Why Hume's Notion of Demonstration Must Reduce to Probability: A Prelude to Quine

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Abstract This paper shows that Hume was ultimately forced to conclude in *The Treatise* that all demonstrative and intuited claims *can* in fact, be imagined as otherwise. As a result, he was forced to conclude that all knowledge claims must, ultimately, reduce to probable claims, or in Hume's *own*, and indisputably clear words: "*all knowledge degenerates into probability*" (THN 1.4.1.1; emphasis added). As a further result, it is suggested (briefly) that this anticipates Quine's well-known attack on the analytic/ synthetic distinction (Quine 1953).

Introduction

According to Hume, demonstrative reasoning, along with intuition, may produce knowledge (THN 1.3.1.2, THN 1.3.11.2). In the case of demonstrative reason, we move from one idea to another via a reflective "comparing" process, while in the case of intuition, no reflective process is involved; we immediately "intuit" the given relation between the ideas at hand. Regardless of this difference, in places in *The Treatise*, Hume claims that neither demonstrative nor intuitive claims can be imagined as otherwise without creating a contradiction, and so, are certain (THN 1.3.3.3, THN 1.3.6.1, THN 1.3.6.5, THN 1.3.7.3, THN 1.3.9.10, THN 1.3.14.13, and THN Abstract 11, 18).

But why, exactly, can't we imagine a demonstrative or intuited claim as otherwise?[1] Did Hume think that they constitute "deductive inferences?" (Stove 1973), (Millican 1995, 2002), (Mackie 1979), (Beauchamp and Rosenberg 1981))? No. Hume was ultimately forced to conclude in *The Treatise* that all demonstrative and intuited claims *can* in fact, be imagined as otherwise. As a result, he was forced to conclude that all knowledge claims must, ultimately, reduce to probable claims, or in Hume's *own*, and indisputably clear words: "*all knowledge degenerates into probability*" (THN 1.4.1.1; emphasis added). In this paper, I show why this is the case. As a further result, I show—in general terms due to length restraints—how this anticipates Quine's well-known attack on the analytic/synthetic distinction (Quine 1953).

To do this, I have divided the paper into five sections. In §1 we review Hume's notion of an "object," which is a summary of analyses that I have discussed elsewhere (Rocknak 2013, 2019, 2020). Here, we see that all objects are ideas of imagined causes, and so, may be imagined as otherwise. In §2, we examine Hume's notion of mathematical objects, specifically, numbers and geometrical objects, where we see that both kinds of objects—consistent with what we saw in §1—are ideas of imagined causes, and so, may be imagined as otherwise. In §3, we analyze the passages where Hume makes a distinction between demonstration vs probability. Here, we see that because all ideas of objects (including mathematical objects) may be imagined as otherwise, the distinction between demonstration and probability fails. In §4, we examine the passages in *The Treatise* where Hume clearly expresses his *own* doubts pertaining to the distinction between demonstrations vs probable claims. Finally, in §5, I show—in general terms—why Hume's reduction of demonstration to probability anticipates Quine's attack on the analytic/synthetic distinction.

§1 General Overview of Hume's Notion of Objects

Elsewhere, I have argued that in Book I of *The Treatise*, Hume thinks that objects are ideas of imagined causes, and so, we imagine the external world (Rocknak 2013, 2019, 2020). Here, I present a general summary of this interpretation of Hume, where the reader will be directed to the relevant details in the footnotes.

According to Hume, objects are complex ideas that are imagined to be the causes of our resembling but interrupted impressions. ~~In the course of doing so,~~ we imagine that these ideas have a "perfect identity" (THN 1.4.2.30, THN 1.4.6.6) i.e., they are imagined to be uninterrupted and invariable, or (roughly) equivalently, that they are continuous and distinct (Rocknak 2013).[2]

There are three instances in Book I of *The Treatise* where Hume argues that imagining objects with a perfect identity is a condition of possibility for ordinary experience. These occur in: 1.) "Of probability; and of the idea of cause and effect." (THN 1.3.2.1–2), where Hume discusses objects in terms of "secret causes," 2.) "Of skepticism with regard to the senses" (THN 1.4.2.15–24), which involves a discussion of the role that the two levels of constancy and coherence of our impressions play in regard to our conception of objects and 3.) "Of skepticism with regard to the senses" (THN 1.4.2.25–30) which comprises part 1 of 1.4.2's four-part system. Here, Hume discusses his "*principium individuationis*," i.e., the principle of identity.[3]

Ideas of objects that we imagine to admit of a perfect identity are similar to abstract ideas, although the former represent particular objects and the latter represent general objects. We may see why this is the case by first noting that, according to Hume, an idea of a general object is actually an idea of a particular object with a "certain term" (THN 1.1.7.1) attached to it. For instance, after experiencing a set of resembling ideas (say, of cars), we tend to refer to these ideas by the same name, despite any variations (THN 1.1.7.7). After doing so, when we hear the name 'car,' we remember, or "revive" (THN 1.1.7.7) one of the particular ideas of the set of resembling car ideas which allows us to mentally access the other ideas that it resembles.

However, we don't call to mind every idea of a resembling particular car when hear the name 'car' (THN 1.1.7.7). Instead, we employ a "power" or "custom" to bring to mind as many as "we have occasion" (THN 1.1.7.7) to, i.e., we bring to mind as many as we need to at any given moment. This means that abstract, general ideas do not *exactly* represent particular ideas, because if they did, they would merely be particular ideas, rather than particular ideas accompanied with the power to recall members of the set of resembling perceptions, as needed.

Hume has a somewhat similar process in mind in the three sets of passages noted above regarding how we imagine *particular* objects—e.g., the idea of a particular car. We must experience a set, i.e., a "species" (THN 1.3.2.2) or a "number" (THN 1.4.2.26–28) of "resembling" (THN 1.3.2.2) perceptions, where these perceptions are interrupted. For instance, as I direct my gaze outside, I may have a number of resembling blue Honda impressions, which are interrupted as I blink my eyes, or turn away for an instant (THN 1.4.2.27). Upon experiencing this set of resembling, although interrupted perceptions, we bring to mind one of the particular ideas that belongs to the set and imagine that it is the invariable and uninterrupted (or continuous and distinct) "secret cause" (THN 1.3.2.2) of the set. As a result of doing so, we imagine that this "secret cause" has a perfect identity (c.f. Rocknak 2013, 2019)).

Thus, just as was the case with the idea of a general, abstract object, we employ a particular perception from a set of resembling ideas to represent the object. Moreover, as was also the case with an abstract idea, this particular perception (e.g., a car perception) does not *exactly* represent any specific sense impression. This is the case in regard to ideas of particular objects because a.) We imagine the idea at hand to have the properties of invariability and uninterruptedness (or continuity and distinctness), where we never perceive any of these properties with our senses (THN 1.4.2.15–24, THN 1.3.2.2, see also Rocknak (2013, Part II)). And b.) We imagine that this idea is a *cause* of the set of resembling, interrupted perceptions, e.g., my blue Honda perceptions. We never "see or feel [these imagined causes]" (THN 1.3.2.3), so we cannot have had any perceptions of them being "constantly conjoined" (THN 1.1.1.8) with another impression. It is precisely in this respect they are "*secret* cause[s]" (THN 1.3.2.2; emphasis added, see also (Rocknak 2013, Part II)). As such, Hume explains, our imagined idea of an object that admits of perfect identity is only "oblique[ly]" and "indirect[ly]" grounded in our experience (THN 1.4.2.21).

With this in mind, we realize that no idea of an abstract object can come about without first imagining the particular ideas that comprise it. For instance, in order to imagine an abstract idea of a car, we must first imagine a set of resembling ideas of particular cars, where each idea of a particular car, as noted above, is an imagined cause.

Moreover, according to Hume, any of these particular car ideas *could be imagined as otherwise*. For any cause, according to Hume, may be imagined as otherwise (THN 1.3.7.3–4) "though I cannot always believe it." (THN, Abstract, 18) This is the case, generally speaking, because the way in which we conceive of causes and effects is entirely based on custom, specifically, the repetitive experience of impressions as being "constantly conjoined" (THN 1.1.1.8). However, although some constant conjunctions are more regular than others, all of our experience is, in principle, *variable* (THN 1.3).[4] As a result, it is possible to imagine that any experience may be otherwise, and so, it is possible to imagine the conjunction of any combination of impressions as otherwise. Thus, according to Hume, *any* causal relation may be imagined as otherwise with no contradiction (THN 1.3.3.3, THN 1.3.6.1, THN 1.3.6.5, THN 1.3.7.3, THN 1.3.9.10, THN 1.3.14.13 and THN Abstract 11,18).

This means that no idea of a particular object is, in principle, fixed, i.e., is invariable, because it is an imagined cause, despite the fact that we might *imagine* it to be invariable (and uninterrupted). For instance, I could imagine that the resembling impressions that I initially imagined to be caused by a white 2023 Audi are being caused by 2018 white Volkswagen, although I might be stunned that I changed my mind (THN Abstract, 18).

Thus, according to Hume, any idea of an *abstract* object may also be imagined as otherwise. For instance, my abstract idea of a car might initially be represented by an idea of 2023 white Audi—with what we imagine to be a gas engine—which calls to mind a set of resembling ideas of other particular cars—with what I imagine to be gas engines. However, I could, conceivably, imagine an idea of 2023 white Audi with what I imagine to be an electric engine, and in turn bring to mind a set of similar imagined ideas of particular cars, with what I imagine to be electric engines. And so, my abstract idea of a car has *changed*—it now has an electric engine, not a gas engine.

§2 Mathematical Objects

With Hume's conception of objects in general in mind, let us now consider his notion of mathematical objects, beginning with geometrical objects.

2.1 Geometrical Objects

Consider the following passage where Hume discusses the nature of geometrical objects:

> It appears, then, that the ideas which are most essential to geometry, viz. those of equality and inequality, of a right line and a plain surface, are far from being exact and determinate, according to our common method of conceiving them. Not only we are incapable of telling, if the case be in any degree doubtful, when such particular figures are equal; when such a line is a right one, and such a surface a plain one; but we can form no idea of that proportion, or of these figures, which is firm and invariable. (THN 1.2.4.29)

In other words, two of the most important ideas that we use to evaluate geometrical objects—equality and inequality—are "far from being exact and determinate." As a result, no evaluation of equality and inequality is either "firm or invariable." This is the case, Hume tells us immediately after the passage cited above, because the objects of our comparison—geometrical objects—are based on the senses, and, are *imagined*:

> Our appeal is still to the weak and fallible judgment, which we make from the appearance of the objects, and correct by a compass or common measure; and if we join the supposition of any farther correction, 'tis of such-a-one as is either useless or imaginary. In vain shou'd we have recourse to the common topic, and employ the supposition of a deity, whose omnipotence may enable him to form a perfect geometrical figure, and describe a right line without any curve or inflexion. As the ultimate standard of these figures is deriv'd from nothing but the *senses and*

imagination, 'tis absurd to talk of any perfection beyond what these faculties can judge of; since the true perfection of any thing consists in its conformity to its standard. (THN 1.2.4.29; emphases added)

This is entirely consistent with what we saw to be the case in regard to particular objects in §1 of this paper: ideas of geometrical objects are imagined, based on the senses; they are "deriv'd from nothing but the *senses and imagination*" (THN 1.2.4.29; emphases added). And although Hume is not explicit here, it makes interpretive sense to conclude that they are imagined in the same way that the ideas of any other particular object are imagined, i.e., ideas of geometrical objects are imagined to be the uninterrupted and invariable (or continuous and distinct) causes of sets of interrupted and resembling perceptions (e.g., triangular perceptions). For if we did not imagine that these ideas were uninterrupted and invariable (or continuous and distinct) causes of our resembling and interrupted perceptions, we would be unable to conceive of them as "objects"—they would have no identity (Rocknak 2013, 2019).

For instance, we might imagine that a particular uninterrupted and invariable (or continuous and distinct) triangular rock causes our interrupted but resembling triangular impressions. Moreover, we could imagine this particular shape to be otherwise. For instance, as I walk closer, my impressions change, and the sides no longer appear to me as equal. In fact, now it appears as though it has 4 sides. As a result, I imagine that these impressions are being caused by a rock with 4 unequal sides, although I might still call it a 'triangle'.

As a result, because an idea of an abstract triangle is comprised of particular ideas of triangles, it too, could be imagined as otherwise. For instance, although we might initially imagine an abstract idea of a triangle has having 3 sides, we could, in principle, imagine a number of ideas of objects that have 4 unequal sides, and call them triangles. Thus, our abstract idea of a "triangle," could, in principle, fundamentally change, just as our abstract idea of a car could change.

2.2 Numbers

With Hume's conception of geometrical objects in mind, let's now consider what he has to say about numbers. In T 1.1.7 *Of Abstract Ideas,* he writes:

> First then I observe, that when we mention any great number, such as a thousand, the mind has generally no adequate idea of it, but only a power of producing such an idea, by its adequate idea of the decimals, under which the number is comprehended. (THN 1.1.7.12).

In other words, a "great," i.e., a large number, is an idea. However, we do not have an "adequate" idea of a large number, but we "produ[ce] such an idea, by its adequate idea of the decimals." This is somewhat vague, but given that it occurs in the course of T 1.1.7, where Hume is explaining abstract ideas, we can conclude that he has an abstract idea in mind. More specifically, we "produce" an idea of a large number, e.g., the number 1,000, by using an "adequate idea of the decimals, under which the [large] number is comprehended." That is, we bring to mind an idea of a particular smaller number, e.g., the number 10, which, somehow—perhaps by thinking of the equation 10 x 10 x 10— "produces" the number 1,000. As such, this would be an abstract idea in the respect that it is a particular idea of a number accompanied with ideas of other particular numbers.

With this in mind, now consider Hume's understanding of our ideas of smaller numbers. In T 1.2.2 he writes*:* "'Tis evident, that existence in itself belongs only to unity, and is never applicable to number, but on account of the unites, of which the number is compos'd. (THN 1.2.2.3). In other words, "number"—particularly, smaller numbers—only exist in what appears to be an extensional sense, but not in an intensional sense.

Recall that the distinction between extension and intension is, generally speaking, as follows: An extensional definition consists of all of the members of a given set. Or, in Hume's words, it is "compos'd" of the members, i.e., what he refers to as the "unites" in the passage cited immediately above. For instance, the extensional definition of "chair" would consist of a set of particular chairs. Meanwhile, an intensional definition of a chair would consist of a description of a chair, e.g., “Something that is sat on.”

With this distinction in mind, now consider what Hume has to say immediately after the line cited above:

> Twenty men may be said to exist; but 'tis only because one, two, three, four, *&c.* are existent; and if you deny the existence of the latter, that of the former falls of course. 'Tis therefore utterly absurd to suppose any number to exist, and yet deny the existence of unites (THN 1.2.2.3)

That is, if we deny the existence of the twenty particulars (the “unites”), we deny the existence of the “number” twenty. It would be absurd to claim otherwise. In the immediately following passage, Hume employs the term ‘extension’ in the sense explained above:

> and as extension is always a number, according to the common sentiment of metaphysicians, and never resolves itself into any unite or indivisible quantity, it follows, that extension can never at all exist. 'Tis in vain to reply, that any determinate quantity of extension is an unite…For by the same rule these twenty men *may be consider'd as an unite*. The whole globe of the earth, nay the whole universe *may be consider'd as an unite*. That term of unity is merely a fictitious denomination, which the mind may apply to any quantity of objects it collects together; nor can such an unity any more exist alone than number can, as being in reality a true number (THN 1.2.2.3).

Hume’s thoughts here may be parsed as follows:

1. Some metaphysicians claim that because “extension” is a collection of particular things (is a “number”), it cannot be a “indivisible quantity,” and so “extension can never at all exist”
2. But “Tis in vain” to reply to this claim that extension *does* in fact exist, but as a unified, single thing, i.e., as a “unite.”
3. For this would mean that a collection of twenty men is actually one thing, i.e., a “unite,” and as such, does not consist of twenty individual men. Indeed, the “earth” and “the whole universe” would, in this respect, be thought of as a unity, which, Hume implies, would deny the existence of the particular things that respectively comprise the earth and the universe.

4. Instead, we must realize that such a unity is "merely a fictitious denomination" which cannot exist without the twenty men that comprise it.

This means that we could imagine the idea of any smaller number as otherwise—e.g., the number "twenty"— because the particular objects that comprise smaller numbers can be imagined as otherwise. For instance, we could imagine any of the particular "objects," i.e., the ideas of "men" that comprise the ideas of twenty "men" as otherwise. For instance, we could imagine the idea of one man as ideas of *two* "men," or ideas of *three* "men," etc. This could easily occur if the nature of our impressions shift, e.g., we move to the right or the left, or perhaps the light changes, so what we previously imagined to be an idea of one "man," we now imagine as an idea of one "man" standing directly in front of another "man." As a result, we could imagine that the number "twenty" is the number "twenty-one."

Concomitantly, this means that our ideas of larger numbers may be imagined as otherwise as well. For instance, the number 1,000—which, as we saw, is "produc[ed]" (THN 1.1.7.12) by smaller numbers, e.g., an equation comprised of ideas smaller numbers, e.g., 10 x 10 x 10—could shift. For as we just saw, our idea of the number 10 depends on our impressions, which could shift, and so our idea of the number 10 could shift to become an idea of the number 11 which in turn shifts the equation that "produce[s]" (THN 1.1.7.12) the idea of the number 1,000 to 11 x 11 x 11, which would give us the idea of the number 1,331 instead.

§3 Knowledge vs Probability

With Hume's notion of ideas of objects in mind—particularly, ideas of geometrical and arithmetical objects—let us now consider those passages in *The Treatise* where he makes a distinction between "comparisons of ideas" and "matters of fact." As noted in the Introduction to this paper, this comparison may occur as the result of "intuition" or a "demonstrative" process. Moreover, "matters of fact" can be imagined as "being otherwise" while "comparison[s] of ideas" cannot be imagined as otherwise without generating a contradiction (THN 1.3.7, THN Abstract, 33, 34, 35). Concomitantly, intuition and demonstration produce knowledge claims while "matters of fact" may only produce "probable" claims (THN 1.3.1, T 1.3.11.2). Arithmetical knowledge is the best example of demonstrative knowledge (THN 1.3.1.5–6).

We begin by taking a look at THN 1.3.1.1:

[philosophical] relations may be divided into two classes; into such as depend entirely on ideas, which we compare together, and such as may be chang'd without any change in the ideas. 'Tis from the idea of a triangle, that we discover the relation of equality, which its three angles bear to two right ones; and this relation is invariable, as long as our idea remains the same. On the contrary, the relations of *contiguity* and *distance* between two objects may be chang'd merely by the alteration of their place, without any change on the objects themselves or on their ideas; and the place depends on a hundred different accidents, which cannot be foreseen by the mind. 'Tis the same case with *identity* and *causation.* (THN 1.3.1.1).

Hume's thoughts here may be parsed as follows, where we will also draw on what Hume has to say shortly after this paragraph:

1.There are two classes of philosophical[5] relation:

a.) Those relations that "depend entirely on ideas," which we compare together, where these relations remain the same "as long as our idea remains the same." For instance, we may "discover the relation of equality" from a triangle (by comparing it's three angles to two right angles), but only if our idea of a triangle does not change. Thus—as long as these ideas do not change—these relations are *invariable*. They consist of: resemblance, contrariety, degrees in quality and proportions of quantity and number (THN 1.3.1.2)

b.) Those relations that change because the "place" in which the "objects" are located may change. For instance, two "objects" that are initially located next to each other, may not be located next to each other in the following instant, thanks to "a hundred different accidents." This means that these relations are *variable*, and include contiguity and distance.

2. The relation of identity is also variable. This is the case, Hume explains shortly after the paragraph cited above, because two "objects" may be "perfectly resembling" (T 1.3.1.1), and so, it seems, be *identical* to each other, and even may occasionally appear at the exact same place, but at different times. Nevertheless, Hume tells us, they are "numerically different" (THN 1.3.1.1) simply because there are two of them. Thus, Hume suggests here, identity can only occur when an "object" is identical to itself, where, as such, it is only *one* "object." As soon as there are two or more perfectly resembling "objects," identity is lost. As a result, the relation of identity is *variable*.
3. Meanwhile the relation of cause and effect is a relation "of which we receive information from experience" (THN 1.3.1.1). As a result, it is also *variable* because experience is variable.

There is much to be discussed in regard to this summary, but first, let's call attention to the fact that the invariability that holds of "comparisons of ideas" is a function of the invariability of the ideas involved in the comparison, e.g., the idea of a triangle (recall 1 a. above). Yet, as we saw in §1–2 of this paper, we can imagine the idea of *any* object as otherwise, including the idea of a triangle, particular or abstract. Immediately then, we see that this presents a serious problem for the invariability of demonstrative reasoning.

Moreover, note that here, specifically in 2. above), Hume acknowledges that the relation of identity is *variable*; one "object"—that is identical to itself—can become two or more "perfectly resembling" "objects," and so, lose its identity. For ease of reference, we will henceforth refer to the identity that Hume discusses here as identity *simpliciter*. This is different from the "perfect identity" that we discussed in §1, where we saw that an idea of an object admits of *perfect* identity when we imagine it to be the uninterrupted and invariable (or continuous and distinct) cause of a set of resembling perceptions. As such, as we saw, perfect identity is also variable because any cause may be imagined as otherwise. Regardless, it is clear that identity of any idea of an object—*simpliciter* or perfect—is variable, and so, once again, presents a serious problem for the invariability of demonstrative claims.[6]

Finally, here Hume makes it clear that any causal claim is variable because it is based on experience, which is also variable. This reinforces the fact that any idea of an object that admits of perfect identity is variable, because it is an imagined cause.

Now consider THN 1.3.1.5, which occurs shortly after the passage examined above:

> [A]lgebra and arithmetic [are] the only sciences, in which we can carry on a chain of reasoning to any degree of intricacy, and yet preserve a perfect exactness and certainty. We are possest of a precise standard, by which we can judge of the equality and proportion of numbers; and according as they correspond or not to that standard, we determine their relations, without any possibility of error. (THN 1.3.1.5)

In other words: Algebra and arithmetic are the only "sciences" that admit of "perfect exactness and certainty." This is the case because we can judge the "equality and proportion of numbers" with an "exact standard" that allows us to "determine" the relations that hold in algebra and arithmetic "without any possibility of error." However, as we saw in §2 of this paper, even this "standard" must be variable. For if our imagined idea of any number should vary, then the "exact standard" that we use to judge the "equality and proportion" of that number in relation to other numbers, or even to itself, would correspondingly vary. For instance, as explained in §2 of this paper, our idea of the number 20 could shift to an idea of the number 21, and so, any equation that involved the idea of number 20 would correspondingly shift, e.g., "20 = (10 x 2)" would be false, for the equation would have shifted to "21 = (10 x 2)."

Now consider THN 1.3.7.3:

> nor is it possible for the imagination to conceive anything contrary to a demonstration. But as in reasoning from causation, and concerning matters of fact, this absolute necessity cannot take place, and the imagination is free to conceive of both sides of the question (THN 1.3.7.3).[7]

However, as we saw in §2 of this paper, and above in regard to THN 1.3.1.1 and 1.3.1.5, it is only impossible to "conceive anything contrary to a demonstration" if the ideas involved do not vary, i.e., if the identity of any of the "objects" involved in the relation do not vary. Yet, as we have also seen, according to Hume, any idea of any object may, in principle, vary because no identity is invariable, whether identity *simpliciter*, or perfect identity. Thus, in principle, any demonstration may be imagined as otherwise.

§4 Hume's Doubts

Having illustrated the problems regarding Hume's distinction between knowledge and probability in light of the fact that all ideas of objects are variable, thanks to Hume's conception of identity—*simpliciter* and perfect—we must now analyze the textual evidence comprising Hume's *own* doubts regarding this distinction. We will see that these doubts are not only extensive, but that they are entirely consistent with the problems that I have pointed out above.

§4.1 Geometry: Hume's Doubts

First consider THN 1.2.4.10:

> [T]he objects of geometry, those surfaces, lines and points, whose proportions and positions it examines, are mere ideas in the mind; and not only never did, but never can exist in nature. They never did exist; for no one will pretend to draw a line or make a surface entirely conformable to the definition: They never can exist; for we may produce demonstrations from these very ideas to prove, that they are impossible (THN 1.2.4.10).

Hume's remarks here are fairly straightforward: We do not experience the "objects" of geometry "in nature." That is, we do not have sense impressions of perfect "surfaces, lines and points" etc. Instead, these "objects" are "mere ideas of the mind."

As a result, Hume continues, geometry is not a "precise" or exact" science (THN 1.2.4.17). This means, Hume writes—in no uncertain terms—that the *relations* that hold of the ideas of geometrical "objects" are inexact as well:

> It appears, then, that the ideas which are most essential to geometry, viz. those of equality and inequality, of a right line and a plain surface, are far from being exact and determinate, according

to our common method of conceiving them. Not only we are incapable of telling, if the case be in any degree doubtful, when such particular figures are equal; when such a line is a right one, and such a surface a plain one; but we can form no idea of that proportion, or of these figures, which is firm and invariable. Our appeal is still to the weak and fallible judgment, which we make from the appearance of the objects, and correct by a compass or common measure; and if we join the supposition of any farther correction, 'tis of such-a-one as is either useless or imaginary (THN 1.2.4.29).

That is:

1. "Equality and inequality," which are "most essential to geometry" are "far from being exact and determinate."
2. This is the case because these relations obtain of ideas of "objects" that are based on sense impressions: "Our appeal is still to the weak and fallible judgment which we make from the appearance of objects."
3. For instance, we might have impressions of what we imagine to be two triangular rocks ("figures"), and, by sight, determine that they are "equal," where this equality can only be a rough equivalence which we then attempt to "correct by a compass or common measure."
4. We may correct this idea of equality still further, but this correction is "either useless or imaginary."[8]

Hume makes very similar points in THN 1.3.1.4:

I have already observ'd, that geometry, or the art, by which we fix the proportions of figures; tho' it much excels both in universality and exactness, the loose judgments of the senses and imagination; yet never attains a perfect precision and exactness. It's first principles are still drawn from the general appearance of the objects; and that appearance can never afford us any security, when we examine the prodigious minuteness of which nature is susceptible …'Tis the same case with most of the primary decisions of the mathematics. (THN 1.3.1.4)[9]

His thoughts here may be parsed as follows:

1. Although geometry does excel “both in universality and exactness” it “never attains a perfect precision and exactness.”
2. This is the case because the “first principles” of geometry are based on the senses, i.e., “the general appearance of the objects.”
3. These appearances “can never afford us any security.”
4. Hume concludes: “’Tis the same case with most of the primary decisions of mathematics.”

Thus, in sum, because all of the ideas of objects of geometry are imagined, based on “appearances[s]” i.e., impressions, and all impressions, are, in principle, variable, all of our ideas of the relations that hold between these ideas of objects are variable as well. Thus, although Hume does not explicitly mention that geometrical objects are imagined *causes* here, his doubts concerning the “precis[ion]” and “exact[ness]” (THN 1.3.1.4) of geometry are consistent with the problems outlined in §2.1 of this paper. Specifically, we saw that, according to Hume, the “objects” of geometry—if they are to have an identity—must be imagined to be the uninterrupted and invariable causes of our interrupted and resembling impressions. As such, these ideas of geometrical objects must be variable, explaining in more precise terms, why Hume claims that they—and the relations that hold of them—are variable in the passages cited above.

§4.2 Arithmetic: Hume’s Doubts

Hume’s doubts about demonstrative knowledge are not limited to just geometry. Rather, he has clear concerns about the relations that hold between numbers as well. But before we discuss those concerns, we must take into account an argument he makes immediately before he discusses the probability inherent in algebra and arithmetic. In particular, in THN 1.4.1 he writes:

In all demonstrative sciences the rules are certain and infallible; but when we apply them, our fallible and uncertain faculties are very apt to depart from them, and fall into error…Our reason must be consider'd as a kind of cause, of which truth is the natural effect; but such-a-one as by the irruption of other causes, and by the inconstancy of our mental powers, may frequently be prevented. By this means all knowledge degenerates into probability; and this probability is greater or less, according to our experience of the veracity or deceitfulness of our understanding, and according to the simplicity or intricacy of the question. (THN 1.4.1.1)

Hume's argument here is as follows:

1. The rules that obtain of all demonstrative sciences are "certain and infallible."
2. However, the "faculties" that we apply to these rules are "fallible and uncertain."
3. Thus, our "reason" is a "kind of cause" and its effect is the "truth."
4. But this cause (i.e., our reason) is often interrupted by other causes and our faulty mental powers.
5. Thus, "*all knowledge degenerates into probability*," (emphases added).

Thus, in short, all demonstrations (knowledge) reduce to matters of fact (probability) thanks to our inability to correctly follow the "rules." This problem is distinct from the problems noted above concerning demonstrations and intuited claims, specifically, the fact that because any "object" can be imagined otherwise, any demonstration or intuited claim can be imagined as otherwise. However, in the immediately following paragraph, Hume begins to acknowledge that this too, is a problem, and, confusingly, he begins to retract his claim that the rules that obtain of "all demonstrative sciences," particularly, arithmetic, are actually "certain and infallible" (THN 1.4.1.1):

There is no Algebraist nor Mathematician so expert in his science, as to place entire confidence in any truth immediately upon his discovery of it, or regard it as any thing, but a mere probability. Every time he runs over his proofs, his confidence encreases; but still more by the approbation of his friends; and is rais'd to its utmost perfection by the universal assent and applauses of the learned world. Now 'tis evident, that this gradual encrease of assurance is nothing but the addition of new probabilities, and is deriv'd from the constant union of causes and effects, according to past experience and observation (THN 1.4.1.2).

That is:

1. All Algebraists and/or Mathematicians initially regard their conclusions to be "mere probability[ies]."
2. But the more they "run over" their proofs, the more confident they become and the less probable they seem.
3. In fact, as friends praise the proofs and the "learned world" lauds them, they become still more confident, to the point where the proofs are "rais'd to [their] utmost perfection."
4. However, this is a sham. This increase in confidence is "nothing but the addition of new probabilities" which are based on causes and effects, and so on "experience and observation."

Here, Hume does *not* seem to be claiming that knowledge reduces to probability due to our inability to follow the rules. Rather, he seems to be claiming that the rules themselves are probable. For the application of more rules, i.e., the more we "run over" them, is merely the addition of "new probabilities."[10]

Hume makes it clearer that this is what he means in the next paragraph:

In accompts of any length or importance, Merchants seldom trust to the infallible certainty of numbers for their security; but by the artificial structure of the accompts, produce a probability beyond what is deriv'd from the skill and experience of the accomptant. For that is plainly of itself some degree of probability; tho' uncertain and variable, according to the degrees of his experience and length of the accompt. Now as none will maintain, that our assurance in a long numeration exceeds probability, I may safely affirm, that there scarce is any proposition concerning numbers, of which we can have a fuller security (THN 1.4.1.3).

That is:

1. Merchants rarely think or trust that numbers are "infallib[ly] certain."
2. Instead, they trust the "artificial stricture" (structure) of their accounts.
3. This structure produces a high degree of "probability."
4. Thus, no one will "maintain" that "our assurance" involving any "long numeration exceeds probability."

5. Thus, it's safe to conclude that *no* "proposition concerning numbers" exceeds probability.

Here, Hume makes it clear that the "stricture" (structure) of our accounting system—i.e., the *rules* that we use to keep our accounts—is "artificial" and probable. Thus, the problem does not, in fact, lie with the inability to apply the rules. Additionally, he claims, no merchant trusts the "infallible certainty" of "numbers," which seems to mean that they don't trust the nature of "numbers" themselves.

This doubt is more pronounced in the immediately following paragraph:

For 'tis easily possible, by gradually diminishing the numbers, to reduce the longest series of addition to the most simple question, which can be form'd, to an addition of two single numbers; and upon this supposition we shall find it impracticable to shew the precise limits of knowledge and of probability, or discover that particular number, at which the one ends and the other begins. But knowledge and probability are of such contrary and disagreeing natures, that they cannot well run insensibly into each other, and that because they will not divide, but must be either entirely present, or entirely absent. Besides, if any single addition were certain, every one wou'd be so, and consequently the whole or total sum; unless the whole can be different from all its parts. I had almost said, that this was certain; but I reflect, that it must reduce itself, as well as every other reasoning, and from knowledge degenerate into probability (THN 1.4.1.3).

We may parse this argument as follows:

1. We could reduce any long series of numerical additions to smaller components, e.g., to "an addition of two single numbers." As such, we could attempt to eradicate the probability inherent in a long (probable) series of additions to a series of short (seemingly certain) additions of just "two single numbers."
2. But this would mean that it is "impracticable" to show "the precise limits of knowledge and probability." For where, precisely, Hume seems to mean, would the probability that admits of long numerations end and the certainty that allegedly admits of short numerations begin?
3. But if we can't discover where probability ends and certainty begins, we can't conclude that long numerations are somehow both probable and certain. For knowledge and

probability are mutually exclusive: "they cannot well run into each other [and so] they must be entirely present, or entirely absent."

4. Moreover, if one "single addition were certain," then the whole lengthy series of additions would be certain, and we wouldn't run into the problem noted in the passage cited just above this one, i.e., the conclusion that long numerations are probable.
5. Thus, we have no choice but to conclude that the numeration, "*as well as every other reasoning… must reduce itself, as well as every other reasoning, and from knowledge reduce to probability*." (emphases added)
6. Hume repeats his conclusion that all demonstrations reduce to probability twice in the next paragraph: "*all knowledge resolves itself into probability*, and becomes at last of the same nature with that evidence, which we employ in common life … "*demonstration is subject to the controul of probability*" (THN 1.4.1.5; emphases added).

In sum, then, in the course of Hume's analysis of arithmetic, we see Hume retract his initial claim that the problem with the demonstrations lies with our inability apply rules, to claim instead that all cases of arithmetical reasoning, even cases where we compute short, easy equations (where there would be no problem applying the rules), are probable. As a result, it seems that the problem must lie with the nature of numbers themselves—they are variable, as we saw to be the case in §2.2. of this paper.[11]

Thus, at this point, it should be clear that ultimately—despite his claims to the contrary (recall §3 of this paper)—Hume did not believe that there was a mutually exclusive distinction between demonstrations and matters of fact; i.e., a mutually exclusive distinction between knowledge and probability, where the former is certain and the latter is not.[12] Rather, as Hume clearly tells us in THN 1.4.1—on four separate occasions—demonstrative claims "degenerate" (THN 1.4.1.3) into probable claims.[13]

§5 Quine's attack on the Analytic/Synthetic distinction: A Brief Overview

One might attempt to defend Hume's notion of demonstrations from the fact that all ideas of objects may be imagined as otherwise as follows: Simply stipulate that certain meanings, e.g., the meaning of the idea of "triangle," or the meaning of the idea of the number "twenty," never change. In other words, establish a *convention* that these meanings are fixed, and so, are not variable. As a result, the relations that hold between these meanings would not be variable either, and so, it seems, would remain certain. Some might argue that this is what Hume was trying to do when he wrote in T 1.3.1.1 (discussed in §3 of this paper): "'Tis from the idea of a triangle, that we discover the relation of equality, which it's three angles bear to two right ones; *this relation is invariable, as long as our idea remains the same*" (emphases added)

However, this was simply not an option for Hume. Merely stipulating that, e.g., a triangle, has a necessary nature such that our ideas of it never change would have stood in direct conflict with Hume's opposition to metaphysics and the "certainty" that it touts (THN Intro 7, 9–10).[14] Moreover, and relatedly, this would stand in direct conflict with Hume's notion of the variable nature of our ideas of objects, explained above—according to Hume, no idea of an object is invariable, regardless if we insisted otherwise, i.e., tried to establish it by "convention."

As a result, Hume anticipates the contemporary epistemologist Quine, who famously argued that there is no distinction between necessary and probable claims. Rather, like Hume, he argued that all knowledge claims are probable. More specifically, he claimed that there is no distinction between "analytic" and "synthetic" claims, where analytic claims are alleged to be true in virtue of their meanings and synthetic claims are true in virtue of empirical facts. For instance, an example of an analytic claim is "all bachelors are unmarried men," which is necessarily true because of the meanings of the terms 'bachelor' and 'unmarried men.' Meanwhile, an example of a synthetic claim is "The sky is blue." As such, this claim is merely probable because experience is variable, and so, empirical facts are variable (Quine (1953);[15] see also Rocknak (2010) (2013b)).

In brief, and in very general terms, Quine argued that all analytic claims reduce to synthetic claims because, like Hume, he claims that all meanings are variable—because we cannot stipulate an invariable meaning without being circular. For instance, in order to stipulate what the meaning of a word is, e.g., ‘bachelor’, we must first know what ‘bachelor’ means.[16]

As a result, as we saw in §1–4 of this paper, Hume clearly anticipated Quine, albeit in a general, but very powerful sense: All allegedly certain claims reduce to probable claims. So why didn’t Quine credit Hume?[17] He certainly read Hume, even teaching a semester-long course on him at Harvard (Quine 1985, 194). However, Quine admits he was simply not interested in figuring out what Hume thought (Quine 1985, 194). But it’s clear that he and Hume were epistemological bedfellows; Hume dismantled the distinction between necessary and probable claims long before Quine would make his career by claiming to be the first empiricist to do so.

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[1] For the sake of brevity, I will, hereafter, for the most part, refer to demonstrative and intuited claims as "demonstrative" because the distinction between demonstrations and intuitions does not pertain to the goals of this paper, i.e., it does not matter if we immediately "intuit" the relation between two "objects" or use a reflective comparing process.

[2] For more detail, see (Rocknak 2013, 88–89, 104, 122, 153–155, 181, 182).

[3] Although Hume discusses variants of 1.4.2's principle of identity in THN 1.4.6, the bulk of 1.4.6 consists of giving three separate accounts of the transition from the vulgar perspective to the philosophical perspective. See (Rocknak 2013, 189–217) for more detail. For an extensive account of how Hume's transcendental account of imagining objects differs from the philosophical account and the vulgar account, see Rocknak (2007, 2013, 48-51, 244–45).

[4] However, although our experience is, in principle variable, and so is not certain, some of it is *regular*. See (Rocknak 2020) for more detail.

[5] For the purpose of this paper, we need not broach what Hume means by "philosophical." For more on this term, see (Rocknak 2013, 29–52, 47–51, 92–95, 254–261). See also (DePierris 2002), (Owen 1999, 151–153) and (Schliesser 2007).

[6] We can square Hume's notion of identity, *simpliciter*, with his notion of perfect identity, as follows: We may imagine that, e.g., our idea of a "chair" has a perfect identity in virtue of imagining that it is the cause of our resembling chair perceptions. This idea also admits of identity *simpliciter* if we imagine that it is one chair, i.e., that it is identical to itself. However, if, for example, a friend of mine buys what appears to be the exact same kind of chair, and I experience a set of resembling perceptions from this "chair" that exactly resemble the perceptions that I experience with my "chair," this does not mean that the two imagined ideas of "perfectly resembling" "chairs" are identical. Rather, they are two ideas, and so, are numerically different. See also THN 1.4.2.25–43, where Hume discusses the role that unity and number play in regard to perfect identity, which, as noted above, appears to be distinct from the identity discussed in THN 1.3.1.1. See also (Rocknak 2013, Chapter 7), and for an alternative point of view (Baxter 2008).

[7] See also THN Abstract, 33, 34, 35 for similar claims.
[8] Hume repeats and clarifies this point on THN 1.4.2.22, where he references THN 1.2.4, cited above.
[9] See also THN 1.3.1.6
[10] Indeed, according to Hume, *all* "rules" are probable; see (Rocknak 2020). For an alternative view on Hume's skepticism in *The Treatise*, and how it relates to *The Enquiry*, see (Qu 2020).
[11] See also the conclusion of Book I of *The Treatise* (THN 1.4.7.2) where Hume makes it very clear that he has cast doubt on mathematics, which is supposed to be the bastion of demonstration (THN 1.3.1.5).
[12] This is contrary to many, if not most Hume scholars, including, most notably, (Kemp Smith 1941), (Flew 1961) (Stroud 1977) and Garrett (1998) as well as those scholars noted in the Introduction to this paper, i.e., (Stove 1973), (Millican 1995, 2002), (Mackie 1979) and (Beauchamp and Rosenberg 1981).
[13] It is worth noting that even Hume's claim that all knowledge must degenerate into probability, is, itself, probable. However, a discussion of this point would take us too far afield. See THN 1.4.7 for more detail.
[14] Cf. (Garrett 1997, 30–33), (Millican 2002) and Rocknak (2013, 2020).
[15] Quine discusses this issue in multiple places, in addition to his 1953 article "The Two Dogmas of Empiricism" (originally published in 1951 in *The Philosophical Review*). For instance, see at least, "Two Dogmas in Retrospect" (Quine 1991), "Five Milestones of Empiricism" (Quine 1981) "Truth by Convention" (Quine, 1976), and *Dear Carnap, Dear Van* (Creath 1990).
[16] For more detail, see Quine (1951), i.e., "The Two Dogmas of Empiricism." See also Rocknak (2010) and (2013b), where I explain Quine's argument in detail.
[17] Quine mysteriously did not cite Morton White (1950) either, who attacked the analytic synthetic distinction a year before Quine published "The Two Dogmas of Empiricism" in *The Philosophical Review*.