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Rethinking Identity:

Dialectics, Quasi-Sets, and Metalogic

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Philosophy can never be separated from the rigorous pursuit of the foundations of science, where "science" is understood broadly as a body of rational knowledge organized into a systematic, self-founded discourse. Philosophy is, in a certain sense, the evolving metarationality, and only it is capable of advancing into the inquiry of ultimate presuppositions.

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The problem of the foundation, therefore, far from being secondary, is central to the very notion of philosophy, as well as to the continuity of serious philosophical discussion among peers.

One of the fundamental topics of philosophy is the principle of identity ($A = A$). Reflecting on it requires situating ourselves in an intermediate zone between metalogic, metamathematics, metametaphysics, and metaepistemology. Traditionally, the notion of identity is conceived as a primitive concept—“to be identical to...”—and is qualified by the typical properties of binary relations, namely: reflexivity ($(\forall x)\ x = x$), symmetry ($(\forall x, y)\ x = y \rightarrow y = x$), transitivity ($(\forall x, y, z)\ (x = y) \land (y = z) \rightarrow (x = z)$), and substitutivity ($\forall x\ \forall y\ ((x = y \rightarrow A(x)) \rightarrow A(y))$).

The problem with its intuitive notion is that identity ("to be identical to...") depends on and is supported by the principle of the indiscernibility of identicals/identity of indiscernibles in classical logic; however, to begin with, identity does not exactly and strictly correspond to indiscernibility, leaving room for imprecision and error. The principle of identity asserts that if two objects are identical, then they are exactly the same object in all respects. The principle of the identity of indiscernibles, in turn, asserts that two objects cannot be identical (i.e., cannot be the same object) if they do not share exactly the same properties. In other words, if two objects share all properties in common, then they are the same object—

this principle addresses the distinction between different objects in terms of their individual properties.

Thus, their differences are as follows:

The principle of identity asserts how objects are identical in terms of their properties. It deals with the relationship between identity and the specific properties of objects.

The principle of the identity of indiscernibles addresses the distinction between distinct objects based on the properties they possess. It investigates when two objects are considered different due to their distinct properties.

The principle of identity (or the principle of the identity of identicals) is often used in logic and metaphysics to discuss the nature of identity and the properties of objects.

The principle of the identity of indiscernibles is used to discuss issues of individuation and the distinct identity of objects in a context where distinguishing between objects is crucial (such as in the discussion of individual entities).

In conclusion, it can be said that identity leads to indiscernibility, but indiscernibility does not necessarily lead us to identity. The first breach in the front of the principle of identity in its classical formulation lies here. And through this breach, we move into the space opened by non-standard models of identity: such models arise when considering structures where identity behaves unusually or violates expected standard properties, which can include situations where distinct objects may be indiscernible through the formulas of identity theory.

Going further, expanding this space of indeterminacy within identity, we can recall the method of partial isomorphisms, developed by José Carlos Cifuentes Vásquez (CIFUENTES VÁSQUEZ, José Carlos. O Método dos Isomorfismos Parciais e a Caracterização Algébrica da Expressabilidade Matemática. Dissertação apresentada ao Instituto de Matemática, Estatística e Ciência da Computação, UNICAMP. Campinas: UNICAMP, 1988.file:///C:/Users/user/Downloads/cifuentesvasquez_josecarlos_m.pdf). In this case, the relationship between partial isomorphisms and identity in logic can be understood through the lens of model theory and identity theory. Let us examine.

A partial isomorphism between two models “M” and “N” is a function: f : M → N, which preserves the relations and functions by which it is defined. Partial isomorphisms are useful for establishing structural equivalence between complex models, where a partial function can be extended to cover all elements of the domains of "M" and "N."

In model theory, identity is a primitive relation that satisfies properties like reflexivity, symmetry, and transitivity. In first-order logic, identity is formalized through the symbol "=". Identity ensures that two identical terms or objects are exactly the same object in all respects. This is formally expressed as "$x = y$", from which it follows that $\varphi(x)$ is true if and only if $\varphi(y)$ is true for any formula φ. Partial isomorphisms are relevant to identity in that they show how structures can be isomorphic over an extension of their parts. This means that two structures can be partially equivalent, preserving certain essential characteristics (such as relations and functions).

On the other hand, in contexts where identity can be interpreted in a non-standard manner (such as in non-standard models of identity theory), partial isomorphisms can be used to show how different interpretations of identity can coexist under certain conditions. The flexibility of partial isomorphisms allows us to explore how different structures can be considered equivalent under less restrictive conditions than those imposed by total isomorphism. This is crucial in investigating non-standard models of identity, where varied conceptions of identity can be explored.

An example of a “non-standard” model of identity would be governed by the introduction of a new element that is not identical to any other element in the domain but is distinguished only by the formulas of identity theory. Let us consider the domain “N” of natural numbers with the following identity theory: identity theory is extended to include a new element “c” in the domain that is not identical to any of the natural numbers. Here, we have the domain “N” of natural numbers, including “c” as an additional element. The interpretation of identity is thus modified to include the identity relation between natural numbers (N) and between “c” and other elements. In this model, all formulas that are true in the standard model of natural numbers remain valid. Additionally, we add the formula stating that “c” is not equal to any natural number, i.e.: $\forall x \neg (c = x)$.

When we challenge the scope of the principle of identity, we implicitly attribute some indeterminacy to the scope of the quantifier “$\forall x$” and the nature of the elements of the domain (x, y, z…). Are they individuals, classes, sets?

For indiscernibility to exist, we must determine what defines identity: properties or relations? Before that: is the definition given by "properties + relations" (conjunction), or "property or relations" (exclusion)? After: what types of properties and relations are we talking about? Spatiotemporal, monadic, dyadic, intrinsic, universal? There will only be a correct resolution of the issue at hand if, first, we consider that there are three versions of identity: a) P1 - identical are those elements that possess the same properties and relations of all kinds; b) P2 - identical are those that possess the same intrinsic or "substantial" properties, excluding those of a spatiotemporal nature; and c) P3 - identical are those that possess the same monadic properties, represented by a unary predicate, such as P(x), where P is the property and x is the individual to which the property is attributed (in mathematical logic, "property" is a formula with a single free variable).

The choice of one version or another of identity also alters the problematic consequences that follow from it. However, in any case, the work of KRAUSE and FRENCH (Identity in Physics: A Historical, Philosophical and Formal Analysis, 2006) shows that, in principle, the model of classical particles in physics and the model of quantum mechanics of particles derogate from the principle of identity in its classical formulation (as mentioned: considering any of the three versions described above!). That is, even if we exclude, for example, the problematic spatiotemporal properties and relations (are we dealing with absolute space-time, general relativistic, or Minkowski space-time? We are stuck in problems here...), the derogation of the principle of identity remains.

And why?

Well, the whole issue here would revolve around the adoption of a classical set theory model (say, Zermelo-Fraenkel based on the axiom of extensionality: $\forall A\ \forall B\ (\forall x\ (x \in A \leftrightarrow x \in B) \rightarrow A = B))$ and classical first-order logic. If we choose to take this step, we are assuming the inevitability of the classical principle of identity. However, pay attention: if we opt for paraconsistent logic and another model of set theory, the situation changes. This is because, according to Krause and French, quantum mechanics shows that the principle of identity does not apply to the quantum physical-mathematical model; thus, we should abandon classical logic and mathematics as references in constructing the principle of identity, going against what Leibniz did in postulating the identity of indiscernibles.

According to these philosophers, in quantum mechanics, particles like electrons are indistinguishable, meaning that there are no intrinsic characteristics that can be used to differentiate them. In a system of multiple particles, therefore, the permutation of two identical particles does not result in a new distinct quantum state; the state remains the same or changes only by a sign (in the case of fermions). Thus, if we cannot distinguish between two electrons based on their properties or spatiotemporal relations, the classical principle of identity seems inadequate for describing the behavior of quantum particles.

This reasoning not only suggests that we must revise the classical principle of identity but also forces us to accept a non-classical logic as a fundamental part of this revision, thus rethinking and limiting the classical notion of "identity." This is the crucial point in French and Krause's thesis.

They thus propose quasi-set theory as a mathematical framework for dealing with collections of entities that are not individuals in the traditional sense. In quasi-set theory, it is possible to handle collections of indistinguishable entities without assigning specific identities to each of them. This thesis has profound implications for the interpretation of the foundations of quantum mechanics, suggesting that the nature of quantum reality challenges classical logic and requires new approaches to understanding entities and their interactions.

Let us imagine a system with two electrons, “e1” and “e2”. In classical mechanics, we could identify each electron individually and track its trajectory. However, in quantum mechanics, electrons are indistinguishable, meaning there is no intrinsic property that allows us to differentiate them. In the classical world, the formula “$e1 \neq e2$” holds, indicating that “e1” and “e2” have distinct identities, even if they are identical in all other respects. At the quantum level, the wave function of a two-electron system must be either symmetric or antisymmetric with respect to particle exchange, depending on whether we are dealing with bosons or fermions. For electrons, which are fermions, the wave function is antisymmetric: $\psi(x1, x2) = -\psi(x2, x1)$. This implies that swapping e1 and e2 results in the same wave function (except for a sign), meaning we cannot distinguish which electron is in which position. Therefore, e1 and e2 do not have their own distinct identities. In this sense, in a quasi-set, it makes no sense to ask "which one is e1?" because e1, e2, and e3 do not have distinct identities. We can only say that we have three indistinguishable "e" particles.

We believe that the debate and tug-of-war surrounding the principle of identity, with the battles waged between logicians and mathematicians for the prevalence of their preferred models, can be circumvented by a structural paradigm shift in addressing the issue. To explain our thesis, let us first return to Hegelian dialectics.

Let us start from the beginning: the Concept generates its own self-evolution by harboring within itself its own contradiction (Widerspruch—coherence). The coherence or reconciliation of opposites, thus, from the contradictory engine, creates the space for Difference in the self-determination of Being, when universal maximality returns upon itself, reaching its concreteness through dialectical self-movement.

The caution we must take here is the following: the totalizing temptation of dialectics often seduces philosophers with its irresistible centrifugal force. Dialectics often presents itself as total and universal. However, quite the contrary, it is self-limiting and deficient. An example of this is that the attempt at full application of the work of the negative results in an irrecoverable regression to infinity.

Nevertheless, dialectics can explain and effectively clarify a strange possibility that we have of thinking about something determined (“x”) as already imbued with difference (“Dx”). According to this thesis, the “identity-difference” circuit simply collapses if we attempt to expel difference from its development.

Let us see.

There is a heavy philosophical taboo around the principle of identity. We believe that current confrontations and challenges from the philosophy of science, especially in the field of quantum physics (we have already discussed KRAUSE and FRENCH), show that this time-worn principle still needs to be philosophically reconfigured to reach its maximum adequacy and intelligibility.

We believe that the new key to be considered here is the philosophical necessity of always thinking of identity in reference to difference. If something determined, let’s say “A”, is identical to itself (A = A), it must also be, at the same time, different from itself (IA = DA). Why? Because

“A” is not an individual, a substance, an object, fundamentally, but a complex of relations in interaction.

Thus, we return to physics: instead of considering particles as entities with intrinsic properties, we can treat them as elements of a system defined by their interrelations. This view emphasizes structure and relationships rather than intrinsic attributes of the particles. In this relational view, the focus is on the interactions between the elements of a system rather than the intrinsic properties of these elements. This can be formalized using mathematical structures that capture these relationships: an n-tuple is an ordered sequence of n elements. If we consider a system of n quantum particles, we can describe the state of the system as an n-tuple of relationships and interactions between the particles.

In this way, we can model a quantum system with structures that capture the interactions between particles without assigning individual identities. One way to do this is by using category theory or graphs, where objects are the particles and arrows (morphisms) represent interactions or relationships.

Example:

Consider a system with three particles A, B, and C. Instead of assigning intrinsic properties to each particle, we describe the system in terms of their interactions:

Rab: Relationship between A and B.

Rbc: Relationship between B and C.

Rca: Relationship between C and A.

Here, the focus is on the network of interactions that defines the state of the system.

In the relational interpretation of quantum mechanics, proposed by Carlo Rovelli (see: ROVELLI, Carlo. Helgoland: Making Sense of the Quantum Revolution. New York: Riverhead Books, 2021), the state of a particle is not absolute but depends on its relationship with other particles. This fits perfectly with the idea of using n-tuples to describe quantum systems, in which we have:

(1) The state of a particle is relative to the observer or the system with which it is interacting.

(2) Properties emerge from interactions and are not intrinsic to the particles.

We can use graphs or category theory to formalize these ideas. A quantum graph could have vertices representing particles and edges representing interactions. The system's properties would be derived from the graph's properties, not from the individual vertices:

Let us consider a quantum system with n particles. We can represent this system using a quantum graph (a) = (V, E), where:

$V = \{v1, v2, ..., vn\}$ are the graph’s vertices, representing the n particles of the system.

$E \subseteq V \times V$ are the graph’s edges, representing the interactions between the particles.

Each edge $\{(vi, vj) \in E\}$ represents an interaction between particles “vi” and “vj.” The presence of an edge indicates that there is a relationship or interaction between the corresponding particles.

The properties of the quantum system can be derived from the structure of the graph.

Connectivity and Interactions: The connectivity of the graph indicates which particles are interacting directly with one another.

Cycles and Configurations: Cycles in the graph may indicate specific interaction configurations that influence the system's behavior.

Functionals and Compositions: In category theory, morphisms between quantum graphs can represent transformations or compositions of quantum states.

Consider a simple system with three quantum particles A, B, and C. We can represent the interactions between them with the following graph:

Vertices: $V = \{A, B, C\}$.

Edges: $E = \{(A, B), (B, C), (C, A)\}$.

This approach suggests a relational (indeed, structural) ontology, where fundamental reality consists of relations and not objects with intrinsic properties. Such a proposal is compatible with certain interpretations of quantum mechanics and can resolve impasses like the indistinguishability of particles. If particles have no intrinsic properties and are defined solely by their relations, then the classical notion of identity is eliminated. Particles are not individuals in the traditional sense; they are nodes in a network of interactions. This approach is consistent with contemporary physics, where quantum particles are described by states that depend on the system's configurations rather than individual identities. The wave function of a particle system incorporates all the information about the interactions without distinguishing between identical particles.

Krause's thesis suggests that quantum particles are not individuals in the traditional sense, as they lack distinct identities. The proposal to emphasize interrelations instead of intrinsic properties reinforces this idea by eliminating the need for individual characteristics to distinguish particles.

If particles are defined only by their interactions, then quantum indistinguishability is a natural consequence. This corroborates the view that quantum particles do not possess individual identity. The theoretical support for such a conclusion would, in summary, be the following:

By using n-tuples to describe quantum systems, where particles are represented solely by their relations, we provide a sufficient mathematical structure that supports this thesis. Thus, instead of thinking of particles as entities with fixed identities, they can be seen as nodes in a network of interactions, aligning with the idea of non-individuality.

From this point on, we can return to dialectics and metalogic:

The dialectic of “identity-difference” adopted here suggests that the only possible identity for an entity is intrinsically linked to difference. Philosophically, this can be seen as a direct critique of the classical logic of identity (A = A) by emphasizing that something can only be identical to itself if it is also, in some way, different from itself. Hegel, as we know, one of the main proponents of dialectics, argued that the identity of a concept is always mediated by its difference. For him, contradiction is essential for the development of any concept or entity.

If we add this idea to the theoretical context of the thesis we have been developing above, we have: the identity of a quantum particle is not a fixed and absolute property but an apparent property that emerges from its interactions and differences. A quantum particle is identical to itself not because it possesses immutable intrinsic properties but because its interactions and relations define it in each specific context. Therefore, its identity is always a function of its differences and interactions.

Krause's thesis on non-individuals suggests that quantum particles do not have fixed, distinctive, and identifiable identities. The introduction of the identity-difference dialectic reinforces this idea by arguing that identity is always linked to difference.

The proposal to emphasize interrelations instead of intrinsic properties reinforces this idea by eliminating the need for individual characteristics to distinguish particles.

If particles are defined solely by their interactions, then quantum indistinguishability becomes a natural consequence. This supports the view that quantum particles do not possess individual identities. The theoretical support for this conclusion can be summarized as follows:

By using n-tuples to describe quantum systems, where particles are represented solely by their relations, we provide a sufficient mathematical structure that supports this thesis. In this way, instead of thinking of particles as entities with fixed identities, they can be seen as nodes in a network of interactions, aligning with the idea of non-individuality.

From this point onward, we can trace our way back to dialectics and metalogic:

The adopted dialectic of “identity-difference” suggests that the only possible identity of an entity is intrinsically tied to difference. In philosophical terms, this can be seen as a direct critique of the classical logic of identity (A = A) by emphasizing that something can only be identical to itself if it is also, in some way, different from itself. Hegel, as we know, one of the main proponents of dialectics, argued that the identity of a concept is always mediated by its difference. According to him, contradiction is essential for the development of any concept or entity.

If we add this idea to the theoretical context of the thesis we have been developing in the lines above, we have: the identity of a quantum particle is not a fixed and absolute property but an apparent property that emerges from its interactions and differences. A quantum particle is identical to itself not because it possesses immutable intrinsic properties, but because its interactions and relations define it in each specific context. Therefore, its identity is always a function of its differences and interactions.

Krause’s thesis on non-individuals suggests that quantum particles do not have fixed and distinctive identifiable identities. The introduction of the identity-difference dialectic reinforces this idea by arguing that the identity of a particle is a dynamic process that depends on its interactions and differences. This means that quantum particles are not individuals with their own identities but entities whose identities are constantly shaped by the differences in their interactions.

In structural-relational ontology, the identity of a particle is defined by its relations with other particles. The introduction of the identity-difference dialectic implies that these relations are dynamic and that the particle's identity is always in flux, depending on specific interactions and contexts. This aligns with the idea that identity cannot be thought of without difference, as the relations that define a particle also introduce differences that shape its identity.

Consequently, if we try to exclude difference from the development of identity, the "identity-difference" circuit would collapse, as identity would become a fixed and immutable property, something that, besides being incompatible with quantum mechanics, also results in an internal contradiction: identity cannot be defined without being referenced to difference. Without difference, there is no context or contrast to establish what something is, rendering identity meaningless and empty, especially since "being something" is precisely defined as the situation of "being-in-relation-with-other." Quantum mechanics shows that particles are in constant interaction and change, suggesting that identity must be seen as something emergent and relational, always mediated by difference.

In short: Identity and Difference are like two sides of the same mirror…

and as such, they are identical and, therefore, distinct from themselves.