

FORMALITY IN LOGIC: FROM LOGICAL TERMS TO SEMANTIC CONSTRAINTS

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ABSTRACT

In this paper I discuss a prevailing view by which logical terms determine forms of sentences and arguments and therefore the logical validity of arguments. This view is common to those who hold that there is a principled distinction between logical and nonlogical terms and those holding relativistic accounts. I adopt the Tarskian tradition by which logical validity is determined by form, but reject the centrality of logical terms.

I propose an alternative framework for logic where logical terms no longer play a distinctive role. This account employs a new notion of *semantic constraints*. The paper includes some preliminary definitions and results in the new framework.

Formality is considered by many to be a central, even definitional feature of logic. Following Tarski, many understand formality to imply that the logical validity of an argument turns on its form. A large part of contemporary discussions on formality is dedicated to the notion of *logical term*, as logical terms are assumed to determine the form of sentences and arguments.

In this paper I challenge the centrality of logical terms assumed in the contemporary conception of logical consequence. I focus on the model-theoretic tradition, and refer to both principled and relativized accounts of logical terms. In the main part of the paper (Section 2), I propose an alternative framework for logic where logical terms no longer play a distinctive role.

1. Logical Terms

1.1. *Principled and Relativistic Accounts*

Take a first order language without identity. To make things simpler, take a first order language with negation, implication and the existential quantifier

* This paper was presented at the University of St Andrews, LMU Munich and the Hebrew University of Jerusalem. I thank the audiences there for many helpful comments. I am also grateful to Lloyd Humberstone, David Kashtan, Aviv Keren, Ran Lanzet, Hannes Leitgeb, Oron Shagrir, Stewart Shapiro and two reviewers for valuable comments and suggestions on earlier versions of this paper.

as the only logical terms. Which are the logical terms of this language? Indeed, they were just listed. The logical terms of formal languages are predefined as such. But notice that the phrase “logical term” is used in two ways in the logic literature. Sometimes it’s a system relative notion, meaning “the terms for which the interpretation is held fixed”.¹ And sometimes the ascription of logicality to a term is meant in a non-relative way, implying that the term satisfies some conditions that entitle it to be characterized as logical. Let us distinguish between *logical terms in the shallow sense* and *logical terms in the deep sense*. While logical terms in the shallow sense are stipulated in and relative to a system (they are simply defined by it as such), the notion of terms being logical in the deep sense, “by their nature”, needs further analysis and specification.²

Simply fixing a term will not make it logical by its nature (or so goes a common conception): the question of logical terms is which terms *should* get fixed and be considered as logical — what is the correct criterion for choosing the terms that are to be fixed in the system. It will be useful to spell out the underlying assumption driving the search for a criterion for logical terms:

PD. There is a principled distinction between logical and nonlogical terms.

Alongside proposals for criteria for logical terms in recent literature (Peacocke, 1976; McCarthy, 1981; Sher, 1991; Warmbröd, 1999; Feferman, 1999; Bonnay, 2008), doubts have been raised as to the prospects of such a project.³ It seems as if all proposed criteria are highly controversial. This doesn’t yet prove that a correct criterion does not exist, but it would reasonably lead to skepticism in the project (Etchemendy, 1990; Dutilh Novaes, 2012) and give rise to relativistic approaches (Varzi, 2002; Tarski, 1936). It has been suggested that although there are various nice logical properties terms may have and for which they can be considered logical, these different properties need not coincide, and logical terms might not form a “natural kind” (van Benthem, 1989, p. 317). I will not provide here any new criticism of PD or specific criteria for logical terms, and for that I refer the reader to the works mentioned above.

¹ That is, in a model-theoretic setting, logical terms are defined at the outset, and their interpretation in a model is a function of the domain of the model.

² A similar distinction can be made from a proof-theoretic perspective, where logical terms understood in the shallow sense would be relative to a proof system, and understood in the deep sense they satisfy some criterion for logicality.

³ See (MacFarlane, 2009) for a survey and discussion of the various approaches to logical terms.

The view that logical consequence is relative to the set of logical terms has been termed *Tarskian Relativism* (Varzi, 2002), following the skepticism Tarski raised at the end of (Tarski, 1936):

TR. Logical validity is relative to a choice of logical terms, and there is no principled distinction between logical and nonlogical terms.

Tarskian relativism employs the shallow meaning of “logical terms”. Yet, logical terms still play a major role: they are an important parameter that must be fixed for logical consequence to be determined.⁴

1.2. *Logical Terms and Formality*

The thesis of the centrality of logical terms follows from two underlying assumptions, which I will refer to as *the two tenets of formality*:

TF1. The logical validity of an argument is determined by its form.

TF2. The form of an argument is determined by the logical vocabulary and the arrangement of all terms in the argument.⁵

I concentrate on arguments at this point, but of course TF2 can be formulated with respect to forms of sentences as well. “Determined” may be read as *is a function of*. By “arrangement” I mean the pattern of repetitions of terms and their separation by auxiliary devices such as parentheses.⁶

From TF1 and TF2 follows this formulation of the thesis of the centrality of logical terms:

(*) The logical validity of an argument is determined by the logical vocabulary and the arrangement of all terms in the argument.

There may be various conditions a valid argument should satisfy (first and foremost: truth-preservation). Whatever these conditions are, (*) states that

⁴ We should note that in (Tarski, 1936) it is not suggested that any choice of logical terms would be as good as another, but merely that there is not one correct choice.

⁵ Cf. (Dutilh Novaes, 2012), where eight “tenets of formality” are listed.

⁶ Dutilh Novaes indicates that the arrangement of the logical terms in the sentence affects logical validity (Dutilh Novaes, 2012, n. 14). But note that the arrangement of *all* terms in an argument affects logical validity, nonlogical terms as well as logical. Take the two following sentences in first-order logic, which differ only in the arrangement of their nonlogical terms:

(1) $\forall x(Rxa \rightarrow Rxa)$

(2) $\forall x(Rxa \rightarrow Rxb)$

(1) is a logical truth and (2) isn't, and this example can be easily used to show that logical *validity* of arguments does not depend solely on the logical vocabulary.

whether an argument satisfies them is determined by the logical vocabulary (and the arrangement of all terms).

TF1 is one of the ways by which logical consequence can be said to be formal, as in (Tarski, 1936). Tarski connects TF1 to an epistemic motivation: formality allows the independence of logical consequence from empirical knowledge or knowledge of the objects that are referred to in an argument. I will follow the Tarskian tradition by assuming TF1 for the rest of the paper, and specifically that the notion of formality in logic pertains to forms of arguments (and sentences). My account will sit well with the epistemic motivation for TF1, but elaborating on that issue would take us too far afield.

TF1 assumes the notion of a form of an argument. In what follows I challenge the conventional understanding of this notion, which is expressed by TF2, and relates form to logical terms. TF2 is generally assumed rather than argued for. I will give up this assumption in favor of a better motivated, weaker one.

TF2 (and so its conclusion (*)) can be understood in two ways, according to whether logical terms are understood in the shallow or in the deep sense. As noted earlier, taking logical terms in the deep sense presupposes PD, that there is a principled distinction between logical and nonlogical terms. Were one to reject PD and endorse Tarskian Relativism (TR), it would still be possible to maintain TF2 and (*) when logical terms are understood in the shallow sense. The notions of *form* and *logical consequence* would then be relative as well.

One can reject the view of logic as formal (Read, 1994) or reject TF1 as a correct articulation of that view (see (MacFarlane, 2000)). But one can accept TF1 and still question the thesis of the centrality of logical terms — a possibility that has been neglected in contemporary literature. The thesis that has mainly been up for debate is PD, that there is a principled distinction between logical and nonlogical terms. TR (and its relative, the pragmatic approach) is a neat way of rejecting PD while holding on to the two tenets of formality — but it still assumes the centrality of logical terms. I propose to consider alternatives to the thesis of the centrality of logical terms, without giving up on formality or on TF1. This means we should let go of TF2 and that the form of an argument (or a sentence) requires a strict division between logical and nonlogical terms, without altogether giving up on the notion of a form.

Thus, at this point I propose to take a step back and see what can be achieved without assuming a division between logical and nonlogical terms. We can start by looking into a possible motivation for the logical/nonlogical dichotomy. We noted earlier that logical terms are those terms that *are* held fixed in the system (this is the shallow sense of “logical terms”) or those that *should* be held fixed (understood in the deep sense). Analogously, nonlogical terms *are* variable in the system, or are those terms that *should*

be variable. Whichever reasons we may have for fixing some terms and not others might possibly also justify fixing some terms to *some extent*, keeping them also variable to some extent. Let's take an example. One might claim that logical consequence has "epistemic virtues" such as certainty or apriority and would require that only terms that yield an "epistemically virtuous" consequence relation should be fixed under all interpretations. These might be terms whose extensions we know with complete certainty. But in the same vein, one can fix other terms *partially*, to an extent that keeps logic "epistemically virtuous". Or, in other words, we can limit the variability of some terms without fixing them completely.

This line of thought leads to the following, if somewhat vague, alternative to TF2:

TF2'. The form of an argument is determined by what is held fixed in the argument under all interpretations.

We note that the syntactic categorization of terms (into predicates, connectives etc.) and their overall arrangement in the argument are some of the things that are held fixed. It is clear from the discussion above that TF2' is in line with the epistemic motivation for TF1 implied by Tarski.

If TF2' is to give us a better understanding of the notion of a form of an argument, we need to first define the class of (admissible) interpretations. Usually, this is done by specifying the logical terms. TF2', which is no longer centered on logical terms, may now inspire us to search for new ways of fixing things in an argument or a sentence. We can go beyond the limiting way of looking at form through the logical/nonlogical distinction between terms and adopt a more general view instead. In what follows, I describe a formal framework and provide examples of ways terms can be fixed in different manners and to various extents. We will then be able to restate the idea behind TF2' in a clearer and more precise manner.

2. Semantic Constraints

2.1. *The Framework*

Acknowledging that within the standard conception of logic what matters to the form of an argument is what is considered to be fixed opens the subject to a range of new possibilities. In what follows I explore one of the options in that range. I devise a notion of form that does not rely on a distinction between logical and nonlogical terms: the terms of the language will not be strictly divided to those that have to do with its form and those that haven't. Instead of trying to distinguish between fixed and non-fixed

terms, I consider fixing terms partly to various extents, or more generally, fixing or constraining parts of the language in different ways.

Fixing a term as logical may be viewed as restricting the admissible models for the language.⁷ Now take, for instance, the terms *allRed* and *allGreen*.⁸ These are paradigmatic cases of nonlogical terms in mainstream logic. There are good reasons for not fixing the extensions of color terms completely. But we could fix their mutual dependencies, and have a rule in our system that says that their intersection is empty in all models. A rule like this, I contend, is not essentially different from a rule fixing the interpretation of a logical term. In both cases, there is a rule that consists in restricting admissible models. We may say that whereas the interpretations for logical terms are completely fixed, other terms may be *partly* fixed.

Let us refer to all such rules on the admissible models for a language as *semantic constraints*. Logical terms (or more precisely, rules defining logical terms) are merely a special case of semantic constraints, while *all* the semantic constraints in a system are involved in determining logical consequence.

I will assume that a language *L* consists of a set of terms (the atomic expressions in *L*) and a set of phrases (the set of all meaningful expressions in *L*). A phrase consists of a string of terms and perhaps auxiliary devices such as parentheses. Phrases are interpreted by the models for the language. I will assume that all phrases are finite.

In a model-theoretic framework, semantic constraints for a language *L* will be sentences in the metalanguage, usually with a universal quantification ranging over all models (domains and interpretation functions) which I will omit. We also make use of the constants *T* and *F* for the truth values. Let a model *M* be a pair $\langle D, I \rangle$ where *D*, the domain, is a non-empty set, and *I* an interpretation function for *L*. In this general, nonstandard setting, what we mean by an interpretation function is a function that takes as arguments phrases in *L* and whose values are objects in the set-theoretic hierarchy with $D \cup \{T, F\}$ as ur-elements.

The previous paragraph characterizes semantic constraints in a *shallow* sense, as constituents in a formal system. My aim in what follows is to lay out the framework of semantic constraints, not give a demarcation of the “correct” semantic constraints. Thus, I will not offer solutions to the

⁷ See e.g. (Varzi, 2002). Varzi contends that *all* terms receive their meaning through the restriction of admissible models, and there is therefore no principled distinction between logical and nonlogical terms: “What are the reasons for maintaining that the distinction between the logical and the extra-logical is up for grabs? Broadly speaking, my reasons stem from the consideration that all bits of language get their meaning fixed in the same way, namely, by choosing some class of models as the only admissible ones.” (Varzi, 2002, p. 199).

⁸ Consider *allRed* and *allGreen* as primitive terms in the object language which stand for *red all over* and *green all over*.

questions of logicality that have been broached in the term-based approach. But moving to a more general framework may dissolve some of the previous questions, and change the considerations in others. The principal benefit is the removal of an unfounded assumption. The proposed framework still allows for the more restricted, term-based, systems but in this setting, the exclusivity of such systems in defining logical consequence is not taken for granted.

Moving on to some examples, the semantic constraint for standard conjunction of sentences will be:

$$(\wedge): I(\varphi \wedge \psi) = T \Leftrightarrow I(\varphi) = T \text{ and } I(\psi) = T$$

where \wedge is in L, and φ and ψ range over sentences in L.⁹ From now on I will assume that a term to which an interpretation function is applied in a semantic constraint is in L. A way of reading the above definition is: a model $\langle D, I \rangle$ is admissible in the semantics only if it satisfies (\wedge) . We may also wish to restrict the class of models with respect to terms in the language without fixing their extension absolutely, for instance, by:

- $0 \in I(\textit{naturalNumber})$,
restricting the term *naturalNumber*, or by relating them to other terms:
- $I(\textit{even}) \cap I(\textit{odd}) = \emptyset$,
restricting *even* and *odd*.

We can also treat the division of terms into syntactic categories as semantic constraints, so that the constraint

$$\bullet I(\textit{big}) \subseteq D$$

tells us that *big* should be interpreted as a unary predicate. This may seem more like a syntactic rather than a semantic constraint, but there is no need for hard and fast distinction between syntax and semantics here. The above item is a constraint on interpretations of the language, and is thus considered here as a semantic constraint.

The semantic constraint mentioned informally previously can be formulated thus:

$$\bullet I(\textit{allRed}) \cap I(\textit{allGreen}) = \emptyset.$$

We can likewise have such a constraint for any pair of color terms.

⁹ The semantic constraint that includes conjunction of open formulas as in standard first-order logic is somewhat more elaborate and is not presented for ease of exposition.

More examples for semantic constraints include:

- $I(\textit{bachelor}) \subseteq I(\textit{unmarried})$
- $I(\textit{wasBought}) = I(\textit{wasSold})$
- $I(\textit{John}) \in D$

A recursive definition of truth or satisfaction can be incorporated in a system of semantic constraints. From now on I will assume that L is a first-order language, and that the systems of constraints we are dealing with include the usual first-order semantics, augmented with further constraints as those above.¹⁰

The consequence relation for L is determined by a collection of semantic constraints. Let Δ be a set of constraints including those mentioned above. Δ fixes a class of models for L : those satisfying all the semantic constraints in Δ . An argument $\langle \Gamma, \varphi \rangle$ (where Γ is a set of sentences in L and φ is a sentence in L) is *valid* in Δ if for any Δ -model (an admissible model for Δ , satisfying those constraints), if all the sentences in Γ are true, then φ is true as well. Let \vDash_{Δ} be the associated consequence relation, so, for instance we have: $\textit{bachelor}(\textit{John}) \vDash_{\Delta} \textit{unmarried}(\textit{John})$.

On this view, the collection of semantic constraints represents, in a very general sense, the form, or structure, of the language. We might not commit to giving the exact extension of *allRed* in every model, and yet commit to its extension having no common member with the extension of *allGreen* in all models. The narrow logical/nonlogical view of terms doesn't allow for such a mutual half-fixing of terms. But just as we constrain *allRed* to be a predicate, we may also constrain it to be disjoint from *allGreen*. Even if a term is not to be completely fixed and considered as a logical term in a system in the standard sense, its range of interpretations can still be limited, and this option should have place in a theory of logical reasoning.

We receive a view where *form* is treated far more generally than it usually is, and yet the generalization is a very natural one.¹¹ Logical terms still

¹⁰ By L being a first-order language I mean that the language consists of individual constants, predicates of each finite arity, individual variables, first-order quantifiers, sentential connectives and auxiliary symbols (parentheses and comma). I also assume that well-formed formulas can be defined recursively. In present terms, only closed expressions (closed formulas and singular expressions) are properly considered as phrases. Note that in a recursive definition of well-formed formulas there is no need to fix the logical terms of the language. For example, there can be a clause saying that if φ and ψ are well-formed formulas and C is a binary sentential connective, then $\varphi C \psi$ is a well-formed formula, regardless of whether C is fixed as a logical term. For elaboration, see (Sagi, 2013).

¹¹ Views of formality that are similar in their generality can be found in the literature. We might not go so far as Chomsky and claim that "To say that a relation is formal is to say nothing more than that it holds between linguistic expressions" (Chomsky, 1955, p. 39). We might require, in the spirit of Carnap, that a formal relation must be determined by explicitly given rules of the language (see Carnap, 1937, p. 186), e.g. in a system of constraints.

have a special place in our system, as they lie at the end of a spectrum — they are the terms whose denotations are completely fixed (i.e. their interpretation is a function of the domain of the model). But formality is no longer subsumed under the issue of logical terms. The notion of form I am proposing has to do with constraints on the language and on the interpretation of arguments rather than with specific terms.

So our final reformulation of TF2 will be:

(TF2*) The form of an argument in a language L is determined by a set of semantic constraints for L and the arrangement of terms in the argument.

Together with TF1, we obtain the following conclusion:

(*)* The logical validity of an argument in a language L is determined by a set of semantic constraints for L and the arrangement of terms in the argument.

Semantic constraints resemble ideas already existing in the literature. Let us take a brief look at the concept of *meaning postulates* in the work of Carnap and later in Montague.¹² In a formal system, meaning postulates are sentences in the object language, fixing the relation between terms. That is, models (*state-descriptions* in the case of Carnap) considered for logical consequence (*L-implication* or *analyticity* for Carnap) are only those that satisfy the meaning postulates (Carnap, 1947, p. 226), (Montague, 1974, pp. 263-264). Let us observe an example from Montague. The meaning postulate

$$\Box [seek'(x, P) \leftrightarrow try - to'(x, [find'(P)])]$$

fixes the relation between the expressions *seek* and *try-to find*, as can be observed without going into the details of Montagovian notation.

Notably, the semantic constraints suggested above look like a variation of meaning postulates. However, for both Carnap and Montague meaning postulates and logical terms are two separate issues. Carnap introduces meaning postulates for his explication of *analyticity*, which he distinguishes from a narrower concept of *logical truth* that does not depend on meaning postulates. For his notion of *logical truth*, Carnap relies on a distinction between logical and nonlogical (descriptive) terms.¹³ Montague, too, relies

¹² Semantic constraints also have much in common with *cross-term restrictions* discussed in (Etchemendy, 1990). For lack of space I cannot compare here the two ideas, and the reader is invited to consult Etchemendy's work.

¹³ In (Carnap, 1947), logical terms are given in lists, and the distinction is considered to be a matter of convention. Previously, in (Carnap, 1937), logical terms were given in a

on a distinction between logical and nonlogical terms. But by contrast to Carnap, he lets meaning postulates affect logical consequence as well. Yet, he does not consider the rules for logical terms among the meaning postulates, although both alike determine the extension of logical consequence. By contrast to both Carnap and Montague, in the framework presented here there is no fundamental distinction between restrictions for logical terms and restrictions for other terms. Consequently, Carnap's distinction between *analyticity* and *logical validity* is presently discarded. The question of whether there is place for an interesting notion of analyticity, different from Carnap's, that can be contrasted to logical validity on this approach, will have to be left out of the current discussion.

So we can sum up two important differences between semantic constraints and meaning postulates as they appear in Carnap and Montague. First, semantic constraints are formulated in the metalanguage and meaning postulates in the object language. This means that while semantic constraints will usually have more expressive power, meaning postulates might be more "manageable" in the system and can be considered as axioms. Secondly, and relatedly, by contrast to Carnap and Montague's systems, there is no fundamental difference between the rules or definitions of logical terms and other semantic constraints. Semantic constraints mark the form of a sentence or an argument by holding parts of it fixed, and therefore are justifiably a factor in a language's logical consequence relation. They are a wide category under which logical terms are also subsumed.¹⁴

2.2. *Some Concepts in the Theory of Semantic Constraints*

When taking all such constraints as those above to be determining the form of sentences and arguments, it seems we might defy a basic intuition, according to which form can be represented by a *schema*.

Let us leave arguments aside for the moment, and concentrate on sentences. In the case where we have a standard language augmented with semantic constraints for color terms as the one above, we may adjust the syntax and have a special set of schematic predicate symbols (metalinguistic

syntactic fashion as such that sentences constructed by them are "determinate". Both cases are of a system-relative notion of logical terms (see *ibid.*, p. 178f).

¹⁴ I have only referred to meaning postulates *vis-à-vis* their role in the formal system, and disregarded their role in giving a semantic theory for natural language, where they are typically used by linguists. Semantic constraints can be used in formal systems for various purposes, among which is the investigation of natural language, but I am not arguing here for the efficacy of using semantic constraints in linguistics. The point here is that form need not be determined through a strict dichotomy between two types of terms, the logical and the nonlogical.

variables) for color terms. For simplicity we confine ourselves in this example to a simplified array of colors, where no two colors overlap. So a sentence such as “There are no green giants” will be schematized as: $\neg \exists x(\mathcal{G}x \wedge Gx)$ where upper-case letters in calligraphy range over color terms, and the rest of the symbols are as usual. While the previous schema is invalid in our system, the following one is valid: $\neg \exists x(\mathcal{G}x \wedge \mathcal{R}x)$.¹⁵ I show below that in any system of semantic constraints it is possible in principle to represent all forms of sentences by schemas. To this end, I use the definitions that follow.

Let π be a permutation on the terms of L. π can be extended to the set of phrases of L in a natural way. π can be further extended to apply to models: for $M = \langle D, I \rangle$, $\pi(M) = \langle D, I^* \rangle$ where for each phrase p , $I^*(s) = I(\pi(p))$. Let us now define the notion of a *category* of terms, which will eventually give us the proper substitution classes. First, however, we need to define when two terms in a language are *interchangeable*:

Definition 1 (Interchangeability). Two terms a and b are *interchangeable* (w.r.t. Δ) if for any permutation π on $\{a, b\}$ and Δ -model M , $\pi(M)$ is a Δ -model.

Interchangeability of terms with respect to a set of semantic constraints is an equivalence relation. So for any set Δ of semantic constraints for a language L there is a partition of the terms of L into maximal interchangeability classes. Those will be the Δ -categories:

Definition 2 (Category). A set of terms A is a *category* (w.r.t. Δ) if every two terms in A are interchangeable, and no term $a \in A$ is interchangeable with a term $b \notin A$.¹⁶

An example of a category is that of color terms in the set of constraints presented above. Note that the categories here are semantic in their nature, and do not in general coincide with what are usually thought of as syntactic or grammatical categories. In a system with constraints for color terms as the one above, *predicate* is not a category by our definition, since not all predicates behave alike in the system (e.g. *big* and *allRed* behave differently).

¹⁵ We should add that we need to give up the convention that the same term may be substituted for two different schematic letters (given they are of the right category). Otherwise the sentence above is comes out invalid when *allRed* is substituted for both schematic letters.

¹⁶ The concept of *semantical category*, to which my concept of *category* is akin, is usually assumed and not defined. Tarski gives a strictly weaker definition of *semantical category* than the one I provide above, though the two definitions coincide in the case of non-logical terms of standard languages (Tarski, 1933, pp. 215-217).

On the basis of the previous definitions, we can now define a *schema*. The basic idea is that each category will have its own pool of schematic letters. Each schema has as instances sentences that are received by substituting a term from the right category for each schematic letter in a uniform way. In a system of constraints where color terms form a category, and all other predicates form a category, the sentence $\neg \exists x(Green(x) \wedge Giant(x))$ is an instance of the schema $\neg \exists x(\mathcal{G}x \wedge Gx)$ but not of the schema $\neg \exists x(\mathcal{G}x \wedge \mathcal{R}x)$, where \mathcal{G} and \mathcal{R} are schematic letters for color terms and G is a schematic term for the category of all other predicates.¹⁷

More rigorously, we can define a *schema* in our system as follows:

Definition 3 (Schema). Let Δ be a set of semantic constraints for a language L . Let $\{T_1, T_2, \dots\}$ be a partition of the terms of L into maximal categories for Δ (which can be viewed as *substitution classes* for the logic) and let $\{S_1, S_2, \dots\}$ be pairwise disjoint sets of schematic letters. A *schema* Φ in $\langle L, \Delta \rangle$ is a string of schematic letters and auxiliary symbols (e.g. parentheses) that has an instance φ . φ is an *instance* of Φ if all the following conditions hold:

1. φ is a sentence.
2. Φ and φ are of the same length (including auxiliary symbols in both).
3. Let $\Phi = A_1 \dots A_n$ and $\varphi = a_1 \dots a_n$. For all $i \in \{1, \dots, n\}$:
 - i) $A_i \in S_k \Leftrightarrow a_i \in T_k$.
 - ii) $A_i = A_j \Leftrightarrow a_i = a_j$.
 - iii) If a_i is an auxiliary symbol, then $A_i = a_i$.

Definition 4 (Validity of schemas). A schema is *valid* if all its instances are valid. A schema is *invalid* if all its instances are not valid.

The validity of sentences is defined as usual: a sentence is *valid* if true in all models.

Proposition 1. *Every schema is either valid or invalid.*¹⁸

¹⁷ Terms such as \neg , \exists and \wedge can be used as schematic letters themselves where (as in the standard setting) each of their categories is a singleton.

¹⁸ Similarly, for any schema, either all its instances are *invalid* (i.e. logical falsehoods) or none are. The proof is similar to the one that follows in the main text. We can thus divide all schemas to those for which all instances are valid (i.e. logical truths), those for which all instances are invalid (i.e. logical falsehoods) and those for which all instances are neither valid nor invalid. The basic idea of the proof is that for every two instances of a schema in $\langle L, \Delta \rangle$ and a Δ -model for one of the instances there is a Δ -model for the other. Note that the notion of *schema* defined here diverges from the standard notion, by which invalid schemas may have a valid instance. On this account, I take it that my notion tracks better the notion of *logical form*. For discussion of this issue, see (Smiley, 1982).

Proof. Let Φ be a schema in $\langle L, \Delta \rangle$. Let φ and ψ be two instances. Let $\varphi = a_1 \dots a_n$, $\psi = b_1 \dots b_n$ (they are of the same length as Φ so their lengths are equal).

First assume that φ and ψ have no terms in common. Assume toward contradiction that φ is valid and ψ isn't. Then there is a Δ -model $M = \langle D, I \rangle$ such that $M \not\models \psi$. We know that for all i , if a_i and b_i are terms (as opposed to auxiliary symbols), then they are of the same category, so they are interchangeable in Δ .

Let π be a permutation over the terms in L such that for all $i \in \{1, \dots, n\}$, $\pi(a_i) = b_i$ and $\pi(b_i) = a_i$ (where those are terms), and for any other term a , $\pi(a) = a$. π can be extended to all symbols in φ and ψ , so that it will be constant on auxiliary symbols, and we have $\pi(a_i) = b_i$ and $\pi(b_i) = a_i$ for all $i \in \{1, \dots, n\}$. Since (when restricted to terms) π is the result of a finite composition of permutations, each of which switches two terms of the same category, $\pi(M)$ is a Δ -model. However, $\pi(M) \not\models \varphi$: note that $\pi(\varphi) = \pi(a_1) \dots \pi(a_n) = b_1 \dots b_n = \psi$ so $\pi(M) \models \varphi$ iff $M \models \psi$. But $M \not\models \psi$, so $\pi(M) \not\models \varphi$, and from this follows a contradiction to our assumption that φ is valid in Δ .

Assume now that φ and ψ have some terms in common: for some $i, j \in \{1, \dots, n\}$, $a_i = b_j$. Assume that φ is valid. We prove that ψ is valid. For each $i \in \{1, \dots, n\}$ such that b_i is a term, add to L the new term b'_i such that $b'_i = b'_j$ iff $b_i = b_j$. For every phrase p in L where b_i occurs for some i , add the phrase p' which is received from p by replacing all b_i s with corresponding b'_i s. Add to Δ the constraint ' $I(p') = I(p)$ ' for every phrase p in L and every phrase p' obtained by replacing some b_i s with corresponding b'_i s. Call the new language L' and the new set of constraints Δ' . Note that the extension made is conservative in the sense that every sentence χ with terms only from L is valid in $\langle L', \Delta' \rangle$ iff it is valid in $\langle L, \Delta \rangle$. Obviously, each b'_i is in the same Δ' -category as b_i , and other than the addition of the b_i s the categories in Δ' are the same as in Δ .

For each i such that b_i is an auxiliary symbol, let $b'_i = b_i$. Now let $\psi' = b'_1 \dots b'_n$. The process of schematization can be done with respect to L' and Δ' , and it can be easily verified that there is a schema of which ψ' , ψ and φ are instances. Since φ is valid in $\langle L, \Delta \rangle$, then φ is valid in $\langle L', \Delta' \rangle$. ψ' and φ have no terms in common. So, according to the first part of the proof, ψ' is valid in $\langle L', \Delta' \rangle$. Also, clearly, for any Δ' -model M for L' , $M \models \psi'$ iff $M \models \psi$. So ψ is valid in $\langle L', \Delta' \rangle$. Because the extension of the language was conservative as explained above, it follows that ψ is valid in $\langle L, \Delta \rangle$. \square

The proposition shows that schemas can serve as or represent forms of sentences: a sentence will be valid if and only if it is an instance of a valid schema. A sentence will not be valid if and only if it is an instance of an

invalid schema. In a similar manner, we can schematize *arguments*, and define validity for argument schemas. It will then follow that an argument is valid if and only if it has a valid argument-schema.

Nonetheless, in many cases the use of schemas may be impractical. We may have a very large number of categories, and in cases in which most categories are singletons, most schemas will have only one instance and there will be no benefit from using them. Expanding the notions of category and schema to deal with *sequences* of terms might be more useful. One would then look at interchangeable sequences of terms. Those will be sequences of terms that exemplify the same dependency relations between terms. I define the notion of *dependency* below, and leave the generalizations of *category* and *schema* that deal with sequences of terms for another occasion.

There are some relevant notions in the logic literature, such as *definability* and related notions that pertain to the relation between terms, that can be adapted to the framework of semantic constraints. One can speak of definability in a semantic or a syntactic setting. Let us consider Tarski's work, in which he dealt with different restrictions on terms in a syntactic setting (Tarski, 1934).¹⁹ There, notions of *definability* and *independence* are considered with respect to extra-logical terms. Tarski defines *definability* as follows:

Let 'a' be some extra-logical constant and *B* any set of such constants. Every sentence of the form:

$$(I) (x) : x = a. \equiv .\phi(x; b', b'', \dots),$$

where ' $\phi(x; b', b'', \dots)$ ' stands for any sentential function which contains 'x' as the only real variable, and in which no extra-logical constants other than 'b', 'b'', ... of the set *B* occur, will be called [...] a *definition of the term 'a' by means of the terms of the set B*. We shall say that the term 'a' is *definable by means of the terms of the set B on the basis of the set X of sentences*, if 'a' and all the terms of *B* occur in the sentences of the set *X* and if at the same time at least one possible definition of the term 'a' by means of the terms of *B* is derivable from the sentences of *X*. (Tarski, 1934, p. 299)

For our purposes we can define an analogous notion of determinateness, relating it more generally to phrases in *L*.

Definition 5 (Determinateness). A phrase *p* is *determined by the set of phrases B* (w.r.t. Δ) if for any two Δ -models $M = \langle D, I \rangle$ and $M' = \langle D', I' \rangle$, if $I(r) = I'(r)$ for all $b \in r$ then $I(p) = I'(p)$.²⁰

¹⁹ The basic framework for Tarski is the system of the *Principia*. The definitions below can easily be transformed to a model-theoretic setting, where "derivability" is changed to "logical consequence" and the proper adjustments are made.

²⁰ The notion of *determinateness* is a modified version of the familiar notion of *implicit definition*.

Note that if a term a is definable (in Tarski's sense) by the terms in the set B on the basis of the set X of sentences, then a is determined by B in a system of constraints that includes the constraint $I(\varphi) = T$ for all $\varphi \in X$,²¹ but not necessarily vice versa (definability requires expressibility in the object language that is not required by determinability).

For convenience, we shall add to each language a (quasi) term \mathcal{D} for the domain. The significance of this treatment is that the definitions and propositions about terms below will then apply to the domain as well. For this purpose, we postulate that for any language L and every model $\langle D, I \rangle$ for L , $I(\mathcal{D}) = D$. The domain will of course keep its status as providing the building blocks of the interpretations of all other terms.

On the basis of the previous definition, we can define the *logical terms* (the "completely fixed" terms of the system) and *compositionality*.

Definition 6 (Logical term). A term a is a *logical term* (w.r.t. Δ) if it is determined by the domain, i.e. by $\{\mathcal{D}\}$ (w.r.t. Δ).²²

Definition 7 (Compositionality). A language L is *compositional* (w.r.t. Δ) if each phrase p is determined by $\{a : a \text{ is a term in } p\} \cup \{\mathcal{D}\}$ (w.r.t. Δ).

It is easy to see that these definitions are in accord with the general use of the notions of logical terms and of compositionality. A first order-order language with a system of semantic constraints that includes the constraints for standard first-order logic will of course be compositional.

Giving place to cases of less than full determination of the extension of terms, we may define notions of *dependency*.

Definition 8 (Dependency). A set of phrases A *depends* on the set of phrases B (w.r.t. Δ) if there are Δ -models $M = \langle D, I \rangle$ and $M' = \langle D, I' \rangle$ sharing the same domain D such that for any Δ -model $M^* = \langle D, I^* \rangle$, if $I^*(r) = I(r)$ for all $r \in B$, then $I^*(p) \neq I'(p)$ for some $p \in A$ (that is, fixing the phrases in B in a certain way excludes some interpretation for the phrases in A that can otherwise be realized).

²¹ Assuming of course the proof system is sound with respect to the system of semantic constraints. In the case of semantic definability we do not need such a provision. Tarski cites a very near result, attributing it to Padoa: "In order [...] to show that a term ' a ' cannot be defined by means of the terms of a set B on the basis of a set X of sentences, it suffices to give two interpretations of all extra-logical constants which occur in the sentences of X , such that (1) in both interpretations all sentences of the set X are satisfied and (2) in both interpretations all the terms of the set B are given the same sense, but (3) the sense of the term ' a ' undergoes a change" (Tarski, 1934, p. 300).

²² Note that this is a definition of logical terms in the *shallow* sense: it is system relative, pertaining to logical terms functional role.

A set of phrases A is *independent* of the set of phrases B if it does not depend on it. We say that a phrase p depends on (is independent of) the set of phrases B if $\{p\}$ depends on (is independent of) B .²³

So, for instance, by the constraint $I(\textit{bachelor}) \subseteq I(\textit{unmarried})$ *bachelor* depends on $\{\textit{unmarried}\}$: let $I(\textit{unmarried}) = \{\textit{John}, \textit{Mary}\}$, $I'(\textit{bachelor}) = \{\textit{John}, \textit{Jim}\}$, so for any I^* such that $I^*(\textit{unmarried}) = I(\textit{unmarried})$, $I^*(\textit{bachelor}) \neq I'(\textit{bachelor})$.

Note that if a is determined by the terms in B , then it depends on the terms in B if and only if it is not a logical term.

Note also that dependency is a symmetric relation. Nonetheless, the set of terms cannot be partitioned into maximal classes of mutually dependent terms, as dependency is *not* an equivalence relation: it fails transitivity (to show this take the constraints: $I(\textit{big}) \subseteq I(\textit{extended})$ and $I(\textit{allYellow}) \subseteq I(\textit{extended})$. *big* and *extended* depend on each other, and so do *allYellow* and *extended*, but *big* and *allYellow* are independent).

Let us consider for example a system Δ with semantic constraints for standard first order logic augmented with semantic constraints for all color terms as formulated above. The color terms form a category: nothing in the constraints we formulated makes a difference between the terms, they all function in the same way in Δ .²⁴ What was noted before, that we can have schematic letters for the color terms, follows from Proposition 1 above. Each color term depends on each of the other ones. Take *allGreen* and *allRed*. For any $M = \langle D, I \rangle$, if $I(\textit{allRed})$ is a non-empty set A , then $I(\textit{allGreen}) \neq A$ although there is a different interpretation I' for the same domain such that $I'(\textit{allGreen}) = A$.

There are categories where all terms are independent of each other (such are the usual predicates of the same arity in standard first order logic with no additional constraints), but sets of dependent terms in general do not form categories: dependent terms need not be interchangeable (such is the set $\{\textit{bachelor}, \textit{unmarried}\}$ in a system like the one above where the only constraint involving those terms is $I(\textit{bachelor}) \subseteq I(\textit{unmarried})$).

This concludes our description of the framework. These preliminary investigations make it clear that there is much more to develop in the theory of semantic constraints²⁵. For instance, modal logic can be incorporated. Furthermore, investigations of the semantics of various philosophical concepts may enjoy the results received from this general framework.

²³ Here we diverge from Tarski's notion of *independence*, which is weaker — for a term $\{a\}$ to be independent of a set of terms B he requires only that a will not be definable by the terms in B (*ibid.*).

²⁴ Of course we could have a more elaborate system where the relations between color terms is more nuanced and not symmetric.

²⁵ For further details, see (Sagi, 2013).

3. Conclusion

Contemporary debates in logic rely on the assumption that arguments have forms, and that an argument's form is determined by its logical vocabulary: the so called "tenets of formality". In this paper I proposed a view of logic that relies on forms, but takes a step towards breaking free of the conception of form as constituted by logical terms.

I contend that a false dichotomy results when taking logical terms to be those that have to do with form, and nonlogical terms to be those that do not. Form has to do with everything that is held fixed in a sentence or an argument. From this standpoint, new logical frameworks become available. Any *semantic constraint* on a language, a way of explicitly fixing terms of the language in some manner or to some degree, contributes to the form of sentences in the language. We may hope that the general view of formality that emerges will open the field of logic to a variety of systems that have not hitherto been considered.

Is there one correct system of semantic constraints for logical consequence? The framework I propose has the permissive spirit of relativism, but one can accept the theory of semantic constraints and deny relativism. It can be claimed that there is one set of constraints which determines logical consequence. Alternatively, one can be skeptical of the search for the "correct" set of constraints as much as one can be of the "correct" set of logical terms. This question is left open, but we should note that even in a relativistic framework there could be theoretical preferences for formal systems that fix certain terms to some degree and not further.²⁶ Naturally, settling these issues will require further philosophical investigation.

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²⁶ For instance, we may fix the dependencies between color terms but refrain from fixing their extensions completely. There may be principled reasons for this (along the lines of the epistemic motivation mentioned earlier), but also various pragmatic reasons. We might not want to commit to a fixed extension and would like to consider different possibilities, we might want to keep things simple, etc.

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