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New theories for new instruments: Fabrizio Mordente's proportional compass and the genesis of Giordano Bruno's atomist geometry

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HIGHLIGHTS

- Giordano Bruno might have envisioned the concept of infinitesimal quantity.
- Bruno's mathematics is best understood in the context of mathematical practices.
- Bruno's idea of geometric minimum originated from the controversy with Mordente.

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ABSTRACT

The aim of this article is to shed light on an understudied aspect of Giordano Bruno's intellectual biography, namely, his career as a mathematical practitioner. Early interpreters, especially, have criticized Bruno's mathematics for being "outdated" or too "concrete". However, thanks to developments in the study of early modern mathematics and the rediscovery of Bruno's first mathematical writings (four dialogues on Fabrizio's Mordente proportional compass), we are in a position to better understand Bruno's mathematics. In particular, this article aims to reopen the question of whether Bruno anticipated the concept of infinitesimal quantity. It does so by providing an analysis of the dialogues on Mordente's compass and of the historical circumstances under which those dialogues were written.

1. Introduction

In October 1585, Giordano Bruno (1548–1600) returned to Paris from London in the entourage of his patron, the French ambassador Michel de Castelnau. So ended the two and a half years that Bruno spent in England, a period in which he wrote eight works (including his six Italian dialogues), and had faced criticism and hostility from the English intellectual environment. Bruno's stay in England has been the subject of several studies, and yet we do not know with certainty the reasons that led to his return to France.[1] Ricci (2000, pp. 191–192) proposes that Bruno was not the only one who failed in England. The diplomatic mission of Castelnau also turned out to be unsuccessful. With no one willing to support him in England, Bruno had no choice but to follow Castelanau when this latter decided to move back to France. What is certain is that Bruno did not feel welcome in Paris either, so much so that he left for Germany less than one year from his arrival.

Two events made it impossible for Bruno to stay in Paris longer: the controversy with the Italian mathematician Fabrizio Mordente (1532–1608), and the dispute against Aristotelian philosophy in which Bruno took part at the Collège de Cambrai (now part of the Collège de France) in May 1586. Bruno scholars have already provided a historical reconstruction of both these events.[2] However, less attention has been paid to the impact that the controversy with Mordente had on Bruno's mathematics. To my knowledge, De Bernart (2002) is the only scholar who has addressed this issue. In writing this article, I have greatly benefited from her work.

There may be several reasons why the Bruno-Mordente controversy has been neglected so far. First of all, this may be the result of the low esteem in which scholars have held Bruno's mathematics in general. For instance, Cassirer notices how the "concrete" character of Bruno's mathematics prevented him from seeing those "laws and ideal relations whose value is independent from the nature of the existing things and of matter" (Cassirer, 1961, p. 345). Likewise, Védrine, borrowing Bachelard's terminology, speaks of a "realistic obstacle" hindering Bruno's mathematics (Védrine, 1976, p. 247). As a result of these criticisms, Bruno has been viewed as a poor mathematician. But when it comes to

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[1] The most important studies on Bruno in England are: Gatti (1989); Aquilecchia (1995); Bossy (2002); Feingold (2004); Pirillo (2010).

[2] For a historical reconstruction of the Bruno-Mordente controversy, see Yates (1951). On the dispute at the Collège de Cambrai, see Perfetti (1992).

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the Bruno-Mordente controversy, there are other factors to be considered, starting with the fact that this controversy remained unknown until the 1950s. Up to that time scholars were aware that Bruno and Mordente met in Paris in 1586, but they did not know about the controversy. It was thanks to the textual discoveries made by Yates (1951) and Aquilecchia (whose findings are published in Bruno, 1957), to which we shall return below, that the controversy became known thus opening a new chapter in Bruno's already long history of conflicts and disagreements.

But what was the bone of contention between Bruno and Mordente? Mordente was the inventor of one the first proportional compasses (also known as sectors), an instrument constructed according to the principles of trigonometry to solve arithmetic and geometric problems (such as calculating the square root of a number or squaring a curved figure). Mordente's compass was almost unknown until the late 1800s, as its existence was overshadowed by that of another proportional compass, invented by a better-known Italian scientist: Galileo Galilei.[3] However, Mordente's compass did not go completely unnoticed by his contemporaries, catching the eye of technicians and mathematical practitioners, but also of speculative thinkers like Bruno. Puzzled by the novelty of Mordente's invention, Bruno offered to write a Latin exposition of the compass in the form of two dialogues (entitled *Mordentius* and *De Mordenti circino*). Mordente, however, must not have liked what Bruno had to say about his compass, as he tried to acquire and burn as many copies of Bruno's dialogues as possible. In response, Bruno wrote two more dialogues (entitled *De idiota triumphans* and *De somni interpreatione*), in which he accused Mordente of plagiarism and stupidity.[4]

The Bruno-Mordente controversy will be discussed at length below. Here, I would like to focus on what happened in the aftermath of that controversy. In the years from 1586 to 1591, Bruno would go on to develop the idea that geometric objects are composed of indivisible parts, turning it into a fully-fledged atomist geometry. Ultimately, this project would result in the publication of *De triplici minimo et mensura* (Bruno, 1889 [1591]), where Bruno theorized his atomist geometry based on the concept of minimum. But if *De minimo* marked the end of Bruno's mathematical odyssey, its starting point was found in the Italian dialogues that Bruno published in London in 1584, in particular in *De la causa, principio e uno* (*On Cause, Principle and Unity*) and *De l'infinito, universo et mondi* (*On the Infinite Universe and Worlds*).[5] Indeed, as Bönker-Vallon (1995; 1999) and Seidengart (2000) demonstrate, it is in these dialogues that Bruno, building on the work of the German philosopher Nicholas of Cusa (1401–1464), laid the foundation for his atomist geometry. The importance of Cusanus for Bruno's mathematics cannot be overestimated. Bruno himself acknowledged his debt to Cusanus, whom he hailed as the "inventor of geometry's most beautiful secrets" (Bruno, 1998, p. 97). Compared with other sources that Bruno had at his disposal, Cusanus offered without doubt the most comprehensive discussion of the concept of 'minimum'. Furthermore, like Cusanus, Bruno argued for the coincidence of minimum and maximum, which provides further evidence for his dependence on the work of the German philosopher.

Nevertheless, although Bruno first dealt with the issue of the composition of the continuum in *De la causa* and *De l'infinito,* Seidengart (2000, p. 63) notices that in these works, Bruno's atomism was not yet fully developed. The term "atoms" was mentioned several times especially in *De l'infinito*, and yet Bruno did not specify how these atoms came together to form an object, or if he subscribed to a specific kind of atomism (e.g., Democritean or Epicurean). Moreover, in the Italian dialogues, Bruno did not seem to conceive of the existence of a geometric minimum, as the atomic structure was only attributed to physical entities. It was not until the controversy with Mordente and the publication of the dialogues on his compass that Bruno claimed that geometric objects were composed of infinitely small indivisible parts. For this reason, an analysis of Bruno's dialogues on Mordente's compass may offer new insights into the development of Bruno's mathematical thinking. Also, it may show how Bruno's mathematics changed over time, highlighting the differences between the theory developed in the dialogues on Mordente's compass, and the theory presented in the later *De minimo.*

From a historical perspective, the Bruno-Mordente controversy is important for another reason. We have seen that early interpreters such as Cassirer and Vedrine argue against the modernity of Bruno's mathematics on account of its being more of a concrete than an abstract knowledge. On the one hand, the fact that Bruno's mathematics grew out of efforts to illustrate the use of Mordente's compass seems to corroborate this opinion, showing that indeed Bruno had an early interest in mathematical practices. On the other hand, if Bruno's mathematics is to be criticized for being outdated, we need to clarify what is meant by modern mathematics. Bruno's critics seem to assume that modern mathematics was characterized by its high level of abstractedness and theoretical speculation. In other words, their concept of modern mathematics seems to coincide with what has been called 'pure knowledge' as opposed to 'applied knowledge'.[6]

But did Renaissance mathematics fall squarely within the domain of pure knowledge? Certainly, Renaissance mathematics was in the process of becoming pure, as one of the goals of those defending the certitude of mathematics at that time was to ensure the independence of mathematics from other forms of knowledge, especially physics.[7] However, one may argue that alongside 'theoretical' mathematicians (such as Cardano, Tartaglia, and Regiomontanus), there was a wide range of mathematical practitioners whose activities have gone almost unnoticed until recently. To be fair, exploring the world of mathematical practitioners is not an easy task, since their work was rarely converted into printed books or formalized in mathematical theories. Nevertheless, especially thanks to the pioneering studies of Taylor (1954), scholars have gradually become aware of the importance of mathematical practitioners in establishing mathematics as a leading discipline during the Renaissance.[8] The Bruno-Mordente controversy provides yet another example of the interaction between theoretical and practical mathematics in this period.

Last but not least, the Bruno-Mordente controversy may shed new light on the question of to what extent Bruno's concept of 'minimum' may be considered a forerunner of the modern notion of 'infinitesimal'. As is well known, the introduction of infinitely small quantities marked a turning point in early modern mathematics, leading to the development of the calculus.[9] Bruno's 'minimum' stood for the smallest quantity which geometric objects were composed of. Despite this, scholars have been reluctant to draw even the slightest connection between Bruno and the infinitesimals. The reasons for this behavior are well explained by Olschki (1927). Arguably one of Bruno's harshest critics, Olschki claims that Bruno was prevented from seeing "the most basic version of the infinitesimal principle" by his denial of the coincidence of the

[3] On Galileo's proportional compass, see Favaro (1907); Rose (1968); Rosen (1968); Schneider (1970); Valleriani (2010), pp. 27–40.

[4] All of the four dialogues on Mordente's compass are now published in Bruno (1957 [1586]).

[5] For an introduction to Bruno's *De la causa* and *De l'infinito*, see respectively Leinkauf (2007) and Bönker-Vallon (2007).

[6] For a discussion of the distinction between pure and applied knowledge in early modern mathematics, see Roux (2010).

[7] Here I am referring to the so-called *Quaestio de certitudine mathematicarum* which took place in the sixteenth century. Among those who made a case for the certitude of mathematics, there were Francesco Barozzi (1537–1604) and Christophorus Clavius (1538–1612). For more information on the *Quaestio,* see De Pace (1993); Mancosu (1996), pp. 10–33; Sergio (2006), pp. 11–52.

[8] For a more recent analysis of early modern mathematical practitioners, see Cormack, Walton, and Schuster (2017).

[9] For the history of the calculus, see Boyer (1949) and Edwards (1994).

minimum arc and the minimum chord (Olschki, 1927, p. 75). In other words, the problem with Bruno's geometry, in Olschki's opinion, was that it envisaged two kinds of minima, one for the straight line and one for the curved line. On the contrary, in the geometry of infinitesimals, every line—no matter whether straight or curved—was considered as composed of infinitely small, straight lines. Moreover, Olschki adds, “Bruno's concrete geometry would have taken on an evident significance, if it had been connected to a theory of motion” (Olschki, 1927, p. 81).

Writing in 1927, Olschki could not have read Bruno's last two dialogues on Mordente's compass, which were rediscovered only in 1957. If he could have done so, he would have probably realized that both his objections to Bruno's geometry were unwarranted. In fact, in *De idiota triumphans* (Bruno's third dialogue), he argued that both straight and curved lines were composed of the same minima (see § 4). Furthermore, in *De Mordenti circino* and more extensively in *De somni interpretatione* (respectively, Bruno's second and fourth dialogue) he envisioned the possibility of a law of motion that could account for both circular and non-circular motions.[10] The main field of application of this law of motion was the study of planetary orbits. Thus, Bruno had an answer for both the objection raised by Olschki, although this latter could not have known it. This opens the possibility for a new assessment of Bruno's mathematics, which is what this article aims to carry out.

In the following, I will first give a brief description of Mordente's compass (§ 1) and of the controversy that originated from Bruno's decision to write about it (§ 2). Then I will move on to analyze Bruno's first and third dialogues (§§ 3–4), where he explained how, working on Mordente's compass, he came to discover the geometric minimum. Since the analysis of the second and the fourth dialogue (those in which Bruno developed the idea of a law of planetary motion based on his geometry of minima) would require a separate article, this analysis is left for future study (see note 9).

2. Mordente's proportional compass: some historical remarks

To fully appreciate the value of Mordente's compass and its importance for the history of science, we need to understand the difference between the reduction and proportional compass in the first place. The reduction compass (Fig. 1) was at least as old as the ancient Romans, one of its first examples having been discovered in the archeological site of Pompeii.[11] The main goal of a reduction compass was to reduce or enlarge a drawing. The proportional compass, on the other hand, allowed to perform several mathematical operations, such as dividing a segment or a circumference into equal parts, or squaring an irregular figure. It did so by exploiting the geometric property that similar triangles have proportional corresponding sides. As such, the proportional compass may be considered the first calculating instrument of the modern age. For a long time, Mordente's compass (Fig. 2) had been seen as a reduction compass, a simplistic view that had not done justice to the Italian mathematician. Instead, as recently demonstrated by Camerota (2000, pp. 5–7), Mordente's compass was a proportional compass in its own right.

Fig. 1. An example of a reduction compass from the Medici Collections (Museo Galileo, Florence – Photography by Franca Principe).

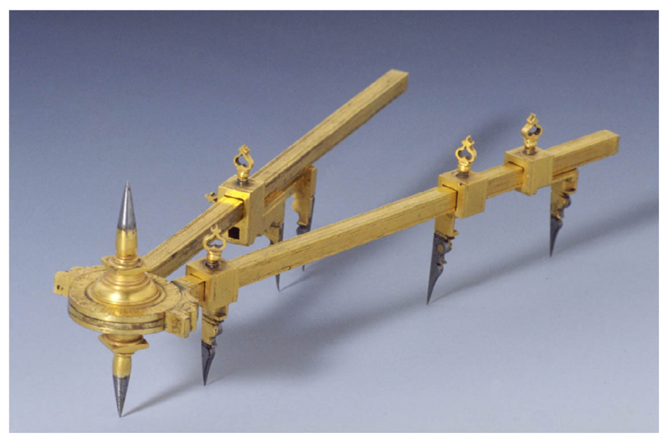

Fig. 2. The compass of Mordente (Museo Galileo, Florence – Photography by Franca Principe).

The rediscovery of Mordente's compass in the late 1800s reopened the question of the authorship of the proportional compass. This question has been debated ever since Galileo published *The Operations of the Geometric and Military Compass* (Galilei, 1606). Galileo claimed to have constructed the first version of his compass (Fig. 3) in 1597. However, there is evidence that other examples of proportional compass circulated in Europe even before 1597. A compass similar to that of Galileo had been constructed by the Flemish mathematician Michel Coignet as early as the 1580s.[12] Coignet in turn was familiar with Mordente's compass, which he had helped promote through the publication of several treatises (Coignet, 1608a; Coignet, 1608b; Coignet, 1626) In light of this intricate network of acquaintances and information exchanges, several reconstructions have been proposed to explain the genesis of the proportional compass. Despite all these efforts, however, it remains unclear whether Mordente may be considered the inventor of the proportional compass—as sustained by Boffito (1931)—or whether Mordente's, Coignet's and Galileo's compass had different stories—as advocated, among others, by Favaro (1907) and

[10] Bruno (1957), p. 58. “Is it not necessary that, in those things that are connected and related, the certain law of what moves away from and towards a center follows from the certain law of what rotates around a fixed center?” (All translations are my own). Considering that in the same text Bruno speaks of planetary motions, and that he mentions the fact that “the stars happen to approach and move back from the sun and the earth” (Bruno, 1957, p. 57), one may argue that here Bruno seems to postulate the existence of non-circular, planetary orbits. This claim would need to be substantiated by a thorough inquiry into Bruno's astronomy, a task that is beyond the purpose of this article. For this reason, as mentioned below, I will not address this issue here.

[11] The reduction compass found in Pompeii is now kept at the Museo Archeologico Nazionale of Naples (inv. 76684). For more information, see Camerota (2000), p. 14.

[12] On Coignet's compass, see Meskens (1997).

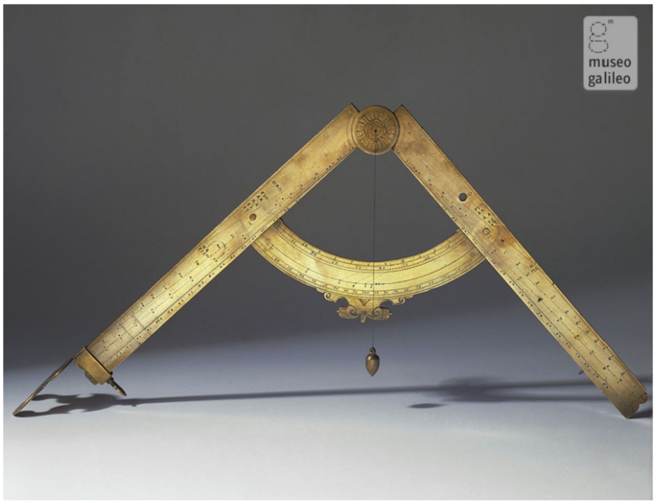

Fig. 3. The proportional compass of Galileo (Museo Galileo, Florence – Photography by Franca Principe).

Rose (1968).

But who was Fabrizio Mordente and under what circumstances did he invent his proportional compass? Born in Salerno in 1532, Mordente spent his youth travelling the world. During his explorations, he spent several months aboard Portuguese ships, an experience that would be crucial for the invention of the compass (Camerota, 2000, pp. 25–26). At the end of the sixteenth century the Portuguese empire was still one of the largest colonial empires in the world, and its fleet was equipped with the most common astronomical instruments. The success of long-distance journeys across the oceans depended on instruments like the astrolabe, which were used by the sailors to determine their position in the open sea. By closely studying these instruments, Mordente would have realized that their precision depended on the number of parts into which the degree of arc was divided. The more the parts of the degree, the more the precision of the instrument. Theoretically, Mordente's compass was capable of dividing the degree of arc into an infinite number of parts. For this reason, when Mordente published his first treatise (Mordente, 1567), he presented the compass as a way to increase the precision of astronomical instruments.

As already mentioned, Mordente's compass would rapidly fall into oblivion, only to be rediscovered in the second part of the nineteenth century. Ironically, it is thanks to Bruno, a great admirer of Mordente at first but then one of his most severe critics, that modern-day scholars turned their attention to Mordente's compass. It all started with Berti (1868) who drew attention to Bruno's first two dialogues on Mordente's compass, those in which Bruno praised Mordente for his invention. Several decades later, Yates (1951) published the letters of Jacopo Corbinelli to Gian Vincenzo Pinelli, in which the Bruno-Mordente controversy was reported in detail. This was six years before Aquilecchia published Bruno's last two dialogues (Bruno, 1957), those written after Mordente tried to burn all the copies of the first two dialogues. Hence, the encounter of Mordente and Bruno was important not only for the development of Bruno's atomist geometry, but also for the history of the proportional compass.

3. The Bruno-Mordente controversy

In a letter to Gian Vincenzo Pinelli dated September 29, 1585, Jacopo Corbinelli reported that Fabrizio Mordente had arrived in Paris (Calderini De Marchi, 1914, p. 240). In his letter, Corbinelli mentioned two printings by Mordente: a single-sheet treatise showing an illustration of the compass (Mordente, 1585), and another work that has not yet been identified. Sheets like that mentioned by Corbinelli were distributed during the public demonstrations that Mordente organized to promote his compass. It is during one of these demonstrations that Bruno, who returned to Paris from London in October 1585, first became acquainted with Mordente's compass. From the beginning, he was very enthusiastic about the invention of his fellow countryman, as reported by the librarian of the abbey of Saint Victor Guillaume Cotin in his diary on February 2, 1586 (Spampanato, 1933, p. 655). According to the librarian, Bruno hailed Mordente as the "god of geometers." Furthermore, since Mordente did not know Latin, Bruno offered to write a Latin exposition of his compass, as mentioned above.

The two dialogues entitled *Mordentius* (*Mordente*) and *De Mordenti circino* (*On Mordente's Compass*) were published shortly thereafter. For a long time, these two dialogues were the only known texts where Bruno spoke of Mordente's compass. This inevitably influenced early interpretations of the relationship between Bruno and Mordente, giving the impression that Bruno's opinion about Mordente was overall positive. For example, writing in 1927, Olschki argued that Bruno praised Mordente "more than any other thinker or mathematician, more than Paracelsus and Copernicus, Cusanus and Plato" (Olschki, 1927, pp. 76–77). Olschki could not know that the two Italians engaged in a discussion as soon as Bruno started writing the first two dialogues, as recorded by Corbinelli in a letter to Pinelli dated February 16, 1586:

> I send you these two writings; our Fabritio is in a brutal rage against the Nolan [i.e. Bruno, who was originally from Nola, near Naples] and wishes to avenge himself in every way: but it does not seem to me that he has all the right on his side because, although the Nolan honors himself with the discourse of his, at the same time he also celebrates, and makes the author, him who is the author. The other writing is considered mad by those who know and there are not many of them to be found. Of such, patience (Yates, 1951, p. 178).

The above letter shows that Bruno must have been on good terms with Pinelli, who defended him, contending that Mordente did not have "all the right on his side." The letter also offers insights into the reasons that led to the discussion between Bruno and Mordente. Pinelli had entrusted Corbinelli with the task of supplying books and manuscripts for the library he was establishing in Padua. In fulfilling this task, Corbinelli attached two writings to the above letter. Undoubtedly, the first writing had triggered the discussion between Bruno and Mordente. However, we cannot be sure that the writing in question was a printed copy of Bruno's first two dialogues, or only a draft of them, as assumed by Yates (1951, pp. 178–179). Be that as it may, Mordente possessed the same writing and certainly did not appreciate its content. The letter provides no information about the second writing that Corbinelli sends to Pinelli. Yates proposes that "Corbinelli is here being purposely vague and mystifying, as often in these letters when he is sending his employer something which he does not want to fall into inquisitorial hands in Italy" (Yates, 1951, p. 179). Hence, if Yates is correct, the second writing was unrelated to the discussion between Bruno and Mordente.

The first two dialogues that Bruno wrote on Mordente's compass must have been published prior to April 14, 1586. On that date, indeed, Corbinelli wrote to Pinelli:

> The Nolan has printed I know not what in which he extols to heaven Fabritio's compass, but as a philosopher it seems that he wants to regulate the judgement and the expression of the said Fabritio, as though to show him that he has need of someone who should expound his arguments better (that he can himself). Fabritio fulminated with rage and wanted to print, but he gets muddled both when he speaks and when he writes. And the Nolan, who knew this, was prepared to scold him well in the second dialogue. It seems to me that the affair is over, and that both of them are content to go no further. It has cost Fabritio several crowns to buy up the Nolan's dialogue and have it burned. If I can get hold of a copy I will send it to your excellence (Yates, 1951, pp. 179–180)

When Yates first published this letter, she could not know that Bruno published two more dialogues on Mordente's compass in addition to the first two. For this reason, she assumed that the first two dialogues were published separately, and that Bruno was preparing the second dialogue by the time this letter was written. Yates's hypothesis was corrected by Aquilecchia once he rediscovered the other two dialogues (Bruno, 1957, p. xix). As can be seen from the dates of Corbinelli's letters, the tension between Bruno and Mordente escalated very quickly. Within less than two months (from February 16 to April 14) Bruno published the first two dialogues on Mordente's compass. It was then that Mordente, annoyed by what Bruno had to say about his compass, sought to acquire and burn all the copies of Bruno's dialogues. As a response, Bruno started working on two new dialogues to defend himself from Mordente's attacks.

The publication of Bruno's last two dialogues on Mordente's compass, titled *De idiota triumphans* (*The Triumphant Illiterate*) and *De somni interpretatione* (*The Interpretation of a Dream*), must have occurred before June 6, 1586, on which date Corbinelli wrote to Pinelli:

> The Nolan still against Mordente, and new dialogues. Now he is engaged in destroying the whole of the peripatetic philosophy, and, from what little I understand of it, it seems to me that he delivers his arguments very well. I think that he will be stoned by this University. But soon he is going to Germany. Enough that in England he has left very great schisms in those schools. He is a pleasant companion, an Epicurean in his way of life. (Yates, 1951, p. 182, p. 182)

Compared to the earlier letters, Corbinelli here says little about the Bruno-Mordente controversy, except that it is still going on. Rather, he draws attention to another event concerning Bruno, his public dispute against Aristotelian philosophy held at the Collège de Cambrai on May 28–29, 1586.[13] In contending that during the dispute Bruno had "deliver[ed] his arguments very well" and in calling him "a pleasant companion, an Epicurean in his way of life", Corbinelli once again expressed his sympathy for Bruno. He also spoke of Bruno's stay in England, which had caused "very great schisms in those schools." As noted by Yates (1951, p. 183), Corbinelli was probably referring to the university of Oxford, where, in 1583, Bruno had taught for a few weeks before being charged of plagiarism. Corbinelli foresaw that Bruno would also be removed from the university of Paris because of the great clamor that had accompanied his anti-Aristotelian dispute. Bruno himself seemed to be aware of this, which explains why he was planning to go to Germany. However, the polemic with the Aristotelians may not be the only reason for Bruno's departure from Paris. Mordente may also have played a role, having decided to abandon the circle of Henry of Navarre to support the Duke of Guise (Yates, 1951, p. 186). Bruno, on the other hand, had remained faithful to Henry of Navarre. Suddenly, the polemic between Bruno and Mordente had taken a political turn, and Bruno may have decided to retreat rather than engage in this sort of fight. He would be safe in Germany by the time the War of the Three Henrys broke out in 1587.

That of Corbinelli is the only extant account of the Bruno-Mordente controversy. Unfortunately, this account provides little information on how the controversy started, or why Mordente was outraged by Bruno. From the Pinelli-Corbinelli correspondence, one gains the impression that Mordente did not accept Bruno's interpretation of the compass. This is also confirmed by the 1591 treatise written by the two Mordente brothers wherein Bruno is defined as a "shadow of a philosopher" because of his failure to understand the theory underlying the use of the compass.[14] In addition, Corbinelli informs us that the controversy started as soon as Bruno's first two dialogues began to circulate. Thus, Mordente's anger must have been provoked by something that Bruno had written in the first two dialogues. Given the lack of other documents, we can only turn to Bruno's dialogues to better understand the reasons for Mordente's anger, aware of the fact that the information gathered from Bruno's dialogues will be necessarily biased.

The first two dialogues that Bruno wrote on Mordente's compass were published together at the beginning of 1586 by Pierre Chevillot in Paris. In the preface to the dialogues, Bruno explained his decision to write about Mordente's compass by presenting its inventor as one of those "Mercuries" sent by the divine providence "to remedy the fatigue and indigence of mortals."[15] This was the best compliment that Mordente could receive from Bruno, who also saw himself as a Mercury, a divine messenger entrusted with the mission of revealing the truth.[16] The figure of Mercury, the Roman equivalent of the Greek god Hermes, was central to the Hermetic tradition that influenced many aspects of Bruno's thought—although the importance of this tradition as a Brunian source has been gradually reduced ever since Yates (1964) first drew attention to it.

Having described the divine character of Mordente's invention, Bruno went on to provide a portrait of the man behind the compass. As a matter of fact, this portrait was not entirely flattering. According to Bruno, Mordente was a quite person, who "speaks with facts, teaches by doing, and remaining silent goes further than anyone else could go by reasoning."[17] Bruno, however, was determined to break Mordente's silence, and translate into words what Mordente showed during his public demonstrations of the compass. The innovativeness of the compass, Bruno declared "with all due respect," was such that Mordente himself was not fully aware of it.[18] Probably, Bruno referred to the possibility of using Mordente's compass to demonstrate the existence of the geometric minimum. The fact that Mordente regarded his compass as 'only' a measurement instrument would have prevented him from seeing this possibility. But the truth was that Mordente was not interested in discovering the minimum; as mentioned earlier, his objective was to create an instrument that could measure the degree of arc down to its smallest fractions. This was what fueled the discussion between the two Italians, as Mordente was outraged by Bruno's attempt to impose his interpretation of the compass.

It is sufficient to read these first lines to understand why Mordente tried to destroy all the copies of Bruno's first two dialogues. But there was more to it. Not only did Bruno state that Mordente had not fully understood his own work. He also claimed that what the two Mordente brothers had written on the compass was "so inelegant, so rough, ordered in such a contorted way, and based on such an ignorant doctrine, that one can easily see how it is as if nothing has never been published."[19] By the time Bruno published his dialogues in 1586, three treatises on the compass were already circulating: the first by Fabrizio Mordente published in Venice in 1567, the second by Gasparo Mordente (Fabrizio's brother) published in Antwerp in 1584, the third by Fabrizio published in Paris in 1587. Camerota (2000, p. 90) notes that Bruno's critique of Mordente's writings was all the more unfair, as he heavily relied on the 1584 treatise to describe the operations of the compass.

[13] Cotin's diary informs us that the dispute took place on that date. See Spampanato (1933), pp. 44–46.

[14] See Mordente and Mordente (1591): "Ma se per sorte alcuna ombra di filosofo, per mostrare anch'ella di sapere …" Quoted from Camerota (2000, p. 54).

[15] Bruno (1957, p. 31). "Ut verum, ita et vulgatum satis est, Deum providentem certis quibusdam temporibus Mercurios, quibus mediantibus labori et inopiae mortalium succurrat, e caelo mittere."

[16] For Bruno's self-identification with Mercury, see Ciliberto (2000).

[17] Bruno (1957, p. 31) "[Mordens] actione loquitur, operatione docet: dum eo ipse silendo promovet, quo caeteri universi nunquam ratiocinando potuere."

[18] Bruno (1957, p. 32) "[In Mordentis geometriae partibus] adeo pregnans atque fecunda praxis continetur, ut illum mihi forte (quod citra iniuram dictum velim) plus quam putare et ipse possit invenisse."

[19] Bruno (1957, pp. 32–33). "De circino autem aliquid editum extat, quod (per meam fidem) tam rude, tam crassum, tam contorto ordine, tam ignorante doctrina scriptum constat: ut ipsum certe tanquam non editum ideo quisque facilissime iudicare posset."

4. Bruno's first dialogue: *Mordentius*

Bruno's first dialogue on Mordente's compass was entitled *Mordentius* and was devoted to presenting the method developed by Mordente to measure the smallest fractions of geometric magnitudes. This method was based on two axioms. According to the first axiom, two magnitudes were in the same ratio as their corresponding parts. For example, if two segments were in a ratio of 1:3, this meant that the half of the shorter segment was three times shorter than the half of the longer segment. By the same token, if we knew that a circumference is divided in 16 equal parts, and we wanted to know the value of a fraction that was smaller than one-sixteenth of the circumference, we could take the length of that fraction and apply it 16 times to the circumference (The example is taken from chapter V of the *Mordentius*). Proceeding in this way, we would cover a portion of the circumference, equal to a certain number of entire parts. This number would be the value of the fraction. If there were a remainder, the same operation could be repeated indefinitely until no portion of the line was left over.

If the first axiom of Mordente's method was mathematical in its character, the second axiom was more philosophical and could be traced back to medieval scholasticism:

> The second is the common philosophical axiom that in natural and artificial objects a minimum and a maximum relative to their form are to be determined, which is why those who divide naturally as well as artificially do not happen to go on to infinity.[20]

What Bruno presented as an axiom commonly accepted by philosophers was the cornerstone of the medieval theory of *minima naturalia*. Although different versions of this theory were developed especially in the thirteenth and fourteenth century, the idea of *minima naturalia* had only one source: Aristotle's *Physics*, Book I, Chapter IV (187b13–188a5). There, in arguing against Anaxagoras and his theory that everything is in everything, Aristotle claimed that the form of natural beings was confined with certain limits. The lower limit—the *minimum naturale*—indicated the smallest form that a natural being could assume without losing its essence. As John Philoponus puts it in his commentary on Aristotle's *Physics*: "no man has the size of a fist or a finger or a grain, because if something is too small it cannot receive a form" (Glasner, 2001, p. 15). The corollary of this theory was that, at least as far as their form was concerned, natural beings could not be infinitely divided, otherwise there would be no limit to the smallness of their forms. This corollary is what gave Mordente (the fictional character created by Bruno, not to be confused with the instrument maker) confidence in the success of his method, assuring him that the division of a magnitude into its smallest fractions would come to an end. It is worth stressing that this was not the way Mordente conceived his compass, but it was Bruno who attributed this interpretation to him.

It is most likely that the axiom on *minima naturalia* was not included in the original method developed by Mordente, but it was added as a result of Bruno's intervention, for no reference was made to this axiom in Mordente's previous works. On the other hand, it should be noted that in *De idiota triumphans* Bruno would accuse Mordente of having misunderstood the theory of *minima naturalia,* showing how this theory ran counter to what Mordente aimed to demonstrate. For this reason, De Bernart (2002, p. 173–177) argues that the axiom on *minima naturalia* was the work of Mordente, and that Bruno reported the axiom in the *Mordentius* only to criticize it in the *De idiota triumphans.* In claiming so, De Bernart implicitly assumes that the project of writing *De idiota triumphans* dated back to the time when Bruno was composing the *Mordentius.* Yet this hypothesis is not supported by the Pinelli-Corbinelli correspondence, which instead informs us that Bruno decided to write the last two dialogues in response to Mordente's attacks on the first two. Rather, I believe that Bruno did not notice that Mordente's findings did not sit well with the theory of *minima naturalia* until a later stage, but then he laid the blame on Mordente instead of admitting that he had made a mistake. Thus, the axiom on *minima naturalia* should be regarded as Bruno's addition to Mordente's method, despite the objections that Bruno himself would raise to this axiom in the later *De idiota triumphans*.

As one can see from the *dramatis personae* included in Murdoch (2001, pp. 99–101), several authors contributed to developing the theory of *minima naturalia* over the centuries. Indeed, the theory of *minima naturalia* was discussed as late as the sixteenth century by the likes of Luis Coronel (d. 1531), Benedict Pereira (1536–1610) and Francisco de Toledo (1532–1596). As shown by Murdoch, different definitions of *minima naturalia* were given during the middle ages, each corresponding to a different group of authors. Among them, there were also those who associated the concept of *minima naturalia* with the issue of minimum limits. Bruno himself established this connection in the *Mordentius*, claiming that the existence of a *minimum naturale* set a limit to the division of natural beings. However, it is hard to say whether Bruno was acquainted with what Murdoch calls the "limit decision literature" (Murdoch, 2001, pp. 116–122). Given Bruno's Dominican education, it is more likely that Thomas Aquinas and Averroes shaped his understanding of *minima naturalia.*

Aquinas discussed the theory of *minima naturalia* in his *Summa theologiae* rather than in his commentary on Aristotle's *Physics*. Murdoch (2001, p. 101) notes that this choice reflects Aquinas's awareness of the relation between *minima naturalia* on one hand, and substantial forms on the other hand. As for Averroes, he developed his theory of *minima naturalia* especially in the middle commentary on Aristotle's *Physics*.[21] A Latin translation of this text by Jacob Mantino (d. 1549) was included in the Junta edition of Aristotle's *Opera omnia*, a copy of which was possessed by the monastery of San Domenico Maggiore in Naples where Bruno received his education.[22] Averroes borrowed aspects of his theory of *minima naturalia* from the mutakallimūn, a group of ninth-century Islamic theologians that defended a form of geometric atomism similar to that of Bruno. In particular, Glasner demonstrates that Averorres was indebted to mutakallimūn for his idea of minimum, which he adopted "taking it out of the atomistic context and adjusting it to the Aristotelian environment" (Glasner, 2001, p. 26).

Since the translation by Mantino only covered the first three books of Averroes's middle commentary on Aristotle's *Physics*, Bruno's knowledge of Averroes's theory of *minima naturalia* was bound to be limited. However, references to the theory were made in Book III of the middle commentary, where we read that: "magnitude is infinitely divisible qua matter, not qua form; qua form its divisibility is limited" (Glasner, 2001, p. 18). Likewise, in the long commentary on the *Physics*, which was also included in the Junta edition, Averroes claimed that "a line as a line can be infinitely divided. But such a division is impossible if the line is taken as made of earth" (Glasner, 2001, p. 18). Reading these texts, Bruno would have thought that Averroes's theory was still Aristotelian in that it was grounded in the concept of 'formal' minimum. Seeking the minimum magnitude rather than the formal minimum, Bruno was more in line with the atomist sources of Averroes, although it is unlikely that Bruno could have been familiar with the doctrines of the mutakallimūn.[23]

Again, it should not be forgotten that this discussion on *minima naturalia* had nothing to do with Mordente's compass. Indeed, as already mentioned, Mordente's objective was to create an instrument

[20] Bruno (1957), p. 38. "Secundum commune philosophorum axioma quod in subiectis phisicis et artificialibus determinatum est ad eorum formas maximum atque minimum: unde sicut vnon naturaliter ita nec artificose dividentibus accidit in infinitum facere progressum."

[21] For an overview of Averroes's physics, see Glasner (2009).

[22] For a reconstruction of the library of San Domenico Maggiore, see Canone (1992).

[23] For an overview of the doctrines of the mutakallimūn, see Dhanani (1994).

capable of dividing the degree of arc into a potentially infinite number of parts. As such, Mordente's compass did not challenge the Aristotelian notion of infinite divisibility of the continuum, which in fact provided a theoretical justification for the use of the compass. Nor was Mordente committed to the theory of *minima naturalia*, as we have seen that there was no trace of this theory in Mordente's writings prior to his encounter with Bruno. Rather, it was Bruno who tried to use Mordente's compass against Aristotle, turning it on its head and taking it as an argument in favor of his atomistic view of the continuum.

5. Bruno's third dialogue: *De idiota triumphans*

De idiota triumphans was one of the last two dialogues that Bruno wrote in response to Mordente's attacks on the first two. Meanwhile, the tension between Bruno and Mordente had rapidly escalated, and Bruno's purpose in writing *De idiota triumphans* was to criticize Mordente's method. If, in the first part of *De idiota triumphans,* Bruno's criticism focused on more superficial aspects of Mordente's method (such as the actual number of operations that could be carried out with the compass), in the last part the focus shifted to its foundations. In particular, Bruno took issue with what, in the *Mordentius*, he had presented as the second axiom of Mordente's method according to which natural and artificial beings had a minimum form and thus could not be infinitely divided in relation to their forms. Bruno started by noticing that this argument only applied to natural beings, and it could be extended to artificial beings insofar as these were considered as formal and not as artificial entities. Therefore, Bruno concluded, it was wrong to speak of artificial beings as distinguished from natural beings, because as formal entities they behaved in the same way. This remark tells us that Bruno was familiar with the Aristotelian theory of *minima naturalia*, for in what was considered the source of all the arguments on *minima naturalia* (Physics IV.1), Aristotle referred to natural and not to artificial beings.

For Bruno, the major flaw in the understanding of the theory of *minima naturalia* that he had attributed to Mordente was that it ignored the distinction between formal minimum and minimum magnitude. Bruno noted, and rightly so, that the supporters of *minima naturalia* did not consider "the minimum magnitude or the minimum continuous quantity, which for them cannot be found, but the minimum substance in which the form of each species can be retained."[24] The reason why especially medieval scholars emphasized the formal character of *minima naturalia* was to distinguish between two kinds of divisibility of natural beings, depending on whether they were viewed as continuous or as formal entities. The former case was associated with infinite divisibility, while the latter with finite divisibility. The source of this distinction was Aristotle, who in *Physics* IV (187b13–188a5) claimed that natural beings could not be infinitely divided without losing their form, while in *Physics* VI (231b14-15) he argued for the infinite divisibility of the continuum. As documented by Maier (1966) and Murdoch (2001), medieval scholars rephrased this Aristotelian distinction in different ways, speaking for example of natural beings as divisible in potency or in act.

In the first dialogue on the compass, Bruno had Mordente claim that the theory of *minima naturalia* provided the foundation for his method because this latter led to identify the formal minimum, or more specifically, the minimum fraction of a curved or a straight line. On the contrary, in *De idiota triumphans*, Bruno argued that Mordente's method showed the minimum magnitude, the existence of which was denied by the supporters of *minima naturalia.* Bruno's argument ran as follows:

> If one refers to the line or the surface to be divided, the assumption [of Mordente], which some philosophers accepts as a principle, means that those who divide mechanically happen to lose first the perception of quality and then that of quantity or extension. For this reason, there is no difference in taking the minima or the almost minima of a curved or a straight line, of a regular and irregular figure. Hence, what is determined in its form is not limited in its matter. This is why Mordente should be considered a god.[25]

What Bruno meant was that when dividing a line down to its smallest fractions a point was reached where it was no longer possible to determine the shape of the fractions. As the size of the fractions decreased, we lost the ability to distinguish between curved and straight, and all the fractions ended up having the same indefinite shape to our eyes. This could pose a challenge to Mordente's method, insofar as if a fraction was too small it could not be measured with the compass. As reported by Bruno (1957, p. 42), Mordente solved this problem by simply measuring the remaining fraction, and then subtracting this value to the whole length of the line. Here, the fact that the smallest fraction turned out to have no defined shape was taken as proof that beyond the perceivable forms of curved and straight there was a common, shapeless minimum magnitude. This shapeless minimum was regarded as the matter of the line, which, in the above quotation, was defined as "determined in its form" (i.e. curved or straight) but "not limited in its matter" (Bruno, 1957, p. 15). The merit of Mordente's method was that it revealed this minimum, shapeless magnitude (or "ultimate fraction," as Bruno called it) standing on the threshold of perception, as it "teaches us to divide down to the ultimate sensible element and, with such ease as I have demonstrated in the specific dialogue, leads us to the ultimate fraction."[26]

A remark is in order. It is true that Bruno's argument worked insofar as curved and straight were considered as perceivable forms and not as abstract geometric determinations. For, in classical Euclidean geometry, curved and straight were not reducible to each other. However, it should be noted that one of the ancestors of the modern calculus, the method of exhaustion, was also based on the approximation of curved and straight. Traditionally ascribed to the ancient Greek mathematician Eudoxus of Cnidus, who in turn seemed to have borrowed the idea from Antiphon the Sophist, the method of exhaustion consisted in measuring the area of a circle by inscribing within it a regular polygon, the number of whose sides was progressively increased until the area of the inscribed polygon 'exhausted' that of the circumscribed circle. Yet even when properly carried out this procedure did not make the polygon coincide with the circle, but at best it reduced the difference between the two areas so that it could be neglected. To this extent, the method of exhaustion implied a certain degree of approximation and, as noted by Boyer, "the gap between the curvilinear and the rectilinear still remain [ed] unspanned" (Boyer, 1949, p. 35). The same can be said of the argument used by Bruno in *De idiota triumphans.*

What did Bruno's argument mean in geometric terms? Generally speaking, Bruno posited the existence of geometric minima, i.e., infinitely small quantities, which were extended but indefinitely shaped. Such minima were the building blocks of *all* geometric objects, regardless of whether they were regular or irregular polygons, curved or straight lines. In the years following the controversy with Mordente, Bruno would go on to develop this intuition into an atomist geometry.

[24] Bruno (1957), p. 14. "Non intelligens quod dicit ratione respectuue formarum, declarare sensum illorum philosophorum non respicere minimum magnitudinis seu quantitatis continuae, quod numquam credunt incurri posse: sed minimum subietum in quo possit saluari forma cuiusque speciei."

[25] Bruno (1957), pp. 14–15. "Quinimmo, si ad superficiem vel lineam dividendam respicere velit, illud acceptum pro principio a quibusdam philosophis: significat in proposito, quod mechanice dividentibus prius contingat perdere sensum qualitatis quam molis seu quantitatis, quia tandem non differt accipere minima seu prope minima lineae curvae atque rectae, regularis atque irregularis: et ideo determinatum secundum formam, nondum est terminatum secundum materiam: Unde Mordentius deificetur."

[26] Bruno (1957, p. 15). "[Mordenti] ad ultimum usque sensibile dividere doces, et tanta facilitate, quantam in dialogo proprio explicaui, ita ultimum fractionum insinuas."

However, differently from what he would do in *De minimo* (Bruno, 1889), in the dialogues on Mordente's compass Bruno did not equate geometric minima to extended circular points. This difference was of crucial importance, because claiming that geometric objects were composed of extended points caused several problems in geometry, such as the impossibility of accounting for incommensurable magnitudes. Therefore, in the dialogues on Mordente's compass Bruno developed a theory which, when compared to that set forth in *De minimo*, was more coherent from a geometric viewpoint. This raises the question of why Bruno changed his mind with regard to the status of geometric minima. Although it is beyond the purpose of this article to answer this question, I will limit myself to the observation that the theory developed in *De minimo* was at the same time a geometric, metaphysical and physical theory. Thus, we can assume that Bruno was probably led astray from the geometric path taken in the dialogues on Mordente's compass by the necessity of combining different kinds of theoretical elements.

6. Conclusions

Most of the objections against Bruno's mathematics are raised in response to the version expounded in Bruno's *De minimo*. However, critics of Bruno's mathematics would probably have had a different picture of this theory, if they had also considered the dialogues on Mordente's compass. This is well exemplified by the case of Olschki (1927). In the introduction, we saw that Olschki raises two objections to Bruno's mathematics. The first objection is that it envisaged different kinds of minima, while there should be only one sort of infinitesimal quantity. The second objection is that it was not linked to a theory of motion. For these reasons, Olschki concludes, Bruno's concept of minima cannot be considered a forerunner of the infinitesimals. If one reads *De minimo*, one cannot help but agree with Olschki. Nevertheless, as soon as the dialogues on Mordente's compass are brought in, one is forced to admit that Olschki's criticisms are unfair. In those dialogues, not only did Bruno claim that there was only one kind of minimum magnitude, but the reason why he claimed so was because he aimed to lay the foundations for the law of planetary motion which he had "dreamt of."[27]

Both the idea of infinitely small quantities, and the attempt to account for natural phenomena such as motion, belong to the historical development of the calculus. This, of course, does not mean that Bruno should be regarded as on a par with Leibinz, Newton, Cavalieri and all the other seventeenth-century mathematicians who contributed to the development of the calculus. For Bruno's theory of minima was not substantiated by any mathematical application, and it lacked a rigorous mathematical foundation. Rather, a reading of the dialogues on Mordente's compass shows that, at least at the beginning of his mathematical career, Bruno had a mathematically correct understanding of infinitely small quantities. As Rowland puts it: "he was moving toward the calculus himself, and could already outline what would become some of its fundamental ideas in theory, if he could not yet express them in usable equations" (Rowland, 2009, p. 194).[28] To this extent, claiming that Bruno envisioned the infinitesimals would not be out of place.

[27] As mentioned in the introduction, Bruno first presents the idea of a law of planetary motion in the last part of the second dialogue on Mordente's compass, which is entitled *Insomnium* (*The Dream*). Then he elaborates on this idea, attempting to provide a mathematical foundation for it based on his geometry of minima, in the fourth dialogue, which is entitled D*e somni interpretatione* (*The Interpreation of a Dream*).

[28] See also Rowland (2012).

Declarations of interest

None.

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