ABSOLUTE GENERALITY AS A HIGHER-ORDER IDENTITY

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ABSTRACT. The question of Absolute Generality is whether quantifiers are ever as general as can be. Absolutists claim that quantifiers sometimes are absolutely general, while Relativists claim that quantifiers are never absolutely general. Although diverse philosophers have found the Relativist ethos compelling, it has been hard to articulate a consistent thesis which says what the Relativist seems to want to say. In this paper, I offer Relativists a way forward: I argue that what is needed to successfully state Relativism is a way of generalizing that is non-quantificational. After showing how to define such a device of generalization in terms of identity between properties in a higher-order logical language, I use the device to articulate a form of Relativism which I prove to be consistent and which I argue captures the intuitive vision of the Relativist.

When theorizing about the world, we sometimes try to make claims about it that are as general as can be. When the metaphysician says *everything* is self-identical, it is not as though the range of their quantification is supposed to stop just short of Australia, and fall silent on whether the kangaroos are self-identical, too. The question of Absolute Generality is whether we in fact succeed in this ambition, whether, as Studd (2019) puts it, we ever "use ... quantifiers to make claims that are as general as can be" (p. 1).[1] It has not always been clear, however, what exactly this question is asking. All parties can agree that there are two answers, *Relativism* and *Absolutism*:

Relativism: Quantifiers always fail to be absolutely general

[1]The history of the debate is interesting, if somewhat tortuous. Dummett (1978) is perhaps the first to suggest that there might be something defective about absolutely general quantification, and the theme is taken up in greater detail in Dummett (1981) and Dummett (1991). (What exactly Dummett takes the deficiency to be is a subtle matter; see pp. 529ff. in his Dummett (1981), and Cartwright (1994) and Linnebo (2018) for some discussion.) The idea emerges indepedently (so far as I can tell) in Parsons (1974), also in the context of set theory; Glanzberg (2004) (and the sequels Glanzberg (2006) and Glanzberg (2023)) are further developments of the Parsonian line. Dummett and Parsons seem to advance Relativist positions. Important early skeptical responses to their arguments for Relativism are, respectively, Cartwright (1994) and Boolos (1998). (Cartwright's response is further developed in Rayo and Williamson (2004), McGee (2004), Linnebo (2006), Rayo (2006) and Linnebo and Florio (2021).) The challenges of formulating a coherent version of Relativism are briefly raised and discussed in Lewis (1991) and McGee (2000), but the first, I believe, extended treatment of the issue is due to Williamson (2003), and it is further discussed in Fine (2006), Lavine (2006), Button (2010), and Warren (2017). Recent surveys of the topic are the introduction to Rayo and Uzquiano (2006a) (a collection of 13 essays on the topic), the article Florio (2014) and the book Studd (2019), which features a particularly well developed attempt to articulate what is at stake between Relativism and Absolutism, and to defend Relativism.

Absolutism: Quantifiers are sometimes absolutely general[2]

But these answers are no more clear than the question. They are templates or slogans, awaiting specifications of what "quantifiers" are, what it is for quantifiers to "always" be a certain way, and what it is for quantifiers to be "absolutely general".

It is not uncommon in philosophy to have the positions in a debate characterized by slogans rather than precise statements; part of the philosopher's work is to turn the former into the latter. What is distressing about Absolute Generality is that it has proved distressingly difficult to precisify the slogans in a way which is both faithful to the motivations for the views and does not render Relativism inconsistent or otherwise self-defeating. It would be extremely natural to try to render Absolutism and Relativism in terms of quantification over quantifiers:

Relativism: No quantifier is absolutely general

Absolutism: Some quantifier is absolutely general

But, as we shall see, precisely because the Relativist calls into question the generality of quantifiers, this way of articulating the Relativist position will either be self-defeating or fail to capture the intuitive spirit of the Relativist's view. As a result, some philosophers have been skeptical that there is any coherent way of articulating Relativism—so that, no matter how we try to precisify our slogans, the Absolutist will come out on top—or any coherent question of absolute generality at all.[3]

My aim is to offer the debate a way forward: I will offer novel versions of Relativism (and its negation, Absolutism) stated with the resources of a higher-order logical language. They key idea, I will argue, is that the Relativist needs to recognize that there are ways of generalizing about quantifiers which are not themselves quantificational or defined in terms of quantification. I will show how we can, with the resources of higher-order identity, define such a way of generalizing. Turning to non-quantificational way of generalizing, I argue, allows the Relativist to avoid the pitfalls usually encountered. In particular, the Relativist can avoid the problem of coherence: my statements of Relativism (and Absolutism) is provably consistent.

Other philosophers have suggested ways of articulating what is at stake between the Absolutist and Relativist before, perhaps most prominently Fine (2006) and Studd (2019). These proposals, however, have problems of their own (as I argue elsewhere in my dissertation). For instance, they introduce special "modal" operators purpose-made for the Absolute Generality debate. Those already skeptical that there is a non-trivial debate may well be skeptical that these special-purpose operators are of any more repute; Studd (2019) himself is unsure he grasps the modality that Fine invokes (p. 147). A distinctive virtue of my proposal is that it will spell

[2]Here "quantifiers are sometimes F" and "quantifiers always fail to be F" are intended to be dual constructions: the former is to be equivalent to "it's not the case that quantifiers always fail to be F" and the latter to "it's not the case that quantifiers sometimes are F".

[3]See, for instance, Lewis (1991), McGee (2000), Williamson (2003), and Button (2010).

out Absolutism and Relativism using only the logical vocabulary of higher-order languages. This ideology is not uncontroversial, but it is certainly not parochial: philosophers of diverse stripes have defended the intelligibility of the higher-order devices to which I appeal, and have applied them in many philosophical contexts orthogonal to the Absolute Generality debate.[4]

The plan is as follows. In Section 1, I will introduce the Absolute Generality debate; in Section 2, I will discuss difficulties in articulating Relativism; in Section 4, I will introduce my key idea, non-quantificational generality, for how Relativism is to be successfully articulated and explain in brief how it is to work. The remaining sections develop these ideas from Section 4 in a formal setting, that of higher-order logic.

1. The Idea of Relativism (and Absolutism)

Philosophers sometimes say that absolutely general quantification is quantification over "absolutely everything", and then suggest that Relativism is the view that we cannot quantify that way—i.e., that we cannot quantify over absolutely everything.[5] Such formulations might seem to have the vice of not being particularly informative—what is it to quantify over absolutely everything?—but the more serious worry is that such formulations are just inconsistent or self-defeating. Lewis puts it pithily: If the Relativist proclaims we never quantify over absolutely everything, then it seems we may reply: "Lo, he violates his own stricture in the act of proclaiming it!" (Lewis (1991), p. 68).[6]

[4]See the contributions in Jones and Fritz (2024).

[5]For such formulations, see Linnebo (2006), Lavine (2006), Williamson (2003), or Button (2010). Some of these authors put Relativism this way only as part of an argument that such formulations do not adequately capture Relativism.

[6]There is, however, a perfectly respectable way philosophers and logicians sometimes use the slogan "absolutely general quantification is quantification over absolutely everything" which pertains to the relationship between a quantifier-expression "$\forall$", for instance, in an object-language and our meta-language quantifier "absolutely everything". Philosophers sometimes say that a quantifier-expression $\forall$ is absolutely general just in case "$\forall x Fx$" is true iff for absolutely every o, "Fx" is true on a variable assignment mapping x to o. There are good questions about whether a quantifier-expression "$\forall$" could be absolutely general in this way. And there is a genuine puzzle about how one could do something like standard model-theoretic semantics in such a way that $\ulcorner\forall\urcorner$ would be absolutely general in the meta-semantic sense: standard model theory requires that, to specify the interpretation of a quantifier, we specify a set over which the quantifier is to "range". Since, one might have thought, there is no set comprising all sets, and hence no set comprising all things, we could not specify a set over which an absolutely general $\ulcorner\forall\urcorner$ is to range. As will become clear, however, this is not the sense of absolute generality relevant to the debate I am interested in, although the two issues are related. (This relation explains why much of the initial wave of work in the debate that I am interested in—especially the famous Cartwright (1994)—does seem to deal with the sense of absolutely general quantification I have mentioned in this footnote.)

A better way of getting a sense of what Relativism is supposed to be is to examine the overall philosophical vision motivating self-described Relativists. Philosophers have found diverse grounds for the rejection of absolutely general quantification, but one strand of thought important to many concerns what Studd (2013) calls a "trade off between 'generality' and 'collectability"' (p. 82).[7]

This trade off usually manifests in the realm of sets or similar entities. Classical logic shows that there cannot be a set of all sets that do not contain themselves, given the inconsistency of:

(Russell) $$\exists x \forall y(y \in x \leftrightarrow y \notin y)$$

Sometimes when a set fails to exist, this is because one of its would-be members fails to exist: if Socrates fails to exist, then his singleton must also fail to exist. The failure of there to be a Russell set r of all the non-self-containing sets—call them the ss—is not like that. The ss exist; the problem, rather, is that they somehow are not lassoed together into a single set. The inconsistency of (Russell) seems to reflect a failure in *collectability*.

The Absolutist accepts that what seems to be the case is the case: since $\exists$ is as general as can be, the fact that $\neg\exists x \forall y(y \prec ss \rightarrow y \in x)$ means there is no sense in which a set of the ss has any claim to existing. For the Relativist, by contrast, the failure is merely apparent. Although the Relativist, of course, recognizes (Russell) is inconsistent and so false, they will instead say that:

(Russell$^+$) $$\exists' x \forall y(y \in x \leftrightarrow y \notin y)$$

Here, the initial quantifier, $\exists'$ and its dual $\forall'$ are *distinct* from our original $\forall$ and $\exists$. Putting things roughly, the Relativist recognizes that r does exist according to $\exists$ ($\neg\exists x(x = r)$), but asserts that it does exist according to some other quantifier $\exists'$. The Relativist is keen to ensure that the s's can be collected, and since $\exists' x(x = r)$, collectability is achieved.[8]

The achievement, though, comes at the price of generality lost. An absolutely general universal quantifier, one would think, would be one such that it would surpass in generality whatever other universal quantifier one might compare it to. But $\forall$ plainly doesn't meet this standard: *all even numbers* is not as general as *all numbers*, because not every number is an even number. Likewise, $\forall$ is not as general as $\forall'$, since r exists according to the latter but not the former. So $\forall$ is not absolutely

[7]Other motivations have to do with the alleged links between quantification and sortals, and general, Carnapian worries about ontology. I myself am not convinced that all these motivations really lead to one common philosophical ethos.

[8]Why is the Relativist so keen not to give up on collectability? The point is often put in terms of explanation: consider all the sets which do not contain themselves—not under that description as it were, but just *those sets*, the plurality of them. What stops there from being a set of them? What is the difference between these sets, and, say, Earth, Mars, and Venus, a trio of which there is surely a set? How precisely this explanatory challenge is supposed to work is an important question. A classic discussion is Dummett (1991) (pp.315–316), other discussions include: Studd (2013) (pp. 698–701), Fine (2006) (pp. 23–25), Linnebo (2010), Yablo (2004) (pp. 148–150), and Soysal (2020).

general, according to the Relativist. And, of course, not just $\forall$—the Relativist is just as keen to say that there should be, in some sense, a set of all (according to $\forall'$) the sets which do not contain themselves; but this new Russell set for $\forall'$ itself can only exist according to yet another quantifier $\forall''$. And now we are off to the races: there will be an ever-extending hierarchy of quantifiers, each more general than the last, never reaching a maximum.

2. Filling in the Slogans

This is the sort of philosophical vision that I aim to capture with the slogan:

Relativism: Quantifiers always fail to be absolutely general

The opposing vision, according to which the hierarchy hits some upper limit, is:

Absolutism: Quantifiers are sometimes absolutely general

These slogans involve three moving parts: they involve a notion of (i) of being a "quantifier"; (ii) of what it is for quantifiers to always be a certain way; and (iii) what it is for a quantifier to be absolutely general. This third notion might seem itself to subdivide into two parts: it is natural to think that for a quantifier to be "absolutely general" is for it to be completely general, that is, maximally general by some ordering of generality—the question then is what (iiia) this notion of maximality is and (iiib) what the relevant ordering of generality might be.

By "quantifiers", I do not mean quantificational expressions but rather the sort of thing those expressions mean. More or less following Frege, and modern linguistics in the tradition of generalized quantifier theory, let us think of the meanings of quantifier-expressions as being certain properties of properties: just as "Bessie is kind" says that the property of being kind applies to Bessie, so "every cow is kind" says that the property *every cow*—that is, the property of applying to all cows—applies to the property of being kind.[9]

Quantifiers come in many varieties: there are existential and universal quantifiers, of course, but also proportional quantifiers, like "most moose" or "all but finitely many mice". For any given kind of quantifier, we could debate between versions of Relativism and Absolutism about quantifiers of that kind: are universal quantifiers ever absolutely general? Are "most" quantifiers? In the set-theoretic motivations for Relativism, though, it would seem to be universal and existential quantifiers which feature most prominently, and so it is Absolutism and Relativism

[9] I use "property" where Frege would have used (the German equivalent of) "function" or "concept"; my properties of properties are his second-level concepts. (See, for instance, Frege (1997b), Frege (1997a), and the discussion in Klement (2024).) Linguists and logicians, by contrast, sometimes say quantifiers are properties of sets (as, for instance, Peters and Westerståhl (2006), p. 12) or sets of properties (as, for instance, Szabolcsi (2010), p. 7) or just sets of sets (as in Barwise and Cooper (1981)). In these cases, the invocation of sets is a simplifying assumption, one which allows us to ignore intensional aspects of language. Linguists also often distinguish between so-called "global" and "local" quantifiers; I mean here to be speaking of the latter.

about those kinds of quantification which I wish to articulate. (I tackle the other cases elsewhere in my dissertation, building on the approach here.)

In fact, I will focus only on universal quantifiers, but only as a matter of convenience. Existential and universal quantifiers occur naturally in pairs, each member of the pair being definable in terms of the other and negation (given classical principles about duality). It is hard to see how one member of such a pair could be maximally general without its twin being maximally general also. Absolutism about existential quantifiers, therefore, seems to stand or fall with Absolutism about the universal, and so we may without loss of generality focus on the latter. It will be helpful sometimes to say talk about existential quantifiers being absolutely general or not, but this is to be understood as shorthand for claims about the universal counterparts being absolutely general.[10]

Later in the paper, I will say more about universal quantifiers, and try to define what it is to be one in logical terms. For now, we should turn our attention to the parts of the slogans which are most problematic: "always" and "absolutely general".

The standard canons for translating these terms would dictate that we understand them in terms of quantification over quantifiers. Just as "Even numbers are always divisible by two" usually means that *all* even numbers are divisible by two, so Relativism would be the view that *every* quantifier is not absolutely general.[11] I have already suggested, being "absolutely general" is a matter of being maximally general along some ordering of generality; but maximality, also, is usually cashed out with quantification: to be maximally tall is to be at least as tall as anything else. For a universal quantifier Q to be absolutely general, then, is just for it to be at least as general as every quantifier.

And so the standard canons would tell us that this is how Relativism and Absolutism ought to be understood:

Q-Relativism: No quantifier is at least as general as every quantifier

Q-Absolutism: Some quantifier is at least as general as every quantifier

Is Q-Relativism a good way for a Relativist to regiment their view? Even without pinning down precisely what ordering "at least as general as" is supposed to

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[10] It is customary in the Absolute Generality literature to focus not on quantifiers (in the sense I understand them), but rather on domains of quantification. Thus Rayo and Uzquiano (2006b) frames the question of Absolute Generality as one about whether there is "an all-inclusive domain of discourse" (p. 2). On the face of it, however, the debate about Absolute Generality would seem to concern quantifiers, not domains. When trying to describe the Relativist's position, we say things like "The Relativist says that $\forall$ is not absolutely general because $\forall'$ is not a restriction of it". This is a statement about quantifiers; perhaps we could paraphrase the point in terms of domains, but why should we? More importantly, though, I think that all reasonable ways of cashing out Domain Relativism and Domain Absolutism are going to be close variants of the quantifier-first theses that I will state, as I argue elsewhere in my dissertation.

[11] See Lewis (1975) for a view on which adverbs like "always" are in general quantificational.

express, one may already feel some creeping queasiness about stating Relativism in these terms. Doesn't the Relativist claim that quantifiers are never maximally general? Presumably, then, the Relativist will claim that *no quantifier* doesn't generalize over quantifiers in a maximal way, but rather is just another quantifier which is surpassed in generality by another quantifier further up in the hierarchy.

Accordingly, Q-Relativism will seem a little parochial: so what if no quantifier is at least as general as every quantifier, for some particular, parochial quantifiers *no quantifier* and *every quantifier*? This is compatible with there being some more expansive quantifier over quantifiers Q such that "there is some quantifier which is absolutely general" is true, when "some quantifier" expresses this more expansive quantifier Q. By the Relativist's own lights, Q-Relativism, the sentence "No quantifier is at least as a general as every quantifier", fails to say what the Relativist wishes to say.

The Relativist might object that denying quantification can be maximally general *simpliciter* doesn't mean that it cannot be absolutely general over a limited range: no matter what we think about sets, perhaps maximally general quantification over donkeys (or quantifiers) is left unharmed. But there are more acute problems for Q-Relativism. Consider the following property of properties Π:

being a property to which all universal quantifiers apply

It is plausible that Π itself is a quantifier. One way to informally argue this point is to conceive of Π as the "conjunction" of all universal quantifiers, and then observe that conjunctions of universal quantifiers seem themselves to be universal quantifiers: if *every cow* and *every cat* are universal quantifiers, then so is *every cow and every cat*.

If, however, Π is a universal quantifier, then it is hard to see how it could fail to be absolutely general, since being an absolutely general is understood as being at least as general as every quantifier. On any plausible understanding of this ordering of generality, Π will be at least as general as every quantifier, just as *every cow and cat* is at least as general as *every cow* on any plausible understanding, so

Π is the source of real trouble for the Relativist. Suppose some quantifier is identical to Π; then Q-Relativism is simply false. For on that supposition, we can infer from the truth:

- Π is a quantifier and is at least as general as every quantifier

That:

- Some quantifier is at least as general as every quantifier

Which is Q-Absolutism, Q-Relativism's negation. In Section 8, when we have introduced a suitable formal language, we will return to this argument and tighten it up.[12]

[12]Parallel arguments are to be found in Fine (2006) and Linnebo and Florio (2021).

If Q-Relativism is to be consistent, then the Relativist cannot accept that some quantifier is identical to Π. This, however, is not a stable position for the Relativist either. Perhaps Π is not in the range of the quantifier which "every quantifier" in Q-Relativism designates. The Relativist spirit, nonetheless, would seem to suggest that there ought to be a more expansive quantifier in whose range it does fall, just as the Russell set r for a quantifier $\forall$ only existed by the lights of a more expansive quantifier $\forall'$. If there is a more expansive sense of "every quantifier", it would seem that we are back to the problem of parochiality: Q-Relativism only gets at what the Relativist would like it to if the quantifiers over quantifiers really say something about quantifiers as a whole.

Not everything, at least, is doom and gloom for the Relativist—they at least can make some good sense of what it is for one quantifier to be at least as general as another. One might have worried that this notion, too, would require quantification of some kind to be stated: $\forall$ is at least as general as $\forall'$ just in case *everything* which $\forall$ ranges over is something that $\forall'$ also ranges over.

Luckily, we can do better: say that $\forall$ is at least as general as $\forall'$ just when

$$\forall' x \exists y (y = x)$$

Where $\exists$ is the dual of $\forall$. It will sometimes be helpful to have a locution for the converse relation: when $\forall$ is at least as general as $\forall'$, we can also say that $\forall'$ is a *restriction* of $\forall$, and we can abbreviate this as $\forall' \subseteq \forall$. This notion of restriction—which only requires the two quantifiers we are comparing themselves—seems to do the job we want. *Every cow* is a restriction of *every ungulate*, since every cow is identical to some ungulate, for instance. Likewise, every set is identical to some^{+} set, where *some*$^{+}$ ranges over the sets that *every set* does and also the Russell set for *every set*.

3. Interlude: Articulating Absolutism?

Relativism, then, seems to be rather hard to state in a coherent way. One might think that Absolutism is bound for a similar, ineffable fate. Yet many philosophers have thought that Absolutism is on firmer ground than Relativism when it comes to being articulable.[13] This might seem surprising: given that Absolutism is supposed to be the negation of Relativism, one would think that either both theses could be articulated or neither could: given a statement of Absolutism, just throw a negation in front of it to obtain a statement of Relativism.

The solution to this little puzzle turns on the important insight of Stalnaker (1978): what proposition a sentence expresses depends on what facts obtain. Had the word "water" meant gold and the rest of the English language been left undisturbed, "water is H20" would have expressed the proposition that *gold* is H20, not the proposition it in fact expresses, that *water* is H20.

[13]See Williamson (2003) p. 433, and Studd (2019), Section 5.1. My conclusions in this section are largely in line with theirs, with the exception of the final paragraph.

Take for granted that there are propositions A and R which correspond to the Absolutist and Relativist positions. The problem the Relativist faces is not one of inexpressibility *simpliciter*, but inexpressibility *by their own lights*: if R were true, it is not clear that we could use the words "No quantifier is at least as general as every quantifier" to state that would-be truth, nor that we could use the words "Some quantifier is at least as general as every quantifier" to state its negation. The challenge for the Relativist is that their position undermines attempts to state it: by denying the possibility of quantification that is as general as can be, they seem to foreclose the possibility of stating a claim which would adequately capture that denial.

The sense in which Absolutism is on sounder footing than the Relativist is that, given Absolutism's own truth, Absolutism is not so difficult to state. If "every quantifier" really expressed an absolutely general quantifier—and if Absolutism is true, what could stop it from expressing that?—it is difficult to see what more one would want from the status of being absolutely general than whatever is expressed by "being at least as general as every quantifier", and so difficult to see what more one could want in a statement of Absolutism than whatever is expressed by "some quantifier is at least as general as every quantifier".

But this may not mean that they are on ground that is sound *simpliciter*. From the difficulties of articulating Relativism, one might draw the lesson that Relativism, and Relativism alone, is a defective position. One might, however, draw the more radical lesson that the very debate between Relativists and Absolutists itself is defective, as Button (2010) does.[14] Self-described Relativists and Absolutists presuppose there is sense to be made of of the notions of "absolutely general" quantification and the like, but the difficulties in articulating Relativism, the thought goes, reveal that there is no sense to be made of these notions. Genuine debates, one might think, can be articulated in a neutral way. To answer this kind of skeptic is as much the task of the Absolutist as it is of the Relativist.

4. Beyond Quantification

Suppose, then, we want an articulation of the Absolute Generality debate which is acceptable to Relativist and Absolutist alike, and which could convince a skeptic that there is sense to be made out of the debate's proprietary notions. We will need to augment the quantificational locutions of Q-Relativism and Q-Absolutism or leave them behind altogether and generalize about quantifiers in some other way.

Some philosophers, such as Fine (2006) and Studd (2019), have thought the right way to articulate Absolutism and Relativism is by appealing to special modal operators. We need not run away from quantification, but we must augment it in

[14]Button (2010) sketches a position according to which "any putative doctrine whatsoever about "unrestricted quantification" fails in its ambitions, whether that doctrine is [Absolutist] or [Relativist]" (p 395).

order to get at what is issue. These philosophers take seriously the "ability" in "collectability": the lesson of Russell's paradox is that, given $\forall$, we *can* produce a new quantifier $\forall'$ which is not a restriction of $\forall$, and so they introduce a new modal operator $\Diamond$ (and its dual $\Box$) to latch on this sense of potentiality. The modal relativist's view is that, *necessarily*, any quantifier is such that, *possibly*, there is another which is not a restriction of the first:

M-Relativism: *Necessarily*, for any quantifier X, there is *possibly* a quantifier Y such that Y is not a restriction of X.

But M-Relativism falls victim to the same problem of Π. So long as truths of logic are necessary in the relevant sense, it appears that we will have that Π will be absolutely general in the relevant sense, since we will have:

Necessarily, every quantifier is a restriction of Π

Moreover, as I mentioned already, there is some worry about the intelligibility of such operators.

Others philosophers, adopting a strategy that hearkens back to Russell, have thought we should leave quantification behind and turn instead to *schemas*. Schemas are a metalinguistic means of characterizing a usually infinite set of sentences in the object-language. To take an example from set theory:

Separation: $\exists y \forall x(x \in y \leftrightarrow \phi(y))$ for any formula $\ulcorner \phi(y) \urcorner$ of the language

In committing to the schema, we commit to every instance, and thereby attain a certain kind of generality—without, at least in the object language, recourse to quantification.

On this model, to say that "quantifiers are always F" is actually not to say one sentence, but rather to affirm a whole schema of instances. For $\forall$ to be maximally general, then, will be for the following schema's instances to be true:

If X is a quantifier, then X is at least as general as $\forall$

A problem with schemas, and a well known one at that, is that they cannot be embedded in more complex constructions. What if we wished (as the Relativist does) to deny, say, a particular quantifier $\forall$ is maximally general. If we express $\forall$'s maximal generality via schema, then we would need, in some sense, to negate the schema—but what exactly does that mean? We cannot deny the schema in the relevant sense by accepting the negation of each instance, since $\forall$ is at least as general as itself. Do we say that some instance is negated? If so, which one—what claim is our theory to include?[15]

And even if we overcome this problem, the Relativist still must say that quantifier always fail to be absolutely general; this would seem to require that they must

[15] See Williamson (2003) for more criticism of schemas, and Lavine (2006) for a rejoinder on the schematicist's behalf. Studd (2019) responds to Lavine's rejoinder.

embed their denial of a schema—this is how they say of a particular quantifier fails to be absolutely general—within another schema—this how they say that something is true of quantifiers in general. How might this be done?

The schematicist is right that we need a non-quantificational way of generalizing if we are to articulate Relativism; but they err in absconding to the meta-language to achieve this generality. What we need is a device of non-quantificational generality that lives in the object-language. One might think that there are no such devices to be found: quantifiers are our mode of expressing generality in the object-language, and our only mode—there is no way of saying that some feature F holds of quantifiers in general which does not involve the resources of quantification.

Such a view, I think, underestimates the diversity of our philosophical resources. Consider, for instance, the claim:

To be a whale is to be a swimming mammal

Following Dorr (2016), we might regiment this sentence using our higher-order language, and in particular higher-order identity, as the following claim:

$$\text{Whale} = \lambda x.\text{Swimming}(x) \wedge \text{Mammal}(x)$$

Where $\lambda x.Fx \wedge Gx$, for properties F and G, is the property of being F and G.

This claim seems to entail a certain kind of general connection between being a whale and being a mammal, that being a mammal somehow is implied by being a whale. In particular, given a very basic logic, it follows from this identity that we should deem as true, for any constant c, the sentence "Whale$(c) \rightarrow$ Mammal(c)". If one regards schemas as conveying a generality of sorts, then, it appears, one should think this identity does as well.[16]

Despite having such generalizing import, it is far from clear that this claim of higher-order identity is any way grounded in, reducible to, or identical to a claim of quantification. There are views on which claims of identity, including first-order ones, are identical to claims of indiscernibility—for x and y to be identical is just for them to share all their properties—but all I claim is that it is not unreasonable to adopt a position which rejects such identification or reduction.

Quite a few of our philosophical idioms might be taken to both have generalizing import and be non-quantificational. Claims of essence—for instance, that it is in the essence of whales to be mammals—may have this sense, as well as claims of "mereological" containment between properties—such as part of being a whale is being a mammal. Let me call relations in this family *entailments*. To say that something holds of Fs in general, we say that being F *entails* being G.

[16]Other philosophers have also been attracted to the idea that there is some connection between such identities and generality. Linnebo (2022), for instance, briefly suggests that such an identity might be a ground or truthmaker for the universal claim $\forall x(\text{Whale}(x) \rightarrow \text{Mammal}(x))$.

Entailment relations, as I have said, seem to be non-quantificational ways of generalizing. We generalize about whales *as a whole* by seeing what being a whale entails. They also seem to be ways of generalizing that are, in an intuitive sense, as general as can be. If it is in the essence of whales to be mammals, is there any important sense in which there could be a whale which wasn't a mammal? Or, to generalize from the first example, consider the following entailment relation, defined in terms of identity between properties. Say that F entails G just in case F is identical to the property of being F and G; or, using the Dorr-like notation:

$$F = \lambda x.Fx \wedge Gx$$

Again, if to be F is just to be F and G, is there any important sense in which an arbitrary F-thing might fail to be a G-thing?

If entailment relations have these two features of being (i) non-quantificational and (ii) as general as can be, then they will allow us to state a form of Relativism which is both (i) coherent in a way that Q-Relativism was not and (ii) faithful to the intuitive spirit of the view. In particular, the proposal is that "quantifiers are always F" is to be understood as the claim that being a quantifier entails being F. $\forall$ will be absolutely general just in case it is a quantifier and being a quantifier entails being a restriction of $\forall$. And so Relativism and Absolutism will be the following theses:

Entailment Relativism: Being a quantifier entails being an property of properties such that being a quantifier does not entail being a restriction of it

Entailment Absolutism: Being a quantifier does not entail being an property of properties such that being a quantifier does not entail being a restriction of it

We can put this more lucidly by separating out from this the definition of "absolutely general" that entailment gives. To be absolutely general is to be an property of properties X such that being a quantifier entails being a restriction of X. Then we have:

Entailment Relativism: Being a quantifier entails being not absolutely general

Entailment Absolutism: Being a quantifier does not entail being not absolutely general

Because entailment gives us a way of generalizing about quantifiers in a maximally general way, Entailment Relativism captures the intuitive ambition of Relativism to say that quantifiers as a whole fail to be absolutely general. For the same reason, one might suspect that Entailment Relativism is self-defeating, just as Q-Relativism was when the quantifiers in it were understood as maximally general. Doesn't the Relativist wish to claim that one cannot generalize in a maximally general way? And isn't this what I say entailment allows us to do?

Here we must keep in mind the distinction between entailment and quantification. The Absolute Generality debate, as I understand it and think other philosophers have understood it, concerns whether *quantifiers* ever attain maximal generality, not whether entailment or other devices of generality may have this status. (In light of this, it might be better to say the debate is about "absolutely general quantification" rather than "absolute generality".) It does not contravene the spirit of Relativism to maintain that we can generalize about quantifiers as a whole, so long as we remember that to generalize about quantifiers as a whole is not necessarily to quantify over quantifiers as a whole.[17]

All this, of course, is a little sketchy. The remainder of this paper attempts to implement these informal thoughts in the more regimented setting of a particular higher-order logical language. The technical work, while it raises interesting issues on its own, serves mostly to support the philosophical vision laid out here. So that we do not lose sight of how the technical work is supporting that vision, let me give a road map of the sections to come.

First, I will introduce the higher-order logical language that we will use to regiment all this talk of properties, identity between properties, and entailment (Section 5. We'll use that language to define a particular notion of entailment—in fact, the one where F entails G just in case $F = \lambda x.Fx \wedge Gx$. This entailment relation, as one can see, is definable without special ideology purpose-made for the Absolute Generality debate, but rather purely logic resources, along with property identity. I will then argue that this relation can do all the work that I have said entailment relations can do (Section 7).

To maintain that Entailment Relativism does not fall inconsistency like Q-Relativism, we will have to maintain a firm line between quantification and entailment in my sense. To ensure my chosen relation doesn't illicitly inch across this boundary, I will also give an account of what universal quantifiers are (Section 8). The idea will be that to be a universal quantifier is just to have the right kind of logical behavior. This, too, will ultimately be understood in terms of entailment. With the account of quantification in hand, I will be able to prove that my chosen notion of entailment is not quantificational (Section 9). Indeed, we will be able to prove that Relativism, on my construal, is a consistent position. By adopting turning to the machinery of entailments, then, we may articulate a form of Relativism which is provably consistent and which, arguably, captures the intuitive spirit of the view.

5. Going Higher-Order

At this point, it will be helpful to introduce some formalism to regiment and precisify our theorizing about quantifiers and entailment. If quantifiers are properties of properties, then this framework will have to be one for theorizing about

[17]The issue is delicate. I am motivated by the idea that the best considerations in favor of Relativism rely on special features of quantifiers, as opposed to other ways of generalizing.

properties and properties of them. We need to be careful here. The lesson of Russell's paradox is that it is not long before naive property-talk spoils: consider the property of being a property that doesn't instantiate itself.

I will formalize property-talk by adopting a *higher-order* language.[18] Many kinds of language, formal and natural alike, have different syntactic categories to which their expression belong. This is true no less of the first-order languages with which logicians and philosophers often work; these languages have singular terms (constants c and variables x), as well as sentences, and predicates which combine with singular terms to form sentences. They also include sentential connectives and operators, like $\wedge$ and $\neg$. Higher-order languages, by contrast, are more liberal. They may include higher-order predicates, which combine with predicates of the usual kind to form sentences, just as predicates of the usual kind combine with singular terms. A widely accepted hypothesis in linguistics, in fact, is that English itself is a higher-order language of this kind.

There are other ways one might go—if one has a well enough worked out formal first-order theory of properties, then that would do the job.[19] Theorists so inclined are welcome to translate my higher-order view into their own first-order theoretic terms.

The higher-order language $\mathcal{L}$ that I will use is defined as follows. $\mathcal{L}$ has two basic syntactic categories: type e, the type of singular terms, and type t, the type of formulae. The rest we define recursively: when σ and τ are types and $\tau \neq e$, then $(\sigma \to \tau)$ is a type, the type of an expression which, when composed with an expression of type σ, returns an expression of type τ. Thus $e \to t$ is a type: the type of unary predicates which, when combined with a singular term, return a formula. And $t \to t$ is a type: this is the type of sentential operators, such as $\Box$ from modal logic, or $\neg$.

The terms at each type may be complex or simple. The simple terms are constants and variables—we assume an infinite stock of variables at each type, and sometimes use superscripts to indicate a variable's type. Our simple terms will include familiar logical constants:

- $\wedge$, of type $t \to t \to t$[20]
- $\vee$, of type $t \to t \to t$

[18]See Fritz (2023), Chapter 1, for an argument for the conclusion that higher-order languages are our best way of consistently theorizing in the way property-talk is supposed to allow us to do.

[19]That is not to say that there has not been much interesting work on developing consistent and sufficiently strong first-order theories of properties: see especially Fine (2005) and Linnebo (2006) for recent attempts. My own view is that the limitations these theories impose upon property-talk to avoid paradox make them ill suited to serve as a metaphysical framework. (Both Fine's and Linnebo's theories take place against a background of classical logic, but there is also a literature in developing first-order theories of properties in the non-classical tradition as well: see for instance Field (2004).)

[20]Properly put, this should be $(t \to (t \to t))$. But I'll drop parantheses in types when possible, assuming a convention of associating types to the right.

- $\neg$, of type $t \to t$

In addition to these, I will also assume that, for each type σ, we have an identity-expression $=^\sigma$ of type $\sigma \to \sigma \to t$, which indeed can be seen as a generalization of identity on type e to higher types—for instance, $=^\sigma$ will have a higher-order version of Leibniz's Law that goes along with it.[21]

Complex terms can be made in two ways: application and λ-abstraction. Application is the generalization to arbitrary types of the way the first-order logician combines predicates and singular terms to form formulae: when A is an expression of type σ (which we may notate as $A : \sigma$) and $B : \sigma \to \tau$, $(BA) : \tau$.

λ-abstraction, as I've already indicated, is our device for creating terms for complex properties. In natural languages like English, it is easy for us to form predicates with complex internal structure. For instance, "is red and round" is a complex predicate in some sense formed from "is red" and "is round". In our formal language, we accomplish a similar task with λ-abstraction: when we have some formula A, in which a variable x may or may not be free, then $\lambda x.A$ is a complex predicate which we might gloss in English as "being such that A". Thus we may write $\lambda x.\text{Red}(x) \wedge \text{Round}(x)$ as our formal gloss for "is red and round". Though I have emphasized λ-abstraction's use in creating complex monadic predicates, λ-abstraction may be used to create complex predicates of arbitrary type. In general, when $A : \tau$ and $x : \sigma$ is a variable (and $\tau \neq e$), $\lambda x.A$ is a term of type $\sigma \to \tau$.

As usual, I will be using the language of "propositions" and "properties" to gloss higher-orderese (propositions are entities at type t, properties and relations entities at type $\sigma_1 \to (\sigma_2 \to \ldots \to (\sigma_n \to t) \ldots)$, and so on).

With all this in place, we regiment some of what I have said about Entailment and Relativism. For F to entail G—I will notate this as $F \leq G$—is defined, in $\mathcal{L}$ like so:

$$F = \lambda x.Fx \wedge Gx$$

Let me introduce a predicate $\mathsf{Quant} : ((e \to t) \to t) \to t$ which is intended to express the property of being a quantifier. (In Section 8, we will also define this term in terms of identity and the logical constants.) The property of being absolutely general, then, is:

$$\lambda X.\mathsf{Quant}(X) \wedge (\mathsf{Quant} \leq (\lambda Y.Y \subseteq X))$$

Where $Y \subseteq X$, recall, means Y is a restriction of X. (In our formal language, $Y \subseteq X$ is defined as $Y(\lambda x.\neg X(\lambda y.y \neq x))$. Plug in $\forall$ and $\forall'$ for X and Y to see how this corresponds to what was said before.) We can let AbsGen abbreviate this expression.

Finally, then, we have that Relativism and Absolutism would be stated as:

HO Relativism: $\mathsf{Quant} \leq \lambda X.\neg\mathsf{AbsGen}$

[21] See Dorr (2016) for more discussion and defense of the legitimacy of using such devices.

HO Absolutism: $\mathsf{Quant} \not\leq \lambda X.\neg\mathsf{AbsGen}$

Or, in more expansive form:

HO Relativism: $\mathsf{Quant} \leq \lambda X.\neg(\mathsf{Quant}(X) \land (\mathsf{Quant} \not\leq \lambda Y.Y \subseteq X))$

HO Absolutism: $\mathsf{Quant} \not\leq \lambda X.\neg(\mathsf{Quant}(X) \land (\mathsf{Quant} \not\leq \lambda Y.Y \subseteq X))$

6. Interlude: Higher-Order Languages and Absolutism

$\mathcal{L}$ contains no primitive quantifier symbols at all. This we can see as a matter of philosophical hygiene: if Relativism and Absolutism are to be articulated without recourse to quantification, let us use a language without quantification baked in.

It is sometimes claimed that to use higher-order languages is to give up on absolutely general quantification or the spirit of Absolutism.[22] That objection goes something like this: using a typed language goes hand in hand with a view of reality and its contents being somehow stratified in the same way as the typed language in question. There will be the individuals, corresponding to type e, and the properties, corresponding to type $e \to t$, and so on. In this typed language, however, any quantifier-expression is of type $(\sigma \to t) \to t$ for some type σ, and so the corresponding quantifier would only range over the entities at type σ. The alleged problem is that a truly absolutely general quantifier would range over not just some limited collection of entities, such as those at a particular type, but entities at all types. This raises a worry for my approach: if just using $\mathcal{L}$ already gives up on the spirit of Absolutism, how can we hope to articulate a thesis which really captures the spirit of the view?[23]

Just as a dialectical matter, it is worth noting that Williamson (2003), after arguing at length of the horrors of Relativism, embraces higher-order languages and Absolutism both. But we need not hide behind Williamson's embrace of the higher-order; we can meet the objection head-on. First: one may worry that the objection overgenerates. First-order languages also have numerous syntactic categories (nominal terms, predicates, sentential connectives and operators), but include only quantifiers over nominal terms. Doesn't this use of numerous syntactic categories go hand in hand with a view of reality and its contents being somehow stratified in the same way as the syntactic categories in question? Won't the nominal quantifier fail to range over entities corresponding to terms, predicates and sentential operators, and therefore fail to be truly absolutely general?

[22] See Linsky (1992), p. 262, for instance.

[23] See Krämer (2017), Florio and Jones (2019), and Florio and Jones (2023) for more discussion about higher-order languages and absolute generality.

If there is an objection that applies to higher-order languages alone, it will have to be one based on something these languages do not share with first-order languages. The most natural diagnosis is that it is the presence of higher-order quantifier expressions in the language that gets the objection going. Even with the objection thus modified, however, $\mathcal{L}$ seems to escape it: by design, $\mathcal{L}$ excludes quantifier-expressions as a whole, and so higher-order quantifier expressions in particular. There may be other good objections to the ideology of $\mathcal{L}$, but the objection that it is at odds with Absolutism is not one of them.

7. A Theory of Higher-Order Identity

Our entailment relation, $\leq$, then, is defined as follows:

$$\lambda XY. X = (\lambda x.Xx \wedge Yx)$$

In other words, a property F entails a property G just when to be F is just to be F and G. Of course, there is not really one category of "properties" in the higher-order framework: for each type σ, roughly speaking, there is the type $\sigma \to t$ of properties of entities of type σ. Accordingly, there is really a notion of entailment $\leq^{\sigma \to t}$ for each type σ. [24]

Now, this notion of entailment will be of some use only with a sufficiently coarse-grained theory of higher-order identity. On theories of higher-order identity according to which propositions and properties are structured like the sentences and predicates we use to express them, it is doubtful whether we could find properties F and G such that F entails G. On coarser-grained theories such as that according to which, roughly, propositions and properties form Boolean algebras, $\leq$ will be better behaved. So my starting point will be the development of a sufficiently coarse-grained theory of higher-order identity. I should note, however, that there are other candidate definitions of $\leq$ which would work just as well for my purposes and would require a weaker background logic; I focus on the present definition and logic because they are especially simple and helpful for conveying the general idea of my approach.

A theory is just a set of formulae in a given language, and a theory is true just in case every *sentence*, i.e., closed formula, in the theory is true. Take first the relatively uncontroversial fragment of higher-order logic defined by the following schemas and rule (here, $\vdash P$ just means P belongs to our theory):

[24] In fact, we will be able to generalize further the applicability of the idea of entailment later in this section.

PC $\vdash P$ where P is a substitution instance of a theorem of classical propositional logic

Id $\vdash a = a$

LL $\vdash a = b \rightarrow Fa \rightarrow Fb$

β $\vdash P[(\lambda x.A)B] \leftrightarrow P[A[B/x]]$, where $A[B/x]$ is the result of replacing every occurence of x in A with B, so long as this can be done without any free variable in B becoming bound[25]

η $\vdash A[\lambda x.Fx] \leftrightarrow A[F]$, where x is not free in F

MP If $\vdash P \rightarrow Q$ and $\vdash P$, then $\vdash Q$

We can call this theory B (for "basic") and write $\mathsf{B} \vdash P$ when P is a formula included in B (i.e., a theorem of B). Any theory which includes B and is closed under Modus Ponens is a *basic theory*. Already from B, one can start making the case that the notion of entailment I have chosen is a linking relation. For we can prove the schema:

$$F \leq G \rightarrow Fc \rightarrow Gc$$

For any F, G and c. For suppose $F \leq G$ and Fc. If $F \leq G$, then $F = \lambda x.(Fx \wedge Gx)$. And if Fc, then $(\lambda X.Xc)F$ by β. So, by LL, we have $(\lambda X.Xc)(\lambda x.Fx \wedge Gx)$. And so, by appealing to β again, we derive $Fc \wedge Gc$ and so Gc.

The theory I want to adopt, however, is an extension of B, and is in particular to be the least basic theory closed under the following rule:

Logical Equivalence: If $\mathsf{B} \vdash P \leftrightarrow Q$, then $\vdash \lambda x_1 \ldots x_n.P = \lambda x_1 \ldots x_n.Q$

The resulting theory I will call $\mathcal{C}(\mathsf{B})$, the *closure* of B.[26]

The general ethos behind Logical Equivalence is something like logical equivalence suffices for worldly identity. Consider, for instance, sentences $A \wedge B$ and $B \wedge A$, which are provably logically equivalent in B. One might think that these sentences must express the same proposition: one might think they will be true in all the same possible circumstances or worlds (in what scenario is the one true but not the other?), and so one will want to commit to the propositional identity $A \wedge B = B \wedge A$.

Logical Equivalence extends this general feeling that logical equivalence suffices for identity from type t to higher types. Consider, for instance, the complex predicates $\lambda pq.p \wedge q$ and $\lambda pq.q \wedge p$. Just as one felt that there was no difference between the conjunction of A and B in one order (say, $A \wedge B$) and conjoining them in the other ($B \wedge A$), so one may feel that there is no difference in general between *conjoining* propositions in one order ($\lambda pq.p \wedge q$) and conjoining them in another ($\lambda pq.q \wedge p$), and so want to identify them—a claim of identity between properties. Or to put it another way: just as one felt that $A \wedge B$ and $B \wedge A$ do not differ in what proposition

[25] Some philosophers will take issue with β. See Dorr (2016), Section 5 for discussion.

[26] This theory is a weakening of the theory *Classicism* of Bacon and Dorr (2024). Note that it is not simply the fragment of Classicism which contains only sentences of $\mathcal{L}$ (Bacon and Dorr work in $\mathcal{L}(\forall)$)—that fragment is inconsistent with my version of Relativism, but $\mathcal{C}(\mathsf{B})$) is consistent with it.

they express, so one may feel that $p \wedge q$ and $q \wedge p$ are also somehow expressively identical: propositions satisfying the one formula will also satisfy the other. Hence we should identify the complex predicates $\lambda pq.p \wedge q$ and $\lambda pq.q \wedge p$ formed via λ-abstraction from the properties.

Logical Equivalence ensures $\mathcal{C}(\mathsf{B})$ makes this identification: $p \wedge q \leftrightarrow q \wedge p$ is a theorem of B—here p and q are propositional variables—so that $\lambda pq.p \wedge q = \lambda pq.q \wedge p$ is one of $\mathcal{C}(\mathsf{B})$. In general, then, a provable equivalence $P \leftrightarrow Q$, with variables $x_1, \ldots, x_n$ free in P and Q, will correspond to an identity between the complex predicates formed from P and Q by λ-abstraction on these free variables.

The following theorems of $\mathcal{C}(\mathsf{B})$, which I will call the *Boolean Identities*, are the result of "upgrading" familiar equivalences of propositional logic to higher-order identities via the Logical Equivalence:

$$\begin{array}{cc} \lambda pq.p \wedge q = \lambda pq.q \wedge p & \lambda pq.p \vee q = \lambda pq.q \vee p \\ \lambda pqr.p \wedge (q \vee r) = \lambda pqr.(p \wedge q) \vee (p \wedge r) & \lambda pqr.p \vee (q \wedge r) = \lambda pqr.(p \vee q) \wedge (p \vee r) \\ \lambda pq.p \wedge (q \vee \neg q) = \lambda pq.p & \lambda pq.p \vee (q \wedge \neg q) = \lambda pq.p \end{array}$$

The Boolean Identities, in some sense, imply that the propositions form a Boolean algebra, and indeed that at each type other than e, the entities at that type form a Boolean algebra. For each such type $\tau \neq e$, we can define a notion of conjunction, disjunction, and negation $\wedge_\tau$, $\vee_\tau$, and $\neg_\tau$ by induction. Taking $\wedge_t$, $\vee_t$ and $\neg_t$ to be $\wedge$, $\vee$, and $\neg$, we say:

- $\neg_{\sigma\to\tau}$ abbreviates $\lambda X^{\sigma\to\tau} z^\sigma.\neg_\tau X z$
- $\wedge_{\sigma\to\tau}$ abbreviates $\lambda X^{\sigma\to\tau} Y^{\sigma\to\tau} z^\sigma.X z \wedge_\tau Y z$
- $\vee_{\sigma\to\tau}$ abbreviates $\lambda X^{\sigma\to\tau} Y^{\sigma\to\tau} z^\sigma.X z \vee_\tau Y z$

  (Often, I'll omit the superscript.)

It is then easy to confirm that from the Boolean identities we can derive identities parallel to them for each non-e types: for instance, we can derive $\lambda X^\sigma Y^\sigma.X = \lambda X^\sigma Y^\sigma.X \wedge^\sigma (X \vee Y^\sigma)$.

Using this notation, we can also rewrite $\leq^{\sigma\to t}$ as:

$$\lambda XY.X = (X \wedge^{\sigma\to t} Y)$$

And so we can see that entailment can be generalized to an arbitrary type $\tau \neq e$. We have:

$$\leq^\tau := \lambda XY.X = (X \wedge^\tau Y)$$

The Boolean identities then imply that $\leq^\tau$ will be a non-trivial relation for each type τ. In a Boolean algebra with conjunction $\sqcap$, disjunction $\sqcup$ and complementation $\cdot^C$, we can always define an ordering relation $\sqsubseteq$ by $a \sqsubseteq b$ just in case $a \sqcap b = a$. On this ordering, $a \sqcap b$ is the greatest lower bound of a and b, and $a \sqcup b$ the greatest upper bound. Entailment, as I have defined it, is no more than this ordering as applied to the Boolean algebra of properties of a given type. Hence we will have that $F \leq F \vee^{\sigma\to t} G$, and $F \wedge^{\sigma\to t} G \leq F$, for instance.

The parallel to Boolean algebra also suggests one last bit of ideology that will be useful. Since the propositions are, intuitively speaking, a Boolean algebra, there is an element $\top$ which is entailed by an arbitrary proposition. We can define $\top$ as $\wedge = \wedge$, or by any other theorem of $\mathcal{C}(\mathsf{B})$. If there is any proposition that deserves to be called a trivial proposition—that is, the proposition which is automatically the case, true no matter the circumstances—it is $\top$. After all, every instance of the following schema is a theorem of $\mathcal{C}(\mathsf{B})$:

$$p \to \top$$

What goes for the propositions goes for other types as well; at each type we may also define a special $\top$ element. In the case of type $\sigma_1 \to \sigma_2 \to \ldots \to \sigma_n \to t$, that element is none other than $\lambda x^{\sigma_1} \ldots x^{\sigma_n}.\top$.

Let us fix some type σ. If there is any property of entities of type σ that deserves to be called the trivial property—that is, the property that is trivially or automatically had, had no matter the circumstances—it is $\lambda x^{\sigma}.\top$. After all, every instance of the following schema is a theorem of $\mathcal{C}(\mathsf{B})$:

$$Fy \to (\lambda x.\top)y$$

As I have said, $\leq$ gives us what seems to be a maximally general way of generalizing over the Fs, for any property F whatever. To generalize over the F's, we use the operator $\lambda X.F \leq X$: G holds of the Fs in general, just in case this operator holds of G. And suppose, for instance, $\mathsf{Quant} \leq F$. Then to be a quantifier is just to be a quantifier and additionally F. If $\mathsf{Quant} \leq F$, then to be a quantifier which is not F, $\mathcal{C}(\mathsf{B})$ proves, is to be something which is both F and not F—that is, to be a property of which a contradiction holds. What greater generality over quantifiers could one wish for?

What if we wanted to generalize not just over quantifiers or whales, but things in, well, general? We just consider the status of being a property entailed by $\lambda x.\top$, $\lambda X.(\lambda x.\top \leq X)$. If we can gloss "$F \leq G$" as "an arbitrary F-thing is G", then we can gloss "$\lambda x.\top \leq F$" as an arbitrary $\lambda x.\top$-thing is G. But if $\lambda x.\top$ really is in a sense a trivial property, one that is automatically had, then this is to say that an arbitrary thing, full stop, is G—that G holds in a general way not of just the F's, but simply, well, in general.

The notion of being identical to the trivial proposition and the trivial properties will play a starring role for us in what is to come, so it will be helpful to have some (rather suggestive) notation for them. Following some recent literature in higher-order metaphysics (which itself is following older literature in higher-order logic), I will abbreviate $\lambda p.p = \top$ as $\Box$, and let $\Diamond$ abbreviate $\lambda p.\neg\Box\neg p$, as usual.[27] The

[27]See especially Bacon (2018), Bacon and Dorr (2024), and the citations therein.

reason is that, given the principles already laid down, $\Box$ acts like an S4 modality—it satisfies $\Box(p \to q) \to \Box p \to \Box q$, for instance, and $\Box p \to p$, and so on. If $\Box p$, then I will say that p is "broadly necessary".[28]

In a similar manner, abbreviate $\lambda x_1 \ldots x_n.P = \lambda x_1 \ldots x_n.\top$ by $\Box_{x_1 \ldots x_n} P$, and $\neg\Box_{x_1,\ldots,x_n}\neg P$ by $\Diamond_{x_1 \ldots x_n} P$.[29] Modal logic also provides a helpful heuristic for reasoning with these operators: their inferential behavior is rather like that of *necessitated* universal quantifiers ($\Box\forall$) and "possibilized" existential quantifiers ($\Diamond\exists$), respectively—not like that of universal quantifiers and existential quantifiers.

If $\lambda X.\Box_x Xx$—that is, $\lambda X.(\lambda x.\top \leq X)$—can be seen as a necessitated universal quantifier, so $\lambda X.(F \leq X)$ can be seen as a necessitated F-restricted universal quantifier, in view of the following theorem of $\mathcal{C}(\mathsf{B})$:

$$F \leq G \leftrightarrow \Box_x(Fx \to Gx)$$

In Section 9, we'll reprise this theme.

For now, though, let me note that $\mathcal{C}(\mathsf{B})$ actually lets us simply our statements of Relativism and Absolutism. Given $\mathcal{C}(\mathsf{B})$, we can write:

HO Relativism: $\mathsf{Quant} \leq \lambda X.(\mathsf{Quant} \not\leq \lambda Y.Y \subseteq X)$

HO Absolutism: $\mathsf{Quant} \not\leq \lambda X.(\mathsf{Quant} \not\leq \lambda Y.Y \subseteq X)$

Which we can rewrite with our suggestive notation as follows:

HO Relativism: $\Box_X(\mathsf{Quant}(X) \to \Diamond_Y(\mathsf{Quant}(Y) \wedge \neg Y \subseteq X)$

HO Absolutism: $\Diamond_X(\mathsf{Quant}(X) \wedge \Box_Y(\mathsf{Quant}(Y) \to Y \subseteq X)$

8. Being a Quantifier

HO Relativism, then, lives up to the ambition of generalizing over quantifiers in a maximally general way. Again, however, we must check that the generalization offered by entailment is not-quantificational. It is time, then, to develop an account of what being a (universal) quantifier consists in. With this account in hand, I will argue that entailment is not quantificational. The proposal is this: to be a (universal) quantifier is just to have the right logical behavior, in a sense we will make precise using the resources of $\mathcal{C}(\mathsf{B})$.

The idea that the logical constants are defined by their logical or inferential behavior is not a new one.[30] Thus we find Prior in "The Runabout Inference-Ticket":

[28]Whether $\Box$ counts as a "genuine" modality in some sense or other is not a question I will pursue here. See Bacon (2018) for an extended discussion of $\Box$ and its logic, and a defense of its status as a genuine modality, all couched in a more Absolutist-friendly context.

[29]This is a variation of a notation due to Peter Fritz.

[30]Gentzen (1964) seems to have had such a view, for instance.

> For if we are asked what is the meaning of the word ‘and’, at least in the purely conjunctive sense (as opposed to, e.g., its colloquial use to mean ‘and then’), the answer is said to be *completely* given by saying that (i) from any pair of statements P and Q we can infer the statement formed by joining P to Q by ‘and’ (which statement we hereafter describe as ‘the statement P-and-Q’), that (ii) from any conjunctive statement P-and-Q we can infer P, and (iii) from P-and-Q we can always infer Q. (Prior (1960), p. 38)

Prior is sketching a view on which the *meaning* of the word “and” is exhausted by its inferential role.[31] But there is a metaphysical thought in the vicinity, and that is what I am after here. The thought is not about what it is for a word “R” to be conjuctive, but what it is for a relation R of propositions to be conjunctive. To be a conjunctive relation is just to satisfy the (metaphysical analogue) of $\wedge$’s inferential role, or to have the logical behavior of $\wedge$.

Let us take for granted that logicians have isolated the logical behavior of $\wedge$—that is, the operation of conjunction, not the term—with any one of their standard axiomatizations of the logic of conjunction; say with the schemas:

$\wedge$-Intro: $A \to B \to (A \wedge B)$

$\wedge$-Elim1: $A \wedge B \to A$

$\wedge$-Elim2: $A \wedge B \to B$

Under what conditions could a relation R be said to have the logical behavior of $\wedge$? One might think it would suffice if every instance of the following schemas were true:

R-Intro: $A \to B \to RAB$

R-Elim1: $RAB \to A$

R-Elim2: $RAB \to B$

But we might want a stronger condition on R. When we take a schema deemed logically true or valid, such as:

$$A \vee \neg A \text{ for all sentences } A$$

It is natural to think that there is something common to all the instances of the schema which underwrites their logical goodness; it is not as though it just so happens that each instance is a logical truth. According to an orthodox thought, this common thing is the (logical) form of the sentence—that which remains when we abstract from a particular instance of the schema.

We can operationalize the notion of form in the present setting, again, by appeal to complex predicates formed via λ-abstraction: roughly, we replace constituent

[31] Prior of course raises the view only to raise trouble for it with his “tonk” operator.

sentences by propositional variables and bind via λ. The form of $A \vee \neg A$ is $\lambda p.p \vee \neg p$; of $A \to A$ it is $\lambda p.p \to p$; of $A \vee (A \wedge B) \leftrightarrow A$ it is $\lambda pq.p \vee (p \wedge q) \leftrightarrow p$.

What is the good status that forms like $\lambda p.p \to p$ have? A natural answer would be that $\lambda p.p \to p$ holds of an arbitrary proposition—that is, $\Box_p(p \to p)$. Indeed, this is something $\mathcal{C}(\mathsf{B})$ proves.

Just as with the schema $A \vee \neg A$, the good status of $\wedge$-Intro and the Elim schemas, we might think, is underwritten by a good status which their forms enjoy. Indeed, all of the following are theorems of $\mathcal{C}(\mathsf{B})$:

$\top$-$\wedge$-Intro: $\Box_{p,q}(p \to q \to (p \wedge q))$

$\top$-$\wedge$-Elim1: $\Box_{p,q}(p \wedge q \to p)$

$\top$-$\wedge$-Elim2: $\Box_{p,q}(p \wedge q \to q)$

The sense in which $\wedge$ satisfies the Intro and Elim axioms in not just that every instance of them is true, but that the forms of the schemas hold of arbitrary propositions. If a relation R is to count as having the same logical behavior as $\wedge$, then R-Intro ought to hold with the same generality. We must have that $p \to q \to Rpq$ holds of arbitrary propositions p, q, in the sense that $\Box_{p,q}(p \to q \to Rpq)$, and likewise for the Elimination axiom schemas.

Let us say, then, that:

- R has the *Intro Property* just in case $\Box_{p,q}(p \to q \to Rpq)$
- R has the *Elim1 Property* just in case $\Box_{p,q}(Rpq \to p)$
- R has the *Elim1 Property* just in case $\Box_{p,q}(Rpq \to q)$

To obtain terms standing for the properties, we need only λ-abstract on R.[32]

As for conjunctive operators, so for universal quantifiers. What universal quantifiers are by nature is exhausted by (the metaphysical analogue) of their inferential role; to be a universal quantifier is just to have the right logical behavior.

Again, let us take our cue from the logicians who have tried to axiomatize the logic of universal quantification. I submit that the inferential role of universal quantifiers is pretty well captured by the axiom schemas and rules for $\forall$ that we find in so-called "free logic", or at least one form of it:

[32]This definition of conjunction, and the one I am about to give for universal quantifiers, brings together a few strands in recent work in higher-order metaphysics and logic. The idea that we should see logical validities as underwritten by some kind of higher-order identities is one that I borrow from Dorr (2014) and Goodman (2016). The idea that certain classes of logical operator can be defined by their logical behavior is present in Bacon (2018), but is worked out in terms of quantification. The contribution of the current section is to bring these two ideas together.

Free Instantiation-$\forall$	$\forall x(\forall y A \to A(x/y))$
K-$\forall$	$\forall x(A \to B) \to \forall x A \to \forall x B$
Vacuity-$\forall$	$A \to \forall x A$, where x is not free in A
Gen-$\forall$	If $\vdash A$, then $\vdash \forall x A$

Classical quantification theory is usually characterized by the K and Vacuity axiom schemas, along with the Gen Rule and a schema of Unrestricted Instantiation ($\forall x A \to A(c/x)$, for any A, c). In Free Logic, by contrast Unrestricted Instantiation is abandoned in favor of free instantiation ($\forall x(\forall y A \to A(x/y))$, for any A). This departure from the classical theory is necessary if we are to have a notion of being a universal quantifier that the Relativist can accept. If $\forall$ is a universal quantifier, then the Relativist should not want to accept the instance of Unrestricted Instantiation given by $\forall x \exists y(y = x) \to \exists y(y = r)$, where r is the "Russell set" for $\forall$.

We are now able to give a definition of Quant, that is, to say what it is to be a universal quantifier. Following the pattern with conjunction, say that O is a universal quantifier just in case it possess the following three properties, each one derived from one of the above axiom schemas (we will return to the Gen Rule in a moment).

- O has the *Free Instantiation Property* just in case
$$\Box_R O(\lambda x.O(\lambda y.Rxy) \to Rxx)$$
- O has the *K Property* just in case
$$\Box_{F,G}(O(\lambda x.Fx \to Gx) \to OF \to OG)$$
- O has the *Vacuity Property* just in case
$$\Box_p(p \to O(\lambda x.p))$$

As before, we obtain terms standing for the properties themselves by λ-abstracting on O.

The way I have "translated" the Free Instantiation schema into the corresponding property deserves some comment. One might have expected that O should have this property just in case:

$$\Box_F O(\lambda x.O(F) \to Fx)$$

Where, instead of the two-place relation R, we have instead the one-place predicate F. The version of the Free Instantiation property with F instead of R, though, is inadequate; operators which are intuitively not universal quantifiers (such as *all but finitely many natural numbers*, for instance) may possess the K and Vacuity properties, as well as the weaker Instantiation property.[33] This takes us, then, to the aforementioned Rule of Generalization. Is there a metaphysical equivalent of

[33] The reason it is inadequate is that the Free Instantiation schema allows x to occur free in A. An instance of the schema is:

$$\forall x(\forall y(y \neq x) \to x \neq x)$$

the rule of Generalization that an operator must satisfy if it is to be considered a universal quantifier? I suggest not. This is because the Generalization rule is an artifact of a particular convention governing the use of formulas with free variables in specifying theories, not a manifestation of real pattern in the logical behavior of quantifiers.

A theory, recall, is just a set of formulae. To accept a theory is to commit to its *sentences*—the closed formulae—being true. While open formulae may belong to a theory that one accepts, they themselves are in no way deemed true or false. We include them only because when we characterize a theory as the least set of formulae containing such and such axioms and closed under such and such rules, it is often expedient to allow open formulae. It can make matters of axiomatization easier—for instance, rather than having to lay down as a separate axiom $\forall x(Fx \to Fx)$, we can simply say to include every formula of the form $A \to A$ in the theory, and then demand that our theory be closed under the Generalization rule.

The Generalization rule is a manifestation of a particular convention we are happy to adopt regarding open formulas and universally quantified sentences. Conventions could be otherwise, and in fact have been: there are presentations of quantified logics which eschew the Generalization rule in favor adding more axioms. Instead of the Generalization rule, one stipulates that any *universal closure* of any axiom also belongs to the theory, where a universal closure of a formula is the result of prefixing that formula by any string of universal quantifiers.[34] It seems hard to accuse such a logician of losing out on something important about the nature of quantification; but, on this way of proceeding, Generalization has no role, and hence the issue of the metaphysical equivalent of Generalization does not even arise.

We may therefore take the conjunction of the Free Instantiation, K, and Vacuity properties to be our account of Quant. That is, Quant is to be taken as an abbreviation for this monstrous λ term:

$$\lambda O.\Box_R O(\lambda x.O(\lambda y.Rxy) \to Rxx) \wedge \Box_{F,G}(O(\lambda x.Fx \to Gx) \to OF \to OG) \wedge \Box_p(p \to O(\lambda x.p))$$

One interesting corollary of this definition of Quant is what I take to be a rather natural account of what it is to be an absolutely general quantifier. Recall that AbsGen, the property of being absolutely general, is defined from Quant and $\leq$ as follows:

$$\mathsf{AbsGen}(O) := \mathsf{Quant}(O) \wedge \mathsf{Quant} \leq \lambda X.X \subseteq O$$

We have, then, the following fact:

This would not seem to be an instance of the schema, if understood to require x not to be free in A. There is a general issue here in the proper axiomatiziation of Free Logic. Leblanc (1995) at least seems to claim, if I read him correctly, that one can derive, just using a version of Free Instantiation in which x cannot occur free in A, the claim $\forall x \exists y(y = x)$. But the proof of his Lemma 10 would seem to require the version of the schema in which x can occur in A free.

[34]This is the approach of Quine (1940).

Lemma 8.1. $\mathcal{C}(\mathsf{B})$ proves: If O is a quantifier, then: O is an absolutely general quantifier iff $E_O = (\lambda x.x = x)$

Where E_O is the "existence property" for O: $\lambda x.\neg O(\lambda y.y \neq x)$. (When O is $\forall$, this may be written, given natural dualities, as $\lambda x.\exists y(y = x))$.) To exist according to an absolutely general quantifier is to be self-identical—or, in view of the fact that $\lambda x.(x = x) = \lambda x.\top$, to be such that $\top$.

We can think of E_O, when O is a quantifier, as O's "domain of quantification". The domain of a quantifier is the collection of entities over which a quantifier "ranges"; whether $O(F)$ is true or not depends only on the behavior of F upon the entities in O's domain. Although domains of quantification are often treated as collections like sets or pluralities, it is perhaps more natural—as Stanley and Gendler Szabó (2000) remind us—to treat them as properties or similar entities, in view of the ways quantifiers interact with intensional phenomena like modalities or temporal operators. And given that O's domain is taken to be a property, it seems very natural that that property would have to be E_O. Presumably, an entity over which O ranges ought to exist according to O; likewise, if an entity exists according to O it seems that whether O applies to F or not will in part depend on whether F holds of x. What the lemma says, then, is that an absolutely general quantifier, in my sense, has as its domain the trivial property $\lambda x.x = x$ (i.e., $\lambda x.\top$). If there were any domain that deserved to be called absolutely general, it would surely be this one—what domain could be more comprehensive?—and so what quantifier could be more deserving of being called absolutely general than one with such a domain?

Now, officially Quant is a term of type $((e \to t) \to t) \to t$. Sets are entities of type e, and so Relativism, in the first place, is about whether quantifiers over those sorts of entity are ever maximally general. The quantifiers involved in Relativism, intuitively understood, are therefore entities of type $(e \to t) \to t$, and so Quant, being a property of them, is thus of type $((e \to t) \to t) \to t$.

Clearly, however, the definition of Quant we have given can easily be generalized to give a notion Quant_σ of being a quantifier over entities of type σ for any type σ. We may say $\mathsf{Quant}_\sigma(O)$ just in case, now disambiguating the types of the various variables:

- O has the *σ-Free Instantiation Property* just in case

$$\Box_{R^{\sigma \to \sigma \to t}} O(\lambda x.O(\lambda y.Rxy) \to Rxx)$$

- O has the *σ-K Property* just in case

$$\Box_{F^{\sigma \to t}, G^{\sigma \to t}} (O(\lambda x.Fx \to Gx) \to OF \to OG)$$

- O has the *σ-Vacuity Property* just in case

$$\Box_{p^t} (p \to O(\lambda x^\sigma.p))$$

Quant as originally defined is just the special case when $\sigma = e$.

One way in which having definitions of being a quantifier at higher types is helpful for present purposes is that it allows us to put on firmer formal footing the informal arguments against Q-Relativism that were considered earlier. We can regiment Q-Relativism and Q-Absolutism in higher-order formal language as follows:[35]

Q-Relativism: $\forall X\,(\mathsf{Quant}_e(X) \to \exists Y(\mathsf{Quant}_e(Y) \wedge \neg Y \subseteq X))$

Q-Absolutism: $\exists X\,(\mathsf{Quant}_e(X) \wedge \forall Y(\mathsf{Quant}_e(Y) \to Y \subseteq X))$

The intuitive problem for Q-Relativism came from Π, the property of being a property to which every quantifier applies. We can now define Π, using λ notation, as follows:

$$\lambda F.\forall X(\mathsf{Quant}_e(X) \to X(F))$$

I argued that it was plausible before that Π itself was a quantifier, but with our definition of Quant in hand, we can dispense with appeal to plausibility. $\mathcal{C}(\mathsf{B})$ in fact proves: if $\mathsf{Quant}_{(e\to t)\to t}(\forall)$, then $\mathsf{Quant}_e(\Pi)$.

$\mathcal{C}(\mathsf{B})$ also proves:

$$\forall X(\mathsf{Quant}_e(X) \to X \subseteq \Pi)$$

And so, granted the assumption $\exists X(X = \Pi)$, we have an argument that Q-Relativism is simply false.

Suppose, on the other hand, $\neg\exists X(X = \Pi)$. In this case, we can define a new quantifier $\forall^+_{(e\to t)\to t}$ which is strictly more general than $\forall_{(e\to t)\to t}$. Set:

$$\forall^+_{(e\to t)\to t} := \lambda F^{e\to t}.\forall F \wedge F(\Pi)$$

$\mathcal{C}(\mathsf{B})$ proves, again on the assumption $\mathsf{Quant}_{(e\to t)\to t}(\forall_{(e\to t)\to t})$, that $\mathsf{Quant}_{(e\to t)\to t}(\forall^+_{(e\to t)\to t})$. Since we assume $\neg\exists X(X = \Pi)$, we have then that $\forall^+$ is at least as general as $\forall$, but not vice versa. This yields the formal version of the problem of parochiality: why do we care about a thesis stated in terms of $\forall_{(e\to t)\to t}$, when that quantifier is surpassed in generality by another, $\forall^+$?

9. The Vindication of Relativism

The problem for the Relativist seemed to be there was no way of articulating their position that was both (i) faithful to the spirit of the view and (ii) consistent. We just saw how Q-Relativism foundered at the challenge of accomodating (i) and (ii). Given maximally general quantification over quantifiers, Q-Relativism is inconsistent; but without maximally general quantification over quantifiers, Q-Relativism will fail to capture the spirit of the Relativist view.

Turning to my versions of Relativism and Absolutism:

[35] This would need to be a language that adds quantifiers for each type—not $\mathcal{L}$.

HO Relativism: $\mathsf{Quant}_e \leq \lambda X.(\mathsf{Quant}_e \not\leq \lambda Y.Y \subseteq X)$

HO Absolutism: $\mathsf{Quant}_e \not\leq \lambda X.(\mathsf{Quant}_e \not\leq \lambda Y.Y \subseteq X)$

We may ask how HO Relativism fares. If entailment gives us, as I have argued, a maximally general way of generalizing about quantifiers, then HO Relativism and HO Absolutism, I think, do capture the intuitive spirit of the Relativist and Absolutist views.

It remains only to argue, then, that HO Relativism is consistent. Let me begin with the headline: we can prove the consistency of both HO Relativism and HO Absolutism, against the background of $\mathcal{C}(\mathsf{B})$:

Theorem 9.1.

- $\mathcal{C}(\mathsf{B})$ and is consistent with HO Absolutism:
$$\mathsf{Quant}_e \not\leq \lambda X.(\mathsf{Quant}_e \not\leq \lambda Y.Y \subseteq X)$$
- $\mathcal{C}(\mathsf{B})$ and is consistent with HO Relativism:
$$\mathsf{Quant}_e \leq \lambda X.(\mathsf{Quant}_e \not\leq \lambda Y.Y \subseteq X)$$

The proof of this theorem may be found here. [36])

I conclude that in HO Relativism, we have a vindication of intuitive idea of Relativism about quantification, expressed in a consistent way.

The reason that HO Relativism is consistent is that entailment will not, given my definition of universal quantification, have the logical behavior of universal quantifier—at least if we want to avoid some rather extreme and, to my mind, implausible metaphysical positions.

Consider the following sentence, where $\bot = \neg\top$:

$$\Box_p(p = \top \vee p = \bot)$$

Intuitively, this sentence says that (necessarily) there are only two propositions—$\top$ and its negation. $\mathcal{C}(\mathsf{B})$ proves that $\lambda X.\mathsf{Quant}_\sigma \leq X$ is a quantifier if and only if (necessarily) there are only two propositions in this sense.

Lemma 9.2. $\mathcal{C}(\mathsf{B})$ proves:

$$\mathsf{Quant}_{(\sigma\to t)\to t}(\lambda X.\mathsf{Quant}_\sigma \leq X) \leftrightarrow \Box_p(p = \top \vee p = \bot)$$

It is not *surely* absurd to maintain there are only two propositions (Frege thought so), but what is not surely absurd may still be probably absurd. Presumably we think there are propositions which are metaphysically contingent: such a proposition will have to be distinct from $\top$ or $\bot$. For $\top$, being expressed by tautologous

[36]Or at this link: `https://tinyurl.com/QMTProof`.

sentences, is presumably metaphysically necessary, and hence $\bot$, being its negation, metaphysically impossible. $\lambda X.\mathsf{Quant}_\sigma \leq X$ thus fails to be a quantifier, at least on any plausible metaphysical view. We can thus vindicate the claim that entailment gives a a non-quantificational form of generality. HO Relativism may *generalize* over quantifiers by means of entailment, but it does not thereby *quantify* over quantifiers.

The same goes for $\lambda X.\Box_x Xx$ in general:

Lemma 9.3. $\mathcal{C}(\mathsf{B})$ proves:

$$\mathsf{Quant}_\sigma(\lambda X.\Box_{x^\sigma} Xx) \leftrightarrow \Box_p(p = \top \vee p = \bot)$$

But if $\lambda X.\Box_x Xx$ is not a universal quantifier, but rather only some device of generalization, then what sort of device of generalization is it?

Earlier, I suggested that its logical behavior is like that of a *necessitated* universal quantifier—of an operator like $\lambda X.\Box\forall X$—more than a bare universal quantifier. To substantiate this, let me list some of the principles that we would expect a necessitated universal quantifier to obey:

Lemma 9.4. $\mathcal{C}(\mathsf{B})$ proves: if $\mathsf{Quant}(\forall)$, then:

Instantiation: $\vdash \Box\forall x(\Box\forall y A \rightarrow A(x/y))$

K: $\vdash \Box\forall x(A \rightarrow B) \rightarrow \Box\forall x A \rightarrow \Box\forall x B$

Vacuity: $\vdash \Box\forall x(\lambda x.\top)$

4: $\vdash \Box\forall x A \rightarrow \Box\Box\forall x A$

Gen: If $\vdash A$, then $\vdash \Box\forall x A$

And, indeed $\mathcal{C}(\mathsf{B})$ proves the result of substituting $\lambda X.\Box_x Xx$ for $\Box\forall x$ in any of these claims. What suggests even more that $\lambda X.\Box_x Xx$ behaves like a necessitated universal quantifier is the following theorem:

Lemma 9.5. $\mathcal{C}(\mathsf{B})$ proves: $\mathsf{AbsGen}_\sigma(\forall_\sigma)$ iff $\lambda X.\Box_x Xx = \lambda X.\Box\forall_\sigma X$

What this lemma says, one might suggest, is that not only does $\lambda X.\Box_x Xx$ *act* like a necessitated universal quantifier; it would be identical to the necessitation of the absolutely general universal quantifier if there were such a quantifier.

Now, if HO Relativism is right, then there is no absolutely general universal quantifier. Even in this case, though, there may still be a way to regard $\lambda X.\Box_x Xx$ as a necessitated universal quantifier, and an absolutely general one at that, even in the absence of an absolutely general universal quantifier of which it is the necessitation.

If we think of necessitated universal quantification as a way of generalizing at all, we tend to think of it in terms of the combination of universal quantification with some kind of necessitation. But perhaps we need not think of it this way. After all, in its beginnings in Aristotle's modal syllogistic, formal modal logic was not stated in terms of a sentential operator and separate quantificational component, but rather a copula which fused the two. What I am suggesting is that we see necessitated universal quantification (despite its name) as a self-standing form of generalization, one that is accomplished usually by means of separate devices of quantification and necessitation—and named for those separate devices—but which can be accomplished even without them.

It is natural to wonder how we could understand being a necessitated-universal operator (note the hyphen—this is my term for the members of this would-be primitive class of generalizing devices) except as the result of joining together $\Box$ and some universal quantifier? But here again we might appeal to the idea that kinds of generalizing devices are defined by their logical behavior. Just as we defined universal quantifiers as operators with a certain logical behavior, so, perhaps, the necessitated-universal quantifiers can be seen as operators defined by their own logical profile. An operator like $\lambda x.\Box_x Xx$ may thus come to be a necessitated-universal operator not because there are $\Box$ and $\forall$ which come together to produce it, but rather because it satisfies the right logical role. Unfortunately, so far as I know, logicians since Aristotle have not been so interested in developing the logic of operators which combine quantificational and modal force. (This is an interesting question of pure logic in its own right, which I am addressing in other work.) The principles I listed above, however, I think give a good start on an axiomatization, but there is more work to be done.

Let me take it for granted, however, that we can understand the necessitated-universal operators as a class of generalizing operators of their own, one defined by fitting a certain logical profile (which at least includes the principles listed above), and that a given operator, like $\lambda x.\Box_x Xx$ could be such an operator even if there is no quantifier of which it could be the necessitation. In this case, there is a good argument to be made that $\lambda x.\Box_x Xx$ is absolutely general among such operators.

All this, I think, provides more support for the claim that HO Relativism does capture the intuitive spirit of Relativism. Suppose Abel the Absolutist and Riley the Relativist are debating and explaining their views to each other. Able smugly says, "I understand the intuitive picture, I understand what you are trying to say—you say that *no quantifier* is absolutely general, where *no quantifier* is itself absolutely general. You probably even want to say something stronger, that *necessarily*, no quantifier is absolutely general, where *no quantifier* is itself absolutely general. After all, there's not a whiff of contingency about the considerations you adduce in support of your view. Such a pity that by your own lights, you can't say this!"

Riley, intrepid higher-order logician that she is, then responds: "Dear Abel, I *can* say exactly that. I do want to say necessarily every quantifier fails to be absolutely general, where my generalization is as general as possible. But I have no need for an unrestricted universal quantifier to do it. Consider my claim (written with the helpful abbreviations of AbsGen and $\Box_X$):

HO Relativism: $\Box_X(\mathsf{Quant}(X) \rightarrow \neg\mathsf{AbsGen}(X))$

This is how I say, after all, what I want to say: *necessarily*-for-any quantifier X, X fails to be absolutely general, where *necessarily-for-any* is a maximally general necessitated-universal-quantifier. You might have thought that to express such a thought, I would need both a necessity operator and a maximally general universal quantifier; what we see now is that through $\lambda X.\Box_x Xx$, I can capture the thought without recourse to quantification.

"You and I agree, in fact," Riley may continue to say, "that this is what I wish to say: if you really do grasp an absolutely general quantifier with your term $\ulcorner\forall\urcorner$, then the sentence $\ulcorner\Box\forall X(\mathsf{Quant}(X) \rightarrow \neg\mathsf{AbsGen}(X))\urcorner$ will express the same proposition, by your own lights, as the HO Relativist sentence I have written. This is a consequence of our dear friend's Lemma 9.5. But, if I am right, you do not so express, and your sentence involving $\forall$ fails to get at what I mean. So let us debate with my sentence instead, since it gets at what is at issue without prejudging that issue."

10. Conclusion

I have argued, then, for a construal of the debate about Absolute Generality based on the tools of higher-order logic. Each of Absolutism and Relativism, on my construal, is consistent, and each captures the intuitive spirits of those views. My proposal is not totally neutral—it involves a controversial theory of higher-order identity—but the tools used, at least, are not parochial to the Absolute Generality debate. What allowed us to articulate a coherent form of Relativism was the key idea that quantification is not the only way we can generalize. Once we recognize Relativism about quantification is compatible with maximal generality obtained by other means, we can use those means to devise reasonable forms of Relativism.

I have shown my version of Relativism to be consistent, but this is not yet to have shown it to be plausible. Much of our theorizing, in mathematics and metaphysics, seems to depend for its significance and import on being absolutely general; if we can't achieve that generality via quantification, how are we to achieve it? The results of the last section suggest a way forward: if we cannot have absolutely general quantification, perhaps we can still have absolutely general "modalized" quantification, and use these "modalized-quantifiers" to do our theorizing. When it comes to mathematics and (certain parts of) metaphysics—in which our theories are, plausibly, necessarily true if true at all— modalized-quantification may well be enough for our purposes. Of course, we will likely want to do more than necessarily-universally-quantify, but this may not be an obstacle either. In other work, I suggest that we can define not just absolutely general necessitated-universal quantification, but also absolutely general necessitated-existential quantification, and necessitated quantification of various other kinds, all with only the resources of $\mathcal{L}$. It may seem strange at first that we might be able to modally-quantify in an unrestricted way without being able to quantify unrestrictedly, but in time, once we internalize the idea that quantifiers are but one star in a vast galaxy of ways of generalizing, that strangeness may yet fade.

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