A Problem in the Frege-Church Theory of Sense and Denotation*

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There is an inconsistency among claims made (or apparently made) in separate articles by Alonzo Church concerning Frege's distinction between sense and denotation taken together with plausible assertions by Frege concerning his notion of ungerade Sinn—i.e., the sense that an expression allegedly takes on in positions in which it has ungerade Bedeutung, denoting its own customary sense.¹ As with any inconsistency, the difficulty can be avoided by relinquishing one of the joint assumptions from which contradiction may be derived. Yet what seems the most plausible resolution, in light of Church's own arguments, involves abandoning the theory of sense and denotation, in the form in which it has been staunchly advocated by both Frege and Church. That theory has come under sustained criticism from several quarters.² Whereas the present difficulty supplements the case against the theory, it is unlike more familiar criticisms, since it is not framed from the perspective of an alternative theory (or picture) but is based instead on an internal conflict, and therefore has special force.

I

The difficulty can be illustrated by assuming (solely for the sake of the illustration) that the word 'brother' has exactly the same customary sense in English as the phrase 'male sibling', and that, consequently, so do the following two sentences:

(1) Holmes has an older brother

(2) Holmes has an older male sibling.
One version of the so-called Paradox of Analysis can then be set out by noting that the following analysis of the proposition that Holmes has an older brother is informative, or can be, even for someone who correctly understands both of the English sentences (1) and (2):

\[(PA) \text{ The proposition that Holmes has an older brother is the proposition that Holmes has an older male sibling.}\]

Extrapolating from remarks in Church’s review of the famous Black/White controversy concerning the Paradox of Analysis, he would evidently claim that the informativeness of (PA) is explained, along familiar lines, by the fact that the English phrases ‘the proposition that Holmes has an older brother’ and ‘the proposition that Holmes has an older male sibling’, while they have the same (customary) denotation, differ in (customary) sense.

Consider now the following English sentences:

(3) Watson believes that Holmes has an older brother

(4) Watson believes that Holmes has an older male sibling.

According to the Frege-Church theory of sense and denotation, relative to their positions in these sentences, respectively, (1) and (2) denote their (shared) customary English sense—the proposition that Holmes has an older brother—rather than their truth value. Both (3) and (4) thereby attribute to Watson belief of the very same proposition, and therefore cannot differ in their truth value in English.

Furthermore, according to the theory, sentence (1) denotes its customary sense in the English sentence (3) by there expressing its indirect sense, which is a concept of (i.e. which determines) the proposition that Holmes has an older brother. In his seminal “Über Sinn und Bedeutung,” Frege says that the indirect sense (in English) of a sentence like (1) is the customary sense (in English) of the phrase ‘the proposition that Holmes has an older brother’. Taken together with Church’s proposed solution to the Paradox of Analysis, this entails that, whereas they have the same customary English sense, the English sentences (1) and (2) above differ in indirect sense, i.e. they express differing senses in English when positioned within the scope of an “ungerade” operator, as in (3) and (4) or in (PA).

Some philosophers—notably Tyler Burge—criticize the assertion that (3) and (4) cannot differ in truth value in English. Imagine that Watson readily accepts the truth of (1), as a sentence of English, yet because of uncertainty as to the exact meaning in English of ‘sibling’, hesitates over (2). Whereas the truth in English of (3) is beyond reasonable doubt, the philosophers in question, on careful reflection of the issue, express doubt concerning (4) (under the current supposition), in consideration of Watson’s hesitation over (2). Adding an interesting wrinkle to the debate, Benson Mates brought the very existence of this controversy to bear on itself, arguing, in effect, that the mere possibility of the
stance taken by philosophers like Burge demonstrates that expressions sharing the same English sense cannot always be inter-substituted in attributing propositional attitude.\(^8\) We consider for this purpose the following complex sentences:

(5) Burge doubts that Watson believes that Holmes has an older brother

(6) Burge doubts that Watson believes that Holmes has an older male sibling.

Many philosophers, in addition to Burge, would follow Mates in his suggestion that even if (1) and (2) in fact share the same English sense, (6) is true in English whereas (5) is false. Indeed, perhaps the most natural way of reporting Burge’s position in English is precisely to assert (6) while denying (5).

In criticism of Mates, Church argued (in effect) to the contrary that (5) and (6) can both be correctly translated (preserving sense) from English into a language lacking a single-word translation for ‘brother’ by means of a single sentence of that language employing translations for the English words ‘male’ and ‘sibling’—thereby establishing that (5) and (6) are alike in (customary) sense, and hence, contrary to Mates and company, alike in truth value.\(^9\)

Church’s criticism of Mates is plausible and seems true to the spirit of Frege’s theory. It is impossible, though, to be certain how Frege himself would have responded to the Mates problem. One of Frege’s explicit doctrines is directly relevant. Frege held that any expression already having ungerade Bedeutung (by occurring within the scope of an operator like ‘Watson believes that’), when positioned within the scope of a further ungerade operator (‘Burge doubts that’), will shift still further to denoting its indirect sense instead of its customary sense. Frege would claim that the sentences (1) and (2), as they occur embedded in (5) and (6), thus denote their indirect senses in English, and must therefore express yet other senses, their doubly indirect senses, which are concepts of their (singly) indirect senses. Church shares Frege’s doctrine that an expression occurring in a doubly ungerade context denotes a sense other than its customary sense and expresses yet a third sense. It is precisely this that leads to the infinite hierarchies of senses of the sort Church explored and clarified in his classical papers on the logic of sense and denotation.\(^10\)

The doctrine of infinite hierarchies of sense is not sufficient by itself, however, to yield an answer to the question of whether substitution of customarily synonymous expressions (i.e. those sharing the same customary sense) will fail in English in doubly embedding ungerade sentences like (5) and (6). It must first be settled whether customarily synonymous expressions are also fully synonymous in the sense that they share the same entire hierarchy of indirect senses (the same indirect sense, the same doubly indirect sense, the same triply indirect sense, and so on). If expressions alike in customary sense differ in the senses they express in ungerade contexts, then those same expressions differ in denotation when occurring in a doubly embedded ungerade context.\(^11\)
The inconsistency in Church’s theory of sense and denotation is now apparent: Given that the sentences (1) and (2) denote their indirect senses in (5) and (6), respectively, the claim that (5) and (6) are customarily synonymous is incompatible with the earlier claim that (1) and (2) do not have the same indirect sense. If (1) and (2) differ in indirect sense, it should be incorrect to translate both (5) and (6) by means of a single sentence of another language. Any sentence of that language that might be put forward as a translation for both (5) and (6) could preserve denotation, within the sentence, of at most only one of the differing indirect senses for (1) and (2).

Doubly embedding ungerade sentences like (5) and (6) are not required to generate the contradiction. If Church’s apparent solution to the Paradox of Analysis, as set out above, is correct, and Frege’s claim about the indirect senses of sentences like (1) and (2) is also correct, then those very sentences, as they occur embedded in the singly embedding ungerade English sentences (3) and (4), differ in the senses they express there. Yet, on the assumption that (1) and (2) have the same customary English sense, Church also claims that one may correctly translate not only (4) but also (3), while preserving its English sense, into a language lacking a single-word translation for ‘brother’ simply by using translations for ‘male’ and ‘sibling’. Since the sentences (3) and (4) would thus both be translated into the very same sentence of the target language, this would establish their customary synonymy in English. But (3) and (4) cannot express the same proposition in English if those sentences contain corresponding components that differ in the senses they express in those sentences. Whereas both (3) and (4) attribute to Watson belief of the same proposition—that Holmes has an older brother—if (1) and (2) differ in the senses they express therein, then (3) and (4) specify the attributed proposition by means of differing senses, and must therefore themselves express differing propositions (one of which may be doubted while the other is not). 12

Consider also translating the English sentence (PA) above, preserving its sense. If one translates in the manner suggested, the translation will have the logical form of a reflexive identity, ‘α = α’, and will therefore lack (PA)’s status as a potentially informative analysis.

II

The problem can be posed in the form of an apparently inconsistent triad: (i) Frege’s assertion that the indirect sense in English of a sentence like (1) is the customary sense of the corresponding phrase ‘the proposition that Holmes has an older brother’; (ii) a Fregean solution to the Paradox of Analysis, along the lines suggested by Church, according to which the English phrases ‘the proposition that Holmes has an older brother’ and ‘the proposition that Holmes has an older male sibling’ differ in customary sense; and (iii) Church’s claim that the English sentences (1) and (2), even as occurring in positions in which they express their indirect senses, can be correctly translated, preserving sense, by means of a single sentence of another language. (Other assumptions are involved, of course,
but they are plausible to the point of being beyond reasonable dispute.) A Fregean resolution of this problem could shed light on a controversial aspect of orthodox Fregean theory that has long seemed obscure.

Resolution requires the rejection of one (or more) of the triad’s components. Properly understood, perhaps the first component is the most compelling of the three—or at least, so is the conjunction of the particular instances involved in deriving the present contradiction. It amounts to the claim that the English sentences (1) and (2), as they occur embedded in (3) and (4), respectively, express the same senses there that the English phrases ‘the proposition that Holmes has an older brother’ and ‘the proposition that Holmes has an older male sibling’ express, respectively, in (PA). This claim involves Frege’s well-known relativization of the semantic relation between an expression and the sense it expresses (in a given language) to a position within a sentence. One and the same univocal (unambiguous) expression, if it recurs in a single sentence, may, according to Frege, express one sense in one of its positions and another sense in another of its positions—as, for example, the two occurrences of (1) in the English sentence ‘Holmes has an older brother, although Moriarity does not realize that Holmes has an older brother’.13 Frege’s relativized notion is not employed in contemporary philosophical semantics; instead, one speaks in absolute terms of the sense (or “content,” “intension,” etc.) of an expression (in a language L). Nevertheless, it seems likely that Frege’s more discriminating notion may be explained or understood in terms of concepts familiar in the philosophy of semantics—for example, as the ternary relation that obtains among an expression, a semantic content, and a position of the expression in a particular sentence, when the second of these relata is the component contributed (in language L) by the occurrence of the expression in the position in question to the proposition expressed (in L) by the sentence in question. In the particular case at hand, the proposition expressed in English by (3) will essentially involve the semantic content (customarily) expressed in English by its complement clause ‘that Holmes has an older brother’. The latter must also be essentially involved in the sense (customarily) expressed in English by the phrase ‘the proposition that Holmes has an older brother’, and hence in the proposition expressed in English by (PA).14 Once Frege’s notion of expressing in such-and-such position (in language L) is properly defined, and once the ungerade operator ‘that’ occurring in both (PA) and (3) is properly seen as merely a sense-quotation device, exactly analogous to a pair of ordinary quotation marks (except grammatically restricted in its application to whole sentences), (i) seems to emerge as something that is trivially true.

Church’s own defense of (iii) is also very persuasive. That defense turns on strong and widely shared intuitions concerning what a particular language, given its resources, is or is not capable of semantically expressing. If the word ‘brother’ expresses exactly the same sense in English as the phrase ‘male sibling’, then, as Church points out, it seems that whatever can be expressed in English by means of the former can also be expressed in another language lack-
ing a single-word translation for ‘brother’ simply by using translations for ‘male’ and ‘sibling’. Assuming the customary synonymy of the English expressions ‘brother’ and ‘male sibling’, the presence of the former surely does not extend the expressive capabilities of English; anything that can be expressed in English by its means—including the proposition expressed by (5)—can also be expressed using ‘male sibling’ instead. As far as the expressive resources of English are concerned, the word ‘brother’ seems to be entirely superfluous, “a dispensable linguistic luxury.” Taking multiply embedded ungerade contexts into account, this observation seems to have the consequence that the customary sense of an expression determines its $n$-fold indirect sense, for $n \geq 1$. With indirect senses of any level above customary sense, there is a “road back” from denotation to sense. This is made all the more plausible by the further observation that knowing the customary sense of an expression is both necessary and sufficient for understanding occurrences of that expression in multiply embedded ungerade contexts, assuming one already understands the surrounding context.

My own favored resolution of the inconsistency is to reject (ii). Although either of the other components of the inconsistent triad might be rejected instead, such rejection seems significantly less plausible than accepting that if the expressions ‘brother’ and ‘male sibling’ share exactly the same semantic content in English, then so likewise do the phrases ‘the proposition that Holmes has an older brother’ and ‘the proposition that Holmes has an older male sibling’. The customary synonymy of the two phrases is supported by the now familiar translation test.

Rejecting (ii) does carry a certain price: it requires one to seek an alternative solution to the Paradox of Analysis. I have suggested such a solution elsewhere, while rejecting the Frege-Church theory of sense and denotation.

If Church were also to resolve the inconsistency by rejecting (ii) and seeking an alternative solution to the Paradox of Analysis, his doing so would immediately raise the question of why he does not embrace an exactly parallel alternative solution to Frege’s original puzzle about the informativeness of ‘Hesperus is Phosphorus’—a solution that, by hypothesis, does not involve distinguishing co-denotational proper names as regards semantic content. Adopting such an alternative solution would seriously threaten the Frege-Church theory. For once it is agreed that the apparent informativeness of an identity statement like (PA) is to be explained in some way other than by postulating a distinction between the senses of the singular terms involved, Church’s and Frege’s original argument for the theory of sense and denotation collapses. The apparent inconsistency in the Frege-Church theory thus goes to the heart of that theory.

Notes
*I am grateful to Anthony Brueckner and Matthew Hanser for their helpful comments.

1I follow Church in translating Frege’s use of ‘Bedeutung’ as ‘denotation’ rather than ‘meaning’, and Frege’s use of ‘Gedanke’ as ‘proposition’ rather than ‘thought’. I also use the word ‘concept’ in Church’s sense, which is very different from Frege’s use of ‘Begriff’.
I have summarized and integrated the most influential aspects of that criticism in part I of my book, Reference and Essence (Princeton University Press, 1981).

The alleged informativeness of analyses might be seen more vividly in the case of nontrivial philosophical analyses, such as have been proposed for the concepts of knowledge, perception, nonnatural meaning, and for propositions expressed by means of definite descriptions, etc.


Terence Parsons recently shared with me a brief note that Church wrote, while the present article was in press, in response to a query by Parsons whether Church had invoked Frege’s notion of indirect sense in the 1946 review of the Black/White controversy. (See note 4 above.) Church says that the notion of indirect sense is, at best, a very vague notion which, as far as it makes sense, is concerned with natural language and not with the formalized language of his Logic of Sense and Denotation (LSD). He explicitly recognizes that the sense of a given expression may be denoted by another expression which will have its own sense, and so on ad infinitum (as is the case in LSD), but adds that this is not the same as saying that a single expression has in addition to its denotation and its sense a third semantic value, which one might want to call its “indirect sense.”

The exact import of Church’s highly compressed remarks is somewhat obscure. It is an essential component of Fregean theory that any expression will denote its own sense in a variety of English constructions, such as the phrase ‘Holmes’s older brother’ in each of the following: ‘Watson believes that Holmes’s older brother lives in London’; ‘Watson is seeking Holmes’s older brother’; ‘The property of being more intelligent than Holmes’s older brother has no instances’.

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All use subject to http://about.jstor.org/terms
etc. Even if this much is regarded as settled, there is a question, over which Fregean theorists differ, whether the phrase ‘Holmes’s older brother’ expresses a sense in English other than its customary sense in such constructions. If the answer is that it does (as Frege himself held), there remains a further question whether the phrase expresses in English the very same sense in all of these constructions. And even if it does, it might still be possible to add an artificially stipulated device to the English lexicon, designed to insure the result that a single expression does not always express the same sense in the expanded language in constructions where it is induced to denote its own sense. There is no clear indication in his reply to Parsons that Church no longer holds, with Frege, that an expression occurring in an English ungerade context (such as (1) as it occurs in (3)) there expresses as its sense a concept of the expression’s customary sense, and that such a concept is denoted by the expression when it occurs in a suitable doubly ungerade context ((1) as it occurs in (5)). I believe that what Church is concerned to deny is instead that there is for any (unambiguous) expression of English a single concept which determines (i.e. which is a concept of) the expression’s sense and which can appropriately serve as the sense expressed when the expression occurs in an ungerade context (or as the denotation when the expression occurs in a doubly ungerade context). Rather, as I interpret his remarks, Church contends that for any expression there may be a plurality of (perhaps even infinitely many) such concepts, one of which can be correctly singled out as the indirect sense in English of the expression—i.e., as the only sense the expression expresses in English when it is induced by the context to denote its own sense. If this is so, it would be better to speak in a relativized manner of the indirect sense in the language of a given expression with respect to a particular ungerade construction, or with respect to a particular choice from among various paths up a tree of Fregean hierarchies rooted in the expression’s customary sense, etc. (Such, for example, might be the sense of the expression’s “first ascendant” in LSD.) But on Church’s view (as I interpret him), one should not allow such a relativized manner of speaking to promote the confusion that there is for any given expression a level of semantic content beyond sense. (Thanks to C. Anthony Anderson for correspondence concerning this issue.)

If my interpretation is at least roughly correct, then Church agrees with Frege that the English sentence (1) occurring in (5) there denotes in English neither its truth value (customary denotation) nor its sense but a concept of its sense. In any event, it is nearly certain that Church would agree that the English phrase ‘that Holmes has an older brother’ expresses the same sense in (PA) that it expresses in (3), and similarly that the English phrase ‘that Holmes has an older male sibling’ expresses the same sense in (PA) that it expresses in (4). This is sufficient to generate the contradiction to be discussed. Indeed, the same problem can be reinstated by substituting for the expression ‘the indirect sense in English of the sentence ‘...’” instead a suitably relativized phrase, or even the phrase ‘the sense in English of ‘that ...’’ (mutatis mutandis).

11Parsons, op. cit., supports the view that customarily synonymous expressions are invariably fully synonymous. Owens, op. cit., argues instead that phenomena like the Mates problem may be seen as indicating that expressions sharing the same customary sense will typically differ in indirect sense. C. Anthony Anderson defends the position taken by Owens. See his in-depth critical review of George Bealer’s Quality and Concept, Journal of Philosophical Logic, 16 (1987), pp. 115-164, at 141-143.

12Owens, op. cit. at p. 381n26, argues that Church failed to recognize that his proposed solution to the Paradox of Analysis amounts to the claim that (1) and (2) differ in indirect sense, and that this failure is shown by his claim that the doubt that gives rise to the temptation to assert (6) is actually a metalinguistic doubt. (Owens uses another example.) The point I am making is different: Church’s apparent solution to the Paradox of Analysis not only makes his rejection of (6) unnecessary. (Indeed, rejection of (6) is unnecessary regardless; see note 9 above.) Given the plausible Fregean doctrine cited, the solution to the Paradox of Analysis actually contradicts Church’s independent claim that (3) and (4) have the same English sense. That solution does indeed make it unnecessary to treat (5) and (6) as equivalent; it does so precisely by making the assertion of the strict synonymy of (3) and (4) logically impossible. I learned while the present article was already in press that Anderson, op. cit. at p. 162n27, also recognizes the tension between Church’s review of the Black/White exchange and his later critique of Mates’s discussion of substitution.

13Frege’s relativization of the relation between an expression and the sense it expresses to positions within a sentence yields perhaps the most straightforward interpretation of his earlier principle that “it is only in the context of a sentence that words have any meaning” (Grundlagen der Arithmetik, J. L. Austin, ed., Blackwell, 1950, 1958, at p. 71, 73).
14 Cf. my “Reference and Information Content: Names and Descriptions,” in D. Gabbay and F. Guenthner, eds., Handbook of Philosophical Logic IV: Topics in the Philosophy of Language (Dordrecht: D. Reidel, ), pp. 409-461, at pp. 440-441. One may assume that the words ‘the proposition’ in the phrase ‘the proposition that’ stand in grammatical apposition to the ‘that’ clause that follows them. The ‘that’ operator may be regarded as the paradigm ungerade operator. Cf. my Frege’s Puzzle (Atascadero, Ca.: Ridgeview, 1986, 1991), at pp. 5-6. Indeed, (PA) could be reformulated more succinctly (albeit somewhat more peculiarly) as ‘That Holmes has an older brother = that Holmes has an older male sibling’, with no effect on its status as a (potentially) informative analysis of the proposition that Holmes has an older brother. There can be little doubt that if the two expressions flanking ‘=’ differ in the senses they express in English, then those same expressions likewise differ in their contributions to the propositions expressed in English by (3) and (4). (Cf. note 10 above.)

15 The present writer regards Church’s arguments in this connection as nearly decisive (modulo the qualification described in note 9 above). Those arguments are, however, philosophical in nature, and not everyone is persuaded. (See note 11 above.) Burge, op. cit., argues that Church’s reliance on translation is inappropriate, on the following grounds:

Good translation should preserve truth value. But Church and Mates disagree about the truth value of [r(6) but not (5)]. Church takes it to be a contradiction, whereas Mates regards it as true. Until this difference is resolved, one surely cannot decide whether Church’s proposed translation of [r(6) but not (5)] (in the relevant context) as an explicit contradiction is correct. Obvious truths and falsehoods are normally used in arriving at a translation—not vice versa. (p. 122)

Perhaps obvious truths and falsehoods are appropriately relied upon in Quine’s special project of so-called radical translation. It is important therefore to note that Church is engaged in an entirely different enterprise, one that explicitly relies on antecedent knowledge of inter-language synonyms among sub-sentential words and phrases. Church’s concern in the present context is to translate in such a manner as to preserve the proposition semantically expressed—rather than preserving, for example, the point being illustrated (if necessary at the expense of preserving sense).

Church calls this ‘literal translation’. Not only is it possible to provide such translation without first determining truth value; given the sole objective of preserving semantic content, it would be highly irregular to proceed as Burge seems to suggest, only by first settling the question of truth value. When one correctly understands each of two languages, one can normally provide a literal translation for a given sentence, in the sense of one preserving semantic content, even without having any opinion as to its truth value. Indeed, contra Burge, knowledge of truth value seems irrelevant to the task. Providing a literal translation for (5) into a language containing literal translations for the English words ‘male’ and ‘sibling’ poses no special problems (on the operative assumption that ‘brother’ has exactly the same English sense as ‘male sibling’); a radical departure from normal practice would not be justified.

Burge further dismisses as begging the question Church’s argument that if the target language can properly translate the English words ‘male’ and ‘sibling’, then the absence of an alternative translation for the English word ‘brother’ cannot be regarded as a deficiency that makes literal translation of (5) into that language impossible (ibid., p. 122n4). The present writer fails to see that Church’s argument genuinely begs the question (Burge does not elaborate beyond the passage quoted above); indeed, it would seem closer to the facts to say instead that the argument genuinely refutes Mates by exposing an implausible consequence of his claims.

16 This might be taken to indicate that Church’s remarks pertaining to the Paradox of Analysis have been misinterpreted. (See note 4 above.) If they have been, then the difficulties raised in the next two paragraphs are particularly pressing for Church.

17 Frege’s Puzzle, especially around pp. 57-60, 78-79. The neo-Russellian theory defended there avoids infinite hierarchies of indirect sense while also affording an alternative explanation for the fact, mentioned above, that knowing the customary semantic content of an expression is both necessary and sufficient for understanding occurrences of that expression in multiply embedded ungerade contexts: The semantic content of an expression in any ungerade context, no matter how deeply embedded, is identical with the customary content.