



# Alethic modalities

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## Abstract

It is widely held that metaphysical modality is the broadest non-epistemic, alethic modality, and that a posteriori modal essentialist truths, like that gold has atomic number 79, enjoy the necessity of the broadest alethic modality. One prominent argument for these conclusions—given by Cian Dorr, John Hawthorne, and Juhani Yli-Vakkuri—rests upon an extremely dubious premise: that certain pairs of properties—e.g., being gold and being made of atoms containing 79 protons—are one and the very same property. The two properties are seen to be distinct on independent philosophical grounds. Metaphysical modality is in fact a restricted alethic modality. In particular, mathematical modality is broader than metaphysical modality. Arguably, the broadest alethic modality is logical modality, which is distinct from metaphysical modality. Even if it is metaphysically necessary that gold have atomic number 79, there is no logical/analytical inconsistency in the supposition that it does not.

**Keywords** Alethic modality · Broadest modality · Impossible world · Logical modality · Metaphysical modality

## 1 1

A type of necessity  $\Box$  (e.g., mathematical necessity,  $\Box_{Math}$ ) is *alethic* iff a proposition's having  $\Box$  logically entails that proposition itself, i.e., iff for all propositions  $p$ ,  $\Box p \models p$ .<sup>1</sup> Among truths, some are alethically necessary of one variety or other, while the rest are contingent of that same variety. The class of worlds that respect a particular set of laws (e.g., the laws of biology), perhaps together with initial or boundary conditions, characterize a particular variety of modality. Some varieties

<sup>1</sup> I take it throughout that alethic necessity and alethic possibility are primarily properties of propositions. Derivatively a sentence is said to have a modality of a particular variety (e.g., to be *mathematically necessary*) iff the proposition it expresses has that variety of modality.

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of alethic possibility are broader or wider than others (less constrained, less narrow, less restricted). A proposition is *logically possible*,  $\Diamond_{Logic}$ , iff it does not logically entail a contradiction. Logical necessity is the variety of necessity which is such that an argument is valid iff it is necessary in that way that if the premises are true then so is the conclusion. A proposition is *mathematically possible*,  $\Diamond_{Math}$ , iff it does not lead to mathematical falsehood. Mathematical possibility is broader than natural (sometimes called ‘nomological’) possibility,  $\Diamond_{Nature}$ , in that the class of naturally possible worlds is a proper subclass of the class of mathematically possible worlds. Correspondingly, the class of naturally possible propositions is a proper subclass of the class of mathematically possible propositions. (Each naturally possible proposition determines its natural intension, the class of naturally possible worlds in which the proposition is true.) Contrapositively, the class of mathematically necessary truths is a proper subclass of the class of naturally necessary truths. That is, some naturally necessary truths (e.g., that nothing travels at a velocity greater than  $c$ ) are mathematically contingent, whereas every mathematically necessary truth is *ipso facto* naturally necessary.

Consequently, and not surprisingly, the propositional logic of one variety of alethic modality may differ from that of another. If there is a unique unrestrictedly broad alethic possibility—the broadest non-epistemic alethic possibility  $\Diamond$ —its propositional logic is at least as strong as  $S4$ , which extends the weak normal alethic modal logic  $T$  by including the logical axiom schema 4:  $\Box \phi \rightarrow \Box \Box \phi$ . For if it is alethically possible in some way  $m_1$  for a proposition  $p$  to be alethically possible in some way  $m_2$  (e.g., if it is mathematically possible for  $p$  to be physically possible) then that way itself—being  $m_1$ -possibly  $m_2$ -possible—is another way that  $p$  is alethically possible. In fact, the propositional logic of the broadest alethic possibility  $\Diamond$ —i.e., of alethic possibility *in some way or other* (assuming there is such a status)—would appear to be  $S5$ , which extends  $T$  with the axiom schema  $E$ :  $\Diamond \phi \rightarrow \Box \Diamond \phi$ . For it would appear that the very same worlds are alethically possible (in some way or other) according to every world that is alethically possible—i.e., according to every naturally possible world, every metaphysically possible world, every mathematically possible world, etc. If so, then if a proposition  $p$  is true in some alethically possible world  $w$ , then in every world  $w'$  that is alethically possible, there will be at least one world, viz.  $w$ , that is alethically possible according to  $w'$  and in which  $p$  is true.

Undoubtedly, the variety of alethic necessity of greatest interest to metaphysicians is metaphysical necessity,  $\Box_M$ , and the propositional modal logic of greatest interest is the propositional logic of metaphysical modality. I have argued elsewhere, following Chandler (1976), that the propositional logic of metaphysical modality is weaker than  $S4$ . (I believe it is  $T$ .) At bottom, the argument is that the 4 axiom, by contrast with the  $T$  axiom, is not analytic.

We may say that a variety of alethic modality  $m$  is a sub-modality of a variety  $m'$  (e.g., that natural modality is a sub-modality of metaphysical modality) iff the  $m$ -possible worlds are a subclass of the  $m'$ -possible worlds. One variety of alethic modality  $m$  is a sub-modality of another  $m'$  iff whatever is  $m$ -possible is  $m'$ -possible, equivalently iff whatever is  $m'$ -necessary is  $m$ -necessary. For example, natural modality is a sub-modality of biological modality, since whatever is biologically necessary (required by laws of biology) is naturally necessary (required by natural

law). In fact, the naturally possible worlds form a proper subclass of the biologically possible worlds. I say that one variety of modality  $m$  is a *restriction on* another variety  $m'$  iff  $m$  is a proper sub-modality of  $m'$ , i.e.,  $m$  is a sub-modality of  $m'$  and not vice versa. I say that  $m_1$  is *broader than*  $m_2$  iff  $m_2$  is a restriction on  $m_1$ , so that  $m_1$ 's notion of possibility  $\Diamond_1$  is broader than  $m_2$ 's notion  $\Diamond_2$ . Natural modality is a restriction on biological modality and also on chemical modality; equivalently, biological modality and chemical modality are each broader than natural modality.

If one variety of possibility  $\Diamond_1$  is broader than another  $\Diamond_2$ , then the dual  $\Box_1$  of  $\Diamond_1$  is necessity in a narrower sense than the dual  $\Box_2$  of  $\Diamond_2$ , and the  $\Box_1$  truths form a proper subclass of the  $\Box_2$  truths. Since biological possibility  $\Diamond_{\text{Biology}}$  is broader than natural possibility  $\Diamond_{\text{Nature}}$ ,  $\Box_{\text{Biology}}$  is a narrower notion than  $\Box_{\text{Nature}}$ . Biological necessity concerns not natural laws in general, but more specifically the laws of biology. Correspondingly, the class of biologically necessary truths is a subclass of the class of naturally necessary truths. Any proposition that has  $\Box_{\text{Biology}}$  *ipso facto* has  $\Box_{\text{Nature}}$ .<sup>2</sup>

It could be that one variety of modality is *conceptually* (*de jure*) a sub-modality of another, whereas each modality variety is *in fact* (*de facto*) a sub-modality of the other, so that neither is broader than the other. There is, for example, the 1960s *All You Need is Love* motif that loving people are capable of whatever it is logically possible for someone to do.<sup>3</sup> The converse is given; whatever someone is capable of doing, it is logically possible for someone to do. Kripke suggested that it could turn out that whatever is metaphysically possible is also physically possible. He says, "Physical necessity, *might* turn out to be necessity in the highest degree. But that's a question which I don't wish to prejudge. At least for this sort of example, it might be that when something's physically necessary, it always is necessary *tout court*" (*Naming and Necessity*, p. 99). The prospect that whatever is physically necessary is metaphysically necessary obtains if, and only if, metaphysical modality is a sub-modality of physical. Some philosophers hold that physical modality is conceptually a sub-modality of metaphysical. It could be, then, that physical modality and metaphysical is each a sub-modality of the other. (I contend they are not.)

## 2 II

Besides metaphysical modality, one other variety of alethic modality (or alleged variety) of special philosophical interest is unrestricted alethic modality—i.e., the broadest variety of alethic modality. Numerous varieties of modality form a complex web in which one variety is conceptually a proper restriction on another. It is widely

<sup>2</sup> Some writers (including Dorr, Hawthorne, and Yli-Vakkuri, *The Bounds of Possibility*; see p. 200n2) say instead that one variety of necessity  $\Box_1$  is "broader than" another variety  $\Box_2$  iff  $\Box_2$  is broader in the present sense than  $\Box_1$ —precisely the reverse of the present usage. The alternative usage is confusing and to some extent encourages the myth that metaphysical necessity  $\Box_M$  is the broadest alethic necessity ■. (See also footnote 4 below.).

<sup>3</sup> This is a plausible interpretation of John Lennon's otherwise puzzlingly vacuous lyric: "There's nothing you can do that can't be done. ...".

posited that metaphysical modality is the limiting case of unrestricted modality, so that all varieties of modality are conceptually sub-modalities of metaphysical. One branch within the web, for example, might include the following sequence, where ‘ $\subset$ ’ indicates the sub-modality relation:

natural modality  $\subset$  physical modality  $\subset$  chemical modality  $\subset$  bio-chemical modality  $\subset \dots \subset$  metaphysical modality.

In fact, many philosophers define metaphysical possibility  $\Diamond_M$  as the broadest alethic possibility  $\Diamond$ . David Lewis so defined metaphysical possibility.<sup>4</sup> Those who take metaphysical modality to be the broadest variety of alethic modality generally assume—erroneously in my judgment—that the propositional logic of metaphysical modality is S5.

Lewis’s contention that metaphysical modality is the broadest alethic modality was based on interconnected errors: First was a mistaken conflation. In *Counterfactuals* (1973), Lewis defined a ‘possible world’—i.e., a world that is possible *tout court*—as a “way things could have been” (p. 84). If a possible world is a maximal way all things *could have been*, then a world *simpliciter* is a maximal way for all things to be, whether metaphysically possible or not. Lewis failed to draw this conceptually crucial distinction between a world (in the metaphysician’s sense) and a world that is possible. A second error was Lewis’s conception of worlds (in the relevant sense) as parallel universes. This misconception was based partly on a serious misinterpretation of the propositional-attribution locution ‘In  $w$ ,  $x$  is such-and-such’. The locution means simply that  $w$  represents (depicts)  $x$  as being such-and-such, so that the proposition that  $x$  is such-and-such obtains according to  $w$ . Lewis interpreted the locution as meaning instead that there is something in  $w$  that: (i) represents  $x$  (through resemblance); and (ii) genuinely *is* such-and-such—i.e., not merely *is such-and-such according to  $w$* , but *is such-and-such* period, full stop. These errors in thought supported Lewis’s equivalence thesis that  $p$  is possible (*tout court*) iff there exists a parallel universe (“possible world”) in which a  $p$ -variant obtains. For example, Lewis inferred from ‘There might have been talking donkeys’ the existence

<sup>4</sup> In an important graduate seminar on possible worlds given jointly by Lewis and Saul Kripke at Princeton University, fall 1979. In keeping with his view that metaphysical modality is the broadest alethic modality, in *On the Plurality of Worlds* (1986) Lewis erroneously complains (pp. 246–248) that metaphysically-modal accessibility—which legitimately generates the prospect of alethic modalities broader than metaphysical modality—amounts to nothing more than an entirely unexplained restriction whereby one unjustifiably ignores unwanted (“obnoxious”) possible worlds.

Oddly, just a dozen pages earlier (p. 234) Lewis also asserts that what we call ‘metaphysical accessibility’ is a certain kind of qualitative resemblance or similarity among possible worlds. Although erroneous, Lewis’s assertion provides an explanation of the very sort that a few pages later he will complain is lacking. Lewis’s assertion is misleading. Foremost, it betrays a serious misunderstanding of the philosophical phrase ‘metaphysically possible’, and reflects his idiosyncratic interpretation. *Contra* Lewis, genuine metaphysical accessibility—what Kripke would have called ‘possibility *tout court*’—is not a matter of qualitative resemblance or similarity. Kripke introduced accessibility not in terms of resemblance but in overtly modal terms. Despite his assertion, Lewis did not share my view that genuine metaphysical modality—modality *tout court*—is a restriction on the broadest alethic modality (and is in this regard just like physical modality, etc.). Even in the earlier passage, Lewis emphasizes that for him, possibility *tout court* is the broadest alethic possibility.

of genuine talking donkeys in a parallel universe. These various blunders resulted in Lewis's serious misunderstanding of modal locutions, especially those involving nested modality. By contrast, a conception of metaphysically possible worlds as maximal scenarios that metaphysically might have obtained makes clear that metaphysical possibility is restricted logical possibility.<sup>5</sup>

The pioneering philosopher/logician Saul Kripke writes the following in the preface to his masterpiece *Naming and Necessity* (1972, p. 19): “the notion of all states of the entire world that are possible in the broadest (metaphysical) sense involves a certain amount of idealization”. This appears to be an endorsement of the thesis that  $\Diamond_M = \Diamond$ . Kripke's actual stance, however, is much more cautious and nuanced. In the text of *Naming and Necessity* he explains what I call ‘metaphysically necessary truths’ as “not contingent truths but necessary truths, and here of course I don't mean just physically necessary, but necessary in the highest degree—whatever that means ... necessity *tout court*.” (p. 99). It is clear that Kripke has in mind metaphysical necessity here—as opposed to full-on logical necessity, properly so called.<sup>6</sup> Kripke later expressed doubts concerning the logic of metaphysical modality. Following up on a conversation regarding my then unpublished paper “The Logic of What Might Have Been”, he wrote this to me in a letter dated February 3, 1987 concerning his early papers on the semantics for modal logic (1987):

As you see, there is no suggestion that S5 is basic and the weaker systems come from some restricted conception. *R* [accessibility] is characterized in terms of truth and possibility of propositions in worlds. ...

One thing I do is, I now think, somewhat misleading. I should have stressed that the use of *R* does not make “possible” (as applied to worlds) into a two-place predicate, any more than, as you say, “is bald” is. Probably I only noticed this afterwards. Also, I should have stressed that strictly speaking, many of the worlds are not “possible” but only “possibly possible”, and so on, unless we have S4.

By the time I gave the seminar I talked to you about [see footnote 4] I had definitely thought these points through, having seriously considered whether the conventional presupposition that the basic modal logic is S5 is justified.

I am getting closer to thinking that your treatment of the ship is the correct solution. ...

Kripke's remarks about his accessibility relation *R* allude to my urging that *R* be defined in terms of possibility (of worlds), as ‘ $\lambda w_1 w_2$  [According to world  $w_1$ , world  $w_2$  is a possible world]’, or more briefly ‘ $\lambda w_1 w_2$  [In  $w_1$ ,  $w_2$  is possible]’. If the

<sup>5</sup> In addition to the immediately preceding footnote, see Lewis's *On the Plurality of Worlds* and my critical review, “Empire of Thin Air” (1988). I wrote “The Logic of What Might Have Been” largely in response to several authoritative but erroneous proclamations that Lewis made as if canonical in the 1979 Kripke-Lewis seminar at Princeton. It is likely that some (or all) of Lewis's philosophical errors are not idiosyncratic and are perhaps widespread.

Whereas Lewis argued that the propositional logic of metaphysical modality is S5, his counterpart theory does not in fact support S5, or even S4.

<sup>6</sup> The ‘necessity’ in the title of Kripke's monograph refers to metaphysical necessity, not logical.

4 axiom schema has false instances, then some possibly possible worlds are impossible worlds. The metaphysically possible worlds form a restricted subclass of the metaphysically possibly possible worlds if, and only if, the propositional logic of metaphysical modality is weaker than *S4*. In that case, metaphysical modality is a restricted alethic modality.

The picture of the various varieties of restricted modality forming a complex web with metaphysical modality being the limiting case of unrestricted modality, however tempting, sharply clashes with the facts. Some varieties of alethic modality are indeed restrictions on metaphysical modality. So-called physical modality places a restriction on metaphysical modality. Physical possibility  $\Diamond_{\text{Physics}}$  is sometimes characterized as whatever is compossible with the actual laws of physics. Physical modality could then be described as “restricted metaphysical modality”. (I believe this characterization of physical modality to be incorrect.) Certainly, metaphysical modality places no like restrictions on itself; it is unrestricted metaphysical modality. Metaphysical modality lies along a spectrum of varieties of alethic modality. It lies on the broader half of the spectrum, but not at the extreme endpoint. There are broader varieties of alethic possibility than  $\Diamond_M$ —for example, mathematical possibility  $\Diamond_{\text{Math}}$  and logical possibility  $\Diamond_{\text{Logic}}$ . Insofar as physical possibility is restricted metaphysical possibility, metaphysical possibility is in turn restricted mathematical possibility, and also restricted logical possibility. It is my contention that metaphysical modality—what Kripke calls ‘modality *tout court*’—is *logical modality restricted by the laws of metaphysics and by initial or boundary conditions* (such as that gold is in fact the chemical element with atomic number 79). In my considered judgment, one variety of alethic possibility that lies on the spectrum between  $\Diamond_M$  and  $\Diamond_{\text{Logic}}$  is metaphysically possible possibility, i.e., the iterated alethic possibility  $\Diamond_M \Diamond_M$ .

If, as Kripke seriously considers, metaphysical is a sub-modality of physical, then the received picture has things exactly reversed: physical modality is broader than metaphysical. For even if it should turn out that whatever is physically necessary is also metaphysically necessary, some other things that are equally metaphysically necessary nevertheless fail in some physically possible worlds. It is metaphysically necessary, for example, of the table Woody that it is not a vase.<sup>7</sup> The laws of physics apply not so much to Woody as to the hunk of wood that makes Woody up. In particular, it is perfectly consistent with the laws of physics, even together with the fact that Woody was built to be a table (and has even been used as such), that Woody is an oversized, table-shaped vase. Sometimes, whereof metaphysics makes demands, thereof physics is silent. If whatever is physically necessary is metaphysically necessary, then things are quite the other way around from the received picture.

A complex structure of the envisioned sort may be maintained on the cheap, by reinterpreting ‘physical modality’ to mean a narrower variety of alethic modality (i.e., a narrower variety of possibility and a correspondingly broader variety of necessity),

<sup>7</sup> Thus Kripke writes that “if the very block of wood from which the table was made had instead been made into a vase, the table would never have existed. So (roughly) *being a table* seems to be an essential property of the table”, *Naming and Necessity*, p. 115n57.

e.g., physical-*cum*-metaphysical modality. However, any such reinterpretation is misinterpretation. In any event, other varieties of alethic modality are both conceptually and extensionally broader than metaphysical. This is decisively established by the nearly indisputable (not to say undisputed) fact that there are metaphysically necessary truths that are neither analytic nor mathematically necessary. The variety of alethic modality that derives from the analytic/synthetic dichotomy is logical modality, not metaphysical: analytically necessary sentences express logically necessary truths; synthetically contingent sentences express logically contingent truths. (See footnote 1.) One of the most significant results that emerges from *Naming and Necessity* is that, contrary to many philosophers (especially empiricists), metaphysical necessity is not reducible to analyticity. Whereas analyticity entails metaphysical necessity, not all metaphysically necessary truths are analytic. One straightforward counterexample is that of Woody. Although it is both logically and mathematically possible, it is metaphysically impossible for Woody to have originated from entirely different matter, for example from a block of frozen water from the Thames River (*Naming and Necessity*, pp. 113–114). In particular, the sentence ‘Woody did not originate from a block of ice’, though metaphysically necessary, is synthetic. The metaphysically impossible prospect that Woody originated from frozen Thames water is perfectly consistent, and is therefore logically possible. Logical modality is an extremely broad alethic modality, significantly broader than metaphysical modality.<sup>8</sup>

There are innumerable many examples that are even more definitive. It is metaphysically necessary that Julius Caesar is not a number. Nothing that is a number in a metaphysically possible world is Caesar, since metaphysically Caesar could not have been a number. However, it is both mathematically possible and logically possible that Caesar is a number. The surreal hypothesis that Caesar is, say, the number one—metaphysically impossible though it is—leads to no mathematical or logical contradiction. In particular, the sentence ‘Julius Caesar is not a number’ is synthetic. Similarly, although it metaphysically impossible that England is the direction of a line, the prospect of England being a line’s direction is logically consistent, hence logically possible. Likewise, it is logically possible that Nathan Salmón is a Shakespearean sonnet in iambic pentameter, though the prospect of my being a poem of any sort is metaphysically quite impossible.<sup>9</sup>

<sup>8</sup> The synthetic sentence ‘Woody did not originate from a block of ice’, though metaphysically necessary, is logically contingent. Since logical possibility is broader than metaphysical possibility, logical necessity (analyticity) is narrower than metaphysical necessity. See footnote 1 above concerning the objects of logical modality. See also footnote 2.

Any logical truth is logically necessary and vice versa. Any mathematical truth is mathematically necessary and vice versa. Any logical truth is mathematically necessary. If logicism is incorrect—i.e., if (as is widely believed) mathematics is not reducible to pure logic—then the irreducibly mathematical truths are not logically necessary. Their mathematically impossible denials are logically possible.

<sup>9</sup> I have discussed these and related issues in a number of pieces, primarily in *Reference and Essence* (1981), at pp. 229–252; “Impossible Worlds” (1984); “Modal Paradox: Parts and Counterparts, Points and Counterpoints” (1986a); “The Logic of What Might Have Been” (1989); “This Side of Paradox” (1993); and “Modal Paradox II: Essence and Coherence” (2021). The examples of Caesar being a number and of England being a direction are from Frege’s masterpiece, *The Foundations of Arithmetic* (1884). (Frege likely held the mistaken view that ‘Caesar is not a number’ is analytic, at least in a great many idiolects.).



Arguably, the broadest variety of possibility  $\blacklozenge$  is in fact logical possibility, and the broadest modality is in fact logical modality. To be sure, there are modality-like concepts that are even broader than logical. Consistency in the first-order logic of universal and existential quantification is broader than logical possibility, and consistency in truth-functional propositional logic is even broader still. Propositional-logical truth might be called ‘tautological necessity’. Analytic sentences express logically necessary truths, but some do not express truths of propositional logic, e.g., ‘If some women are billionaires, then some billionaires are women’. On the other hand, mere truth-functional consistency is at least arguably not a genuine variety of *possibility*. Whereas the prospect that some women are billionaires while no billionaires are women is truth-functionally consistent, it is also quite impossible in any standard sense and does not seem “possible” in any genuine and robust sense. It is at least very plausible that  $\blacklozenge$  is not  $\lozenge_M$  but  $\lozenge_{Logic}$ . On the other hand, where the word ‘possible’ occurs in philosophical English without a qualifier—so-called possibility *tout court*—plausibly it is  $\lozenge_M$  rather than  $\blacklozenge$  (or  $\lozenge_{Logic}$ ) that is intended.

### 3 III

In *The Bounds of Possibility*, Cian Dorr, John Hawthorne, and Juhani Yli-Vakkuri reject the notion that metaphysical modality is a restricted variety of modality.<sup>10</sup> They proffer arguments to the effect that metaphysical modality—as exemplified in Kripke’s examples of metaphysically-modally essentialist truths—is the broadest variety of alethic modality (i.e., the alethic modality with the broadest non-epistemic alethic possibility  $\blacklozenge$ ), and that its propositional logic is *S5*. Dorr et al. dismiss considerations supporting the restrictedness of metaphysical modality as “simply bad arguments,” ones that make “precisely the sort of mistake that Kripke was trying to purge,” *to wit*, the fallacious inference from a-posteriority to metaphysical contingency (§8.1, pp. 200–202, 207–211). I shall expose a fundamental error in what I take to be their central arguments.

Pared to its essentials, their initial argument, which I shall call ‘A’, proceeds as follows, where  $\blacksquare$  is the dual of the broadest non-epistemic alethic possibility  $\blacklozenge$ :

A1:  $\blacksquare$  (Hesperus = Hesperus).

A2: Hesperus = Phosphorus.

Therefore,

A3:  $\blacksquare$  (Hesperus = Phosphorus).

<sup>10</sup> Cian Dorr and John Hawthorne, with Juhani Yli-Vakkuri, *The Bounds of Possibility: Puzzles of Modal Variation* (Oxford University Press, 2021).



This is a simplified reconstruction of the core argument in Dorr et al., §8.1, pp. 204–207.<sup>11</sup> The deduction of A3 from A1 and A2 consists entirely of a single valid application of Leibniz's law (substitution of equality). Dorr et al. argue that the proposition that Hesperus is Hesperus has the necessity of the broadest variety of alethic modality (i.e., the dual of the broadest possibility  $\blacklozenge$ ). I do not dispute argument A. The issue of whether A1 is true might in fact be simply terminological. The term 'modality' may be used to mean: *modality-like notion no broader than logical modality*. So interpreted, A1 is beyond reasonable doubt; it is indeed logically necessary that Venus is itself. Though there are branches of logic (e.g., propositional logic) that do not require that Venus be itself, there is no robust sense in which it is genuinely possible that Venus is not itself.

Things go awry when Dorr et al. apply a variation on argument A to the case of gold having atomic number 79 (§8.1, pp. 200, 207–211). Let us call the following simplified version 'B', wherein the second-order predicate '='<sup>2</sup> is a term for identity of *n*-ary attributes (regarding a proposition as a 0-ary attribute):

B1:  $\blacksquare$  (Every golden thing is golden).

B2:  $(\lambda x[x \text{ is golden}]) =^2 (\lambda x[x \text{ is made of atoms containing 79 protons}])$ .

Therefore,

B3:  $\blacksquare$  (Every golden thing is made of atoms containing 79 protons).<sup>12</sup>

It should be noted that the analog of premise B2 for Hesperus/Phosphorus is significantly more complex than A2. In fact, the deduction of the conclusion of argument B from the premises involves more than Leibniz's law. (See the next section below.) However, I agree with Dorr et al. that B is, like A, perfectly valid. In fact, any controversy over the question of validity is a red herring. The argument that results by strengthening B2 to  $\ulcorner \blacksquare(B2) \urcorner$  is certainly valid and serves Dorr et al.'s purposes equally well. In particular, I grant that if B2 then  $\blacksquare(B2)$ .

Dorr et al. assert that their argument B can be modified to address the case of a wooden artifact like Woody (p. 200–201), and they provide a first approximation of the analog of the second premise (in their (6), §7.3, pp. 185–186). Their analogous argument, which I shall call 'C', is evidently along the lines of the following:

<sup>11</sup> Their argument A is aimed specifically at Justin Clarke-Doane (2021), who asserts that it is logically possible (in one sense) that Hesperus  $\neq$  Phosphorus. Contrary to Dorr et al., Clarke-Doane's dubious assertion is not representative of those who recognize metaphysical modality as a restricted modality.

<sup>12</sup> The predicate-forming ' $\lambda$ ' abstraction operator is employed here not in its standard use as a function-abstraction operator (which does not suit Dorr et al.'s purposes), but instead, in combination with parentheses, as a non-extensional operator of attribute-abstraction, analogous to a gerund or infinitive phrase, 'being golden' or 'to be golden'. (Parentheses may here be regarded as a device of indirect quotation. Dorr et al. use 'golden' to mean a *hunk of gold*.)

C1: ■ (Woody = Woody).

C2:  $(\lambda x[x = \text{Woody}]) =^2 (\lambda x[x = \text{the first table-shaped object to have originated in manner } m])$ .

Therefore,

C3: ■ (Woody = the first table-shaped object to have originated in manner  $m$ ).

The relevant particular manner  $m$  is not specified, but the under-specificity is not problematic given Dorr et al.'s purposes. They presumably believe that there is some particular manner  $m$  that fleshes out the argument in such a way that premise C2 is true. I shall also grant, for what it is worth, that the definite description 'the first table-shaped object to have originated in manner  $m$ ' (once the relevant manner  $m$  is fleshed out) designates Woody with respect to every metaphysically possible world in which Woody exists (and does not designate anything else with respect to worlds in which Woody does not exist). In that sense, the description is a rigid designator.<sup>13</sup>

Analogous arguments may evidently be given for each of Kripke's interesting examples of a posteriori but non-trivially metaphysically necessary truths, e.g., that water is H<sub>2</sub>O.

#### 4 IV

Argument *C* is, like both *A* and *B*, perfectly valid. Its weakness lies rather in its premise C2, which is highly controversial. I shall argue that it is in fact dubious in the extreme.

I judge the analogous premise B2 also erroneous. Dorr et al. provide a brief defense of B2. They write:

if you want to insist that it is obviously [possible in the broadest sense that *not*] every golden thing is made of atoms containing seventy-nine protons, you had better think that the identity B2 is obviously false. But it is obviously *not* obviously false! The investigation of identities like B2 seems to play a central role in science. Something has gone terribly wrong if we start dismissing them out of hand based on philosophical arguments having to do with a priori knowledge and the like. (p. 209, labeling altered to match the present labeling)

<sup>13</sup> It is significant that with respect to some logically possible worlds in which Woody exists the description fails to designate—Woody and everything else. By contrast, the name 'Woody' is an obstinately rigid designator; it designates Woody with respect to every possible world, whether Woody exists there or not. It follows that the name and the description differ in semantic content. Cf. *Reference and Essence*, chapter 3.

Let us stipulate that the sentence 'Anything that originated in manner  $m$  did not originate from a block of ice' is analytic. Dorr et al. write: "But any variant of (6) simple enough for us to write down will feel artificial and over committal, even if it manages to avoid clear counterexamples. ... the relevant sort of difficulty in spelling things out crops up wherever there is vagueness" (§7.3, p. 186) I am not persuaded that there is any relevant vagueness here, but I do not here dispute the claim.

It is of course a scientific finding that a substance sample is gold if it is made entirely of atoms containing 79 protons. Very plausibly, it is even metaphysically necessary that gold is the chemical element with atomic number 79. For arguably, any element in another metaphysically possible world that has a different atomic number simply is not gold, and is instead the element that *actually* has that atomic number (if any does). It does not follow, however, that the property of being gold is literally one and the very same property as—strictly numerically identical with—the structural property of being made of atoms containing 79 protons. On the contrary, there are logically possible worlds, albeit metaphysically impossible, in which gold has atomic number 78, or 80, or no atomic number at all. In any such logically possible world, the two properties come apart; they are not co-extensive there. In such a world, something is a hunk of pure gold yet is not made there entirely of atoms having exactly 79 protons. The two properties are therefore numerically distinct.

This sort of consideration against *B2* cannot be applied also to the reflexive pair of a property and itself. It is logically impossible for a single self-same property to differ in extension from itself; logically, only distinct properties can have different extensions. Though there are logically possible worlds in which gold lacks atomic number 79, there is no logically possible world in which gold lacks the property of being identical with gold. This difference between them is sufficient to establish that being gold and being made of atoms containing 79 protons are distinct properties.

Science is interested in the atomic analysis of gold. Science does not have a similar interest in the philosophical analysis of the property of being gold. The issue of obviousness is beside the point. Mere falsity of *B2*, obvious or not, is sufficient to block argument *B*. The falsity of *B2*, obvious or not, is established on philosophical grounds. Those same grounds establish not only that *B2* is false; they also establish that *B3* is false, assuming there is a broadest modality.

A number of distinguished philosophers—including Dorr et al., David Chalmers, Jaakko Hintikka, David Lewis, Richard Montague, Robert Stalnaker, and Timothy Williamson—are sympathetic to a conception of propositions on which they are no more fine-grained than proposition “intensions”, i.e., classes of metaphysically possible worlds. This radical conception of propositions can embrace *B2* and *C2*, by construing properties as property intensions, i.e., as functions from metaphysically possible worlds to extensions (classes).<sup>14</sup> On the other hand, a good many philosophers reject these coarse-grained conceptions of propositions and properties, precisely because of their extremely implausible consequences. It is a merit of Dorr

<sup>14</sup> See for example Williamson (2021a) and (2021b). I mount a case against the intension conception of propositions in “Synonymy” (2024a) and more extensively in “Singular Concepts” (2024b). Arguably, the claim that science seeks to answer *what* being gold *is* (answer: being the element whose atoms contain 79 protons) gains credence insofar as, and to the extent that, the relevant issue concerns the “property” of being gold in the sense of its metaphysical intension. Unlike *B2*, the weaker premise that the intension of  $(\lambda x[x \text{ is golden}]) =^2$  the intension of  $(\lambda x[x \text{ is made of atoms containing 79 protons}])$  is true, but the argument that results by replacing *B2* with the latter is invalid.

On my view, what science seeks to answer initially is which chemical property coincides in metaphysical extension with being gold. It is science together with metaphysics that yields the stronger result that being gold coincides in intension with being the element whose atoms contain 79 protons. Cf. *N&N*, p. 138.

et al.'s argument that their defense of *B2* is neutral regarding the proper way to conceive of properties. It is in the nature of the dialectic that an argument that metaphysical modality is unrestricted cannot rest on a radical and counterintuitive conception of properties or propositions as merely intensions.

Whatever merit Dorr et al.'s defense of *B2* might have, the analogous defense for *C2* has next to none. The property  $(\lambda x[x = \text{Woody}])$  is Woody's haecceity, its *thisness*. As Morgan Davies has noted (unpublished), an object's haecceity is a very different sort of property from that of being the first such-and-such with a particular origin. Physics has no special interest in the analysis of Woody's haecceity into its two components: Woody itself; and the relation of identity (the smallest equivalence relation). Nor does physics have any particular interest in Woody's manner of origin. Analogously, the property  $(\lambda x[x \text{ is golden}])$  is the intrinsically relational property involving the chemical element gold of being a hunk of *that* particular substance kind, in contrast to the structural property of being composed of atoms with 79 protons. As with gold, there are logically possible (albeit metaphysically impossible) worlds in which Woody has its actual haecceity but lacks the origin property of having been made in manner *m*. This is sufficient to establish that the two properties are just that: not one property but *two*. In fact, it establishes that *C2* and *C3* are each false.

On the assumption that  $\blacksquare = \Box_{\text{Logic}}$ , the arguments just given are, in effect, inversions of arguments *B* and *C*, treating each instead as a *reductio* of the identity premise. The inverted arguments can be significantly strengthened in a way that *B* and *C* cannot. One may sidestep disputes about whether issues that philosophy handles also appear to occupy a central role in science. Arguments *B* and *C* can be recast as arguments that lie outside the province of science and squarely within the jurisdiction of philosophy of logic and analytic metaphysics. The dual of the broadest alethic possibility  $\blacklozenge$  is, like every alethic necessity, a property of propositions. (See footnote 1.) Dorr et al.'s argument *C* aims to establish that the proposition that Woody is the first table-shaped object to have originated in manner *m* has the particular property  $\blacksquare$ , precisely by identifying Woody's haecceity with the property of being the first table-shaped object to have originated in manner *m*. That is the very point of premise *C2*. The validity of argument *C* is secured because the identification of these two properties yields a further and more directly relevant identification: between the proposition that Woody is Woody and the proposition that Woody is the first table-shaped object to have originated in manner *m*. A direct argument for *C3* results by modifying argument *C*, replacing *C2* with the following:

*C'2* (Woody = Woody) =<sup>2</sup> (Woody = the first table-shaped object to have originated in manner *m*).

Here the terms flanking the identity predicate ' $=$ '<sup>2</sup> designate the two relevant propositions. (See footnote 12.) Let us give the name '*C*' to the modified argument  $\ulcorner C1. C'2 \therefore C3 \urcorner$ .

Arguments *C* and *C'* stand or fall together. Each is valid iff the other is. Each has true premises iff the other does. More to the point, if *C2* is true, *C'2* is as well. This

conditional is a special case of a more general principle connecting properties and propositions:

For any properties  $F$ ,  $G$ , and any individual  $x$ , if  $F =^2 G$ , then  $(Fx) =^2 (Gx)$ .<sup>15</sup>

Some such principle or inference rule is also involved in the deduction of  $B3$  from  $B1$  and  $B2$ . An exactly analogous variant  $B'$  of argument  $B$  results by replacing  $B2$  with.

$B'2$ : (Every golden thing is golden)  $=^2$  (Every golden thing is made of atoms containing 79 protons).

If  $B2$  is true,  $B'2$  is as well. Like arguments  $C$  and  $C'$ ,  $B$  and  $B'$  stand or fall together.

Whether proposition identifications like that made in  $C'2$  “seem to play a central role in science” or not—I believe they clearly do not, but even if they did—there are well-known philosophical considerations establishing that  $C'2$  is false, regardless of how the unspecified manner  $m$  might be specified. Some considerations of this sort were emphasized by both Frege and Russell. The sentences ‘Woody = Woody’ and ‘Woody = the first table-shaped object to have originated in manner  $m$ ’, once the latter is fully fleshed out, obviously differ in *Erkenntniswert* or semantic value. The former does not extend knowledge. It is (given that Woody exists) uninformative, a priori, and, according to Kant, to be labelled ‘analytic’. By contrast, suitably fleshed out, the latter provides a valuable extension of knowledge. It is informative, a posteriori, and synthetic.<sup>16</sup> Likewise, even if George IV had wondered whether Woody was the first table-shaped object to have originated in manner  $m$ , an interest in the issue of whether Woody is Woody could hardly be attributed to the first gentleman of Europe. Analogous considerations likewise refute  $B'$ .

Kripke (among others) raised other, very forceful considerations against  $C'2$ , even more forceful than Frege’s. Kripke does not advance a positive theory of propositions, but his negative arguments concern the semantic contents of proper names and definite descriptions. The semantic content of the definite description ‘the first table-shaped object to have originated in manner  $m$ ’ is something like a Millian

<sup>15</sup> Special care must be taken here. Blue is the color of the sky, but the proposition that Woody is blue is not the proposition that Woody is the color of the sky. In a possible world in which the sky is red, if the former proposition is true the latter is false. In the actual world, a thing is blue in color iff it is the color of the sky (in color), but the property of being blue is not the property of being the color of the sky. Because of the non-extensionality of the current use of parentheses (see footnote 12), it is illegitimate to instantiate the predicate-variables of  $C2$  to a second-order definite-description predicate like ‘is the color of the sky’.

The converse principle that if  $(Fx) =^2 (Gx)$  then  $F =^2 G$  is more controversial, but the present response does not depend on it. (See the immediately preceding footnote.).

<sup>16</sup> Frege famously raised these considerations in connection with (German versions of) ‘Hesperus is Hesperus’ and ‘Hesperus is Phosphorus’. I argue extensively in *Frege’s Puzzle* (1986b) that Frege is mistaken that ‘Hesperus is Phosphorus’ is informative (a posteriori, synthetic, etc.) in the relevant sense. (See especially chapter 6, pp. 77–85.) The arguments I give there do not extend to ‘Woody = the first table-shaped object to have originated in manner  $m$ ’. The latter sentence clearly is informative in the relevant sense.

connotation, a conceptualization of something as the sole table-shaped object to have originated in a particular manner. If Millians like myself are right, the semantic content of the name ‘Woody’ is Woody itself—a table and not a concept, made of wood and not of concepts. Even if Millianism is mistaken, the semantic content of ‘Woody’ is certainly not a conceptualization by means of the property of being the first table-shaped object to have originated in a particular manner. Whatever proclamations the physicist might make on the matter, a high degree of philosophical immersion is required to confuse the logical truth that Woody is Woody for the proposition that Woody is the first table-shaped object to have originated in manner *m*.<sup>17</sup>

## 5 V

Dorr et al. will undoubtedly demur. They write:

While there are seductive arguments that [Kripke’s examples that gold has atomic number 79 and that Woody did not originate from ice are alethically contingent in the broadest sense] based (inter alia) on claims about a priori knowledge, these arguments should already have been ringing alarm bells for anyone familiar with *Naming and Necessity*. Our discussion has confirmed that such arguments can be independently seen to be problematic. And for what it’s worth, we think that Kripke’s celebrated warnings about inferring possibility claims from various kinds of epistemic status are most needed when the topic is broad possibility. For a restricted modality, it is a lot harder to see why anyone would have been tempted to make such inferences in the first place. For example, there wouldn’t have been much point in writing a book inveighing against inferring *nomological* possibility from premises about lack of a priori knowability. Interpreting Kripke as talking about some narrower modality not only doesn’t fit with his explicit invocations of ‘absoluteness’ and fails to attend to the crucial dialectical role of Leibniz’s Law inferences, but makes his central theses much less interesting. We suggest that those who still think that there is a good argument from failures of a priori knowledge (or analyticity, or logical truth, ...) to broad possibility haven’t taken to heart the central lessons of *Naming and Necessity*. (p. 210)

I argue that being gold and being made of atoms containing 79 protons are distinct properties, and that being Woody and being the first table-shaped object with a particular origin are distinct properties. I argue likewise that the pair of truths—that Woody is Woody, and that Woody is the first table-shaped object to have originated in manner *m*—are distinct propositions. I also argue likewise that the pair of truths—that every golden thing is golden, and that every golden thing is made of atoms containing 79 protons—are distinct propositions. Dorr et al. castigate unnamed readers of *Naming and Necessity* who allegedly infer that a pair

<sup>17</sup> These considerations against *C’2* play a dominant role in Kripke’s *Naming and Necessity*.

of objects are distinct from the fact that a claim of identity between the objects is not a priori. No such inference is involved in the preceding arguments. I argue—building upon Kripke’s powerful arguments in *Naming and Necessity*—that the sentence ‘Woody is Woody’ differs in semantic content from ‘Woody is the first table-shaped object to have originated in manner *m*’. I also argue for this conclusion on the ground that the former (given that Woody exists) is a priori, relevantly uninformative, etc., whereas the latter is a posteriori, relevantly informative, etc. I make no argument that since the latter is a posteriori, it is therefore metaphysically contingent. On the contrary, I fully agree with Kripke that it is necessary a posteriori. Rather, the inference involved in my objection is a straightforward Leibniz’s-law substitution inference (or the contrapositive of Leibniz’s law)—the same sort of inference invoked in arguments A, B, and C, which Dorr et al. proffer. If one likes, one may take it that the major premise of my relevant objection is a special instance of Leibniz’s law: If  $p =^2 q$ , then  $p$  is a priori iff  $q$  is. Applications of *modus tollens* deliver rejections of B’2 and C’2.

Whereas I do not argue from the a-posteriority of the co-extensiveness of a pair of properties to the numerical distinctness of those properties, it should be noted that, contrary to the impression Dorr, et al. create, *Naming and Necessity* does not reject all such arguments. Kripke says that “such theoretical identification as ‘heat is molecular motion’ are *necessary*, though not a priori. The type of property identity used in science seems to associated with *necessity*, not with a *prioricity*, or *analyticity* ... The philosophical notion of attribute, on the other hand, seems to demand a priori (and analytic) coextensiveness as well as necessary coextensiveness” (N&N, p. 138). Kripke’s thought appears to be that whereas science invokes a relatively coarse-grained conception of a property, philosophy invokes a more fine-grained notion, on which, for example, the “attribute” of heat—or better, the *concept* of heat—is distinct from that of molecular motion. Even if it is metaphysically necessary that something is hot iff its molecules have a high degree of kinetic energy, it does not follow that the proposition that  $x$  is hot is the proposition that  $x$ ’s molecules have a high degree of kinetic energy. (See notes 14 and 15 above.)

Numerous philosophers, both before and after Kripke, have failed to distinguish sharply between metaphysical modality and logical modality.<sup>18</sup> This failure has contributed to the popularity of the mistaken thesis that metaphysical modality is the broadest alethic modality. (See footnote 5 concerning another source.) The philosophical term ‘logical space’ is often a misnomer. A better term for the restricted space of metaphysically possible worlds (or its power class) is ‘metaphysical space’, reserving ‘logical space’ for the space of logically possible worlds. Kripke’s observations in *Naming and Necessity* about the modal essentialism concerning chemical elements, chemical compounds, material artifacts, and creatures born from sexual

<sup>18</sup> Alvin Plantinga in *The Nature of Necessity* (Oxford University Press, 1974) dubs metaphysical necessity ‘broadly logical necessity’. He clarifies that “the sense of necessity in question is wider than that captured in first order logic. On the other hand, it is narrower than that of *causal* or *natural* necessity” (p. 2). He thus recognizes in his alternative (and misleading) terminology that metaphysical possibility is less broad than logical possibility. See footnote 2 above.



reproduction all concern metaphysical modality. There is no legitimate basis for interpreting his examples of a posteriori metaphysically necessary truths as being logically necessary truths. The sentence ‘Gold has atomic number 79’ is not analytic. It is not logically or mathematically necessary that gold have atomic number 79, or even that gold be an element at all. It is not logically or mathematically impossible that Woody originated from water from the Thames.

Contrary to Dorr et al., many philosophers before *Naming and Necessity* did in fact draw an inference from the a-posteriority of a sentence to its contingency with respect to a restricted alethic modality. This inference has rested upon a variety of errors. Before Kripke, many philosophers believed that the only alethic necessity is analyticity, and the only alethic contingency syntheticity. Many, especially empiricists, believed that the only a-priority is analyticity. As just noted, many philosophers have also mistaken metaphysical necessity for logical necessity. Some empiricists committed this confusion even while rejecting metaphysical necessity as conceptually bankrupt or illegitimate. Kripke argued compellingly to the contrary that some true sentences are metaphysically necessary despite being a posteriori. Kripke convinced many, including myself. From the mere fact that a given sentence, such as ‘Gold has atomic number 79’ or ‘Water is two parts hydrogen, one part oxygen’, is synthetic and a posteriori (or informative, etc.), it does not follow that the sentence is metaphysically contingent. On the other hand, if one sentence is a posteriori *in the relevant, semantic sense* while another is a priori, it *does* follow that those sentences do not semantically express the same proposition. It is a serious mistake to read *Naming and Necessity* as lending support to the hypothesis that a pair of sentences one of which is a priori and the other a posteriori can share the very same semantic content, or that a single proposition can have incompatible epistemological properties or can differ in epistemological status from itself.

Premise *C'2* is certainly false, on philosophical grounds. Anything entailing *C'2* is just as certainly false, on those same grounds. In fact, each of arguments *B*, *B'*, *C*, and *C'* fail, all for the same reason. In each case, the argument is valid and its first premise is true while the conclusion is false. It does not matter whether the conclusion is obviously false. It does matter that the conclusion can be seen to be false on philosophical grounds. Likewise, its second premise, and anything entailing it, is seen to be false on philosophical grounds.

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February 3, 1987

Professor Nathan Salmon  
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Dear Nathan,

Enclosed are two papers, including the one in the Linsky volume too. The treatment of the relation R is on pp. 69-70 (section 2.1) of "Semantical Analysis" and on p. 64 of "Semantical Considerations". The same characterization of R is given in both places. As you see, there is no suggestion that S5 is basic and the weaker systems come from some restricted conception. R is characterized in terms of truth and possibility of propositions in worlds. Notice also the discussion of the reduction axioms on p. 70 of "Semantical Analysis", and in particular of transitivity and S4.

One thing I do is, I now think, somewhat misleading. I should have stressed that the use of R does not make "possible" (as applied to worlds) into a two-place predicate, any more than, as you say, "is bald" is. Probably I only noticed this afterwards. Also, I should have stressed that strictly speaking, many of the worlds are not "possible" but only "possibly possible", and so on, unless we have S4.

By the time I gave the seminar I talked to you about I had definitely thought these points through, having seriously considered whether the conventional presupposition that the basic modal logic is S5 is justified.

I am getting closer to thinking that your treatment of the ship is the correct solution. Certainly it is a very good piece of work. I am sorry if almost everyone is unable to see its virtues (you don't say quite that in the paper). As far as I can see, their counterarguments, as presented, are confused or circular. It was good talking to you. Talk to you about Russell, etc., some time.

Best,

  
Saul Kripke

Enc.



Nathan Salmon and Saul Kripke, the CUNY Graduate Center, mid-September 2011