Discussion Note: Positive Relevance Defended*

Sherrilyn Roush†‡

This paper addresses two examples due to Peter Achinstein purporting to show that the positive relevance view of evidence is too strong, that is, that evidence need not raise the probability of what it is evidence for. The first example can work only if it makes a false assumption. The second example fails because what Achinstein claims is evidence is redundant with information we already have. Without these examples Achinstein is left without motivation for his account of evidence, which uses the concept of explanation in addition to that of probability.

The positive relevance view of evidence, in which e is evidence for h if and only if p(h/e) is greater than p(h), that is, if and only if e raises the probability of h above what it was when e was not taken into account, is held in high esteem by Bayesians and others who view probability as the sole concept needed to analyze the concept of evidence. One regularly hears that positive relevance is not sufficient for e to be evidence for h, since e may raise the probability of h without raising it high enough to make h as much as plausible, in which case one may not want to say that one has evidence for h. However, Peter Achinstein has objected, on the basis of putative counterexamples, that positive relevance is not even *necessary* for evidence, as a plank of his argument that probability alone cannot capture the concept of evidence (Achinstein 1983, 2001). I will argue that these examples are ineffectual for making his point, the first because in it he makes a false assumption, the second because what he

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†To contact the author write to Rice University, Department of Philosophy, MS-14, 6100 Main Street, Houston, Texas, 77005; e-mail: roush@rice.edu.

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wants us to count as evidence is redundant with evidence we are already taking into account. Without examples showing that positive relevance is not necessary for evidence, there is no reason to change to Achinstein's explanation-based view of evidence, because, as is familiar, the purported insufficiency of positive relevance can be addressed by adding a condition requiring high posterior probability for the hypothesis.

Achinstein's first example involves a lottery:

e_1 = The *New York Times* (NYT) reports that Bill Clinton owns all but one of the 1000 lottery tickets that exist in the lottery.
e_2 = The *Washington Post* (WP) reports that Bill Clinton owns all but one of the 1000 lottery tickets that exist in the lottery.
b = This is a fair lottery in which one ticket drawn at random will win.
h = Bill Clinton will win the lottery.

e_1 and e_2 are both pieces of evidence, b is background knowledge, and h is the hypothesis. Achinstein submits that given e_1 and b, e_2 is strong evidence for h yet, he claims:

$$p(h/e_1.e_2.b) = p(h/e_1.b) = 999/1000. \text{ (Achinstein 2001, 70)}$$

In other words, he claims that once e_1 is incorporated as evidence, e_2 does not change the probability of the hypothesis, and therefore, on the positive relevance view, e_2 is not evidence for h in the circumstance described. Ergo, the positive relevance condition is too strong.

A little reflection shows that there is something amiss in the assignment of probabilities in this case. The probability that Clinton will win is .999 only if the probability that he owns 999 out of 1,000 tickets is 1. How are we supposed to get that? The only way to get from the claim that it was reported that Clinton owns 999 tickets to the claim that the probability he will win is .999 is to assume that a report in the NYT that c (where c = Clinton owns . . .) makes the probability of c equal to 1, that is, that the NYT is a perfect transmitter. This assumption makes the probability of a Clinton win as high as it could get (.999) on the basis of any report that he owns 999 out of 1000 tickets, and thereby prevents any other such report from raising the probability. The NYT is a respectable newspaper, but this assumption is inappropriate, not just because it is false—the existence of a 'corrections' section is sufficient to show this—but also because it automatically removes from consideration what goes on with probabilities when you have two imperfect sources of information, which is where all of the interest of this example lies.

It is obvious that if there exists evidence that we have and that makes the probability of a hypothesis equal to 1, then on the positive relevance view nothing else will count as evidence, because nothing can change a probability of 1. The question is whether there are any examples of that sort where the verdict strikes us as wrong. It is very hard to come up with examples where evidence makes the probability of a hypothesis equal to 1 where that probability was not already 1, that is, by raising it from a lower value, unless the hypothesis itself is taken as evidence. It is not enough simply to *assume* that a given case is one where evidence has made the probability of something, here c, equal to 1. That would have to be argued for, since it is the crucial, and difficult, premise of the argument against positive relevance. It is obvious that c in this example does not start out with probability 1, but it is also obvious that having the NYT report that c does not make $p(c) = 1$ either. What Achinstein would need to prove is implausible in this case, and that is enough to ruin this counterexample.

However, there is more to say. Even though the NYT report does not make the probability of c equal to 1, we may assume that it makes that probability high, and therefore the posterior probability of the hypothesis high. If one thinks being evidence is a matter of positive relevance, one may also think that the degree of relevance is proportional to the strength of the evidence, although this is not implied. That is, one may think that the strength of the evidence is measured by the degree to which it positively changes the probability of the hypothesis. If so, then the example looks strange, because it looks as if how good the NYT or WP report is as evidence on the positive relevance measure, depends on whether it was discovered first, since the one discovered second has little room to change the probability of h once the other evidence has been registered. If the two reports are not independent, if, say, they both got their information from Reuters, then that seems far less strange. But the two reports could have been independent, and then there seems to be a problem. Nevertheless, this is not a reason to think positive relevance is not necessary for evidence. It is only a reason to think that degree of relevance—measured as the difference between $p(h/e)$ and $p(h)$—does not measure the degree to which one thing is evidence for another.

It is instructive to compare the likelihood ratio method as applied to this example, since this method makes it harder to slip into presuming that a report of c makes $p(c) = 1$. On this view, to determine whether e is evidence for h we compare $p(e/h)$ to $p(e/\text{-}h)$ and if the first is greater than the second, then e fulfills what a likelihood conception takes to be necessary for one thing to be evidence for another. (Fulfilling the likelihood condition implies fulfilling positive relevance.) In our case, to decide whether e_2 is evidence for h when e_1 is already in the stock of evidence, we compare $p(e_2/h.e_1)$ to $p(e_2/\text{-}h.e_1)$; $p(e_2/h.e_1)$ is clearly greater than $p(e_2/\text{-}h.e_1)$, since in the second case the given fact that Clinton does not win the lottery casts doubt on the veracity of the NYT report that he owns 999 out of 1000 of the tickets. If that report was false then, unless we can assume that the WP always copies the NYT, there is less reason to believe

e_2 than there is if Clinton does win the lottery and the NYT report is the same. This means that the likelihood ratio is greater than 1 if we can assume that there is some chance the WP report is independent of the NYT report, which is anyway the only case where not counting e_2 as evidence is counterintuitive.

Achinstein's second example involves an intervening cause:

e_1 = On Monday at 10 a.m. David, who has symptoms S, takes medicine M to relieve S.
e_2 = On Monday at 10:15 a.m. David takes medicine M′ to relieve S.
b = Medicine M is 95% effective in relieving S within 2 hours; medicine M′ is 90% effective in relieving S within 2 hours, but has fewer side effects. When taken within twenty minutes of having taken M medicine M′ completely blocks the causal efficacy of M without affecting its own.
h = David's symptoms S are relieved by noon on Monday.

In familiar form, Achinstein claims that given e_1 and b, information e_2 is strong evidence for h, because medicine M′ is 90% effective in relieving symptoms S.[1] Yet the positive relevance account of evidence does not render this verdict, for:

$p(h/e_1.b) = .95$
$p(h/e_2.e_1.b) = .90$. (Achinstein 2001, 70–71)

e_2 not only does not increase h's probability over what it was when e_2 was not taken into account, it decreases that probability.

In this case it is obvious that the probabilities are right. It is much less obvious that they yield counterintuitive judgments about evidence. When the example is first introduced Achinstein says we should believe that e_2 is strong evidence for h when e_1 and b are given "since medicine M′ is 90% effective in relieving symptoms S" (Achinstein 2001, 71). However, the fact cited would justify the claim that e_2 is strong evidence for h given e_1 only if supported by one of two assumptions that are false, and that Achinstein has disavowed.

The first is that a sufficient condition for e to be evidence for h is that p(h/e.b) is high. Achinstein rightly rejects this high probability condition as sufficient for evidence because it does not require that e be relevant to h, or, intuitively, that e 'made' the probability of h high (Achinstein 2001, 71). It would count the fact that a man consumed birth control pills as evidence that he will not get pregnant. The problem is that the probability that he would not get pregnant was already high, so his consumption of

1. It is not stated but must be assumed in the example that the symptoms S are such that if David did not take medicine then he would not recover.

birth control pills has no work to do supporting it. The same is true of e_2 in the case of the medicines. The probability that David would recover was already high when the second medicine came along. e_2's role in making the probability that he would recover high is redundant with work that would have been done by e_1.[2] (I will have more to say in support of this claim below.) By Achinstein's own lights the mere fact that the probability of h is high after e_2 is taken into account does not make e_2 evidence for h when e_1 is given.

The other assumption that would make "medicine M′ is 90% effective in relieving symptoms S" a justification for the claim that e_2 is evidence for h once e_1 is taken into account is, as the quoted clause strongly suggests, that e_1 has not been taken into account! It is clear that if David had never taken medicine M, but had taken medicine M′, then the fact that he had taken M′ would be strong evidence that he would recover, because that medicine is 90% effective. The probabilities also conform to that judgment, since $p(h/e_2.b) > p(h/b)$. However, these are not the probabilities we have to do with in Achinstein's claim that e_2 is evidence for h once e_1 has been taken into account, and do not obviously bear a helpful relation to $p(h/e_1.b)$ and $p(h/e_2.e_1.b)$.

We get a different justification when Achinstein later considers the same example (with minor changes):

> Isn't the fact that I am taking [M]′ after having taken [M] evidence that my pain will be relieved, even though the probability that it will be relieved has decreased from 9[5]% to 90%? (Achinstein 2001, 84)

The phrase "the fact that I am taking M′ after having taken M" is notably ambiguous. The reading that invites itself is "the fact that that I have taken M′ and taken M′. It is obvious that this conjunction *is* evidence for h on intuitive grounds, but it is also clear that the corresponding positive relevance condition is fulfilled: $p(h/e_1.e_2.b) > p(h/b)$. That this conjunction is evidence is not the claim Achinstein needs to defend for his conclusion, but it is the claim the words suggest when he asks for our intuitions.

Achinstein's phrase could, and should, mean the fact that I am taking M′ *given* that I have taken M. This would mean that the question is whether given the background and the fact that I have taken M, my having taken M′ seems like further evidence, new information, that I will recover. Consider a concerned friend who knows that I have taken M. Would she be convinced if we told her we had new evidence that I was going to recover,

2. This example is analogous to another Achinstein example dealt with by Patrick Maher (1996, 172), who drew the same conclusion I have as to whether evidence is present. My reply has the advantage of not relying on Maher's particular view of confirmation, some of whose assumptions Achinstein defended himself by attacking (Achinstein 1996).

namely that I had taken M′? She would undoubtedly be less confident that I would recover than she was before, if only by a little, and furthermore annoyed at the misleading advertisement. It is clear that when we have e_1, and we acquire e_2, we do *have* evidence that I will recover. It is not at all clear that given e_1, e_2 is evidence that I will recover.

Any lingering confusion in our intuitions about this case comes, I think, from the fact that if I do recover then it will have been due to the causal efficacy of M′ alone, the medicine the taking of which is reported by e_2. In this the example differs from previous Achinstein examples of similar form, such as one in which there is a lottery and the first piece of evidence that Freddy will win is that he owns 999 of 1,000 tickets, and the second piece of evidence tallied says that by the next day 1,001 tickets have been sold of which Freddy still owns 999 (Achinstein 1983, 152). The probability of Freddy winning is still very high after the second report is in, but it has dropped from what it was with the first report, so e_2 does not count as evidence in this context on the probabilistic relevance view. Nor should it, on the basis of what I have argued above, since e_2 is redundant with part of e_1.

One might think that although e_2 has no right to count as evidence in this case, that is because there is no sense in which someone else's buying another ticket *causes* Freddy to win if he wins. The case of the medicines is different since if I recover then M′, and not M, will be the cause of that. The latter claim is true but of no avail, I think, since other things are true as well. For example, I would have had an at least 90% probability of recovery even if I had not taken M′. This is related to the peculiar fact that the only reason that M is not the actor in bringing about my recovery is a secondary action of M′. If I had not taken M′, then I also would not *need* M′, for recovery. M′ did not do anything relevant to whether I recover that M would not have done, supporting the claim that e_2, the report that I have taken M′, is in the most important sense redundant with part of e_1. When we learn e_2, we learn something about the mechanism of recovery, but we learn nothing new about *whether* I will recover—the point at issue in the hypothesis—except the negative news that my chances have gone down, which does not change the fact that my chances are very good.

Each of M and M′ raises the probability of recovery when acting alone, and when acting together they raise the probability over what it was with neither. This conforms to the fact that M and M′ are jointly and each individually evidence for my recovery. However, once I know that M has been taken, learning that M′ has been taken does not increase my confidence in recovery, and is not evidence for that recovery, since the information it gives about whether I will recover is redundant with information we already had.

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