**Spinoza’s Ethics as a Mathematical Object**

**by**

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**Abstract**

Spinoza’s geometrical approach to his masterwork, the *Ethics*, can be represented by a digraph, a mathematical object whose properties have been extensively studied. The paper describes a project that developed a series of computer programs to analyze the Ethics as a digraph. The paper presents a statistical analysis of the distribution of the elements of the *Ethics*. It applies a network statistic, betweenness, to analyze the relative importance to Spinoza’s argument of the individual propositions. The paper finds that a small percentage of the propositions greatly exceed the majority in this importance. The paper then describes two logical structures that appear respectively in the *Ethics* and argues that they result in redundancy in the sense that about ten percent of the propositions could have been eliminated. The appendices list these structures and describes how the resources of the study can be made available to readers.

**Spinoza’s Ethics as a Mathematical Object**

*“…But don’t you think that life with a statistician would be rather, shall we say, humdrum.”*

*“It’s not the numbers themselves,” she said finally, “it’s what you do with them that matters[[1]](#footnote-1).”*

**1. Introduction**

Centuries before networks were formalized as mathematical objects, Spinoza represented the logical structure of the *Ethics* with a type of network, a digraph (directed graphs). Consequently, the *Ethics* belatedly inherited the mathematical properties of digraphs. Although others[[2]](#footnote-2) have analyzed the *Ethics* using mathematically oriented tools rather than traditional philosophic analysis, more can be learned by applying recently developed software and statistical tools to the *Ethics* digraph.

Beginning in the the 1950’s the use of the theories and methods of networks became pervasive in the social sciences. The mathematical properties of social networks were formalized, and social network analysis became an accepted methodology in the social sciences. The study of social networks stresses the properties of the relationships between social entities rather than the properties of the entities themselves. Some of the relationships studied are power, communication, influence, etc. Social scientists have developed non-parametric statistical and software tools for the analysis of social networks. Since the logic of the *Ethics* can be represented as a digraph, a type of network, I believe these tools can be usefully applied to the *Ethics.*

Some of the most useful statistics in social network analysis measure the degree of connection of a unit to other units in the network. This measure is called the unit‘s *centrality* and is often taken to correspond to the “strength” of the unit. Depending on the context, strength can refer to influence, status, communication ability, etc. For example, units with strong influence tend to have their opinions promulgated to more units in the network. Such centrality statistics measure a unit’s influence on units both immediately connected to it and also connected through other units. Unit A may influence unit B which passes A’s influence to unit C, although A has no direct contact with C.

Obviously the meaning of the strength of a unit’s characteristics in a social network does not apply to the elements, definitions, axioms, propositions, etc., in the *Ethics* network. But the statistics used to measure strength in social networks are strictly mathematical computations that depend solely upon the number of connections between units, not the meaning of these connections. Consequently, such statistics may be useful for understanding the relationships between digraph elements in the *Ethics* and the structure of Spinoza’s rhetoric. Since the elements in the *Ethics* digraph are logically dependent on one another, the “logical strength” of an element is a measure of the logical interdependence of that element with others in the digraph. Mathematical details will be provided later in the paper, but the logical strength of a network element is related to the number of elements to which it is either directly or indirectly connected.

The statistics presented in this paper are purely descriptive. They apply solely to the *Ethics* and are not comparable to those of social science. The only links to the social sciences are the use of similar statistics and software developed for that field. Plainly, the association of degree of logical strength with the statistics of the *Ethics* network is a normative judgment. Moreover, whether logically stronger network entities are more *philosophically* significant requires further argument beyond the scope of this paper. The hope is that the statistical analysis of logical strength will help to elucidate Spinoza’s methodology, rhetoric and philosophy.

This paper first provides a brief review of the technical background required for the rest of the paper. The second section uses basic descriptive statistics to make some observations on the distribution of the digraph elements over the five Parts of the *Ethics*. This section makes some observations on the logical strength of the propositions of the *Ethics* by Part and the subject matter of the strongest propositions. The final section identifies two network structures that appear frequently in the *Ethic*s. This section shows that propositions appearing in these structures, which are redundant in the sense that they can be incorporated into other propositions, amount to about ten percent of all propositions. The section concludes by identifying clusters of these structures that are final results of the network. Appendix A provides a list of the propositions included in these structures, and Appendix B describes the resources developed for this study that are available on my website.

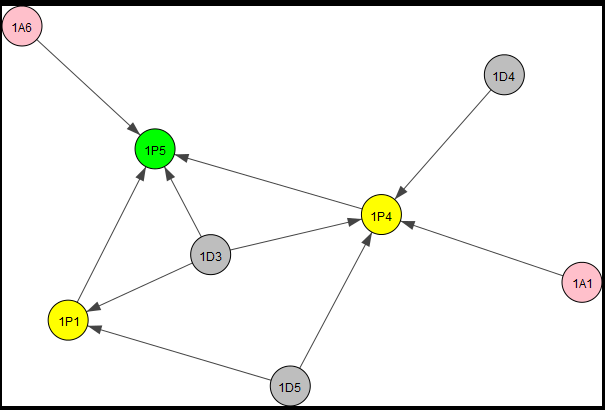


Figure 1-A Small Section of the Ethics Network

**2. Technical Background**

*Spinoza’s Ethics as a Digraph[[3]](#footnote-3)*

The sub-network comprising the first six propositions of the *Ethics* (Figure 1-produced by my program spin\_explode) can be used to explain the basic principles of digraphs. The mathematical definition of a graph is a set of vertices (or nodes) with a relationship on the vertices that can be graphically depicted by lines called “edges”. For a digraph the relationship is asymmetric, non-reflexive and is graphically depicted by lines terminated with arrows called “arcs”. In the figure the vertices model definitions (D), axioms (A) and propositions[[4]](#footnote-4) (P), while the arcs model the relationship “is necessary for the demonstration of.” The order of a graph or a digraph is the number of vertices. The order of the depicted digraph is 8 (3 definitions, 2 axioms and 3 propositions). Another important concept is degree. The degree of a vertex is the number of arcs terminating in the vertex. Degree can be broken down to in-degrees and out-degrees depending on the direction of the arcs relative to the vertex. P4 has degree 5, with in-degree 4 and out-degree 1. Axioms and definitions have in-degree 0 and are called transmitters, while propositions with out-degree of 0 are called “leaves” or receivers. In the *Ethics* leaves are not used to prove other propositions so, in a sense, they are Spinoza’s ultimate thoughts. Nevertheless leaves may not be the most important propositions; for example, the important metaphysical proposition that God necessarily exists (1P11) is not a leaf.

A “walk” in a graph is defined as a sequence of vertices connected by arcs, and a closed walk is a sequence that begins and ends at the same vertex. In a digraph each arc in the sequence must be traversed consistently with its direction. In the *Ethics* a walk represents the sequence of network elements that is logically required to prove a proposition given an earlier network element. The walk includes both the given element and the proven proposition.

An acyclic graph has no closed walks. Spinoza insured that the digraph for the *Ethics* was acyclic through his numbering system. Recall that a proposition number comprises a Part number concatenated with a proposition number. An arc from P1 to P2 is permitted if the following rules are satisfied: the Part number of P2 is greater than or equal to the Part number of P1, *and* If the Part numbers are equal, then the proposition number of P2 must be greater than the proposition number of P1. There are additional rules that control the arcs of the remaining network elements but are not essential to this paper. We can thank Spinoza for the acyclic nature of the *Ethics* because a cycle would result in an unresolvable logical problem. This point will be seen to be significant later on in this essay.

*Network Statistics[[5]](#footnote-5)*

Statistics that apply to the network as a whole are edge count, vertex count, density, the ratio of the number of actual to possible arcs, diameter, the maximum walk in the digraph in number of arcs, and degree distribution, the percent of elements for each degree.

*Centrality Statistics*

Measures of centrality apply to individual vertices and measure the connectedness of a vertex to the rest of the vertices in the network. For the *Ethics* vertices with higher measures of centrality are more frequently used in Spinoza’s deductions. In general, vertices with higher centrality contribute more to the argument of the *Ethics.* They are, in this sense, stronger with respect to the logical development of Spinoza’s arguments. There are three measures of centrality most applicable to the *Ethics*.

* Degree Centrality

This is the simplest measure of centrality and was defined above, along with in-degree and out-degree. In-degree is a count of the number of immediate elements needed to establish logically the proposition modeled by a vertex, whereas out-degree is a count of vertices with direct logical dependence on that vertex. Thus, vertices with higher degrees are directly logically connected to more vertices. Spinoza’s text, with some exceptions, specifies only the vertices associated with the in-degree. Since every arc uniquely joins two vertices, out-degrees can be computed from in-degrees. The statistical data base developed for this study provides both the in-degree and out-degree for each vertex. The Python class for the study lists the propositions comprising both in-degree and out-degree.

* Closeness

Closeness measures the distance in network steps from one vertex to all others connected to it in the network. The game *Six Degrees of Kevin Bacon* illustrates closeness. People with a low degree of Kevin Bacon to say Barak and Michelle Obama, would tend to have fewer intermediaries between their selves and the Obamas. Mathematically, closeness is the sum of the reciprocals of the shortest walk in network steps from the vertex to all vertices connected to it either directly or indirectly. For large order networks closeness will tend to be small, and distinctions between the closeness of vertices will be slight. I have computed closeness for the vertices in the *Ethics* digraph and have foundlittle difference for the vertices modeling propositions, axioms and definitions. Consequently, closeness has not proved to be a useful statistic for the analysis of the *Ethics*. For completeness I have included this description of closeness and included the closeness for each vertex in the statistical data base that is available for download from my web site (See Appendix B.). I have excluded other commonly used network statistics that have no relevance to the *Ethics.*

* Betweenness.

Betweenness is a more useful measure of centrality for the *Ethics*. The first step in computing betweenness is to find the shortest walks in arc steps between all of the network vertices. The betweenness of a vertex is the number of shortest walks in the network which include that vertex. I have scaled the measurement by dividing by the number of possible node parings [ (n-1)\*(n-2)] and multiplying by 100. This enables comparison of betweenness in similarly scaled networks with different orders.

In the Spinoza network the shortest walk can be interpreted as the number of logically necessary propositions required to include a network entity in the proof of a target proposition. Note that both the shortest walk and target propositions may require other entities not on the walk for their proof. Propositions with higher betweenness are logically necessary for the proof of a larger number propositions than those with low betweenness.

**3. Descriptive Statistics for the *Ethics***

The mathematical object constructed from the *Ethics* can be statistically described on two levels: the network as a whole and the elements comprising the network. The former is an overall statistical description of the scope and structure of Spinoza’s masterwork, and the latter describes the relationships of the vertices to one another.

*Network Statistics*

As computed by *igraph*, a computer system that is used widely in social network analysis, the digraph for the *Ethics* comprises 408 vertices connected by 1,082 arcs. The diameter of the network is 12. The meaning of the diameter in the *Ethics* is the greatest number of propositions required to link an element to the proof of a proposition. Density, the simplest description of network structure, density is .0065 arcs per possible arc, for the *Ethics* digraph. To put this in perspective, Spinoza’s network is orders of magnitude less dense than such real world networks as peer-to-peer file sharing (density = .0514), social networks (density =.179), email networks (density =.0112) and yeast/protein interactions (density =.0303). See Melancon for a more detailed discussion of network density along with a several more real world examples.

Density measures the volume of relationships in the *Ethics* but provides no information about their structure. There are two structures that appear frequently in the *Ethics*. The first structure, which I have called triangular, is illustrated by the relationship between D3, P1 and P5 in Figure 1 above: D3 is required to prove P1 and P5, and P1 is also required to prove P5. Note that P1 is used twice. This triangular structure appears 231 times in the *Ethics*. Second, the quadrilateral structure, which adds an additional proposition to the triangular structure while retaining the multiple use of one proposition, appears 152 times. These structures will be discussed in more detail below where it will be argued that a subset of both structures include a redundant proposition.

*Elements of the Network*

Network Element Counts

Table 1 provides a breakdown of the counts of the different types of vertices in the *Ethics* digraph. Note that the table further classifies propositions as (1) intermediate: both in-degree and out-degree are greater than zero, or (2) leaf: out-degree equals zero. The table shows that there are 408 vertices in the network. Thirty-five elements in the *Ethics* are not in the table because their degree equals zero and therefore are not connected to the network. These elements include two axioms (1A2, 2A10), five definitions, two postulates and 26 affects. Because it is mentioned frequently, I have added the general definition of an affect to the network even though Spinoza does not give it a number. This brings the total number of vertices listed in the Elwes translation to 442.

In the final two Parts of The *Ethics* Spinoza significantly altered the distribution of network elements from that in the first three Parts*.* In the first three parts Spinoza concentrated on intermediate propositions, while he increased his focus on the derivation of leaves in the final two Parts. Close to 60 percent of the leaves are in the final two parts, and leaves as a percentage of vertices increases from the about 15 percent in the first three parts to about 35 to 40 percent in that last two parts. Since leaves do initiate arcs to any other propositions, they are the ultimate conclusions of the *Ethics.*

The statistical evidence elucidates Spinoza’s rhetorical strategy. If the most significant message of the work is to be ethical, justification of the ultimate ethical conclusions must be supported by a superstition-free foundation in metaphysics, epistemology, psychology and anthropology. Spinoza starts from relatively self evident premises to create this foundation in Parts 1 through 3, while in Parts 4 and 5 he generates the ultimate ethical conclusions. The statistical effect of this strategy is the increased percentage of leaves in the latter two parts of the *Ethics.*

Interestingly Spinoza frequently employed the modern spoke-hub network paradigm to connect intermediate propositions and leaves. Propositions are connected like a wire wheel: several leaves are joined by arcs, the spokes, to an intermediate proposition in the center, the hub. Examples of spoke-hubs are 4P26 and 3P7 which originate arcs to 11 and 24 propositions respectively. Thus our seventeenth century lens grinder/philosopher invented network concepts that were rediscovered by companies such as Delta Airlines and FedEx to route and schedule aircraft more efficiently. This structure can be seen vividly using the Spinoza\_Explosion program described in Appendix B.

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| --- | --- | --- | --- | --- | --- | --- |
|  | **Table 1-Percentage of Vertex Types By Part** | | | | |  |
|  | **Part of the *Ethics*** | | | | |  |
|  | **1** | **2** | **3** | **4** | **5** | **Total** |
| **Count** | **65** | **94** | **100** | **97** | **52** | **408** |
| Definitions | 12.3 | 5.3 | 3.0 | 6.2 | 0.0 | 5.4 |
| Axioms | 9.2 | 9.6 | 0.0 | 1.0 | 3.8 | 4.4 |
| Inter. Propositions | 61.5 | 58.5 | 57.0 | 53.6 | 61.5 | 57.8 |
| Leaves | 16.9 | 12.8 | 16.0 | 39.2 | 34.6 | 23.3 |
| Lemmas | 0.0 | 8.5 | 0.0 | 0.0 | 0.0 | 2.0 |
| Postulates | 0.0 | 5.3 | 1.0 | 0.0 | 0.0 | 1.5 |
| Affects | 0.0 | 0.0 | 23.0 | 0.0 | 0.0 | 5.6 |
| **Total** | **100.0** | **100.0** | **100.0** | **100.0** | **100.0** | **100.0** |
| Leafs/Inter. Props. | 0.28 | 0.22 | 0.28 | 0.73 | 0.56 | 0.40 |

Centrality

Table 2 summarizes the centrality statistics for the *Ethics* network. Measures of centrality in a social network are related to influence, communication, power, etc. of the entities modeled by the vertices. Analogically, in Spinoza’s network the measures of centrality can be related to the logical significance of the propositions, axioms, etc. modeled by the vertices. Two measures appear in the table: degree centrality and betweenness. Both were computed by *igraph.*

Degree centrality is a local measure of centrality, while betweeness is a global measure. Degree centrality, used in previous technical studies of the *Ethics* network, measures only direct logical connections, while betweenness counts walks between both directly and indirectly connected vertices. The lengths of the walks range from one arc to the diameter of the network. Thus, betweenness is a more encompassing measure than degree centrality.

For the *Ethics*, betweenness and degree centrality are not closely related statistically. The Spearman rank correlation coefficient[[6]](#footnote-6) of degree to betweenness is .28 for all network elements and .21 for propositions only. These rank correlations indicate that there is a weak linear relationship linking degree to betweenness.

Examination of Table 2 shows that betweenness is a more sensitive statistic than degree centrality. Comparing the average degree centrality of Parts 1-3 to Parts 4-5, the former is about 20 percent higher than the latter. The same comparison for average betweenness shows Parts 1-3 is 84 percent higher than Parts 3-4. Statistically, betweenness is therefore the more appropriate statistic for exploring Spinoza’s methodology.

The differentials in both statistics in Parts 1-3 versus Parts 4-5 emphasizes the modifications in Spinoza’s approach demonstrated in Table 1. While both statistics show a decrease in Parts 4 and 5 as compared to Parts 1-3, the difference is more striking for betweenness. The dramatic reduction in betweenness (an order of magnitude) results from (1) the asymmetric dependence of propositions in Parts 4-5 on propositions in Parts 1-3, and (2) the higher percentage of leaves in Parts 4 and 5 results in a larger number of vertices which are connected by inward arcs only and therefore can only be on a walk to itself.

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| --- | --- | --- | --- | --- | --- | --- |
| **Table 2-Centrality of Propositions by Part of the *Ethics*** | | | | | | |
|  | **1** | **2** | **3** | **4** | **5** | **Total** |
| ***Degree*** |  |  |  |  |  |  |
| **Average** | 5.47 | 6.91 | 6.55 | 5.22 | 4.82 | 5.83 |
| **St. Dev** | 4.23 | 4.65 | 5.80 | 3.63 | 2.56 | 4.24 |
| ***Betweenness*** |  |  |  |  |  |  |
| **Average** | 0.18 | 0.22 | 0.16 | 0.06 | 0.04 | 0.13 |
| **St. Dev** | 0.26 | 0.36 | 0.38 | 0.13 | 0.07 | 0.24 |
|  |  |  |  |  |  |  |

Table 2 also shows that the standard deviation for betweenness is larger than the mean for all parts of the network. The distribution of the betweenness of the propositions is far from the normal distribution that one might expect. Instead the distribution is highly skewed as is shown in Figure 2. The figure shows that betweenness of the great majority of the propositions is small (between 0 and .3). In addition, there are a small number propositions whose betweenness is an *order of magnitude* larger than this majority. The highly skewed distribution shows that Spinoza chose to focus on a small number of propositions as the logical foundation for his derivations. These propositions may not be most significant philosophically, but they are most essential to Spinoza’s derivations. *Things equal to the same thing are equal to each other* may not be Euclid’s most important *geometric* insight, but his entire argument fails without this axiom

The content of the most logically significant propositions is shown in Table 3, the ten propositions with the highest betweenness listed in order of decreasing betweenness. The text of the propositions has been highly abbreviated to save space, but they recall Spinoza’s full text. The table demonstrates three features of Spinoza’s rhetorical method. First, the betweenness of the propositions declines rapidly with the rank order of betweenness: the betweenness of the tenth ranked proposition (2P29.1) is 62 percent smaller than the first ranked proposition (3P3). Inspection of Figure 2 indicates that this rapid decline will continue for the 442 network elements until a betweenness of zero is reached. The betweenness of the proposition with rank 50 (4P38, betweenness = .26) is an order of magnitude less than the betweenness of the first ranked proposition and almost 70 percent less than the tenth ranked proposition. Second, four of the first five ranked propositions and six of the ten propositions in Table 3 are involved with the metaphysics or epistemology of mind. Third, nine of the ten propositions in the table are in the foundational parts of the *Ethics*, Parts 1-3. It is difficult to characterize Spinoza’s rhetorical methodology precisely, but the statistics show that he focused on a small number of network elements to provide the logical foundation of his arguments. The betweenness numbers of the most important propositions indicate that the primary category used in the foundational parts of the *Ethics* is mind.

***Figure 2. Distribution of Betweenness***

To summarize, the analysis of betweenness indicates the following three features of the *Ethics*. First, Spinoza’s logical approach differed in Parts 1-3 versus Parts 4-5. In the former group of parts Spinoza concentrated on building the logical foundation in the latter on deriving ethical conclusions. Second, Spinoza’s logical structure is based upon on a small number of intermediate propositions. There are 236 intermediate propositions in the work but most are logically connected to few propositions. Only a small (on the order of 30) number of propositions have a high level connectivity to other propositions. Third, propositions with highest connectivity are concentrated on the category of mind.

|  |  |  |
| --- | --- | --- |
| **Table 3-Ten Propositions with Highest Betweenness** | | |
| **Prop.**  **No.** | **Betweenness** | **Abbreviated Proposition** |
| 3P3 | 2.25 | The activities of the mind arise solely from adequate ideas. |
| 2P11 | 2.14 | The being of the human mind, is the idea of something existing. |
| 3P9 | 1.90 | The mind endeavors to persist in its being for an indefinite period. |
| 2P12 | 1.34 | If the idea of the human mind be a body, nothing can take place in that body without being perceived by the mind. |
| 3P7 | 1.11 | The endeavor to persist in its own being is the actual essence of the thing. |
| 3P13 | 1.08 | When the mind conceives things which diminish the body's power of activity, it remembers things which exclude the first--named things. |
| 4P37 | .98 | The good which every man desires for himself he will also desire for other men. |
| 1P21 | .93 | All things which follow from any attribute of God must always exist and be infinite. |
| 2P9 | .87 | The idea of an individual thing actually existing is caused by God. |
| 2P29.1 | .85 | The human mind has a confused knowledge of itself, its body and external bodies. |

**4. Reasoning Structures**

Redundant Propositions

The above section on network statistics gave the number of occurrences of two reasoning structures in the *Ethics:* triangular and quadrilateral. For the triangular structure at least two adjacent propositions are required to prove a target proposition. One of these adjacent propositions, termed the “Direct/One-Step” proposition, is also required to prove the other adjacent proposition, termed the “Direct” proposition. The Direct proposition may or may not be required to prove additional propositions in the network. In the former case the Direct proposition is an intermediate result used in further argument, in the latter case the Direct proposition is redundant. There are 231 instances of triangular reasoning structures in the *Ethics[[7]](#footnote-7)* of which 22 include a redundant Direct proposition. In all of these instances Spinoza prevents circularity by observing the numbering rules defined in the technical background. The triangular structures containing redundant propositions are listed in Table A1 in Appendix A. An example of a triangular structure is given in Figure 3 below (The arrows in the figure depict arcs.).

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Direct/One-Step |  | Target |  | Direct |  | Direct/One-Step |
| 1P14.1 | **→** | 2P4 | **←** | 1P30 | **←** | 1P14.1 |

**Figure 3- Example of Predecessors in a Triangular Structure**

In the example, the target proposition, 2P4, *God’s idea must be unique*, *(*Again, I have abbreviated the propositions to allow for a concise presentation.) follows from 1P30, *An actual intellect must comprehend God’s attributes and affections* and 1P14.1, *God is unique*. (1P14.1 is the first corollary of 1P14.)But1P30 also follows from 1P14.1 along with 1A6 and 1P15. Thus, 2P4 could have been proven from 1P14.1 (used only once), 1A6 and 1P15, and 1P30 could have been omitted from the *Ethics*. 1P30 is redundant in that sense. I have written a computer program that shows that 1P30, along with 21 other direct propositions, are included in triangular structures and are not required for the proof of any other proposition in the *Ethics.* The triangular structures involving these redundant propositions are listed in Table A1 in Appendix A.

Spinoza’s quadrilateral structure adds an additional proposition to the triangular structure. Like the triangular structure at least two propositions are required to derive the target. Here the repeated proposition is both direct and two network steps removed from the target. The “Direct/Two-Steps” proposition is required to prove a proposition termed the “One Step” which is required to prove the Direct proposition. Like the triangular case the numbering rules are observed for these structures. If the Direct proposition is used solely to derive the target, it is redundant. The quadrilateral structure appears 152 times in the *Ethics*, and thirteen have a redundant direct proposition. (See Table A2 in Appendix A.)

An example of this structure is given in Figure 4. For brevity, I have omitted the text of the supporting propositions, but like the triangular case a computer program showed that 1P35, *Whatever we conceive to be in God’s power exists*, is used to prove 2P3 but is not used to prove any other proposition in the *Ethics.* Thus, 2P3 could have been proved by 1P34 and 1P16 used once, and 1P35 is redundant in that sense.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Direct/  Two  Steps |  | Target |  | Direct |  | One  Step |  | Direct/  Two  Steps |
| 1P16 | **→** | 2P3 | **←** | 1P35 | **←** | 1P34 | **←** | 1P16 |

**Figure 4- Example of Predecessors in a Quadrilateral Structure**

Of course, Spinoza did not have the luxury of a computer with a very convenient programming language; consequently, it would have been a tedious clerical task for him to determine that a Direct proposition was not used further in the *Ethics*. Since the *Ethics* only lists predecessors, Spinoza would have had to check manually the text of all succeeding propositions to determine if a proposition was used to prove another proposition. The Python class (Appendix B) developed for this project includes *both* the predecessors and successors of each network element so it was relatively easy to locate these redundant cases.

Spinoza may have chosen to include these redundant propositions for rhetorical or stylistic reasons. He may indeed have been aware of the redundancy but included these redundant Direct propositions because he thought the argument would be more effective if it explicitly emphasized some intermediate results. That said, there are 35 of these redundant propositions comprising more than ten percent of all propositions (35/331). Another way of understanding the redundancy is that the proposition used twice in either structure could have been used only once by collapsing the deduction of the target into a single proof. Thus the citing of propositions in proofs could have been reduced by more than ten percent. Karl Jaspers states “that the constant references to previous theorems makes continuous reading [of the *Ethics*] very difficult.” (32) Readability and Occam’s razor suggests that some of these propositions ought to have been omitted.

Clusters

The section on technical background argued that leaves are Spinoza’s ultimate philosophic conclusions because they are not used in any further argument in the *Ethics*. Some of the quadrilateral and triangular structures listed in Tables A1 and A2 extend this idea to a cluster of propositions. These clusters, which include one or more of the structures, are connected to the network by a single arc. The clusters may include additional leaves, but they must be connected by arcs *within* the cluster. A metaphor which will help the reader to visualize clusters in the *Ethics* network is a single twig connecting several smaller twigs and leaves on a tree.

|  |  |  |
| --- | --- | --- |
| **Table 4- Terminal Groups of Propositions** | | |
| **Target** | **Type** | **Additional Leaves** |
| 2P49.1 | Triangle |  |
| 5P20 | Triangle | 3P31.1 |
| 5P36 | Triangle | 5P36.1, 5P42 |
| 5P39 | Triangle |  |
| 5P36 | Quad. | 5P36.1, 5P42 |

Like the leaves, these clusters (Table 4) are also Spinoza’s ultimate conclusions, but these conclusions are expressed by a group of interrelated propositions rather than a single proposition. The cluster related to proposition 2P49.1, “the will and the intellect are one and the same”, defines Spinoza’s position on free will. This cluster occurs early in the foundational portion of the *Ethics* and gives Spinoza’s view of an extremely important philosophical questions, but the propositions defining this view are not used as predecessors in any other proposition in the *Ethics*.

The distribution of these clusters over the Parts of the *Ethics* follows that of the leaves: most are in the latter two sections of the text. This reinforces the argument that Spinoza stresses development of the ultimate principles of his ethics in the final two Parts. Finally, note that 5P36 appears in a cluster of both triangular and quadrilateral structures, and two leaves are added to the cluster. The two structures are central to Spinoza’s formulation of the relation of our minds to the intellectual love of God and God’s love for himself. It is understandable that this complex set of ideas requires the merging of two interacting structures into a single cluster rather than a set of independent leaves.

**5. Conclusion**

Virtually all of the essays written on Spinoza’s *Ethics* apply the traditional tools of philosophic analysis to this complex and difficult text. None of the results given in this essay would be possible unless the *Ethics* was treated as a mathematical object, digitally codified and analyzed with computer programs. The statistical tools described variations in the organization of the *Ethics* over its Parts and showed that a small percentage of the elements dominate Spinoza’s argument. Observation of displays of the *Ethics* digraph together with computer programs demonstrated that the *Ethics* contains repetitive structures that involve three or four digraph elements.

This essay certainly does not close the book on this type of analysis. There are many more statistical studies that can be performed on the data, and I am confident that there are data structures in the digraph that I have not yet observed. The data base needs to be upgraded to include references to Scholia used in Spinoza’s arguments, and most important the philosophic implications of these statistical findings should be investigated.

In conclusion, I hope I have demonstrated that it is fruitful to apply mathematical techniques to this venerable text. Furthermore, I hope the resources that I have provided on my web site will promote further study.

**Acknowledgment**

This paper has relied heavily upon *igraph*, an excellent suite of software which addresses the depiction and analysis of networks. The program is used extensively by academics in network science and related fields such as social network analysis. The program is complex and was sometimes difficult for me, a new user, to understand its workings. The people responsible for developing this software were patient and extremely helpful in answering my questions. I couldn’t have produced this paper without the responsive resolution of what I am sure were sometimes trivial, repetitive questions by Tamás Nepusz and his colleagues.

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**Appendix A**

**Triangular and Quadrilateral Network Structures with Redundant Direct Predecessors**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Table A1**  **Triangular Structures\*** | | | | | | | |
| **S Number\*\*** | **Direct/ One-Step Predecessor** |  | **Target** |  | **Redundant**  **Direct Predecessor** |  | **Direct/ One-Step**  **Predecessor** |
| 77 | 1P14.1 | **→** | 2P4 | **←** | 1P30 | **←** | 1P14.1 |
| 132 | 2P25 | **→** | 2P29.1 | **←** | 2P28 | **←** | 2P25 |
| 134 | 1P28 | **→** | 2P31 | **←** | 2P30 | **←** | 1P28 |
| 158 | 2P48 | **→** | 2P49.1 | **←** | 2P49 | **←** | 2P48 |
| 166 | 2P13 | **→** | 3P3 | **←** | 2P15 | **←** | 2P13 |
| 195 | 3P13 | **→** | 3P27.3 | **←** | 3P22 | **←** | 3P13 |
| 231 | 3P13 | **→** | 3P55.2 | **←** | 3P24 | **←** | 3P13 |
| 235 | 3P1 | **→** | 3P59 | **←** | 3P58 | **←** | 3P1 |
| 297 | 4P22.1 | **→** | 4P26 | **←** | 4P25 | **←** | 4P22.1 |
| 297 | 3P7 | **→** | 4P26 | **←** | 4P25 | **←** | 3P7 |
| 297 | 3P6 | **→** | 4P26 | **←** | 4P25 | **←** | 3P6 |
| 299 | 3P1 | **→** | 4P28 | **←** | 4P23 | **←** | 3P1 |
| 316 | 4P38 | **→** | 4P42 | **←** | 4P39 | **←** | 4P38 |
| 381 | 3P31 | **→** | 5P20 | **←** | 3P35 | **←** | 3P31 |
| 388 | 5P24 | **→** | 5P27 | **←** | 5P25 | **←** | 5P24 |
| 388 | 4P28 | **→** | 5P27 | **←** | 5P25 | **←** | 4P28 |
| 399 | 5P32.1 | **→** | 5P36 | **←** | 5P35 | **←** | 5P32.1 |
| 402 | 5P29 | **→** | 5P38 | **←** | 5P37 | **←** | 5P29 |
| 403 | 5P15 | **→** | 5P39 | **←** | 5P16 | **←** | 5P15 |

**\*** The arrows indicate logical dependency.

**\*\*** The S (Spinoza) number is the sequence of the appearance of the target in Elwes’ translation of the *Ethics.*

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
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|  |  |  | |  |  | | |  |  | |  | |  | |  | |
| **Table A2**  **Quadrilateral Structures\*** | | | | | | | | | | | | | | | |
| **S**  **Number\*** | **Direct/**  **Two**  **Steps** |  | **Target** | | |  | **Redundant**  **Direct**  **Predecessor** | |  | **One**  **Step** | |  | | **Direct/**  **Two**  **Steps** | |
| 76 | 1P16 | **→** | 2P3 | | | **←** | 1P35 | | **←** | 1P34 | | **←** | | 1P16 | |
| 166 | 2P13 | **→** | 3P3 | | | **←** | 2P29.1 | | **←** | 2P29 | | **←** | | 2P13 | |
| 166 | 2P11 | **→** | 3P3 | | | **←** | 2P15 | | **←** | 2P13 | | **←** | | 2P11 | |
| 231 | 3P11 | **→** | 3P55.2 | | | **←** | 3P24 | | **←** | 3P21 | | **←** | | 3P11 | |
| 235 | 3P11 | **→** | 3P59 | | | **←** | 3P58 | | **←** | 3P53 | | **←** | | 3P11 | |
| 297 | 3P6 | **→** | 4P26 | | | **←** | 4P25 | | **←** | 3P7 | | **←** | | 3P6 | |
| 307 | 3P3 | **→** | 4P35 | | | **←** | 4P33 | | **←** | 3P56 | | **←** | | 3P3 | |
| 365 | 3P48 | **→** | 5P6 | | | **←** | 5P5 | | **←** | 3P49 | | **←** | | 3P48 | |
| 388 | 4P28 | **→** | 5P27 | | | **←** | 5P25 | | **←** | 4P36 | | **←** | | 4P28 | |
| 399 | 5P32 | **→** | 5P36 | | | **←** | 5P35 | | **←** | 5P32.1 | | **←** | | 5P32 | |
| 402 | 5P23 | **→** | 5P38 | | | **←** | 5P37 | | **←** | 5P29 | | **←** | | 5P23 | |
| 403 | 5P14 | **→** | 5P39 | | | **←** | 5P16 | | **←** | 5P15 | | **←** | | 5P14 | |
| 407 | 3P3 | **→** | 5P42 | | | **←** | 5P3.1 | | **←** | 5P3 | | **←** | | 3P3 | |

**\*** The arrows indicate logical dependency.

**\*\*** The S (Spinoza) number is the sequence of the appearance of the target in Elwes’ translation of the *Ethics.*

**Appendix B**

**General Description of the Computer Programs**

About two years ago I became interested in exploring the logic of Spinoza’s *Ethics* as a mathematical object, a directed network. I was aware that the statistical study of social networks had become an important research area of the social sciences over the past decade and that there were interesting developments in both the statistics of networks and the software for the depiction and analysis of these networks. The first step was to represent the network in a form that could be accessed by a programming language. Using the Elwes translation, available at Gutenberg.org, I created five files, one for each Part of the *Ethics.* Each record in the filescomprises: (1) Spinoza’s network element designation (e.g. 1D4, 2P6, etc.), (2) the text of the network element and (3) the predecessors specified by Spinoza. With a few exceptions the records are exactly as represented in the Elwes translation. The exceptions are:

(1) Arabic instead of Roman numerals,

(2) Designators for the definition and axioms after the lemmas in Part 2,

(3) Addition of propositions as predecessors to their corollaries when Spinoza did not do so,

(4) Addition of designators F for affects and T for postulates,

(5) Inclusion of the general definition as an affect as a network element (3F0), and

(6) Addition of predecessors in cases in which Spinoza says something like “Proved in the same way as Proposition XPYY.”

This was a major clerical effort. It was double and triple checked but still may have errors. I believe it is substantially correct.

The next step was to write a series of programs that develop a Python class that creates an ordered Python dictionary of the network entities when included in a Python program. The dictionary key is the element designation. The list accompanying the key contains: (1) the Spinoza number, (2) the key as a data element, (3) a list of the direct predecessors as they appear in the *Ethics*, (4) a list of the direct successors of the element and (5) the text of the element as it appears in the Elwes translation. The Spinoza number is the order of appearance of the element in the *Ethics.* (For example 1D1 and 1A1 have Spinoza numbers 1 and 9). The Spinoza number enables presentation of the network elements in the same sequence as in the *Ethics*. The successors, which as a set are logically equivalent to the predecessors, were developed by a program which inverted the predecessors listed in the Elwes translation.

I wrote programs that used this class and Python classes with a system called *igraph* to produce graphical representations of the first three parts of the *Ethics*. *igraph* is widely used software to create and manage networks in academic research in social science, network analysis and related fields.

I found that large number elements and interconnections in the *Ethics* made these graphs too complex to follow visually. (I have included these graphs on my website for those with more visual capacity than I.) Consequently, I created a system, Spinoza\_Explosion, that explodes any element in the network either forward, to graphically represent all elements directly and indirectly dependent logically on the element, or backward to represent all direct and indirect elements required to derive the element. This system is available on my website as an msi file and can be downloaded to any Windows computer. These smaller, more readable graphs revealed the structures present in the *Ethics* and Spinoza’s anachronistic use of hub and spoke network structure.

I also used *igraph* to load the statistics for each element in the *Ethics* (in-degree, out-degree, degree, betweenness, and closeness) onto an *Excel* worksheet, which can also be downloaded from my website. The statistical studies were created from this workspace. Finally, I have created two text files listing all of the triangular and quadrilateral structures, also available on my website.

I consider all of these programs to be open source. I will send any program of interest to any requester. The only thing I will restrict is write access to is the initial files that I created from the Elwes translation. This will enable me to ensure that all corrections that are discovered be applied to a single set of files in order to create ultimately an error free Python class for the Spinoza network.

My website is: sites.google.com/site/hroseman.

To access the files, navigate to the page marked Spinoza Project. The following files are available for download:

1. Part1.pdf-- Graphical representation of the *Ethics*, Part 1
2. Part2.pdf-- Graphical representation of the *Ethics*, Part 2
3. Part3.pdf-- Graphical representation of the *Ethics*, Part 3
4. Spin\_Explode-0.1-win32.msi—installer package file for the Spinoza Explosion
5. Spinoza\_Stats.xlsx—Excel spreadsheet with statistics for each element
6. ofinal\_props.py—Python class that creates the ordered dictionary for the *Ethics*

To store the dictionary in props\_dict: (1) import final\_props (2) props\_dict = final\_props.p

1. Quads.txt--List of quadrilateral structures
2. Triangles.txt—List of triangular structures

1. K.A.C. Manderville*, The Undoing of Lamia Gurdleneck*. I have substituted “numbers” for “figures” in the original. [↑](#footnote-ref-1)
2. See the websites of R, Bombardi, J. Gautier and T. Doppett listed in the references. [↑](#footnote-ref-2)
3. An excellent, readable introduction to networks can be found in Trudeau’s *Introduction to Graph Theory*. [↑](#footnote-ref-3)
4. Although not depicted in Figure 1, in the *Ethics* the vertices also model lemmas, affects and postulates. [↑](#footnote-ref-4)
5. The network statistics defined in this section conform to what is generally found in the literature. Since a digraph is a type of network the definitions apply to digraphs. There are many network statistics used in the social sciences that do not apply to the *Ethics*. A readable introduction to social network statistics can be found in Hanneman and Riddle’s on-line text *Introduction to Social Network Methods.* [↑](#footnote-ref-5)
6. The Spearman rank correlation coefficient is a measure of the monotonic relationship between two ranked variables. A value of .20-.29 indicates a “weak” relationship. This statistic is described in almost any text covering non-parametric statistics such as J.S. Maritz, *Distribution-Free Statistical Methods*. [↑](#footnote-ref-6)
7. A full list of the triangular and quadrilateral reasoning structures can be found in two text files on my website. See Appendix B for details. [↑](#footnote-ref-7)