ABSOLUTE GENERALITY AS A HIGHER-ORDER IDENTITY

ETHAN RUSSO

ABSTRACT. The question of Absolute Generality is whether quantifiers are ever as general as can be. Absolutists claim that quantifiers sometimes are absolutely general, while Relativists claim that quantifiers are never absolutely general. Although diverse philosophers have found the Relativist ethos compelling, it has been hard to articulate a consistent thesis which says what the Relativist seems to want to say. In this paper, I offer Relativists a way forward: I argue that what is needed to successfully state Relativism is a way of generalizing that is non-quantificational. After showing how to define such a device of generalization in terms of identity between properties in a higher-order logical language, I use the device to articulate a form of Relativism which I prove to be consistent and which I argue captures the intuitive vision of the Relativist.

When theorizing about the world, we sometimes try to make claims about it that are as general as can be. When the metaphysician says *everything* is self-identical, it is not as though the reach of their claim is supposed to stop just short of Australia and fall silent on whether the kangaroos are self-identical, too. The question of Absolute Generality is whether we in fact succeed in this ambition— whether, as Studd (2019) puts it, we ever "use ... quantifiers to make claims that are as general as can be" (p. 1).[1] But what exactly is this question asking? All parties can agree that there are two answers, *Relativism* and *Absolutism*:

Relativism: Quantifiers always fail to be absolutely general

[1]The history of the debate is interesting, if somewhat tortuous. The idea that there might be something defective about absolutely general quantification is perhaps implicit already in Russell's ramified theory of types, but it surfaces explicitly for the first time perhaps in Dummett (1978). (The theme is taken up in greater detail in Dummett (1981) and Dummett (1991). What exactly Dummett takes the deficiency to be is a subtle matter; see pp. 529ff. in his Dummett (1981), and Cartwright (1994) and Linnebo (2018) for some discussion.) The idea emerges independently of Dummett (so far as I can tell) in Parsons (1974), also in the context of set theory; Glanzberg (2004) (and the sequels Glanzberg (2006) and Glanzberg (2023)) are further developments of the Parsonian line. Dummett and Parsons seem to advance Relativist positions. Important early skeptical responses to their arguments for Relativism are, respectively, Cartwright (1994) and Boolos (1998). (Cartwright's response is further developed in Rayo and Williamson (2004), McGee (2004), Linnebo (2006), Rayo (2006) and Linnebo and Florio (2021).) The challenges of formulating a coherent version of Relativism are briefly raised and discussed in Lewis (1991) and McGee (2000), but the first, I believe, extended treatment of the issue is due to Williamson (2003), and it is further discussed in Fine (2006), Lavine (2006), Button (2010), and Warren (2017). Recent surveys of the topic are the introduction to Rayo and Uzquiano (2006a) (a collection of 13 essays on the topic), the article Florio (2014) and the book Studd (2019), which features a particularly well developed attempt to articulate what is at stake between Relativism and Absolutism, and to defend Relativism.

Absolutism: Quantifiers are sometimes absolutely general[2]

These answers, though, are no more clear than the question. They are templates or slogans, awaiting specifications of what "quantifiers" are, what it is for quantifiers to "always" be a certain way, and what it is for quantifiers to be "absolutely general".

Often in philosophy, the positions in a debate are characterized by slogans rather than precise statements; part of the philosopher's work is to turn the former into the latter. With the debate about Absolute Generality, however, it has proved distressingly difficult to precisify the slogans in a way which both is faithful to the motivations for the views and does not render Relativism inconsistent or otherwise self-defeating. It would be extremely natural to try to render Absolutism and Relativism in terms of quantification over quantifiers:

Relativism: No quantifier is absolutely general

Absolutism: Some quantifier is absolutely general

But, as we shall see, precisely because the Relativist calls into question the generality of quantifiers, this way of articulating the Relativist position will fail to capture the intuitive vision of the Relativist or else be self-defeating. As a result, some philosophers have been skeptical that there is any coherent way of articulating Relativism—so that, no matter how we try to precisify our slogans, the Absolutist will come out on top—or even skeptical that there is any coherent question of absolute generality at all.[3]

My aim is to offer the debate a way forward: I will offer novel versions of Relativism (and its negation, Absolutism) stated with the resources of a higher-order logical language. The Relativist, I will argue, needs to recognize that there are ways of generalizing about quantifiers which are not themselves quantificational or defined in terms of quantification. I will show how we can, with the resources of higher-order identity, define such a way of generalizing, and so state a form of Relativism which avoids the usual pitfalls. In particular, the Relativist can avoid the problem of coherence: my statements of Relativism (and Absolutism) is provably consistent.

Other philosophers have suggested ways of precisifying what is at stake between the Absolutist and Relativist before. We shall see problems with other attempts in what is to come, but it is worth noting a difference between my version of Relativism and important proposals from Fine (2006) and Studd (2019), which appeal to special modal operators, purpose made for the Absolute Generality debate, to suitably precisify Relativism.

---

[2]Here "quantifiers are sometimes F" and "quantifiers always fail to be F" are intended to be dual constructions: the former is to be equivalent to "it's not the case that quantifiers always fail to be F" and the latter to "it's not the case that quantifiers sometimes are F".

[3]See, for instance, Lewis (1991), McGee (2000), Williamson (2003), and Button (2010).

Now, those already skeptical that there is a non-trivial debate may well be skeptical that these special-purpose modal operators are of any more repute (Studd (2019) himself is unsure he grasps the modality that Fine invokes (p. 147)). A distinctive virtue of my proposal is that it will spell out Absolutism and Relativism using only the logical vocabulary of higher-order languages. This ideology is not uncontroversial, but it is certainly not *parochial*: philosophers of diverse stripes have defended the intelligibility of the higher-order devices to which I appeal, and have applied them in many philosophical contexts orthogonal to the Absolute Generality debate.[4]

The plan is as follows. In Section 1, I will introduce the Absolute Generality debate; in Section 2 and 3, I will discuss difficulties in articulating Relativism; in Section 4, I will introduce my key idea, non-quantificational generality, for how Relativism is to be successfully articulated and explain in brief how it is to work. The remaining sections develop these ideas from Section 4 in a formal setting, that of higher-order logic.

1. The Idea of Relativism (and Absolutism)

Let me begin by sketching the sort of philosophical vision that I aim to capture with the slogan:

Relativism: Quantifiers always fail to be absolutely general

Philosophers have found diverse grounds for the rejection of absolutely general quantification, but one strand of thought important to many concerns what Studd (2013) calls a "trade off between 'generality' and 'collectability'" (p. 82).[5]

This trade off usually manifests in the realm of sets or similar entities. Classical logic shows that there cannot be a set of all things that do not contain themselves, given the inconsistency of:

(Russell) $$\exists x \forall y (y \in x \leftrightarrow y \notin y)$$

Sometimes when a set fails to exist, this is because one of its would-be members fails to exist: if Socrates fails to exist, then his singleton must also fail to exist. The failure of there to be a Russell set r of all the non-self-containing things—call them the ss—is not like that. The ss exist; the problem, rather, is that they somehow are not lassoed together into a single set. The inconsistency of (Russell) seems to reflect a failure in *collectability*.

The Absolutist accepts that what seems to be the case is the case: since $\exists$ is as general as can be, the fact that $\neg \exists x \forall y (y \prec ss \to y \in x)$ means there is no sense in which a set of the ss has any claim to existing. For the Relativist, by contrast, the

[4]See the contributions in Jones and Fritz (2024).

[5]Other arguments have to do with the alleged links between quantification and sortals, and Carnapian worries about ontology. I am not convinced there is in fact one notion of absolute generality at issue in all these argument.

failure is merely apparent. Although the Relativist, of course, recognizes (Russell) is inconsistent and so false, they will instead say that:

(Russell$^+$) $\exists' x \forall y (y \in x \leftrightarrow y \notin y)$

Here, the initial quantifier, $\exists'$ and its dual $\forall'$ are *distinct* from our original $\forall$ and $\exists$. Putting things roughly, the Relativist recognizes that r does exist according to $\exists$ ($\neg\exists x(x = r)$), but asserts that it does exist according to some other quantifier $\exists'$. The Relativist is keen to ensure that the s's can be collected, and since $\exists' x(x = r)$, collectability is achieved.[6]

The achievement, though, comes at the price of generality lost. An absolutely general universal quantifier, one would think, would be one such that it would surpass in generality whatever other universal quantifier one might compare it to. But $\forall$ plainly doesn't meet this standard: *all even numbers* is not as general as *all numbers*, because not every number is an even number. Likewise, $\forall$ is not as general as $\forall'$, since r exists according to the latter but not the former. So $\forall$ is not absolutely general, according to the Relativist. And, of course, not just $\forall$—the Relativist is just as keen to say that there should be, in some sense, a set of all (according to $\forall'$) the sets which do not contain themselves; but this new Russell set for $\forall'$ itself can only exist according to yet another quantifier $\forall''$. And now we are off to the races: there will be an ever-extending hierarchy of quantifiers, each more general than the last, never reaching a maximum.

2. Filling in the Slogans

This is the sort of philosophical vision that I aim to capture with the slogan:

Relativism: Quantifiers always fail to be absolutely general

The opposing vision, according to which the hierarchy hits some upper limit, is:

Absolutism: Quantifiers are sometimes absolutely general

These slogans involve three moving parts: they involve a notion of (i) of being a "quantifier"; (ii) of what it is for quantifiers to always be a certain way; and (iii) what it is for a quantifier to be absolutely general. This third notion might seem itself to subdivide into two parts: it is natural to think that for a quantifier to be "absolutely general" is for it to be completely general, that is, maximally general by some ordering of generality—the question then is what (iiia) this notion of maximality is and (iiib) what the relevant ordering of generality might be.

[6]Why is the Relativist so keen not to give up on collectability? The point is often put in terms of explanation: consider all the sets which do not contain themselves—not under that description as it were, but just *those sets*, the plurality of them. What stops there from being a set of them? What is the difference between these sets, and, say, Earth, Mars, and Venus, a trio of which there is surely a set? How precisely this explanatory challenge is supposed to work is an important question. A classic discussion is Dummett (1991) (pp.315–316), other discussions include: Studd (2013) (pp. 698–701), Fine (2006) (pp. 23–25), Linnebo (2010), Yablo (2004) (pp. 148–150), and Soysal (2020).

As for (i): by "quantifiers", I do not mean quantificational expressions but rather the sort of thing those expressions mean.[7] More or less following Frege, and modern linguistics in the tradition of generalized quantifier theory, let us think of the meanings of quantifier-expressions as being certain properties of properties: just as "Bessie is kind" says that the property of being kind applies to Bessie, so "every cow is kind" says that the property *every cow*—that is, the property of applying to all cows—applies to the property of being kind.[8]

In addition to the usual existential and universal quantifiers, there are quantifiers of other kinds, like "most moose" or "all but finitely many mice". For any given kind of quantifier, there is version of the debate about that kind (Are quantifiers of kind K ever absolutely general?). However, since the set-theoretic considerations above seem to bear most upon on the absolute generality of universal and existential quantifiers, it is on these kinds that I will focus, and of these two mostly on the universal. For existential and universal quantifiers occur naturally in pairs, each member of the pair being definable in terms of the other and negation (given classical dualities), and it is hard to see how one member of such a pair could be maximally general without its twin being so too. Consequently, Relativism for the two kinds stand and fall together, and so we may without loss of generality focus only on one. [9]

Later in the paper, I will say more about universal quantifiers, and try to define what it is to be one in logical terms. For now, we should turn our attention to the parts of the slogans which are most problematic: "always" and "absolutely general".

[7]It is customary in the Absolute Generality literature to focus not on quantifiers (in the sense I understand them), but rather on domains of quantification. Thus Rayo and Uzquiano (2006b) frames the question of Absolute Generality as one about whether there is "an all-inclusive domain of discourse" (p. 2). On the face of it, however, the debate about Absolute Generality would seem to concern quantifiers, not domains. When trying to describe the Relativist's position, we say things like "The Relativist says that $\forall$ is not absolutely general because $\forall'$ is not a restriction of it". This is a statement about quantifiers; perhaps we could paraphrase the point in terms of domains, but why should we? More importantly, though, I think that all reasonable ways of cashing out Domain Relativism and Domain Absolutism are going to be close variants of the quantifier-first theses that I will state, as I argue elsewhere in my dissertation.

[8]I use "property" where Frege would have used (the German equivalent of) "function" or "concept"; my properties of properties are his second-level concepts. (See, for instance, Frege (1997b), Frege (1997a), and the discussion in Klement (2024).) Linguists and logicians, by contrast, sometimes say quantifiers are properties of sets (as, for instance, Peters and Westerståhl (2006), p. 12) or sets of properties (as, for instance, Szabolcsi (2010), p. 7) or just sets of sets (as in Barwise and Cooper (1981)). In these cases, the invocation of sets is a simplifying assumption, one which allows us to ignore intensional aspects of language. Linguists also often distinguish between so-called "global" and "local" quantifiers; I mean here to be speaking of the latter.

[9]It will be helpful sometimes to say talk about existential quantifiers being absolutely general or not, but this is to be understood as shorthand for claims about the universal counterparts being absolutely general.

The standard canons for translating these terms into more precise language would dictate that we render them by quantification over quantifiers. Just as "Even numbers are always divisible by two" usually means that *all* even numbers are divisible by two, so Relativism would be the view that *every* quantifier is not absolutely general.[10] I have already suggested, being "absolutely general" is a matter of being maximally general along some ordering of generality; but maximality, also, is usually cashed out with quantification: to be maximally tall is to be at least as tall as anything else. For a universal quantifier Q to be absolutely general, then, is just for it to be at least as general as every quantifier.

And so the standard canons would tell us that this is how Relativism and Absolutism ought to be understood:

Q-Relativism: No quantifier is at least as general as every quantifier

Q-Absolutism: Some quantifier is at least as general as every quantifier

Is Q-Relativism a good way for a Relativist to regiment their vision? Even without pinning down precisely what ordering "at least as general as" is supposed to express, one may already feel some creeping queasiness about stating Relativism in these terms. Doesn't the Relativist claim that quantifiers are never maximally general? Presumably, then, the Relativist will claim that *no quantifier* doesn't generalize over quantifiers in a maximal way, but rather is just another quantifier which is surpassed in generality by another quantifier further up in the hierarchy.

Accordingly, Q-Relativism will seem a little *parochial*: so what if no quantifier is at least as general as every quantifier, for some particular quantifiers *no quantifier* and *every quantifier*? This is compatible with there being some more expansive quantifier over quantifiers Q such that "there is some quantifier which is absolutely general" is true, when "some quantifier" expresses this more expansive quantifier Q. By the Relativist's own lights, Q-Relativism, the sentence "No quantifier is at least as a general as every quantifier", fails to capture their own vision of an unending quantifier hierarchy.

As it stands, this argument against Q-Relativism's faithfulness to the Relativist vision fails. The Relativist may deny that universal quantifiers ever absolutely general *simpliciter*, but that does not mean they must deny that quantifiers are ever as general as can be *over a certain range*: just because the hierarchy of quantifiers over sets, and *a fortiori* things in general, is unbounded, does not mean that the quantifiers over just donkeys must be as well. Whatever we think about sets, no one should deny we can quantify maximally generally over a group consisting of Tom, Dick, and Harry, for instance. Perhaps, then, it is possible to quantify maximally generally over quantifiers, even if it is not possible to quantify maximally generally *simpliciter.*

[10]See Lewis (1975) for a view on which adverbs like "always" are in general quantificational.

Here is an argument that the Q-Relativist must deny even this. Consider the following property of properties Π:

being a property to which every universal quantifier applies

It is plausible that Π itself is a quantifier. One way to informally argue this point is to conceive of Π as the "conjunction" of all universal quantifiers, and then observe that conjunctions of universal quantifiers seem themselves to be universal quantifiers: if *every cow* and *every cat* are universal quantifiers, then so is *every cow and every cat*. It is even more plausible that it is at least as general as every quantifier (on any reasonable understanding of the generality ordering): just as *every cow and every cat* is at least as general as *every cow*, so Π ought to be at least as general as any quantifier.

But now Q-Relativism seems to simply be false. If *some quantifier* and *no quantifier* in Q-Relativism and Q-Absolutism really are maximally general, then it would seem that they should range over Π. But then from:

- Π is a quantifier and is at least as general as every quantifier

Which I have just argued for, one can infer by Existential Generalization:

- Some quantifier is at least as general as every quantifier

Which is Q-Absolutism, Q-Relativism's negation. In Section 7, when we have introduced a suitable formal language, we will formalize and strengthen this argument, including giving more justification for the claim that Π itself is a quantifier.[11]

We can see Π as posing the Q-Relativist with a sort of dilemma: either *some quantifier* and *no quantifier* do not range over Π, and so fail to be absolutely general (this is the problem of Parochiality); or they do (that is, it is true that some quantifier is identical to Π), and Q-Relativism is false.

Not everything, at least, is doom and gloom for the Relativist—they at least can make some good sense of what it is for one quantifier to be at least as general as another. Say that $\forall$ is at least as general as $\forall'$ just when

$$\forall' x \exists y (y = x)$$

Where $\exists$ is the dual of $\forall$. It will sometimes be helpful to have a locution for the converse relation: when $\forall$ is at least as general as $\forall'$, we can also say that $\forall'$ is a *restriction* of $\forall$, and we can abbreviate this as $\forall' \subseteq \forall$. This notion of restriction—which only requires the two quantifiers we are comparing themselves—seems to do the job we want. *Every cow* is a restriction of *every ungulate*, since every cow is identical to some ungulate, for instance. Likewise, every set is identical to some^{+} set, where *some*$^{+}$ ranges over the sets that *every set* does and also the Russell set for *every set*.

[11] Parallel arguments are to be found in Fine (2006) and Linnebo and Florio (2021).

3. Beyond Quantification

Suppose, then, we want an articulation of the Absolute Generality debate which is acceptable to Relativist and Absolutist alike. We will need to augment the quantificational locutions of Q-Relativism and Q-Absolutism or leave them behind altogether and generalize about quantifiers in some other way.

Some philosophers, such as Fine (2006) and Studd (2019), have thought the right way to articulate Absolutism and Relativism is by appealing to special modal operators. We need not run away from quantification, but we must augment it in order to get at what is issue. These philosophers take seriously the "ability" in "collectability": the lesson of Russell's paradox is that, given $\forall$, we *can* produce a new quantifier $\forall'$ which is not a restriction of $\forall$, and so they introduce a new modal operator $\Diamond$ (and its dual $\Box$) to latch on this sense of potentiality. The modal relativist's view is that, *necessarily*, any quantifier is such that, *possibly*, there is another which is not a restriction of the first:

M-Relativism: *Necessarily*, for any quantifier X, there is *possibly* a quantifier Y such that Y is not a restriction of X.

In addition to the problem of the intelligibility of these operators, M-Relativism falls victim to the same problem of Π which doomed Q-Relativism. Recall that Π is the property of being a property to which all (universal) quantifiers apply. It follows, then, by only very basic quantificational reasoning that every quantifier is a restriction of Π. So long, then, as even these very basic truths are logic are necessary according to this special modality (and neither Fine nor Studd give indication that this would not be the case), it will follow that:

Necessarily, every quantifier is a restriction of Π

If Π is in the range of "some quantifier", then we may conclude by Existential Generalization that:

Some quantifier X is such that necessarily every quantifier Y is a restriction of X

Which contradictions M-Relativism. If Π is not in the range of "some quantifier", the problem of parochiality rearises.

Others philosophers, adopting a strategy that harkens back to Russell, have thought we should leave quantification behind and turn instead to *schemas*. Schemas are a metalinguistic means of characterizing a usually infinite set of sentences in the object-language. To take an example from set theory:

Separation: $\exists y \forall x (x \in y \leftrightarrow \phi(y))$ for any formula $\ulcorner \phi(y) \urcorner$ of the language

In committing to the schema, we commit to every instance, and thereby attain a certain kind of generality—without, at least in the object language, recourse to quantification.

On this model, to say that "quantifiers are always F" is actually not to say one sentence, but rather to affirm a whole schema of instances. For $\forall$ to be maximally general, then, will be for the following schema's instances to be true:

If X is a quantifier, then X is at least as general as $\forall$

A problem with schemas, and a well known one at that, is that they cannot be embedded in more complex constructions. What if we wished (as the Relativist does) to deny, say, a particular quantifier $\forall$ is maximally general? If we express $\forall$'s maximal generality via schema, then we would need, in some sense, to negate the schema—but what exactly does that mean? We cannot deny the schema in the relevant sense by accepting the negation of each instance, since $\forall$ is at least as general as itself. Do we say that some instance is negated? If so, which one—what claim is our theory to include?[12]

And even if we overcome this problem, the Relativist still must say that quantifier always fail to be absolutely general; this would seem to require that they must embed their denial of a schema—this is how they say of a particular quantifier fails to be absolutely general—within another schema—this how they say that something is true of quantifiers in general. It is not at all clear how this might be done.

4. Entailments

The schematicist is right that we need a non-quantificational way of generalizing if we are to articulate Relativism; but they err in absconding to the meta-language to achieve this generality. What we need is a device of non-quantificational generality that lives in the object-language. One might think that there are no such devices to be found: quantifiers are our mode of expressing generality in the object-language, and our only mode—there is no way of saying that some feature F holds of quantifiers as a whole which does not involve the resources of quantification. Such a view, I think, underestimates the diversity of our philosophical resources: we have ways of generalizing over quantifiers, sets, donkeys or what have you, that do not involve *quantifying* over them.

Here is one way to warm up to that thought. Suppose someone asked you why all whales were mammals—"why" in the sense of what grounds that truth, or what makes it the case that all water molecules are so. One thing you might say is that being a mammal is just in the essence of whales or whalehood: part of what it is to be a whale is just to be a mammal.

Your claim, about the essence of whales, is not on the face of it a claim of quantification. To say that it is in the essence of whales to be F is not equivalent to saying that all whales are F, and in general "it is in the essence of Fs to be G" is not equivalent to "Every F is G": it seems perfectly coherent to maintain a position on which both (i) all whales live in the Atlantic ocean while (ii) it is not in the

[12] See Williamson (2003) for more criticism of schemas, and Lavine (2006) for a rejoinder on the schematicist's behalf. Studd (2019) responds to Lavine's rejoinder.

essence of being a whale to live in the Atlantic ocean or that living in the Atlantic is not part of what it is to be a whale. Nonetheless, your essence claim still seems to have a certain generalizing force: that, after all, is why it might have seemed like such a good explanation for the quantificational claim in the first place.[13]

Of course, some philosophers might reject the ideology of essence—but claims of essence are not the only ones that might seem to have this non-quantificational, generalizing force. Even skeptics of essence may need an ideology of properties, and so will be more comfortable with claims of identity between properties—"the property of being a whale is identical to a swimming mammal"—or closely related constructions, like "to be a whale is to be a swimming mammal" (cf. Dorr (2016)). But each of these claims, too, seem to imply some kind of general connection between whalehood and mammalhood, that being a mammal is somehow implied by being a whale. (They too might be good explanations for the truth of "all whales are mammals".)

Let me call relations of this sort—which imply some kind of general connection between their relata which is not a quantificational connection—*entailments*. Entailments, I submit, do not just underwrite the truth of universally quantified generalizations about whales or sets or quantifiers, but are themselves ways of generalizing. We say something about whales *as a whole* when we say that it is in the essence of whales to be mammals: namely, that they are mammals.

Crucially, this ideology of entailments seems to be a non-quantificational not just in the sense that "F entails G" is not equivalent to "Every F is G", but non-quantificational in the more general sense that these notions of entailment do not depend on their good standing for any notion of absolutely general quantification—and hence should be perfectly acceptable even to someone deeply enamored of the Relativist vision of an unending hierarchy of quantifiers. This is a vision about the impossibility of a certain kind of quantification; why should that impugn the ideology of essence or identity between properties? Surely the property of being a set is identical to the property of being a set, no matter what sort of hierarchy of quantifiers over the sets there may be.

If that is so, then, I submit, entailments may be just what we need to fashion a reasonable and coherent form of Relativism. For entailments, as I have just suggested, seem to provide us with ways of generalizing—when we say that to be a water molecule is to have a hydrogen atom and two oxygen atoms arranged in some fashion, we say something about water molecules *as a whole*—and indeed, a way of generalizing that seems in an intuitive sense to be as general as can be. If it

[13]Other philosophers have also been attracted to the idea that there is some connection between such identities and generality. Linnebo (2022), for instance, briefly suggests that such an identity might be a ground or truthmaker for the universal claim $\forall x(\text{Whale}(x) \rightarrow \text{Mammal}(x))$.

is in the essence of whales to be mammals, is there any important sense in which there could be a whale which wasn't a mammal?[14]

My proposal is that "quantifiers are always F" is to be understood as the claim that being a quantifier *entails* being F. $\forall$ will be absolutely general just in case it is a quantifier and being a quantifier *entails* being a restriction of $\forall$. And so Relativism and Absolutism will be the following theses:

Entailment Relativism: Being a quantifier entails being an property of properties such that being a quantifier does not entail being a restriction of it

Entailment Absolutism: Being a quantifier does not entail being an property of properties such that being a quantifier does not entail being a restriction of it

Or more succinctly:

Entailment Relativism: Being a quantifier entails being not absolutely general

Entailment Absolutism: Being a quantifier does not entail being not absolutely general

Because entailment gives us a way of generalizing about quantifiers in a maximally general way, Entailment Relativism captures the intuitive ambition of Relativism to say that quantifiers as a whole fail to be absolutely general. Since entailments give us a non-quantificational way of generalizing, however, the hope is that Entailment Relativism avoids the pitfalls faced by Q-Relativism before, such as the problem of Π.

The remainder of this paper is an attempt at showing that the hope of avoiding the pitfalls of Q-Relativism by recourse to entailments is not illusory. I will do so by developing what I take to be a particularly compelling and interesting version of Entailment Relativism, with a particular notion of entailment, defined in terms of identity between properties.

We will develop this notion of entailment in the more regimented setting of a higher-order logical language. In an area as ripe for inconsistency as Relativism, the structure and clarity that a formal language brings will be helpful. One benefit of this formalism is that we will be able to give a proof that Relativism, as I state it, is consistent after all—there are no problems like that of Π lurking.

[14]Objection: Wait, doesn't it contravene the spirit of the Relativist to admit that there are maximally general ways of generalizing? Response: Keep in mind the distinction between entailment and quantification. The Absolute Generality debate, as I understand it and think other philosophers have understood it, concerns whether *quantifiers* ever attain maximal generality, not whether entailment or other devices of generality may have this status. (In light of this, it might be better to say the debate is about "absolutely general quantification" rather than "absolute generality".) It does not contravene the spirit of Relativism to maintain that we can generalize about quantifiers as a whole, so long as we remember that to generalize about quantifiers as a whole is not necessarily to quantify over quantifiers as a whole.

In Section 5, I will introduce the higher-order logical language that we will use to regiment all this talk of properties, identity between properties, and entailment. We'll use that language to define the notion of entailment (in terms of property identity) which I will then argue can do all the work that I have said entailment relations can do (Section 4). With these pieces of philosophical machinery in place, we will then come to what in many ways is the heart of this project: an account of what it is to be a universal quantifier (Section 7 and 8). It is only with an account of what it is to be a quantifier in place that I'll be able to vindicate my claims that the notion of entailment I've chosen isn't somehow quantificational (Section 9) and be able to prove that Relativism, on my construal, is a consistent position.

By adopting turning to the machinery of entailments, then, we may articulate a form of Relativism which is provably consistent and which, arguably, captures the intuitive spirit of the view.

5. Going Higher-Order

I will now introduce some formalism to regiment and precisify our theorizing about quantifiers and entailment. If quantifiers are properties of properties, then this framework will have to be one for theorizing about properties and properties of them. Some caution is necessary here—the lesson of Russell's paradox is that even theories of properties or classes which seem very plausible can easily fall prey to contradiction.

I will formalize property-talk by adopting a *higher-order* language.[15] Many kinds of language, formal and natural alike, have different syntactic categories to which their expression belong. This is true no less of the first-order languages with which logicians and philosophers often work; these languages have singular terms, as well as sentences, and predicates which combine with singular terms to form sentences. The number, though, is relatively limited. Higher-order languages, by contrast, are more liberal. They may include higher-order predicates, which combine with predicates of the usual kind to form sentences, just as predicates of the usual kind combine with singular terms. A widely accepted hypothesis in linguistics, in fact, is that English itself is a higher-order language of this kind.

While I will be using a higher-order language for my property-theorizing, there are other ways one might go—if one has a well enough worked out formal first-order theory of properties, then that would do the job.[16] Theorists so inclined

[15]See Fritz (2023), Chapter 1, for an argument for the conclusion that higher-order languages are our best way of consistently theorizing in the way property-talk is supposed to allow us to do.

[16]That is not to say that there has not been much interesting work on developing consistent and sufficiently strong first-order theories of properties: see especially Fine (2005) and Linnebo (2006) for recent attempts. My own view is that the limitations these theories impose upon property-talk to avoid paradox make them ill suited to serve as a metaphysical framework. (Both Fine's and Linnebo's theories take place against a background of classical logic, but there is also a literature in developing first-order theories of properties in the non-classical tradition as well: see for instance Field (2004).)

are welcome to translate my higher-order view into their own first-order theoretic terms.

The higher-order language $\mathcal{L}$ that I will use is defined as follows. $\mathcal{L}$ has two basic syntactic categories: type e, the type of singular terms, and type t, the type of formulae. The rest we define recursively: when σ and τ are types and $\tau \neq e$, then $(\sigma \to \tau)$ is a type, the type of an expression which, when composed with an expression of type σ, returns an expression of type τ.[17] Thus $e \to t$ is a type: the type of unary predicates which, when combined with a singular term, return a formula. And $t \to t$ is a type: this is the type of sentential operators, such as $\Box$ from modal logic, or $\neg$.

The terms at each type may be complex or simple. The simple terms are constants and variables—we assume an infinite stock of variables at each type, and sometimes use superscripts to indicate a variable's type. Our simple terms will include familiar logical constants:

- $\wedge$, of type $t \to t \to t$
- $\vee$, of type $t \to t \to t$
- $\neg$, of type $t \to t$

In addition to these, I will also assume that, for each type σ, we have an identity-expression $=^\sigma$ of type $\sigma \to \sigma \to t$, which indeed can be seen as a generalization of identity on type e to higher types.footnoteSee Dorr (2016) for more discussion and defense of the legitimacy of using such devices.

Complex terms can be made in two ways: application and λ-abstraction. Application is the generalization to arbitrary types of the way the first-order logician combines predicates and singular terms to form formulae: when A is an expression of type σ (which we may notate as $A : \sigma$) and $B : \sigma \to \tau$, $(BA) : \tau$.

When $A : \tau$ and $x : \sigma$ is a variable (and $\tau \neq e$), $\lambda x.A$ is a term of type $\sigma \to \tau$. λ-abstraction is our device for creating terms for complex terms and predicates. In natural languages like English, it is easy for us to form predicates with complex internal structure. For instance, "is red and round" is a complex predicate in some sense formed from "is red" and "is round". In our formal language, we accomplish a similar task with λ-abstraction: when we have some formula A, in which a variable x may or may not be free, then $\lambda x.A$ is a complex predicate which we might gloss in English as "being such that A". Thus we may write $\lambda x.\text{Red}(x) \wedge \text{Round}(x)$ as our formal gloss for "is red and round".

Finally, while $\mathcal{L}$ is our official language, as is conventional I will often informally gloss it in English, availing myself of terms like "proposition" to talk about type

[17] I will often drop parentheses in the names for types, assuming a convention of associating type-arrows to the right: thus, e.g., $t \to t \to t$ is really $(t \to (t \to t))$.

t and terms like "property" and "relation" to talk about type $\sigma_1 \to (\sigma_2 \to \ldots \to (\sigma_n \to t)\ldots)$.

With all this in place, we can regiment some of what I have said about Entailment and Relativism. For F to entail G—I will notate this as $F \leq G$—is defined, in $\mathcal{L}$ like so:

$$F = \lambda x.Fx \wedge Gx$$

Let me introduce a predicate $\mathsf{Quant} : ((e \to t) \to t) \to t$ which is intended to express the property of being a quantifier. (In Section 7, we will also define this term in terms of identity and the logical constants.) The property of being absolutely general, then, is:

$$\lambda X.\mathsf{Quant}(X) \wedge (\mathsf{Quant} \leq (\lambda Y.Y \subseteq X))$$

Where $Y \subseteq X$, recall, means Y is a restriction of X. (In our formal language, $Y \subseteq X$ is defined as $Y(\lambda x.\neg X(\lambda y.y \neq x))$. Plug in $\forall$ and $\forall'$ for X and Y to see how this corresponds to what was said before.) We can let AbsGen abbreviate this expression.

Finally, then, we have that Relativism and Absolutism would be stated as:

HO Relativism: $\mathsf{Quant} \leq \lambda X.\neg\mathsf{AbsGen}(X)$

HO Absolutism: $\mathsf{Quant} \not\leq \lambda X.\neg\mathsf{AbsGen}(X)$

Or, in more expansive form:

HO Relativism: $\mathsf{Quant} \leq \lambda X.\neg(\mathsf{Quant}(X) \wedge (\mathsf{Quant} \not\leq \lambda Y.Y \subseteq X))$

HO Absolutism: $\mathsf{Quant} \not\leq \lambda X.\neg(\mathsf{Quant}(X) \wedge (\mathsf{Quant} \not\leq \lambda Y.Y \subseteq X))$

6. A Theory of Higher-Order Identity

Our entailment relation, $\leq$, is:

$$\lambda XY.\, X = (\lambda x.Xx \wedge Yx)$$

Note that, when we disambiguate the types of variables, it becomes clear that there is really a notion of entailment for each type of the form $\sigma \to t$, $\leq^{\sigma \to t}$, given by $\lambda XY.X = \lambda x^{\sigma}.(Xx \wedge Yx)$. [18]

If $F \leq G$ in this sense, then very plausibly G holds of Fs as a whole: if to be F just is to be F and G, then is there any reasonable sense in which an F thing could fail to be G? But will there ever be F and G such that $F \leq G$ in this sense? Clearly, this notion of entailment will be of some use only with a sufficiently coarse-grained theory of higher-order identity. On theories of higher-order identity according to which propositions and properties are structured like the sentences and predicates we use to express them, it is doubtful whether we could find properties F and G such that F entails G. On coarser-grained theories such as that according to which, roughly, propositions and properties form Boolean algebras, $\leq$ will be better

[18] In fact, we will be able to generalize further the applicability of the idea of entailment later in this section.

behaved. So my starting point will be the development of a sufficiently coarse-grained theory of higher-order identity. I should note, however, that there are other candidate definitions of $\leq$ which would work just as well for my purposes and would require a weaker background logic; I focus on the present definition and logic because they are especially simple and helpful for conveying the general idea of my approach.

Now,

A theory is just a set of formulae in a given language, and a theory is true just in case every *sentence*, i.e., closed formula, in the theory is true. Take first the relatively uncontroversial fragment of higher-order logic defined by the following schemas and rule (here, $\vdash P$ just means P belongs to our theory):

- PC $\vdash P$ where P is a substitution instance of a theorem of classical propositional logic
- Id $\vdash a = a$
- LL $\vdash a = b \rightarrow Fa \rightarrow Fb$
- β $\vdash P[(\lambda x.A)B] \leftrightarrow P[A[B/x]]$, where $A[B/x]$ is the result of replacing every occurence of x in A with B, so long as this can be done without any free variable in B becoming bound[19]
- η $\vdash A[\lambda x.Fx] \leftrightarrow A[F]$, where x is not free in F
- MP If $\vdash P \rightarrow Q$ and $\vdash P$, then $\vdash Q$

We can call this theory B (for "basic") and write $\mathsf{B} \vdash P$ when P is a formula included in B (i.e., a theorem of B). Any theory which includes B and is closed under Modus Ponens is a *basic theory*.

The theory I want to adopt, however, is an extension of B, and is in particular to be the least basic theory closed under the following rule:

Rule of Equivalence: If $\vdash P \leftrightarrow Q$, then $\vdash \lambda x_1 \ldots x_n.P = \lambda x_1 \ldots x_n.Q$

The resulting theory I will call $\mathcal{C}(\mathsf{B})$, the *closure* of B.[20]

The general ethos behind the Rule of Equivalence is something like logical equivalence suffices for worldly identity. Consider, for instance, sentences $A \wedge B$ and $B \wedge A$, which are provably logically equivalent in B. One might think that these sentences must express the same proposition: one might think they will be true in all the same possible circumstances or worlds (in what scenario is the one true but not the other?), and so one will want to commit to the propositional identity $A \wedge B = B \wedge A$.

[19] Some philosophers will take issue with β. See Dorr (2016), Section 5 for discussion.

[20] This theory is a weakening of the theory *Classicism* of Bacon and Dorr (2024). Note that it is not simply the fragment of Classicism which contains only sentences of $\mathcal{L}$ (Bacon and Dorr work in $\mathcal{L}(\forall)$)—that fragment is inconsistent with my version of Relativism, but $\mathcal{C}(\mathsf{B})$) is consistent with it.

The Rule of Equivalence extends this general feeling that the logical equivalence suffices for identity from type t to higher types. Consider, for instance, the complex predicates $\lambda pq.p \wedge q$ and $\lambda pq.q \wedge p$. Just as one felt that there was no difference between the conjunction of A and B in one order (say, $A \wedge B$) and conjoining them in the other ($B \wedge A$), so one may feel that there is no difference in general between *conjoining* propositions in one order ($\lambda pq.p \wedge q$) and conjoining them in another ($\lambda pq.q \wedge p$), and so want to identify them—a claim of identity between properties. Or to put it another way: just as one felt that $A \wedge B$ and $B \wedge A$ do not differ in what proposition they express, so one may feel that $p \wedge q$ and $q \wedge p$ are also somehow expressively identical: propositions satisfying the one formula will also satisfy the other. Hence we should identify the complex predicates $\lambda pq.p \wedge q$ and $\lambda pq.q \wedge p$ formed via λ-abstraction from the properties.

The Rule of Equivalence ensures $\mathcal{C}(\mathsf{B})$ makes this identification: $p \wedge q \leftrightarrow q \wedge p$ is a theorem of B—here p and q are propositional variables—so that $\lambda pq.p \wedge q = \lambda pq.q \wedge p$ is one of $\mathcal{C}(\mathsf{B})$. In general, then, a provable equivalence $P \leftrightarrow Q$, with variables $x_1, \ldots, x_n$ free in P and Q, will correspond to an identity between the complex predicates formed from P and Q by λ-abstraction on these free variables.

The following theorems of $\mathcal{C}(\mathsf{B})$, which I will call the *Boolean Identities*, are the result of "upgrading" familiar equivalences of propositional logic to higher-order identities via the Rule of Equivalence:

$$\begin{array}{cc} \lambda pq.p \wedge q = \lambda pq.q \wedge p & \lambda pq.p \vee q = \lambda pq.q \vee p \\ \lambda pqr.p \wedge (q \vee r) = \lambda pqr.(p \wedge q) \vee (p \wedge r) & \lambda pqr.p \vee (q \wedge r) = \lambda pqr.(p \vee q) \wedge (p \vee r) \\ \lambda pq.p \wedge (q \vee \neg q) = \lambda pq.p & \lambda pq.p \vee (q \wedge \neg q) = \lambda pq.p \end{array}$$

The Boolean Identities, in some sense, imply that the propositions form a Boolean algebra, and indeed that at each type other than e, the entities at that type form a Boolean algebra. For each such type $\tau \neq e$, we can define a notion of conjunction, disjunction, and negation $\wedge_\tau$, $\vee_\tau$, and $\neg_\tau$ by induction. Taking $\wedge_t$, $\vee_t$ and $\neg_t$ to be $\wedge$, $\vee$, and $\neg$, we say:

- $\neg_{\sigma\to\tau}$ abbreviates $\lambda X^{\sigma\to\tau} z^\sigma.\neg_\tau X z$
- $\wedge_{\sigma\to\tau}$ abbreviates $\lambda X^{\sigma\to\tau} Y^{\sigma\to\tau} z^\sigma.X z \wedge_\tau Y z$
- $\vee_{\sigma\to\tau}$ abbreviates $\lambda X^{\sigma\to\tau} Y^{\sigma\to\tau} z^\sigma.X z \vee_\tau Y z$

  (Often, I'll omit the superscript.)

It is then easy to confirm that from the Boolean identities we can derive identities parallel to them for each non-e types: for instance, we can derive $\lambda X^\sigma Y^\sigma.X = \lambda X^\sigma Y^\sigma.X \wedge^\sigma (X \vee Y^\sigma)$.

Using this notation, we can also rewrite $\leq^{\sigma\to t}$ as:

$$\lambda XY.X = (X \wedge^{\sigma\to t} Y)$$

And so we can see that entailment can be generalized to an arbitrary type $\tau \neq e$.

In a Boolean algebra with conjunction $\sqcap$, disjunction $\sqcup$ and complementation $\cdot^C$, we can always define an ordering relation $\sqsubseteq$ by $a \sqsubseteq b$ just in case $a \sqcap b = a$. On this ordering, $a \sqcap b$ is the greatest lower bound of a and b, and $a \sqcup b$ the greatest upper bound. Entailment, as I have defined it, is no more than this ordering as applied to the Boolean algebra of properties of a given type. Hence we will have that $F \leq F \vee^{\sigma \to t} G$, and $F \wedge^{\sigma \to t} G \leq F$, for instance.

The parallel to Boolean algebra also suggests one last bit of ideology that will be useful. Since the propositions are, intuitively speaking, a Boolean algebra, there is an element $\top$ which is entailed by an arbitrary proposition. We can define $\top$ as $\wedge = \wedge$, or by any other theorem of $\mathcal{C}(\mathsf{B})$. If there is any proposition that deserves to be called a trivial proposition—that is, the proposition which is automatically the case, true no matter the circumstances—it is $\top$. After all, every instance of the following schema is a theorem of $\mathcal{C}(\mathsf{B})$:

$$p \to \top$$

What goes for the propositions goes for other types as well; at each type we may also define a special $\top$ element. In the case of type $\sigma_1 \to \sigma_2 \to \ldots \to \sigma_n \to t$, that element is none other than $\lambda x^{\sigma_1} \ldots x^{\sigma_n}.\top$.

Let us fix some type σ. If there is any property of entities of type σ that deserves to be called the trivial property—that is, the property that is trivially or automatically had, had no matter the circumstances—it is $\lambda x^{\sigma}.\top$. After all, every instance of the following schema is a theorem of $\mathcal{C}(\mathsf{B})$:

$$Fy \to (\lambda x.\top)y$$

As I have said, $\leq$ gives us what seems to be a maximally general way of generalizing over the Fs, for any property F whatever. To generalize over the F's, we use the operator $\lambda X.F \leq X$: G holds of the Fs in general, just in case this operator holds of G. And suppose, for instance, $\mathsf{Quant} \leq F$. Then to be a quantifier is just to be a quantifier and additionally F. If $\mathsf{Quant} \leq F$, then to be a quantifier which is not F, $\mathcal{C}(\mathsf{B})$ proves, is to be something which is both F and not F—that is, to be a property of which a contradiction holds. What greater generality over quantifiers could one wish for?

What if we wanted to generalize not just over quantifiers or whales, but things in, well, general? We just consider the status of being a property entailed by $\lambda x.\top$, $\lambda X.(\lambda x.\top \leq X)$. If we can gloss "$F \leq G$" as "an arbitrary F-thing is G", then we can gloss "$\lambda x.\top \leq G$" as an arbitrary $\lambda x.\top$-thing is G. But if $\lambda x.\top$ really is in a sense a trivial property, one that is automatically had, then this is to say that an arbitrary thing, full stop, is G—that G holds in a general way not of just the F's, but simply, well, in general.

Now being entailed by $\top$ or $\lambda x.\top$ is in fact the same as being identical to $\top$ or $\lambda x.\top$. Since this notion of being identical to the trivial proposition and the trivial properties will play a starring role for us in what is to come, it will be helpful to have some (rather suggestive) notation for them. Following some recent literature in higher-order metaphysics (which itself is following older literature in higher-order logic), I will abbreviate $\lambda p.p = \top$ as $\Box$, and let $\Diamond$ abbreviate $\lambda p.\neg\Box\neg p$, as usual.[21] The reason is that, given the principles already laid down, $\Box$ acts like an S4 modality—it satisfies $\Box(p \to q) \to \Box p \to \Box q$, for instance, and $\Box p \to p$, and so on. If $\Box p$, then I will say that p is "broadly necessary".[22]

In a similar manner, abbreviate $\lambda x_1 \ldots x_n.P = \lambda x_1 \ldots x_n.\top$ by $\Box_{x_1 \ldots x_n} P$, and $\neg\Box_{x_1,\ldots,x_n}\neg P$ by $\Diamond_{x_1 \ldots x_n} P$.[23] Modal logic also provides a helpful heuristic for reasoning with these operators: their inferential behavior is rather like that of *necessitated* universal quantifiers ($\Box\forall$) and "possibilized" existential quantifiers ($\Diamond\exists$), respectively—not like that of universal quantifiers and existential quantifiers.

Finally, let me note that $\mathcal{C}(\mathsf{B})$ lets us simply our statements of Relativism and Absolutism. Given $\mathcal{C}(\mathsf{B})$, we can write:

HO Relativism: $\mathsf{Quant} \leq \lambda X.(\mathsf{Quant} \not\leq \lambda Y.Y \subseteq X)$

HO Absolutism: $\mathsf{Quant} \not\leq \lambda X.(\mathsf{Quant} \not\leq \lambda Y.Y \subseteq X)$

Which we can rewrite with our suggestive notation as follows:

HO Relativism: $\Box_X(\mathsf{Quant}(X) \to \Diamond_Y(\mathsf{Quant}(Y) \wedge \neg Y \subseteq X)$

HO Absolutism: $\Diamond_X(\mathsf{Quant}(X) \wedge \Box_Y(\mathsf{Quant}(Y) \to Y \subseteq X)$

7. Being a Quantifier

HO Relativism, then, lives up to the ambition of generalizing over quantifiers in a maximally general way. Again, however, we must check that the generalization offered by entailment is non-quantificational. It is time, then, to develop an account of what being a (universal) quantifier consists in. With this account in hand, I will argue that entailment is not quantificational. The proposal is this: to be a (universal) quantifier is just to have the right logical behavior, in a sense we will make precise using the resources of $\mathcal{C}(\mathsf{B})$.

The idea that the logical constants are defined by their logical or inferential behavior is not a new one.[24] Thus we find Prior in "The Runabout Inference-Ticket":

> For if we are asked what is the meaning of the word 'and', at least in the purely conjunctive sense (as opposed to, e.g., its colloquial use to mean 'and then'), the answer is said to be *completely* given by saying that (i) from any pair of statements P and Q we can infer

[21] See especially Bacon (2018), Bacon and Dorr (2024), and the citations therein.

[22] Whether $\Box$ counts as a "genuine" modality in some sense or other is not a question I will pursue here. See Bacon (2018) for an extended discussion of $\Box$ and its logic, and a defense of its status as a genuine modality, all couched in a more Absolutist-friendly context.

[23] This is a variation of a notation due to Peter Fritz.

[24] Gentzen (1964) seems to have had such a view, for instance.

> the statement formed by joining P to Q by 'and' (which statement we hereafter describe as 'the statement P-and-Q'), that (ii) from any conjunctive statement P-and-Q we can infer P, and (iii) from P-and-Q we can always infer Q. (Prior (1960), p. 38)

Prior is sketching a view on which the *meaning* of the word "and" is exhausted by its inferential role.[25] But there is a metaphysical thought in the vicinity, and that is what I am after here. The thought is not about what it is for a word "R" to be conjuctive, but what it is for a relation R of propositions to be conjunctive. To be a conjunctive relation is just to satisfy the (metaphysical analogue) of $\wedge$'s inferential role, or to have the logical behavior of $\wedge$.

Let us take for granted that logicians have isolated the logical behavior of $\wedge$—that is, the operation of conjunction, not the term—with any one of their standard axiomatizations of the logic of conjunction; say with the schemas:

$\wedge$-Intro: $A \to B \to (A \wedge B)$

$\wedge$-Elim1: $A \wedge B \to A$

$\wedge$-Elim2: $A \wedge B \to B$

Under what conditions could a relation R be said to have the logical behavior of $\wedge$? One might think it would suffice if every instance of the following schemas were true:

R-Intro: $A \to B \to RAB$

R-Elim1: $RAB \to A$

R-Elim2: $RAB \to B$

But we might want a stronger condition on R. When we take a schema deemed logically true or valid, such as:

$$A \vee \neg A \text{ for all sentences } A$$

It is natural to think that there is something common to all the instances of the schema which underwrites their logical goodness; it is not as though it just so happens that each instance is a logical truth. According to an orthodox thought, this common thing is the (logical) form of the sentence—that which remains when we abstract from a particular instance of the schema.

We can operationalize the notion of form in the present setting, again, by appeal to complex predicates formed via λ-abstraction: roughly, we replace constituent sentences by propositional variables and bind via λ. The form of $A \vee \neg A$ is $\lambda p.p \vee \neg p$; of $A \to A$ it is $\lambda p.p \to p$; of $A \vee (A \wedge B) \leftrightarrow A$ it is $\lambda pq.p \vee (p \wedge q) \leftrightarrow p$.

[25] Prior of course raises the view only to raise trouble for it with his "tonk" operator.

What is the good status that forms like $\lambda p.p \to p$ have? A natural answer would be that $\lambda p.p \to p$ holds of an arbitrary proposition—that is, $\Box_p(p \to p)$. Indeed, this is something $\mathcal{C}(\mathsf{B})$ proves.

Just as with the schema $A \vee \neg A$, the good status of $\wedge$-Intro and the Elim schemas, we might think, is underwritten by a good status which their forms enjoy. Indeed, all of the following are theorems of $\mathcal{C}(\mathsf{B})$:

$\top$-$\wedge$-Intro: $\Box_{p,q}(p \to q \to (p \wedge q))$

$\top$-$\wedge$-Elim1: $\Box_{p,q}(p \wedge q \to p)$

$\top$-$\wedge$-Elim2: $\Box_{p,q}(p \wedge q \to q)$

The sense in which $\wedge$ satisfies the Intro and Elim axioms is not just that every instance of them is true, but that the forms of the schemas hold of arbitrary propositions. If a relation R is to count as having the same logical behavior as $\wedge$, then R-Intro ought to hold with the same generality. We must have that $p \to q \to Rpq$ holds of arbitrary propositions p, q, in the sense that $\Box_{p,q}(p \to q \to Rpq)$, and likewise for the Elimination axiom schemas.

Let us say, then, that:

- R has the *Intro Property* just in case $\Box_{p,q}(p \to q \to Rpq)$
- R has the *Elim1 Property* just in case $\Box_{p,q}(Rpq \to p)$
- R has the *Elim1 Property* just in case $\Box_{p,q}(Rpq \to q)$

To obtain terms standing for the properties, we need only λ-abstract on R.[26]

As for conjunctive operators, so for universal quantifiers. What universal quantifiers are by nature is exhausted by (the metaphysical analogue) of their inferential role; to be a universal quantifier is just to have the right logical behavior.

Again, let us take our cue from the logicians who have tried to axiomatize the logic of universal quantification. I submit that the inferential role of universal quantifiers is pretty well captured by the axiom schemas and rules for $\forall$ that we find in so-called "free logic", or at least one form of it:

Free Instantiation-$\forall$	$\forall x(\forall y A \to A(x/y))$
K-$\forall$	$\forall x(A \to B) \to \forall x A \to \forall x B$
Vacuity-$\forall$	$A \to \forall x A$, where x is not free in A
Rule of Generalization-$\forall$	If $\vdash A$, then $\vdash \forall x A$

[26]This definition of conjunction, and the one I am about to give for universal quantifiers, brings together a few strands in recent work in higher-order metaphysics and logic. The idea that we should see logical validities as underwritten by some kind of higher-order identities is one that I borrow from Dorr (2014) and Goodman (2016). The idea that certain classes of logical operator can be defined by their logical behavior is present in Bacon (2018), but is worked out in terms of quantification. The contribution of the current section is to bring these two ideas together.

One might have thought the inferential role of universal quantifiers is better captured by *classical* quantification theory, which replaces Free Instantiation-$\forall$ with the schema of Unrestricted Instantiation ($\forall x A \to A(c/x)$, for any A, c). But when we remember that universal quantifiers are to include properties like *being a property had by every cow*, it becomes clear that classical quantification theory is inadequate: it does not follow from every cow being a cow that New York City is a cow.

We are now able to give a definition of Quant, that is, to say what it is to be a universal quantifier. Following the pattern with conjunction, say that O is a universal quantifier just in case it possess the following three properties, each one derived from one of the three above axiom schemas (we will return to the Gen Rule in a moment).

- O has the *Free Instantiation Property* just in case

$$\Box_R O(\lambda x.O(\lambda y.Rxy) \to Rxx)$$

- O has the *K Property* just in case

$$\Box_{F,G}(O(\lambda x.Fx \to Gx) \to OF \to OG)$$

- O has the *Vacuity Property* just in case

$$\Box_p(p \to O(\lambda x.p))$$

As before, we obtain terms standing for the properties themselves by λ-abstracting on O.

The way I have "translated" the Free Instantiation schema into the corresponding property deserves some comment. One might have expected that O should have this property just in case:

$$\Box_F O(\lambda x.O(F) \to Fx)$$

Where, instead of the two-place relation R, we have instead the one-place predicate F. The version of the Free Instantiation property with F instead of R, though, is inadequate; operators which are intuitively not universal quantifiers (such as *all but finitely many natural numbers*, for instance) may possess the K and Vacuity properties, as well as the weaker Instantiation property.[27] To be a quantifier, then,

[27]The reason it is inadequate is that the Free Instantiation schema allows x to occur free in A. An instance of the schema is:

$$\forall x(\forall y(y \neq x) \to x \neq x)$$

This would not seem to be an instance of the schema, if understood to require x not to be free in A. There is a general issue here in the proper axiomatiziation of Free Logic. Leblanc (1995) at least seems to claim, if I read him correctly, that one can derive, just using a version of Free Instantiation in which x cannot occur free in A, the claim $\forall x \exists y(y = x)$. But the proof of his Lemma 10 would seem to require the version of the schema in which x can occur in A free.

is just to have the Free Instantiation, K, and Vacuity properties. Indeed, we will take Quant to be an abbreviation for this monstrous λ term:

$$\lambda O.\Box_R O(\lambda x.O(\lambda y.Rxy) \to Rxx) \wedge \Box_{F,G}(O(\lambda x.Fx \to Gx) \to OF \to OG) \wedge \Box_p(p \to O(\lambda x.p))$$

Now, officially Quant is a term of type $((e \to t) \to t) \to t$. Sets are entities of type e, and so Relativism, in the first place, is about whether quantifiers over those sorts of entity are ever maximally general. The quantifiers involved in Relativism, intuitively understood, are therefore entities of type $(e \to t) \to t$, and so Quant, being a property of them, is thus of type $((e \to t) \to t) \to t$.

Clearly, however, the definition of Quant we have given can easily be generalized to give a notion Quant_σ of being a quantifier over entities of type σ for any type σ. We may say $\mathsf{Quant}_\sigma(O)$ just in case, now disambiguating the types of the various variables:

- O has the *σ-Free Instantiation Property* just in case
$$\Box_{R^{\sigma \to \sigma \to t}} O(\lambda x.O(\lambda y.Rxy) \to Rxx)$$
- O has the *σ-K Property* just in case
$$\Box_{F^{\sigma \to t}, G^{\sigma \to t}}(O(\lambda x.Fx \to Gx) \to OF \to OG)$$
- O has the *σ-Vacuity Property* just in case
$$\Box_{p^t}(p \to O(\lambda x^\sigma.p))$$

Quant as originally defined is just the special case when $\sigma = e$.

There is room for dispute about the goodness of this definition. One may think that being a quantifier requires something more than having the right logical behavior, for instance. But even supposing nothing more is required than having the logical behavior given by Free Logic, can we be sure that Quant fully captures that logical behavior? Or to put it another way: if we simply added $\mathsf{Quant}(X)$ to our theory $\mathcal{C}(\mathsf{B})$, would we be sure that claims of the resulting theory would be enough to guarantee that X has the logical behavior of quantifiers, as set forth in free logic? These are good concerns, especially since Quant does not obviously contain a metaphysical equivalent for the Rule of Generalization, one of the constituent rules of inference in the axiomatization of Free Logic. But they can be met.

Suppose we augment $\mathcal{L}$ with a symbol $\forall_\sigma$ of type $(\sigma \to t) \to t$. $\forall_\sigma$ is, intuitively, meant to be a quantifier. What would we add to $\mathcal{C}(\mathsf{B})$ to guarantee that it was one? One thing we could do is add all the instances of the free logical axioms discussed above (so that we added, that is, $\forall_\sigma x(\forall y A \to A(x/y))$ for every formula A), and closing the resulting theory under the Rule of Generalization (from the Free Logic axiomatization) and also the Rule of Equivalence. Let us call this theory $\mathcal{C}(\mathsf{FL})$. Since $\mathcal{C}(\mathsf{FL})$ contains all the axioms of free logic and is closed under the Rule of Generalization, it seems hard to deny that this theory suffices to guarantee that $\forall_\sigma$

has the logical behavior of a quantifier (so long as we take that behavior to be given by free logic).[28]

Here, now, is the key fact:

Fact 7.1. $\mathcal{C}(\mathsf{FL})$ is equivalent to the least basic theory containing $\mathcal{C}(\mathsf{B})$ and $\mathsf{Quant}_\sigma(\forall_\sigma)$

In other words: by simply adding $\mathsf{Quant}_\sigma(\forall_\sigma)$ to $\mathcal{C}(\mathsf{B})$, we obtain the same theory as we did by adding in by hand the Free Logic axioms (and closing under the Rule of Generalization): Quant_σ bundles all the relevant behavior up into a neat package. If, then, $\mathcal{C}(\mathsf{FL})$ ensured that $\forall$ had the right logical behavior, then $\mathcal{C}(\mathsf{B})$ plus $\mathsf{Quant}_\sigma(\forall_\sigma)$, despite our worries, should suffice as well, since the theories are in fact identical.

8. Some Facts about Quant

Now that we have a definition of Quant in hand, we can make it do some philosophical work for us.

First, with the definition in hand, we can put on firmer footing the informal arguments against Q-Relativism that we considered earlier. Suppose for each type σ we have a quantifier $\forall_\sigma$ and add to our theory $\mathsf{Quant}_\sigma(\forall_\sigma)$, so that $\forall_\sigma$ really is a quantifier. Then we can regiment Q-Relativism and Q-Absolutism as follows:

Q-Relativism: $\forall X\,(\mathsf{Quant}_e(X) \to \exists Y(\mathsf{Quant}_e(Y) \wedge \neg Y \subseteq X))$

Q-Absolutism: $\exists X\,(\mathsf{Quant}_e(X) \wedge \forall Y(\mathsf{Quant}_e(Y) \to Y \subseteq X))$

The intuitive problem for Q-Relativism came from Π, the property of being a property to which every quantifier applies. We can now define Π, using λ notation, as follows:

$$\lambda F.\forall X(\mathsf{Quant}_e(X) \to X(F))$$

I argued that it was plausible before that Π itself was a quantifier, but with our definition of Quant in hand, we can dispense with appeal to plausibility, in favor of theoremhood: $\mathcal{C}(\mathsf{B})$ in fact proves: if $\mathsf{Quant}_{(e\to t)\to t}(\forall)$, then $\mathsf{Quant}_e(\Pi)$.

$\mathcal{C}(\mathsf{B})$ also proves:

$$\forall X(\mathsf{Quant}_e(X) \to X \subseteq \Pi)$$

And so, granted the assumption $\exists X(X = \Pi)$, we have an argument that Q-Relativism is simply false.

[28]We close under both the Rule of Generalization and the Rule of Equivalence because, just as you might think that part of $\wedge$'s logical behavior is having it be that $\lambda pq.p \wedge q \to p$ should hold of arbitrary p and q, so you might want $\lambda p.p \to \forall_\sigma(\lambda x^\sigma.p)$ (the form corresponding to Vacuity) to hold of arbitrary p. But merely adding the Free Logical axioms and closing under the Rule of Generalization does not suffice for this latter desideratum—only closing under the Rule of Equivalence gets us that.

Suppose, on the other hand, $\neg\exists X(X = \Pi)$. In this case, we can define a new quantifier $\forall^+_{(e\to t)\to t}$ which is strictly more general than $\forall_{(e\to t)\to t}$. Set:

$$\forall^+_{(e\to t)\to t} := \lambda F^{e\to t}.\forall F \wedge F(\Pi)$$

$\mathcal{C}(\mathsf{B})$ proves, again on the assumption $\mathsf{Quant}_{(e\to t)\to t}(\forall_{(e\to t)\to t})$, that $\mathsf{Quant}_{(e\to t)\to t}(\forall^+_{(e\to t)\to t})$. Since we assume $\neg\exists X(X = \Pi)$, we have then that $\forall^+$ is at least as general as $\forall$, but not vice versa. This yields the formal version of the problem of parochiality: why do we care about a thesis stated in terms of $\forall_{(e\to t)\to t}$, when that quantifier is surpassed in generality by another, $\forall^+_{(e\to t)\to t}$?

One interesting corollary of this definition of Quant is what I take to be a rather natural account of what it is to be an absolutely general quantifier. Recall that AbsGen, the property of being absolutely general, is defined from Quant and $\leq$ as follows:

$$\mathsf{AbsGen}(O) := \mathsf{Quant}(O) \wedge \mathsf{Quant} \leq \lambda X.X \subseteq O$$

We have, then, the following fact:

Lemma 8.1. $\mathcal{C}(\mathsf{B})$ proves: If O is a quantifier, then: O is an absolutely general quantifier iff $E_O = (\lambda x.x = x)$

Where E_O is the "existence property" for O: $\lambda x.\neg O(\lambda y.y \neq x)$. (When O is $\forall$, this may be written, given natural dualities, as $\lambda x.\exists y(y = x)$).) To exist according to an absolutely general quantifier is to be self-identical—or, in view of the fact that $\lambda x.(x = x) = \lambda x.\top$, to be such that $\top$.

We can think of E_O, when O is a quantifier, as O's "domain of quantification". The domain of a quantifier is the collection of entities over which a quantifier "ranges"; whether $O(F)$ is true or not depends only on the behavior of F upon the entities in O's domain. Although domains of quantification are often treated as collections like sets or pluralities, it is perhaps more natural—as Stanley and Gendler Szabó (2000) remind us—to treat them as properties or similar entities, in view of the ways quantifiers interact with intensional phenomena like modalities or temporal operators. And given that O's domain is taken to be a property, it seems very natural that that property would have to be E_O. Presumably, an entity over which O ranges ought to exist according to O; likewise, if an entity exists according to O it seems that whether O applies to F or not will in part depend on whether F holds of x. What the lemma says, then, is that an absolutely general quantifier, in my sense, has as its domain the trivial property $\lambda x.x = x$ (i.e., $\lambda x.\top$). If there were any domain that deserved to be called absolutely general, it would surely be this one—what domain could be more comprehensive?—and so what quantifier could be more deserving of being called absolutely general than one with such a domain?

9. The Vindication of Relativism

The problem for the Relativist seemed to be there was no way of articulating their position that was both (i) faithful to the spirit of the view and (ii) consistent. We just saw how Q-Relativism foundered at the challenge of accomodating (i) and (ii). Given maximally general quantification over quantifiers, Q-Relativism is inconsistent; but without maximally general quantification over quantifiers, Q-Relativism will fail to capture the spirit of the Relativist view.

Turning to my versions of Relativism and Absolutism:

HO Relativism: $\mathsf{Quant}_e \leq \lambda X.(\mathsf{Quant}_e \not\leq \lambda Y.Y \subseteq X)$

HO Absolutism: $\mathsf{Quant}_e \not\leq \lambda X.(\mathsf{Quant}_e \not\leq \lambda Y.Y \subseteq X)$

We may ask how HO Relativism fares. If entailment gives us, as I have argued, a maximally general way of generalizing about quantifiers, then HO Relativism and HO Absolutism, I think, do capture the intuitive spirit of the Relativist and Absolutist views.

It remains only to argue, then, that HO Relativism is consistent. Let me begin with the headline: we can prove the consistency of both HO Relativism and HO Absolutism, against the background of $\mathcal{C}(\mathsf{B})$:

Theorem 9.1.

- $\mathcal{C}(\mathsf{B})$ and is consistent with HO Absolutism:
$$\mathsf{Quant}_e \not\leq \lambda X.(\mathsf{Quant}_e \not\leq \lambda Y.Y \subseteq X)$$
- $\mathcal{C}(\mathsf{B})$ and is consistent with HO Relativism:
$$\mathsf{Quant}_e \leq \lambda X.(\mathsf{Quant}_e \not\leq \lambda Y.Y \subseteq X)$$

The proof of this theorem may be found here.[29]

I conclude that in HO Relativism, we have a vindication of intuitive idea of Relativism about quantification, expressed in a consistent way.

One reason HO Relativism is consistent is that $\lambda P.(\mathsf{Quant} \leq \lambda X.(X(P)))$—the property of being a property such that being a quantifier entails applying to you—is provably only a quantifier under some rather implausible circumstances. This is in contrast to Π, $\lambda P.\forall X(\mathsf{Quant}(X) \rightarrow P)$, which is always provably a quantifier.

Consider the following sentence, where $\bot = \neg\top$:

$$\Box_p(p = \top \vee p = \bot)$$

Intuitively, this sentence says that (necessarily) there are only two propositions—$\top$ and its negation. $\mathcal{C}(\mathsf{B})$ proves that $\lambda X.\mathsf{Quant}_\sigma \leq X$ is a quantifier if and only if (necessarily) there are only two propositions in this sense.

[29] Or at this link: `https://tinyurl.com/QMTProof`.

Lemma 9.2. $\mathcal{C}(\mathsf{B})$ proves:

$$\mathsf{Quant}_{(\sigma\to t)\to t}(\lambda X.\mathsf{Quant}_\sigma \leq X) \leftrightarrow \Box_p(p = \top \vee p = \bot)$$

It is not *surely* absurd to maintain there are only two propositions (Frege thought so), but what is not surely absurd may still be probably absurd. Presumably we think there are propositions which are metaphysically contingent: such a proposition will have to be distinct from $\top$ or $\bot$. For $\top$, being expressed by tautologous sentences, is presumably metaphysically necessary, and hence $\bot$, being its negation, metaphysically impossible. $\lambda X.\mathsf{Quant}_\sigma \leq X$ thus fails to be a quantifier, at least on any plausible metaphysical view. We can thus vindicate the claim that entailment gives a a non-quantificational form of generality. HO Relativism may *generalize* over quantifiers by means of entailment, but it does not thereby *quantify* over quantifiers.

10. "Modal Quantification"

The same goes for $\lambda X.\Box_x Xx$ in general:

Lemma 10.1. $\mathcal{C}(\mathsf{B})$ proves:

$$\mathsf{Quant}_\sigma(\lambda X.\Box_{x^\sigma} Xx) \leftrightarrow \Box_p(p = \top \vee p = \bot)$$

But if $\lambda X.\Box_x Xx$ is not a universal quantifier, but rather only some device of generalization, then what sort of device of generalization is it?

Earlier, I suggested that its logical behavior is like that of a *necessitated* universal quantifier—of an operator like $\lambda X.\Box\forall X$—more than a bare universal quantifier. To substantiate this, let me list some of the principles that we would expect a necessitated universal quantifier to obey:

Instantiation: $\vdash \Box\forall x(\Box\forall y A \to A(x/y))$

K: $\vdash \Box\forall x(A \to B) \to \Box\forall x A \to \Box\forall x B$

Vacuity: $\vdash \Box\forall x(\lambda x.\top)$

4: $\vdash \Box\forall x A \to \Box\Box\forall x A$

Gen: If $\vdash A$, then $\vdash \Box\forall x A$

And, indeed $\mathcal{C}(\mathsf{B})$ proves the result of substituting $\lambda X.\Box_x Xx$ for $\Box\forall x$ in any of these claims. What suggests even more that $\lambda X.\Box_x Xx$ behaves like a necessitated universal quantifier is the following theorem:

Lemma 10.2. $\mathcal{C}(\mathsf{B})$ proves: $\mathsf{AbsGen}_\sigma(\forall_\sigma)$ iff $\lambda X.\Box_x Xx = \lambda X.\Box\forall_\sigma X$

What this lemma says, one might suggest, is that not only does $\lambda X.\Box_x Xx$ *act* like a necessitated universal quantifier; it would be identical to the necessitation of the absolutely general universal quantifier if there were such a quantifier.

Now, if HO Relativism is right, then there is no absolutely general universal quantifier. Even in this case, though, there may still be a way to regard $\lambda X.\Box_x Xx$ as a necessitated universal quantifier, and an absolutely general one at that, even in the absence of an absolutely general universal quantifier of which it is the necessitation.

If we think of necessitated universal quantification as a way of generalizing at all, we tend to think of it in terms of the combination of universal quantification with some kind of necessitation. But perhaps we need not think of it this way. After all, in its beginnings in Aristotle's modal syllogistic, formal modal logic was not stated in terms of a sentential operator and separate quantificational component, but rather a copula which fused the two. What I am suggesting is that we see necessitated universal quantification (despite its name) as a self-standing form of generalization, one that is accomplished usually by means of separate devices of quantification and necessitation—and named for those separate devices—but which can be accomplished even without them.

It is natural to wonder how we could understand being a necessitated-universal operator (note the hyphen—this is my term for the members of this would-be primitive class of generalizing devices) except as the result of joining together $\Box$ and some universal quantifier? But here again we might appeal to the idea that kinds of generalizing devices are defined by their logical behavior. Just as we defined universal quantifiers as operators with a certain logical behavior, so, perhaps, the necessitated-universal quantifiers can be seen as operators defined by their own logical profile. An operator like $\lambda x.\Box_x Xx$ may thus come to be a necessitated-universal operator not because there are $\Box$ and $\forall$ which come together to produce it, but rather because it satisfies the right logical role. Unfortunately, so far as I know, logicians since Aristotle have not been so interested in developing the logic of operators which combine quantificational and modal force. (This is an interesting question of pure logic in its own right, which I am addressing in other work.) The principles I listed above, however, I think give a good start on an axiomatization, but there is more work to be done.

Let me take it for granted, however, that we can understand the necessitated-universal operators as a class of generalizing operators of their own, one defined by fitting a certain logical profile (which at least includes the principles listed above), and that a given operator, like $\lambda x.\Box_x Xx$ could be such an operator even if there is no quantifier of which it could be the necessitation. In this case, there is a good argument to be made that $\lambda x.\Box_x Xx$ is absolutely general among such operators.

All this, I think, provides more support for the claim that HO Relativism does capture the intuitive spirit of Relativism. Suppose Abel the Absolutist and Riley the Relativist are debating and explaining their views to each other. Able smugly says, "I understand the intuitive picture, I understand what you are trying to say—you say that *no quantifier* is absolutely general, where *no quantifier* is itself absolutely general. You probably even want to say something stronger, that *necessarily*, no quantifier is absolutely general, where *no quantifier* is itself absolutely general. After all, there's not a whiff of contingency about the considerations you adduce in support of your view. Such a pity that by your own lights, you can't say this!"

Riley, intrepid higher-order logician that she is, then responds: "Dear Abel, I *can* say exactly that. I do want to say necessarily every quantifier fails to be absolutely general, where my generalization is as general as possible. But I have no need for an unrestricted universal quantifier to do it. Consider my claim (written with the helpful abbreviations of AbsGen and $\Box_X$):

HO Relativism: $\Box_X(\mathsf{Quant}(X) \to \neg\mathsf{AbsGen}(X))$

This is how I say, after all, what I want to say: *necessarily*-for-any quantifier X, X fails to be absolutely general, where *necessarily-for-any* is a maximally general necessitated-universal-quantifier. You might have thought that to express such a thought, I would need both a necessity operator and a maximally general universal quantifier; what we see now is that through $\lambda X.\Box_x Xx$, I can capture the thought without recourse to quantification.

"You and I agree, in fact," Riley may continue to say, "that this is what I wish to say: if you really do grasp an absolutely general quantifier with your term $\ulcorner\forall\urcorner$, then the sentence $\ulcorner\Box\forall X(\mathsf{Quant}(X) \to \neg\mathsf{AbsGen}(X))\urcorner$ will express the same proposition, by your own lights, as the HO Relativist sentence I have written. This is a consequence of our dear friend's Lemma 10.2. But, if I am right, you do not so express, and your sentence involving $\forall$ fails to get at what I mean. So let us debate with my sentence instead, since it gets at what is at issue without prejudging that issue."

11. Conclusion

I have argued, then, for a construal of the debate about Absolute Generality based on the tools of higher-order logic. Each of Absolutism and Relativism, on my construal, is consistent, and each captures the intuitive spirits of those views. My proposal is not totally neutral—it involves a controversial theory of higher-order identity—but the tools used, at least, are not parochial to the Absolute Generality debate. What allowed us to articulate a coherent form of Relativism was the key idea that quantification is not the only way we can generalize. Once we recognize Relativism about quantification is compatible with maximal generality obtained by other means, we can use those means to devise reasonable forms of Relativism.

I have shown my version of Relativism to be consistent, but this is not yet to have shown it to be plausible. Much of our theorizing, in mathematics and metaphysics, seems to depend for its significance and import on being absolutely general; if we can't achieve that generality via quantification, how are we to achieve it? The results of the last section suggest a way forward: if we cannot have absolutely general quantification, perhaps we can still have absolutely general "modalized" quantification, and use these "modalized-quantifiers" to do our theorizing. When it comes to mathematics and (certain parts of) metaphysics—in which our theories are, plausibly, necessarily true if true at all— modalized-quantification may well be enough for our purposes. Of course, we will likely want to do more than necessarily-universally-quantify, but this may not be an obstacle either. In other work, I suggest that we can define not just absolutely general necessitated-universal quantification, but also absolutely general necessitated-existential quantification, and necessitated quantification of various other kinds, all with only the resources of $\mathcal{L}$. It may seem strange at first that we might be able to modally-quantify in an unrestricted way without being able to quantify unrestrictedly, but in time, once we internalize the idea that quantifiers are but one star in a vast galaxy of ways of generalizing, that strangeness may yet fade.

References

Bacon, Andrew (2018). "The Broadest Necessity". In: *The Journal of Philosophical Logic*.

Bacon, Andrew and Cian Dorr (2024). "Classicism". In: *Higher-Order Metaphysics*. Ed. by Peter Fritz and Nicholas Jones. OUP.

Barwise, John and Robin Cooper (1981). "Generalized Quantifiers and Natural Language". In: *Linguistics and Philosophy* 4.2, pp. 159–219. DOI: 10.1007/bf00350139.

Boolos, George (1998). "Reply to Charles Parsons' "Sets and Classes"". In: *Logic, Logic, and Logic*. Harvard University Press.

Button, Tim (Oct. 1, 2010). "Dadaism: Restrictivism as Militant Quietism". In: *Proceedings of the Aristotelian Society* 110.3, pp. 387–398. ISSN: 0066-7374. (Visited on 09/28/2024).

Cartwright, Richard L. (Mar. 1994). "Speaking of Everything". In: *Noûs* 28.1, p. 1. ISSN: 00294624. DOI: 10.2307/2215917. URL: https://www.jstor.org/stable/2215917?origin=crossref (visited on 09/28/2024).

Dorr, Cian (2014). "Quantifier Variance and the Collapse Theorems". In: *The Monist*.

– (2016). "To be F is to be G". In: *Philosophical Perspectives*.

Dummett, Michael (1978). "The Philospohical Significance of Godel's Theorem". In: *Truth and Other Engimas*. Harvard University Press.

– (1981). *Frege: Philosophy of Language*. 2nd. Harvard University Press.

– (1991). *Frege: Philosophy of Mathematics*. Harvard University Press.

Field, Hartry (2004). "The Consistency of the Naive Theory of Properties". In: *Philosophical Quarterly*.

Fine, Kit (2005). "Class and Membership". In: *Journal of Philosophy*.

– (2006). "Relatively Unrestricted Quantification". In: *Absolute Generality*. Ed. by Agustin Rayo and Gabriel Uzquiano. OUP.

Florio, Salvatore (2014). "Unrestricted Quantification". In: *Philosophy Compass* 9.7, pp. 441–454.

Frege, Gottlob (1997a). "Function and Concept". In: *The Frege Reader*. Ed. by Michael Beaney. Blackwell.

– (1997b). "On Concept and Object". In: *The Frege Reader*. Ed. by Michael Beaney. Blackwell.

Fritz, Peter (2023). *Foundations of Modality: From Propositions to Possible Worlds*. OUP.
Gentzen, Gerhard (1964). “Investigations Into Logical Deduction”. In: *American Philosophical Quarterly* 1.4, pp. 288–306.
Glanzberg, Michael (2004). “Quantification and Realism”. In: *Philosophy and Phenomenological Research* 69.3, pp. 541–572. ISSN: 1933-1592. (Visited on 09/28/2024).
– (2006). “Context and Unrestricted Quantification”. In: *Absolute Generality*. OUP.
– (June 1, 2023). “Unrestricted quantification and extraordinary context dependence?” In: *Philosophical Studies* 180.5, pp. 1491–1512. (Visited on 09/28/2024).
Goodman, Jeremy (2016). “An Argument for Necessitism”. In: *Philosophical Perspectives*.
Jones, Nicholas and Peter Fritz, eds. (2024). *Higher-Order Metaphysics*. OUP.
Klement, Kevin C. (2024). “Higher-Order Metaphysics in Frege and Russell”. In: *Higher-Order Metaphysics*. Ed. by Peter Fritz and Nicholas Jones. Oxford University Press, pp. 355–377.
Lavine, Shaughan (2006). “Something about Everything: Universal Quantification in the Universal Sense of Universal Quantification”. In: *Absolute Generality*. Ed. by Agustin Rayo and Gabriel Uzquiano. OUP.
Leblanc, Hugues (1995). “On Axiomatizing Free Logic – And Inclusive Logic in the Bargain”. In: *Quebec Studies in the Philosophy of Science*. Kluwer Academic Publishers.
Lewis, David (1975). “Adverbs of Quantification”. In: *Formal semantics of natural language: papers from a colloquium sponsored by the King’s College Research Centre, Cambridge*. Ed. by Edward Louis Keenan. Cambridge University Press, pp. 3–15.
– (1991). *Parts of Classes*. Basil Blackwell.
Linnebo, Øystein (2006). “Sets, Properties, and Unrestricted Quantification”. In: *Absolute Generality*. Ed. by Agustin Rayo and Gabriel Uzquiano. OUP.
– (2010). “Pluralities and Sets”. In: *Journal of Philosophy* 107, pp. 144–164.
– (2018). “Dummett on Indefinite Extensibility”. In: *Philosophical Issues*.
– (2022). “Generality Explained”. In: *Journal of Philosophy*.
Linnebo, Øystein and Salvatore Florio (2021). *The Many and the One*. OUP.
McGee, Vann (2000). “‘Everything’”. In: *Between Logic and Intuition: Essays in Honor of Charles Parsons*. Ed. by Gila Sher and Richard Tieszen. Cambridge: Cambridge University Press, pp. 54–78.
– (2004). “Universal Universal Quantification: Comments on Rayo and Williamson”. In: *Liars and Heaps: New Essays on Paradox*. Ed. by JC Beall. Oxford University Press. (Visited on 09/28/2024).
Parsons, Charles (1974). “Sets and Classes”. In: *Nous*.
Peters, Stanley and Dag Westerståhl (2006). *Quantifiers in Language and Logic*. Oxford, England: Clarendon Press.
Prior, Arthur (1960). “The Runabout Inference-Ticket”. In: *Analysis* 21, pp. 38–39.
Rayo, Agustin (2006). “Beyond Plurals”. In: *Absolute Generality*. OUP.
Rayo, Agustin and Gabriel Uzquiano, eds. (2006a). *Absolute Generality*. OUP.
– (2006b). “Introduction”. In: *Absolute Generality*. Ed. by Agustin Rayo and Gabriel Uzquiano. OUP.

Rayo, Agustin and Timothy Williamson (2004). “A Completeness Theorem for Unrestricted First-Order Languages”. In: *Liars and Heaps: New Essays on Paradox*. Ed. by JC Beall. Oxford University Press.

Soysal, Zeynep (2020). “Why is the Universe of Sets Not a Set?” In: *Synthese*.

Stanley, Jason and Zoltán Gendler Szabó (2000). “On Quantifier Domain Restriction”. In: *Mind & Language* 15.2, pp. 219–261.

Studd, James (2013). “The Iterative Conception of Set: A Bimodal Axiomatization”. In: *Journal of Philosophical Logic*.

– (2019). *Everything, More or Less*. OUP.

Szabolcsi, Anna (2010). *Quantification*. New York: Cambridge University Press.

Warren, Jared (2017). “Quantifier Variance and Indefinite Extensibility”. In: *The Philosophical Review* 126.1, pp. 81–122. ISSN: 0031-8108.

Williamson, Timothy (2003). “Everything”. In: *Philosophical Perspectives* 17, pp. 415–465.

Yablo, Stephen (2004). “Circularity and Paradox”. In: *Self-Reference*. CSLI Publications.