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Abstract

G. W. Leibniz developed a new model for rational decision-making which is suited to complicated decisions, where goods do not rule each other out, but compete with each other. In such cases the deliberator has to consider all of the goods and pick the ones that contribute most to the desired goal which in Leibniz's system is ultimately the advancement of universal perfection. The inclinations to particular goods can be seen as vectors leading to different directions much like forces in Leibniz's dynamics. The vectorial model of rational decision-making is related to Leibniz's work with metaphysical physics and the calculus of variations and is a heuristic tool which helps in finding reasonable combinations – in ideal cases optimums - between competing goods. By applying the model, the decision-maker can map and compare outcomes of combinations of goods in question and practice a kind of pseudo-mechanical arithmetic of reasons. A central feature of the model is the possibility to employ geometrical figures to help the conceptualization. In this paper I present the vectorial model, examine its applications in practical cases from political theory, jurisprudence and ethics Leibniz presented, and compare the model to recent theories of acting under uncertainty, such as bounded rationality and optimizing under constraints.

Keywords: Decision-Making, Models, Moral psychology, Optimization, Calculus of Variations, Practical rationality, Bounded rationality

Introduction

According to G. W. Leibniz, we live in a very rational world. In his system everything hinges on a unique case of rational decision-making, the one performed by God when he chooses the

best of all possible worlds, the actual world, to be created.[1] Men should try to imitate God as far as possible in their practical rationality and make as enlightened decisions as they can.[2] In various ethical, ecumenical, jurisprudical, political and economical writings Leibniz strived to show how one can solve controversies, reconcile between opposing opinions and find solutions which all parties can accept.[3] He was one of the first modern philosophers who applied psychological models to decision-making and can be seen with Arnauld, Nicole and Pascal as one of the Early Modern pioneers of the theory of rational decision-making.

Leibniz applied two kinds of models in human rational decision-making. The first applies to a case of deliberating between goods which are independent of each other. This is an either-or-situation: should I stay or should I go? The traditional metaphor of a pair of scales, widely used as a symbol of jurisprudence and already found in Homer (*Iliad*, book VII), could be used to illustrate these kinds of deliberations: in the former case the weight in the left or right pan decides the case for one or the other. If the weight in the left pan is heavier than that in the right, the option represented by the left one is chosen. The deliberator collects reasons or evidence on both sides, and the weightier alternative wins (for example, whether the accused is guilty or not guilty). Thus, the options exclude each other, and one cannot help but decide between them.[4]

But the pair of scales is often too limited a model to cope with difficult decisions with multiple indivisible goods. In these more difficult cases the goods are in competition with one another.[5] This kind of situation requires a different kind of model for decision-making. Leibniz thought that the deliberator must consider all the goods simultaneously and pick the ones that contribute most to the desired goal. The goods can be seen as inclinations or kinds of vectors leading in different directions and the deliberator should try to optimize between them, applying a novel vectorial model of decision-making Leibniz developed in 1660's and 1670's to deal with difficult legal and political cases.

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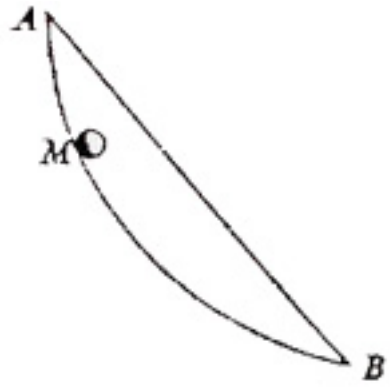
[1] On God's deliberation, see *Essais de theodicée*, §225, G. W. Leibniz, *Philosophische Schriften* I-VII, ed. G. I. Gerhardt, Hildesheim, Germany, Olms, 1961, VI, p. 252.
[2] See, for example, *Discours de metaphysique,* §36, G. W. Leibniz, *Sämtliche Schriften und Briefe*, Reihe I-VII, ed. Berlin-Brandenburgischen Akademie der Wissenschaften und der Akademie der Wissenschaften in Göttingen. Berlin, Akademie, 1923-, VI, 4, p. 1586.
[3] For a collection of texts on Leibniz's efforts to solve controversies of his time, see G. W. Leibniz, *The Art of Controversies*, ed. Marcelo Dascal with Quintiín Racionero and Adelino Cardoso, Dordrect, Springer.
[4] On the model, see M. Dascal, (2005). *The Balance of Reason*, in *Logic, Thought and Action*, ed. D. Vanderveken, Dordrecht, Springer. 2005, pp. 27-47.
[5] I have discussed the differences of these models in M. Roinila, *Leibniz's Models of Rational Decision*, in *Leibniz: What Kind of Rationalist?*, ed. M. Dascal, Dordrecht, Springer, 2008, pp. 357-70.

As Leibniz's views on the pair of scales-model have already been discussed fairly extensively, my focus will be on extending and specifying with Leibniz's own examples the brief descriptions of the vectorial model presented so far in Leibniz scholarship, in particular by J. Hintikka and S. Knuuttila. My discussion is founded on various short remarks in Leibniz's practical writings and correspondence, and I will close with a comparison of the model to contemporary theories of decision-making known as bounded rationality. I will also reflect the uniqueness of the model.

Metaphysical Background

Let us first take a look at the famous brachistochrone problem which was related to 17th - century mathematical physics. As posed by J. Bernouilli in *Acta Eruditorum* (June 1696), the problem is as follows. Let two points A and B be given in a vertical plane. Find the curve that a point M, moving on a path AMB, must follow such that, starting from A, reaches B in the shortest time under its own gravity. While the initial velocity is given, friction and air resistance are neglected (M. Kline, *Mathematical Thought from Ancient to Modern Times*, 1972, pp. 574–75).[6] The solution is shown in the figure below.[7] The optimal curve is not the straight line (AB), because gravity affects the moving point. The solution, called brachistochrone, is represented by a cycloid.



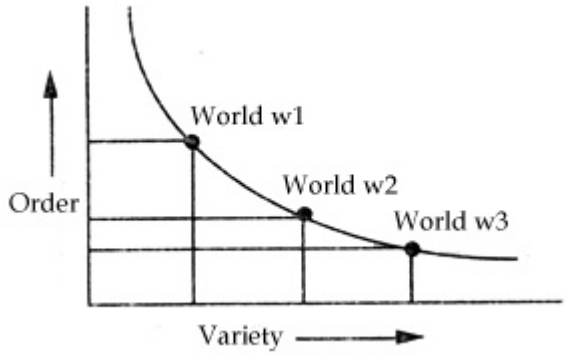
The idea of an optimal path turned out not to be a mere single solution to a particular problem but a metaphysical general rule, which was later reformulated by Maupertuis as the principle

[6] M. Kline, M., *Mathematical Thought from Ancient to Modern Times* I-III. Oxford, Oxford University Press, 1971.

[7] The right solution was found by, among others, the Bernouilli brothers, Newton and Leibniz. For details, see H. H. Goldstine, *A History of the Calculus of Variations from the 17th through the 19th Century*, New York, Springer, 1980, p. 30f.

of least action. Today the doctrine is known as the calculus of variations.[8] The objective of the calculus is to find a unique (easiest, optimal) path among an infinite number of alternative paths that achieves the extremization (maximisation or minimisation) of some specified characteristic (time or distance, for example). An essential feature of these kinds of problems is the fact that the solution is an optimal combination of competing, independent properties. Today similar kind of optimization is often used in deliberations concerning economical enterprises (achieving maximal benefits with minimal costs), cognitive science, biology and so on.[9]

N. Rescher has argued that Leibniz's central metaphysical doctrine, the choosing of the best possible world, can be seen as an application of the calculus of variations. The optimal and most unique combination of minima (order) or maxima (variety) of phenomena (these factors lead in opposite directions) is simply the best of all possible worlds (Rescher, *On Leibniz*, 2003, p. 52).[10]



In the figure above the possible worlds are presented along a curve of feasible order/variety combinations. The first one (w_1) is very orderly, but lacks variety, and $world_3$ has variety but lacks order. Therefore, the best world is $world_2$, which is both orderly and includes variety,

[8] In the 17th century Snell, Fermat and Leibniz, among others found support for the basic idea of the calculus of variations in various phenomena. By the end of the century mathematicians were persuaded to believe that nature did, in fact, try to maximise or minimise some important qualities. For the history of optimality in nature and the principle of least action, see L. Couturat, *La logique de Leibniz d'après des documents inédits*. Hildesheim, Olms, 1985 and Kline, *Mathematical Thought*, cit., p. 580.

[9] However, these deliberations take place under the constraints of limited knowledge unlike in God's choice. For contemporary views concerning these kinds of cases, to be discussed later, see G. Gigerenzer, *The Adaptive Toolbox*, in *Bounded Rationality. The Adaptive Toolbox*, ed. G. Gigerenzer & R. Selten, Cambridge, MA, MIT Press, 2001, pp. 37-50.

[10] N. Rescher, *On Leibniz*, Pittsburgh, University of Pittsburgh Press, 2003. Leibniz presents the deliberation as if it is a mathematical equation with only one possible solution: God finds and creates the best possible world because it is better than all other possible worlds. His main argument to ground this view is his doctrine of principle of sufficient reason, which states that nothing happens without a reason why it should be so and not otherwise (see, for example, *De rerum originatione radicali*, G. W. Leibniz, *Philosophische Schriften*, cit., VII, pp. 302-08).

but neither feature dominates. Rescher argues that Leibniz opposed the traditional *summum bonum*-theories in which perfections are added up, and rather thought that the order/variety combination was, in this world, as large as could be realised within the realm of realisability (Rescher, *On Leibniz*, 2003, p. 58).[11] It is for this reason that Rescher's theory is commonly called the trade-off theory. I will here adopt Rescher's interpretation which fits well with Leibniz's dynamical metaphysics and the various descriptions he gave on the structure of the best world, but it has also been heavily criticized owing to Leibniz's many descriptions of the world as being a maximum.[12]

The Vectorial Model of Decision-Making

The leading idea in what follows is that one can regard Leibniz's vectorial model of rational decision-making as analogous to God's choosing the best world, as interpreted by Rescher. In other words: it is an application of the calculus of variations. There is some plausibility in this hypothesis: Leibniz thought that men in general should follow God's example (*Monadology*, §83) and the divine decision is the most complex deliberation there can be. Men ought to use similar methods. Furthermore, he sometimes used similar descriptions of both divine and human decision-making.[13]

On the other hand, there are evident and unavoidable differences. God is an all-knowing and all-powerful being, while human decision-making takes place almost always in the shadow of uncertainty. In Leibniz's logic, truths of fact such as historical facts require infinite

[11] In this sense the optimal order/variety-combination is the utility God is looking for when creating the world. Leibniz seems to refer to this in *Theodicy*, §208 when he says: «One may…reduce these two conditions, simplicity and productivity, to a single advantage, which is to produce a much perfection as is possible.» G. W. Leibniz, *Philosophische Schriften*, cit., VI, p. 241, G. W. Leibniz, *Theodicy. Essays on the Goodness of God, the Freedom of Man and the Origin of Evil*, ed. A. Farrer & trans. E. M. Huggard, La Salle, Open Court, 1985, p. 257.

[12] In short, while Rescher sees maximum as following from an optimal trade-off, his critics argue that it is not possible for God to bargain on any properties of the best world. For Leibniz's descriptions of the best world, see G. W. Leibniz, *Philosophische Schriften*, cit., I, p. 331 and VI, p. 603 and G. W. Leibniz, *Textes inédits d'après les manuscrits de la bibliothèque provinciale de Hanovre* I-II, ed. G. Grua. Paris, Presses universitaires de France, 1948, p. 267. For criticism of Rescher's view, see D. Blumenfeld, *Perfection and Happiness in the Best Possible World*, in *Cambridge Companion to Leibniz*, ed. N. Jolley, Cambridge, Cambridge University Press, 1995, pp. 382-410, D. Rutherford, *Leibniz and the Rational Order of Nature*, Cambridge, Cambridge University Press, 1995 and L. Strickland, *Leibniz Reinterpreted*, London, Continuum, 2006. While I support Rescher's interpretation and have defended it in M. Roinila, *Leibniz on Rational Decision-Making*, University of Helsinki, 2007, I think P. Rateau is right when he argues that while the best world is essentially an optimum rather than a maximum (agreeing with Rescher), it is not a question of a mere calculation process – God's will is needed to choose and realize the best of all possible worlds. Rateau, *Leibniz on the Problem of Evil*, Oxford, Oxford University Press, pp. 191-96.

[13] See the distinction between antecedent and consequent will, below.

analysis which is only possible for God, whereby man's judgement is furthermore troubled by minute, unconscious perceptions which mingle with clear and distinct ideas.[14] Despite these difficulties, Leibniz thought divine and human decision-making to be analogous concerning competing goods.[15]

It is clear that the vectorial model was designed for complicated situations where the goods are in competition with one another.[16] Leibniz took an interest in cases involving plural values, all of which would affect the decision. In his view, one deliberates between conflicting goods that, in general, advance the universal perfection. The deliberator should pick up the most important goods and try to form an optimum between them. It is rare for men to succeed in this task due to their limited cognitive abilities, but they can strive for a solution which approaches the unique optimum (which always exists) as closely as possible. If it is found, the decision promotes the general good in the best possible way.

Leibniz can be seen as a follower of the perfectionist tradition in the sense that his conception of universal perfection apparently sprang from *eudaimonia*.[17] On the other hand, Leibniz's ethics is also consequentialist, representing a kind of proto-utilitarianism.[18] In our moral deliberations we should strive to find the most relevant good available, but also consider the consequences of the proposed good to the common good.

The vectorial model was not explicated systematically in any of Leibniz's writings that I know of, but there is an interesting description of the soul in *Essais de Theodicée*, §325,

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[14] On the distinction between truths of reason and truths of fact, see Couturat, *La logique de Leibniz*, cit., Ch. VI. On the effect of minute perceptions, see *Nouveaux essais sur l'entendement humain* (1704), Preface, G, W, Leibniz; *Sämtliche Schriften*, cit., VI, 6, pp. 52–8.

[15] Leibniz often lamented the lack of a calculus of probabilities which would have greatly helped human decision-making and strived to create it without clear success (see, for example, G. W. Leibniz, *Philosophische Schriften*, cit. III, pp. 193-94; G. W. Leibniz, *Sämtliche Schriften*, cit., VI, 6, p. 372). For a general discussion on Leibniz's views on analysis of contingent truths and the theory of probability, see Roinila, *Leibniz on Rational Decision-Making*, cit., pp. 122-158.

[16] The model has attracted some brief and general discussions. These include Couturat and J. Elster (L. Couturat, *La logique de Leibniz*, cit., pp. 562–65; J. Elster, *Leibniz et la formation de l'esprit capitaliste*, Paris, Montaigne, 1975, pp. 123–24) and Hintikka who argued that Leibniz developed the model in order to ease difficulties in making rational decisions and to offer an alternative to the Aristotelian practical syllogism (J. Hintikka, *Was Leibniz's Deity an Akrates*?, in *Modern Modalities. Studies of the History of Modal Theories from Medieval Nominalism to Logical Positivism*, ed. S. Knuuttila, Dordrecht, Springer, 1987, pp. 85-108. Knuuttila considered the model Leibniz's most original contribution to practical rationality (p. 333) (S. Knuuttila, *Old and New in Lebniz's View of Rational Decision*, in *Meeting of the Minds. The Relations Between Medieval and Classical Modern European Philosophy. Acts of the International Colloquium held at Boston College, June 14-16, 1996, organized by the Société Internationale pour l'Étude de la Philosophie Médiévale*, ed. S. F. Brown, Turnhout, Brepols, 1998, pp. 333-46. The term vectorial model of rational decision-making was coined by Knuuttila.

[17] On the perfectionist tradition, see J. Passmore, *The Perfectibility of Man*. Duckworth, London, 1970.

[18] On Leibniz's consequentialism, see J. Schneewind, *The Invention of Autonomy*. Cambridge, Cambridge University Press, 1998, pp. 248-250.

which illustrates his way of thinking:

> «As very often there are diverse courses to choose from, one might, instead of the balance, compare the soul with a force that puts forth an effort on various sides simultaneously, but which acts only at the spot where action is easiest or there is least resistance…the inclinations of the soul extend over all the goods that present themselves: they are antecedent volitions; but the consequent volition, which is their result, is determined in the direction of that which touches most closely» (Leibniz, *Die Philosophische Schriften* VI, 1961, p. 309; *Theodicy*, 1985, p. 322).[19]

Here Leibniz argues that the traditional model of pair of scales is not appropriate to describe the soul's movements, for there are several inclinations or efforts which Leibniz defines elsewhere as passive forces.[20] The reference to the easiest action is clearly related to the calculus of variations and Leibniz relates the model to his work in dynamics, where forces have a vector character as Hintikka has observed (Hintikka, *Was Leibniz's Deity an Akrates*?, 1987, p. 89-99).[21]

To my mind, the following picture emerges: in deliberation different goods act as inclinations, forming different paths or vectors in different directions ("effort on various sides simultaneously"). The best choice is the unique optimal path to the good which is reached when all of these inclinations are taken into account and the optimal solution is found ("action is easiest or there is least resistance"). The optimal good is not pre-determined: it is a result of all the goods taken together, that is, it is a trade-off of all the goods involved. This picture is very different from using the traditional practical syllogism of the pair-of-scales-model – it is a result of a number of vector-like forces pulling the agent, so to speak, in different directions (Hintikka, *Leibniz's Deity*, p. 89-99).

The will in the soul is always directed to the good and antecedent volitions are directed to

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[19] The translation is slightly altered. Leibniz presented a similar distinction between antecedent and consequent volitions in his discussion of God's decision-making in §118-119 of the same work which supports that view that he saw human and Divine decision-making as analogous. Leibniz is here arguing against Bayle who presents the soul as comparable to a balance, where reasons and inclinations take the place of weights which makes it clear that Leibniz's approach to the problem is different from the traditional pair-of-scales-model. See also *Primary Truths* for a fairly similar example of a vessel filled with a liquid (in G. W. Leibniz, *Philosophical Essays*, ed. R. Ariew and D. Garber. Indianapolis, Hackett, 1989, p. 33). I thank the external reviewer for pointing this out.

[20] See Leibniz, *Sämtliche Schriften*, cit., VI, 6, pp. 169–170.

[21] On Leibniz's dynamics, see F. Duchesneau, *La dynamique de Leibniz*, Paris, Vrin, 1994.

particular goods or inclinations that represent them. Consequent will or the decision which leads to action is an outcome between these inclinations ("result in the direction which touches most closely"). There is always a single best decision and the more developed our understanding is, the more informed we are of the real goods involved in each case, and the more rationally we can deliberate:

> «As for the rational soul, or mind, there is something more in it than in monads, or even in simple souls. It is not only a mirror of the universe of created things, but also an image of the divinity. The mind not only has a perception of God's works, but it is even capable of producing something that resembles them, although on a small scale…» (*Principes de la Nature et de Grâce, fondés en raison*, §14; Leibniz, *Philosophische Schriften* VI, pp. 604–05; Leibniz, *Philosophical Essays*, 1989, p. 212).

In Rescher's metaphysical interpretation God can see perfectly that the basis for His choice of the best world lies in the optimum between the criteria of order and variety, but men frequently err in their assessments of the good. By adopting good habits and developing one's understanding, however, they can approach it and reach virtue.[22]

Due to the limitations of men's cognitive abilities, the vectorial model can only be a heuristic method which cannot give any certain results. But by applying the model the moral agent can map the situation, evaluate and compare alternative courses of action and, in an ideal case, find the optimum among carefully selected different inclinations to different goods. In a sense the deliberation is a pseudo-mechanical arithmetic of reasons. Instead of adding up reasons, the deliberator applying the vectorial model "multiplicates" the separate continuous and infinite values in order to find a balanced optimum.

> «I came to see that there is a species of mathematics in estimating reasons, where they sometimes have to be added, sometimes multiplied together in order to get the sum. This has not yet been noted by the logicians» (Leibniz to T. Burnett 1/11. 2. 1697; *Philosophische Schriften* III, p. 190).[23]

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[22] Of Leibniz's recommendations for methods of improving our understanding, see *Nouveaux essais*, II, xxi, §35, G. W. Leibniz, *Sämtliche Schriften*, cit., VI, 6, pp. 186–88.

[23] Translations not referring to existing translations of Leibniz are my own. Leibniz refers here to his early treatise on Polish succession, to be discussed later, and to the English economist W. Petty (1623-1687).

Human deliberation concerning competing goods appears to proceed in two stages. First, the deliberator chooses the good to be strived at in the situation, taking into account its consequences to the common good. Second, the deliberator tries to find the optimum between several individual related goods with respect to few appropriate criteria.[24] Often the optimum cannot be found, but one can at least strive to find the apparently best trade-off.

Applying the model is facilitated by the fact that it can be illustrated with geometrical figures in order to map the situation. In mathematical terms the question is one of function.[25] In this Leibniz was influenced by Arnauld and Nicole's *The Art of Thinking*, or the so-called *Port Royal Logic* (Knuuttila, *Old and New in Leibniz's View of Rational Decision*, 1998, p. 340). While discussing lotteries, the authors argue that one should not only think about the good but should also take into account the probability that it will materialise. These values are to be multiplied:

> «In order to decide what we ought to do to obtain some good or avoid some harm, it is necessary to consider not only the good or harm in itself, but also the probability that it will or will not occur, and to view geometrically the proportion all these things have when taken together» (Arnauld and Nicole, *Logic or the Art of Thinking*, 1996, pp. 273–74.[26])

In *New Essays on Human Understanding*, his response to John Locke's *An Essay Concerning Human Understanding,* Leibniz generalised the idea to apply to all assessments of the good in complicated situations:

> «The question of how inevitable a result is, is heterogenous from – i. e. cannot be compared with – the question of how good or bad it is…in this as in other disparate and heterogeneous assessments with more than one dimension (so to speak), the magnitude of the thing in question is made up proportionately of two estimates; it is like a rectangle with two things to be considered, namely its

[24] Leibniz always proposes two – in modern calculus of variations there may be more.
[25] The idea of a function was made popular by Galileo in his *Two New Sciences* (1638). On the history of function, see Kline, *Mathematical Thought*, cit., p. 338.
[26] A. Arnauld, & P. Nicole, P., *Logic or the Art of Thinking. Containing, besides common rules, several new observations appropriate for forming judgement*, ed. and trans. J. V. Buroker, Cambridge, Cambridge University Press, 1996.

> length and its breadth» (NE II, xxi, §66; Leibniz, *Sämtliche Schriften und Briefe*, 1923- VI, 6, p. 206; Leibniz, *New Essays on Human Understanding*, 1996, p. 206[27]).

Here Leibniz applied the functional approach, referring to any quantity varying from one point to another along a curve. The curve could be illustrated by a coordinate system which was first introduced by Descartes. One can compare the different options by drawing vectors to different points of the curve formed by different combinations of related values. The best or easiest vector represents the optimal solution.

However, again it is clear that in human decision-making the values in question are mostly only estimations, and consequently the resulting function is also uncertain. Despite this, the vectorial model could be of great heuristic value in deliberation and by employing it we are able to tentatively compare different proposed options and map them with respect to few criteria. It seems to be clear that Leibniz considered the model as a general model of rational decision-making concerning competing goods, applicable not only to ethical or political deliberations, but also in aesthetics, theology and in all situations where indivisible goods are to be assessed.[28]

Some Examples of Leibniz's Use of the Vectorial Model

I will next present a few instances in Leibniz's writings and correspondence where he seemed to apply the vectorial model. While these examples have featured briefly in earlier accounts, my discussion is more extensive and related to the vectorial model which has not been done before. To be fair, I have to admit that Leibniz's descriptions of the vectorial model are sketchy, and a fair amount of rational reconstruction is required. Despite this, I think these cases are interesting attempts to conceptualize difficult decisions and are therefore worth attention.

In his *La logique de Leibniz* (1901) Couturat was interested in Leibniz's early political memoir on the succession of the king in Poland in 1669 (*Specimen demonstrationum*

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[27] G. W. Leibniz, *New Essays on Human Understanding*, ed. Jonathan Bennett and trans. P. Remnant, Cambridge, Cambridge University Press, 1996.

[28] The model was conceived early in his career (the earliest example of its use is arguably *Specimen demonstratium politicarum* (G. W. Leibniz, *Sämtliche Schriften*, cit., IV, 1, pp. 3–98) (see below)) and he applied it frequently in legal and practical writings at the beginning of the 1670's. However, he also applied the model to both human and Divine deliberation in his later philosophical works such as *Nouveaux essais* (1704) and *Essais de Theodicée* (1710).

politicarum pro eligendo rege Polonorum), which aimed to show with some mathematical arguments that Leibniz's chosen candidate (or rather his employer's, Baron von Boineburg), Philipp Wilhelm von Neuburg, was the best choice as the new King of Poland (Leibniz, *Sämtliche Schriften* IV, 1, pp. 3-98; Couturat *Logique*, pp. 562-564).

In proposition LIII, which concerned the military situation in Poland, Leibniz presented an interesting argument that illustrates his goals: in an optimal case the future King of Poland should be militarily capable but should not be given enough military power to enable him to disturb the political balance in Europe (Leibniz, *Sämtliche Schriften* IV, 1, p. 53). In this sense the proximity of his current territory was essential: if it neighboured Poland, the combined land would constitute a threat to other powers nearby. Thus, it was important to estimate the total power of the candidates, which could be done by multiplying the proximity by the military power as follows:

> «The proximity is in itself a sort of power, and every aspect of power is itself multiplied by its proximity, from which it follows that the total power of a neighbour is a product of its proximity and its power and is also the square of its power. In consequence the relation of simple power to the total power of a powerful neighbour is its square root» (Leibniz, *Sämtliche Schriften* IV, 1, p. 53).

Military power, measured by the number of soldiers, for example, was set against the degree of proximity. If a candidate possessed great military power but was not a neighbour of Poland, he was ideal. However, a candidate from a neighbouring country who had great military power was dangerous and should not be elected. Thus, it was possible to estimate the total power of different candidates and compare them. What is interesting in the argument is that Leibniz strives to compare the candidates with respect to two mutually independent criteria of proximity and military power.

Next, let us take a look at a memoir concerning scientific academies from 1671, entitled *Grundriss eines Bedenckens von Aufrichtung einer Societät in Deutschland*. He describes the problem with powerful princes as follows: «If power is greater than reason, he who possesses it is either a lamb who cannot use it at all, or a wolf and a tyrant who cannot use it well» (Leibniz, *Sämtliche Schriften* IV, 1, p. 531; Leibniz, *Political Writings*, 1988, p. 24[29]). Those

[29] G. W. Leibniz, *Political Writings*, ed. Patrick Riley. 2nd ed, Cambridge, Cambrige University Press, 1988.

who have more power than reason is either unable to use the power or they use it arbitrarily. In the latter case they are bad rulers or tyrants, and in the former they are also bad rulers because they are weak. On the other hand, those with more wisdom than power to use it are "overpowered." Leibniz argued that such people have the right to be counsellors and their princes should listen to them patiently (Leibniz, *Sämtliche Schriften* IV, 1, p. 533).

The ideal prince is an optimal balance between wisdom and power, and it is easy to see that this can be illustrated by a similar geometrical figure as the choice of the best world, as presented by Rescher above. The ideal prince uses his power in proportion to his wisdom and has a beautiful soul: «Those to whom God has given at once reason and power to a high degree are heroes created by God to be the promoters of His will, as principal instruments...» (Leibniz, *Sämtliche Schriften* VI, 1, p. 533; Leibniz, *Political Writings*, p. 24).

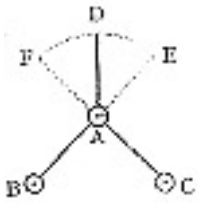
Let us next look at another text in which Leibniz attempted to apply exact values to a similar example of estimating reason and power. He described in a letter to Arnauld (from the same year, 1671) the estimation of a good man (beauty) with respect to Canon law: «Presuming that a man has wisdom of the third degree and power in the fourth, his total estimation would be twelve and not seven, since wisdom be of assistance to power» (Leibniz, *Sämtliche Schriften* II, 1, p. 174).[30] The beauty of a person is not his wisdom and power added up, but a balanced optimum (a product of multiplication) between these properties. This is because on a higher level wisdom could contribute to power ("wisdom may be of assistance to power"). Both related goods have to be taken into account in estimating the overall value of the good man. This passage is something of an exception in Leibniz's writings since this is the only instance, as far as I know, in which he gave definite quantities to such abstract values as wisdom and power.

My next example concerns a reasonable verdict in jurisprudence. It is from a memoir entitled *Doctrina conditionum* (1667-69), which contains Leibniz's most extensive discussion on conditional rights (Leibniz, *Sämtliche Schriften* VI, 1, pp. 368-430).[31] Point 271 concerns a case in which there are equal claims, and the judge has to decide between them. Leibniz

[30] Leibniz gives a more detailed description of the idea in a letter to Arnauld in November 1671 which can also be seen to refer to God's creation in the sense referred above: «To do good means to multiply, not add (whereas to do evil, by contrast, is to divide), as shown by the mathematical example of two numbers, the one greater than the other. When multiplied by the same number, the result is greater in the case of the greater of the two numbers» (G. W. Leibniz, *Sämtliche Schriften*, cit., II, 1, p. 280; translation from Rateau, *Problem of Evil*, cit., p. 57).

[31] It is also known as *Specimen certitudinis seu demonstrationum in Jure*, often also called simply *Specimina juris*, because it appeared in 1669 as a part of a collection of jurisprudical examples. On Leibniz's efforts at conceptualizing jurisprudical problems, see also M. Parmentier, *Concepts juridiques et probabilistes chez Leibniz*, «Revue d'histoire des sciences» 44, 1993, pp. 439-85.

stated that this affair «contains in fact a sort of physical principle which is drawn from the nature of movement» (Leibniz, *Sämtliche Schriften* VI, 1, pp. 392).[32] This physical model is the following:



> «Let there be a body A which is moved uniformly at the same moment by two bodies, B and C. The first is according to the line BA, the second according to the line CA. Let us divide the angle BAC in two equal parts by the line AD and the opposing side in two bodies by the movement B and C and extend the lines BA to E and CA to F. I say that body A will advance following the line AD. …If A was pushed only by B, it would advance following the line AE and, on the contrary, if it was [pushed] by C, it would be according to the line AF» (Leibniz, *Sämtliche Schriften* VI, 1, pp. 368-430).[33]

When the physical principle is applied to jurisprudence, the two opposing parties in the legal case correspond to the movements BA or BE and CA or CF. The effect of movement B or C is greater when the angle with respect to A is smaller, and it is maximal when the angle is zero. This model could be applied to a case of hereditary rights – when only one party is present it will have all, but if the other party or parties appear, its share is diminished depending on the strength of their accepted claims (Leibniz, *Sämtliche Schriften* VI, 1, pp. 393). Depending on the reasons or proofs offered by parties B and C, the judge inclines towards either E or F (which may represent a stronger inheritance claim). The result is a verdict based on the reasoned judgment of the judge. It favours party B or C, but does not give total rights to either, since both have a lawful claim. If the claims are equally strong, the judge ends up with an optimum (marked D), which satisfies both claims. As in the previous examples, the decision takes place by a conflict of two incompatible, but related criteria.

The final example is a slightly different kind of case, despite being an optimality problem as well. It can be found in a draft document which is related to Leibniz's proposed universal

[32] See also G. W. Leibniz, *Logico-Philosophical Puzzles in the Law. Philosophical Questions and Perplexing Cases in the Law*, ed. A. Artosi, B. Pieri and G. Sartor, Dordrecht, Springer, 2013, pp. xxii-xxv.
[33] The figure is from page 392.

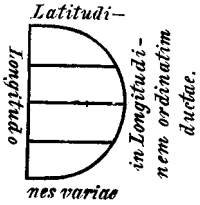
science. There he considered the good, in this case happiness, «*ex ductu bonitatis in durationem*»:

> «If we are to discuss that properly, we must use mathematical operations and say that the whole of the good consists in how long the good can be sustained (*ex ductu bonitatis in durationem*), as in land measurement a field (are) is measured by breadth in length (*ex ductu latitudinum in longitudinem*)» (Leibniz, *Philosophische Schriften* VII, p. 115).

Here are two separate, but related continuous values, the duration and the intensity of the good. If the deliberator is inclined to choose the maximum possible intensity, the result is great happiness, which will last only for a short time. If she chooses maximum duration, the happiness is not very intense (Leibniz, *Philosophische Schriften* VII, p. 115). He reasoned that in the long run strong sensual feelings (passions) were harmful, and if the oil-lamp burned with too great a flame it would soon go out (Leibniz, *Philosophische Schriften* VII, p. 116).

The difference from the previous two examples is that Leibniz is striving to optimize one of the goods, namely duration («how long the good can be sustained»). As we saw, maximal intensity is not a good option due to its short length and thus it has to be optimized with duration. We must strive at a reasonable intensity of good that lasts for some time. In this way the different inclinations (duration and intensity of good) «combine and the volition is the result of the conflict amongst them» (NE II, xxi, §39; Leibniz, *New Essays*, p. 192). One encounters both values to a degree, but the overall result is better than either of the extremes. Sensual temptations might lead us to believe that maximum intensity of the good is the best path to happiness. If we follow this wrong idea, the following action produces something other than the desired effect.

Leibniz illustrated the problem geometrically with the unique figure below (Leibniz, *Philosophische Schriften* VII, p. 115), where longitude represents the duration of the good and latitudo its intensity. The arc of the semicircle shows the corresponding combinations of different goods. When the breadth of the area (*latitudo*) varies, the length (*longitudo*) rises or falls, and vice versa (Leibniz, *Philosophische Schriften* VII, p. 115-16).



The figure is a mirror image compared to the one that describes the optimum between two equal goods such as order and variety or power and wisdom, but one can easily situate the figure to a coordinate system and draw vectors from the points of the arc of the semicircle to two perpendicular lines of the coordinate system. Leibniz is striving to illustrate the maximal intensity of good with respect to duration of good. We can see that intensity is equally strong in options 1 (top line within the semicircle) and 3 (lowest line within the semicircle). If one wishes to choose between these two options, the preferred choice is option 1 where the intensity of the good lasts longer than in option 3. But superior to both of these combinations is option 2 (middle line within the semicircle) where the intensity is at a maximum and it lasts for a reasonable amount of time. Here the optimum can be found by multiplying the middle points of *latitudo* and *longitudo*.

As these examples very well show, the vectorial model is essentially a heuristic device, and we usually have to trust estimations of the appropriate goods which are continuous and infinite in degree. In addition, one is supposed to take into account the goal of our moral action, that is, promoting the universal perfection. One can discuss, for example, whether a different combination of wisdom and power would be optimal within this framework. There is an infinite number of possible combinations of values (variations), of which only one can be the best, the optimum. By applying the vectorial model, it is easier to find possible variations in each case and to compare them with each other. In legal context, which is typical for Leibniz, one forms a presumption which is based on informed view of the case at hand. This presumption holds until contrary evidence is presented.[34]

The Vectorial Model and Contemporary Approaches

[34] A. Blank has argued that in Early Modern and consequently Leibniz's works of legal argumentation the notion of presumption is often used in connection with the notion of easiness (compare *Essais de Theodicée*, §325 above, where the soul is compared with a force that acts where it is easiest or there is least resistance). The basic idea is that in legal cases one should presume what can be presumed most easily or what option is the most possible. See his *Arguing From Presumptions. Essays on Early Modern Ethics and Politics*, Munich, Philosophia, 2019, pp. 173-200.

In this last section I will compare Leibniz's vectorial model to recent approaches to rational decision-making under uncertainty, namely theories of bounded rationality and optimization under constraints. While I do not claim it to be superior to them, the vectorial model can be considered as an interesting Early Modern predecessor to these contemporary theories.

In 20th Century, the optimism concerning rational decision-making of the Enlightenment has been largely rejected and new, less demanding models have been created. One of these approaches is known by H. Simon's term "bounded rationality" – he illustrates it with a metaphor of a pair of scissors, where one blade represents "the cognitive limitations of humans" and another "the structure of the environment" (Gigerenzer & R. Selten, *Rethinking Rationality*, 2001, p. 4[35]). The idea is that with limited resources one can make successful rational decisions in a reasonable amount of time.

While it is tempting to compare Leibniz's vectorial model with models of bounded rationality, as both aim to deal with situations involving limited information, there are some profound differences. Common to both models is the lack of absolute, calculative certainty. But while bounded rationality is dependent on the fast and frugal heuristics, that is to say, the readily available empirical data, Leibniz's vectorial model usually concerns values which are universal and abstract, and the model does not necessarily produce quick decisions.

Basically, Leibniz strives to optimization of a single good from a divine viewpoint by reasoning and logic, without domain-specific properties such as a limited search or adaptation of values according to changing circumstances. The vectorial model is a kind of soft version of the calculus of variations. Leibniz was also optimistic about the realization of his calculus of probabilities which will provide men with a new level of certainty concerning factual truths, enabling them to calculate the right decision.[36] Therefore his ideal decision-maker is a superior man which approaches the infinite knowledge of God.[37]

Because the models of bounded rationality, as presented by Gigerenzer, do not strive at optimization, the difference from Leibniz's model is clear.[38] In Leibniz's case the rationality

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[35] G. Gigenrenzer & R. Selten, *Rethinking Rationality*, in *Bounded Rationality. The Adaptive Toolbox*, ed. G. Gigerenzer & R. Selten, Cambridge, MA, MIT Press, 2001, pp. 1-12.

[36] Gigerenzer refers to this idea as "Leibniz's dream" and sees his adaptive toolbox as a more modest version of it. G. Gigerenzer, *Adaptive Toolbox*, cit., pp. 42-3.

[37] This is precisely what the theory of bounded rationality rejects: «A serious program of bounded rationality needs to emancipate itself from the Christian ideal of an omniscient and omnipotent God, or its secularized version, Laplace's superintelligence» (G. Gigerenzer, *Adaptive Toolbox*, cit., p. 42).

[38] This is acknowledged by Gigerenzer (see *Adaptive Toolbox*, cit., pp. 40–3). However, he does not mention the vectorial model which requires less certainty than the *calculemus!*-model. An external anonymous reviewer remarked that there are also differences concerning the certainty between the different versions above. Simon's

is prior to the specific situation and thus not adaptive and the decision-maker is not a satisficer– one always seeks an optimal solution instead of a satisfactory one as in bounded rationality models. So, whereas the vectorial model is a general tool for estimating combinations of values, Gigerenz's adaptive toolbox model is a domain-specific model (Gigerenzer, *The Adaptive Toolbox*, 2001).

It seems to me that Leibniz's vectorial model is closer to another recent model, applied in theories of optimization under constraints, where there is an unlimited search for information. For example, if a person wants to buy a used car, he searches and searches without limit. He stops searching only when the costs of further search exceed the benefits of the further search (Gigerenzer, *Adaptive Toolbox*, 2001, pp. 38-39). As in Leibniz's model the period of deliberation is not limited and the search concerns mostly internal information such as memory, the constraints are formed by limitations of cognition. But, for Leibniz, there is no stopping rule (optimal or simple) where the costs are exceeding the benefit of further search. Of course, the deliberation has to end at some point, but Leibniz's discussion is often on a general and timeless level, even when he is analysing practical examples. The utility sought is increase of universal perfection rather than immediate personal benefit (although the Leibnizian virtuous person may experience joy as a result of the right action following from rational deliberation).[39]

In fact, this universal and conceptually mathematizing character is, to my mind, what makes Leibniz's vectorial model unique and interesting.[40] Although it can be applied to specific problems, he usually applies it to conceptualizing trade-offs of competing universal goods. It is also important to remember, as Hintikka and Rateau have emphasized, that especially young Leibniz often based his demonstrations on physical models (Hintikka, *Leibniz's Deity*, pp. 89-99; Rateau. *Problem of Evil*, 2019, p. 34).[41] Because of the generality and interpersonality of the vectorial model, it can be applied to cases where facts are not available or relevant, for example in aesthetics, politics, economy, ethics, justice, cosmology etc. The vectorial model is a useful method of tentatively assessing and mapping the goods in complicated decisions and it gives us an understanding of the good rather than a straight route to it. Because of its relative simplicity, it is easy to adopt and the possibility to illustrate

framework is closer to computation and artificial intelligence models than Gigerenzer's more general and uncertain theory of adaptive toolbox. This is certainly a good point.

[39] On Leibniz's moral psychology, see Roinila, *Leibniz on Rational Decision-Making*, cit., pp. 187-229.

[40] Parmentier has also argued that the model was a preliminary stage of differential and integral analysis (M. Parmentier, *Concepts juridiques et probabilistes chez Leibniz*, cit., pp. 473-74).

[41] Compare also the case of *Doctrina conditionum* above.

problems by geometrical figures gives the model an immediate attractiveness.[42]

[42] This work was supported by the Academy of Finland under Grant 1137891.