How Can Mathematical Objects Be Real but Mind-Dependent?

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Abstract Taking mathematics as a language based on empirical experience, I argue for an account of mathematics in which its objects are abstracta that describe and communicate the structure of reality based on some of our ancestral interactions with their environment. I argue that mathematics as a language is mostly invented. Nonetheless, in being a general description of reality it cannot be said that it is fictional; and as an intersubjective reality, mathematical objects can exist independent of any one person's mind.

I can sum up the thesis under discussion here to the idea that mathematics can be thought of as a very abstract theoretical physics, this is an idea that Quine welcomes (Linnebo 2017: 94). Quine (1986: 402) claims that epistemologically, mathematical objects and that of theoretical physics are the same.

I take mathematical objects as non-spatiotemporal. If we agree that space and time are a (transcendental or physical) basis of our reality, it's not so far from tautological to say that all spatiotemporal objects are necessarily a part of the physical reality. However, I argue that mathematical objects despite being non-spatiotemporal are also in a sense real.

Regarding reality in mathematics, one interesting view is that of Wittgenstein in the *Philosophical Investigations*, where he sees this problem as an illness that we should get rid of (PI 2009: 254; Bangu 2012: sec. 4). Before that in the *Tractatus*, Wittgenstein maintained that mathematical propositions are not real propositions (TLP 6.2), for one reason because we can see that their forms are superficial (see Bangu 2012: Introduction). The most important reason for this superficiality is because Wittgenstein thought that mathematics is not independent of human language and practices (Bangu 2012: sec. 4).

So whereas he first thought of the mathematical propositions as unreal, he later claimed that the question of the reality of mathematics is altogether nonsense. My thesis here is in alignment with these two claims from Wittgenstein. However, although I agree that the philosophical question of the reality of mathematics is analogous to a sickness, I think we are compelled to come up with a cure because we cannot just get rid of this sickness only by acknowledging it.

In this paper, I present two ways in which we can talk about the reality of mathematical objects. Firstly, mathematics is a description of reality and in this way, we can affirm that mathematics cannot be fictional. Not only mathematics is a description of reality but it's the transcendental condition of its possibility. Secondly, mathematics can be regarded as an abstract realm in which its reality is independent of our empirical limitations. Here the reality of mathematics is maintained by the activities of the mathematical society, and the existence of mathematical objects is justified in an intersubjective way.

Though the mathematical practice is dependent on human creativity, the enterprise of mathematics is more than just another formal game or fiction, and because the apprehension of reality is intractable without mathematical discourse, we cannot say that mathematics is fictional. This argument can be construed along the same lines as Quine's indispensability argument, that is, our most accurate description of the world is the scientific description which is inseparable from mathematics, and that this inseparability justifies the existence of mathematical entities (see Resnik 1995: 166; Putnam 1971). Now if we take mathematics as an indispensable apparatus to describe reality, we can never hope that it can describe reality purely. Here I adhere to Resnik (1997: 124–26), maintaining that we cannot hope for a discipline that is deprived of presuppositions.

Mathematics is a descriptive apparatus, and at its most fundamental level, it describes matters of the real world—not fantasies. The practices of our ancestors in sorting, arranging, fitting things and constructing geometric templates is the precursor to mathematics (see Resnik 1997: 226–28). As Linnebo (2017: 15) points out the focus on the empirical experience for commencing the epistemology of mathematics is not a new thing, e.g., Meno's slave boy needed some empirical trigger to activate his mathematical intuition. Kant also admits the importance of empirical experiences but eliminates the sort of Platonic basis. With Kant (1998: B1), the source of such knowledge comes built-in in our cognitive faculties.

Resnik (2018: 305) narrate an oral account from Benacerraf that we acquire mathematical knowledge by somewhat abstracting physical reality into properties. Let me put it this way. Mathematics describes reality, and maybe we can call it real in this thin sense: mathematical intuition works as a thin wrap that bundles the phenomenal experience into understandable equivalence classes that show the structure of reality. This wrap abstracts the outer world as to be comprehensible by our cognitive faculties. It packs and forces the components of our perception to fit into their place in a communicable mental structure that is shared between human beings. So mathematical objects are abstractions from the physical reality and their purpose is to describe this reality, in a specific sense.

The general working of the abstraction process can be characterized by what Lewis (1986: 84–85) calls "the Way of Abstraction" that is "abstract entities are abstractions from concrete entities. They result from somehow subtracting specificity, so that an incomplete description of the original concrete entity would be a complete description of the abstraction". Mathematical abstractions tend to generalize reality into an austere and unambiguous language that shows the structure of reality—that is, the general form in which reality can be described—and it's in this sense that I say mathematics shows the structure of reality.

As a way of communicating cognitive abstractions, mathematics is invented. And this is regardless of whether mathematics is an innate cognitive ability or not. So even if mathematics is a priori and transcendental this will not contradict the fact the communication of mathematics can only be superficial. Because after all, in order to communicate something abstract and non-spatiotemporal we can but invent tools and conventions that strive to catch and reify the intangible. And this is the sense in which Wittgenstein thinks that we invent mathematics (LFM 1976: 22). Further, the very act of communicating the cognitive abstractions give rise to further abstractions. These abstractions are not a direct result of our interactions with the physical world. The set-theoretic hierarchy is one example, and it's untenable to say that the reality of this complex comes from the transcendental nature of mathematics nor can we find any use for it in our best sciences. Later, I will argue that such abstractions get their legitimacy in a separate reality.

Wittgenstein mocks platonists by saying that "chess only had to be *discovered*, it was always there!" (PG 1974: 374). Now whereas Wittgenstein maintained that mathematics is another invented formal game like chess I say that this is not so. Although it was not necessary to have chess in order to describe the world, it was necessary to have for example the notion of a geometric line to describe the geography of the environment. Or to talk cogently, we needed to distinguish between quantities; one can note the embedded quantifications in the grammar of many languages, like English that distinguishes between one and more than one, or Arabic that does so for one, two, and more than two. So even by accepting the superficiality of mathematical expressions, these expressions induce a descriptive power that formal games like chess don't have; and an expressive faculty that is not implied in works of fiction.

There's yet another aspect that distinguishes basic mathematical objects from fictional creativities. For once more, take the notion of a geometric line, although this object is not out there in the concrete world, it is naturally inclined by the workings of our minds. That is, it doesn't take much effort to convey the existence of this object to another person. This is so because other people possess the same mental capabilities and our descriptions, gestures and communication strategies will effortlessly trap a mental impression in the mind of the other that is known as a line.

The rules that mathematics is based on are derived from practical empirical experience, which itself is regulated by the regularities of the world. As culture and society develop, some of these regularities are "hardened" into rules. Solidification of rules, is again a Wittgensteinian notion. I am saying that when the rules of mathematics are derived, mathematics starts the life of its own and its truth and expansion becomes independent of the experience and of the physical world. So much so that mathematics becomes the paradigm on which reality itself is measured, not the other way around (cf. RFM 1978: VI, 22).

I assume that we agree that there are things in the world that we specify by names, that these things interact in ways that we can describe, and that there are laws that these objects are based on and rules that they follow. Regardless, we cannot ostensively denote these laws or rules—they are abstract. But this doesn't mean that they do not exist and that they are not real. Think for example of the second law of thermodynamics—we can only observe systems that exemplify its idea. And in doing so we can affirm the fit of the abstract model to what we take as its concrete exemplification. These examples are the doors that enable us to perceive the underlying abstract structure. So similarly, it's not correct to talk about the existence of mathematical objects in the terms of physical spatiotemporality. We should also note that objects of mathematics are ontologically more abstract and general than that of physics.

In platonism, we can view mathematics as an independent reality. In this regard not only does mathematics describe reality, but it's a reality in its own right. Mathematical platonism constitutes of three theses of existence, abstractness, and independence. That is: there are mathematical objects, they are abstract, and they are mind-independent (see Linnebo 2017: 11). The characterizing difference between platonism and its antithesis—nominalism—is that in the former we hold that mathematical objects exist. Now, the locution *existence* can denote a variety of different meanings. From a model-theoretic neutral stance about existence to a substantial denotation of existence analogous to the material mind-independent existence of ordinary things. Here to keep the matters tractable—concerning normal platonism—we take Frege's view. That is, the singular terms of true arithmetical statements refer to numbers, now, these statements can be true only if there exist such things as numbers (see Linnebo 2018: sec. 2.1).

Though I stand for the *existence* of these objects, here I deviate from the normal and take *existence* in a different sense. Mathematical abstracta exist as the necessary result of the interaction of the human mind with the world. In embodied mind theories we have a similar view in which mathematical *abilities* are the result of our interaction with the world. For example, the fact that at least some basic properties of numbers (multitudinous of things) are perceptual attributes (see Jones 2018: 148).

Mathematical objects are mind-dependent as much as the word "table" is mind-dependent in contrast to the intended mind-independent table. So by saying that the word "table" is mind-dependent we do not deny the existence of a pile of particles that we dub as "table"; it just happens to be the case that we have an ability and a practical inclination to discern so much of these particles as a table, but not a handful of electrons and other elementary particles within this pile (cf. Frege 1953: 34). This is to argue that the superficiality of mathematics is not arbitrary, and there is "something" that exists substantially, that *compels* us to invent conventions to target its existence. We are compelled because we live in a mind-independent environment that we share with other thinking beings like us and we need to convey the content of our phenomenological experience: we have to convey the innate impression of objects, patterns and thoughts to other parties. These impressions are the abstract objects. And the texture of these existences is not the same as the concrete particles that we denote by means of words. The mathematical objects are internal abstractions that generalize and regiment the phenomenological experience.

The independent realm of mathematical reality lies in a non-spatiotemporal domain. This reality is the enterprise that rose from the grounds of natural cognitive tendencies of the human mind in communicating abstract proto-mathematical ideas. But ideas in any one person's mind can be vague, ambiguous or prone to blunder. By getting rid of these unwanted properties, via regimentation of abstract mathematical thoughts, a grounded reality can come into being that exists between the interactions of people and is maintained by a collective effort. This reality can be found in the practice of mathematicians. And in this sense, one can say that mathematical objects exist, independent of any one mind. Wittgenstein talks about some behavioural agreement that is not simply an agreement of opinion but rather an agreement on the whole procedure that leads to an agreement about a matter at hand: a consensus "of action" (Bangu 2012: sec. 5). In the *Philosophical Investigations*, Wittgenstein says, that these agreements are not "agreement[s] . . . in opinions, but rather in form of life" (PI 2009: 241). In this sense, there is another layer to the reality of mathematics, which is the consensus of the society, as accepting mathematics as a legitimate tool to describe reality and as a reality in its own right. For example, if we are to run a simulation, in which we populate a physically similar world with people that are mentally similar to us, these people should end up having roughly the same mathematics as we do.

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