relation to Beall & Restall's attack on non-reflexive and non-transitive logics as non legitimate accounts of logical consequence. The main purpose is to not disagree with them in the sense that logical consequence is forwards truth-preserving and that it has the features of necessity and normativity. However, it will be argued against the idea that logical consequence in the way just characterized does necessarily require a reflexive and transitive relation.

3. B-consequence and b-logics

Logical consequence is often thought of as preservation of truth, but we could also reason based on falsity as backwards from conclusion to the premises. A notion of falsity preservation, as suggested in Shramko and Wansing (2011), is definable in the following way: $\Gamma \models^f \alpha$ iff the falsity of the conclusion implies the falsity of at least one of the premises. From this it is possible to see that $\models^t = \models^f$, i.e preservation of truth from the premises to the conclusion is the same as preservation of falsity from the conclusion to at least one of the premises. If logical consequence must be based on the properties of necessity and normativity, then it can be grasped not only as truth-preserving from the premises to the conclusion, but also as backwards falsity-preserving. What is so distinctive of necessity and normativity that can be grasped only by truth-preserving consequence relations? What reasons could one have to explore different forms of logical consequence?

A step done into an abstract way of investigating consequence relations is proposed by Jean-Yves Béziau's idea of *Universal Logic*. Béziau's project of Universal Logic, defended in Beziau (2005) andBéziau (1998), aims to promote a general framework capable to express different features of consequence relations from an abstract point of view. The term universal logic was coined from the necessity to take a step further from the level of abstraction already explored by authors such as Tarski, Suszko and Łos. Through the framework of universal logic one is able to explore logics without any specification of language or logical operators. In this sense a logic is defined through a number of metaproperties that governs the behavior of a *consequence operator*. As put by Béziau (2001)[p. 4]: "I reached the idea that we must throw out all principles altogether, that logic *is not grounded on any principles or laws*." Therefore differently from Beall & Restall's pluralism, Béziau's view is that logic should not be reflexive or transitive since it is not grounded on any particular principle or law. The kind of pluralism entailed by Universal Logic does not commit with any particular notion of logical consequence.

As argued by Béziau, from the point view of universal logic the Tarskian properties can be seen as defining one (abstract) consequence operator which characterizes a certain class of logics. That is also how Beall & Restall thinks about the (GTT) schema. However, on Béziau's view, there is no reason to limit ourselves in characterizing a consequence operator as legitimate only as long as it respects the Tarskian properties. This was accomplished in Malinowski (1990) and Frankowski (2004) through the investigation of logics that were not characterizable by a two-valued semantics and thus were able to evade Suszko's reduction theorem[5]. In the following we introduce these logics:

A q-logic is defined as the structure $L^q = \langle Fr, \vdash^q \rangle$, where $\vdash^q \subseteq \wp(Fr) \times Fr$ is a *q-consequence relation* (or a *Malinowskian consequence relation*), i.e a consequence relation that respects the following properties for every $\varphi \in Fr$ and every $\Delta, \Gamma \subseteq Fr$:

$$\text{If } \Delta \vdash^q \varphi \text{ then } \Delta \cup \Gamma \vdash^q \varphi \text{ (Monotonicity)} \quad (1)$$

$$\Gamma \cup \{\gamma \mid \Gamma \vdash^q \gamma\} \vdash^q \phi \text{ implies } \Gamma \vdash^q \phi \text{ (Weak-cut)} \quad (2)$$

The associated q-entailment relation is constructed in the following way: let $\mathcal{M}^q = \langle \mathcal{V}, \mathcal{D}, \mathcal{R} \rangle$ be called a **logical q-matrix**, where $\mathcal{V}$ is the set of truth-values with $\mathcal{V} \neq (\mathcal{D} \cup \mathcal{R})$ and $\mathcal{D} \subseteq \mathcal{V}$, $\mathcal{R} \subseteq \mathcal{V}$ are called, respectively, the sets of **accepted** and **rejected** values. Call as **valuations** all homomorphisms $v : Fr \to \mathcal{V}$ and define a **semantics** as the set $Val^q = \{v_i\}_{i \in I}$. A **q-entailment relation** $\models^q$ is then defined as

$$\Gamma \models^q \alpha \text{ iff, for every } v \in Val^q, v(\Gamma) \cap \mathcal{R} = \emptyset \text{ implies } v(\alpha) \in \mathcal{D} \quad (3)$$

[5] Suszko's reduction theorem states that every Tarskian logic can be characterized by a two-valued semantics.

In an analogous manner, a p-logic is defined as the structure $L^p = \langle Fr, \vdash^p \rangle$, where $\vdash^p \subseteq \wp(Fr) \times Fr$ is a *p-consequence relation* (or a *Frankowskian consequence relation*), i.e a consequence relation that respects the following properties for every $\varphi \in Fr$ and every $\Delta, \Gamma \subseteq Fr$ [6]:

$$\Delta \cup \{\varphi\} \vdash^p \varphi \text{ (Reflexivity)} \tag{4}$$

$$\text{If } \Delta \vdash^p \varphi \text{ then } \Delta \cup \Gamma \vdash^p \varphi \text{ (Monotonicity)} \tag{5}$$

The associated p-entailment relation is constructed in the following way: let $\mathcal{M}^p = \langle \mathcal{V}, \mathcal{D}, \mathcal{R} \rangle$ be called a **logical p-matrix**, where $\mathcal{D} \cap \mathcal{R} \neq \emptyset$, $\mathcal{V} = \mathcal{D} \cup \mathcal{R}$ and all rest remains the same. The associated semantics is defined by the set of **valuations** $Val^p = \{v_i\}_{i \in I}$. A **p-entailment relation** $\models^p$ is then defined as

$$\Gamma \models^p \alpha \text{ iff, for every } v \in Val^p, v(\Gamma) \subseteq \mathcal{D} - \mathcal{R} \text{ implies } v(\alpha) \notin \mathcal{R} \tag{6}$$

The motivation for the development of q-logics was the idea that acceptance and rejection need not to be complementary. A logical q-matrix leaves room for values that are accepted, rejected and undetermined. According to Malinowski, the assumption that acceptance and rejection need not to be complementary reflected "the mathematical practice that treats some auxiliary assumptions as mere hypothesis rather than axioms. These assumptions may be accounted for by deduction (or not), which results in their justified further occurrence in the place of conclusions"[7]. Thus q-logics would reflect the scientific *modus operandi* of working with hypothesis (i.e non-rejected premises) and evaluating their consequences. From this point of view, the demise of Reflexivity seems natural once we do not want a scientific hypothesis be used to justify itself. In an analogous manner, p-logics were introduced with the intention of formalizing a kind of reasoning that, from accepted premises follows non-rejected conclusions. These logics were dubbed logics of plausible reasoning. Frankowski's influence on the plausibility relation was inspired in Ajdukiewicz's project of a Pragmatic Logic. At the core of Ajdukewicz's project was the idea of analyzing the relation between formal and informal argumentation.

As explained in Section 2, according to Beall & Restall q- and p-logics do not deserve their name. They should be considered as "logics" only by family resemblance to other already familiar systems, but they are not legitimate ac-

[6] Cf. Frankowski (2004)
[7] Cf. Malinowski (1990)

counts of logical consequence. Legitimate accounts of logical consequence are so in virtue of their necessity and normativity. Could we say that q- and p-logics also account for those? I do think so. Necessity and normativity are intricate properties. When an agent evaluates arguments she cannot hold contradictory beliefs, and if she commits to the truth of a given set of premises, then she also must commit to the entailed conclusions. But to say that we always assess arguments only after committed to the truth of a proposition seems to be a too ideal situation, for we also reason and assess arguments based on guesses, hunches, plausible and non-rejected premises. Moreover, we do all that relative to the information that we have access. Many times we do not assert nor deny, we evaluate the reasons we have to reject/to non-reject, or the reasons we have to accept/to non-accept a certain premise.

The B-entailment framework was introduced in Blasio et al. (2018) and Blasio (2017b) as a way to express forms of reasoning beyond the Tarskian conception of consequence. By a *b*-logic we shall call any structure of the form $L^b = \langle Fr, \div|\div \rangle$, where $\div|\div \subseteq \wp(Fr)^2 \times \wp(Fr)^2$ will be called a *b-consequence relation* (or a *bi-dimensional consequence relation*, whenever $\div|\div$ satisfies the following conditions for every $\varphi \in Fr$ and every $\Gamma, \Delta, \Phi, \Psi \subseteq Fr$:

$$\frac{\emptyset}{\varphi}\Big|\frac{\varphi}{\emptyset} \tag{R1}$$

$$\frac{\varphi}{\emptyset}\Big|\frac{\emptyset}{\varphi} \tag{R2}$$

$$\text{If } \frac{\Gamma}{\Phi}\Big|\frac{\Psi}{\Delta} \text{ then } \frac{\Gamma \cup \Gamma'}{\Phi \cup \Phi'}\Big|\frac{\Psi \cup \Psi'}{\Delta \cup \Delta'} \tag{M}$$

$$\text{If } \frac{\Psi}{\Sigma, \Gamma}\Big|\frac{\Delta, \Pi}{\Phi} \text{ for any partition } \langle \Sigma, \Pi \rangle \text{ of any } \Omega \subseteq Fr, \text{ then } \frac{\Psi}{\Gamma}\Big|\frac{\Delta}{\Phi} \tag{C1}$$

$$\text{If } \frac{\Sigma, \Psi}{\Gamma}\Big|\frac{\Delta}{\Phi, \Pi} \text{ for any partition } \langle \Sigma, \Pi \rangle \text{ of any } \Omega \subseteq Fr, \text{ then } \frac{\Psi}{\Gamma}\Big|\frac{\Delta}{\Phi} \tag{C2}$$

Every b-logic is endowed with two kinds of reflexivity and two kinds of transitivity. The associated b-entailment relation is constructed from a **logical b-**

matrix $\mathcal{M}_b = \{\mathcal{V}, \mathcal{D}, \mathcal{R}\}$, where $\mathcal{D} \cap \mathcal{R} \neq \emptyset$ and $\mathcal{V} \neq (\mathcal{D} \cup \mathcal{R})$. Define the semantics Val^b in the usual form, a **b-entailment relation** $\frac{\cdot}{\cdot}\Big\|\frac{\cdot}{\cdot}$ is defined as:

$$\frac{\Psi}{\Gamma}\Big\|\frac{\Delta}{\Phi} \text{ iff for every } v \in Val^b$$
$$v(\Gamma) \subseteq \mathcal{D} \text{ implies } v(\Delta) \subseteq \mathcal{D} \text{ and } v(\Phi) \subseteq \mathcal{R} \text{ implies } v(\Psi) \subseteq \mathcal{R}$$

(7)

The B-entailment framework is based on the idea of k-dimensional logics proposed in Shramko and Wansing (2011), where a logical system can be defined with more than one consequence relation. Under this view each consequence relation represents a different point of view on the arguments taken to be valid. In the way presented above the interaction between accepted and rejected values in B-entailment allows one to obtain the following four notions of entailment[8]:

- $\Gamma \models^t \delta$ iff there is no $v \in Val^b$ such that $v(\Gamma) \subseteq \mathcal{D}$ and $v(\delta) \notin \mathcal{D}$;
- $\Psi \models^f \varphi$ iff there is no $v \in Val^b$ such that $v(\Psi) \subseteq \mathcal{R}$ and $v(\varphi) \notin \mathcal{R}$;
- $\Psi \models^q \delta$ iff there is no $v \in Val^b$ such that $v(\Psi) \subseteq \mathcal{R}$ and $v(\delta) \notin \mathcal{D}$;
- $\Gamma \models^p \varphi$ iff there is no $v \in Val^b$ such that $v(\Gamma) \subseteq \mathcal{D}$ and $v(\varphi) \notin \mathcal{R}$.

What amounts to the following directions:

- $\frac{\cdot}{\Gamma}\Big|\frac{\delta}{\cdot}$ - t-consequence;
- $\frac{\Psi}{\cdot}\Big|\frac{\cdot}{\varphi}$ - f-consequence;
- $\frac{\Psi}{\cdot}\Big|\frac{\delta}{\cdot}$ - q-consequence;
- $\frac{\cdot}{\Gamma}\Big|\frac{\cdot}{\varphi}$ - p-consequence;

For the purpose of showing that b-logics are a natural generalization of tarskian, frankowskian and malinowskian logics, we prove that every model for a b-logic is also a model for tarskian truth-preserving logics, tarskian falsity-preserving logics, malinowskian non-reflexive logics and frankowskian non-transitive logics. Thus every model for a b-logic is also a model for t-, f-, q-, and p-logics.

Theorem 3.1. *Every b-logic can be characterized by a four-valued semantics.*

We prove the result by showing that the logics in each direction of B-entailment can be characterized by a minimal number of truth-values. This

[8] Presented in single-conclusion forms.

is useful to illustrate how the class of models relative to each of these classes of logics are expressible within the b-entailment framework[9]. Let us denote a n-valued semantics, where n is the number of truth-values (with $n > 2$) as $Val = \{v_i | v_i : Fr \to \mathcal{V}_n\}_{i \in I}$. Define also a function $t : \mathcal{V}_n \to \mathcal{V}_4$, where $\mathcal{V}_4 = \{T, F, N, B\}$, in the following way:

$$t(x) = \begin{cases} T, & \text{if } x \in \mathcal{D} \\ F, & \text{if } x \in \mathcal{R} \\ N, & \text{if } x \in \mathcal{V} - (\mathcal{D} \cup \mathcal{R}) \\ B, & \text{if } x \in \mathcal{D} \cap \mathcal{R} \end{cases}$$

Case 1: t-logics

$$\Gamma \models^t \alpha \text{ iff } \Gamma \models_{Val_2} \alpha \tag{8}$$

From the set of valuations and the function $t(x)$, define a *bivaluation* $b_v = t \circ v$ and collect them into the set of bivaluations $Val_2 = \{b_v | v \in Val_n\}$.

Proof. By contraposition, assume $\Gamma \not\models^t \alpha$. From definition of tarskian-entailment we know there is a valuation $v \in Val_n$ such that $v(\Gamma) \subseteq \mathcal{D}$ and $v(\alpha) \notin \mathcal{D}$. From the composition of t and this valuation, it follows $t(v(\Gamma)) \subseteq \mathcal{D}$ and $t(v(\alpha)) \notin \mathcal{D}$. Therefore there is a bivaluation $b_v \in Val_2$ such that $b_v(\Gamma) \subseteq \{T\}$ and $b_v(\alpha) \notin \{T\}$. From this our desired result $\Gamma \not\models_{Val_2} \alpha$ follows.

The other direction follows in an analogous manner.

□

Case 2: q-logics

$$\Gamma \models^q \alpha \text{ iff } \Gamma \models_{Val_3} \alpha \tag{9}$$

For this case define a *trivaluation* $t_v = t \circ v$ and collect them into the semantics $Val_3 = \{t_v | v \in Val_n\}$.

Proof. By contraposition, assume $\Gamma \not\models^q \alpha$. From definition of q-entailment we know there is a valuation $v \in Val_n$ such that $v(\Gamma) \cap \mathcal{R} = \emptyset$ and $v(\alpha) \notin \mathcal{D}$. From the composition of t and this valuation, it follows that $t(v(\Gamma)) \cap \mathcal{R} = \emptyset$ and $t(v(\alpha)) \notin \mathcal{D}$. Therefore there is a trivaluation $t_v \in Val_3$ such that $t_v(\Gamma) \cap \{F\} = \emptyset$ and $t_v(\alpha) \notin \{T\}$. From this our desired result $\Gamma \not\models_{Val_3} \alpha$ follows.

[9] The result is a generalization of Suszko's reduction theorem.

The other direction follows in an analogous manner.

□

Case 3: p-logics

$$\Gamma \models^{p} \alpha \text{ iff } \Gamma \models_{Val_3} \alpha \tag{10}$$

Proof. Analogous to the previous case. □

The result above shows that b-logics are a natural generalization of t-, q- and p-logics. The four-valued models of b-logics are enough to characterize all logics expressible in b-consequence. The move towards a bi-dimensional notion of entailment leaves room for non-Tarskian logics. There seems to be no reason to believe that these logics does not deserve their name, since the models of tarskian logics are just a subset of three-valued and four-valued models.

In the following, it will be introduced, as originally developed in Blasio (2017b), how the semantics of b-logics can be presented in terms of a society of agents and their respective cognitive attitudes of "accepting" (Y), "non-accepting" (⅄), "rejecting" (N) and "non-rejecting" (И).

Definition 1. *Let* $COG = \{$Y, ⅄, N, И$\}$ *be the set of cognitive attitudes, an* ***agent*** s *is a valuation* $s : Fr \to COG$*. A set of agents will be denoted by* Soc*.*

Based on the set of cognitive attitudes, it is possible to represent the set of truth-values $\mathcal{V}_4$ in terms of the cognitive attitudes of the agents.

Definition 2. *Given an agent* $s \in Soc$*,* $c \in COG$ *and* $\alpha \in Fr$*, the situation "The agent* s *entertains the cognitive attitude* c *towards* α*" will be represented as "*$Cs : \alpha$*". In relation to the informational content of* α *relative to the agent* s*, the truth-value assigned to* α *is determined in the following way:*

F, if ⅄$s : \alpha$ *and* N$s : \alpha$
N, if ⅄$s : \alpha$ *and* И$s : \alpha$
B, if Y$s : \alpha$ *and* N$s : \alpha$
T, if Y$s : \alpha$ *and* И$s : \alpha$

Given the above description of truth-values in terms of cognitive attitudes, it is possible to represent the four entailment relations of B-entailment in the following way:

Definition 3. *Let* Soc *be a set of agents. For all* $\Gamma \cup \{\alpha\} \subseteq Fr$*:*

- $\Gamma \models^t \alpha$ *iff there is no agent* $s \in Soc$ *such that* $\mathsf{Y}s : \Gamma$ *and* $\mathsf{⅄}s : \alpha$;
- $\Gamma \models^f \alpha$ *iff there is no agent* $s \in Soc$ *such that* $\mathsf{N}s : \alpha$ *and* $\mathsf{И}s : \Gamma$;
- $\Gamma \models^q \alpha$ *iff there is no agent* $s \in Soc$ *such that* $\mathsf{И}s : \Gamma$ *and* $\mathsf{⅄}s : \alpha$;
- $\Gamma \models^p \alpha$ *iff there is no agent* $s \in Soc$ *such that* $\mathsf{Y}s : \Gamma$ *and* $\mathsf{И}s : \alpha$.

Theorem 3.2. *Every b-logic can be characterized by a set of cognitive attitudes.*

Proof. Corollary of Theorem 3.1. □

4. The normative character of logic

The normativity of logic comes from the fact that the notion of logical consequence constrains our judgements about the arguments we deal and employ in our day-to-day practices. Thus, logic has always been understood as a tool to distinguish good from bad reasoning. The Tarskian conception of entailment states that once the premises are true, the truth of the conclusion is *necessarily* the case. The non-Tarskian conceptions of p- and q-entailment are based on the attempt to express forms of reasoning that deal with non-rejected propositions under the assumption that acceptance and rejection are not complementary notions, what is a key point to understand the kind of reasoning explored by B-entailment.

In the B-entailment framework, as described by C. Blasio in Blasio (2017a), agents are thought of as evaluating the reasons for accepting/non-accepting and for rejecting/non-rejecting the informational content provided by propositions[10]. Therefore b-logics are not to be explained through only the traditional illocutionary forces of assertion and denial, so characteristic of the notion of consequence under the Tarskian approach. In b-logics the agent is thought of as entertaining *cognitive attitudes* towards the information carried by the propositions. These cognitive attitudes, as stated by Blasio (2017b)[p. 23], are not to be confused with truth-values. In fact, as exhibited in section 3, truth-values can be expressed in terms of cognitive attitudes. In Blasio's description of the adequate reading for cognitive attitudes, the agent is thought of facing situations in which she was "told to be true" or "told to be false" relative to the information of a certain proposition. It is in face of such situations that the agents will present their possible cognitive attitudes.

The move to cognitive attitudes is proposed by Blasio as alternatives to the epistemic situations represented by Belnap's four-valued logic. In this situation

[10] In the same paper the author shows how the approach of b-entailment is able to express Belnap's four-valued logic.

the agent is not committed to the truth or falsity of a given proposition, instead the agent is expected to entertain cognitive attitudes of acceptance and rejection towards the informational content provided by propositions. B-logics are then thought as logics of acceptance and rejection within a type of bi-dimensional consequence relation. This move allows one to see all classes of t-,f-,q- and p-logics as single dimension forms of consequence relation.

The fact that acceptance and rejection are not complementary notions allows for the expression of cognitive attitudes that expresses *gappy* and *glutty* states. These states are characteristic of the forms of reasoning explored by p- and q-entailment.

The form of pluralism proposed by B-entailment is one in which more than one conception of validity is acceptable, leaving room for conceptions of validity not captured by accounts through truth-preservation. This view is in disagreement with the views according to which a proper account of logic's normative role should allow one to arrive at the correct consequence relation. The form of pluralism proposed by B-entailment is non-factual in the sense of Field (2009) . According to Field's non-factualism, we evaluate our norms as relative to our epistemic norms, but these norms does not arise from facts. They are the product of the way we deal with interact with the information thrown at us.

If legitimate accounts of logical consequence must be enclosed within the Tarskian properties of logical consequence, then b-logics still preserve their desirable properties. However, by weakening the traditional acts of assertion and denial to cognitive attitudes of acceptance and rejection, one is able to keep the normative character of logical consequence without being restricted to the Tarskian notion of logical consequence.

5. Final remarks

In the present paper it was shown that through the framework of B-entailment one is able to refine the folk view of logic as "what follows from what" to "what follows from what (*in a given perspective/dimension*)", where each dimension is expressed through single dimension forms of entailment. It was also shown that under this conception of validity one is able to accept not only the plurality of logics that are created from the challenge to classical logic, but also the plurality of logics that are raised from the challenge to the Tarskian properties of logical consequence. It remains to be further explored whether the cognitive attitudes expressed by b-logics in terms of acceptance/non-acceptance and rejection/non-rejection can shed some light in debates about uses of sentences from an incompatibility perspective, such as the bilateralism debate[11].

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6

The Special Composition Question and Natural Fusion

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1. Introduction

Philosophical problems about the part-whole relation have been discussed throughout the history of philosophy, at least since Plato and Aristotle[1]. In contemporary philosophy, the understanding of these issues has benefited from the formal tools of Classical Extensional Mereology. This paper aims is to defend mereological restrictivism against some constraints imposed by the vagueness argument. To achieve this, the paper is divided into three parts. In the first, I introduce the *special composition question* (hereafter SCQ) as formulated by [Van Inwagen, 1990] and briefly present the three sets of theories that proffer answers to it. In the second part, I focus on the vagueness argument, that was presented by [Lewis, 1986,] and [Sider, 2007,] to defend mereological universalism. In the third section, I introduce the theory of natural properties by David Lewis. From this theory, I propose a new mereological operation labeled as *natural fusion*. The purpose of this operation is to falsify one of the premises of the vagueness argument. Then, I discuss some examples and counter-examples to natural fusion.

2. The special composition question

[Van Inwagen, 1990] distinguishes philosophical problems associated with the material composition into two questions: the general composition question and the special composition question. According to him, the former question is to

[1] I thank the audience of 3rd FILOMENA Workshop held in UFRN, Natal, for the discussion after I presented this paper. I am also grateful for the anonymous reviewer for the comments who improved the text in many aspects.

know what composition is and it should be answered by mereology, the logical-ontological study of the whole-part relation [Imaguire, 2007, p. 314], within its different axiomatizations and predicates. In other words, for each theory of mereology bears a different notion of composition. Being the Mereological Monism, the philosophical thesis is that there is one only one true, general and exhaustive theory of composition, and that this theory is Classical Extensional Mereology (CEM). The latter question concerns the following

Special Composition Question: under what circumstances for some x's (parts) is there a y (whole), such that x's compose y?

There are three sets of theories that answer the special composition question. These theories are universalism, nihilism, and restrictivism. Differences between them might be considered as differences between the scope of quantification when mereological predicates are being evaluated. Mereological universalism argues that the domain of quantification must be, necessarily, unrestricted. Universalists admit that any two (or more) random parts compose a new object. It does not matter how these parts are distant in spacetime or even in logical space. For instance, a universalist considers as genuine an object composed by the cup that is on my table and the pen in which Pero Vaz wrote his famous letter in the sixteenth century. This is the position known as *mereological universalism* and its leading proponents are [Lewis, 1991] and [Sider, 2007].

Mereological nihilism is in diametrical opposition to mereological universalism. Nihilists argue that necessarily mereological composition never occurs. All that exists are simple parts or mereological atoms that can be organized in different ways. For the nihilist, there is no composition. In this theory, references in language to common sense objects could be replaced by suitable paraphrases. These are the two characteristics of the position that [Tallant, 2014, p. 1512] named as *standard nihilism*. In that sense, for a standard nihilist material objects like chair, tables and bicycles do not exist as a result of a composition operation. The discourse on these objects could be replaced by expressions such as "organized-in-form-of-chair"or "organized-in-form-of-bycicle". This is the position advocated by [Dorr, 2001], [Dorr and Rosen, 2002] and [Sider, 2013].

Mereological restrictivism stands between the previous two theories. Their advocates argue that some restriction is needed to distinguish between objects obtained by genuine composition and freakish objects obtained by non-genuine composition. That is, not all composition would form legitimate objects. This position presents itself as an intermediate between the other two theories.

It is close to a layman's answer about the special composition question since it denies both that nothing is an object (nihilism) and that any every possible composition results in a new an object (universalism). Yet, the problem commonly faced by the mereological restrictivists is to find a precise criterion to restrict composition and distinguish genuine objects from non-genuine objects. Some of the deemed criteria are:

a) the mere *physical contact* between the parts;
b) a *certain junction* between the parts with which to allow them to move;
c) parts form a whole when the parts play a *function* in the whole.

Each of these constraints seems to accommodate common sense intuitions. But at the same time they result in unexpected problems, such as, for (a), to consider that every time there is a handshake between humans there is composition of a new object, human1-handshake-human2. Or, for (b), every time someone fastens the seatbelt on a car seat and drives out into the city, it also results in a new object, the-driver-and-the-car. Or, for c), what would be the criterion for establishing that a part is functional or not? For instance, would the brake system be a functional part of the bicycle, even if it can perform its function without the brakes?

Besides the three restrictions mentioned, there is a fourth (and famous) one advocated by [Van Inwagen, 1990, chapter 9] according to which only living organisms would be genuine compositions and able to respond to the SCQ. In this sense, dogs, cats, plants and humans, for example, are considered common objects as a result of genuine composition, whereas tables, chairs, and computers are not genuine objects.

In this section, I have provided a brief overview of the different theories of mereological composition and some problems related to each one. It is not the scope of this text to make a thorough assessment of the arguments and objections for each theory, but rather to discuss a more precise point, the trueness or falsity of the premise of a well-known argument against mereological restrictivism. This is the argument of vagueness which I shall discuss in the next section.

3. The vagueness argument

The vagueness argument, as formulated by [Lewis, 1986, p. 211-213] and [Sider, 2001, Sider, 1997], might be placed in canonical form this way:

$P1$: If the restricted composition is true, then the occurrence (or non-occurrence) of composition might be vague.

$P2$: The occurrence (or non-occurrence) of the composition might not be vague.
$C1$: Thus, restrictivism is false.

This argument aims to refute mereological restrictivism in a more general way since premise one claims conditionally that any restriction applied to composition would be vague, given the beliefs that there is composition and that any restriction to composition would be vague. Further, that this vagueness is the result of a semantic indecision. Being vague, there will be limited cases in which the composition occurs or does not occur. Since there seems to be composition in the world, the operation cannot be vague. Therefore, if there is composition it cannot somehow be restricted. A more questionable premise of this argument is P1. [Lewis, 1986, p. 212] strengthens it by adding these two extra premises:

$P1a$: If composition is restricted, this restriction might obey intuitive desiderata.
$P1b$: Since intuitive desiderata are vague, they vary according to the adopted theory, then the occurrence or non-occurrence of the composition might be vague.

The above premises and transitivity rule entails P1 as conclusion. Thus, we found a sub-argument to justify P1. The argument that has C1 as conclusion has the form of *modus tollens* and is, therefore, a valid argument. Being a *modus tollens*, if we deny the consequent of the conditional of the first premise, we logically conclude the negation of the antecedent of this conditional: any restrictivist theory about composition is false (C1). Therefore, if one wants to present an objection to the argument, the truth of one of the premises ought to be confronted. This is what we intend to do in this paper.

Meanwhile, that was the first part of the argument. The argument itself is not complete yet. As it is formulated, it is still possible to conclude that nihilism is true. Since the rejection that the occurrence (or non-occurrence) of composition is vague, it is not assured that the composition occurs. Then, to obtain the conclusion desired by the universalist, the falsity of restrictivism and the truth of universalism, the universalist still needs to add an extra argument:

$P3$: There is, in fact, composition.
$P4$: If there is composition, then nihilism is false.
$C2$: So, universalism is true.

From the conjunction of P3 and P4, we get the new conclusion expressed in C2 that would guarantee the truth of universalism. So far, everything seems to be going well for one who does not doubt the truth of P1. The truth of this premise in conjunction with the belief in the conditional sentence "if there is a restriction, it will be inherently vague" leads to the rejection of restrictivism.

Restrictivists can offer, at least, one answer against this argument, as pointed out by [Smith, 2006, p. 364]. The answer is to find a criterion to restrict composition without blind submission to intuitive desiderata. The desired criterion ought to rely on some objective feature of reality. The alternative pointed out by Smith is similar to the natural fusion operation that I first presented in [Rocha, 2017] and have been developing in the sequel. Both options stick on a restriction that relied on perfectly natural properties. [Smith, 2006, p. 365] states that the occurrence of composition would also be a good criterion for attributing naturalness to properties since both composition and natural properties share the same role: to carve out reality in its joints. According to this proposal, P1 could be reformulated as follows:

$P1^*$: If the composition is restricted then it obeys a restriction consisting of perfectly natural properties and relations.

Between this and that, there seems to be light at the end of the tunnel for those who lean towards mereological restrictivism. Or those who somehow believe that intuition might play a noteworthy role when evaluating opposing theories. To shed more light on this path, in the next section, I present a new mereological operation, natural fusion, that aims to thicken reasons for defending restrictivism and avoid universalism.

4. Natural fusion and natural properties

Now, I introduce a new mereological operation, labeled as *natural fusion*. The purpose of this operation is to falsify one of the premises of the vagueness argument. This is an argument offered by universalists against restrictivist and nihilistic responses to the SCQ. It relies on the hard task restrictivists assign themselves to provide a precise and non-vague restriction to the composition. So, the main premise of this argument hinges on the vagueness of constraints to mereological composition. Natural fusion intends to minimize the alleged vagueness of the criteria for restricted composition. Natural fusion largely relies on the notion of natural property. So, any vestige of vagueness found in natural fusion is inherited by the vagueness found in natural properties. Another (broader and pretentious) aim of this new operation is to both assure ontological innocence of mereology as claimed by universalist theory and, at the same time, to accommodate the folk intuition claimed by restrictivists.

If natural fusion operation is successful, it can make the vagueness argument unsound by falsifying the consequent of the first premise. That is, natural fusion might provide a tool that allows us to restrict composition less vaguely, and so, achieve precise intuitive desiderata for restrictivists. Natural fusion strongly relies on my definition [Rocha, 2017] of natural properties, which is widely inspired by the work of [Lewis, 1983] and [Quinton, 1957, p. 36].

Let us start from the beginning. A natural property might be defined as this:

Natural property $=_{df}$ a property is natural, if, and only if, each element of the class defined by the property is sufficiently similar to the other elements of this class and each element of the class is representative of the other elements of the class.

I concede that there are problems surrounding the above definition. There are, at least, two ways to escape the vagueness of the expression "sufficiently similar". The first is to reckon a more precise definition on, for instance, a set-theoretical approach to similarity as proposed by [Tversky, 1977]. Another way is just to assume similarity as a primitive feature in a theory and escape the duty to define it. Those who do it believe that anyone is able to identify similarity when presented with two similar objects. In this sense, similarity would be an eye-catching feature in reality.

The concept of natural properties, I believe, grounds most of David Lewis' metaphysic. Maybe we could even spare our ontology from concrete possible worlds, and most of the theoretical applications of his philosophy would obtain. As [Lewis, 1975, p. 17] himself pointed out, "if you insist on repudiating possible worlds, much of my theory can be adapted to meet your need.". This quotation is previous to the publication of *On the Plurality of Worlds*, the book Lewis in which Lewis fully develops his possible worlds metaphysics and claims that, considering the best-known distinctions of abstract and concrete entities, the possible worlds in his theory would be concrete entities. In fact, the point I stress might sound as too strong a claim. Nevertheless, the intention is just to highlight that there are more on Lewis' philosophy than possible worlds. Much must be done even to those who have incredulous stare on them. Probably, things would be different if we dismiss natural properties. Since they are a kind of elite properties that carve nature at its joints. The analogy is useful here because we could say that natural fusion, also carves genuine composition in the multitude of unrestricted fusions.

Inspired by the definition of natural property, this new operation aims to accommodate ordinary intuitions about composition better. It would be a tool to select from the so-called ontological garbage produced by the unrestricted fusion a subset of compositions that would accommodate ordinary beliefs about genuine objects. Similar to the notion of natural property that selects from the myriad properties, a minority elite, one that carves reality in its relevant articulations, natural fusion would be a minority subset of the class of all fusions. While the fusion of the statue of Christ the Redeemer and the tip of my nose would not be considered a natural fusion, the fusion of the parts of the statue of Christ the Redeemer or the fusion of the Corcovado hill and the statue of Christ the Redeemer would be an example of what I call a natural fusion. A preliminary definition would be something like the following. First, consider that a fusion might be defined as [Sider, 2015,]:

$x\,Fu\,S$ **(*x* is a fusion of the elements of S)** $=_{df}$ each member of S is part of x and each part of x overlaps any member of S.

By conjoining the definitions of natural property and fusion, I propose a new mereological operation, the *natural fusion*:

$x\,Fu^{nat}\,S$ **(x is a natural fusion of the elements of S)** $=_{df}$ each member of S is a part of x, members of S share at least one natural property, and the class of x's also constitute a natural property.

To better understand the role of natural fusion, let us consider a possible distinction between two mereological operations: composition and fusion. Both have upsides and downsides. From one side, composition would be a non-arbitrary complex operation of ordering. It takes some parts as inputs and generates a new object that beyond parts has some order (spatial, temporal, causal) between them. Three parts ordered in different ways would be three distinct objects. For example, take three parts of a glass of wine: the base, the stem, and the cup. Restrictivists would say that only these parts arranged in a specific way form a complex, labeled as a glass of wine. Any other arrangement would not compose a genuine object. In this sense, composition is not ontologically innocent. It generates a new object every time it is applied. On the other hand, some philosophers claim that fusion is ontologically innocent. It is the arbitrary mereological operation that forms a complex. There is a gap between composition and fusion when we understand them in this way. The restrictivist prefer the former since it put aside most of the possible complex objects resulting from fusion and that would be considered as gruesome and ill-demarcated objects. Universalists prefer the latter since they believe that a complex is nothing over above its constituents parts. Nevertheless, the universalist would not agree, *stricto sensu*, that three fusions formed by the same parts and arranged otherwise would be the *same object*. Rather, the universalist recalls four-dimensionalism[2] and say that the three different orderings for the same parts correspond to three temporal parts of the same object.

Natural fusion aims to fulfill the gap between the two extremes of composition and fusion. Since it is a fusion, it accepts that, for any two parts, there is a fusion between them. However, there would not be the case that every fusion would be a natural fusion. To qualify as a natural one, the fusion has to meet the following criteria: members of the fusion must share, at least, one natural property and the whole must be a member of a natural property class. For example, consider these two fusions. First, the fusion between the tip of my nose and the statue of Christ the Redeemer. This fusion would not be classified as a natural fusion since its parts do not share natural properties. Let us assume that the biological constitution would count as natural properties in this case. Soapstone and human body skin cells may share some natural properties, but only very few of them would. Probably, both have some water or carbon atoms in them. In a charitable reading, it would meet the first condition of natural fusion definition, but not the last condition. The class formed by the fusion of the statue and the tip of my nose is not a natural property since each element of the class formed by this fusion is not representative of the whole fusion.

Let us now consider an example of a good candidate to a natural fusion, the fusion of the bones of a human skeleton. Each member of this fusion shares plenty of natural properties between them; they all are made of calcium. The class formed by the bones of the human skeleton is also a natural property, for each member of this class is similar to each one. For example, the *tibia* is similar to the *humerus* and even the *cranium* is similar to the *ribs*, when considering its organic elements rather the external form. So each element of the class of skeletons is somehow representative of each other elements of the class.

This was an attempt to flesh out the notion of natural fusion; I believe that more examples need to be worked on. And, having shed some tiny light on natural fusion, I turn back to the vagueness argument presented in the previous section to reformulate it using the idea presented by natural fusion.

[2] Four-dimensionalism holds, in a few words, that an object persists through time similar its extension through space.

So, rewriting the first premise by adopting natural fusion, we get:

$P1^{nat}$ If composition is restricted, then composition occurs only when it is an outcome of a natural fusion.

So a restrictivist might benefit from natural fusion and present a positive argument for restrictivism. Considering that natural fusion furnishes the criteria to distinguish between the occurrence or non-occurrence of genuine composition and dismiss the vagueness generated by the imprecision of intuitive desiderata argued previously. So, if restricted composition might obey a precise and non-vague constraint, the old vagueness argument against restrictivism becomes unsound.

We can restate it and proceed to argue as:

$P2$ The occurrence of the composition is not vague.
C^{nat} Hence, restrictivism is true.

Henceforth, if natural fusion is able to eliminate the vagueness of restriction criteria the trueness of restrictivism (C^{nat}) would be a consequence of $P1^{nat}$ and $P2$.

5. Final remarks

Lastly, and surprisingly, the natural fusion, largely inspired by the work of David Lewis, is actually being used against one of the theories defended by the philosopher, namely, the mereological universalism and in favor of restrictivism. Nevertheless, despite rejecting universalism in the letter, natural fusion keeps the spirit of its formulation, that is to assure the ontological innocence of mereology. In fact, this is a surprising result and needs to be better understood and sustained through refinements in the definition of natural fusion, from the search for more examples and evaluation against examples to the definition.

Finally, I am aware there yet some work to be done on natural fusion. I hope that the paper achieved its initial objective of introducing the notion and discussing some assumptions related to it. The success natural fusion to properly refute vagueness argument is still to be evaluated and discussed in further work.

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7

Completeness in Quantified Modal Logic and the Interpretation of the Quantifiers

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1. Introduction

James Garson [Garson, 2013b, pag. 264] has remarked a nice feature of propositional modal logics, which consists in the modularity of their completeness results. What characterizes modularity is the fact that for the most part of systems of propositional modal logic, when the completeness of a logic S has been proved with respect to a property of frames $\| F \|$ and the completeness of a logic S′ with respect to frame property F′, then the completeness for the system S+S′, with respect to the conjunction of F and F′ can be given by 'pasting together' the reasoning of the original pair of completeness proofs[1]. For example, from the proof that S4 (which is K+(T)+(4)) is complete for reflexive transitive frames, and the proof that B (which is K + (T) + (B)) is complete for reflexive symmetric frames, one may obtain a proof that S4+B=S5 is complete for reflexive, transitive, and symmetric frames. This kind of modularity may fail for quantified modal logic (QML)[2]. Let us use an example borrowed also from Garson to illustrate this fact[3]. Classical quantificational logic QL is complete for the substitutional and objectual interpretations of the quantifiers, and S4 is complete for reflexive transitive frames, but when the semantical conditions for the two systems are combined in the obvious way, the resulting semantics $\| \mathsf{QL}+\mathsf{S4} \|$ validates the Barcan formula BF: $\forall x \Box A \rightarrow \Box \forall x A$, which is not provable in QL+S4. As Garson also remarks, the situation is oddly asymmetrical, because the converse Barcan formula CBF:

[1] We are following the notation used in [Braüner and Ghilardi, 2001], where + just means 'together with'.

[2] We will be using the denominations 'quantified modal logic, 'first-order model logic' and 'predicate modal logic' indistinctly.

[3] *op. cit. pag. 265*

$\Box\forall xA \rightarrow \forall x\Box A$, is provable in $\mathsf{QL} + \mathsf{S4}$. Modularity may fail even in systems that restore symmetry by adopting BF. Cresswell [Cresswell, 1995] has showed that the propositional modal logic S4.1 is complete, but QL+BF+S4.1 is no longer complete.

In the last years some relational semantics for quantified modal logic have made use of alternative interpretations of the quantifiers in order to fix this kind of incompleteness. Working in this line, James Garson [Garson, 2013b] has introduced, amongst others, a so-called 'sentential' interpretation that is based on a weaker reading of the quantifiers. Garson's sentential interpretation is akin to the well known substitutional interpretation of the quantifiers. This last one was introduced in the early decades of the last century by Ruth Barcan [Barcan Marcus, 1946], [Barcan Marcus, 1961], who advocated for this interpretation in the setting of QML. Notably, the most renowned interpretation used to be the 'objectual' reading of the quantifier, which is at the same time the most problematic concerning quantified modal logic, because it is supposed to highlight the ontological commitments assumed by the language.

The aim of this paper is to reflect on the curious incidence that these alternative readings of the quantifiers have had on the problem of the completeness for normal systems of quantified modal logic. The problem will be addressed here almost exclusively in the setting of relational semantics for modalities, deriving from the pioneer work of Kanger and Kripke. Our interest will be also focused on the sequel of proofs based on the use of canonical models. Admittedly, there are much more alternatives with this concern, but it is not our purpose to deal with all these approaches. Moreover, we are going to confine ourselves to the consideration of the sentential interpretation of the quantifiers as it is described by J. Garson in [Garson, 2013b], who also support it. It does not mean that we prefer this interpretation over the others. it will serve solely as a test-case for attaining our main goal of analysing the philosophical implications that the interpretation of first-order quantifiers has on the completeness results in QML. Finally, we will conclude by observing the way in which current elaborations around the notion of 'linguistic framework', once developed by R. Carnap, and other concepts derived thereof, can also help to shed some light on this issue.

A final warning is also in order here. This paper does not contain outstanding results in modal logic. It is rather conceived as a philosophical reflection based on more technical results concerning QML. It can be also viewed as a technically-informed consideration of philosophical issues related with the interaction of first-order quantifiers and modal operators.

2. Completeness and modularity in QML

Quantifiers make much more difficult the development of completeness proofs in modal logic. It may be conjectured that to achieve completeness proofs in quantified modal logics only calls for putting together those results obtained for propositional modal logic and first order logic respectively. This seems very likely, since quantified modal logic is just an extension of propositional modal logic. Unfortunately, the problem turned out to be much more complex. We will see firstly in this section what happens with the completeness of propositional modal logic in order to be able to appreciate more accurately the complexities that arise in the case of quantified modal logic. We are going to consider only completeness proofs obtained by using canonical models; that is, those proofs which are based on the use of maximal consistent sets of formulas. We are also going to assume some familiarity with this type of proof and the notion of canonical model[4].

The main idea in this type of completeness proofs, inspired by the completeness proof for first order logic given by Leon Henkin in the middle of the Twentieth Century, is to show that any argument of the form $H\backslash C$, where H is a set of premises and C the conclusion, has a counterexample, whenever it is not provable. In order to prove that, it suffices to show that given $H \nvdash C$, the set which consists of H and $\neg C$ is consistent, and then it can be extended to a maximal consistent set **h** by means of the Lindenbaum's Lemma. Additionally, the canonical model is built by using **h** to show that there exists a counterexample to $H\backslash C$.

When the quantifier $\forall$ is added to the language, we can no longer appeal to the assistance of maximal consistent sets. An example from Garson [Garson, 2013a] (chapter 17), helps to clarify what happens in this case. We will employ the rules for $\forall$ under the substitutional interpretation, though the same problems arise with other interpretation too whether we adopt classical or free logic approaches to the quantifiers. Thus, the truth-condition for $\forall$ is formulated through the following clause:

- $\mathbf{a_w}\,(\forall x A x) = \mathbf{T}$ if and only if for all constant $\mathbf{c},\ \mathbf{a_w}\,(\mathbf{Ac}) = \mathbf{T}$

What is most important is to show that this holds in the canonical model, since the canonical model is defined in such a way that $\mathbf{a}_w\,(\mathbf{A}) = \mathbf{T}$ if and only if $\mathbf{w} \vdash \mathbf{A}$, for maximal consistent sets **w**. Thus, the fact that the above condition holds amounts to the following:

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[4] For a comprehensive exposition see [Hughes and Cresswell, 1996]. Our own exposition in this section will follow [Garson, 2013b], (Chapter 16).

- $\mathbf{w} \vdash \forall x A x$ if and only if for all constant $\mathbf{c}$, $\mathbf{w} \vdash \mathbf{Ac}$

As Garson points out, the left-right direction of this bi-conditional follows from the properties of the maximal consistent sets of modal formulas and the rules for the quantifier. The real problem lies in the proof of the other conditional. Plausibly, there are maximal consistent sets **w** which are not $\omega - complete$; i.e., such that one can derive instances of **Ac**, for each constant **c**, even though $\forall x A x$ is not derivable from **w**. That is because the set, which consist of $\neg \forall x A x$ and $A\mathbf{c}$, for each constant **c**, is consistent and can be extended to a maximal consistent set not containing $\forall x A x$. To explain a bit more this requirement, it must be said that its reason lies in the characteristics of Henkin's completeness proof for first order logic. According to the mechanism of Henkin's proof, starting from a set of consistent formulas Γ, it must be extended to a maximal consistent set. But not all maximal consistent sets turn out to be $\omega - complete$. This is a consequence of the process of forming maximal consistent sets from Γ, which requires at each step a choice of the formulas to be included in that set. Consequently, some extensions might fail to satisfy the property of $\omega - completeness$, since the appropriate formulas have not been included. Nonetheless, it can be proved the existence of a maximal consistent extension of Γ, which is $\omega - complete$, whenever Γ is a consistent set of wffs of first order logic with identity. This is tantamount to the 'Saturated Set Lemma' mentioned in the next page[5]. According to Garson, the problem can be solved by appealing to the so-called 'saturated sets' instead of simple maximal consistent sets. Precisely, this saturated sets are maximal consistent set which also satisfy the following condition.

$$If\ \mathbf{w} \nvdash \forall x A x,\ then\ for\ some\ constant\ \mathbf{c},\ \mathbf{w} \nvdash A\mathbf{c}$$

The contrapositive of this proposition guarantee that, whenever $\mathbf{w} \vdash A\mathbf{c}$, for each constant **c**, $\mathbf{w} \vdash \forall x A x$. That means that for saturated sets **w**, the truth-condition for the quantifier $\forall$ is satisfied in the canonical model. Thus, it seems that the solution to the problem above mentioned consists on defining the canonical model in such a way that the set **W** of possible worlds only contains all the saturated sets. Notwithstanding, to achieve that it will be necessary to obtain a new version of the Lindenbaum's Lemma. It must be proved now that all consistent set can be extended to a saturated set, instead of its customary formulation that every consistent set can be extended to a maximal consistent set. Also the

[5] For a more detailed exposition see [Carnielli and Pizzi, 2008, chapter 9.3], and [Garson, 2013a, pag. 370].

method of proving must be changed, since when a formula $\neg\forall x B$ is added to the set M_i to form the set M_{i+1} during the construction of the maximal consistent set, an instance of $\neg B\mathbf{c}$ must be also added, where $\mathbf{c}$ is a new constant. Moreover, this last condition of novelty regarding the constant causes some new difficulties. The most problematic case appears when one intends to build a saturated set from the consistent set M, whose sentences already contain all the constants of the language. Given that there are no more constants to be used, the saturated set cannot be construed in this language. This is not a problem for the completeness proof of quantified non-modal logic, because it suffices to introduce a more comprehensive language in order to obtain the saturated set **h** in this newer language by using the Saturated Set Lemma[6]. As a result, the canonical model for this new language can be construed and also will serve to obtain a counterexample to disprove the original argument. This strategy can be followed to obtain a completeness proof for QML were it not for the fact that it must be proved that the canonical model satisfies the truth-conditions for both, the first-order quantifier and the box, namely,

- $\mathbf{w} \vdash \forall x Ax$ iff $for\ all\ constant\ \mathbf{c}, \mathbf{w} \vdash A$
  $mathbfc$
- $\mathbf{w} \vdash \Box A$ iff $for\ all\ \mathbf{v} \in \mathbf{W}\ if\ \mathbf{w}R\mathbf{v}\ then\ \mathbf{v} \vdash A$

Some previous clarifications are needed to explain this better. To define the set V, let's remember first that **w** denote a maximal consistent set in **W**, the set of worlds of the canonical model, which contains each and every maximal consistent set for system S. Thus, the following condition defines the set V: $\mathbf{B} \in V\ iff\ \Box\mathbf{B} \in \mathbf{w}$[7]. So, for example, if **w** is the set $\{\mathbf{A}, \Box\mathbf{C}, \neg\Box\mathbf{B}, \Box\mathbf{D}, \Box\mathbf{A} \rightarrow \mathbf{E}\}$, then V is $\{\mathbf{C}, D\}$. (See [Garson, 2013a, pag. 198]). Moreover, it is well-known that the Lindembaum's Lemma is required to prove the second clause above in propositional modal logic[8]. Thus, in the case of QML, the saturation lemma should be used twice in order to show that there is a saturated set **v** such that $\mathbf{w}R\mathbf{v}$ and $\mathbf{v} \nvdash$ A. In that case, it turns out that the canonical model satisfies the truth-condition for $\Box$.

[6] Cfr. [Garson, 2013a, pag. 370]: 'If M is consistent and ready, then M has a saturated extension'. On the definition of 'ready set' see [Garson, *idem*, pag. 369].

[7] We can also see V as a list that results from removing $\Box$ from those members of **w** with the shape $\Box\mathbf{B}$.

[8] A proof of $(\nvdash \Box)$ is required:

$$(\nvdash \Box) : If\ \mathbf{w} \nvdash \Box \mathrm{A},\ for\ same\ \mathbf{v} \in \mathbf{W},\ \mathbf{w}R\mathbf{v}\ and\ \mathbf{v} \nvdash \mathrm{A} \quad (1)$$

Assuming $\mathbf{w} \nvdash \Box$A one shows that V, $\neg$A is consistent, where A $\in$ V iff $\Box$A $\in$ **w**. But then the Lindembaum's Lemma is needed to create from V, $\neg$A a maximally-consistent set **v** such that $\mathbf{w}R\mathbf{v}$ and $\mathbf{v} \nvdash$ A.

Turning back to the starting question, the main problem now is that there is no guarantee that a sufficient quantity of constants, which are not in the set of $\mathsf{V}, \neg\mathrm{A}$, would be available. Actually, given that $(P\mathbf{c} \to P\mathbf{c})$ is a theorem, it follows that it is an element of the set of $\mathsf{V}, \neg\mathrm{A}$ for all constant **c** of the language. In consequence, no new constant would satisfy the required condition. The problem cannot be overcome by means of a greater language L' for **v**, because we may be in the grip of a self-perpetuating cycle. We have to prove everything again for the language L' and to define **W** as the set of all saturated sets which can be formed in the language L'. Moreover, the problem would arise again when one intends to prove the second clause above-mentioned. Thus, we come back to the starting point.

Following the presentation of [Garson, 2013a], we have described the main trouble which have to confront the completeness proofs of QML by using the method of canonical models. There are many alternatives to overcome this thorny problem, but -that is the point- each solution works only with respect to some subset of QML. It is worth to mention the following passage from [Garson, 2001] with respect to this limitation (pag. 288-89),

> Each [strategy] has its strengths and weaknesses. Ideally, we would like to find a completely general completeness proof. The proof would demonstrate completeness of the most general semantics we have considered, namely QS. The proof for all less general systems would then fall out of the general proof just as proofs for the stronger propositional modal logics results from the completeness proof for K. This would help to clarify and unify quantified modal logic. (...) However, any such method will face some limitations...

3. QML's Completeness and the Barcan Formulas

Let us turn now to another aspect of the completeness problem of QML related to inclusion of the Barcan Formula in systems of quantified modal logic.

A skeleton is a structure that widen the frames for PML by adding individual's domains. We take the following definitions concerning these structures from [Braüner and Ghilardi, 2001]. A Kripkean-skeleton is a pair $\langle \mathcal{F}, \mathrm{D} \rangle$, where $\mathcal{F} = (W, R)$ is a frame and D is a Kripke's domain for $\mathcal{F}$[9].

[9] A Kripke $\mathcal{F}-domain$ (or simply a Kripke domain) D based on the Kripke frame $\mathcal{F} = (W, R)$ is a collection of (non empty) sets $\mathrm{D} = \{\mathrm{D_w} \mid \mathrm{w} \in W\}$ such that $\mathrm{D_v} \subseteq \mathrm{D_w}$ holds whenever $\mathrm{v}R\mathrm{w}$ (i.e., whenever w is 'accessible' from v). The Kripke domain D is constant iff we have $\mathrm{D_v} = \mathrm{D_w}$ for all $\mathrm{v}, \mathrm{w} \in W$. (See [Braüner and Ghilardi, 2001, pag. 850]).

A Firs-Order system of Modal Logic S is complete with respect to a class $\mathcal{K}$ of Kripkean-skeletons if and only if S is valid in every Kripke-skeleton belonging to $\mathcal{K}$. Moreover, every formula $\varphi \notin \mathbf{S}$ is not true in a Kripke model $\mathcal{M} = (\mathcal{F}, \mathrm{D}, \mathcal{I})$, such that $\langle \mathcal{F}, \mathrm{D} \rangle$ belongs to $\mathcal{K}$. It is said that S is K (Kripke)-complete if and only if it is complete with respect to a class $\mathcal{K}$ of Kripke's skeletons[10].

Having said all that, the basic quantificational systems, such as QK, QT, QK4, QS4, QS5, etc. and their variants with the Barcan Formula, are all of them K-completes. Moreover, in spite of the simplicity of the definitions of a modal system K-complete, it can be shown that K-completeness -contrary to what happens in PML- is not that frequent as one may expect.

We are going to consider now two examples of K-complete systems, whose completeness proofs are not straightforward and remained unsettled till the end of the last century. We know that the system S4 is obtained from the system K by adding the schemes $\Box\varphi \rightarrow \varphi$, $\Box\varphi \rightarrow \Box\Box\varphi$; on the semantic side, this system is valid and complete with respect to all Kripke frames $(W, \leqslant)$, where the accessibility relation $\leq$ is a pre-order, that is, reflexive and transitive. For its part, S4.2 is an extension of S4 with the axiom $\Diamond\Box\varphi \rightarrow \Box\Diamond\varphi$. As is noted in [Braüner and Ghilardi, 2001], the quantificational version of both systems turned out to be K-complete, although K-incompleteness seems to be the rule for many systems extending QS4.

In the beginning of this paper, we have considered an interesting trait of PML stemming from the so-called modularity of their completeness results. This modularity refers to the fact that the completeness proof for the result of putting together several systems of propositional modal logic, can be obtained straightforwardly by combining the completeness proofs for the former systems. QML lacks this kind of modularity. It may happen that we cannot obtain the completeness proof for a logic Q+M, which consists of a quantificational system Q and a propositional modal logic M, just by combining the completeness proofs for both respectively, Q and M. Taking again Garson's example: quantificational classical logic QL is complete for both, the objectual and substitutional interpretation of the quantifiers. In its turn, S4 is complete with respect to transitive and reflexive frames. Nevertheless, when the semantics for these systems are combined in the obvious way, the resulting semantics validates the Barcan Formula $\forall x\Box\varphi \rightarrow \Box\forall x\varphi$, which cannot be proved in QL+S4. This happens when the quantification domain is fixed and the same for all possible worlds. Curiously,

[10] As usual, lower case Greek letters are used here as metavariables over wffs.

the so-called Converse Barcan Formula $\Box\forall x\varphi \rightarrow \forall x\Box\varphi$ can be proved. Modularity is also wiped out from systems that restore symmetry by adopting the Barcan Formula. For example, the propositional modal system S4.1 is complete, but QL+BF+S4.1 is no longer complete[11].

It is likely to give an intuitive interpretation to this bewildering fact concerning the completeness of systems of modal logic with or without the Barcan Formula [Corsi and Ghilardi, 1989], [Hughes and Cresswell, 1996]. That is, why things change so drastically when we add this axiom. The reason is that in these systems if w_0Rw_1 and w_0Rw_2, then previous results from PML guarantee that whenever $\Diamond\Box\varphi \rightarrow \Box\Diamond\varphi$ is a theorem for all formula φ, then $\Lambda = \{\beta : \Box\beta \in w_1\} \cup \{\gamma : \Box\gamma \in w_2\}$ is consistent. What can be no longer guaranteed within a system with BF is that there would be a maximal consistent extension w_3 of Λ with the so-called $\forall$-property. This means that for every formula δ if $\delta\,[x/y] \in w_3$ for all variable *y*, then $\forall x\delta \in w_3$. Without BF, however, we can add new variables in order to get the desired $\forall$-property.

4. Modularity and the Interpretation of the Quantifiers

We have referred above two well-known interpretations of firt-order quantifiers, namely, the objectual and the substitutional interpretations. It is also reasonable to conjecture that the interpretation of the quantifiers have something to do with those issues related with the completeness of QML. Precisely, the consequences of the alternatives readings of the quantifiers shows up more clearly in the modal language. [Garson, 2013b] introduces a 'natural-semantics' method, which is based on valuations, on the lines of those of [Leblanc, 1976]. By using this technique one can obtain modular completeness results in QML. At least, where the properties of relevant frames are expressed. Based on these outcomes, QL+S4 turns out to be complete for this 'natural semantics', by making use also of the sentential interpretation of the first-order quantifiers. It can be predicted now, that BF and CBF will be symmetrical for this semantics. It is not difficult to show also that BF -not demonstrable in QL+S4- is invalid in this semantics, even though CBF is valid therein. This sharply differs from the result obtained by using the objectual or the substitutional interpretation, since both BF and CBF are validated under these interpretations. What happens is that the sentential interpretation is weaker than the objectual and the substitutional readings and so it leaves room for counterexamples to the Barcan formula.

What have been called a quantificational modal asymmetric logic, which

[11] See [Hughes and Cresswell, 1996]. S4.1 is S4$+\Box(\Box\,(\varphi \rightarrow \Box\varphi) \rightarrow \varphi) \rightarrow (\Diamond\Box\varphi \rightarrow \varphi)$.

accepts BF but rejects CBF, turns out to be adequate for a semantics in which the domains of quantification expand and can vary from one world to another. That is, whenever wRw', the domain of w is a subset of the domain of w'. Some authors -like Garson himself- have warned against this ad-hoc stipulation. Arguably, it lacks intuitive appealing and seems to be motivated solely by the desire of getting an adequate semantics for the combination of QML and S4. Having no domains for quantifiers, the semantics of valuations does not need to make these adjustments in order to get the purported results. Accordingly, the sentential interpretation constitute a newer and promissory way of interpreting the quantifiers in the modal setting, which provides in its turn, an adequate semantics to straightforwardly combine quantifiers and modality.

All these more technical achievements contribute to clarify the non-modularity which affects the completeness results of QML. According to this perspective, more traditional interpretations of the quantifiers bestow a stronger meaning upon the rules that govern the first-order quantifiers' inferential behaviour. In a modal context, this inferentialist thesis boils down to the validation of the Barcan formula, which would not be valid in a more natural setting.

5. The Interpretation of First-Order Quantifiers

We have been talking about the interpretation of the first-order quantifiers here and there throughout this pages, it is time to say a bit more about that before continuing with the development of this paper.

There are alternative approaches to the semantics of first-order quantifiers. Moreover, each interpretation makes its own when modal operators are added to first-order languages. Admittedly, possible world semantics for QML extends the standard semantics for classical logic and so it heavily depends on the reading conferred to the quantifiers. For these reasons, it is advisable to review briefly here the main options that have been on the market since many years ago.

5.1. The substitutional interpretation

Arguably, it is the simplest of all alternative readings of the quantifiers, which have been proposed. It gives also a good starting point to understand other approaches, since it avoids many technical conundrums.

Substitutional interpretation is based on the idea that an universal sentence $\forall xAx$ is true exactly when each instance Aa, Ab, Ac, ... is true. In the case of classical logic, this condition amounts to say that $\forall xAx$ is true if and only if

Ac is true for each constant *c* of the language. Concerning free logic, this condition would be the following instead: $\forall x A x$ is true if and only if Ac is true for all constants *c* of the language, which do not fail to refer to an object.

It is likely to develop the semantics for quantified modal logic by introducing adequate modifications into the models for propositional modal logic. If textitS is a propositional modal logic, a truth-value model for *S*, a *model v-S* $\langle W, R, \mathbf{a}\rangle$ contains the following items: a frame $\langle W, R\rangle$, with a set of possible worlds *W* and an accessibility relation *R*, which meets the conditions of the modal logic *S*, and an assignment function that determines the truth values at each world for all sentences belonging to the language of predicate modal logic. Assignment functions also meet the conditions for boolean connectives. For a language with $\perp$, $\rightarrow$, $\Box$ for example, we have the following definitions:

- $\mathbf{a_w}\,(\perp) = F$
- $\mathbf{a_w}\,(A \rightarrow B) = T$ if and only if $\mathbf{a_w}\,(A) = F$ or $\mathbf{a_w}\,(B) = T$
- $\mathbf{a_w}\,(\Box A) = T$ if and only if for all *v* such that wRv, $\mathbf{a_w}\,(A) = T$
- $\mathbf{a_w}\,(\forall x A x) = T$ if and only if for every constant *c*, $\mathbf{a_w}\,(A_c) = T$

Even though this semantics defines truth-conditions that allow assigning truth-values to compound sentences of the language, given the truth-values of their components, it does not allow to give satisfactory explanations in all of the cases. One of this cases concerns equality $s \approx t$, where it is not determined the way in which its value depends on the values of the involved terms and predicates. Actually, atomic sentences constitute a limit for this rather modest semantics. That said, given the definition of a model, other customary semantics notions can be defined as well, such as soundness, counterexample and validity.

Additionally, a domain for quantifiers can be introduced, which allows defining the reference of terms and predicates. A more significant improvement will introduce a domain-structure $\mathcal{D}$, which consists of a set of possible objects $\mathbf{D}$ and a subset $\mathbf{D_w}$ of $\mathbf{D}$ for each world $\mathbf{w}$. A substitutional model or *model-S* $\langle \mathbf{W}, \mathbf{R}, \mathcal{D}, \mathbf{a}\rangle$ includes a domain-structure $\mathcal{D}$ and an assignment function **a**, which confers now adequate extensions to all language expressions, including terms and predicates, by meeting also the conditions relative to the logical terms. Ordinarily, these work is performed in substitutional semantics by listing elements of **D** and the extension for a sentence will be a truth-value. Moreover, intentions of terms and predicates should be also defined. Following Carnap's idea, an intention is defined as a function from possible worlds to objects and a predicate intention, in its turn, is defined as a function from possible worlds to lists of objects.

We are ready to see now how looks the clause that settle the truth-conditions for quantifiers according to this proposal:

- $\mathbf{a_w}(\forall x A x) = T$ if and only if for all constant c if $\mathbf{a_w}(c) \in \mathbf{D_w}$, then $\mathbf{a_w}(Ac) = T$

Substitutional quantification is also related to the well known controversies which maintained Quine and Ruth Barcan about the interpretation of the formulas that combine quantifiers and modal operators[12]. By that time, Ruth Barcan's reply takes that the variables bounded by the quantifier are not in lieu of objects belonging to the universe, but in lieu of terms of a particular language. This quantifier's reading put assay the values of the variables and focus on their substitution instances in a specific language instead.

A more serious trouble with the substitutional interpretation is that it validates the so-called omega rule. Let us consider a version of this rule where substitution instances are variables instead of constants. The rule stipulates an argument with the conclusion $\forall x A$, whose premises comprise all instances of $A(x/y)$ for all variable *y* of the language:

- $\{A(x/y) : y \in Var\} \vdash \forall x A x$

Let us consider now the clause for the evaluation of the quantifier again:

- $\mathsf{v}(\forall x A x) = T$ if and only if for all variable *y* in *Var*, $\mathsf{v}(A(x/y)) = T$

Let us assume that a language *L* endowed with a set of variables *Var* has been fixed. An argument shall be deemed valid if and only if in every model over *L*, for all valuation v, which makes the premisses true, the conclusion is also true. Accordingly, by the preceding clause for the quantifier, the omega rule turns out to be valid. Moreover, it is not valid if a finite set of hypothesis is considered. It means that the notion of validity is not compact now. Compactness means that whenever an argument $H \vdash C$ is valid, there is another valid argument $H' \vdash C$, where H' is a finite subset of *H*. This lack of compactness implies the absence of strong completeness for finitary systems.

5.2. The objectual interpretation

This is no doubt the most renowned interpretation of the first-order quantifiers. This reading carries a more explicit association between quantification and ontology. Notably, it was defended and developed by Quine[13]. According to the

[12] On this long-standing debate see [Barcan Marcus, 1993] (Ch. 14) : 'A Backward Look at Quine's Animadversions on Modalities', and the references therein.

[13] See for example [Quine, 1953]

objectual interpretation, the sentence $\forall x Ax$ claims that every object of the universe has the property expressed by A. An existential sentence says that at least one has this property. Quine coined once the famous phrase 'to be is to be a value of a variable', that became a slogan of this position.

It is precisely a consequence of the objectual thesis the impossibility of combining quantifiers and intesional operators, whenever the first ones are interpreted as ontological indicators. According to the objectual interpretation, modal operators cause a change in the circumstances, moving our attention toward situations other than the actual world. The most remarkable example concerning this interaction between extensional and intensional operators has to do with the question of their scope. If an arithmetical theorem establishes that it is true $9 \geq 7$, it is also necessary true. That is, $\Box\,(9 \geq 7)$. If we say as well that 9 is *the number of planets of the Solar System* -at least, it used to be then- we can conclude *necessarily* $the\ number\ of\ planets\ of\ the\ Solar\ System \geq 7$, solely by changing the numeral by the definite description. The puzzling conclusion seems blatantly false. A first question to be accounted for is that the conclusion looks ambiguous, supporting both de dicto and de re readings. Needless to say, this will change its truth value. Admittedly, the argument is a tough nut to crack as witnessed by the sustained debate surrounding this topic[14].

It is clear that the objectual interpretation is deeper than the substitutional reading. At least, with respect to the ontological compromises which they carry with them. The ordinary reading of the existential quantifiers seems to come closer to the objectual interpretation too. The common-sense understanding of the expression that something is A, $\exists x A$, not only requires the existence of an instance *At*, which turns out to be true for some term *t*. But that there is an *object* having the property expressed by A. According to the objectual interpretation, the truth of $\exists x A$ commits us with the existence of certain thing and not merely with the truth of a sentence.

Additionally, the objectual interpretation gives rise to an interesting question with regard to modal logic. The issue concerns the so-called "rigid designator". S. Kripke in [Kripke, 1972] introduced the distinction between rigid terms (rigid designators) and no-rigid terms. According to this renowned distinction, rigid designators include proper names, like 'Saul' and those used for natural classes, like 'water', 'gold'. Characteristic of a rigid designator is that it always chooses the same object at each possible word (more precisely, worlds where the

[14] See, [Quine, 1953], [Quine, 1943], [Quine, 1976], [Smullyan, 1948] and for a more recent critical study of this problem and its solutions [Tuboly, 2015].

object exists). On the contrary, definite descriptions are supposed to be non-rigid designators, like 'the inventor of bifocal', since their referents vary from one world to another. In the actual world, this last expression refers to Benjamin Franklin, but in others possible worlds it refers to other individuals, such as for example, Thomas Edison. Now, we are ready to see the question about the objectual interpretation. The problem is that the following rules do not give rise to a valid sequence, when the term *t* is not rigid:

- $\forall x Px/Pt$
- $Pt/\exists x Px$

If we consider now the special case of the second rule: $\Box\, t \approx t/\exists x \,\Box\, x \approx t$, we have also that $\exists x \,\Box\, x \approx t$ is a theorem of any quantified modal logic. Nevertheless, this formula is not valid according to the objectual interpretation. This can be shown by using the example above. Suppose that *t* stands for 'the inventor of the bifocals'. Thus, $\exists x \,\Box\, x \approx t$ tells that there is someone such that necessarily he is the inventor of the bifocals. Admittedly, this affirmation is controvertible, since Benjamin Franklin (that creative though) was not *necessarily* the inventor of the bifocals.

It is likely to avoid this problem by getting ride of non-rigid terms. This solution seems to be very costly though. Lacking non-rigid terms will cause a serious shortage, since these expressions have an important role in the language. Other solutions have been explored, as it is the option of switching between classical and free logic [15].

5.3. The sentential interpretation

One can put the interpretation called 'sentential' by [Garson, 2013b] within an inferentialist tradition in philosophy. Garson himself encourages this suggestion. The sentential interpretation is as well akin to the interpretation of modal operators as quantifiers over possible worlds. This reading of the quantifiers is also related to possible world semantics for intuitionistic logic developed by Kripke. It shall be sufficient here to give the definition of the valuation for the quantifiers to be acquainted with this semantics. Here it is,

- $\mathtt{v}\,(\forall x A) = T$ if and only if for all $\mathtt{v}'$ if $\mathtt{v}' \leq_x \mathtt{v}$, then $\mathtt{v}'\,(A) = T$

In this clause, $\mathtt{v}' \leq \mathtt{v}$ holds whenever $\mathtt{v}'$ is an extension of $\mathtt{v}$, except for formulas containing *x* free. More precisely, $\leq_x$ is defined as follows:

[15] See, for example, [Garson, 2013a] and [Corsi, 2002].

- $v' \leq_x v$ if and only if for all wff A, which do not contain *x* free, if $v(A) = T$, then $v'(A) = T$

The sentential interpretation differs from the substitution and objectual interpretations. According to Garson, this interpretation expresses the natural reading of the quantifiers. It takes part in their natural semantics. Garson points out that the difference between these interpretations emerges even more clearly in a modal setting" [Garson, 2013b, pag. 265]. According to this natural semantics, intuitionistic and classical quantified modal logics based on K are complete[16]. Furthermore, these results extend to systems such as S4 that includes T and the axiom (4). It also turns out that BF and CBF are treated asymmetrically is this semantics for QL+S4. It can be proved that BF, which is disprovable in QL+S4, is not valid in this semantics now (even though CBF is valid). As Garson remarks, this contrasts with the symmetrical results obtained when the substitution or objectual interpretations are used, for in those cases both, BF and CBF, are validated. The explanation, as we said before, is that the sentential interpretation introduces a weaker reading of the quantifier that leaves room for counterexamples to BF.

Garson concludes that

> In general, the method of natural semantics provides resources for establishing modular completeness results in quantified modal logic, at least where the relevant frame properties are expressed. In fact it was the recognition that natural semantics would provide a tool for obtaining modular completeness results in QML that first prompted my research in the area[17]

.

It seems clear the advantage of the natural semantics favored by Garson with respect to its competitors concerning modular completeness results in QML. Arguably, a weaker semantics turns out to be more convenient when the quantifiers are added to the propositional modal language and modular completeness results are at stake. In the next section a much broader approach will be setlle in order to canvass the philosophical significance of these blatantly technical results.

6. Integrating these controversies into a more philosophical approach

Likely, this section might look like as a weird digression in this paper. As has been pointed out at the end of the previous section, we strive now to get a more

[16] [*Idem*, pag. 266].
[17] [Garson, 2013b, pag. 266].

philosophical perspective on all these developments and controversies, which are mainly given in the setting of formal semantics. Thus, we shall try to accommodate our considerations about the problem of completeness of QML in the wider issue related to the definition of a proper frame for philosophical logics. We will make use of some philosophical remarks, delivered from a Carnapian point of view, that we attempt to connect with the core tenets of this paper. What we intend to do is to interpret QML as a philosophical language, which also allows multiple reinterpretations that broaden its philosophical possibilities. I will follow the approach by [Damböck, 2009], who has reprocessed Carnap's original ideas about 'tolerance' in logical systems.

According to Damböck, it is convenient to distinguish in logic a mathematical and a philosophical point of view. Traditional metalogical questions are important from a mathematical point of view, but they are less important from a philosophical perspective. So that, in respect of philosophical issues it might be better to count with a simple semantic frame with less complicated metalogical properties. A framework where we can implement the philosophical logic that we want to use. Damböck also advises that, with respect to philosophical considerations, it is more convenient to follow a strategy of 'semantical restriction', rather than a more conservative strategy, which connects each language with its 'natural ontology'. Consequently, philosophical motivations can give a reason to limit the semantics and to proceed in accordance with the situation. Moreover, the expressive capacity of a logic can be considered formally in two different ways, namely, metalogical and metaphysical. In the metalogical sense, the expressive capacity of a logical system is given through model-theoretic considerations, such as the maximal cardinality of the sets characterizable up to isomorphism. In the metaphysical sense, the expressive power is given by all those formal traits of the language, which can be interpreted as expressions of some metaphysical properties. Having second-order quantification, modal operators, multiple truth-values, are these kind or formal traits in the metaphysical sense. In the traditional way of conceiving a logical systems, such as the position customarily ascribed to Quine according to Damböck, these notions of expressive capacity tend to converge. In accordance with this reading, having second-order quantification turns out to be tightly connected with the mathematical property of counting with more expressive power than first-order logic. Accordingly, it makes no sense to appeal to another interpretation of second order logic other than the standard one, since any other specification violates the natural connection between philosophical expressiveness and its mathematical expressive capacity.

Damböck's analysis remarks that a language possesses formal traits of a purely philosophical character, independent of mathematical or metalogical considerations. So that, there is no a-priori connection between the metaphysical expressiveness and the mathematical expressive capacity of a language. In other words, this connection is not a formal question, but a philosophical issue. So that, if we have a formal language with expressive capacity **A** (in a metaphysical sense), we can look for a metalogical specification for this formal language, which equip it with a mathematical expressive power **B**. For example, we can give the second order language the standard interpretation, but we can also specify it within the first-order language. With a philosophical purpose, the procedure goes that way: to stipulate the mathematical expressive power **B** that you need for your language, then to develop the language with metaphysical traits **A** that you want, within the metalogical framework provided by **B**[18].

From a historical point of view, this conception of logic is opposed to Quine's conception and follows Carnap's. Under this criteria, Quine have adhered to a monolithic conception of logic, in which logic and ontology are not differentiated. Alternatively, Carnap made a distinction between 'internal' and 'external' issues about logic[19]. External issues have to do with the determination of a general formal framework, whereas internal questions are those which can be settle within this formal framework. Damböck reinterprets somehow this Carnapian distinction by introducing a more explicit link between syntax and internal questions, on the one hand, and semantics and external issues on the other. At the external level, we aim to develop the main ontology. To settle a formal framework with a particular ontology. Anyway, the internal structure of the language also allows to introduce an ontology, which goes further than the immediate level of our language's natural ontology. It is likely to rewrite this natural ontology, as it were, by inserting powerful metaphysical traits at the internal level, by doing it at the level of syntax. Consequently, the famous 'Principle of Tolerance', once coined by Carnap, does not diverge significantly from this way of considering logic throughout the fundamental distinction between an external and an internal level, between metaphysical and metalogical issues. Thus, if the language's natural ontology can be rewritten, then it is no longer immovable.

[18] This is the key, according to Damböck (*op. et loc. cit.*), of the so-called 'Henkin's trick', which allows to reduce logics with arbitrary philosophical traits **A** to first-order logic

[19] Particularly, see [Carnap, 1950].

7. Concluding remarks

As to the quantified modal logic, it did not turns out to be a simple matter the addition of quantifiers to propositional languages. As it was presented throughout this work, this simple step causes much more problems than the transition from propositional to quantified first-order languages. There are a variety of alternatives semantics for QML that confront this technical and philosophical conundrum. Neither alternative gives convincing arguments that a system of rules is adequate for an intended interpretation.

Particularly, we have considered in this paper completeness proofs for quantified modal logic given in terms of Henkin models. That is, by an adaptation of the technique of canonical models borrowed from Henkin's completeness proofs for first-order logic and type theory. Admittedly, this is an election that can be questioned or rebutted. It can be right, but it was not our intention to defend possible-world semantics and the technique of canonical models from its detractors. We have just assumed this rich tradition and the results that it has produced. Admittedly, it remains an important line of research in current philosophical logic. In the setting of canonical models, the main difficulty that completeness profs for QML have confronted is related to the expansion of the language by means of the introduction of new constants. As we have seen, this fact creates a conflict that impedes the proof of the main Lemma. This difficulty persists whatever interpretation of the quantifiers we decide to use whether we adopt a classical or free logic approach. Different semantical options have been explored in order to account for this thorny problem. However, any method will face some limitations. So that, the interpretation of the quantifiers fall short of contributing to fix this big metalogical shortcoming. One can say that this is a more recalcitrant metalogical framework.

On the contrary, quantifiers' reading appears closely related with the election of the semantics that permits us to keep up modularity results concerning completeness in QML. Precisely, this last observation motivated the final part of this short work. What we have tried to show there, by following a Carnapian-inspired idea of C. Damböck, is that the identification of an independent philosophical framework for a formal language allows to put in a different perspective those technical results that at first glance looks very upsetting. As it was said, in the metaphysical sense, the expressive power is given by all those formal traits of the language, which can be interpreted as expressions of some metaphysical properties. The interpretation of the quantifiers can be counted among these kind or formal traits in the metaphysical sense. Thus, it turns out that a particular reading of the quantifiers can legitimize a philosophical framework that will put further requirements on the metalogical results, such as modular completeness.

Let us close this paper by quoting a passage by Ruth Barcan Marcus reminiscent of our theme. In the early days of the revival of modal logic, She made a clever statement on the difficulties caused by the interaction between quantifiers and modal operators, where the interpretation of the quantifiers was perspicuously remarked:

> The second prominent area of criticism of quantified modal logic involves interpretation of the operations of quantification when combined with modalities. It appears to me that at least some of the problems stem from an absence of an adequate, unequivocal, colloquial translation of the operations of quantification. It is often not quantification but our choice of reading and the implicit interpretive consequences of such a reading that lead to difficulties. Such difficulties are not confined to modal systems. [Barcan Marcus, 1961], reprinted in [Barcan Marcus, 1993, p. 16] .

All these being said, one can disagree with Garson's idea about the existence of a 'natural' reading for the quantifiers in QML. Certainly, we can admit though a more 'natural reading' according to certain philosophical framework, and it seems to suffice anyway[20].

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[20] I want to greatly thank an anonymous referee of this publication for a careful reading of this paper and for making many useful comments to a former version.

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